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344

Examen

1.(a) Alegem valorile:

$a=2$
$b=3$
$c=10$
$d=5$

Deci, $b_0=(8,5)$; $b_1=(6;1)$, $b_2=(2;3)$; $b_3=(10;5)$

Notăm $b_0=b_0^0$
$b_1=b_1^0$
$b_2=b_2^0$
$b_3=b_3^0$

Schema de Casteljau este:

$$\begin{array}{ccccccc}
b_0^0(t) & \searrow & & & & & \\
& & b_0^1(t) & \searrow & & & \\
b_1^0(t) & \rightarrow & & & b_0^2(t) & \searrow & \\
& \searrow & b_1^1(t) & \rightarrow & & & b_0^3(t) \\
b_2^0(t) & \rightarrow & & \searrow & b_1^2(t) & \rightarrow & \\
& \searrow & b_2^1(t) & \rightarrow & & & \\
b_3^0(t) & \rightarrow & & & & &
\end{array}$$

Forma generală de calcul este:

$$b_r^m(t)=(1-t)b_r^{m-1}(t)+t\,b_{r+1}^{m-1}(t)$$

Cu $t=\frac{1}{2}$, vom avea:

$$b_0^1\left(\frac{1}{2}\right)=\frac{1}{2}b_0^0\left(\frac{1}{2}\right)+\frac{1}{2}b_1^0\left(\frac{1}{2}\right)=\frac{1}{2}(8;5)+\frac{1}{2}(6;1)=(7;3)$$

$$b_1^1\left(\frac{1}{2}\right)=\frac{1}{2}\cdot b_1^0+\frac{1}{2}b_2^0=\frac{1}{2}(6,1)+\frac{1}{2}(2;3)=(4,2)$$

$$b_2^1\left(\frac{1}{2}\right)=\frac{1}{2}b_2^0+\frac{1}{2}b_3^0=\frac{1}{2}(2;3)+\frac{1}{2}(10;5)=(6,4)$$

$$b_0^2\left(\frac{1}{2}\right)=\frac{1}{2}b_0^1\left(\frac{1}{2}\right)+\frac{1}{2}b_1^1\left(\frac{1}{2}\right)=\frac{1}{2}(7;3)+\frac{1}{2}(4;2)=\left(\frac{11}{2};\frac{5}{2}\right)$$

$$b_1^2\left(\frac{1}{2}\right)=\frac{1}{2}b_1^1\left(\frac{1}{2}\right)+\frac{1}{2}b_2^1\left(\frac{1}{2}\right)=\frac{1}{2}(4;2)+\frac{1}{2}(6;4)=(5;3)$$

$$b_0^3\left(\frac{1}{2}\right)=\frac{1}{2}b_0^2\left(\frac{1}{2}\right)+\frac{1}{2}b_1^2\left(\frac{1}{2}\right)=\frac{1}{2}\left(\frac{11}{2};\frac{5}{2}\right)+\frac{1}{2}\left(\frac{10}{2};\frac{6}{2}\right)=\left(\frac{21}{4};\frac{11}{4}\right)$$

(b) Forma Bernstein a curbei Bézier asociate poligonului de control (b_0, b_1, b_2) este:

$$b_0^2=(1-t)^2b_0+2t(1-t)b_1+t^2b_2$$

Înlocuind b_0, b_1 și b_2 obținem:

$$b_0^2=(t^2-2t+1)(8;5)+(2t-2t^2)(6;1)+t^2(2;3)$$

$$b_0^2=(8t^2-16t+8+12t-12t^2+2t^2,\ 5t^2-10t+5+2t-2t^2+3t^2)$$

$h_0^{b} = (-2t^2 - 4t + 8, 6t^2 - 8t + 5) \Leftrightarrow$

$h_0^{b} = t^2(-2;6) + t(-4;-8) + 1\cdot(8,5)$

2. (a) a un semnal discret

f un filtru continuu de rază 2

Fie $\alpha = 4,4$

Știm că $(a * f)(x) = \sum_{j \in \mathbb{Z}} a[j] f(x-j)$

Deci:

$(a * f)(4.4) = \sum_{j \in \mathbb{Z}} a[j] f(4.4-j)$

Cum f e un filtru continuu de rază 2, știm că $4.4 - j \in (-2;2)$

$\Leftrightarrow -2 < 4.4 - j < 2$

$-6.4 < -j < -2.4 \quad | \cdot (-1) \Leftrightarrow$

$6.4 > j > 2.4 \Rightarrow j \in \{3, 4, 5, 6\}$

$(a * f)(4.4) = a[3] \cdot f(4.4-3) + a[4] \cdot f(4.4-4) + a[5] \cdot f(4.4-5)$
$+ a[6] \cdot f(4.4-6)$

$\Leftrightarrow (a * f)(4.4) = a[3] \cdot f(1.4) + a[4] \cdot f(0.4) + a[5] \cdot f(-0.6) +$
$a[6] \cdot f(-1.6)$

(b) Fie $\alpha = 40$, $\beta = 50$, $\gamma = 70$

$(f * A)[i,j] = 0 \cdot 50 + 1 \cdot 60 + (-1) \cdot 80 + (-1) \cdot 30 + 2 \cdot 80 + 1 \cdot 20$
$+ 0 \cdot 40 + (-2) \cdot 50 + 3 \cdot 70$
$= 60 - 80 - 30 + 160 + 20 - 100 + 210$
$= 240$

3. (a) Fie $f(x) = (2\cos x + \sin x)^2$

Încercăm să o aducem sub forma:

$f(x) = a_0 + \sum_{k=1}^{\infty} [a_k \cos(kx) + b_k \cdot \sin(kx)]$

$f(x) = 4\cos^2 x + \sin^2 x + 4\cos x \cdot \sin x$
$= \underbrace{(\cos^2 x + \sin^2 x)}_{=1} + 3\cos^2 x + 4\cos x \sin x$

$\cos^2 x = \frac{\cos 2x + 1}{2}$

$\sin x \cdot \cos x = \frac{1}{2} \sin(2x)$

$\Rightarrow f(x) = 1 + \frac{3}{2} \cdot (\cos 2x + 1) + \frac{4}{2} \cdot \sin 2x$

$f(x) = 1 + \frac{3}{2} + \frac{3}{2}\cos 2x + \frac{4}{2}\sin 2x \iff$

$f(x) = \frac{5}{2} + \frac{3}{2}\cos 2x + 2\sin 2x$

Deci, $a_0 = \frac{5}{2}$ $\quad a_1 = 0 \quad b_1 = 0$

$a_2 = \frac{3}{2} \quad b_2 = 2$

$a_K = 0 \quad b_K = 0, \ \forall K \geq 3$

(b) $h = a\cdot\phi_0^2 + b\cdot\phi_1^2 + c\cdot\phi_2^2 + d\cdot\phi_3^2$ în spațiul V^2.

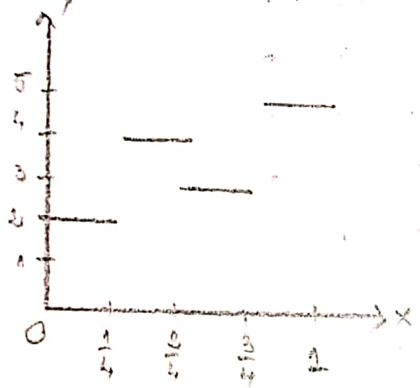
Fie $a = 2$

$b = 4$

$c = 3$

$d = 5$

În spațiul V^2, $\phi_0^2, \phi_1^2, \phi_2^2, \phi_3^2$ sunt funcții constante pe intervalele $[0;\frac{1}{4})$, $[\frac{1}{4};\frac{2}{4})$, $[\frac{2}{4};\frac{3}{4})$, $[\frac{3}{4};1)$. Deci, graficul funcției este:



$h(\frac{1}{6})$: $\frac{1}{6} \in [0;\frac{1}{4}) \Rightarrow h(\frac{1}{6}) = 2$ (deoarece $\phi_0^2(x) = 1$ pe $[0;\frac{1}{4})$, iar $\phi_1^2, \phi_2^2, \phi_3^2$ se anulează)

$h(\frac{2}{6})$: $\frac{2}{6} \in [\frac{1}{4};\frac{2}{4}) \Rightarrow h(\frac{2}{6}) = 4$

$h(\frac{4}{6})$: $\frac{4}{6} \in [\frac{2}{4};\frac{3}{4}) \Rightarrow h(\frac{4}{6}) = 3$

$h(\frac{5}{6})$: $\frac{5}{6} \in [\frac{3}{4};1) \Rightarrow h(\frac{5}{6}) = 5$

$\Rightarrow h(\frac{1}{6}) + h(\frac{2}{6}) + h(\frac{4}{6}) + h(\frac{5}{6}) = 2+4+3+5 = 14$

4.(a)

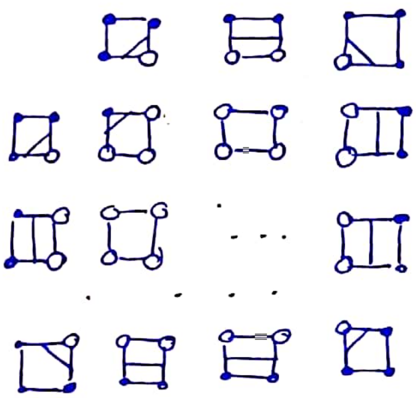
$$\begin{matrix} 26 & 27 & 26 & 29 & 28 \\ 27 & 28 & 41 & 42 & 29 \\ 28 & 42 & 33 & 38 & 28 \\ 26 & 41 & 42 & 45 & 29 \\ 28 & 42 & 35 & 37 & 27 \\ 24 & 27 & 28 & 27 & 26 \end{matrix} \xrightarrow[\text{de prag 31}]{\text{Valoarea}} \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Înlocuim cu • = 0 și o = 1.

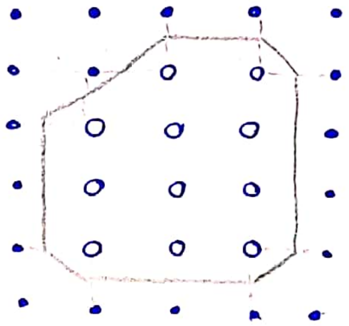
• • • • •
• • o o •
• o o o •
• o o o •
• o o o •
• • • • •

O fereastră glisantă de 2 × 2 va trece peste elemente și va stabili unde trebuie trasată o margine.

Se vor identifica, pe rând, cazurile:



$\Rightarrow$ Conturul va fi:



(b) Direcția de reflexie este:

$\rho = \delta - 2(\delta \cdot s) \cdot s$, unde δ = raza de lumină
s = normala
ρ = raza reflectată

Fie direcția razei de lumină $\delta = (1, 2, 4)$.

$\Rightarrow \rho = (1,2,4) - 2\big((1,2,4)\cdot(0,0,1)\big)\cdot(0,0,1)$

$\Leftrightarrow \rho = (1,2,4) - 8\cdot(0,0,1)$

$\Leftrightarrow \rho = (1,2,-4)$