Finding Viscosity

In this experiment I replicated a falling-ball viscometer, finding, controlling, and changing variables from this equation, to calculate η, the variable for viscosity.

$$\eta = \frac{2r^2 g(p_s - p_f)}{9v}$$

I derived this equation from the fact that the sum of the forces (the resultant force) on the ball in the fluid would be 0 at terminal velocity. The ball is under the forces of weight, up-thrust, and drag. Weight and up-thrust can be substituted by density, volume and gravity, using the equation $W = \rho V g$ which itself is derived by substituting $m = \rho V$ into $W = mg$. The densities involved in each equation are that of the solid dropped into the fluid and the fluid itself. The volume is that of the solid, as the volume of liquid displaced will be equal to the volume of solid put in. You can substitute the equation for the volume of a sphere, $4\pi r^3/3$ in place of V. Gravity is a constant. Drag can be substituted by Stoke's Law, $d = 6\pi r\eta v$, where r and v are the radius and velocity of the solid respectively, and η is the viscosity of the fluid. Combining these makes the equation as such:

$$\frac{4}{3}\pi\rho_s r^3 g = \frac{4}{3}\rho_f r^3 g + 6\pi r\eta v\ (w = u + d)$$

Moving the weight and up-thrust values to one side of the equation, and factoring out common values gets this equation:

$$\frac{4\pi r^3 g(p_s - p_f)}{3} = 6\pi r\eta v$$

Dividing both sides by everything except η on the right, then simplifying the fraction, gives the equation for viscosity:

$$\eta = \frac{2r^2 g(p_s - p_f)}{9v}$$

However, I used the equation in a different form. It is much more precise to get a range of different values and plot them on a graph, using the gradient of a line of best fit as the value I calculate. This means I can plot a straight line through all results, in essence giving a different "importance" to each point based on its distance from a possible trend. To get η as the gradient of a line, I had to rearrange the equation into the form $y = mx$, where m was η, x held my independent variable, y held the dependent variable. The rest of the constants were spread between each side of the equation, as this would split the uncertainty as evenly as possible between the vertical and horizontal axis of the graph, which I believe would make anomalies easier to find, as each point would have a box created by error bars rather than solely vertical or horizontal ones. However, in this case almost all the uncertainty in the experiment was in the terminal velocity of the solid, meaning there was no use in splitting up the equation evenly. I settled on this equation in the end:

$$\frac{2(p_s - p_f)}{9v} = \frac{1}{r^2 g}$$

I chose to vary the radius of the ball bearings as my independent variable and measure the terminal velocity as my dependent variable. This meant I would need to fix the density of the ball and liquid, and the acceleration due to gravity as my control variables. These are easy to keep the same, as I repeated the experiment in the same place with the same room temperature and pressure each time.

I used washing-up liquid as the fluid, as it had a high enough viscosity and was in high enough supply to use a larger measuring cylinder. In hindsight, I should have prioritised the translucency of the liquid over the higher viscosity, as the washing-up liquid was too dark to see the ball bearing clearly enough to point near the start and end of each clip. Along with this, I was using a camera with 240fps, meaning there were plenty of points to determine whether the section of the line captured was accurate. It would have been more beneficial in my case to capture a larger line segment of the ball's drop, in essence focussing more on accuracy than precision, the main flaw of the experiment. The terminal velocity was by far the largest uncertainty of the experiment, as I used precise scales for all measurements, and most of the variables only required one to two different measurements.

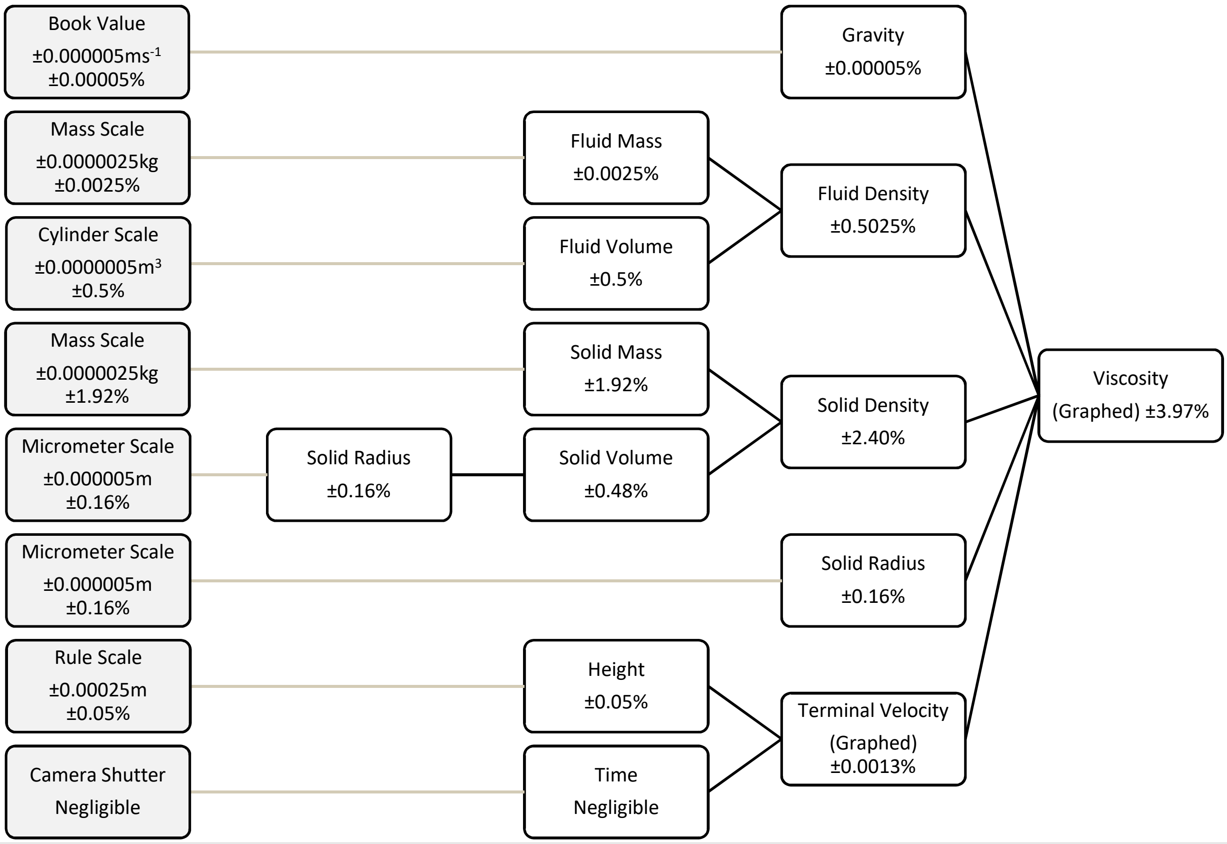
In terms of inaccuracies, there were a few I could think of. Firstly, I used a measuring cylinder, which would very likely affect the path of the solid and introduce lateral flow to the fluid system, and increase the perceived viscosity of the fluid, as liquid molecules would be reintroduced to the path of the sphere by rebounding from the edges of the container. This would render Stokes Law an inappropriate model for the system. I minimised this by using the largest clear container I had available. Another inaccuracy was likely in the way I calculated the density of the solids and fluid. I got the density of the ball bearings after the experiment, meaning they likely still had traces of fluid on them, which would increase their perceived density, and hence increase the viscosity in my results. Another error that would cause inaccuracy is in the parallax of the measuring cylinder. I countered this as much as possible using a ruler for markings in line with where the balls would drop, rather than in line with the markings on the cylinder. However, there is still parallax in that the camera is in a fixed position, with the ball falling to become in-line with it, then move further. This would mean the ball is slightly further from the camera at the start and end of each clip than in the centre. This could be countered by altering the way I detect the height of the ball, such as having the camera move smoothly downwards at the same speed as the ball (determined by a less accurate experiment beforehand). This is unnecessary in my case however, as the parallax effect of the camera is much smaller than other, more important errors, and may cancel out due to being equal before and after the ball comes in-line with the camera. Another error based on the camera was in that I didn't ensure it was perfectly aligned with gravity, meaning the ball could have been accelerating at a slight angle. This would make the ball seem to be falling slower than it was, also increasing the perceived viscosity, and could be simply fixed using a spirit level on top of the camera, or built into it, to correct the footage after the fact.

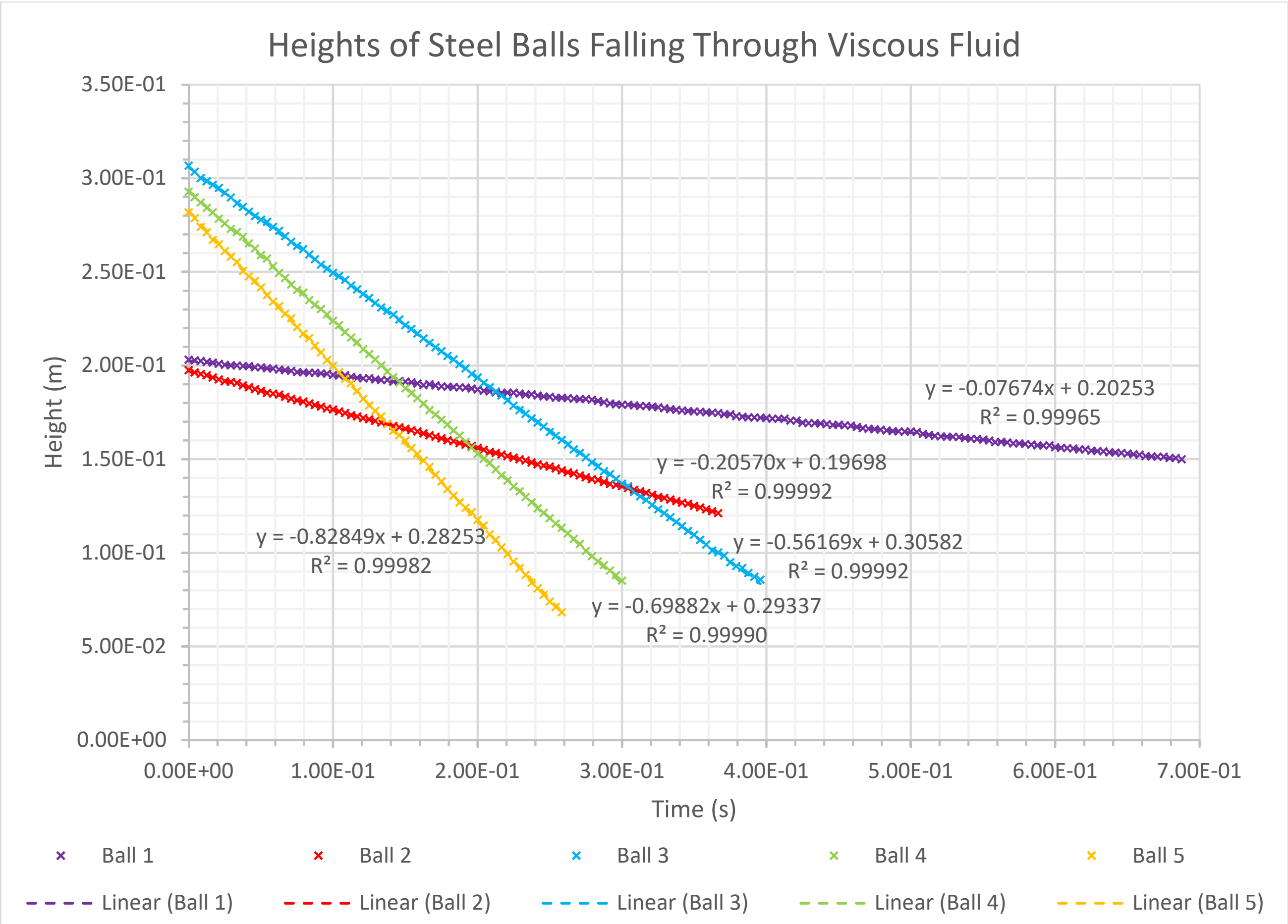
The majority of the systematic errors in the experiment were likely to increase the value for viscosity in my results, however there is no way to account for this, as I do not know to what extent each error was in effect. I also do not have a real value to compare my result to, so I cannot calculate the real percentage error to test the validity of my uncertainty margins.

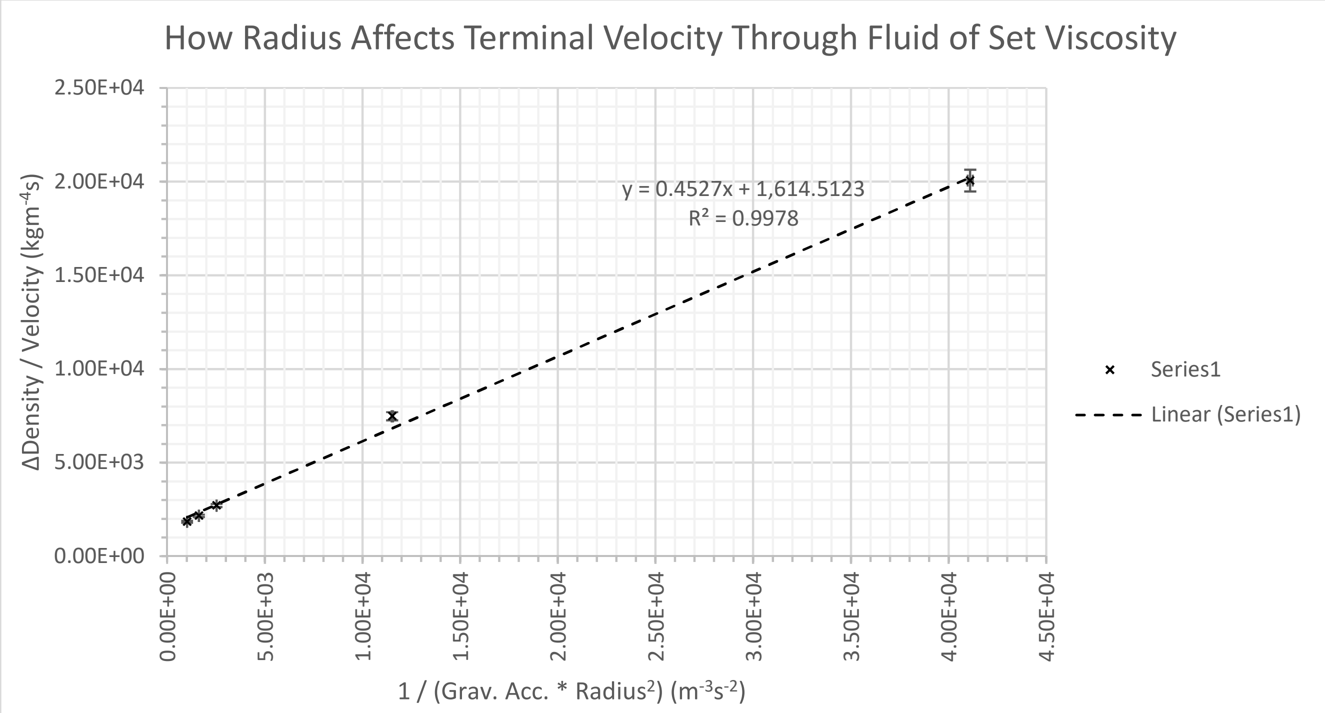
With most of the random error being in finding the terminal velocity of the ball, I set my main goal to minimise that. I calculated the terminal velocity of the ball using another graph, of the y-position of each mass over time, to get their velocity as the gradient. To get each y-position, I used 240 frames of image every second, and used tracking software to store the ball's height every frame, based on an inputted real-world scale in the image. To ensure that I would be able to pinpoint the mass each frame, I trimmed the video to just the frames over which the ball was most visible and used a ruler of half-millimetre scale to locate the exact centre of the circle on each frame, precise to ±0.00025m. I also scaled the image up to 2.77 times its real-world size, to make it much easier to see. Despite this, there were a few things I could have improved. As I mentioned before, the translucency of the liquid could have greatly helped the accuracy of the result set, but it could also have helped improve the precision, as the darkness around the edge of the circle made it hard to align the ruler to be along its diameter. Along with this, the light I used to illuminate the cylinder more had an offset framerate to the camera I used, meaning the visibility of the circle was greatly reduced during times of destructive interference. This pattern can be seen on the graph, where the smaller ball bearings have periods during which they are show as higher up than they would be presumed to be based on the trend. However, this was very minor, and would only raise the y-intercept of the line, not change its gradient, and thus can be ignored.

The value for the viscosity of the washing-up liquid I calculated with was $0.453 kgm^{-1}s^{-1}$, which is a reasonable value, considering the standard citied viscosity is around $0.5-0.8 kgm^{-1}s^{-1}$, and the liquid I used was science lab standard, bulk washing up liquid, which would likely be lower viscosity than industry standard.

I set out to find the value for viscosity uncertainty using the equation I had. That value was derived by:







In my final graph, one point is slightly off the line of best fit, making it an anomaly. Along with this, the intercept for the line of best fit is not 0, as this would greatly decrease the correlation coefficient of the line, making most of the points anomalous.

Viscosity (kgm-1s-1)				
Upper Bound	Value	Lower Bound	Uncertainty	% Uncertainty
4.71E-01	4.53E-01	4.35E-01	1.79E-02	3.97%

Ball	Diameter (m)	Radius (m)	Volume (m^3)	Mass (kg)	Density (kgm^{-3})	Position Uncertainty (m)
1	3.15E-03	1.58E-03	1.64E-08	1.30E-04	7.94E+03	±5.68E-07
2	5.95E-03	2.98E-03	1.10E-07	8.75E-04	7.93E+03	±1.07E-06
3	1.27E-02	6.35E-03	1.07E-06	8.430E-03	7.86E+03	±2.29E-06
4	1.59E-02	7.93E-03	2.08E-06	1.6280E-02	7.81E+03	±2.86E-06
5	2.00E-02	9.99E-03	4.17E-06	3.2835E-02	7.87E+03	±3.60E-06
Unc	±5E-06	±3E-06	±7.79E-11	±2.5E-06	±1.87E+02	% Pos Uncertainty
%Unc	±0.16%	±0.16%	±0.48%	±1.92%	±2.40%	±0.02%

	Volume (m^3)	Mass (kg)	Density (kgm^{-3})	Grav Acc	Ruler Scale (m)	Micrometer Scale (m)
Value	1E-04	0.10164	1E+03	9.80665	5E-04	1E-05
Unc	±5.00E-07	±2.50E-06	±5.11E+00	±5.00E-06	Upscale (w -> s)	Massometer Scale (kg)
%Unc	±0.5%	±0.0025%	±0.5%	±0.00005%	2.77	5E-06

Ball	Terminal Velocity	Uncertainty	x ($m^{-3}s^2$)	x Uncertainty	y ($kgm^{-4}s$)	y Uncertainty
1	-7.67E-02	±0.0002%	4.11E+04	±1.31E+02	2.01E+04	±5.82E+02
2	-2.06E-01	±0.0006%	1.15E+04	±3.66E+01	7.47E+03	±2.17E+02
3	-5.62E-01	±0.0012%	2.53E+03	±8.03E+00	2.71E+03	±7.86E+01
4	-6.99E-01	±0.0019%	1.62E+03	±5.16E+00	2.16E+03	±6.27E+01
5	-8.28E-01	±0.0028%	1.02E+03	±3.25E+00	1.84E+03	±5.34E+01