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344

Nr. Crt 7

Examen - Calcul Numeric - V7

1. Factorizare LU cu pivotare parțială

$$\begin{cases} x_1 + 5x_2 + 3x_3 = 25 \\ 5x_1 + 13x_2 + 12x_3 = 77 \\ x_1 + 2x_2 + x_3 = 8 \end{cases}$$

$$A = \begin{pmatrix} 1 & 5 & 3 \\ 5 & 13 & 12 \\ 1 & 2 & 1 \end{pmatrix}$$

$w = (1; 2; 3)$ - interschimbări de linii

$K = 1$

$$|a_{p1}| = \max_{j=\overline{1;3}} |a_{jk}| = \max\{|a_{11}|, |a_{21}|, |a_{31}|\} = |a_{21}|$$

Interschimbăm $L_1 \leftrightarrow L_2$ și $w_1 \leftrightarrow w_2$

$$A = \begin{pmatrix} 5 & 13 & 12 \\ 1 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 5 & 13 & 12 \\ 0 & \frac{12}{5} & \frac{3}{5} \\ 0 & -\frac{3}{5} & -\frac{7}{5} \end{pmatrix} \overset{L_3 \leftarrow L_3 + \frac{1}{4}L_2}{\sim} \begin{pmatrix} 5 & 13 & 12 \\ 0 & \frac{12}{5} & \frac{3}{5} \\ 0 & 0 & -\frac{5}{4} \end{pmatrix}$$

$w = (2; 1; 3)$ $L_2 \leftarrow L_2 - \frac{1}{5}L_1$

$L_3 \leftarrow L_3 - \frac{1}{5}L_1$

$^{5)}5 - \frac{13}{5} = \frac{12}{5}$

$^{5)}3 - \frac{12}{5} = \frac{15-12}{5} = \frac{3}{5}$

$^{5)}2 - \frac{13}{5} = -\frac{3}{5}$

$^{5)}1 - \frac{12}{5} = \frac{-7}{5}$

$\frac{3}{5} \cdot \frac{5}{12} = \frac{1}{4}$

$^{4)}-\frac{7}{5} + \frac{1}{4} \cdot \frac{3}{5} = \frac{-28+3}{20} = -\frac{5}{4}$

$K = 2$

$$|a_{p2}| = \max_{j=2;3} |a_{jk}| = \max\left\{\left|\frac{12}{5}\right|; \left|-\frac{3}{5}\right|\right\} = |a_{22}| \rightarrow \text{nu sunt interschimbări}$$

Deci $U = \begin{pmatrix} 5 & 13 & 12 \\ 0 & \frac{12}{5} & \frac{3}{5} \\ 0 & 0 & -\frac{5}{4} \end{pmatrix}$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{5} & 1 & 0 \\ \frac{1}{5} & -\frac{1}{4} & 1 \end{pmatrix}$$

$$b' = \begin{pmatrix} 77 \\ 25 \\ 8 \end{pmatrix}$$

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Cum $L \cdot U = A'$ (cu permutarea de linii), $Ax = b$ $\Big\} \Rightarrow L\underbrace{Ux}_{} = b'$

$Ux = y$

Rezolvăm

$Ly = b'$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{5} & 1 & 0 \\ \frac{1}{5} & \frac{-1}{4} & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 77 \\ 25 \\ 8 \end{pmatrix}$$

$y_1 = 77$

$\frac{77}{5} + y_2 = 25 \Rightarrow y_2 = \frac{48}{5}$

$y_3 + \frac{77}{5} - \frac{\cancel{48}^{12}}{5 \cdot \cancel{4}} = 8 \Leftrightarrow y_3 + \frac{\cancel{65}^{13}}{\cancel{5}} = 8 \Rightarrow y_3 = -5$

$$Ux = y \Rightarrow \begin{pmatrix} 5 & 13 & 12 \\ 0 & \frac{12}{5} & \frac{3}{5} \\ 0 & 0 & \frac{-5}{4} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 77 \\ \frac{48}{5} \\ -5 \end{pmatrix}$$

$-\frac{5}{4} x_3 = -5 \Rightarrow x_3 = 4$

$\frac{12}{5} x_2 + \frac{12}{5} = \frac{48}{5} \Big| \cdot 5 \Rightarrow 12 x_2 = 36 \Rightarrow x_2 = 3$

$5 x_1 + 13 \cdot 3 + 12 \cdot 4 = 77 \Rightarrow x_1 = -2$

$$\Rightarrow x = \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix}$$

II Polinomul de interp. Lagrange $P_3(x)$, $y = f(x)$

Newton cu DD

$x = (x_1, x_2, x_3, x_4)$

$f(x) = \sqrt{x}$, $x = (1, 3, 5, 6)$

$f(1) = 1$

$f(3) = \sqrt{3} = 1,732$

$f(5) = \sqrt{5} = 2,236$

$f(6) = \sqrt{6} = 2,449$

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Construim tabelul de diferențe divizate

x_i	DD ord 0	DD ord 1	DD ord 2	DD ord 3
$1 = x_1$	1	///	///	////
$3 = x_2$	1,732	0,366	////	////
$5 = x_3$	2,236	0,252	-0,028	////
$6 = x_4$	2,449	0,213	-0,013	0,003

$$P_3(x) = f[x_1] + f[x_1, x_2](x - x_1) + f[x_1, x_2, x_3](x - x_1)(x - x_2)$$
$$+ f[x_1, x_2, x_3, x_4]\cdot(x - x_1)(x - x_2)(x - x_3)$$

$$f[x_i] = f(x_i)$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{1,732 - 1}{3 - 1} = 0,366$$

$$f[x_2, x_3] = \frac{2,236 - 1,732}{5 - 3} = \frac{0,504}{2} = 0,252$$

$$f[x_3; x_4] = \frac{2,449 - 2,236}{6 - 5} = 0,213$$

$$f[x_1, x_2, x_3] = \frac{0,252 - 0,366}{5 - 1} = -0,028$$

$$f[x_2, x_3, x_4] = \frac{f[x_3; x_4] - f[x_2, x_3]}{x_4 - x_2} = \frac{0,213 - 0,252}{6 - 3} = -0,013$$

$$f[x_1, x_2, x_3, x_4] = \frac{f[x_2, x_3, x_4] - f[x_1, x_2, x_3]}{x_4 - x_1} = \frac{-0,013 + 0,028}{6 - 1} = 0,003$$

$$P_3(x) = 1 + 0,366\cdot(x-1) - 0,028\,(x-1)(x-3) + 0,003\cdot(x-1)(x-3)(x-5)$$
$$= 1 + 0,366\cdot x - 0,366 - 0,028\cdot x^2 + 0,028\cdot 4\cdot x - 3\cdot 0,028 +$$
$$0,003\cdot x^3 - 9\cdot 0,003\cdot x^2 + 23\cdot 0,003\,x - 0,003\cdot 15$$
$$= 0,003\cdot x^3 + x^2(-0,028 - 9\cdot 0,003) + x\,(0,366 + 0,112 + 0,069)$$
$$+ (1 - 0,366 + 0,084 - 0,045)$$
$$= 0,003\cdot x^3 - 0,055\,x^2 + 0,547\cdot x + 0,673$$

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III Spline pătratică S

V7 : $f(x)=\sqrt{x}$; $x=(1;3;5)$

Calculăm:

$f(1)=1$

$f(3)=1,732$

$f(5)=2,236$

$$S(x)=\begin{cases} S_1(x)=a_1+b_1(x-x_1)+c_1(x-x_1)^2, & x\in[x_1;x_2) \\ S_2(x)=a_2+b_2(x-x_2)+c_2(x-x_2)^2, & x\in[x_2;x_3] \end{cases}$$

$$S(x)=\begin{cases} a_1+b_1(x-1)+c_1(x-1)^2, & x\in[1;3) \\ a_2+b_2(x-3)+c_2(x-3)^2, & x\in[3;5] \end{cases}$$

S interpolează f în cele 3 noduri

$S_1(1)=1 \Rightarrow a_1=1$

$S_2(3)=1,732 \Rightarrow a_2=1,732$

$S_2(5)=\underbrace{a_2}_{1,732}+2b_2+4c_2=2,236 \Rightarrow b_2+2c_2=0,252$

S continuă pe $[1;5]$

$S_1(3)=S_2(3) \Rightarrow 1+2b_1+4c_1=1,732$

$b_1+2c_1=0,366$

S' continuă în nodul interior $x_2=3$

$$S'(x)=\begin{cases} b_1+2c_1(x-1), & x\in[1;3) \\ b_2+2c_2(x-3), & x\in[3;5] \end{cases}$$

$S_1'(3)=S_2'(3)$

$b_1+2c_1\cdot 2=b_2 \Leftrightarrow b_1+4c_1=b_2$

Considerăm satisfăcută condiția:

$$\left.\begin{array}{l} S_1'(1)=f'(1) \\ f'(x)=\frac{1}{2\sqrt{x}}=\frac{1}{2} \end{array}\right\} \Rightarrow b_1=\frac{1}{2} \Rightarrow 2c_1=0,366-0,5$$

$b_1=0,5$ $\qquad c_1=-0,067$

$0,5+4\cdot(-0,067)=b_2 \Rightarrow b_2=0,232$

$c_2=\frac{0,252-0,232}{2}$

$c_2=0,01$

$$S(x)=\begin{cases} 1+0,5(x-1)+(-0,067)\cdot(x-1)^2, & x\in[1;3) \\ 1,732+0,232(x-3)+0,01\cdot(x-3)^2, & x\in[3;5] \end{cases}$$

$$S(x)=\begin{cases} -0,067x^2+0,634x+0,433, & x\in[1;3) \\ 0,01x^2+0,172x+1,126, & x\in[3;5] \end{cases}$$

IV. $J = \int_a^b f(x)\,dx$

Integrala exactă

Aproximare numerică

Eroare absolută

V7. $a = 0$ $b = 1$ $f(x) = x^2$

5 noduri, dreptunghi simetric

$J = \int_0^1 x^2 dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3} = 0,333$

$0 = x_1$	x_2	x_3	x_4	$x_5 = 1$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
0	0,25	0,5	0,75	1

$2m+1 = 5 \Rightarrow m = 2$

$h = \frac{1-0}{4} = 0,25$

$f(0) = 0$

$f(0,25) = 0,0625$

$f(0,5) = 0,250$

$f(0,75) = 0,562$

$f(1) = 1$

$J_d = 2 \cdot h \cdot \sum_{K=1}^{m} f(2K)$

$\Leftrightarrow J_d = 2 \cdot 0,25 \cdot \sum_{K=1}^{2} f(2K)$

$\Leftrightarrow J_d = 0,5 \cdot (f(x_2) + f(x_4)) \Leftrightarrow J_d = 0,5\,(0,062 + 0,562)$

$\Leftrightarrow J_d = 0,5 \cdot 0,624$

$\Leftrightarrow J_d = 0,312$

$e_a = |J - J_d| = |0,333 - 0,312| = 0,021$

$e(f) = J(f) - J_d(f) = 0,333 - 0,312 = 0,021$

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V. V7. $P_m(x)$ grad necunoscut

x	0	1	2
$P_m(x)$	2	-1	4

Toate dif div de ord 3 sunt 1

coef lui x^2

Deoarece știm că toate dif div de ord 3 sunt 1, înseamnă că dif. divizați de ord 4 sau mai mare sunt 0. Deci, polinomul este de gradul 3.

$$P_3(x) = f[x_1] + (x-x_1)\cdot f[x_1,x_2] + (x-x_1)(x-x_2) f[x_1,x_2,x_3] + (x-x_1)(x-x_2)(x-x_3) f[x_1,x_2,x_3,x_4]$$

Deoarece polinomul interpolează punctele, rezultă că:

$P_3(0) = f(0) = 2 = f[x_1]$

$P_3(1) = f(1) = -1 = f[x_2]$

$P_3(2) = f(2) = 4 = f[x_3]$

Construim tabelul

	DD ord 0	DD ord 1	DD ord 2	DD ord 3
0	$\boxed{2}$	///	///	////
1	-1	$\boxed{-3}$	///	////
2	4	5	$\boxed{4}$	////
				$\boxed{1}$

$f[x_1, x_2] = \frac{-1-2}{1-0} = -3$

$f[x_2, x_3] = \frac{4+1}{2-1} = 5$

$f[x_1, x_2, x_3] = \frac{5+3}{2} = 4$

Știu că $f[x_1, x_2, x_3, x_4] = 1$

$$P_3(x) = 2 + x\cdot(-3) + x(x-1)\cdot 4 + x(x-1)(x-2)\cdot 1$$
$$= 2 - 3x + 4x^2 - 4x + x^3 - 3x^2 + 2x$$
$$= x^3 + x^2 - 5x + 2$$

Coeficientul lui x^2 e 1.

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