Measuring Resistivity

Resistance is the measure of how much a material resists the flow of charge. There are 3 factors which affect resistance: Length, cross-sectional area, and resistivity. The resistivity also depends on the materials type and its temperature, meaning materials can be shown by their resistivity at a certain zero-temperature, and their temperature coefficient (the amount the resistivity changes based on the change in temperature). For some materials such as copper wire, there is a positive temperature coefficient, meaning the resistivity increases with temperature, but for others such as most LDRs, there is negative temperature coefficient, meaning the opposite. This is because when a current flows through the material, or any photons are shone at the material, there are collisions with the atoms in the material. These collisions have two different effects: Firstly, they vibrate the particles of the material, disrupting the flow of charge through the circuit and thus increasing the resistivity; secondly, transferring energy to electrons in the atoms, causing them to move higher in their energy levels, or release from the atom entirely, and be more easily accessed as free charge carriers, thus decreasing the resistivity. These effects occur in tandem, both to variable degrees. This combination is what determines the temperature coefficient of a material, making its resistivity change in a certain way depending on temperature.

Any values for resistivity I compared against were using a temperature of 20°C, as the temperature of the room was 19.9°C, and it's much easier to find precise values for the resistivity at RTP. It is important to note that some heating may have taken place in the wire during the gathering of data, which would have made the resistivity appear higher than expected. However, due to the cross-sectional area being so low, the current could not be high enough to produce a significant heating effect. With all this in mind, the equation for the resistance is as such:

$$\text{resistance} = \frac{\text{function}(\text{resistivity}_0,\ \text{temperature coefficient},\ \text{temperature}) * \text{length}}{\text{area}}$$

$$R = \frac{f(\rho_0, k, \theta) * l}{A} \text{ or } f(\rho_0, k, \theta) = \frac{R * A}{l} \text{ where } f(\rho_0, k, \theta) \text{ makes up the material's resistivity}, \rho$$

The function that takes in the current temperature is not the same for each material. Being a combination between two likely non-linear effects, deriving this function would be difficult. I have modelled the function as if it were a linear graph in the form $f(\rho_0, k, \theta) = k\theta + \rho_0$, in the form $\mathrm{y} = \mathrm{mx} + \mathrm{c}$, where k and p_0 are constant (which they are for a given material) making it a single-variable function which takes in temperature and returns resistivity. However, there are some problems with this model, as with a linear equation, if the temperature coefficient is negative, this implies that there will be a point at a high enough temperature where the resistivity is zero or negative, which is not possible. In reality, materials with a negative temperature coefficient have reciprocal graphs. The infinite resistivity at zero temperature implies that when no energy is present in the molecules (0°K), no current can flow, which is true for all materials. Hence, all models for function $f(\rho_0, k, \theta)$ should have a reciprocal term added to the graph to account for this. However, there is a common model for this function specific to conductors for a range of temperatures, that resembles the vector equation of a line, commonly cited as $f(\theta, \rho_0, \theta_0, \alpha) = \rho = \rho_0[1 + \alpha(\theta - \theta_0)]$, where the resistivity is ρ and ρ_0 at temperatures θ and θ_0 respectively. This function takes a coordinate (θ_0, ρ_0), and passes a line through it with gradient α.

The formula above can be used to interpret the way variables would change based on the change of other variables. If the length of material wire increases in the circuit, the resistance will increase. If the resistivity of the material increases, the resistance will also increase. However, as the cross-sectional area of the material increases, the resistance will decrease, as there is either more space for the charge carriers to flow, or more overall charge carriers. It's important to note here that any given material for this formula is modelled as a cylinder full of uniform flowing charge carriers/fixed charge-holding atoms, with a fixed potential difference across either "cap" of the cylinder. In reality, the actual matter flows in the opposite direction to the flow of charge, as the slow drift velocity is that of the particles traveling from the negative terminal to the positive one, and the actual "speed" of the electricity is the interactions between the particles causing a space of higher negative charge (where the charge carriers are closer together), which moves the opposite direction in the circuit, at the speed of light (or electromagnetic vibration).

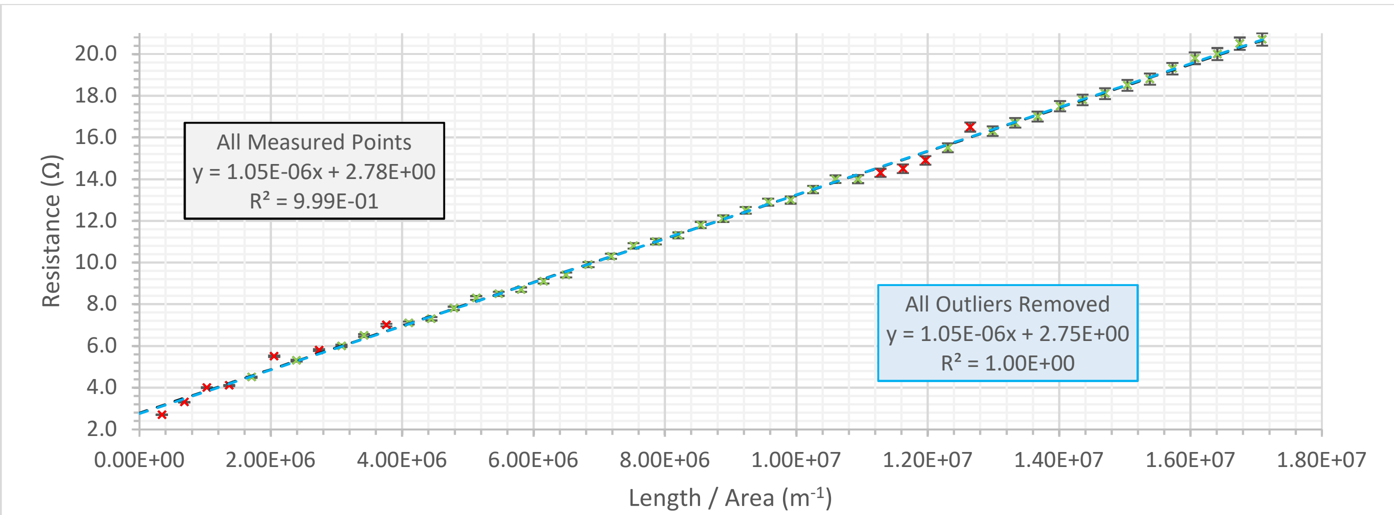
To carry out the experiment, I used a very simple circuit with the wire connected in one branch with a multimeter set to measure ohms connected with a crocodile clip at one end of the wire, and a flying lead which was able to reach any point along the wire between the circuit end (0m) and 0.5m (the other end). The wire was secured to the top of metre ruler (±0.0005m uncertainty) at either end by adhesive putty, as I hoped this would have as little an effect on the circuit as possible. The multimeter works by applying a small current to the circuit, measuring the potential difference created and using $V = IR$ to determine resistance. This small current can have a heating effect at lower resistances, which was seen in some of my results. I used a graphical method to measure the resistivity of the materials, changing the length of wire many times and measuring the resistances at each of these points. In carrying out this experiment, I sought specifically to get as many points of data as I could, and settled for 50 points, each spaced 0.01m apart along the wire. I used the equation for resistance in its original form, as it already represented a graph of $y = mx$, with R being y, l/A as x, and ρ for m, meaning I could plot resistance and length/area as the axis of a linear graph, and read resistivity as the gradient of the graph. There is a "$+c$" in this case, namely the rest of the resistance in the circuit that I haven't accounted for. To deal with this, I added a Res_0 to my Excel document, and plotted $R - R_0$ on the y axis. I measured the zero-resistance by connecting the flying lead to the crocodile clip at the start of the circuit, (excluding the wire from the circuit).

I used the metal wires I had available to me at the time: nichrome, nickel, constantan (and copper and graphite) [and conductive dough], as they were plentiful and came with their respective standard wire gauge noted down, which I could convert to material diameter easily. I also went for dough and graphite as their properties were slightly different to regular wires, dough's diameter being much larger, decreasing resistance, and both of their compositions being different to that of regular metal wire, meaning they would react differently to changes in temperature.

Length (l)	Resistance	l/A	Res-Res_0	Resistivity	Varience	Res Var	Varience 2	Var-Var2	Res_0:	2
0.010	4.7	3.42E+05	2.7	7.90E-06	4.70E-11	5.98E-03	4.69E-11	4.60E-14	Points:	50
0.020	5.3	6.84E+05	3.3	4.83E-06	1.43E-11	1.20E-02	1.43E-11	2.54E-14	Diameter:	1.93E-04
0.030	6.0	1.03E+06	4.0	3.90E-06	8.15E-12	1.80E-02	8.13E-12	1.92E-14	Area:	2.93E-08
0.040	6.1	1.37E+06	4.1	3.00E-06	3.81E-12	2.39E-02	3.80E-12	1.31E-14	Mean Rsty:	1.05E-06
0.050	6.5	1.71E+06	4.5	2.63E-06	2.52E-12	2.99E-02	2.51E-12	1.07E-14	Var Sum:	1.75E-12
0.060	7.5	2.05E+06	5.5	2.68E-06	2.68E-12	3.59E-02	2.66E-12	1.10E-14	Mean R 2:	1.05E-06
0.070	7.3	2.39E+06	5.3	2.22E-06	1.37E-12	4.19E-02	1.36E-12	7.84E-15	Non Outliers	
0.080	7.8	2.73E+06	5.8	2.12E-06	1.16E-12	4.79E-02	1.15E-12	7.21E-15	Total Outliers	
0.090	8.0	3.08E+06	6.0	1.95E-06	8.18E-13	5.39E-02	8.12E-13	6.06E-15	Partial Outliers	
0.100	8.5	3.42E+06	6.5	1.90E-06	7.32E-13	5.98E-02	7.26E-13	5.74E-15	Book Rsty:	
0.110	9.0	3.76E+06	7.0	1.86E-06	6.65E-13	6.58E-02	6.60E-13	5.47E-15	Raymond A.	1.10E-06
0.120	9.1	4.10E+06	7.1	1.73E-06	4.69E-13	7.18E-02	4.64E-13	4.59E-15	Electrical Re	1.1E-06
0.130	9.3	4.44E+06	7.3	1.64E-06	3.56E-13	7.78E-02	3.52E-13	4.00E-15	Resistivity ar	1.00E-06
0.140	9.8	4.79E+06	7.8	1.63E-06	3.41E-13	8.38E-02	3.37E-13	3.91E-15	Table of Elec	1.10E-06
0.150	10.3	5.13E+06	8.3	1.62E-06	3.28E-13	8.98E-02	3.24E-13	3.84E-15	Average:	1.08E-06
0.160	10.5	5.47E+06	8.5	1.55E-06	2.58E-13	9.57E-02	2.55E-13	3.40E-15	% Error:	2.69%
0.170	10.7	5.81E+06	8.7	1.50E-06	2.03E-13	1.02E-01	2.00E-13	3.02E-15		2.38%

Here is the top portion of my table of results for nichrome. All values are in the standard units for each given variable (lengths are in metres, resistances are in ohms, etc.). The values along the right-hand side of the table remained constant for the material used. The first pair of columns in the table shows values I measured directly from my apparatus, the length from the ruler and the resistance from the multimeter. The second pair of columns shows the X and Y axis values respectively, such that I was able to measure the resistivity as the gradient of the graph. Column 2 uses the Res_0 value I measured at the start of the experiment, and column 3 uses the area I calculated using the SWG of the wire. The 5th column shows the resistivity each individual point implies; and I use these values to determine the variance of each point from the mean. You can think of these points as a vector gradient from the origin to each point on a graph, however, these values have a fault I will get into later.

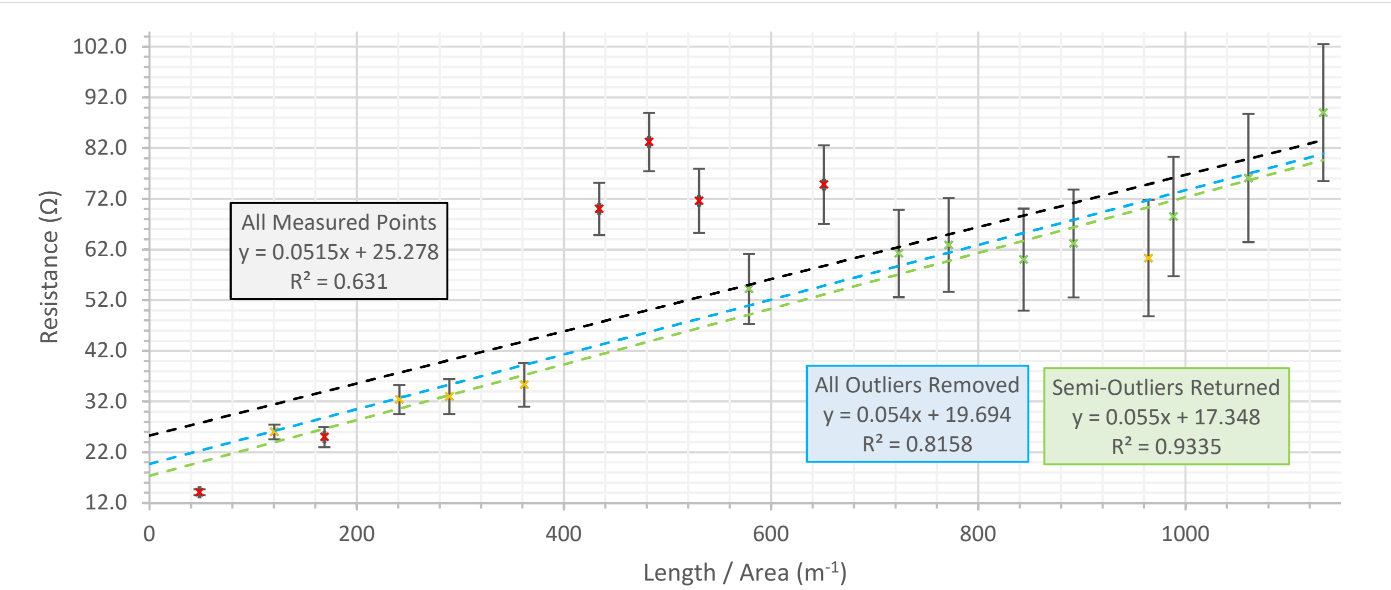
The next 4 columns were used after calculating an initial value for the resistivity, to help determine the accuracy of each point, to find anomalies mathematically. I used Excel's LINEST() function to calculate my "Mean Rsty", as I would read from the line of best fit on a regular graph. The variance of each point is the square of the difference between the resistivity that point implies and the mean resistivity, calculated from the LINEST(). These can be thought of as the "square distance of each point away from the mean". I then got the average resistivity, by summing the variances of each point, then dividing by the number of points (50). Then I put this average variance (of resistivity) back through the equation for resistance for each point, thereby calculating a variance in the resistance of each point, which I plotted on the graph as vertical error bars. The bars were an arbitrary multiple of the variance such that a suitable number of points lay on the line. Here was the graph I made for nichrome:



Using the error bars and black line of best fit through all points and zooming into the graph, I identified and made note of all outliers which I marked red on the table of data. I then made a new line of best fit, going through only the non-anomalous points. In this case, the new line of best fit had a slightly higher gradient and lower intercept.

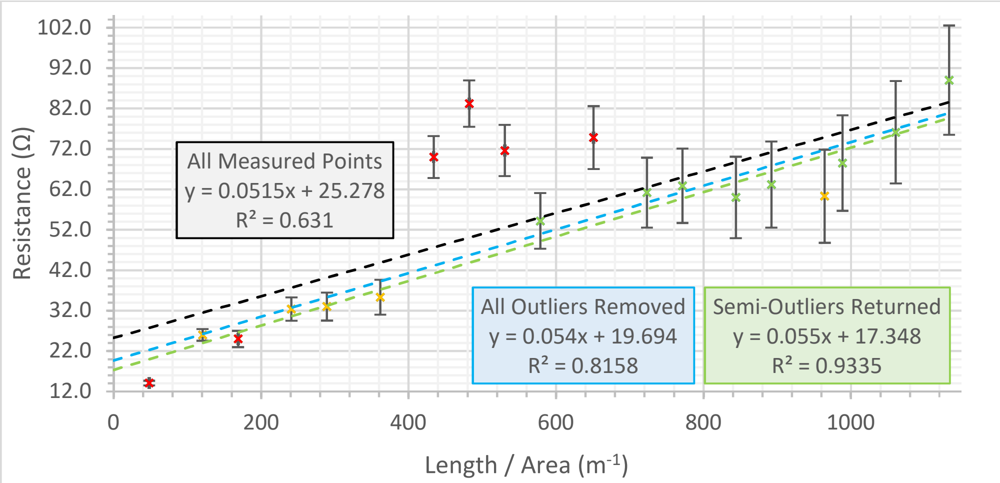
It is important to note here, that my choice of vertical error bars of resistance was totally arbitrary, as I could have used my calculation of average variance to find horizontal error bars, or used the number of significant figures and other factors to calculate an uncertainty from the raw data rather than after I processed each point into a variance. My choice in this case was solely to come to understand how variance works in a set of data, and use it on an experimental data set. In the future I may try using both methods in tandem such that I don't have to choose a coefficient to place an arbitrary number of points on the line of best fit, or calculate both the horizontal and vertical error bars from the variance. In the case of nichrome, I needed to use 10,000 times the variance of each point vertically just to get any points on the line of best fit. This was almost certainly because of how uniform and strong the correlation of my points was on this graph.

Another important thing to consider is that, despite subtracting a measured Res_0 value (2 ohms in this case), there was still a notable intercept on the graph (2.75 more ohms than predicted). Although this may seem like a harmless factor, this is where the issue with using individual calculated resistivities for each point to calculate variance comes in. As each point is treated as the slope of a vector from the origin to the point, and I am calculating the difference between this slope and the slope of a line of best fit, it is clear to see that this line of best fit must also be being treated as going through the origin, which it is not. This means that either the line of best fit should be fixed to the origin, or the intercept of the line of best fit should be subtracted from each point such that they form a line going through the origin. In the former case, this would change the gradient of the line drastically, especially with such a high R^2 value (meaning a low average variance). This new measured resistivity would not represent the trend of the data anywhere near as well as a free line of best fit would. In the latter case, the subtraction would be too arbitrary to justify, as that subtracted value is not accounted for in my model of the system, so my value for resistivity would not truly represent the data I collected. In either case, there would need to be a lot of revising to the sets of anomalous points, meaning hours of extra work for a possibly insignificant change of calculated resistivity in the range of significant figures I am using. With these in mind, I have decided to stick with my original method. I saw an interesting effect when carrying out the same variance process with the dough:



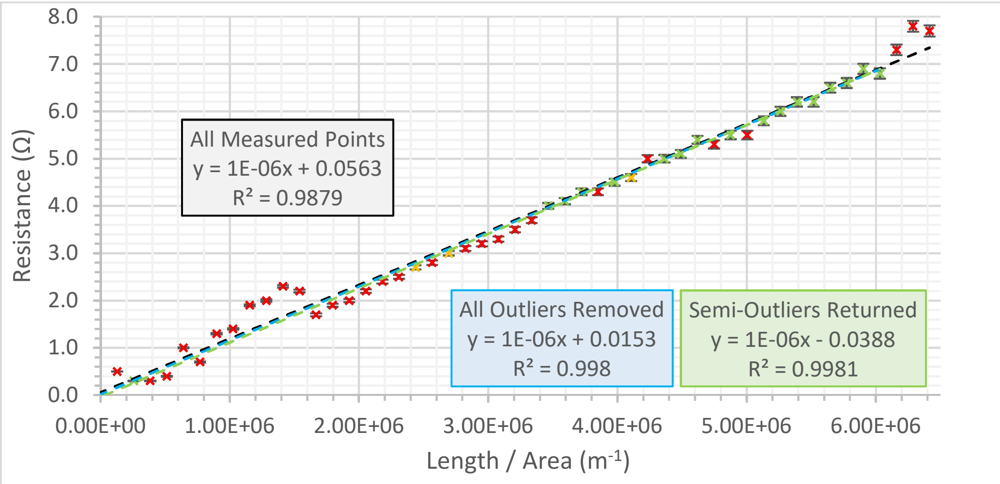
The majority of the points were anomalous using the original line of best fit, as most of the points lay close enough to the line for the total variance to be quite low, but far enough away for their error bars to not cross the black line. Once I took out the outliers, and plotted the new (blue) line of best fit, however, 5 more of the points (yellow) were now able to fit on the line of best fit, which showed that those points were less anomalous than the points that didn't (red). I decided then to reintroduce the "semi-outliers" back into the dataset, and plot a new (green) line of best fit. Surprisingly, the R^2 value for this third line was higher than the second one, despite adding in more points. This process shows a lot about data analysis in physics, as a human looking at the graph would probably derive through intuition that the 4 red points at the top centre of the graph are anomalies. Yet there's nothing to show that they are more anomalous than the yellow points, as with error bars, none of them cross the line. However, after going through one more iteration of forming a line then altering the dataset to fit it, it is now much clearer that the green points towards the right side of the graph "imply" the yellow points to be more correct than the red ones. One shouldn't assume facts about a dataset that give unfair or biased value to some points, without analysing the data in this way.

Here are all the graphs and values for each material I successfully tested:



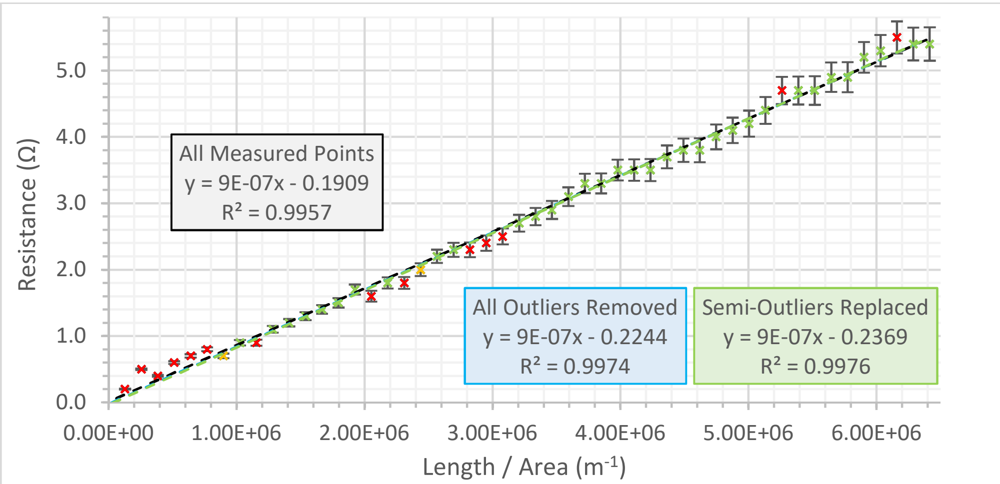
Conductive Dough

Res_0:	7
Points:	19
Diameter:	1.63E-02
Area:	2.07E-04
Mean Rsty:	0.0515
Var Sum:	7.96E-03
Mean R 2:	0.0540
Mean R 3:	0.0535



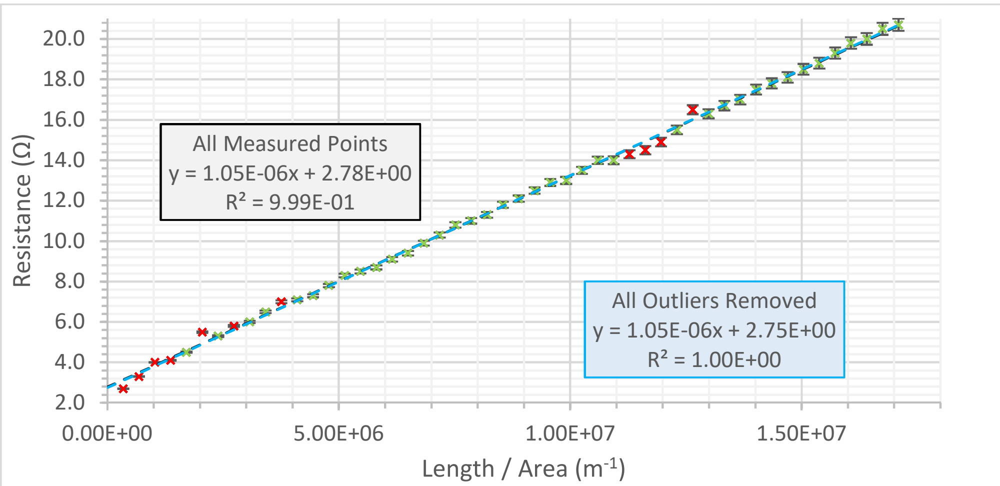
Nickel

Res_0:	1.5
Points:	50
Diameter:	3.15E-04
Area:	7.79E-08
Mean Rsty:	1.14E-06
Var Sum:	1.82E-13
Mean R 2:	1.14E-06
Mean R 3:	1.15E-06
Book Value:	6.99E-08
	1524.64%
% Error:	1530.12%
	1543.28%



Constantan

Res_0:	1.3
Points:	50
Diameter:	3.15E-04
Area:	7.79E-08
Mean Rsty:	8.54E-07
Var Sum:	3.92E-14
Mean R 2:	8.60E-07
Mean R 3:	8.62E-07
Book Value:	4.90E-07
	74.20%
% Error:	75.50%
	76.00%



Nichrome

Res_0:	2
Points:	50
Diameter:	1.93E-04
Area:	2.93E-08
Mean Rsty:	1.05E-06
Var Sum:	1.75E-12
Mean R 2:	1.05E-06
Book Value:	1.08E-06
% Error:	2.69%
	2.38%

The most precise and accurate dataset by far is nichrome. Besides the conductive dough, nichrome also has the lowest resistivity of any of the wires I measured. I believe this demonstrates the heating effect on the wire quite well. Modelling the metals using the oft-cited function I mentioned earlier, with the values of α found online (nickel: 0.005866, constantan: -0.000074, nichrome: 0.00017), with an increase in temperature, nickel's resistivity should increase considerably, constantan's resistivity should decrease slightly, and nichrome's resistivity should increase very slightly. Comparing that to my results, the percentage error for the nickel could be explained by the fact that there may have been considerable heating of the wire during measurements, which would both increase the total resistivity of the wire, and create a convex curve to the results, as I took each measurement increasing in length each time. On average, more points were under the line of best fit in the middle, and above it on either edge of the graph, showing there was likely some heating effect. Along with this, nickel had the lowest resistivity of any wire I measured, except for copper, meaning it could likely have had a higher current passing through it, and a higher heating effect than any other material. Contrary to this, however, the measured value for constantan was much higher than the book value, meaning some other factor must have been involved in the uncertainty of my result for either constantan or both materials. At the lower end of the nickel graph, the resistance was so low that the multimeter reading was fluctuating a large amount. This meant I was only able to get resistance values to 2 significant figures, a factor that I should have included in the error bars, and possibly could have allowed some of the points at that lower end of the graph to be non-anomalous.

The conductive dough helped explain one possible cause of uncertainty, in that it was composed of regular dough mixed in with a conductive liquid. However, the mixing process of the dough would very likely not create a uniform spread of the liquid throughout, meaning there may have been many branches of the liquid formed with different resistances, resulting in a very unpredictable resistance throughout. In changing the length, I couldn't ensure an even quantity of liquid was removed each time, or that I wasn't breaking a branch of low resistance, causing the resistance to raise as the length decreased, something that was actually seen in my data. Along with this, the dough was likely much further from the cylindrical shape of the model it was intended to be, being quite a stiff material; the cross-sectional area wasn't constant throughout the length of dough. Both of these irregularities, although amplified by the use of the dough, are definitely present to a degree in the metal wires, meaning they may well have had an effect on my results.

Another issue that could well have been seen in the wires was due to the tension in the wires being held taut. Generally, the lower the SWG of the wire, the harder it was to keep taut, and the more distance there was past what we had measured on the ruler. This meant as readings increased, the perceived length became shorter than the actual length I was measuring. This would also produce a convex shape to the lines on the graphs, and increase the perceived resistivity. The thickest (and thus most effected) wires were the nickel and constantan, which were both measured to have higher resistivities than the book values. The combination of this issue and the heating effect were likely what caused me to be unable to get accurate results with the copper wire. There were 3 possible SWGs of copper wire available to me. The thickest, and thus most resistive wire, was too thick to get an accurate length reading from, and I had no feasible method of holding it taut to the ruler. However, the thinner two wires had much lower resistances, due to coppers resistivity being considerably lower than any of the other metals. This caused the values for resistance to fluctuate far too much to get accurate or precise readings from, like what was seen with the lower end of the nickel measurements.

The graphite measurement was a separate problem. There were a few main factors that made it very hard to measure the resistance of graphite at different lengths. Firstly, the length of graphite available to me was about 8cm worth, which was way too small to get enough readings to create a graph from. This also meant that it was very hard to fix the rod in place such that a length could be measured reliably, without it breaking due to its brittle composition. Along with this, the only flat precise ruler I had available was made of steel, which was also an electrical conductor, making it hard to measure directly and closely. Lastly, it was hard to connect the flying lead reliably to the rod, even using a knife-edge wire, meaning the measured resistance fluctuated quite a lot as my hand moved even slightly.

I don't believe there would have been merit in expanding the range of lengths of wire over which I measured, as towards either end of each of the graphs, there was always more variance in the data points. However, I could have benefitted from measuring more data points in the range I chose, or using multiple readings over the same points to locate anomalies before plotting graphically. Using different diameter wires of the same material, or carrying out the investigation at different temperatures could also have been beneficial, as I could single out reasons for the errors in my data much more easily using more independent variables, at the cost of time.

Citations

Book or Site	Values Used
Table of Electrical Resistivity and Conductivity https://www.thoughtco.com/table-of-electrical-resistivity-conductivity-608499 11/03/19	Nichrome: 1.10E-06 Nickel: 6.99E-08 Constantan: 4.9E-07
Electrical Resistivity Table for Common Materials https://www.electronics-notes.com/articles/basic_concepts/resistance/electrical-resistivity-table-materials.php 11/03/19	Nichrome: 1.1E-06 Nickel: 7E-08
Resistivity and Temperature Coefficient at 20 C http://hyperphysics.phy-astr.gsu.edu/hbase/Tables/rstiv.html 11/03/19	Nichrome: 1.00E-06 Constantan: 4.9E-07
Raymond A. Serway (1998). *Principles of Physics* (2nd ed.). Fort Worth, Texas; London: Saunders College Pub. p. 602. ISBN 978-0-03-020457-9.	Nichrome: 1.10E-06
Electrical resistivity and conductivity https://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity#Resistivity_and_conductivity_of_various_materials 11/03/2019	Nickel: 6.99E-08
John O'Malley (1992) Schaum's outline of theory and problems of basic circuit analysis, p. 19, McGraw-Hill Professional, ISBN 0-07-047824-4	Constantan: 4.90E-07
Temperature Coefficient of Resistance https://www.allaboutcircuits.com/textbook/direct-current/chpt-12/temperature-coefficient-resistance/ 16/03/19	Nichrome: 0.00017 Nickel: 0.0058866 Constantan: -0.000074

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