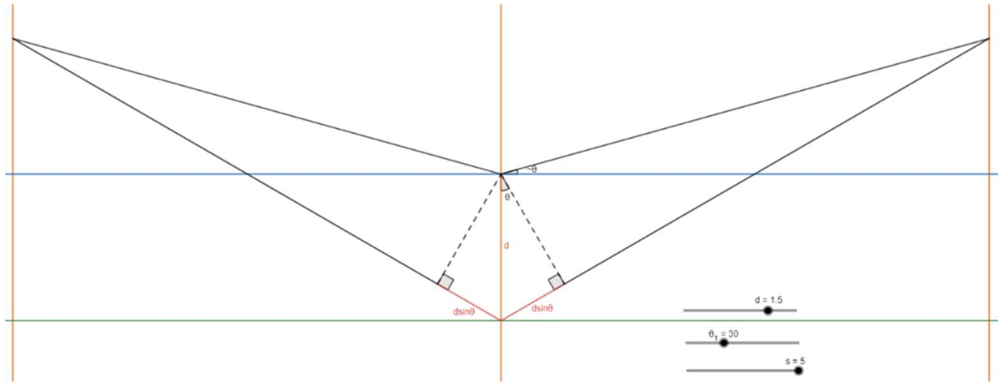
Measuring the Wavelength of Light by Diffraction

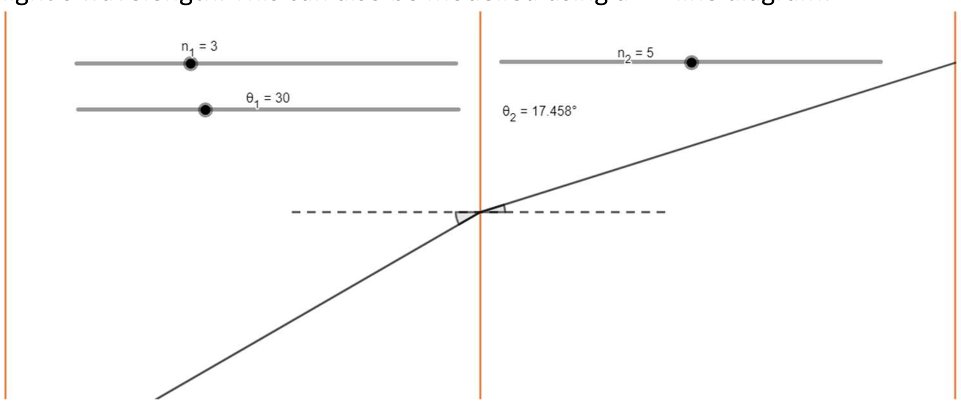
There are 3 fundamental properties of electromagnetic radiation when encountering changes in the environment around it: Reflection, refraction and diffraction.

Reflection is not generally affected by wavelength; however, a property of reflection, namely partial reflection of multiple layers of material boundaries, is. A model with 2 layers, and 2 edges of a beam of light demonstrates this pattern well:

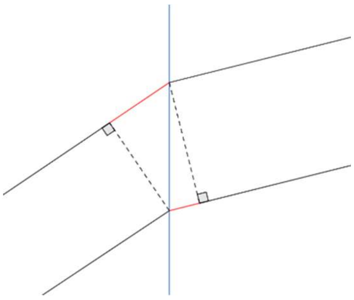


The diagram depicts a point source of light (black) shone at an angle towards two layers of a material (blue and green). Viewing the model from left to right shows that until the first perpendicular dotted line, the two beams of light are in phase; from after the third dotted line, they are also in phase moving to the right-hand side of the diagram. This just leaves the red section of the lower wave, creating the phase difference between the waves. Two right triangles can be formed with the opposite side of the triangles representing the total phase difference. Viewing the waves as superposed at the point on the right-most orange line after the reflection, it is clear that for maximum constructive superposition, the red section of the bottom wave should be equal to an integer multiple of the wavelength of the light, leading to the formula $2dsin\theta = n\lambda$ for the layer spacing, angle, and wavelength of the nth light point reflected off a substance. This formula can be used to tell the spacing between atoms in a specimen. Cited models of this equation online and in text books treat the edges of the ray of light as perpendicular, traveling towards and away from the material, and both reflections with the same angle. This is a safe approximation, as it does not affect the right triangle constructions, as long as only the top edge of the beam is angled down to meet the bottom at some two points equidistant to the point of reflection. The 3 variables I created sliders for in this model were d, the distance between reflective layers, θ, the angle of the bottom edge of the beam of light, and s, the distance of the point source horizontally from the reflection site. In all real-life cases, the value for d will be much lower than s, meaning it is safe to assume in the local site around the reflection that the edges of the beam are perpendicular.

Wavelength is a very important factor in refraction, as the change in speed of a beam of light determines the change in its angle. The frequency of a beam light is fixed regardless of medium. When a boundary is crossed, however, the wavelength does change, meaning the wave speed should also change to satisfy the relationship $v = \lambda f$. The wave speed changes a different amount based on the frequency of the light, causing the beam to bend a different amount depending on the light's wavelength. This can also be modelled using a 2D line diagram:



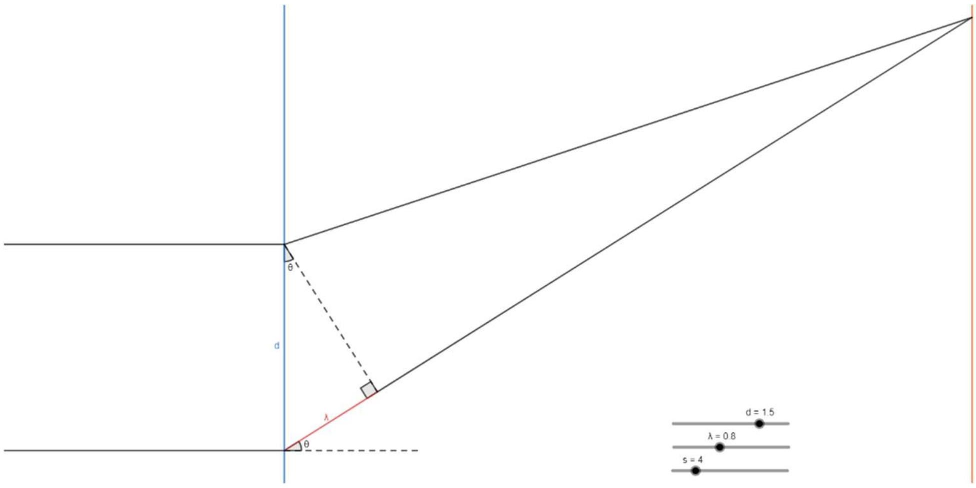
The refractive index, n, of a material determines the speed electromagnetic waves pass through it. The higher the refractive index, the more the material "resists" the passthrough of electromagnetic waves, causing refraction. The light bending itself can be explained by a closer diagram of the ray. For the leftmost and rightmost sections of this diagram, the edges of the ray are parallel and in phase. The red section of either edge of the beam are different lengths,



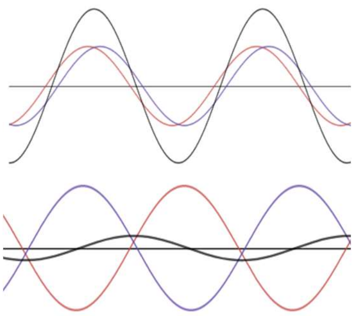
as one edge of the beam has passed into the slower medium before the other. This section is produced by the light during the same period of time, meaning the upper edge of the light "turns" an angle to return to being parallel to the other edge. These correlations mean that there is a simple fractional relationship between $sin\theta$, n and v for mediums 1 and 2:

$$\frac{sin\theta_1}{sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Diffraction is also affected by wavelength, as the angles a beam is diffracted through, and the diffractions intensity, when passing through gaps is dependent on the wavelength of the light. If we think of just the first order of the light diffraction, in only one direction, the system can be modelled very similarly to the reflection example: as the construction of a right triangle with two parameters, d and λ, the slit separation (distance from the centre of one given point to the centre of the next) and wavelength of the beam:



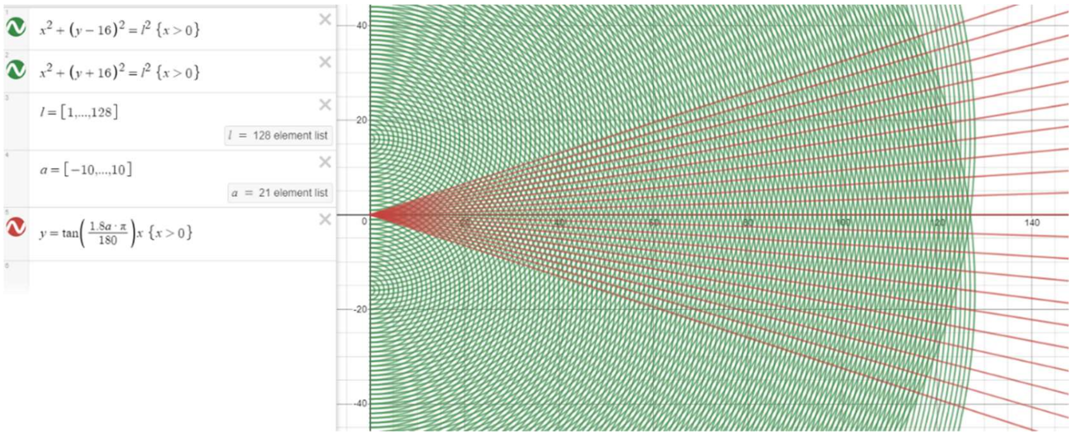
In this model, editing the wavelength changes the $dsin\theta$ side of the triangle, thus affecting the angle, rather than vice versa. As λ approaches the value for d, the angle gets larger and larger until the point where they are equal, and a right triangle can no longer be formed with d as its hypotenuse. In this model, as with the reflection model, the beams of light are represented to collide at some point close to the diffraction grating, despite being explained to leave the grating at one angle. Again, I have taken the bottom edge of the light (the one which includes the construction of the right triangle. In reality, there are infinite possible constructions of right triangles, each with a certain brightness of superimposed light projected from them. The brightest points are from where the opposite edge to the angle of the right triangle is equal to $n\lambda$, and in the least bright points (most destructive interference) the side is equal to $2^{-1}\lambda(2n+1)$. In all realistic cases, the values for d and λ are small enough, s is large enough, and enough gaps and waves are present, that the model can be shown as a series of parallel beams from each point at an angle θ. This model is not to scale, and does not use SI values for its parameters. I made it such so that it better demonstrates the full path of the beams of light, their destination, and the effect of passing through the slits on the beams.



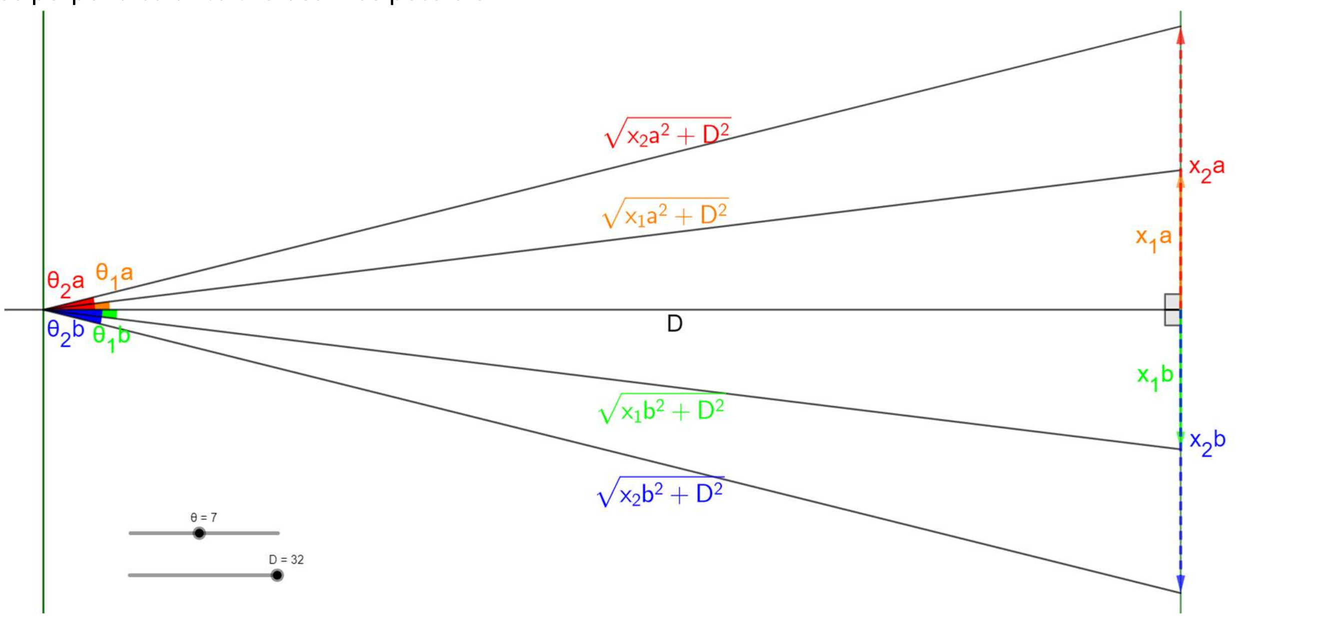
The superposition of the electromagnetic waves is a very important factor in all systems involving waves, as it determines the brightness of light or intensity of radiation at any point in space with any number of wave sources. It occurs when two waves meet in space, and is defined as the summation of their sine waves to create a single new wave, present at that point. The more waves are superposed at a single point, the brighter, and usually sharper, the point of light created if the wave at that point if projected onto a surface. This is because the summation of a sine wave with another always results in another sine wave, with a higher amplitude when constructively superposing.

I produced all of these examples to be interactable to aid in understanding the systems and models, using GeoGebra and Desmos, both free online graphing resources. They are available to be viewed under https://www.geogebra.org/graphing/sqcqujab, https://www.geogebra.org/graphing/wmtaamyh, https://www.geogebra.org/graphing/eykbsvfd, and https://www.desmos.com/calculator/hswmrbpcyh. (All as of 28/03/19) If using this report solely for the understanding of reflection, refraction and diffraction, I strongly advise to have these models open as supplement to my explanations on these pages.

The reason beams of light can be modelled as being projected by any angle from the two slits, is due to electromagnetic waves' radial nature. Two radial wave-fronts meet in points roughly linearly spaced from the midpoint of the two fronts. The closer together the radial points, the more points there are, and the closer together each concentric circle is, the more accurately linearly the points line up. Along with this, each radial projection of points is at an angle from the horizontal original beam. For small angles, and with a low value for d, these can be treated as a multiple of an angle, as I have done for the red lines in this diagram. However, at higher angles, this breaks down, as the elliptical nature of the pair of radial sources is more prevalent nearer the top and bottom of the diagram. Rather than linear beams, the rays of diffracted light could be better modelled as a function like a parabola or hyperbola, however I could not find a function which sufficiently passes through the intersections in this two-source, 128-period example. Also, this would significantly complicate the model I have used in my experiment to find the wavelength, and slit separation of my sample lasers and gauzes. In reality, with the number of significant figures I measured, the distance away from the sources I measured, and the angles I measured the light over, the current model would create no systematic error from reality, making it a safe representation of the real system, for this investigation.



In my experiment, I chose to use the diffraction angles of a few different wavelengths of electromagnetic lasers, and different separations of slits. I only had access to 3 different lasers, which quite different precisions in their "actual value". For the two different-wavelength handheld lasers, I used a clamp stand to hold them secure in place. A second clamp stand was placed about 1 metre away from a flat white planar board, holding the current diffraction grating. The further the grating was from the board, the more spaced apart and easily distinguished the projected points of light would be from each other. The board itself was used to help distinguish between each point, as a plain ruler could be placed flat and horizontal on the board at the centre of the spread of light from each beam. At each point, there was a 2mm diameter of glare from each point. This could be minimised by covering the board in a material that is as planar as possible, with particles whose electrons re-emit absorbed photons sooner and in similar directions as they absorbed them. This will lead to minimum possible spread of the point of light on contact with the material. Along with this, a thinner beam of light from the laser, or a more controlled beam (to ensure all waves are parallel), would help minimise the point diameter before contact with the plane. I could also have taken measures to ensure the board was as perpendicular to the beam as possible.



Using the linear model, I can construct right triangles to use the values for x and D to find θ. Because I am using trigonometry for this, there are 3 different forms of the equation for θ:

$$\theta = asin\left(\frac{x}{\sqrt{x^2+D^2}}\right) \qquad \theta = acos\left(\frac{D}{\sqrt{x^2+D^2}}\right) \qquad \theta = atan\left(\frac{x}{D}\right)$$

Because of the equation of diffraction, $n\lambda = dsin\theta$, I chose to use the expression for theta with $\mathrm{asin}(f(x,D))$, as the arcsine would cancel with the sine, resulting in this final equation:

$$n\lambda = \frac{dx}{\sqrt{D^2+x^2}}$$

I attempted rearrange the equation into a form with λ and x isolated, such that I could form a linear graph (in the form $y = mx$) with solely the input on one axis and the output on the other, however this is not possible due to the Pythagorean calculation involving x. In spite of this, I have created a table with 4 useful forms of the equation and their uses in finding values.

$$n\lambda = \frac{dx}{\sqrt{D^2+x^2}}$$

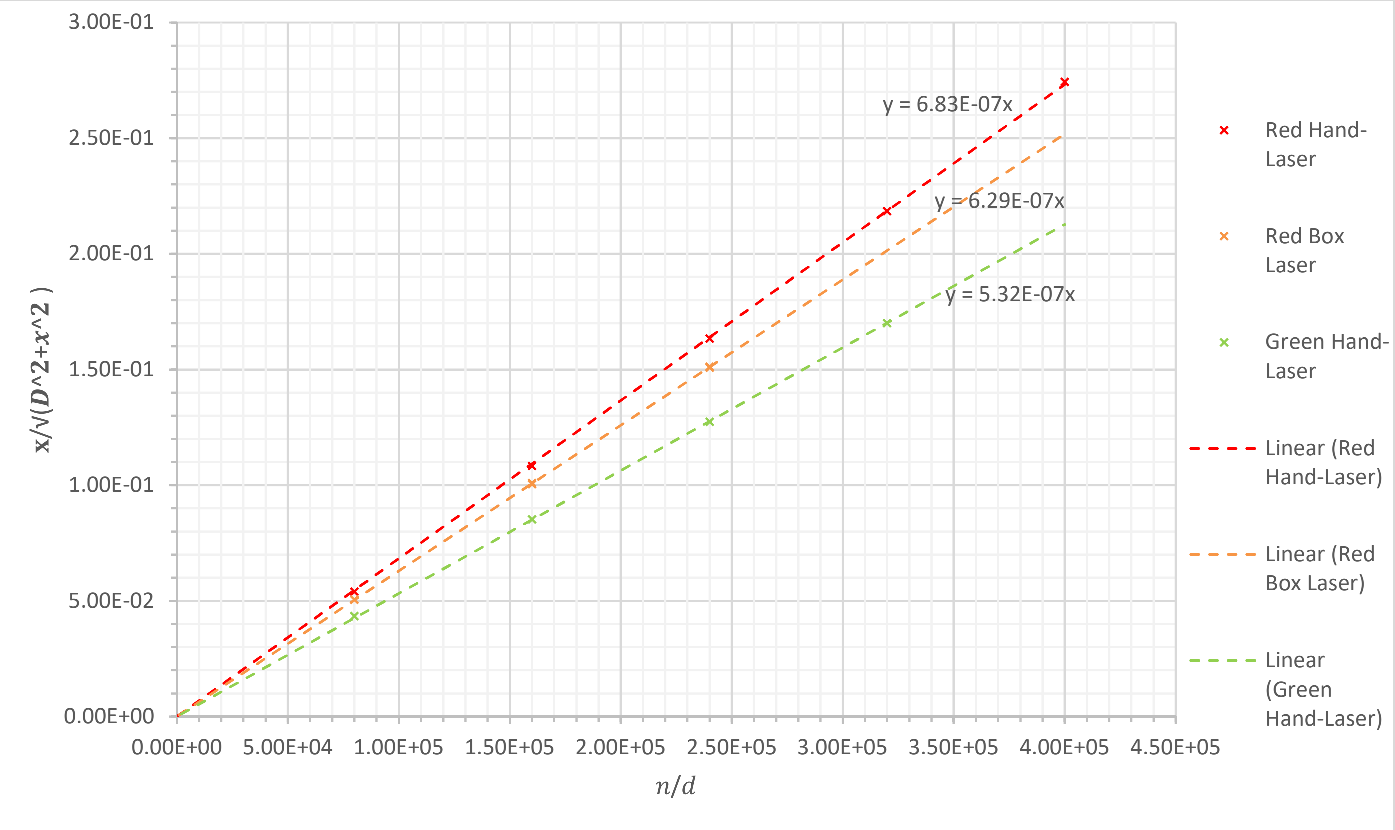
$$n^2\lambda^2D^2 + n^2\lambda^2x^2 = d^2x^2$$

$$x^2 = \frac{n^2\lambda^2D^2}{d^2-n^2\lambda^2}$$

$$x = \frac{n\lambda D}{\sqrt{(d+n\lambda)(d-n\lambda)}}$$

y	=	m	x	Use
$\frac{x}{\sqrt{D^2+x^2}}$	=	λ	$\frac{n}{d}$	Finding wavelength from a set of different slit separations or degrees.
$\frac{\sqrt{D^2+x^2}}{x}$	=	d	$\frac{1}{n\lambda}$	Finding slit separation from a set of different wavelengths or degrees.
λ	=	$\frac{d}{n}$	$\frac{x}{\sqrt{D^2+x^2}}$	Horizontal input as a function with x, and vertical output as wavelength.
d	=	$n\lambda$	$\frac{\sqrt{D^2+x^2}}{x}$	Horizontal input as a function with x, and vertical output as slit separation.

To find the wavelength of each laser, I used 2 methods: Calculating the wavelength by formula for each degree then averaging, and plotting the x and y using the top row formula from the table, plotting a line of best fit through the origin, then finding its slope. The red lasers had values for their wavelength on the device, so I used those for my calculation of percentage error. For the green laser, however, there was no given value, so I found a cited range for green light from https://en.wikipedia.org/wiki/Color as of 28/03/19. The red hand-laser also had a cited range instead of a value. For these, I just calculated a maximum and minimum percentage error using the bounds of the "real" wavelength. For both of the hand lasers, I found the slit separation of some diffraction gratings using the same method, however, I did not have real values for this, so I cannot analyse the efficiency of my calculation, only make rough judgements based on the success of the former part of the investigation.



	n	x (m)	θ	λ (m)	x (axis)	y (axis)
Red Hand-Laser	1	0.0510	3.089	6.74E-07	8.00E+04	5.39E-02
	2	0.1030	6.220	6.77E-07	1.60E+05	1.08E-01
	3	0.1565	9.403	6.81E-07	2.40E+05	1.63E-01
	4	0.2115	12.615	6.83E-07	3.20E+05	2.18E-01
	5	0.2695	15.917	6.86E-07	4.00E+05	2.74E-01
			Average:	6.80E-07	6.83E-07	:Graphed
			%diff max:	7.93%	8.43%	
			%diff min:	-0.01%	0.46%	

D:	0.945
d:	1.25E-05
λ min:	6.30E-07
λ max:	6.80E-07

n	x	θ	d
1	0.0035	0.2122	1.84E-04

D:	0.945
λ:	6.80E-07

n	x	θ	d
1	0.0115	0.6972	5.59E-05

D:	0.945
λ:	6.80E-07

For the red hand-laser, my value for its wavelength was very close to the maximum cited value on the device. With the value calculated as an average giving equal value to each order, the wavelength was 0.01% under the 680nm upper bound, meaning the result was as accurate as possible for the precision and bounds I had access to. However, if given a true value for the wavelength, or even another bound with higher precision, it is very likely, although not certain, that my value would then lie outside the range. Along with this, after plotting my results as points as coordinates forming a linear graph, the value increased such that it then fell outside the bounds. I do not believe this average can be considered more reliable than the original non-graphed value, as there were only 5 points of data on the graph, all of which lay very close to a line of best fit, such that the linear regression of the points from the mean was 1.000 to 4 significant figures. This suggests that any deviation from the line of best fit would have been caused by very minute changes to the environment containing the investigation, such as the table or clamp stand being nudged slightly. Because there is no way to predict such changes, and they can only be minimised by conducting the experiment in a more controlled environment than a classroom, I believe there was equal chance for any method of data compilation and averaging to vary like this, above or below the real value. The change in percentage error between the graphed average and the calculated one was 0.47%, so I believe the actual percentage uncertainty in my experiment to be around 1%.

	n	x (m)	θ	λ (m)	x (axis)	y (axis)
Red Box Laser	1	0.0505	3.059	6.30E-07	8.00E+04	5.04E-02
	1	0.0505	3.059	6.30E-07	8.00E+04	5.04E-02
	2	0.1010	6.101	6.28E-07	1.60E+05	1.00E-01
	2	0.1015	6.130	6.31E-07	1.60E+05	1.01E-01
	3	0.1525	9.167	6.28E-07	2.40E+05	1.51E-01
	3	0.1530	9.197	6.30E-07	2.40E+05	1.51E-01
			Average:	6.30E-07	6.29E-07	:Graphed
			%diff:	-0.48%	-0.55%	

D:	1
d:	1.25E-05
λ:	6.328E-07

For the box laser, I had a much higher degree of precision for the real wavelength, as it was a device more tailored for scientific use. The values I calculated for the wavelength were under 1% error, backing up my estimate for the uncertainty in the investigation. Since this was the highest quality laser, and the values for percentage error were lower than the rest, it leads to believe that much of the uncertainty for the hand-lasers was from that fact. There are some differences in the build of the laser that could have caused such error: The initial angle of the laser, which was harder to control for the hand laser, as it needed to be held in place by a clamp stand, whereas the box laser sat perfectly flat; the monochromaticity of the light, as a higher quality laser would likely be designed with keeping the same wavelength throughout the beam in mind, which would lead to a more precise point projection, whereas with slight changes in wavelength, at higher orders, there will be a horizontal spread of projection, where the lower wavelengths are refracted less. In this case, plotting the results on a graph also resulted in an increase of percentage error, however the difference is still below 1%, so I do not believe the -0.55% to be any more reliable than the -0.48%.

	n	x (m)	θ	λ (m)	x (axis)	y (axis)
Green Hand-Laser	1	0.0435	2.636	5.43E-07	8.00E+04	4.35E-02
	2	0.0855	5.170	5.32E-07	1.60E+05	8.52E-02
	3	0.1285	7.744	5.31E-07	2.40E+05	1.27E-01
	4	0.1725	10.345	5.31E-07	3.20E+05	1.70E-01
			Average:	5.34E-07	5.32E-07	:Graphed
			%diff max:	2.79%	2.26%	
			%diff min:	-4.56%	-5.05%	

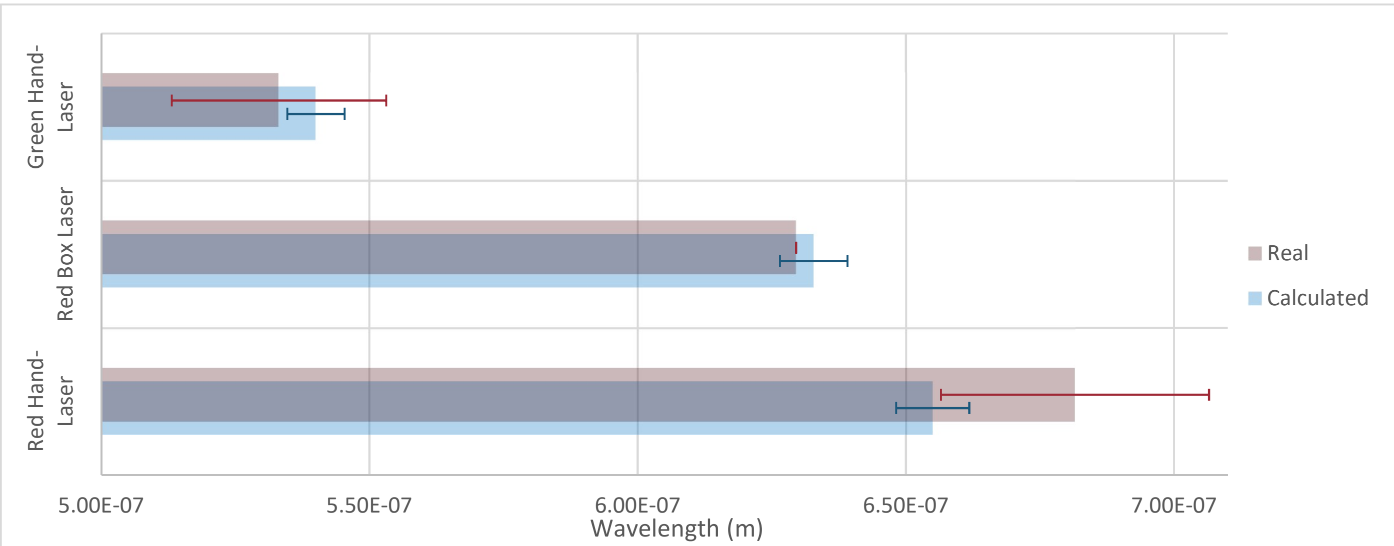
D:	1
d:	1.25E-05
λ min:	5.20E-07
λ max:	5.60E-07

n	x	θ	d
1	0.164	9.8453	3.30E-06
1	0.1645	9.8748	3.29E-06
		Average:	3.30E-06
		%diff:	1.07%

D:	1
λ:	5.34E-07
d:	3.33E-06

Although the green laser appears to have been the least successful value, this mostly comes down to the fact that my values for the real wavelength was the entire green range of the electromagnetic spectrum, rather than an exact value to 4 significant figures, or a more precise range as the red hand-laser was (630nm – 680nm, when the red spectrum is 635nm – 700nm). The actual value fell quite close to the centre of the green range of light, which is why both values for the error appear to be large. There is no way to determine where in the range the actual value fell, so this can be treated as 100% accuracy for the precision I had access to. For this laser, I did a calculation of slit separation for a diffraction grating which I knew the value of d for, and calculated a percentage error of 1.07%. This falls outside the range I theorised for the wavelength, however it is a logical result, as I only took the two values at order 1 for x, meaning they could likely have been anomalous, and there is no way to determine without more results.

For all of my results for the wavelength, I plotted a range-crossover graph, showing the possible range within which each real and calculated value falls. It shows that this investigation was very reliable at the precision under which I executed it, as all but the red hand-laser had a full range overlap, and the red hand-laser itself has close to 50% crossover under my 1% error estimate. This experiment can be made more precise by holding it in a more controlled environment, in darkness, using a more monochromatic laser, increasing the distance from the diffraction gauze to the projection plane, using spirit levels, or more precise positioning equipment to ensure as little unwanted angle as possible, and using a less reflective material for the plane so that the light bleeds into a larger circle less after impact.



A way to analyse individual datapoints would be to plot my 1% error margins as lines onto the graph from which I got my slope values, and ensure all points fall between those two slopes. This can be done for this experiment as all lines of best fit were ensured to pass through the origin. However, I cannot do this with my current equipment, as Excel does not have a feature to plot known equations onto scatter graphs.