Measuring Mass Per Unit Length of String Via Oscillation

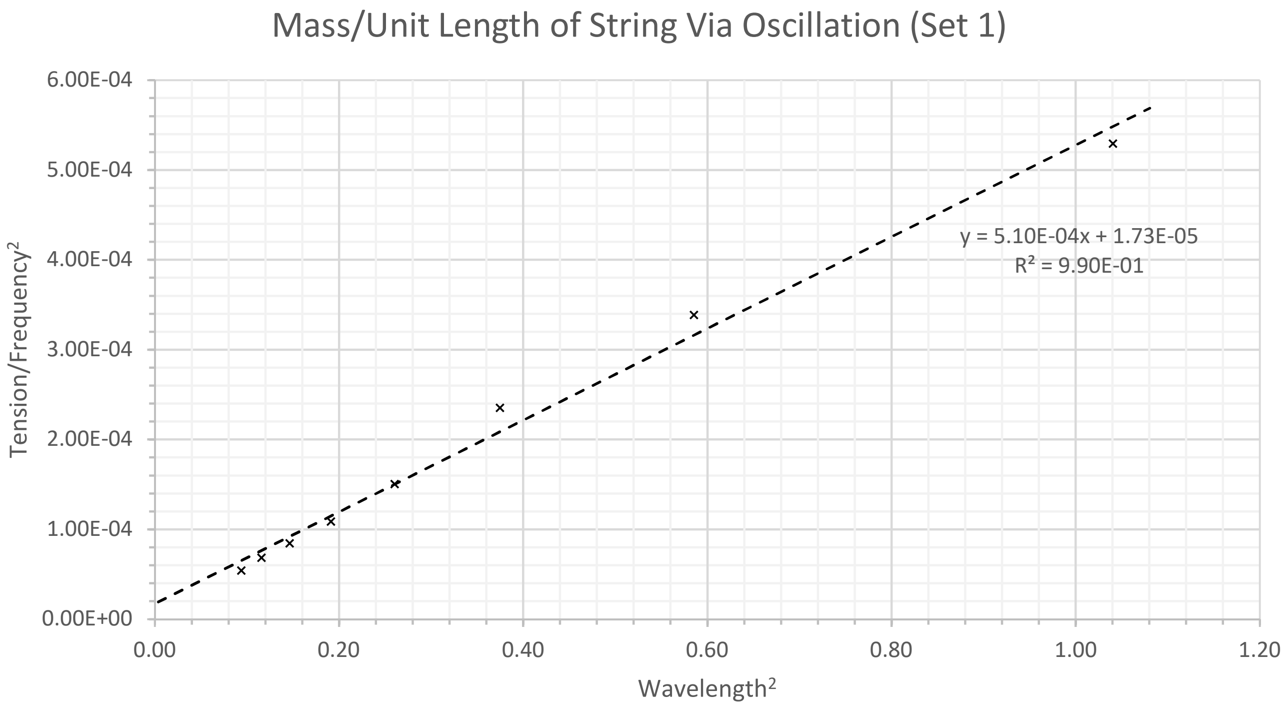
Method

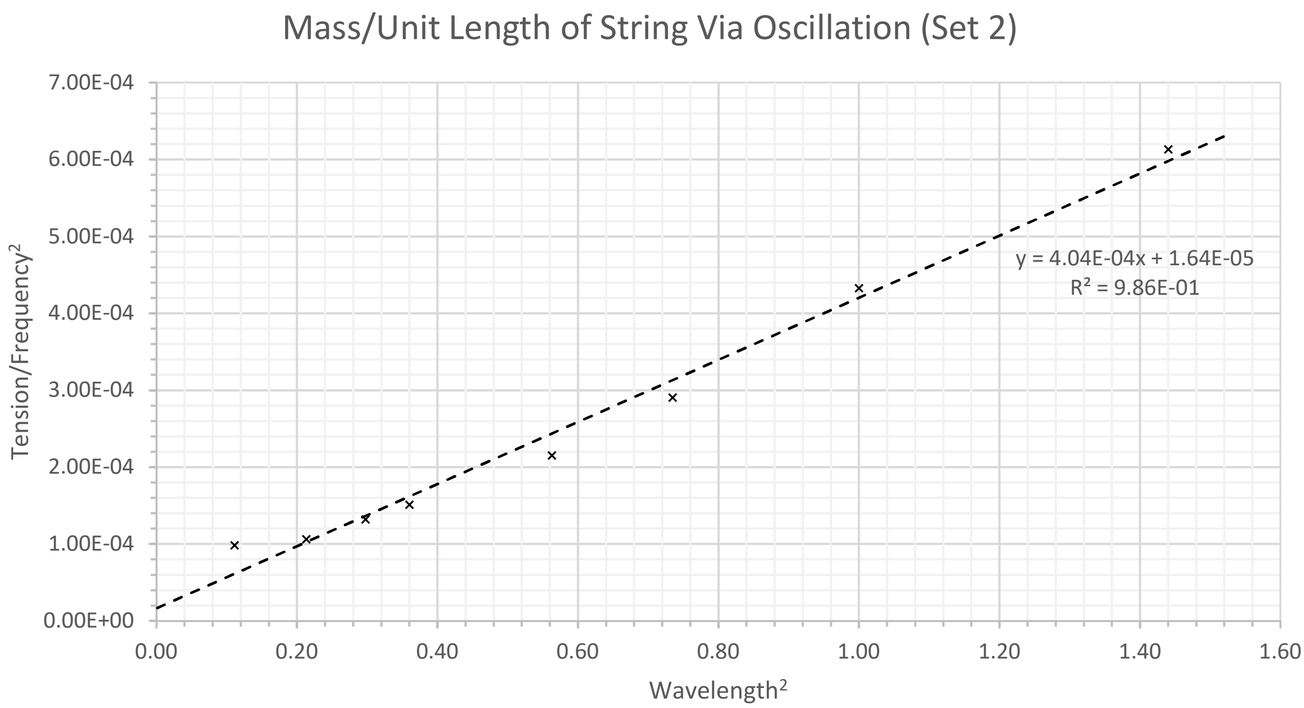
I was tasked to find the value of μ, the mass per unit length (in units kgm^{-1}) of a given piece of elastic string. To do this, I attached the string to a vibration generator, and measured its frequency at certain fixed discrete wavelengths (0.5, 1.0, 1.5, 2.0… etc), along with its tension under this experiment. Because $v = \sqrt{\frac{T}{\mu}}$, I could rearrange this and substitute in $v = f\lambda$ to bring us to $\frac{T}{f^2\lambda^2}$, a formula for μ in terms of values which I could measure. My signal generator was very inaccurate, so I used an oscilloscope to precisely measure the frequency I was setting for each whole wavelength. I finetuned the frequency dial to line up every half a wave to have clear nodes and antinodes so I could count the waves as easily as possible. Each of these frequencies I landed on were multiples of the resonant frequency for the string, meaning the wave and its reflection lined up to produce full destructive and maximum constructive interference, producing a clear standing wave. The second end of the string was attached to a pully system with a mass tied to the dangling end. This allowed me to precisely set the tension on the string, my final controlled variable. I found a range of different frequencies and wavelengths, allowing me to plot a graph of my results, and get a much higher precision for my result.

	Boxes ( )	Frequency (s^{-1})	Waves ( )	Wavelength (m)	T*Freq.$^{-2}$ (Ns^2)	Wavelength^2 (m^2)
Set 1	15.0	33.3	1.5	1.02	5.30E-04	1.04
	12.0	41.7	2.0	0.77	3.39E-04	0.59
Box Scale (s)	10.0	50.0	2.5	0.61	2.35E-04	0.37
0.002	8.0	62.5	3.0	0.51	1.51E-04	0.26
Length (m)	6.8	73.5	3.5	0.44	1.09E-04	0.19
1.53	6.0	83.3	4.0	0.38	8.47E-05	0.15
Tension (N)	5.4	92.6	4.5	0.34	6.86E-05	0.12
0.59	4.8	104.2	5.0	0.31	5.42E-05	0.09
Set 2	12.5	40.0	2.5	1.20	6.13E-04	1.44
	10.5	47.6	3.0	1.00	4.32E-04	1.00
Box Scale (s)	8.6	58.1	3.5	0.86	2.90E-04	0.73
0.002	7.4	67.6	4.0	0.75	2.15E-04	0.56
Length (m)	6.2	80.6	5.0	0.60	1.51E-04	0.36
3.00	5.8	86.2	5.5	0.55	1.32E-04	0.30
Tension (N)	5.2	96.2	6.5	0.46	1.06E-04	0.21
0.98	5.0	100.0	9.0	0.33	9.81E-05	0.11

The darker two columns and the leftmost column of the table represent the values I measured. The boxes were part of the oscilloscope measurement of the frequency. It displayed a still image of the wave projected onto an x-y plane, with x representing a scale of time, and y representing the waves amplitude. There were boxes covering this plane, with the axis dividing each box into 5 subsections, 0.2 boxes. I could adjust the position of the wave on the screen to line it up with the more precise scale on the axis. Getting the frequency from the box value required the box scale on the left of the table. The waves value was just the number of full waves (node → antinode → node → antinode) I saw at each given frequency. Getting the wavelength from the number of waves required the total string length on the left of the table. I then put these all together (along with the single tension value on the left of the table) into a set of x and y values to plot a graph in the form y = mx + c, where m would be μ. This is a very useful method for calculating a single result from a set of data in table form, as it gives varying significance to different values based on their effect on the line of best fit through them. I compared this final value with a simple scale measurement for the mass of 1 meter of the string, called the "actual" value on the results table set, despite being less precise than our measurement.

Results





Set 1	Actual	Measured	Set 2	Actual	Measured
Mass/Len. ML^{-1} (kgm^{-1}):	5.0E-04	5.10E-04	Mass/Len. ML^{-1} (kgm^{-1}):	4.00E-04	4.04E-04
Uncertainty (±kgm^{-1}):	5.00E-06	2.84E-05	Uncertainty (±kgm^{-1}):	5.00E-06	1.98E-05
Upper Bound (kgm^{-1}):	5.05E-04	5.39E-04	Upper Bound (kgm^{-1}):	4.05E-04	4.24E-04
Lower Bound (kgm^{-1}):	4.95E-04	4.82E-04	Lower Bound (kgm^{-1}):	3.95E-04	3.84E-04
	Base	Max		Base	Max
% Error:	2.10%	8.86%	% Error:	0.98%	7.26%
R^2:	0.9903	-	R^2:	0.9862	-

Analysis / Evaluation

I used the formula $f^2\lambda^2 = T/\mu$ to calculate μ, the mass per unit length of a piece of string. This meant I needed to calculate 3 values: The frequency a piece of string oscillates at, it's wavelength at that frequency, and the tension exerted on it. This meant I could use a vibration generator to act as a node on one end of the string, and a pulley force-system on the other end to fix the tension. I measured the frequency using an oscilloscope to increase precision and accuracy, as the signal generator was very inaccurate. To measure the wavelength, I counted the number of waves after lining up the nodes and divided the total length by this value. I improved the second investigation by increasing the total length of the string to both fit more waves and decrease the ratio between the scale marking uncertainty and the length measured. I also used a higher tension the second time, which may not have been helpful, as it decreased the value we measured, although the tension was a negligible proportion of the total uncertainty.

The "actual" value on the table was only measured precise to one significant figure, meaning I have no clear way to test how correct our value is. In place of this, I have calculated the bounds of both our measured value, and the "actual" value we calculated with a set of scales. I used $\pm 5\text{x}10^{-5}$ for the actual value, as my value was $n\text{x}10^{-4}$ for both sets of data, and the value for uncertainty calculated for the measured value. I used these bounds to calculate a **"**maximum" percentage error, which is the largest of the two values that satisfy **ABS((a-b)/b)** where **a** is either the upper or lower bound of the measured value, and **b** is the opposite bound for the "actual" value. This gives a worst-case scenario for the percentage error, and the furthest point apart the two values can be. Most of the uncertainty in this investigation was in my scale measurement for the value, which shows a benefit to using the method I used, as opposed to a simple set of scales, as they give a much lower precision than the combination of the oscilloscope, standing wave and tension values.

Deriving Uncertainty

$$mass\ per\ unit\ len = \frac{tension}{frequency^2 \times wavelength^2}\left(kgm^{-1} = \frac{kgms^{-2}}{(s^{-1})^2 \times (m)^2}\right)$$

$$\therefore \frac{T}{f^2} = \mu\lambda^2 + c\ (in\ the\ form\ y = mx + c)$$

Frequency:

$$\frac{\pm half\ scale\ div}{\downarrow measured\ range} = \uparrow \%\ uncertainty$$

$$\%\ uncertainty \times \downarrow frequency = freq.range$$

Set 1:

$$\frac{\pm 0.1}{4.8} = \pm 2.1\%\ (units\ in\ boxes\ per\ wave)$$

$$\pm 2.1\% \times 33.3Hz = \pm 0.69Hz$$

Set 2:

$$\frac{\pm 0.1}{5.0} = \pm 2.0\%\ (units\ in\ boxes\ per\ wave)$$

$$\pm 2.0\% \times 40.0Hz = \pm 0.80Hz$$

Wavelength:

$$\frac{\sim \pm uncertain\ range}{\downarrow measured\ range} = \sim \uparrow uncertainty$$

$$\%\ uncertainty \times \downarrow wavelength = w.len\ range$$

Set 1:

$$\frac{\sim \pm 0.01}{1.5} = \pm 0.67\%\ (units\ in\ waves)$$

$$\sim \pm 0.67\% \times 1.02m = \pm 0.0068m$$

Set 2:

$$\frac{\sim \pm 0.01}{2.5} = \pm 0.40\%\ (units\ in\ waves)$$

$$\sim \pm 0.40\% \times 1.20m = \pm 0.0048m$$

Combination:

$$u = (2 \times u_f) + (2 \times u_\lambda) + (1 \times u_T)\ (2\ degrees\ of\ f\ \&\ \lambda\ used, 1\ degree\ of\ T\ used)$$

Set 1:

$$(2 \times \pm 2.083\%) + (2 \times \pm 0.67\%) + (1 \times 0.06\%) = \pm 5.560\%$$

Set 2:

$$(2 \times \pm 2.000\%) + (2 \times \pm 0.40\%) + (1 \times 0.10\%) = \pm 4.900\%$$

Locating Anomalies

Combination:

$x = 2 \times u_\lambda, \quad y = (2 \times u_f) + (1 \times u_T)$

Set 1:

$x = (2 \times \pm 0.67\%) = \pm 1.3\%$

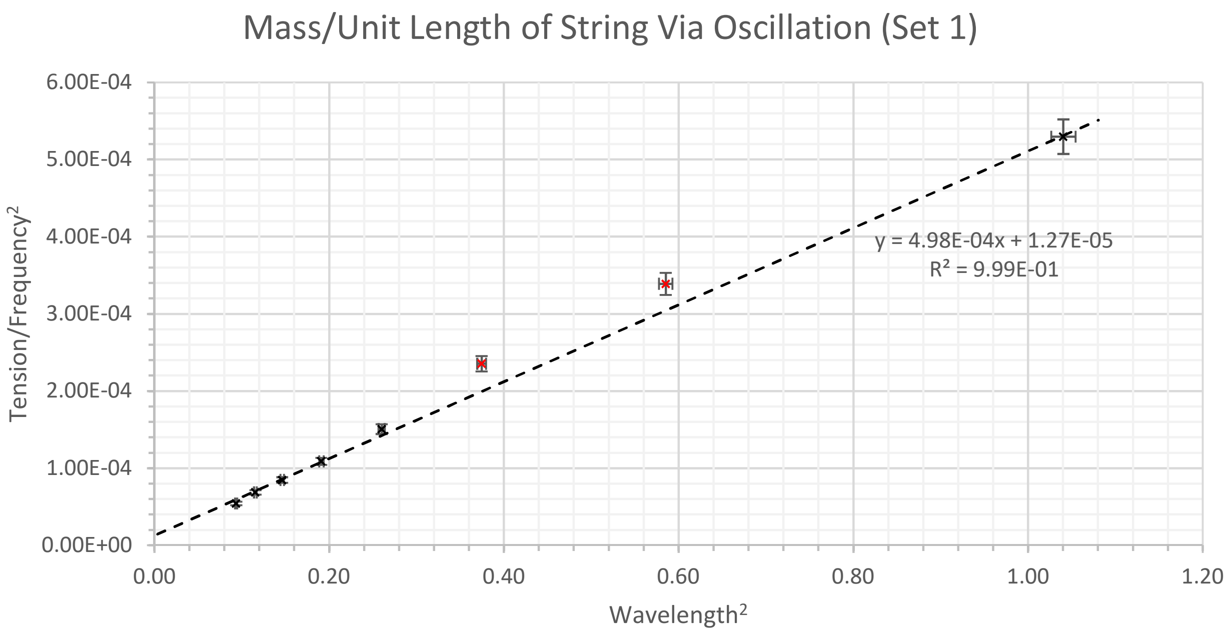
$y = (2 \times \pm 2.083\%) + (1 \times 0.06\%) = \pm 4.2\%$

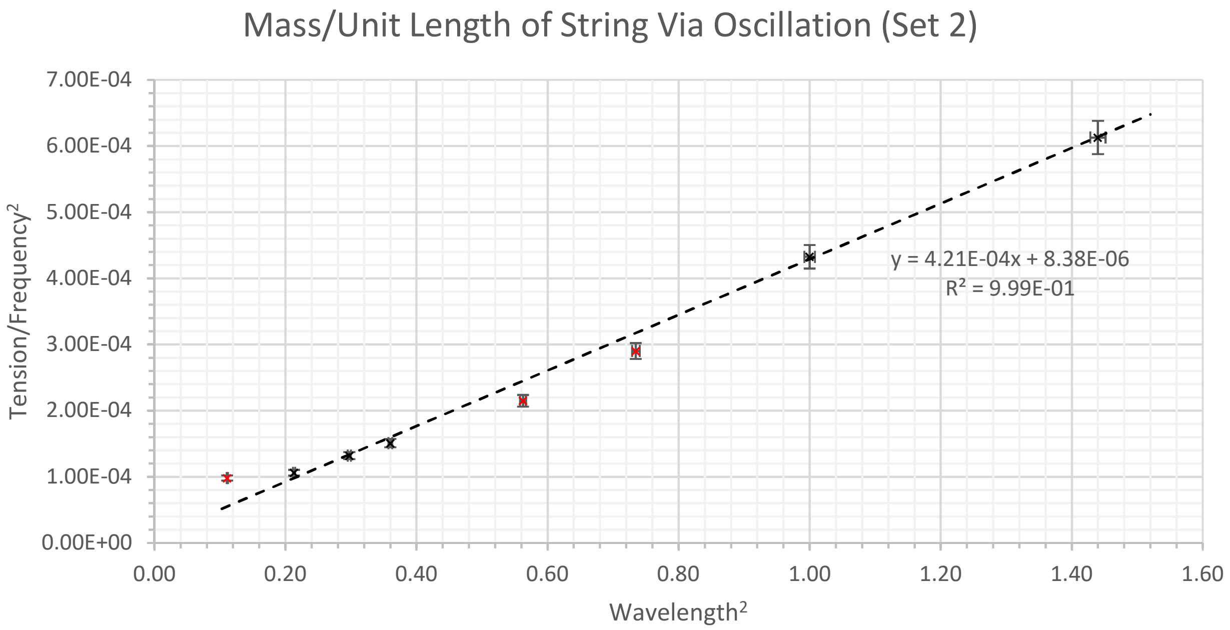
Set 2:

$x = (2 \times \pm 0.40\%) = \pm 0.8\%$

$y = (2 \times \pm 2.000\%) + (1 \times 0.10\%) = \pm 4.1\%$

I used these uncertainty percentages as error bars on the graph and removed the points whose boxes formed by their error bars didn't pass through the line of best fit. I then plotted a new line of best fit and created a new set of results.





The Results

Set 1	Actual	With Anomaly		Without Anomaly	
Mass/Len. ML^{-1} (kgm^{-1}):	5.0E-04	5.10E-04		4.98E-04	
Uncertainty ($\pm kgm^{-1}$):	5E-06	2.84E-05		2.84E-05	
Upper Bound (kgm^{-1}):	5.05E-04	5.39E-04		5.27E-04	
Lower Bound (kgm^{-1}):	4.95E-04	4.82E-04		4.70E-04	
Error		Upper	Lower	Upper	Lower
Accuracy:	1 - % Error	97.90%	91.14%	99.63%	93.03%
Precision:	R^2	99.03%	-	99.94%	-
Set 2	**Actual**	**With Anomaly**		**Without Anomaly**	
Mass/Len. ML^{-1} (kgm^{-1}):	4.0E-04	4.04E-04		4.21E-04	
Uncertainty ($\pm kgm^{-1}$):	5E-06	1.98E-05		1.98E-05	
Upper Bound (kgm^{-1}):	4.05E-04	4.24E-04		4.40E-04	
Lower Bound (kgm^{-1}):	3.95E-04	3.84E-04		4.01E-04	
Error		Upper	Lower	Upper	Lower
Accuracy:	1 - % Error	99.02%	92.74%	94.86%	88.52%
Precision:	R^2	98.62%	-	99.92%	-

In the results table, I calculated the uncertainty for all values I used, including the “actual” value I compared the results to. I used this to calculate an upper and lower bound for each of the values, and thus, an upper and lower bound for the percentage error of the results. The percentage error represents the accuracy of the experiment, and the R^2 value (which I took from the Excel graph of each set of data), which uses the average distance of each point from the line of best fit. To calculate the lower bound for the accuracy, I found the minimum of the two percentage errors, the upper bound of the actual value with the lower bound of the measured one, or lower of actual with upper of measured. The upper bound was just using the original measurements in the ABS((a-b) / b) formula. In set 1, I removed values 2 and 3 as anomalies. This increased the precision value, as the line of best fit had to go through fewer points and thus, could fulfil the criteria more closely. In the first set, removing the 2 anomalies increased the accuracy by roughly 2 percent. This is because both anomalies are near the top right of the line of best fit, meaning in general they would increase the gradient of the line. The value for the mass per unit length was above the value from the weighing scales, meaning that the value without anomalies was more accurate. This was not the case for the second set of results where I removed results 3, 4 and 8. The 3 and 4 results were below the line quite high on the scale, and the 8 result was above the line, low down on the scale. This served to increase the gradient of the line when I removed the points. However, the value calculated from the graph was already larger than the value measured from the weighing scales, so this only made my result less accurate.

If I had more time to, I would try different methods of calculating error bounds on the graphs, such as standard deviation from the line of best fit. I would also go back through my results and fix the intercept of each line of best fit to 0, as that would change how removing outliers is handled.

My results showed curves rather than the straight lines they were predicted to, meaning there was likely another variable at play that I didn’t anticipate, such as possible plastic deformation in the string under such a high tension, especially in the second set. Interestingly, the curves on the first and second sets of results were decreasing and increasing respectively. The two factors we changed between these two sets of results were the tension and length of the string. These directly effect the mass per unit length, but could serve to enhance the effects of unknown variables causing the curve.

Experimental write-up by Laura Hannah – Prince William School