

# From $\Lambda$ CDM to EDE

## Lecture 1: Theory

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CosmoVerse@Corfu May 13 - 18, 2024

# Overview

Today:

- **Lecture 1:** Theory from  $\Lambda$ CDM to EDE (1:30h)
- **Hands-on session 1:** From theory to predictions (1h)

Tomorrow:

- **Lecture 2:** Observation – Can EDE solve the Hubble tension? (1h)
- **Hands-on session 2:** Let's analyse EDE with cosmic microwave background and supernovae (1:30h)

# Outline: From $\Lambda$ CDM to EDE

1. Introduction: basic equations in a homogeneous & isotropic universe  
(Friedmann equations, matter content in the universe, defining distances)
2. Hubble tension: How to solve the tension? How does CMB constrain  $H_0$ ?
3. Early Dark Energy: Idea behind EDE, background dynamics of EDE

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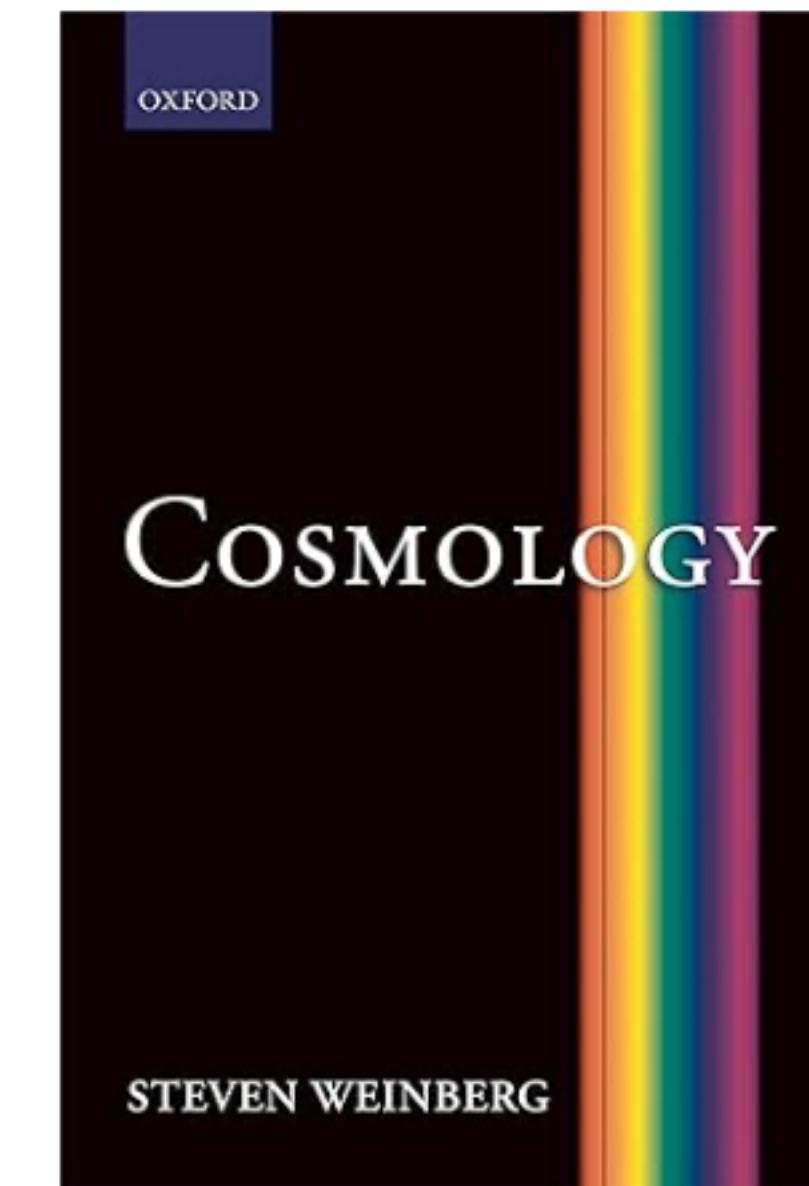
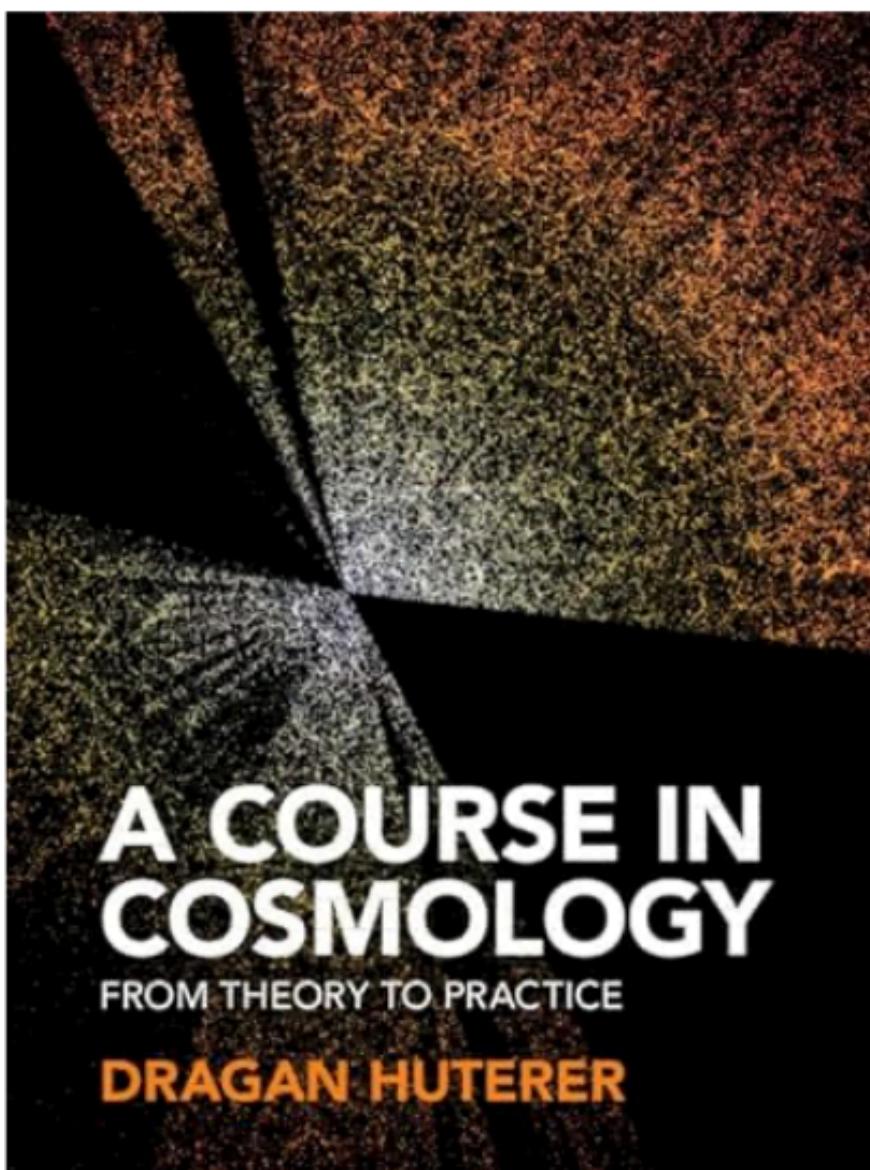
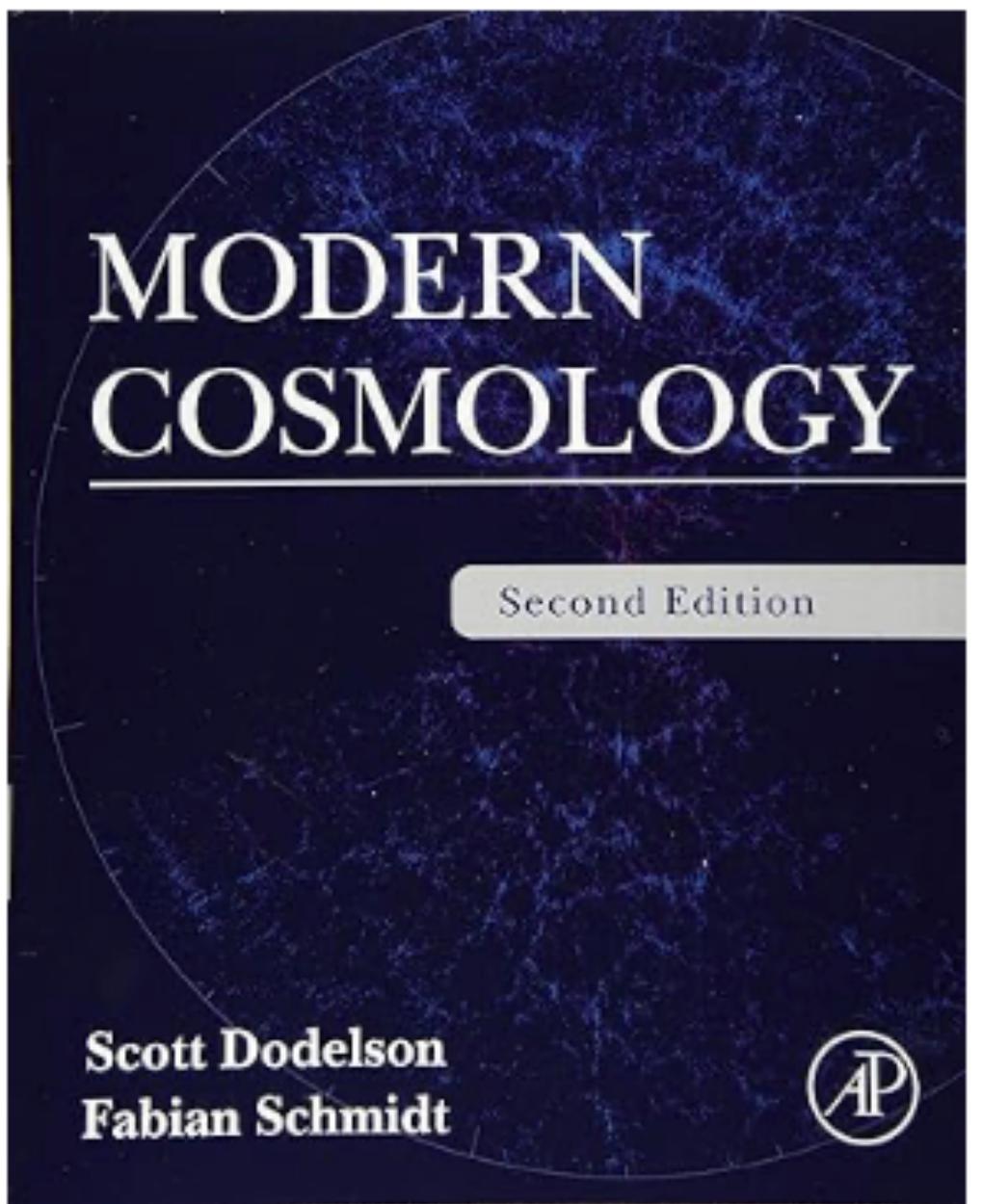
Please ask questions any time!

# Introduction

Short “crash course” to fix the notation

Natural units:  $c = 1$

# Resources



# General Relativity

- Let's imagine it was 100 years ago:
  - We didn't know about dark matter (DM)
  - We didn't know about dark energy (DE)
  - But a few years ago, Albert Einstein had published the theory of General Relativity (GR)

1916.

Nº 7.

ANNALEN DER PHYSIK.  
VIERTE FOLGE. BAND 49.

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1. *Die Grundlage  
der allgemeinen Relativitätstheorie;  
von A. Einstein.*

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie“ bezeichneten Theorie; die letztere nennt

# General Relativity

- We will not go into details here but only sketch the rough idea

For more about  
GR, see Matteo  
Martinelli's lecture

# General Relativity

- We will not go into details here but only sketch the rough idea
- Einstein Equations:

$$R^{\mu\nu} - \frac{R}{2}g^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu},$$

Ricci curvature tensor      metric      cosmological constant      energy-momentum tensor

The diagram illustrates the Einstein field equations. The equation is  $R^{\mu\nu} - \frac{R}{2}g^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu}$ . Four green arrows point to the terms: one to the Ricci curvature tensor  $R^{\mu\nu}$ , one to the metric tensor  $g^{\mu\nu}$ , one to the cosmological constant  $\Lambda g^{\mu\nu}$ , and one to the energy-momentum tensor  $8\pi G T^{\mu\nu}$ .

**“Matter tells space how to curve, space tells matter how to move”**  
**(Misner++ 1973)**

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# General Relativity

- Solution for Einstein Equations for the universe as a whole?
- Cosmological Principle:

*“On sufficiently large scales, the properties of the universe are the same for all observers.”*

*or*

*“The universe is spatially **homogeneous and isotropic** on large scales.”*

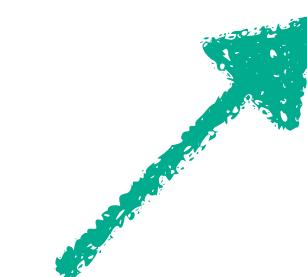
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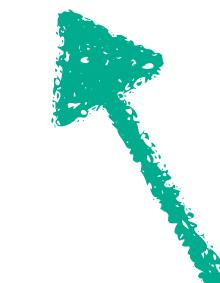
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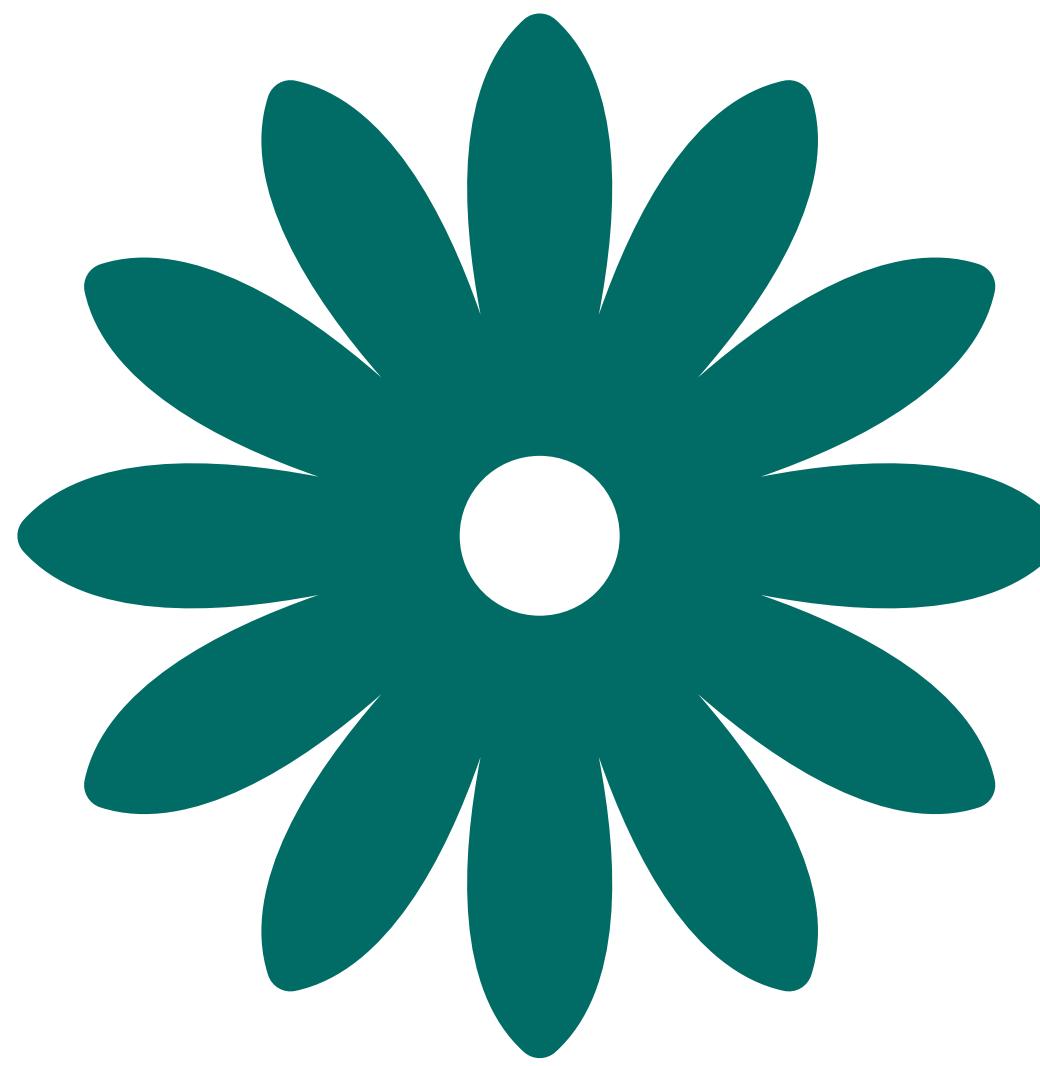
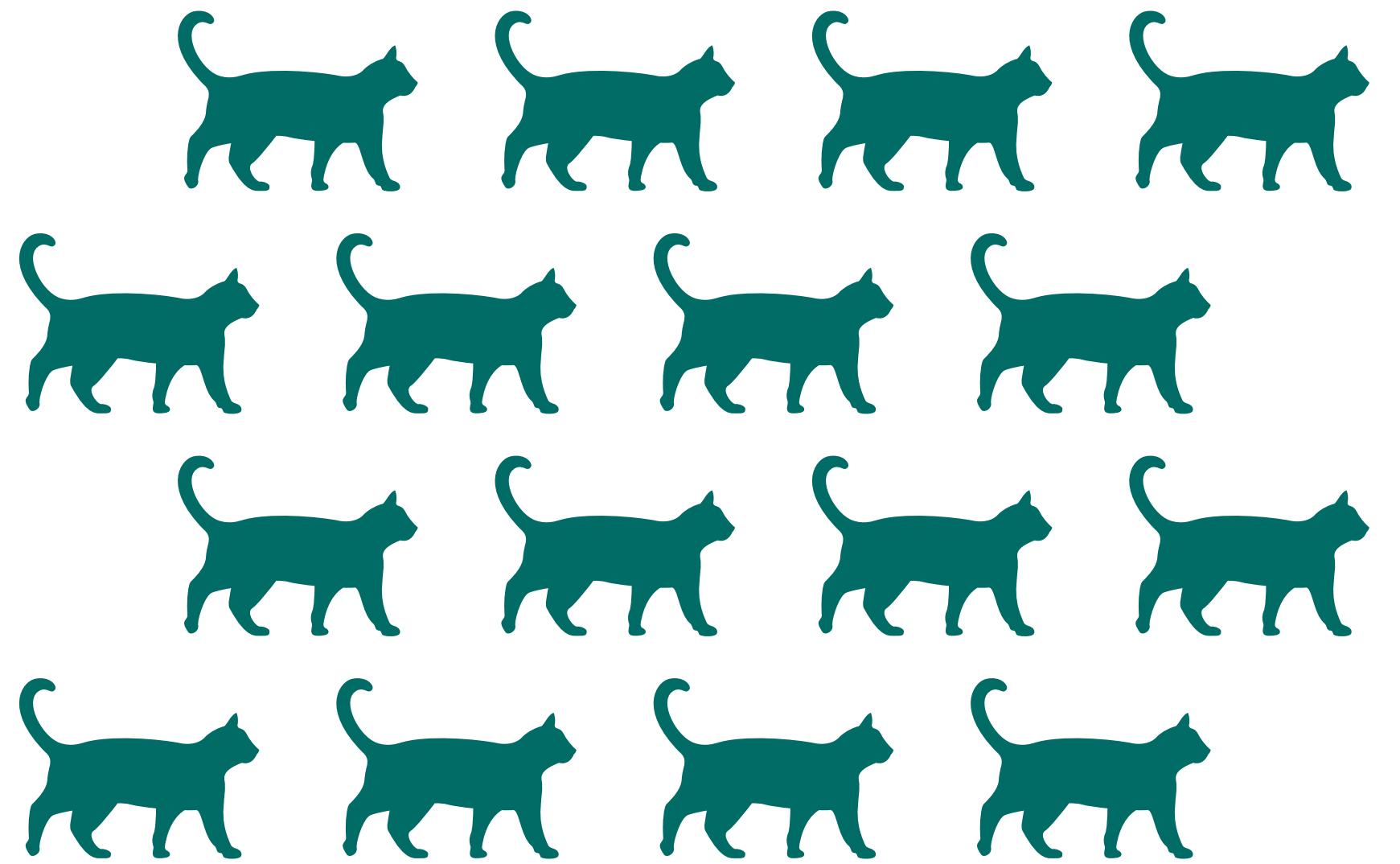


**“the same in every point”**

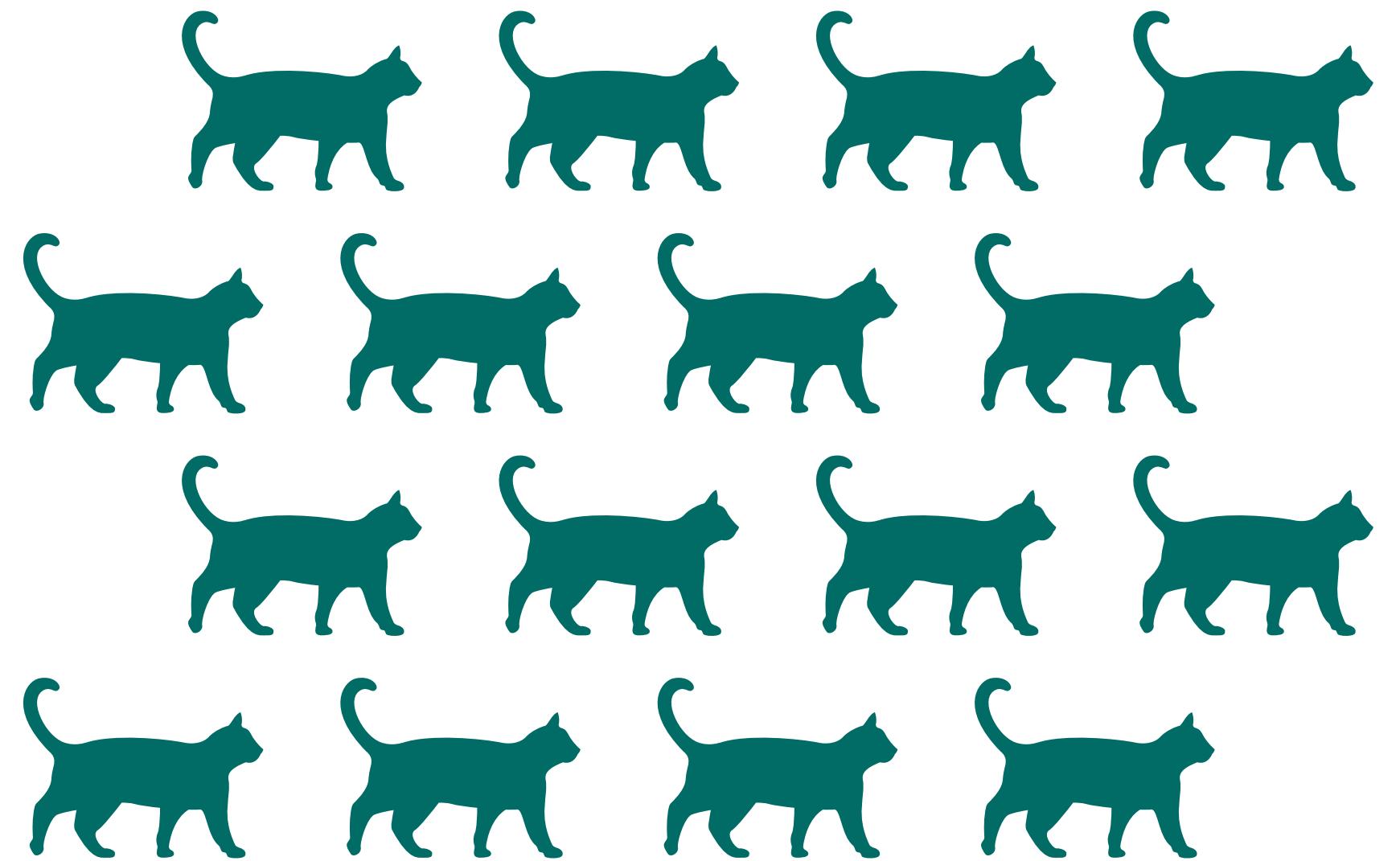


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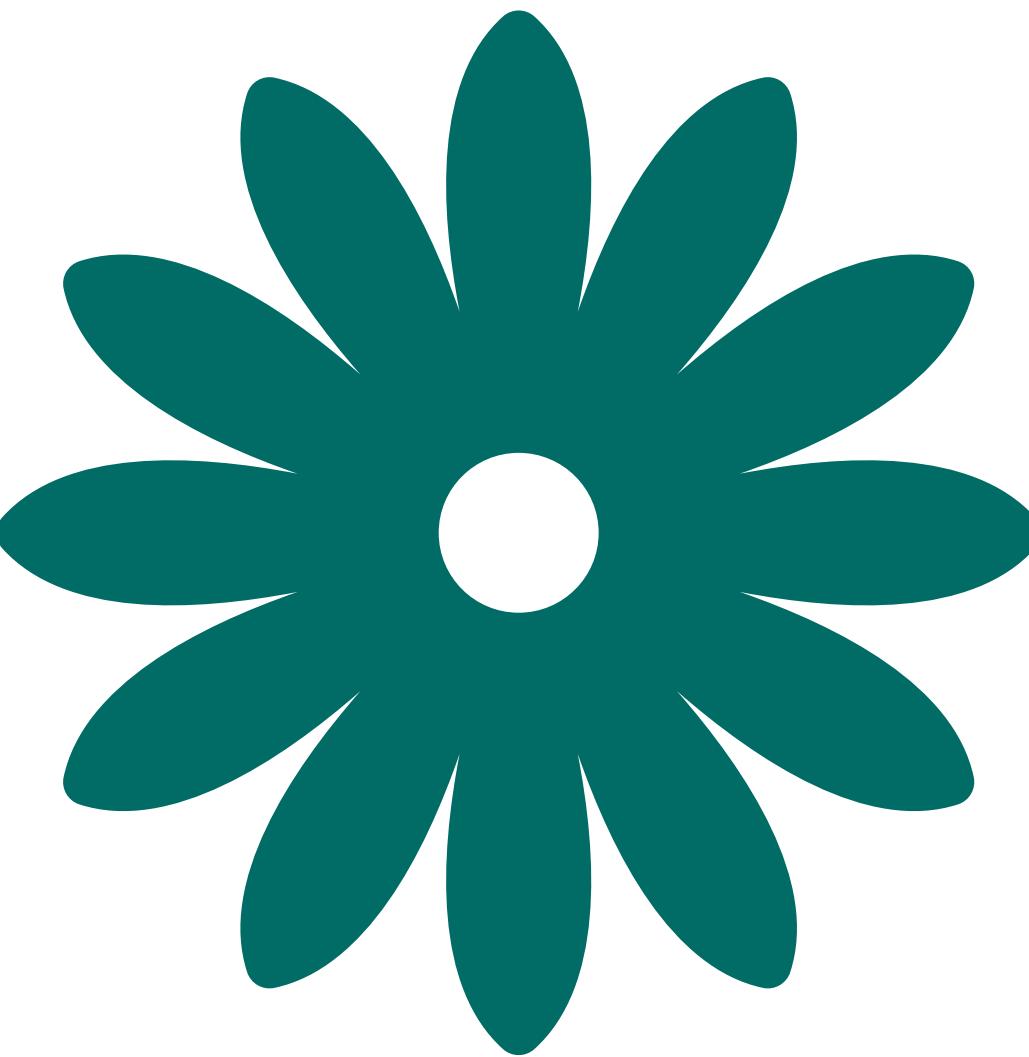
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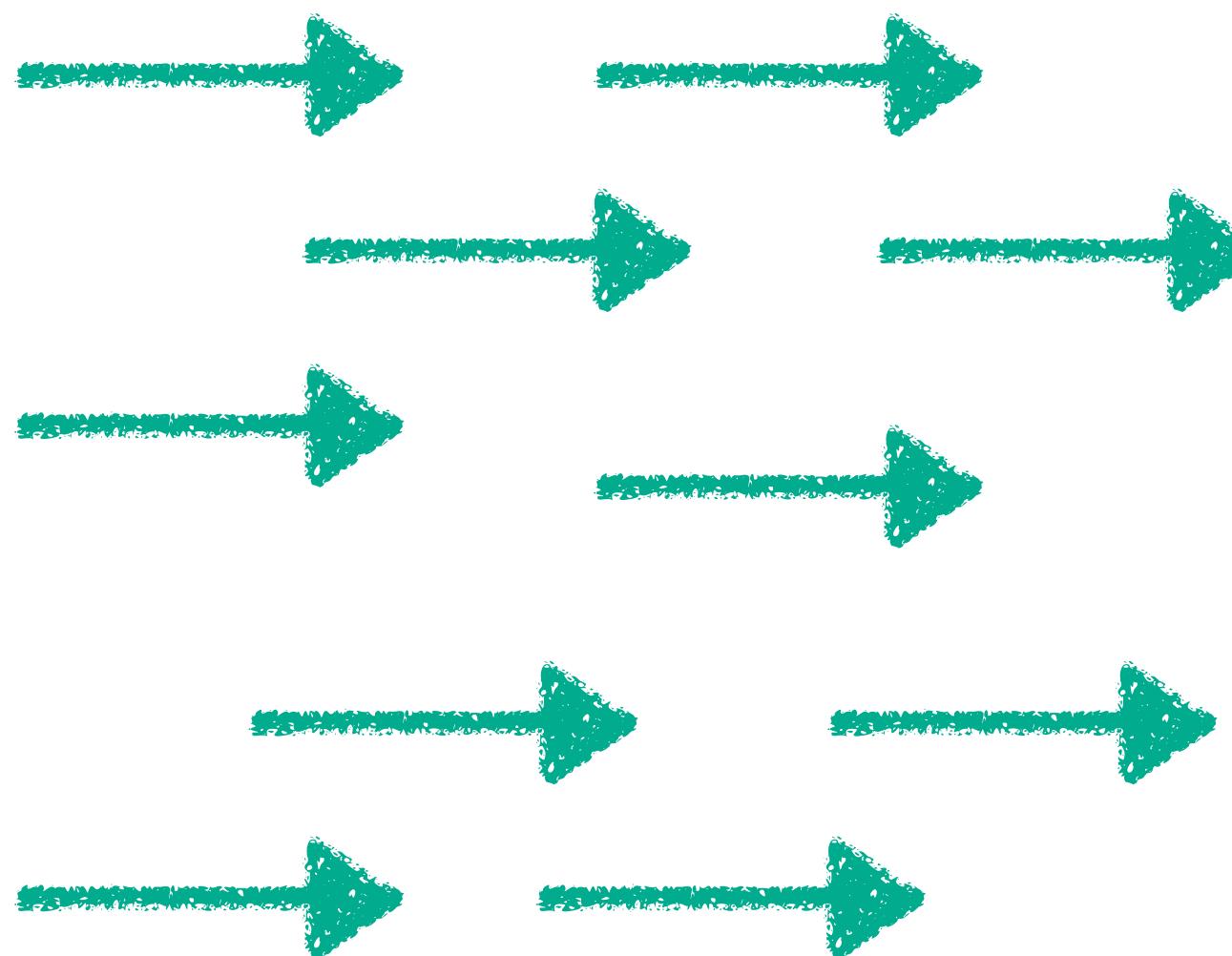


homogenous but not isotropic

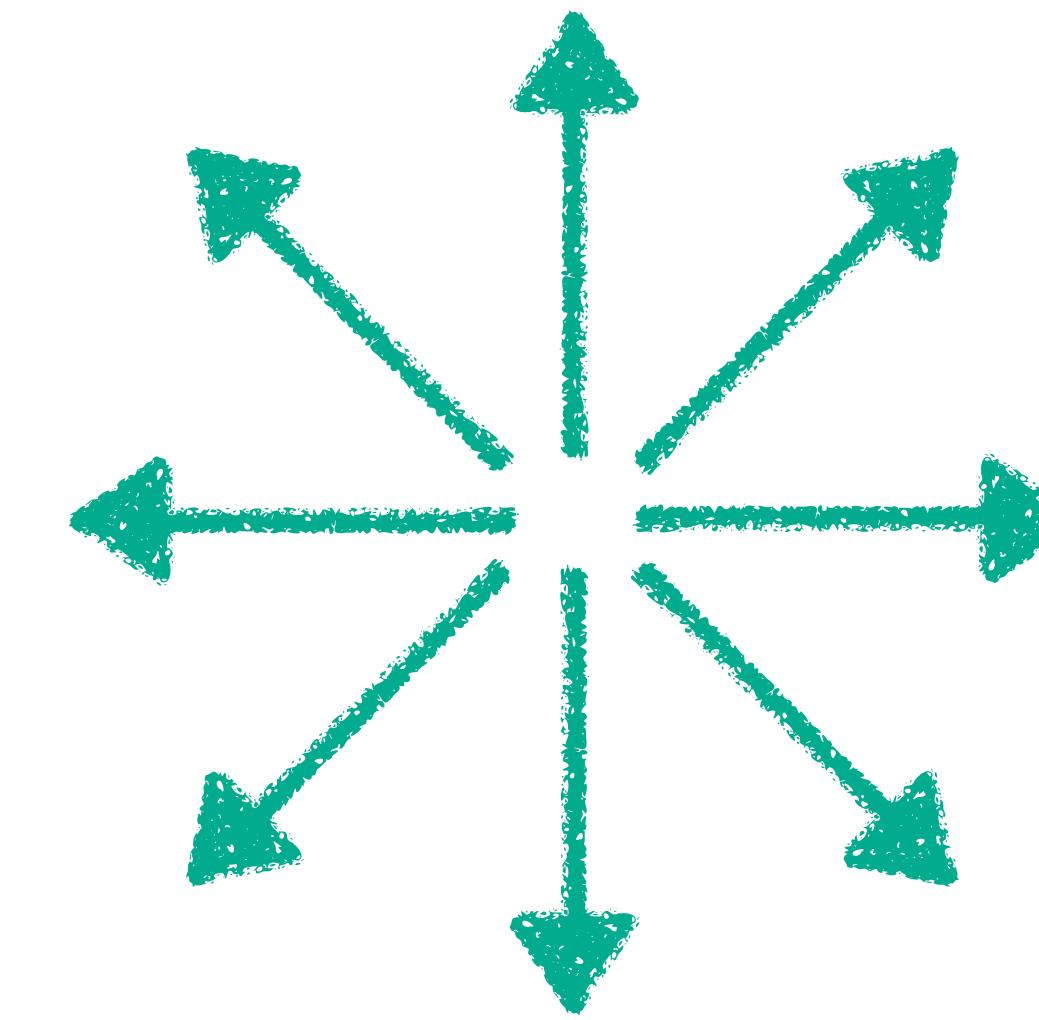


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# Isotropic or homogeneous?



homogenous but not isotropic



isotropic but not homogenous

# Friedmann Equations

- Friedmann 1922, Robertson 1935, Walker 1937: For a spatially homogeneous and isotropic universe, the metric simplifies to

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]$$

spacetime line element

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spacetime line element

scale factor



$$a(t_{\text{today}}) = a_0 = 1$$

Figure credit: Bianchi, Rovelli, Kolb

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spacetime line element      scale factor      curvature parameter

$$k = \begin{cases} -1 & \text{hyperbolic} \\ 0 & \text{flat} \\ +1 & \text{spherical} \end{cases}$$

# Friedmann Equations

- A perfect fluid is a fluid, which can be completely characterised by its (energy) density and pressure
- The energy momentum tensor of a perfect fluid is

$$T_{\mu\nu}^{\text{pf}} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$$


energy density      pressure

- $u_\mu = \dot{x}_\mu$  is the 4-velocity of the observer
- Inserting this in the Einstein Equations, yields the Friedmann Equations

# Friedmann Equations

- Inserting the FLRW-metric and the energy momentum tensor of the perfect fluid into the Einstein equations, yields the **Friedmann equations**:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \quad (\text{i})$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad (\text{ii})$$

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- Inserting (ii) into the temporal derivative of (i), yields the **continuity equation**:

$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a} \quad (\text{iii})$$

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- Where  $\rho = \rho_{\text{tot}}$  is the energy density of the universe.

- If we define  $\rho_k = -\frac{3}{8\pi G}\frac{k}{a^2}$  and  $\rho_\Lambda = \frac{\Lambda}{8\pi G}$ , one can rewrite (i) as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{tot}} \quad (\text{i})$$

$$\Lambda\text{CDM model} \quad \rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda + \rho_k$$

$\Lambda$ CDM model

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# $\Lambda$ CDM model

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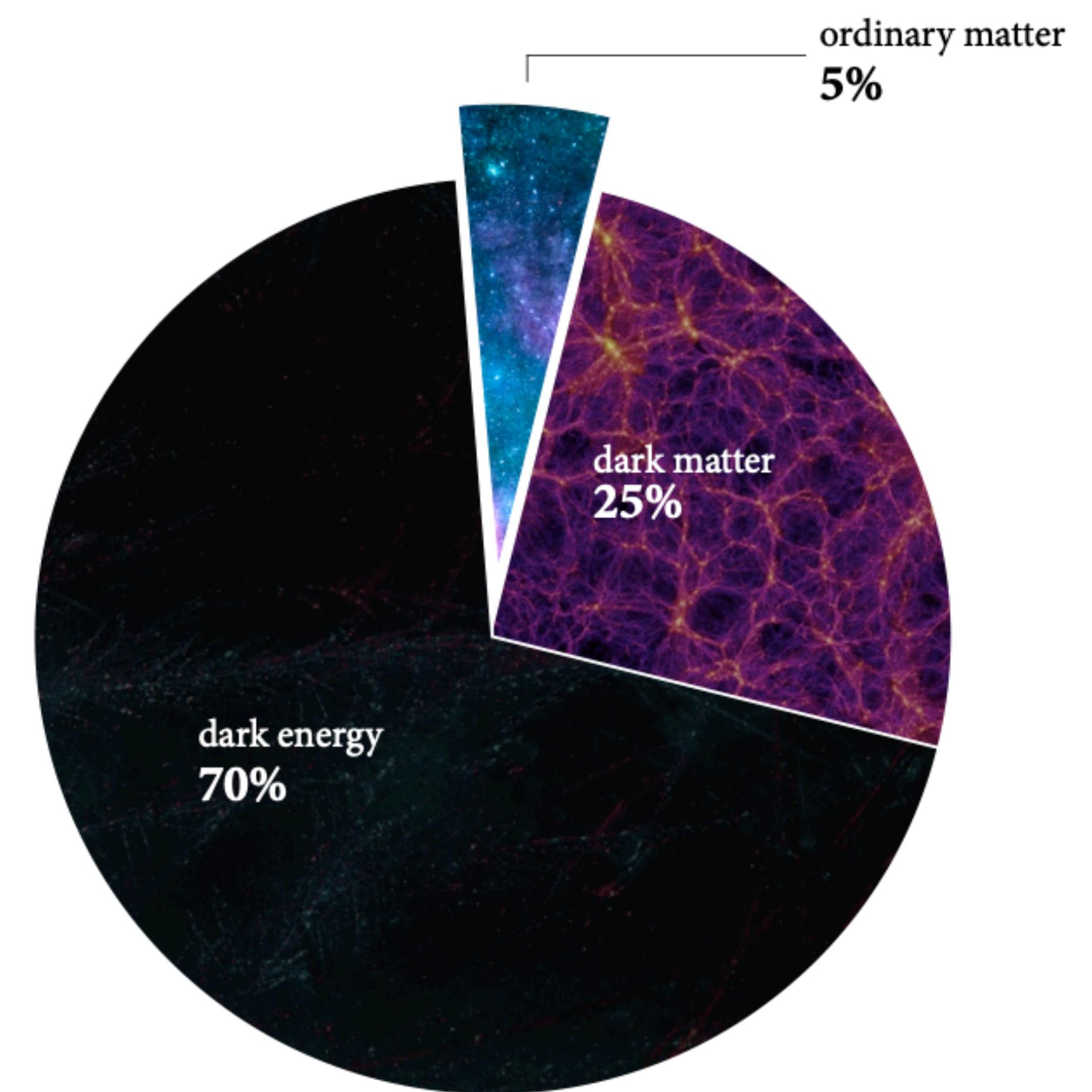


Figure credit: Florian Wolz

# $\Lambda$ CDM model

- Equation of state:

$$\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda$$

$$p = w \cdot \rho$$

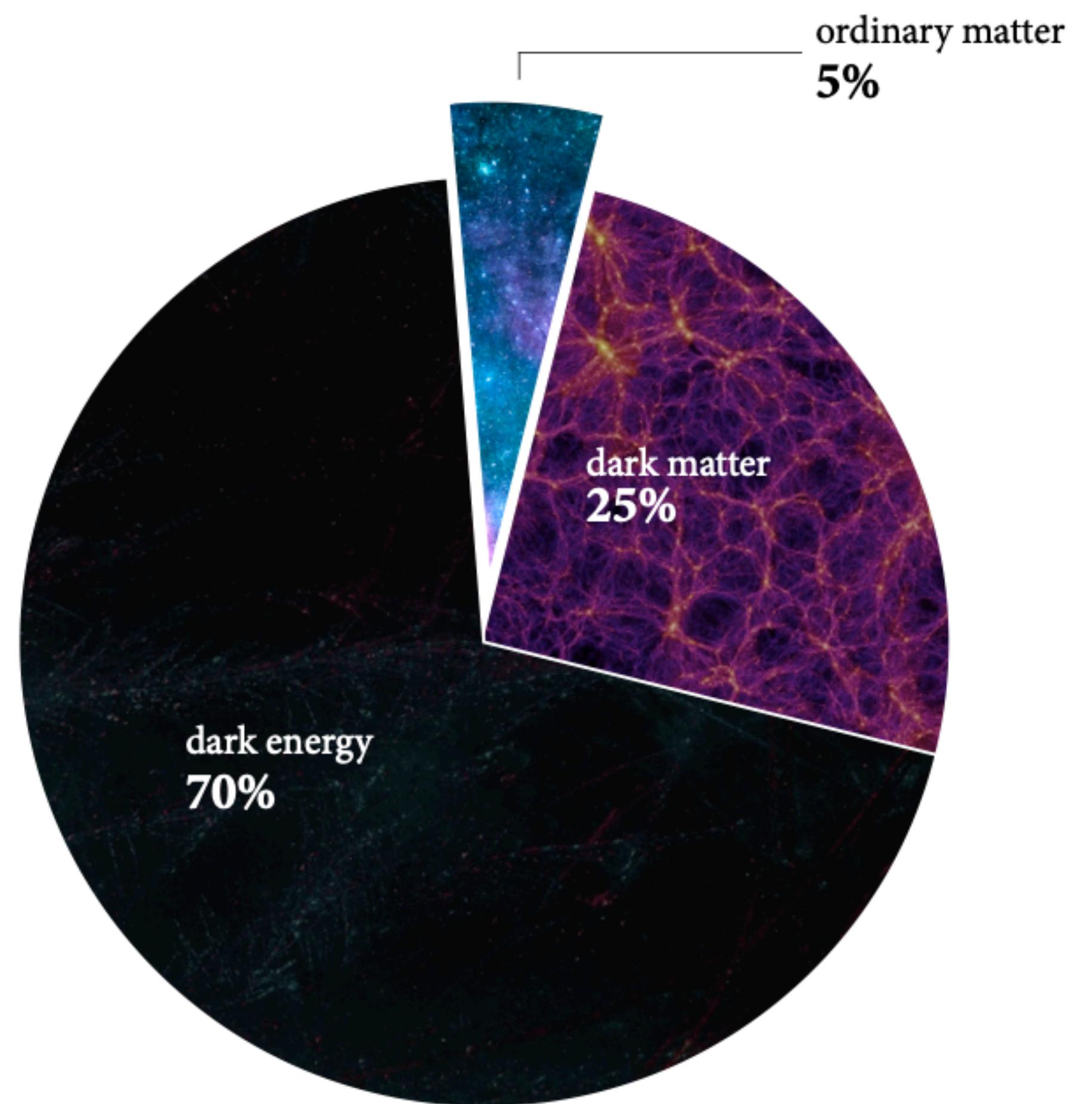


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  - Dark matter and baryonic matter:  $w = 0$

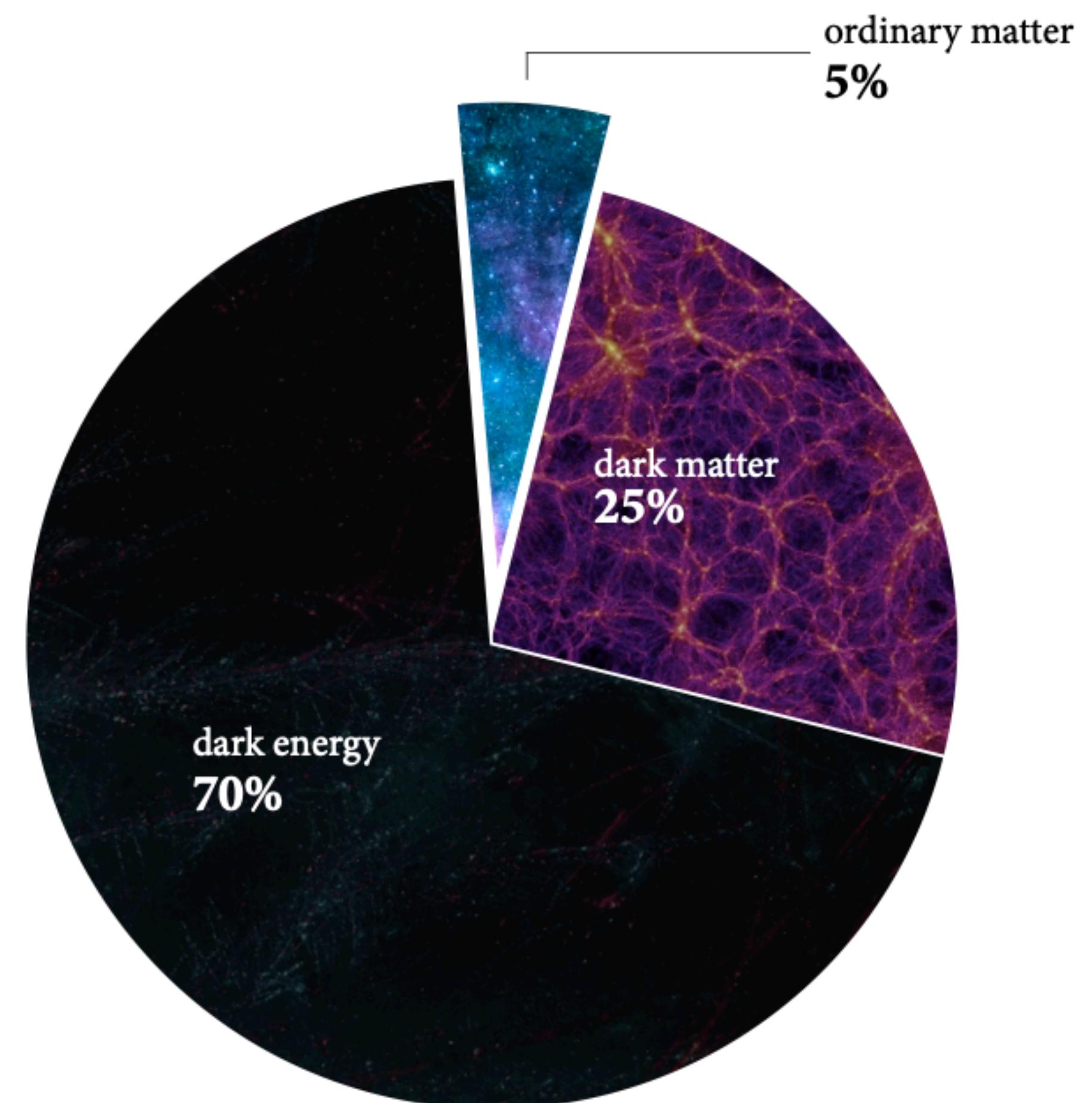


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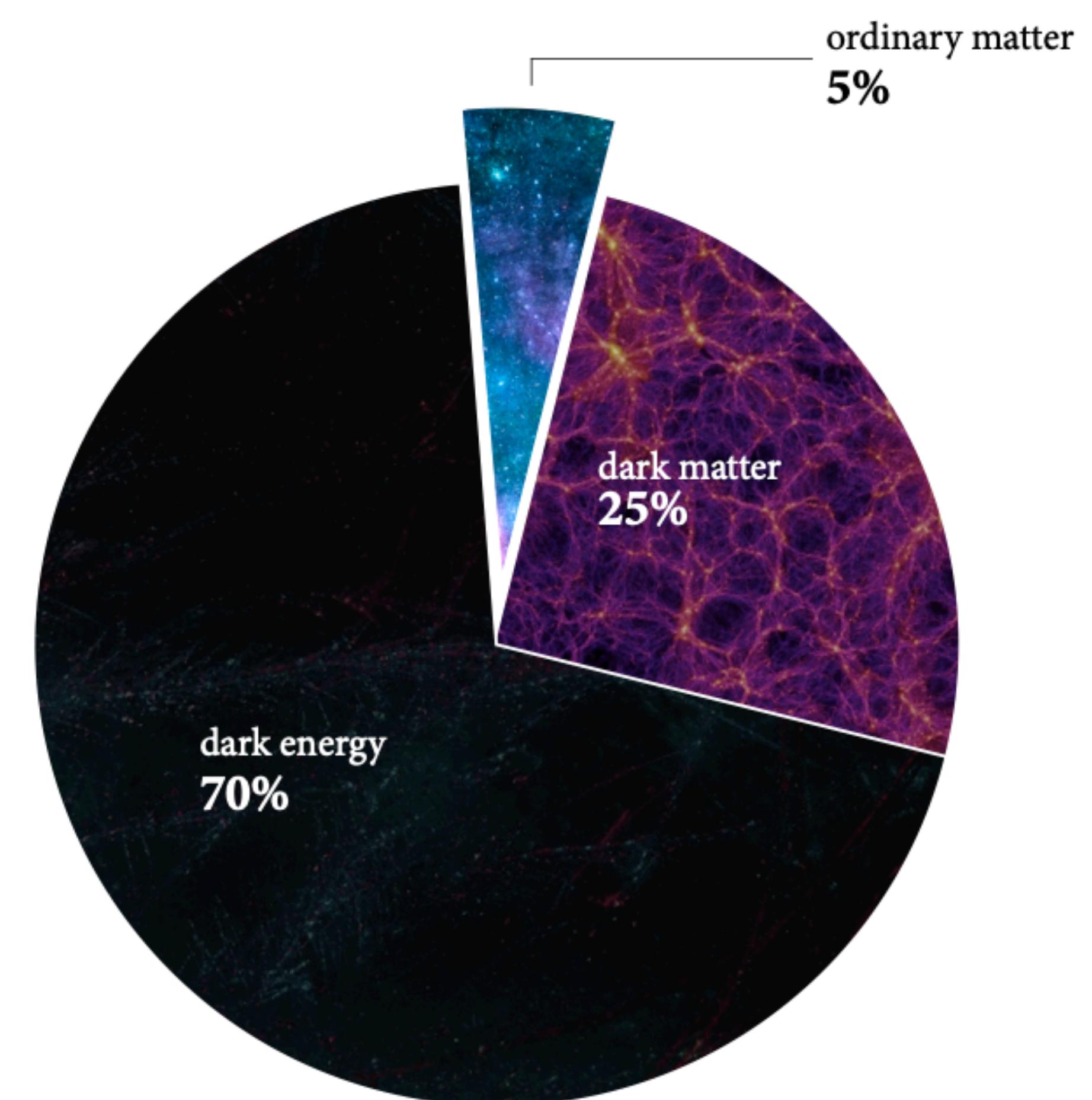


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  - Dark energy:  $w = -1$

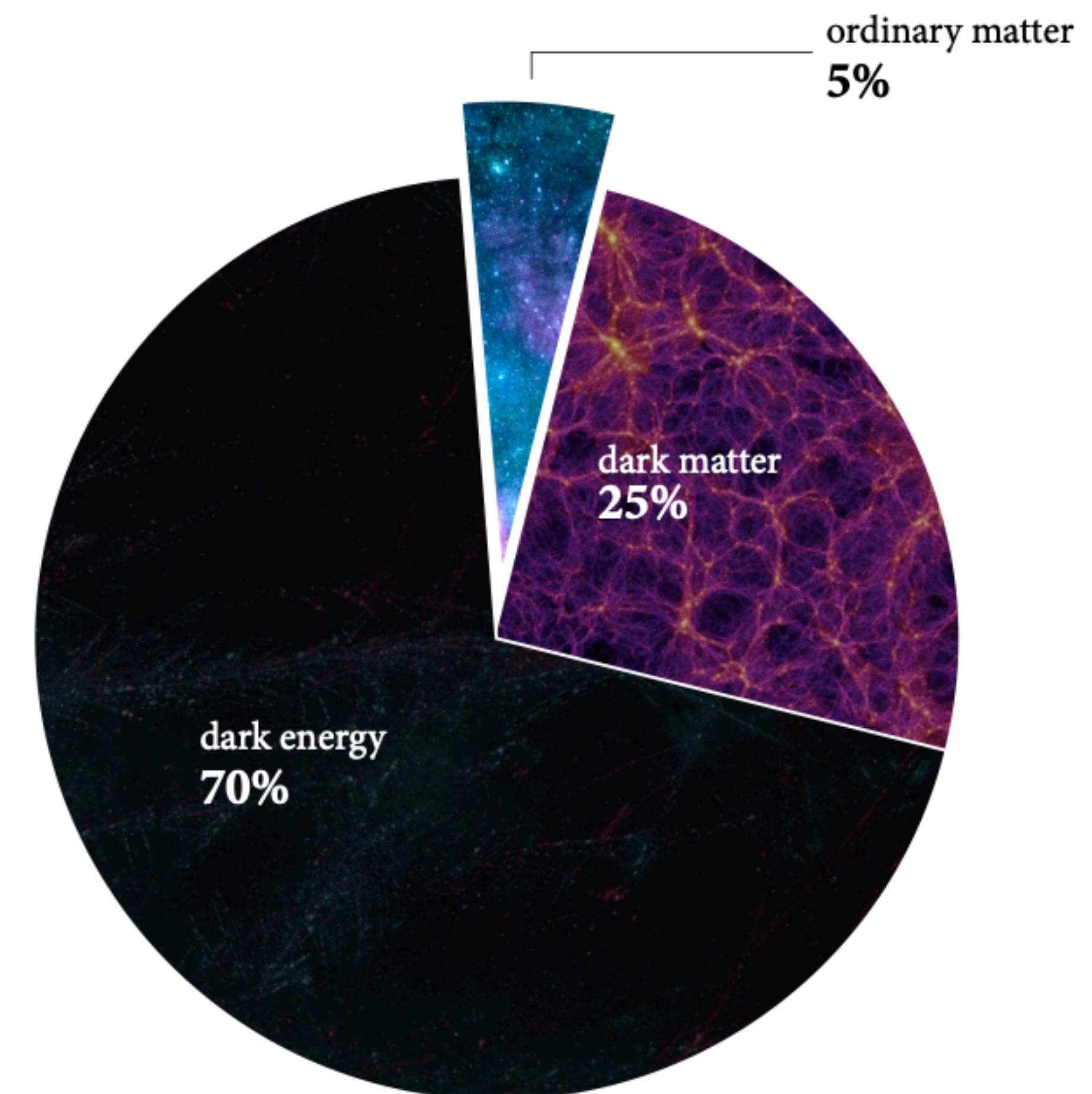


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# Friedmann Equations

- Equation of state:  $p = w \cdot \rho$
- Inserting the E.O.S into the continuity equation  $\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}$  yields:  
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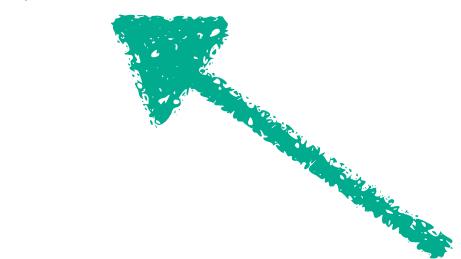
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... (your exercise later)

$$\frac{\rho(a)}{\rho_0} = \left(\frac{a}{a_0}\right)^{-3(1+w)}$$



The subscript 0 refers to the current time “today”

# The Hubble parameter

- The first Friedmann equation describes the rate of expansion as a function of the energy content  $\rho$  of the universe:

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$$H_0 = H(t = t_0)$$


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- 
- density of flat universe

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$$\Omega_{I,0} = \Omega_I(t_0) = \Omega_I$$

# Distances in an expanding universe

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- The angular diameter distance  $D_A$  is defined such that it holds

$$\theta = \frac{s}{D_A}$$

angle

physical size of the object

ang. diam. distance

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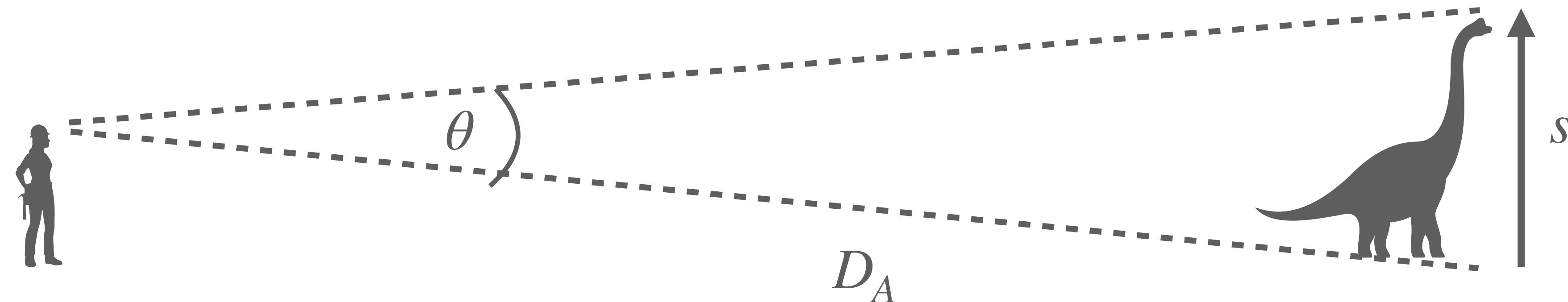
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$$\theta = \frac{s}{D_A}$$

$$D_A(t) = a(t)\chi(t) = \frac{1}{1+z(t)} \int_0^{z(t)} \frac{dz'}{H(z')}$$

\*To derive an equation for  $D_A$ , note that the proper size,  $s$ , of the object can also be expressed as  $s = \chi(t)\theta \cdot a(t)$ , where  $\chi(t)\theta$  corresponds to the comoving size of the object.

# Distances in an expanding universe

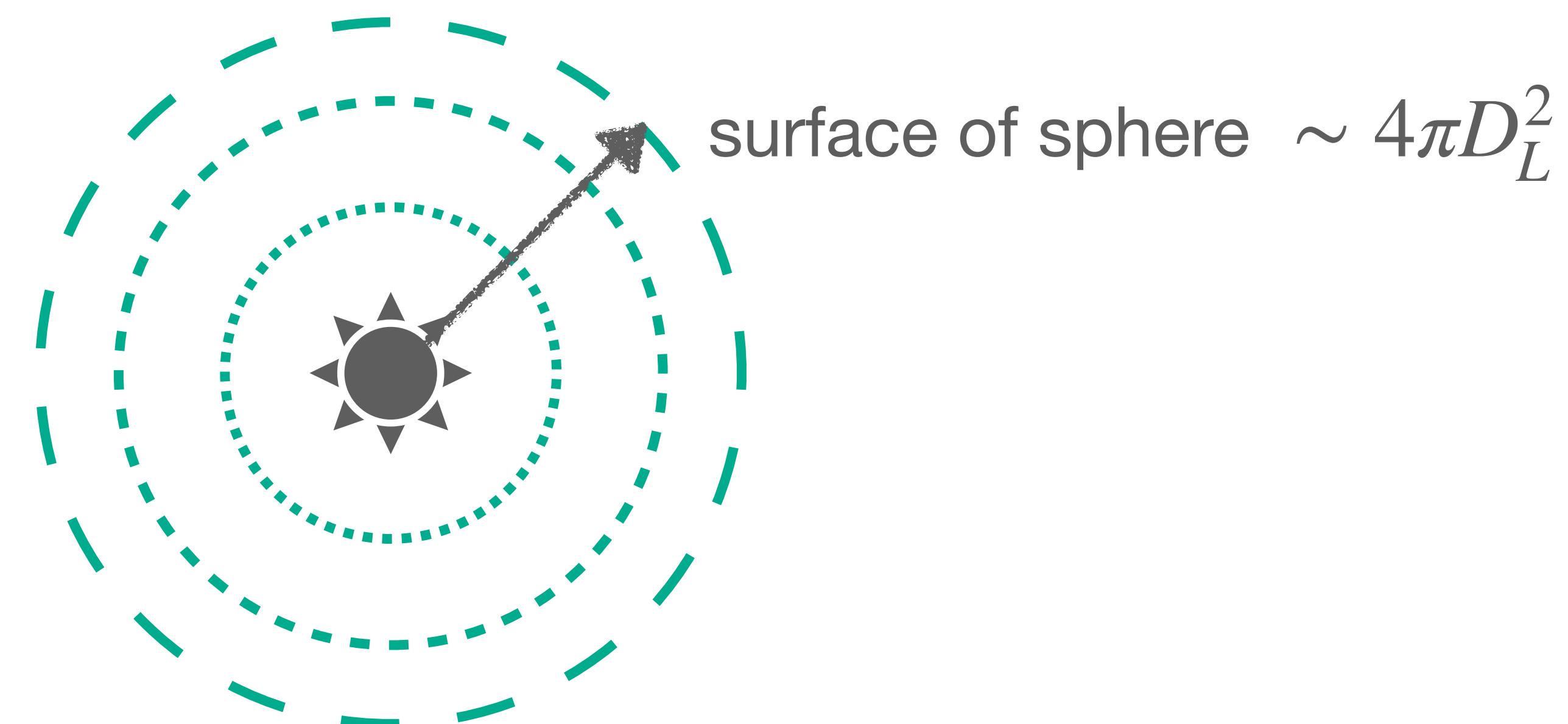
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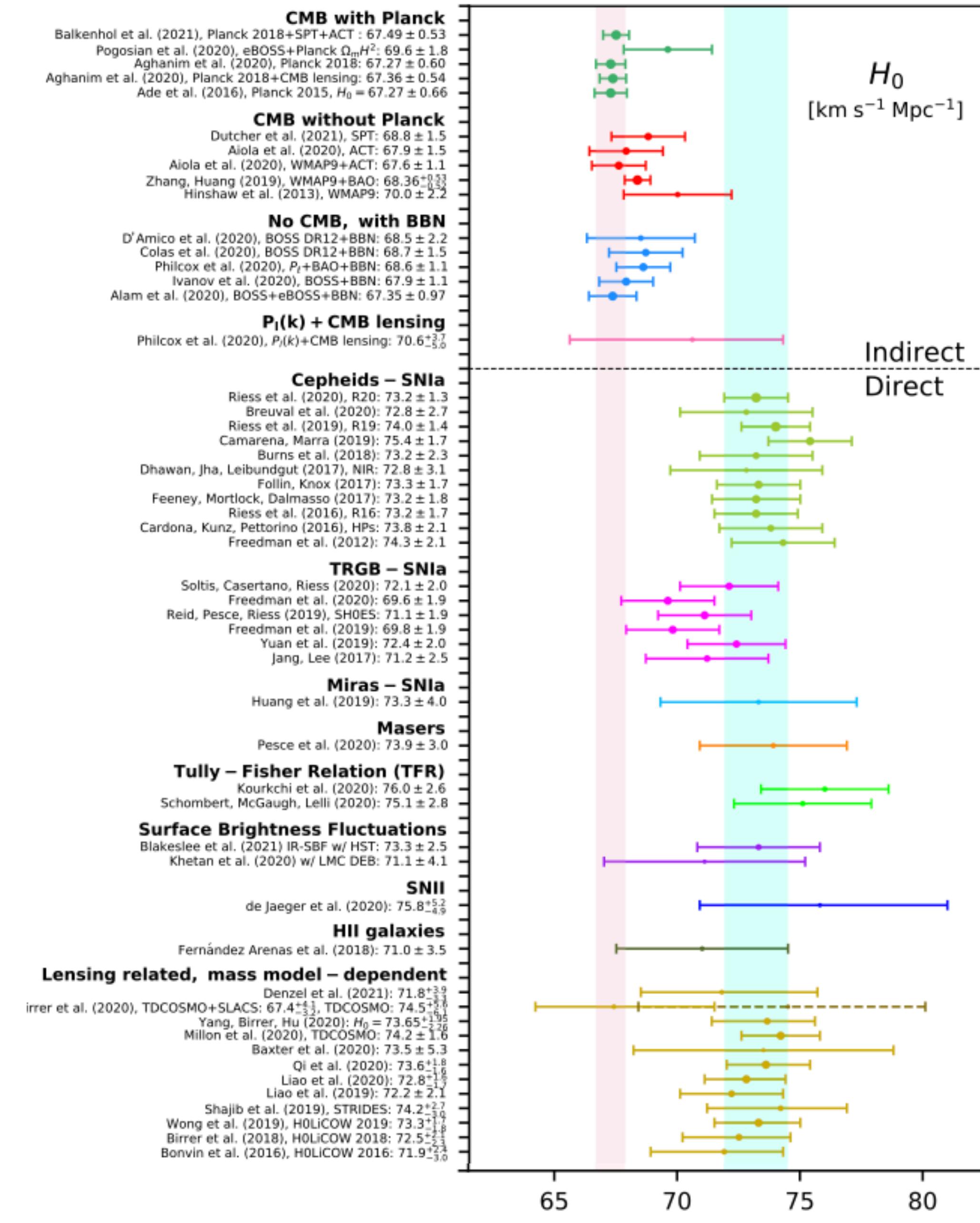
$$D_L(t) = \frac{\chi(t)}{a(t)} = [1 + z(t)] \int_0^{z(t)} \frac{c \, dz'}{H(z')}$$

\*Since the Universe is expanding, it holds that  $F = \frac{L a^2}{4\pi \chi^2(a)}$ , where the additional factor of  $a^2$  comes from the fact that the expansion of the Universe leads to a dilution of photons ( $\propto a$ ) and to an increase in wavelength ( $\propto a$ ).

# How to solve the Hubble tension?

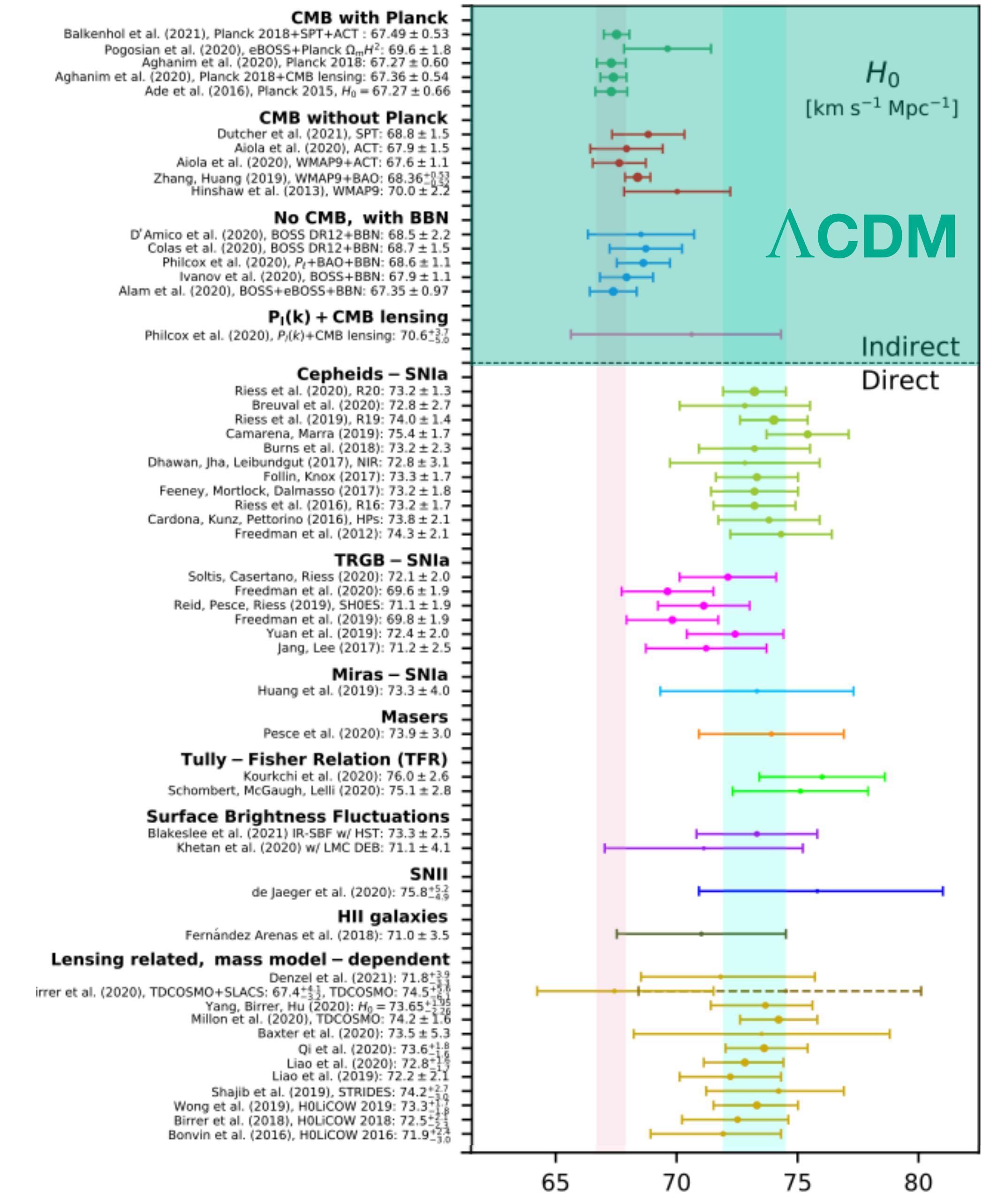
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- **Direct measurements:** distance ladder (Cepheids, TRGB, SNe, ...), gravitational lensing, ...



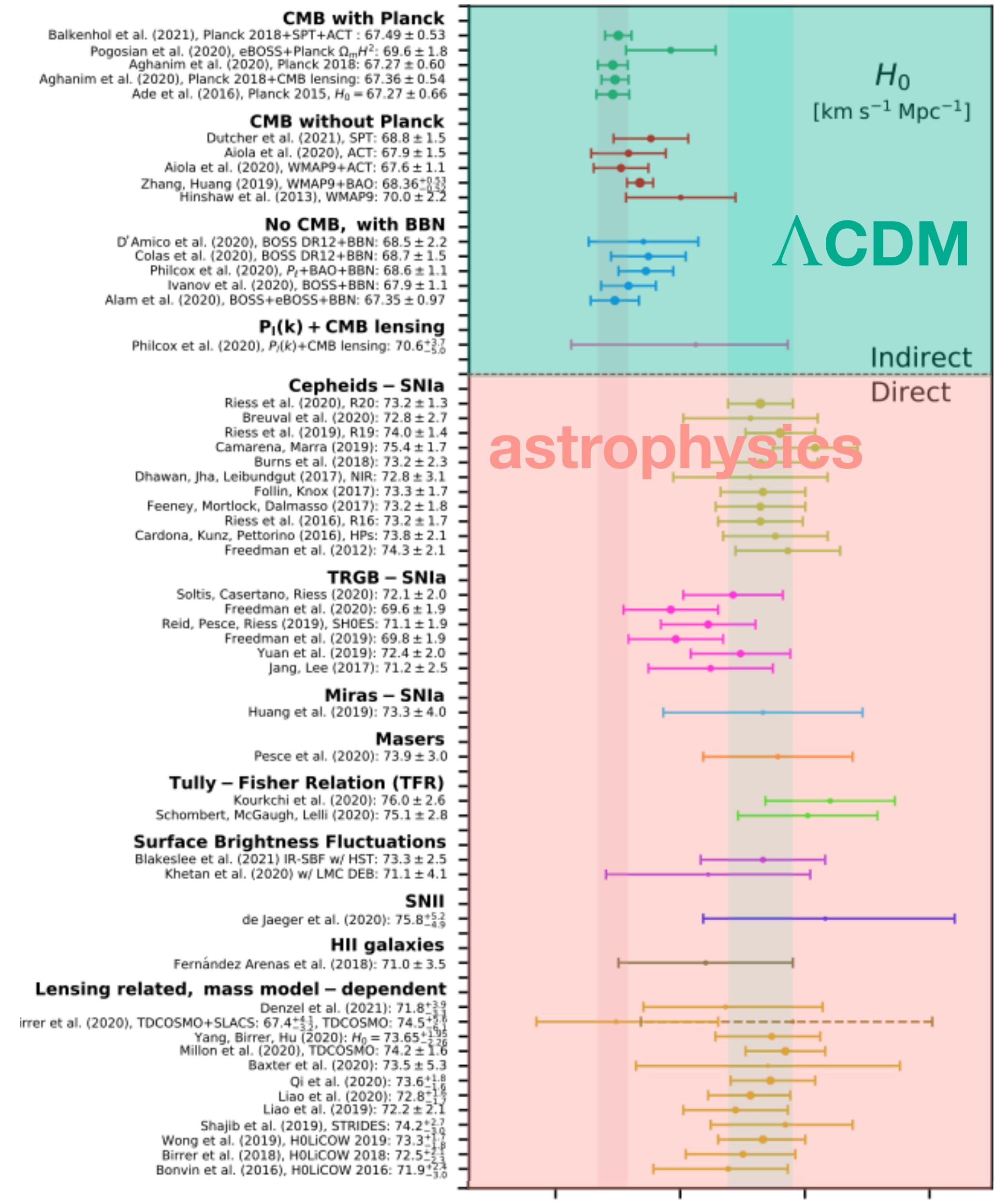
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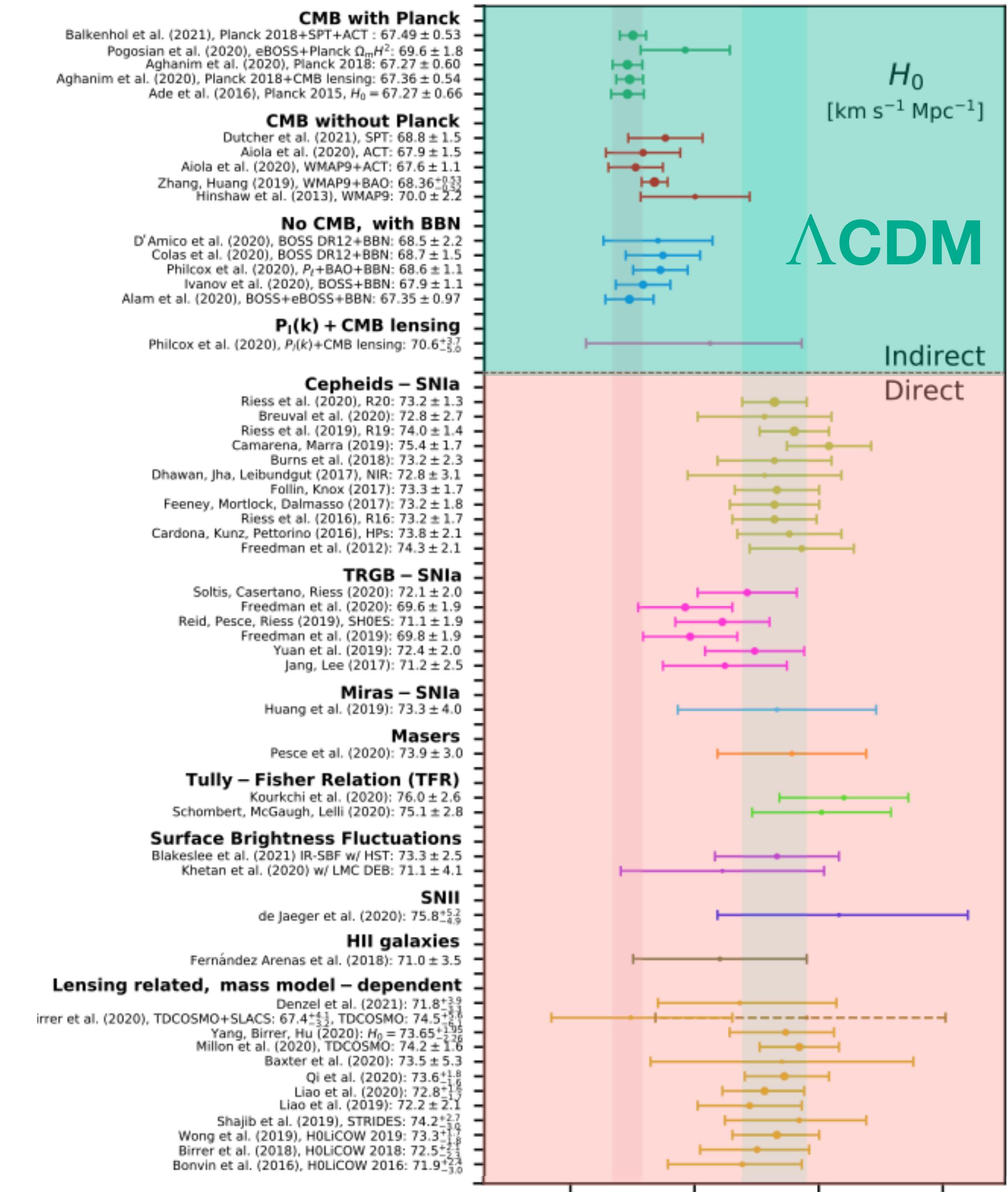
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Expansion rate  $H_0$  [km/s/Mpc]

# The Hubble tension

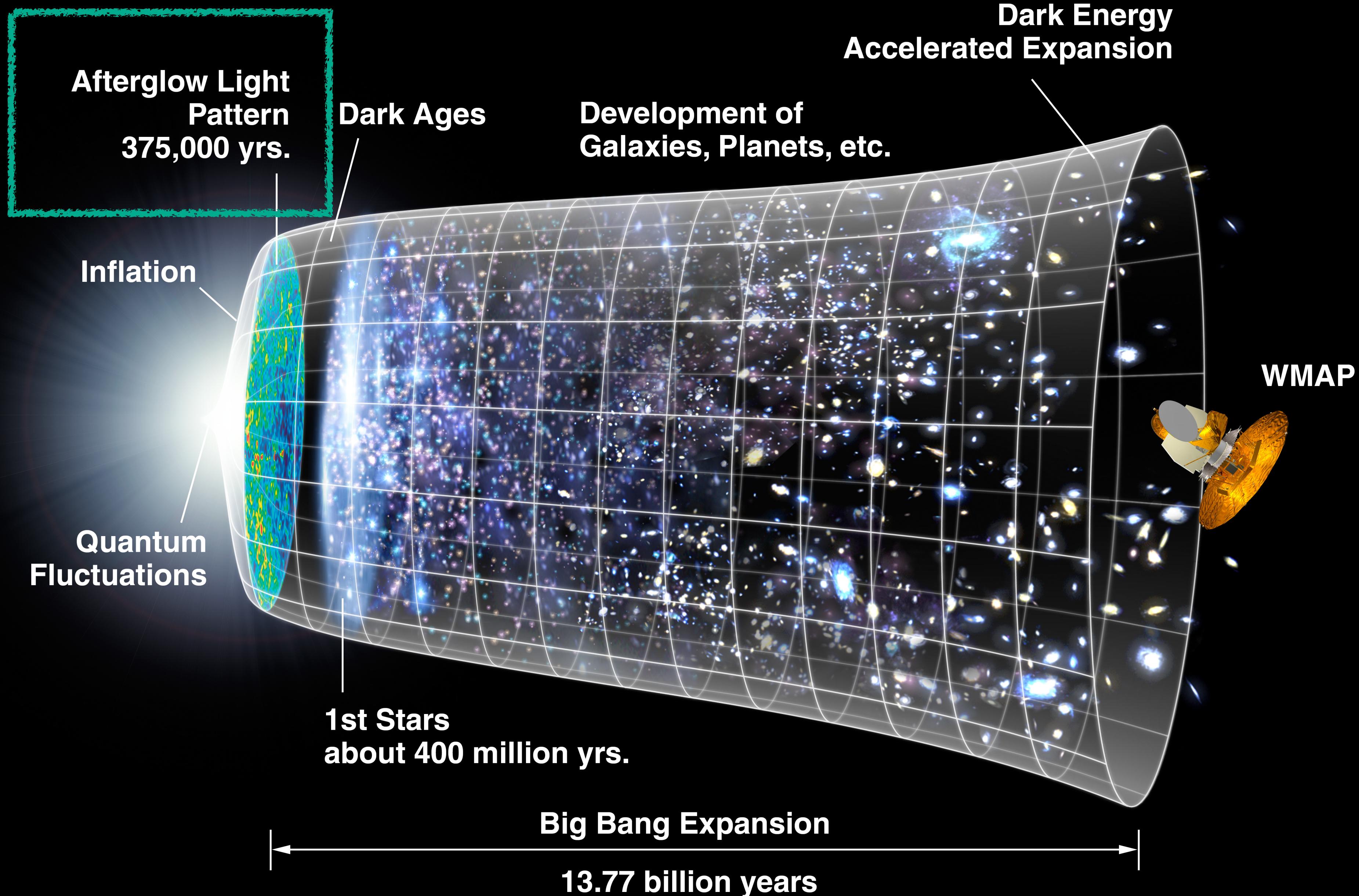
- General strategy to solve the  $H_0$  tension:  
Assume direct measurements are correct  
and **change cosmological model** in order  
to *infer* a higher  $H_0$
- Goal: “get CMB- $H_0$  to  $\sim 73$  km/s/Mpc”



Expansion rate  $H_0$  [km/s/Mpc]

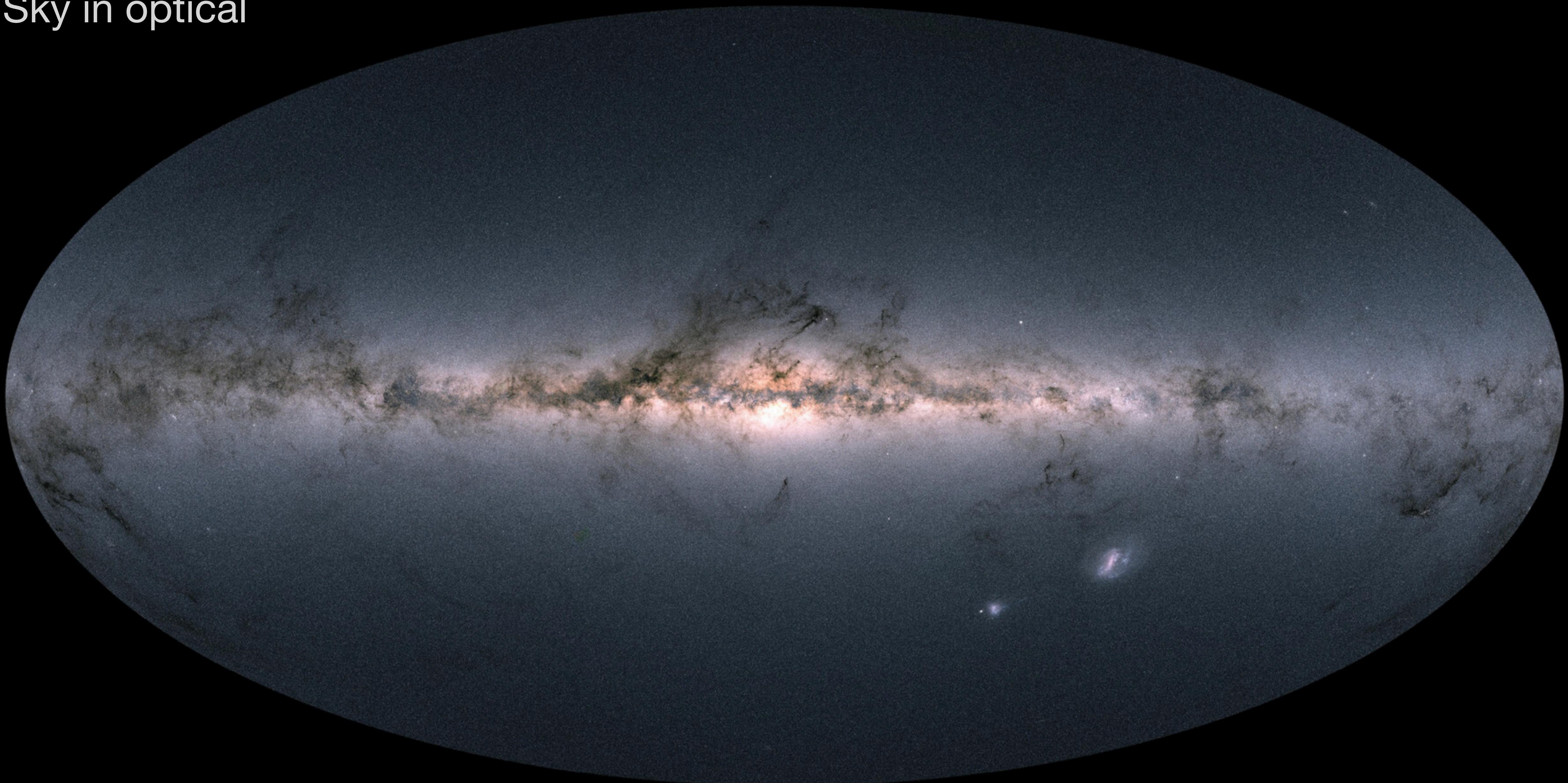
# How does the CMB constrain $H_0$ ?

- In order to understand how we can solve the Hubble tension, we need to understand how the CMB constrains  $H_0$
- The CMB provides the most important cosmological probe when it comes to constraining the parameters of the cosmological model with high accuracy
- However, it is an indirect probe of  $H_0$ : **it depends on the cosmological model** that is assumed → the CMB constrains the universe **at early times** and **predicts  $H_0$  today**



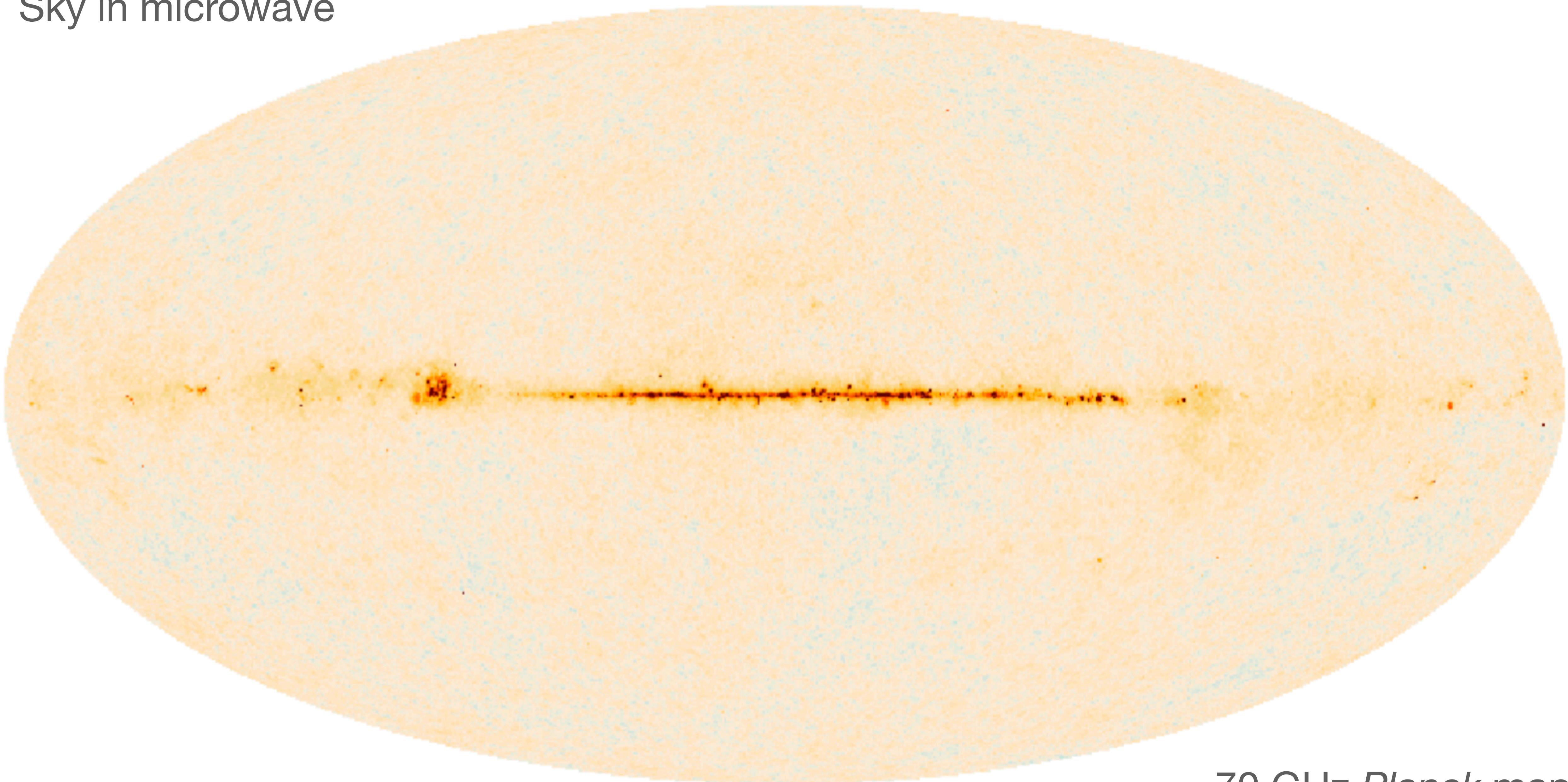
Credit: WMAP Collaboration

Sky in optical



Credit: Gaia collaboration

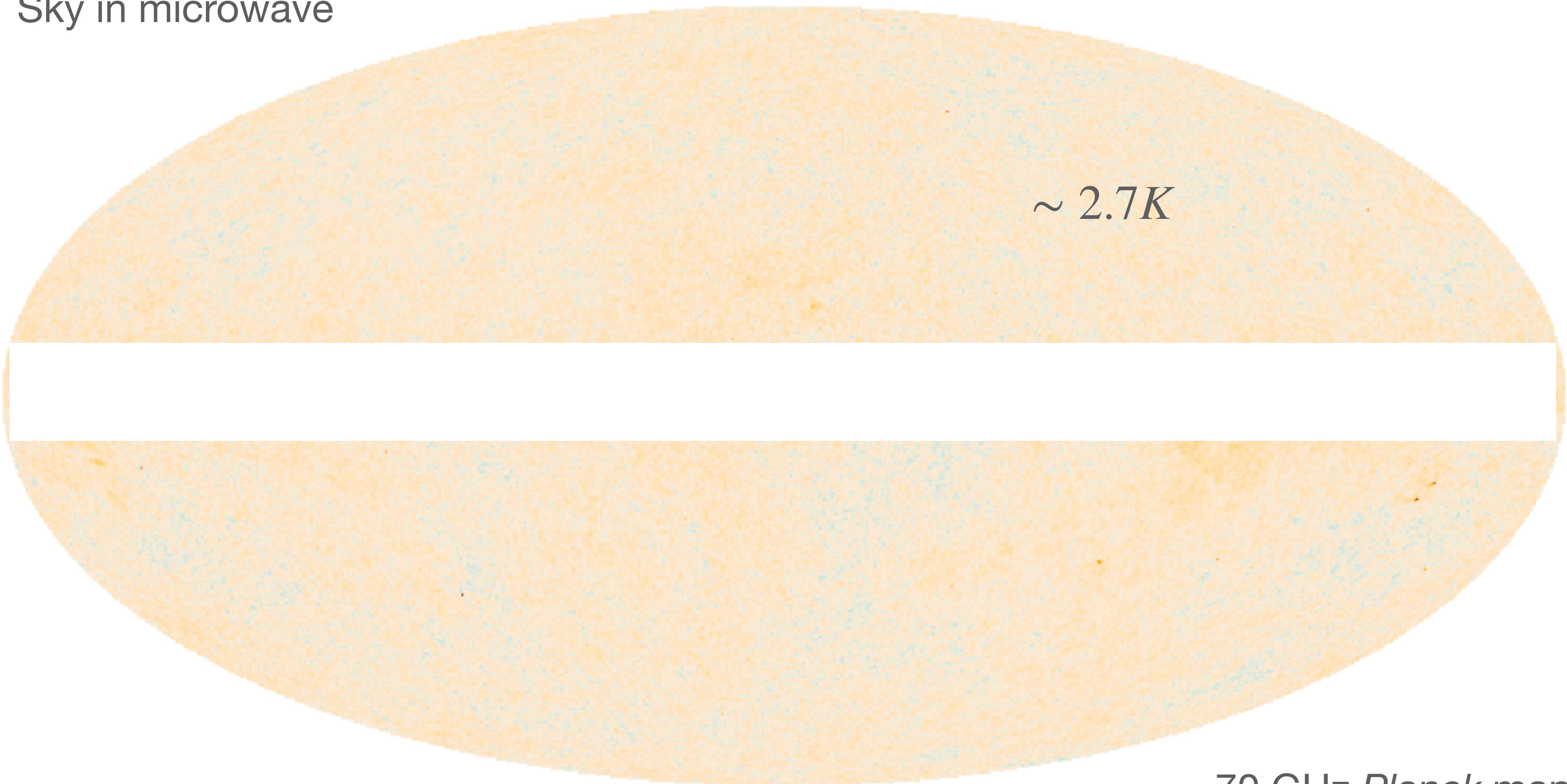
Sky in microwave



70 GHz *Planck* map

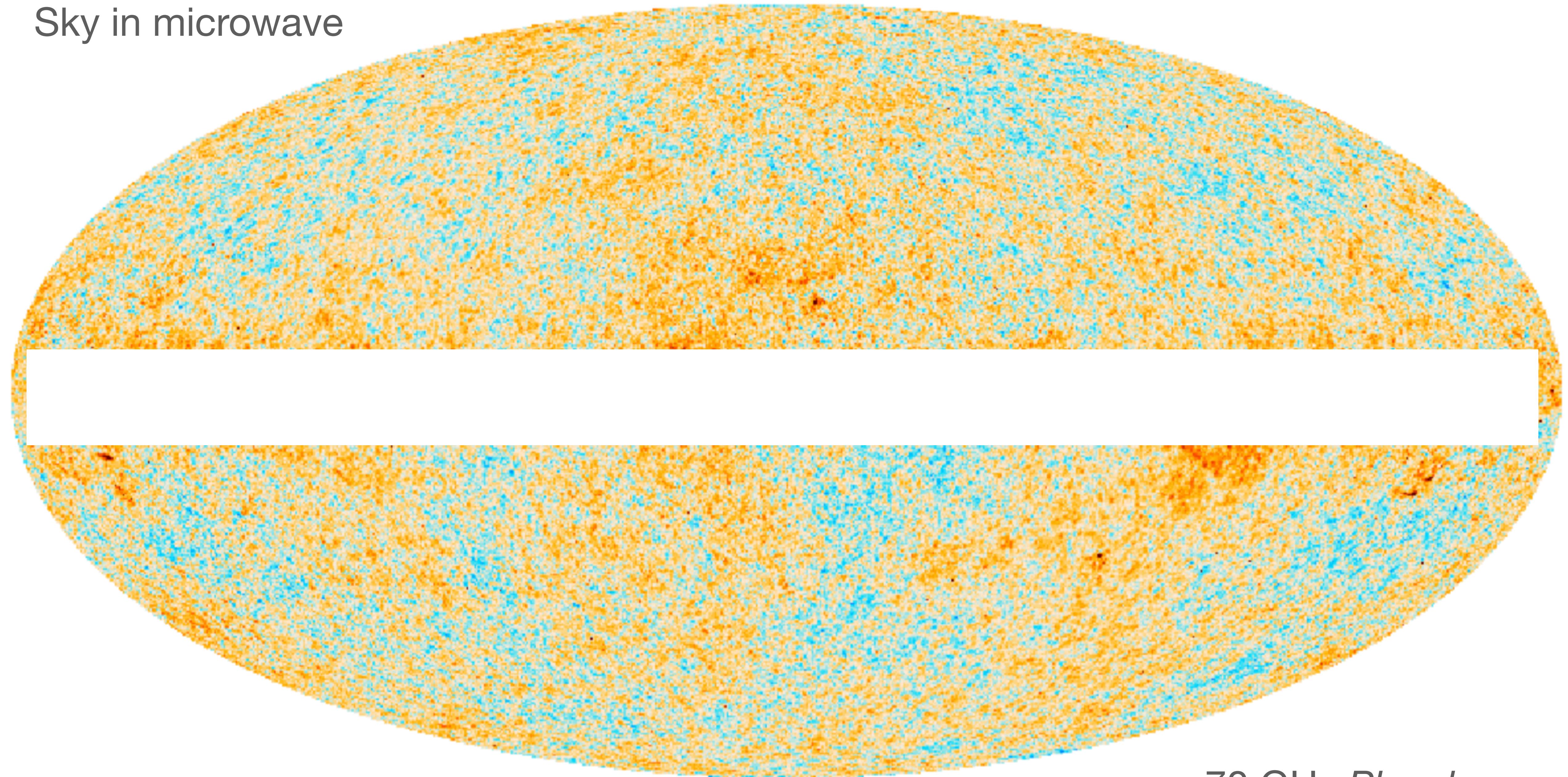
Sky in microwave

$\sim 2.7K$



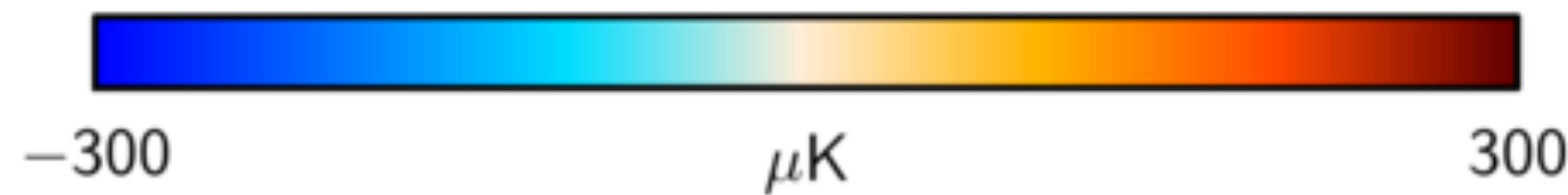
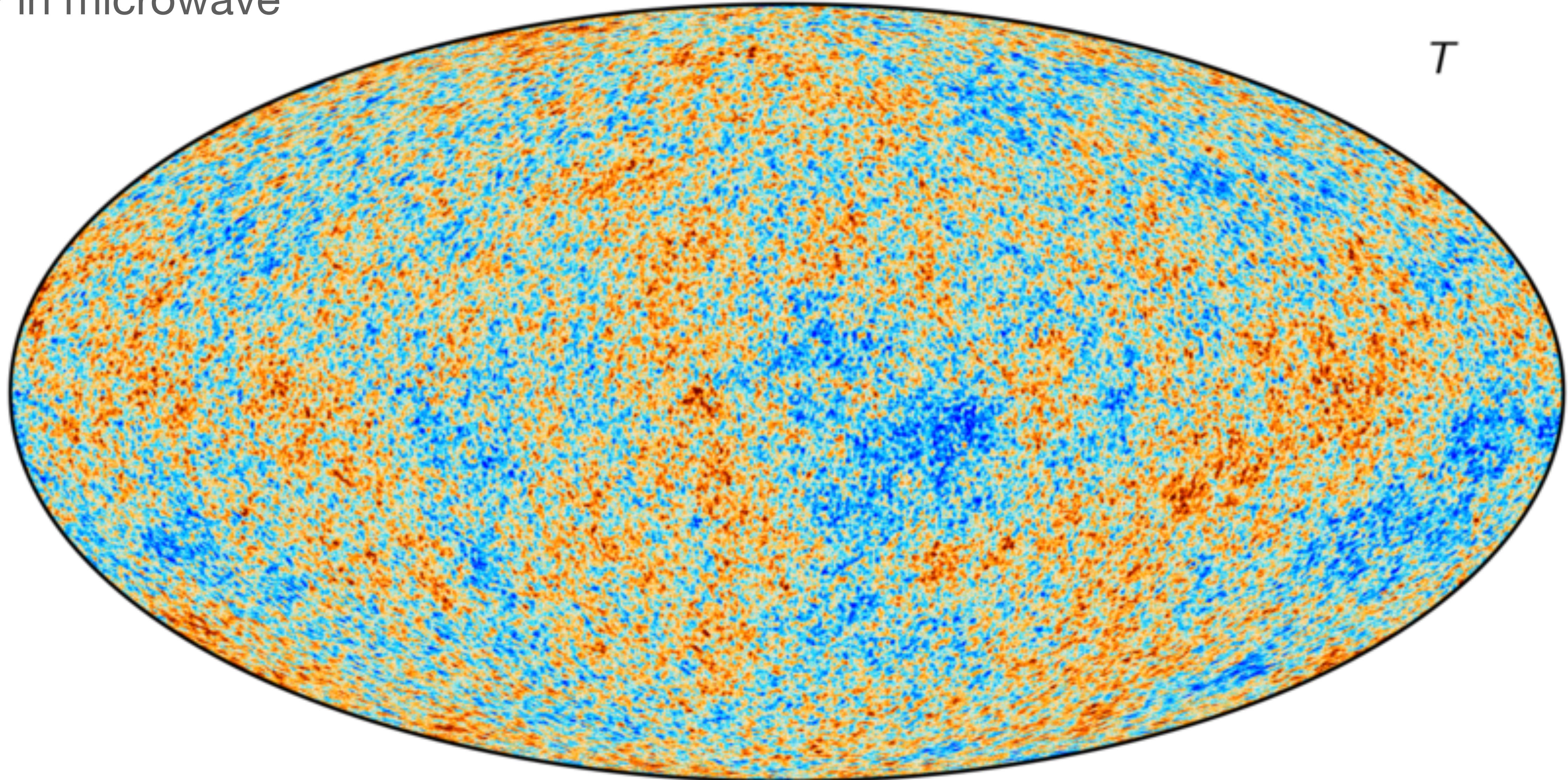
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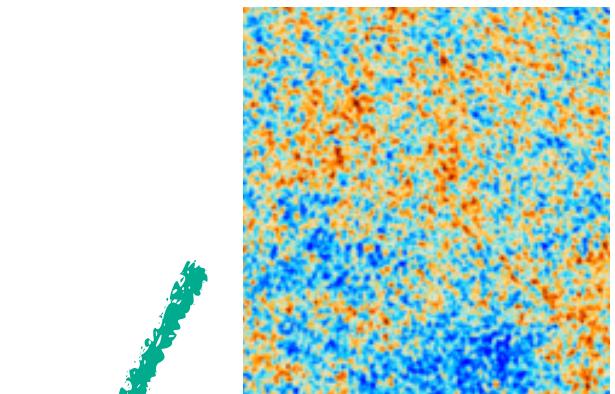
Credit: Planck  
Collaboration

# The CMB

- How to analyse the CMB map with temperature fluctuations?
  - 1. Decompose the temperature fluctuations into a set of waves with various wavelength
  - 2. Plot the strength of each wavelength: **Power spectrum**
- In 1 dimension: decomposing into waves = Fourier transform
- On the unit sphere: decomposing into waves = **Spherical harmonics** decomposition

# The CMB

- Decompose temperature fluctuations:  $\Delta T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$

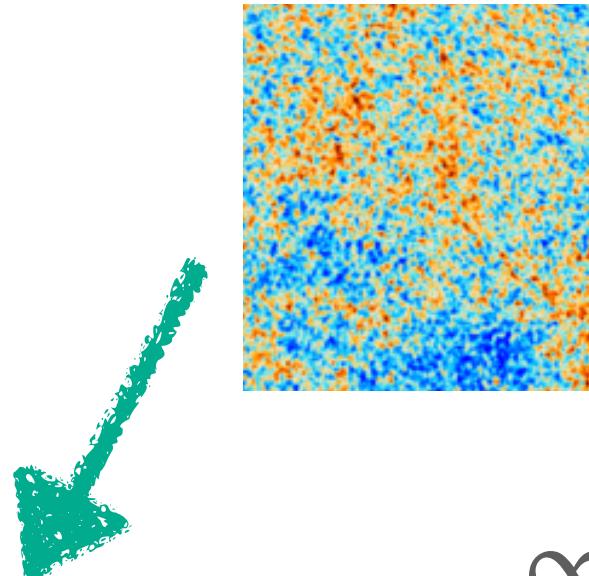


coefficients

spherical harmonics

# The CMB

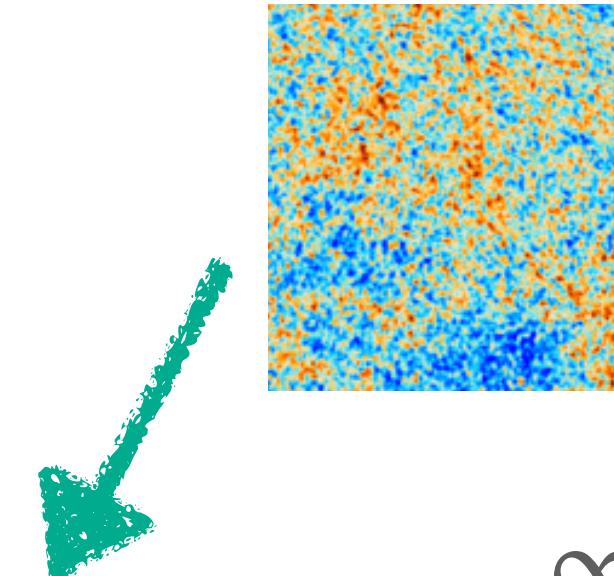
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spherical harmonics

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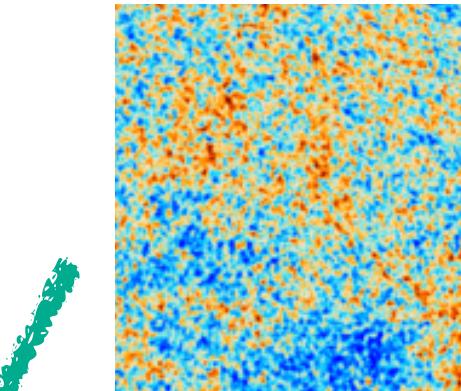
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spherical harmonics

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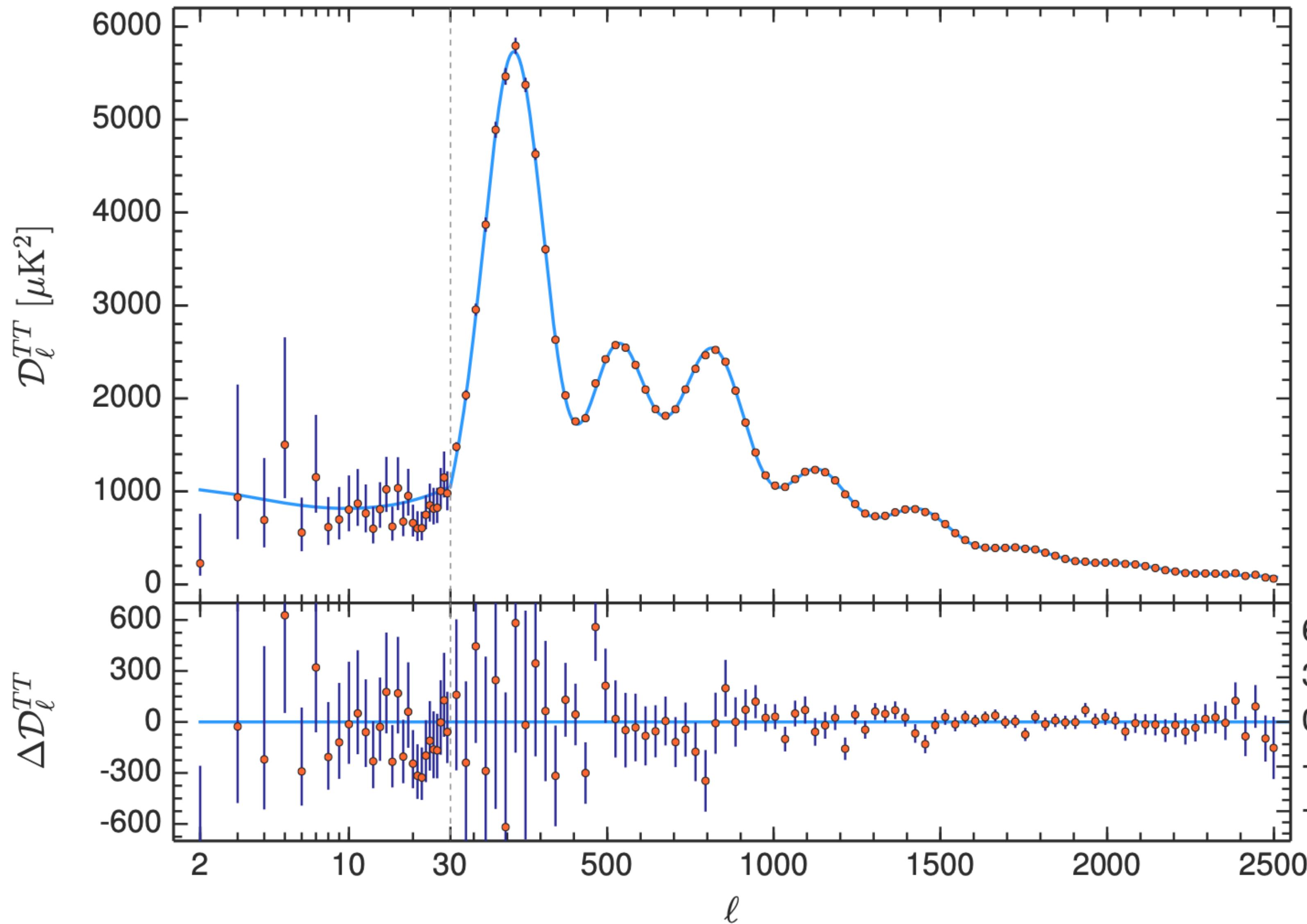
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- The  $a_{\ell m}$  contain the same information as the map itself
- $\ell$  defines the multipole, the higher  $\ell$  the smaller the scale
- The power spectrum is then given by:

$$C_{\ell} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell,m} a_{\ell,m}^*$$

spherical harmonics

# The CMB

Planck Collaboration

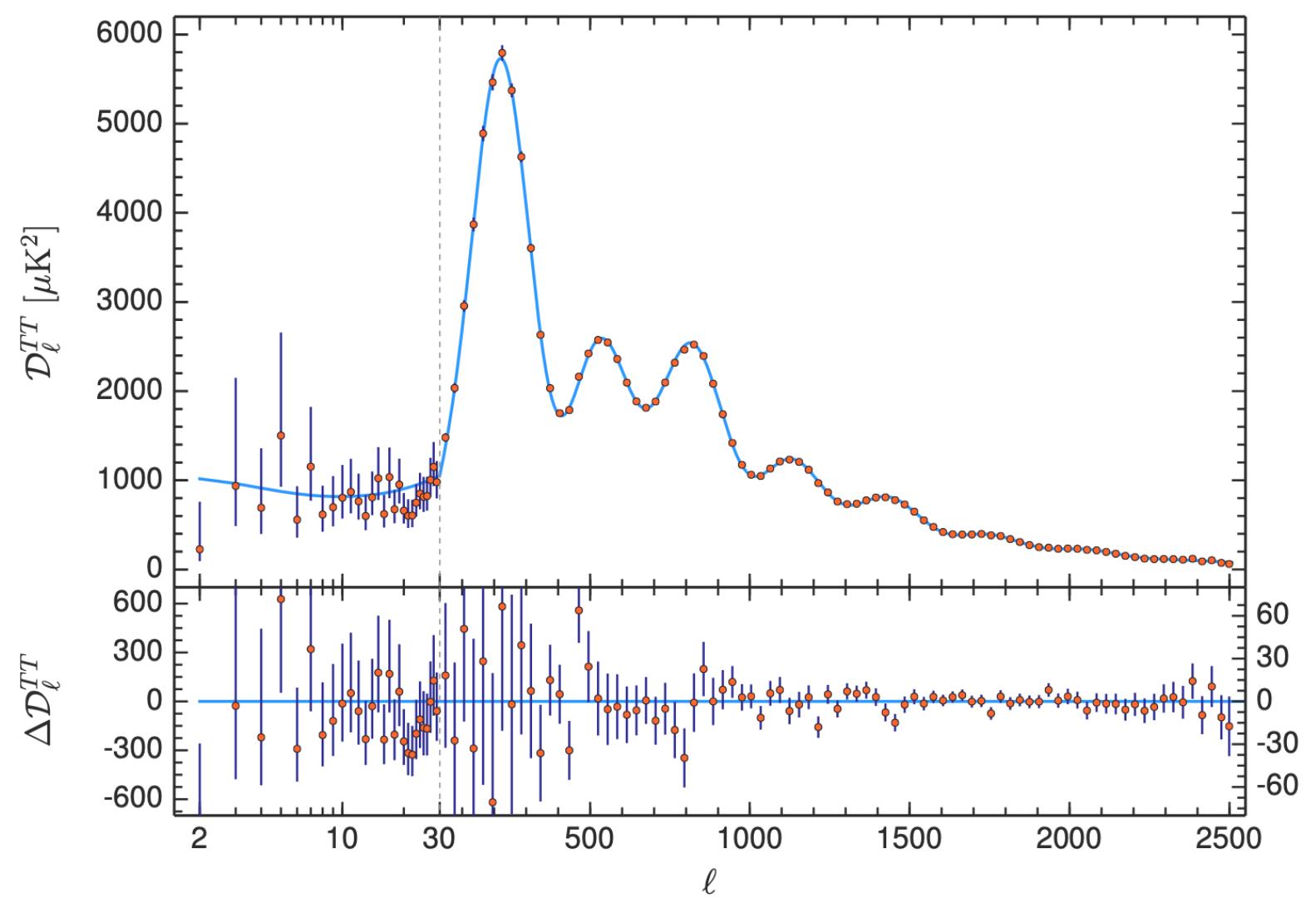


$$D_\ell = \frac{\ell(\ell + 1)}{2\pi} C_\ell$$

\*You will need this in the hands-on session later

# The CMB

How to analyse data like this?

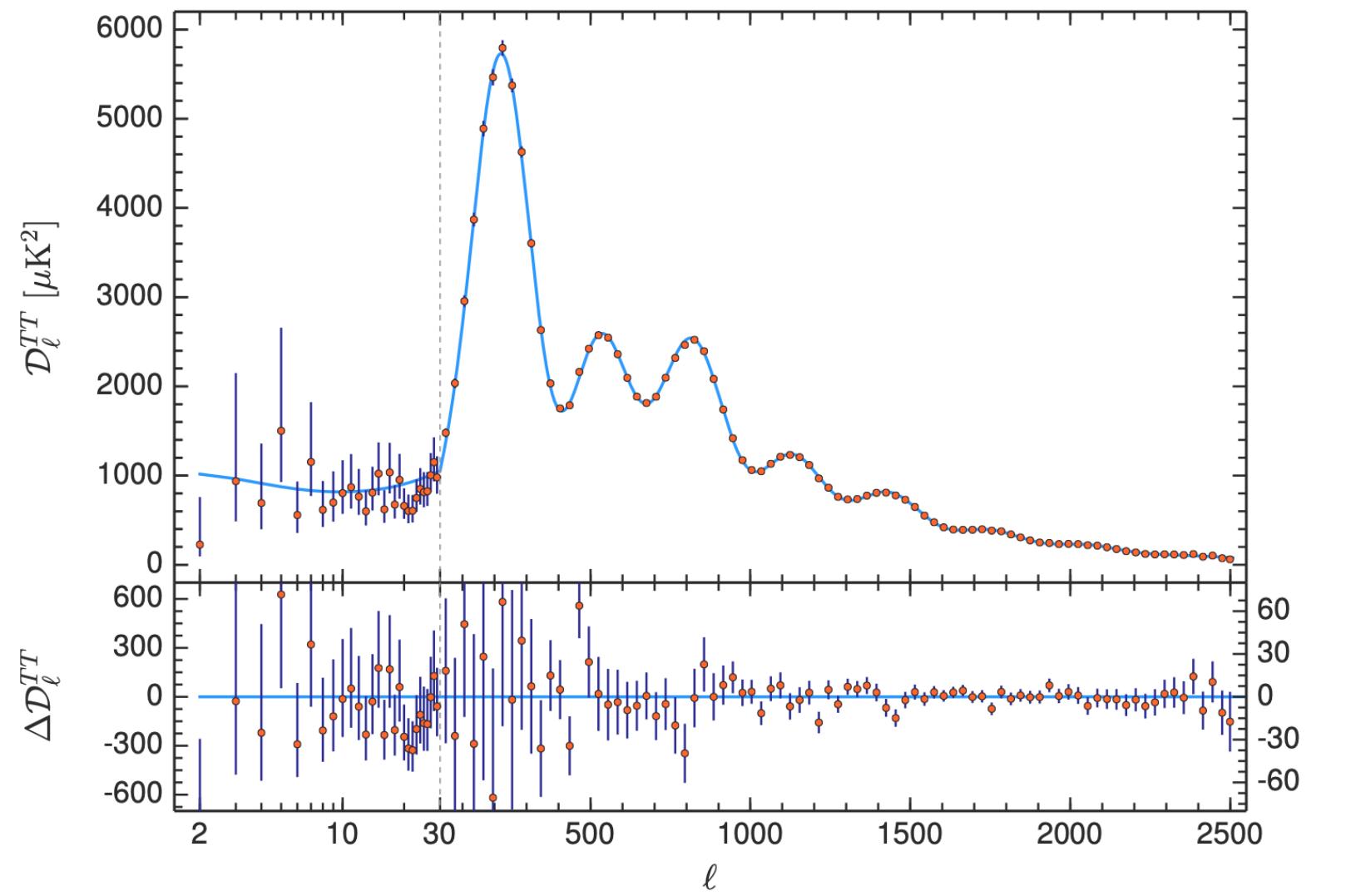


- To predict the CMB power spectrum: one needs to solve full fluid-equations (the Boltzmann equations) for all components of the universe
- This can be done with cosmological Boltzmann solvers like **CAMB** (Lewis&Bridle 2002) and **CLASS** (Blas, Lesgourgues, Tram 2011)
- These Boltzmann solvers take as input the  $\Lambda$ CDM parameters and can be sampled within MCMC samplers

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# The CMB

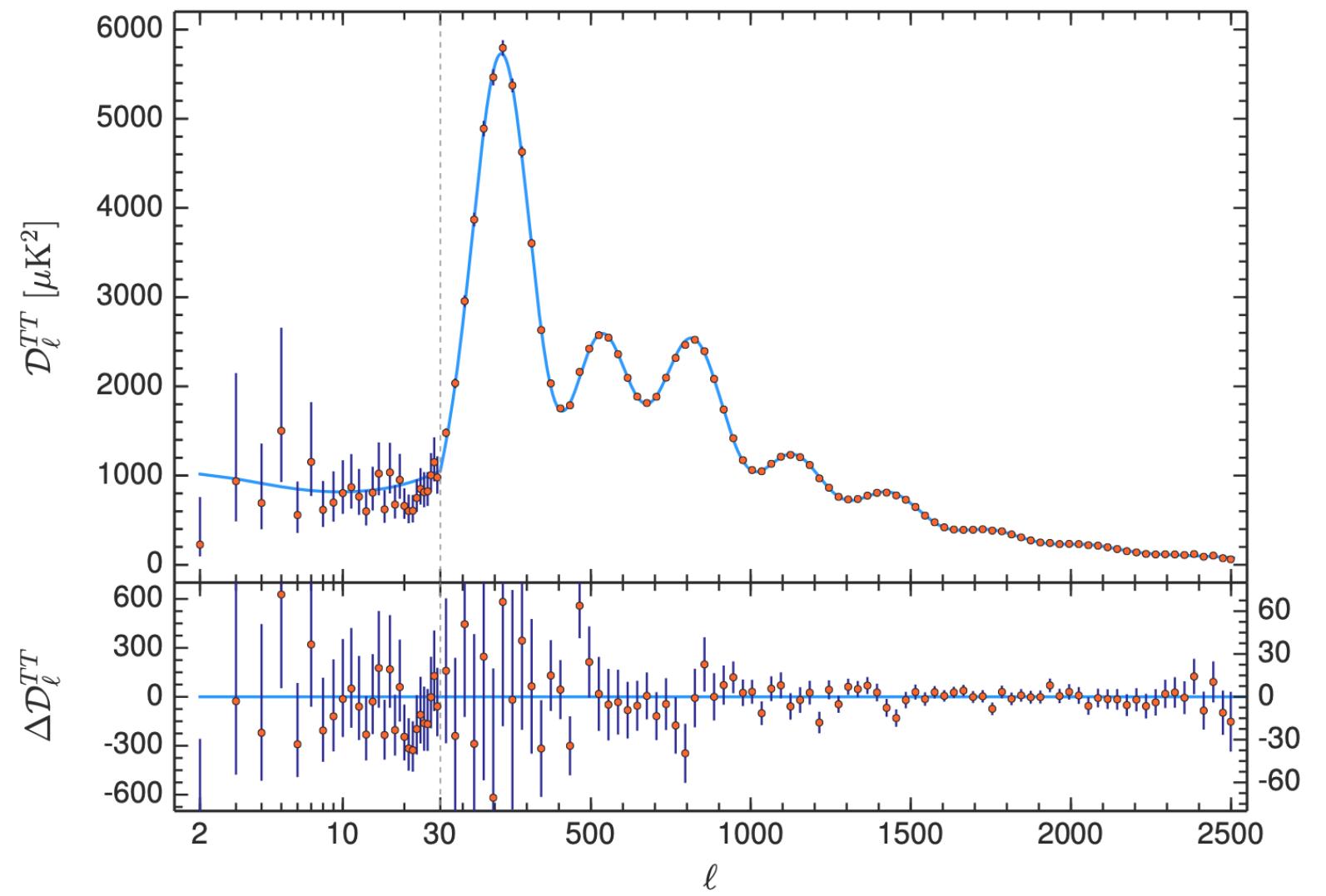
$\Lambda$ CDM model: 6 parameters



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# The CMB

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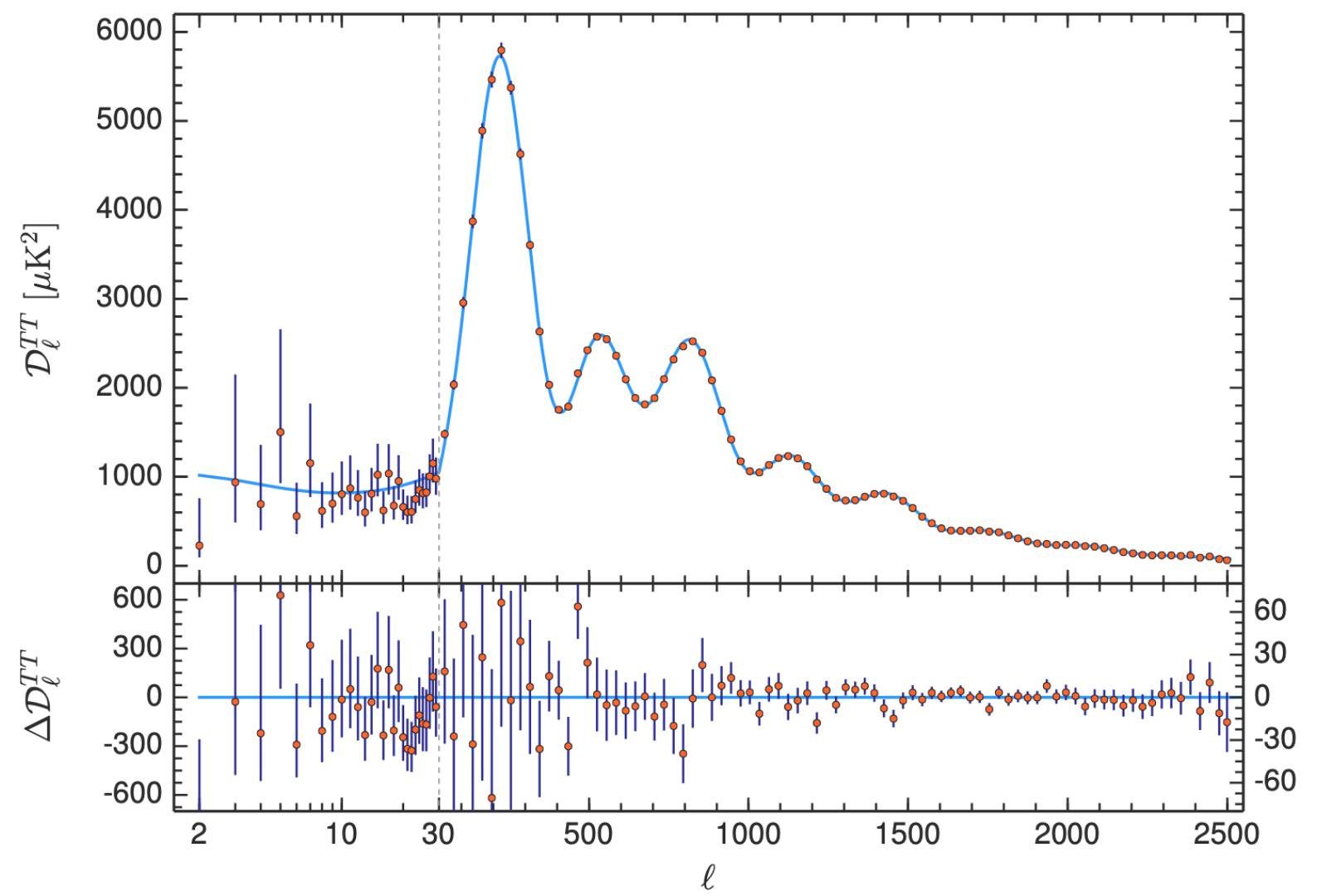


- $\omega_{\text{cdm}} = h^2 \Omega_{\text{cdm}}$ :
- $\omega_b = h^2 \Omega_b$ :
- $h = H_0 / (100 \text{ km/s/Mpc})$ :
- $\tau_{\text{reio}}$ :
- $A_s$ :
- $n_s$ :

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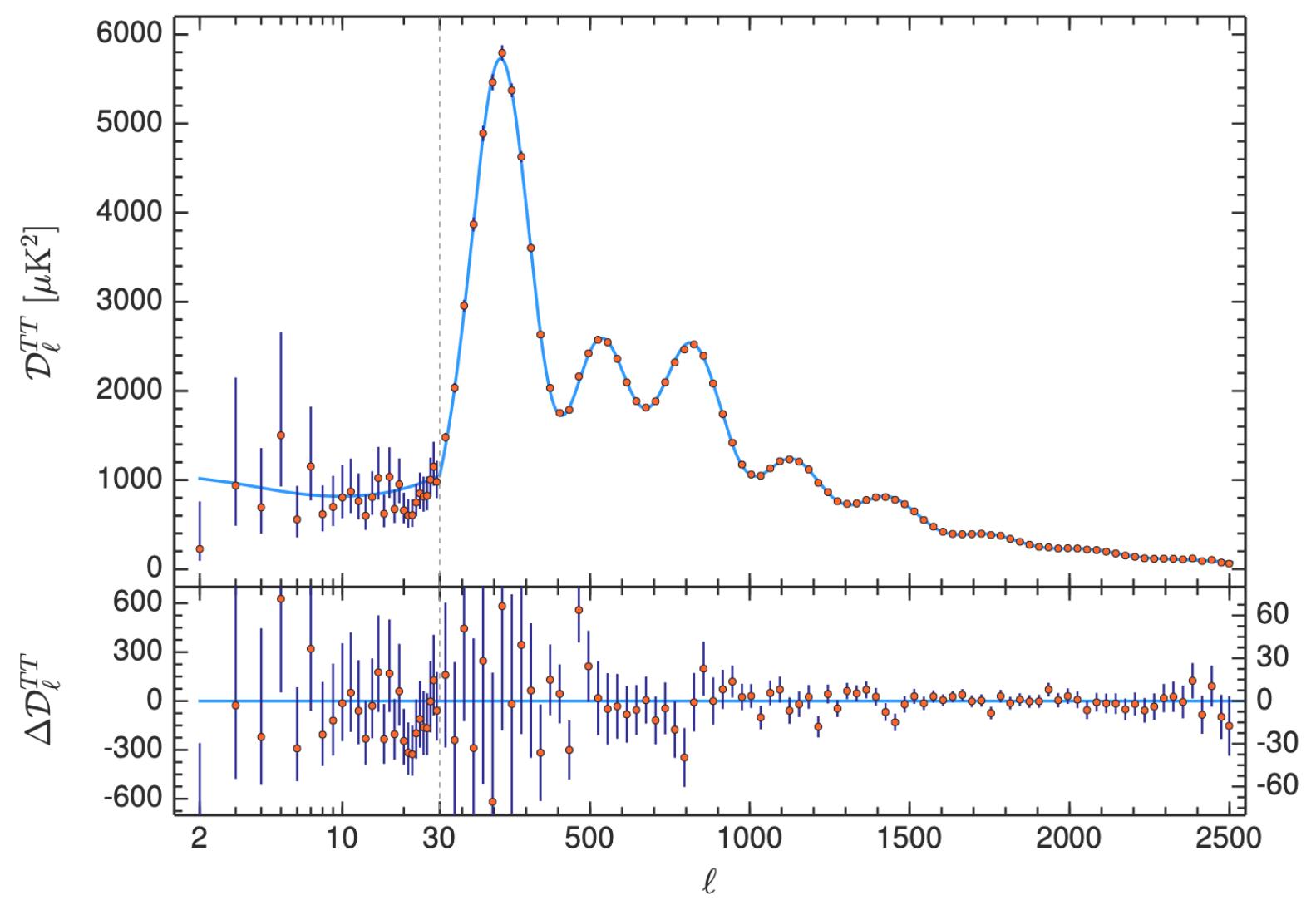


- $\omega_{\text{cdm}} = h^2 \Omega_{\text{cdm}}$ : physical energy density in CDM
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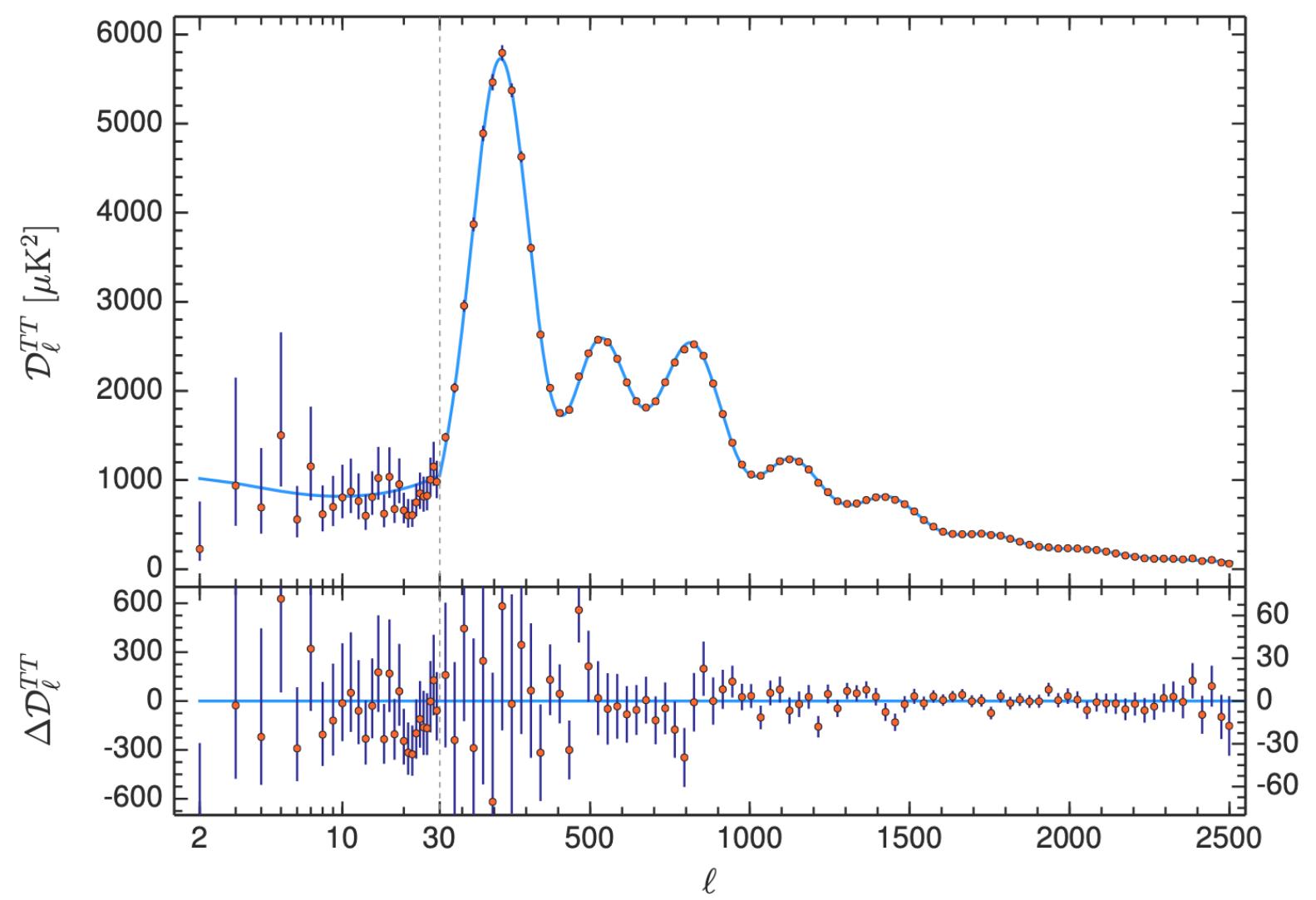


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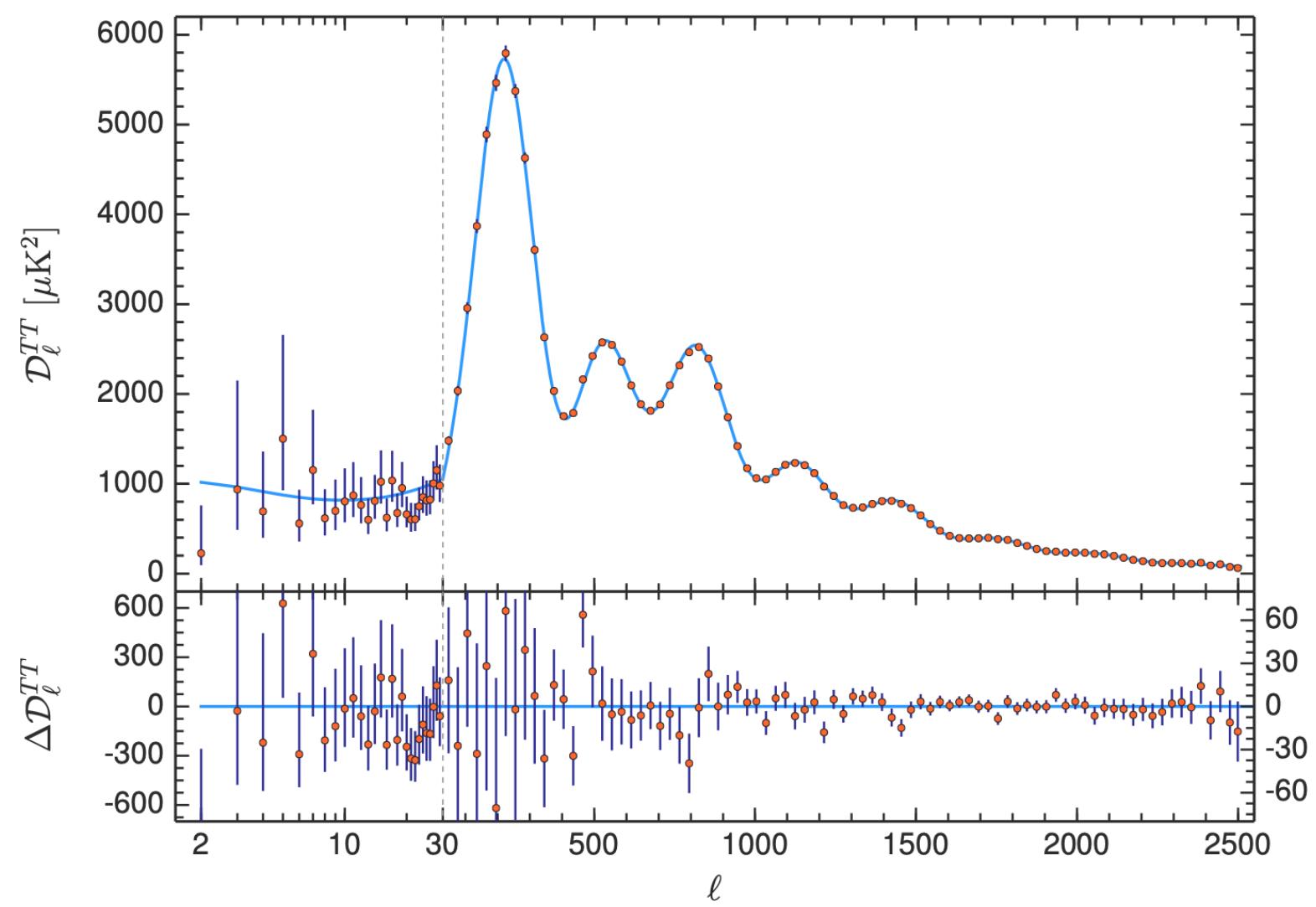


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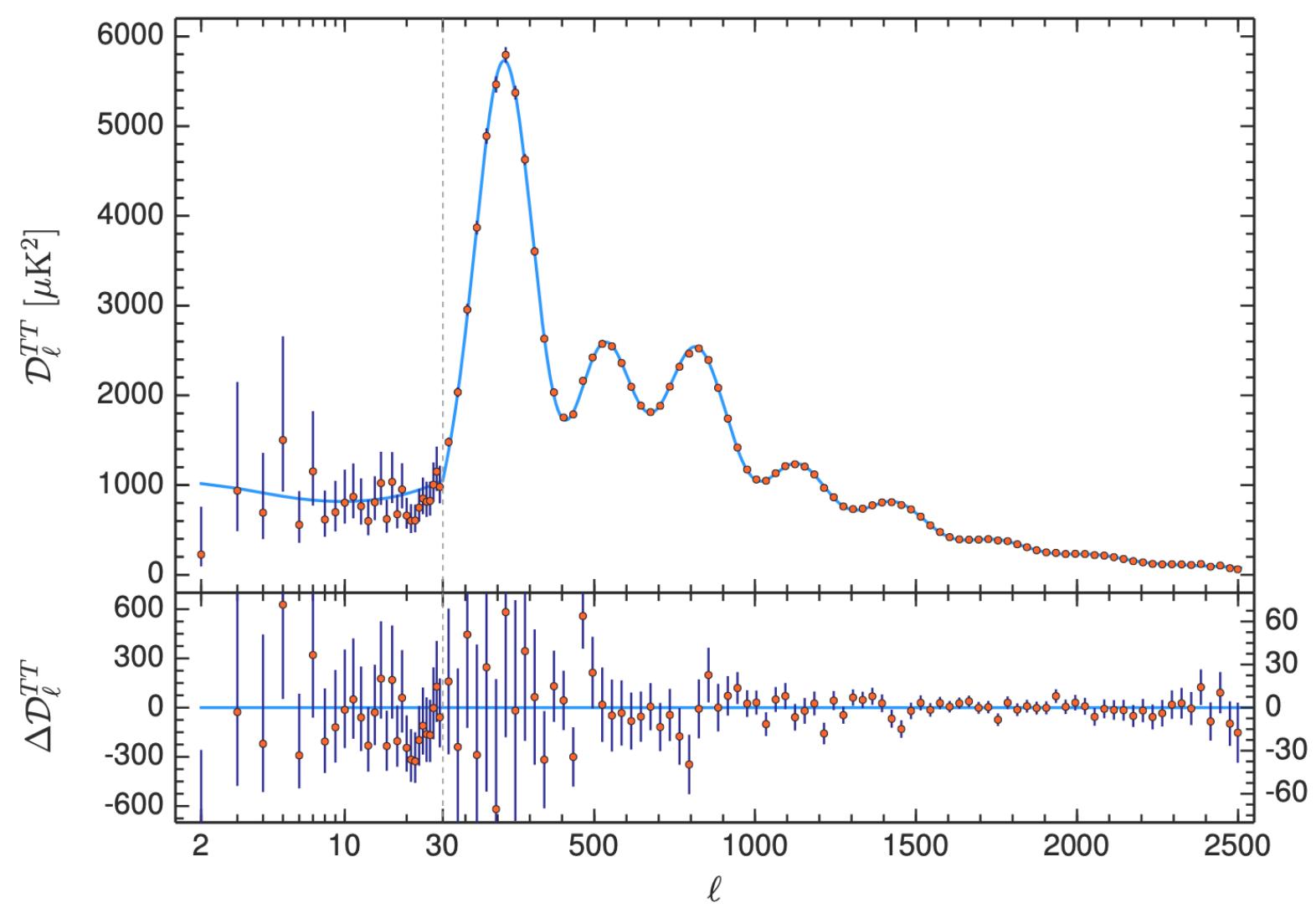


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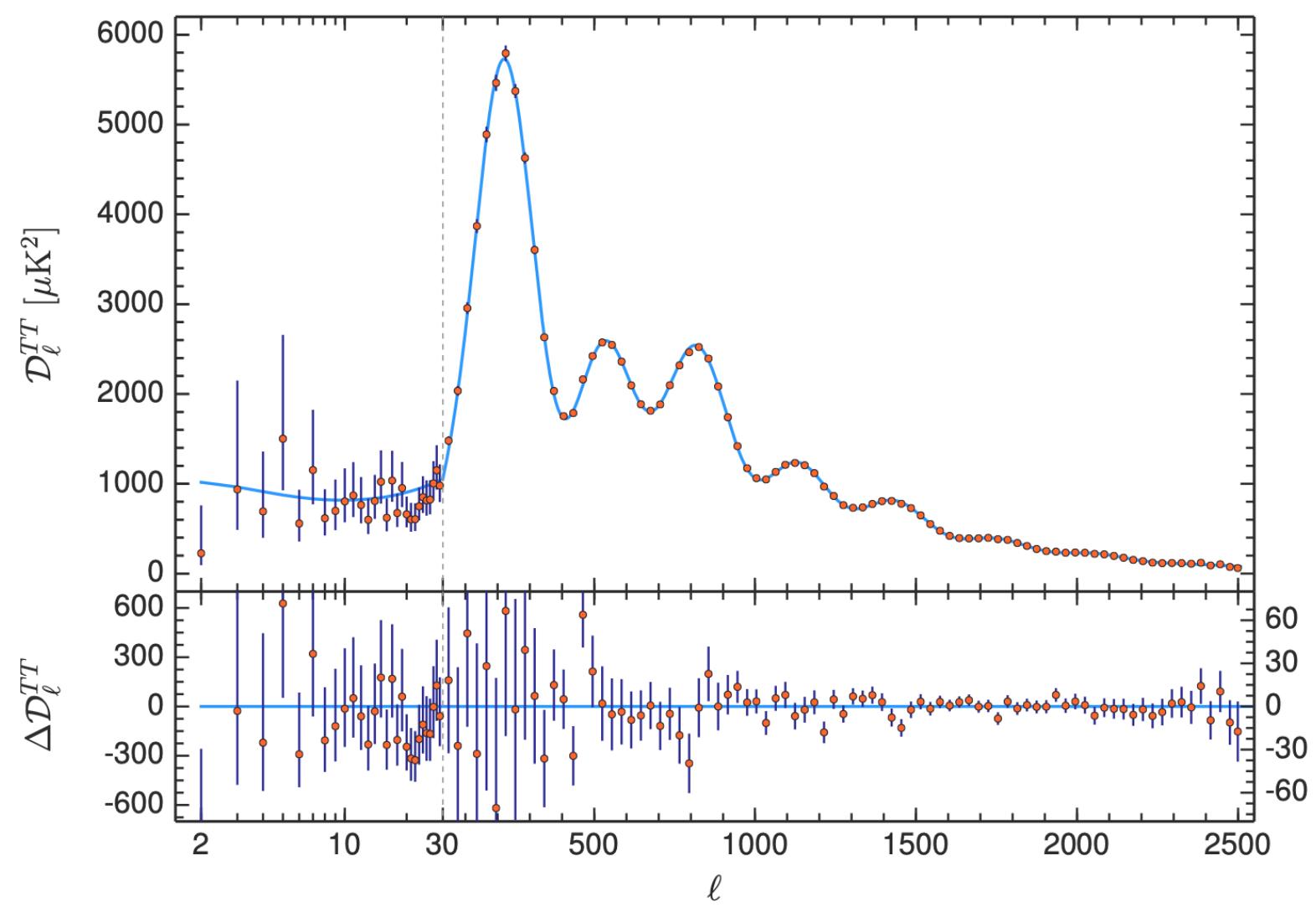


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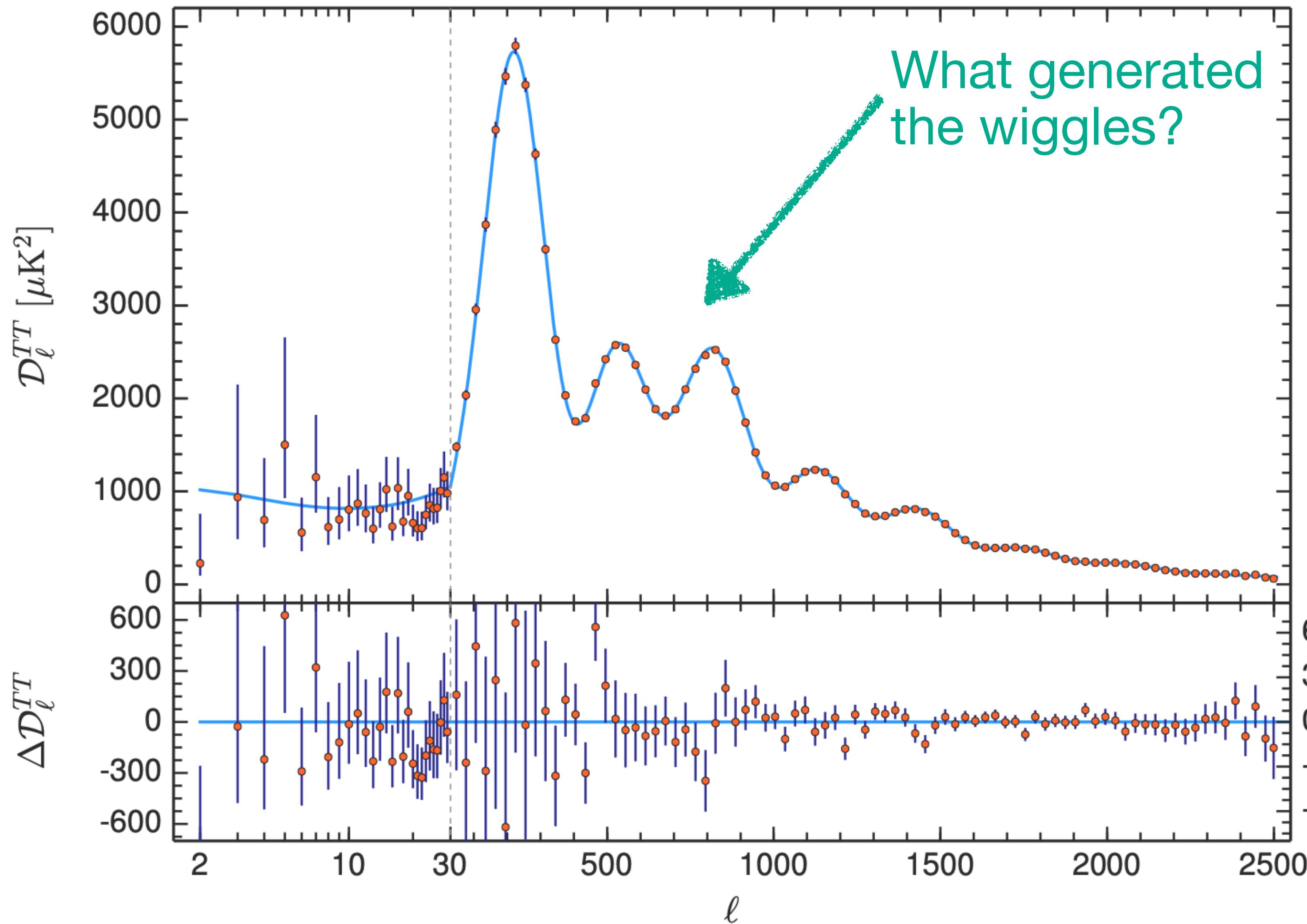


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- $n_s$ : tilt of the primordial power spectrum

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# The CMB

Planck Collaboration



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# The baryon acoustic oscillations (BAO)

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- Like rain drops on the water surface many waves overlap
- Once electrons and protons recombine, photons can travel freely → the density waves freeze



Credit: Mabel Amber



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(when protons and electrons combine into atoms):  $r_s = \int_0^{t^*} \frac{c_s(t)}{a(t)} dt$

- While the sound waves travel, the universe expands. This slows down the sound waves in the comoving coordinate frame

# The baryon acoustic oscillations (BAO)

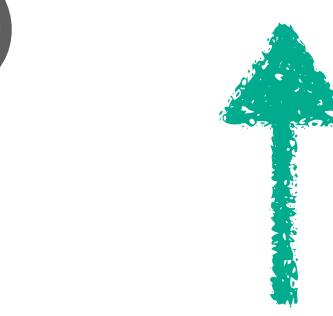
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$$\frac{da}{dt} = \dot{a}$$

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$$\frac{dz}{da} = \frac{d(1/a)}{da} = -\frac{1}{a^2}$$

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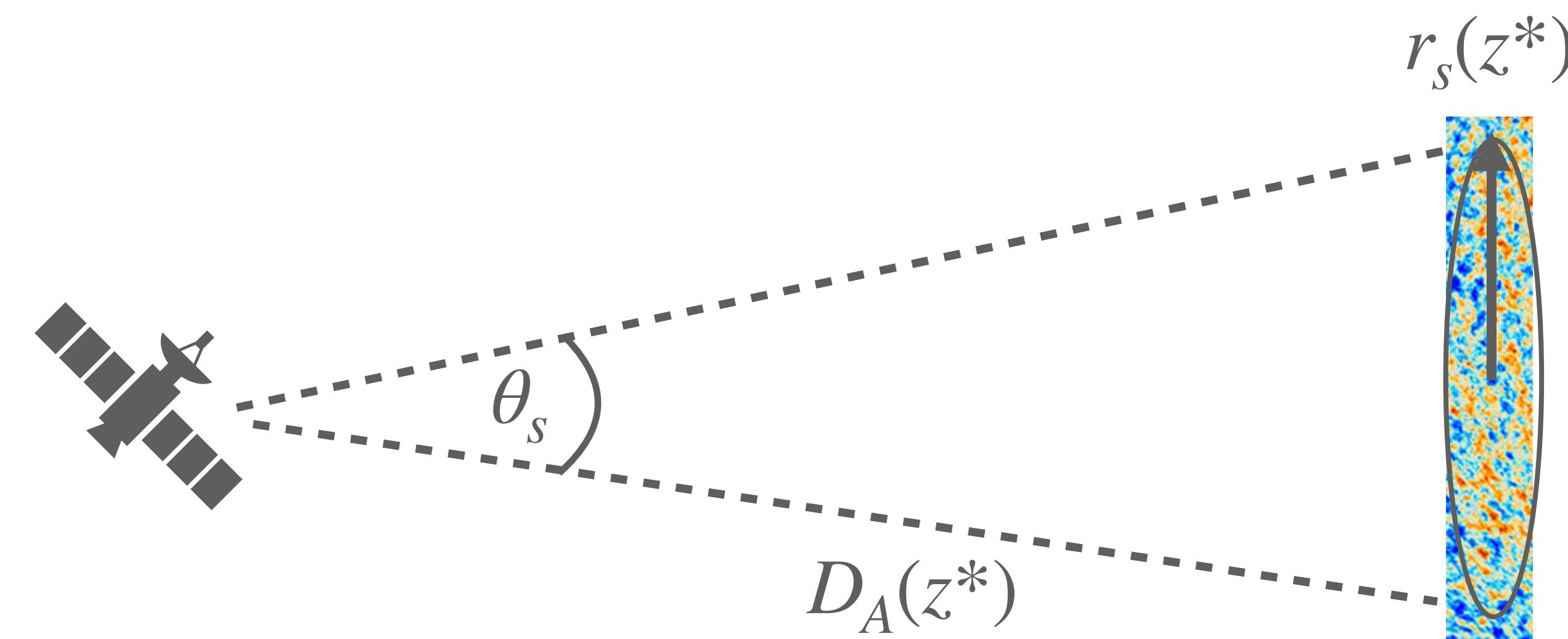
- The sound speed can be expressed as (e.g. Dodelson&Schmidt, 2020):

$$c_s(z) = \frac{1}{\sqrt{3(1 + R(z))}}$$

- With  $R$  being the baryon-photon ratio:

$$R(z) = \frac{3\omega_b}{4\omega_\gamma} \frac{1}{1+z}$$

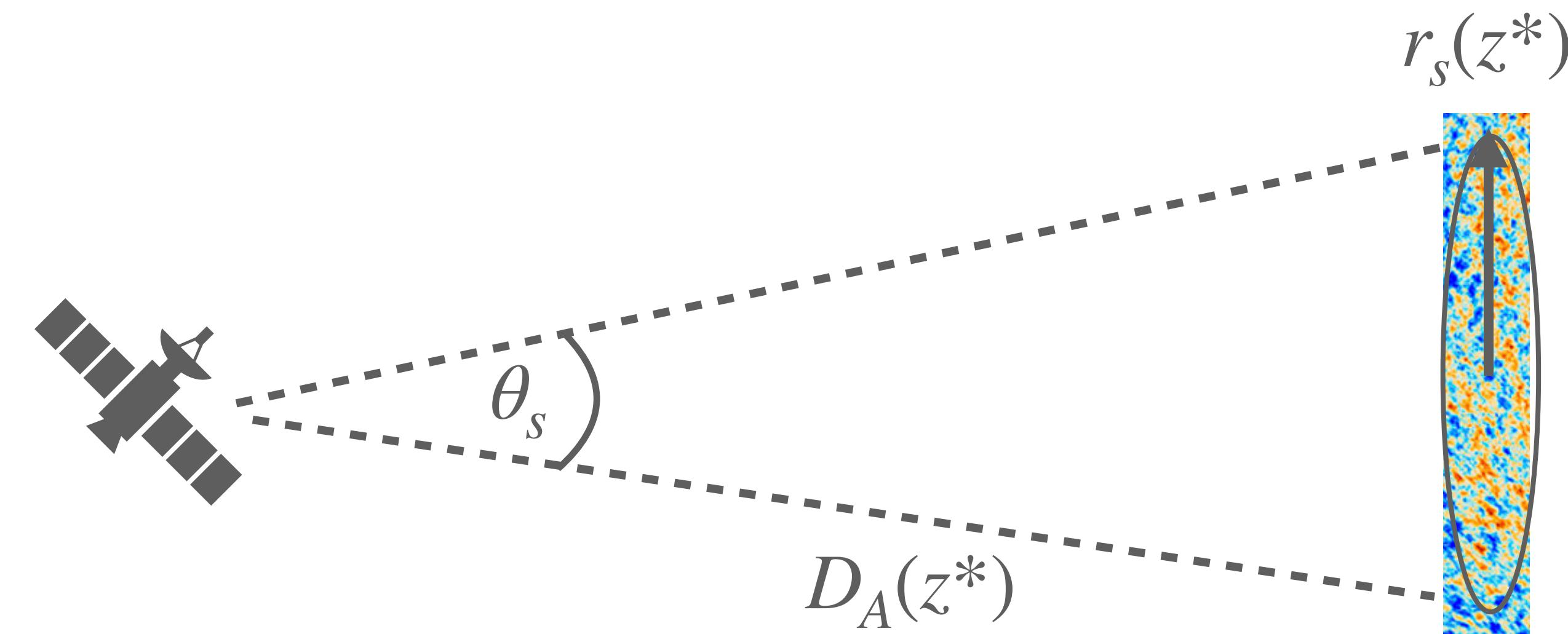
# The baryon acoustic oscillations (BAO)



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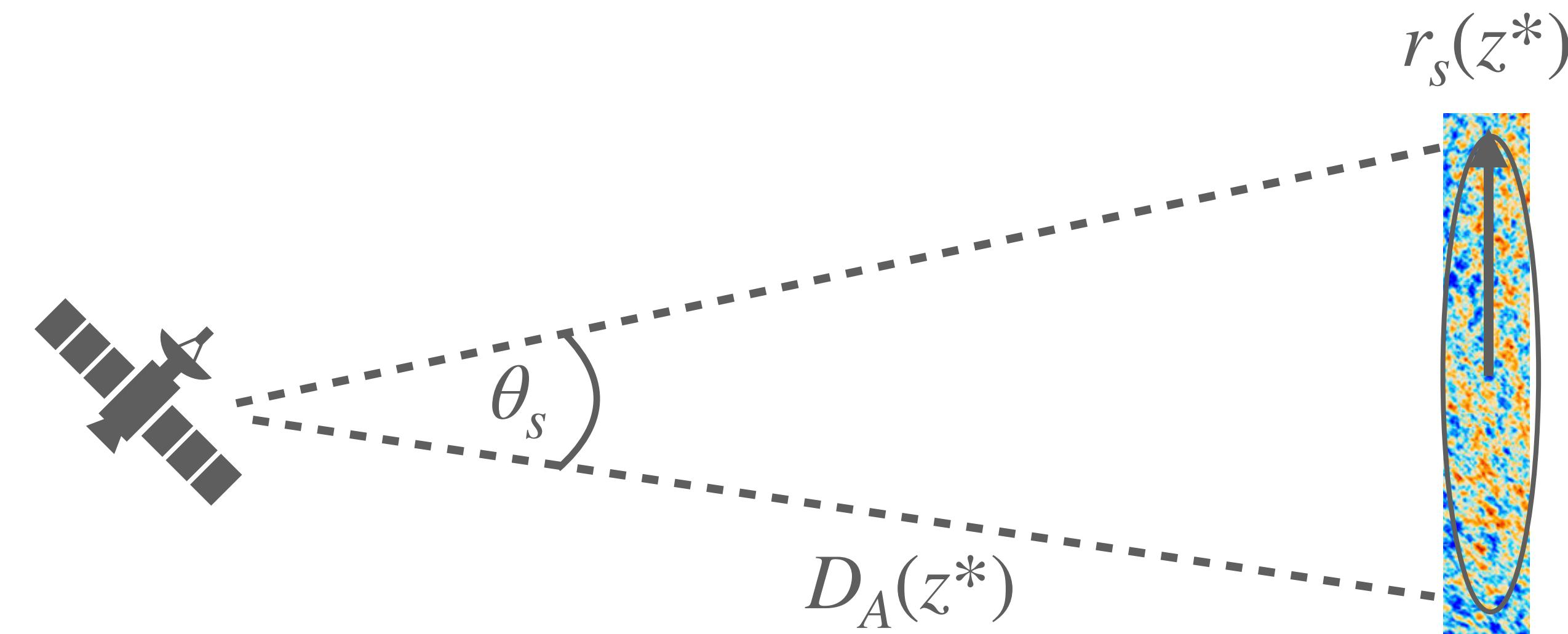


- What we observe is the *angular size of the sound horizon*  $\theta_s$
- In the small-angle approximation:

$$\theta_s = \frac{r_s^{\text{phys}}(z^*)}{D_A(z^*)}$$

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- What we observe is the *angular size of the sound horizon*  $\theta_s$

- In the small-angle approximation:

$$\theta_s = \frac{r_s^{\text{phys}}(z^*)}{D_A(z^*)}$$

- where the angular diameter distance is (see slide earlier):

$$D_A(t) = \int_0^{z(t)} \frac{dz'}{H(z')}$$

$$r_s^{\text{phys}}(z^*) = a(z^*) r_s(z^*)$$

# How does the CMB constrain $H_0$ ?

- We know that the CMB directly constrains

$$\theta_s = \frac{r_s^{\text{phys}}(z^*)}{D_A(z^*)} = \frac{\int_{z^*}^{\infty} c_s(z) dz / H(z)}{\int_0^{z^*} dz / H(z)}$$

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- If we get  $\Omega_r$ ,  $\Omega_m$ ,  $\Omega_\Lambda$  from somewhere else, this becomes an implicit equation for  $H_0$  – How does the CMB constrain  $\Omega_r$ ,  $\Omega_m$ ,  $\Omega_\Lambda$ ?

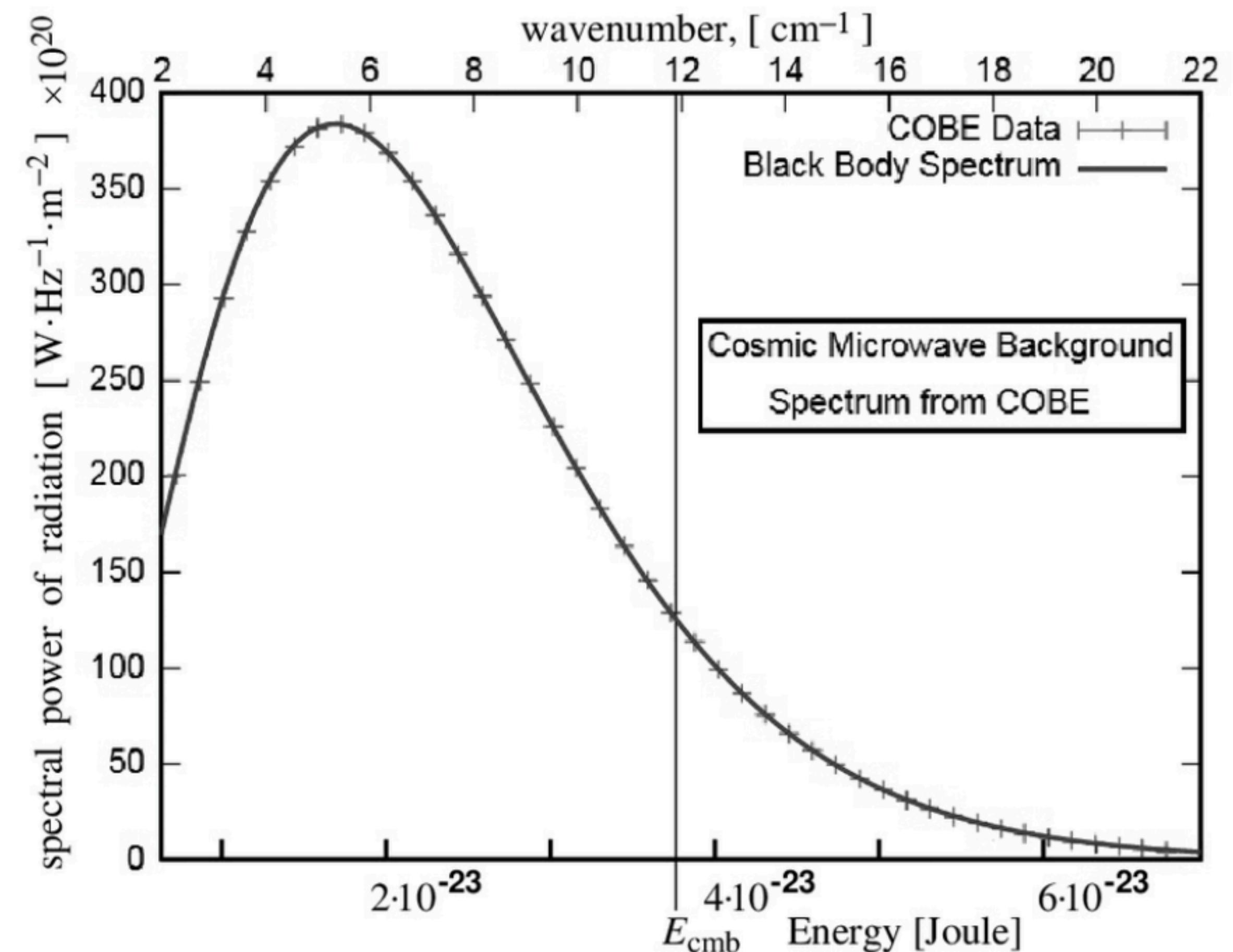
# How does the CMB constrain $\Omega_r$ ?

- The radiation density  $\Omega_r$  is precisely measured by the CMB temperature

$$T = 2.725 \pm 0.002 \text{ K} \text{ (COBE - FIRAS measurement)}$$



Credit: Berkeley Center for Cosmological Physics



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- For a black-body spectrum, the energy density in radiation is then given by (e.g. Weinberg 2008, Ch. 2.1):

$$\rho_{0,\text{CMB}} = \int_0^{\infty} h\nu \cdot n(\nu) d\nu = \frac{8\pi^5 k_B^4}{15h^3 c^3} T^4 = 4.64 \cdot 10^{-34} \text{ g/cm}^3$$

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- Taking into account the earlier time of decoupling of neutrinos, one finds:

$$\rho_\nu = 0.4 \rho_{0,\text{CMB}}$$

- Since the CMB and neutrinos are by far the dominant contribution:

$$\Omega_r = 1.4 \rho_{0,\text{CMB}} / \rho_{\text{crit}} = 4.15 \cdot 10^{-5} h^{-2}$$

For more about neutrinos, see Olga Mena's lecture

# How does the CMB constrain $\Omega_m$ ?

- It does not constrain  $\Omega_m$ , but  $\omega_m = h^2\Omega_m$ , where  $\omega_m = \omega_{\text{cdm}} + \omega_b$

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- It does not constrain  $\Omega_m$ , but  $\omega_m = h^2 \Omega_m$ , where  $\omega_m = \omega_{\text{cdm}} + \omega_b$
- $\omega_m$  is constrained by the height of the acoustic peaks:
  - a smaller  $\omega_m$  leads to a later time of matter-radiation equality
  - this leads to a stronger decay of the gravitational potentials at recombination → **early integrated Sachs-Wolfe effect (eISW)**
  - Since the eISW adds in phase with the BAO, this leads to a boost of all peaks, particularly the first peak.

# How does the CMB constrain $\Omega_\Lambda$ ?

- The sum of the energy densities has to satisfy:

$$\Omega_r + \Omega_m + \Omega_\Lambda = \Omega_k$$

- For a flat universe:

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1$$

- Hence, if we know  $\Omega_r$  and  $\omega_m = h^2\Omega_m$ , we can compute

$$\Omega_\Lambda = 1 - \Omega_r - \omega_m/h^2$$

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- Since we can constrain  $\Omega_r$  and  $\omega_m$  independently, this becomes an implicit equation for  $h$

$$\frac{H^2(z)}{(100 \text{ km/s/Mpc})^2} = h^2 \Omega_r (1+z)^4 + \omega_m (1+z)^3 + (h^2 - \omega_m - h^2 \Omega_r)$$

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\*  $c_s(z)$  and  $z^*$  also depend on cosmology but we neglect that here

- Cosmology enters mainly over the Hubble parameter here:

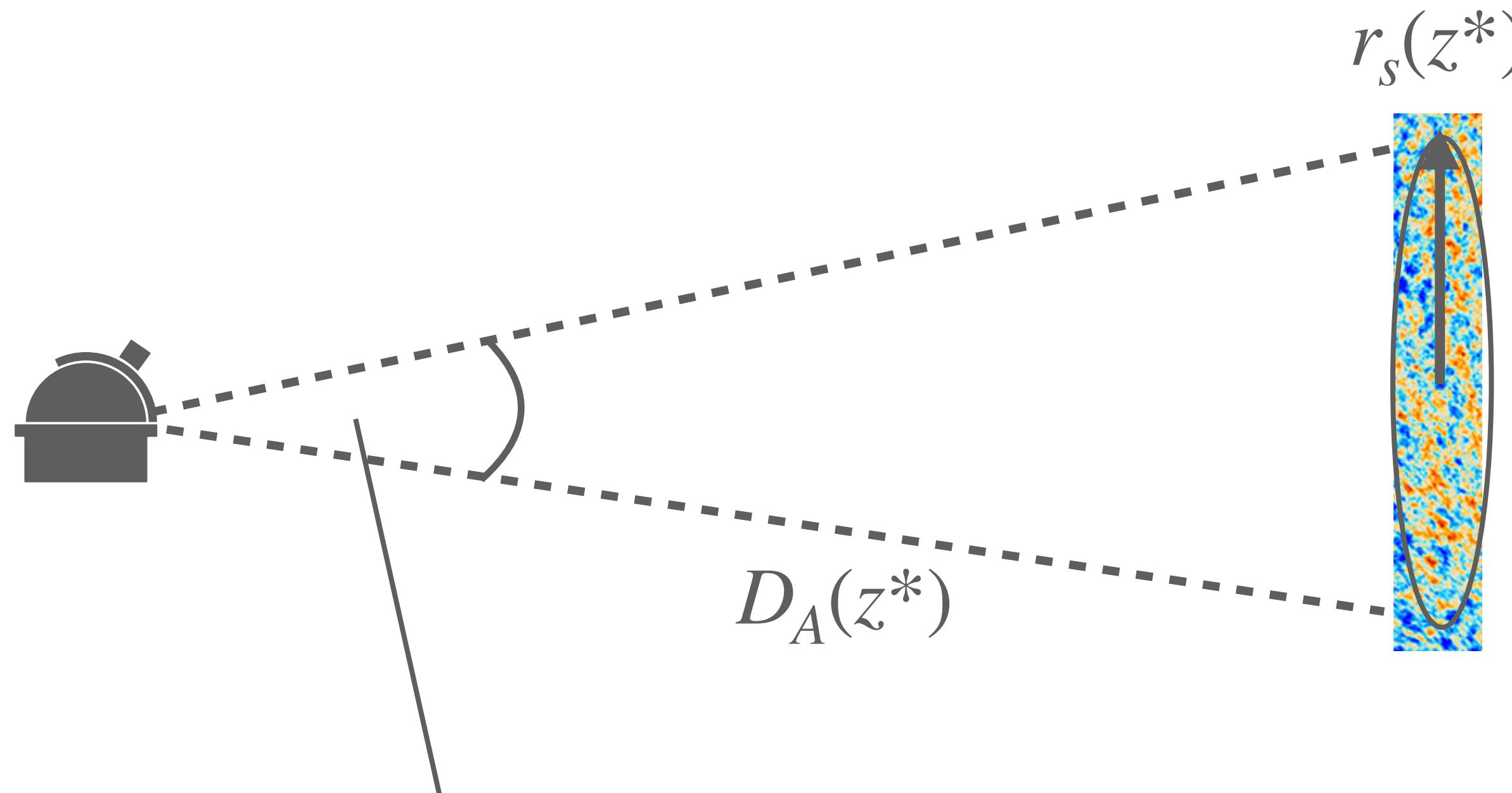
$$H(z) = H_0 \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$

- Since we can constrain  $\Omega_r$  and  $\omega_m$  independently, this becomes an implicit equation for  $h$

$$\frac{H^2(z)}{(100 \text{ km/s/Mpc})^2} = h^2 \Omega_r (1+z)^4 + \omega_m (1+z)^3 + (h^2 - \omega_m - h^2 \Omega_r)$$

# Solutions to the Hubble tension

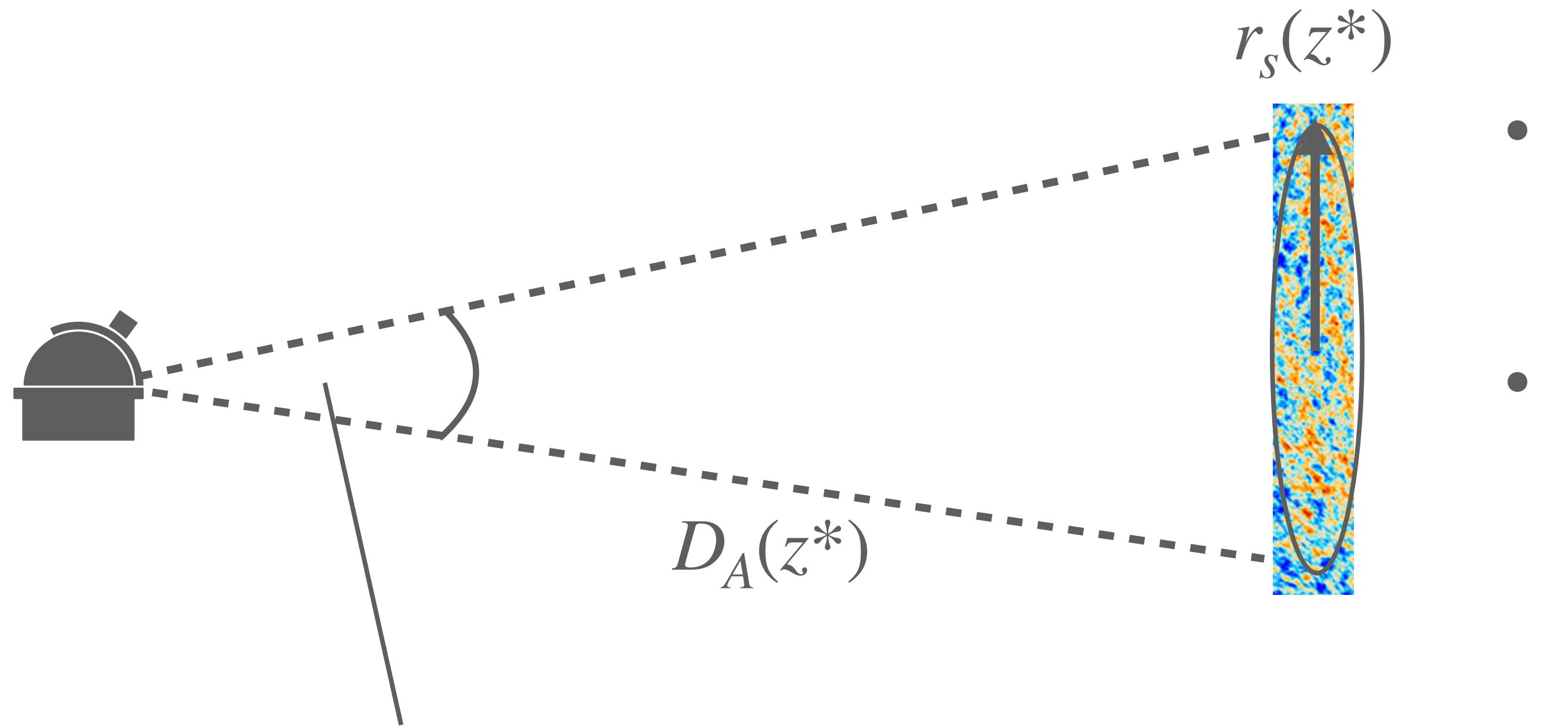
# Solutions to the Hubble tension



- $\theta_s$  is measured precisely by CMB  
→  $\theta_s$  fixed
- Two options to solve the Hubble tension:

$$\theta_s = \frac{r_s(z^*)}{D_A(z^*)} = \frac{\int_{z^*}^{\infty} c_s(z) dz / H(z)}{\int_0^{z^*} dz / H(z)}$$

# Solutions to the Hubble tension

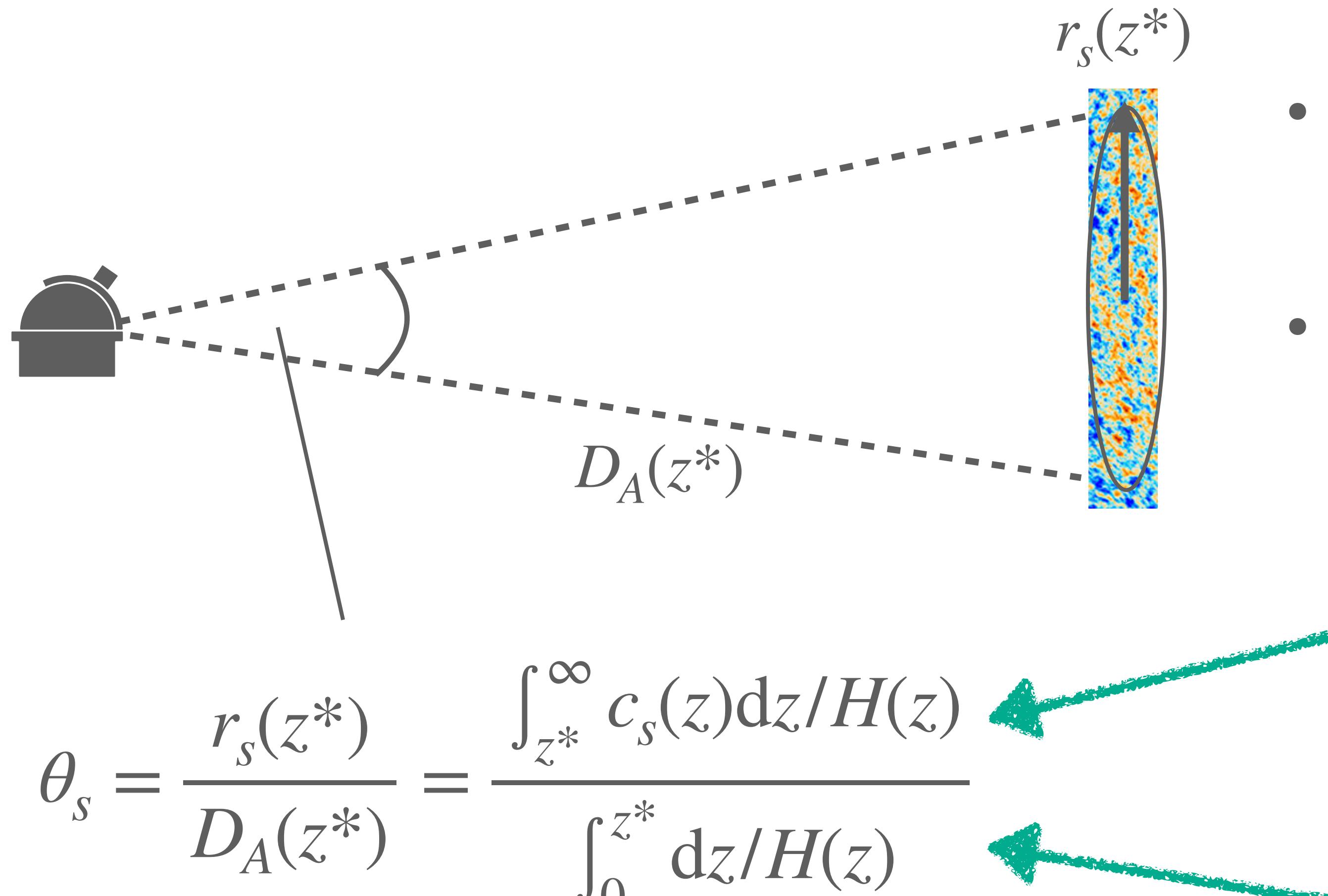


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- Two options to solve the Hubble tension:

Early-time solutions  
modify  $r_s$

# Solutions to the Hubble tension



- $\theta_s$  is measured precisely by CMB  
→  $\theta_s$  fixed
- Two options to solve the Hubble tension:

Early-time solutions  
modify  $r_s$

Late-time solutions  
modify  $D_A$

# Late-time solutions

- Modify  $D_A(z^*) = \int_0^{z^*} \frac{dz}{H(z)}$  by modifying the expansion rate between today (0) and recombination  $z^*$  (since  $\Omega_r \approx 0$  at times after  $z^*$ ):

$$H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}$$

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- However,  $H(z)$  is well constrained by galaxy BAO data and supernova data  
→ It is challenging to increase  $H_0$  enough
- But there are some promising approaches (see other lectures)

# Early-time solutions

- Modify  $r_s = \int_{z^*}^{\infty} \frac{c_s(z) dz}{H(z)} \rightarrow$  3 options:
  - ▶ modify  $z^*$
  - ▶ modify  $c_s(z)$
  - ▶ modify  $H(z)$

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- Modify  $r_s = \int_{z^*}^{\infty} \frac{c_s(z) dz}{H(z)}$  → 3 options:

► **modify  $z^*$**

- modify  $c_s(z)$
- modify  $H(z)$



E.g. by modifying the mass of the electron  $m_e$ :

- This shifts the energy levels of the atoms,
- and changes the ionization energy,
- which changes the time of recombination  $z^*$

# Early-time solutions

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  - ▶ modify  $c_s(z)$
  - ▶ **modify  $H(z)$**  →

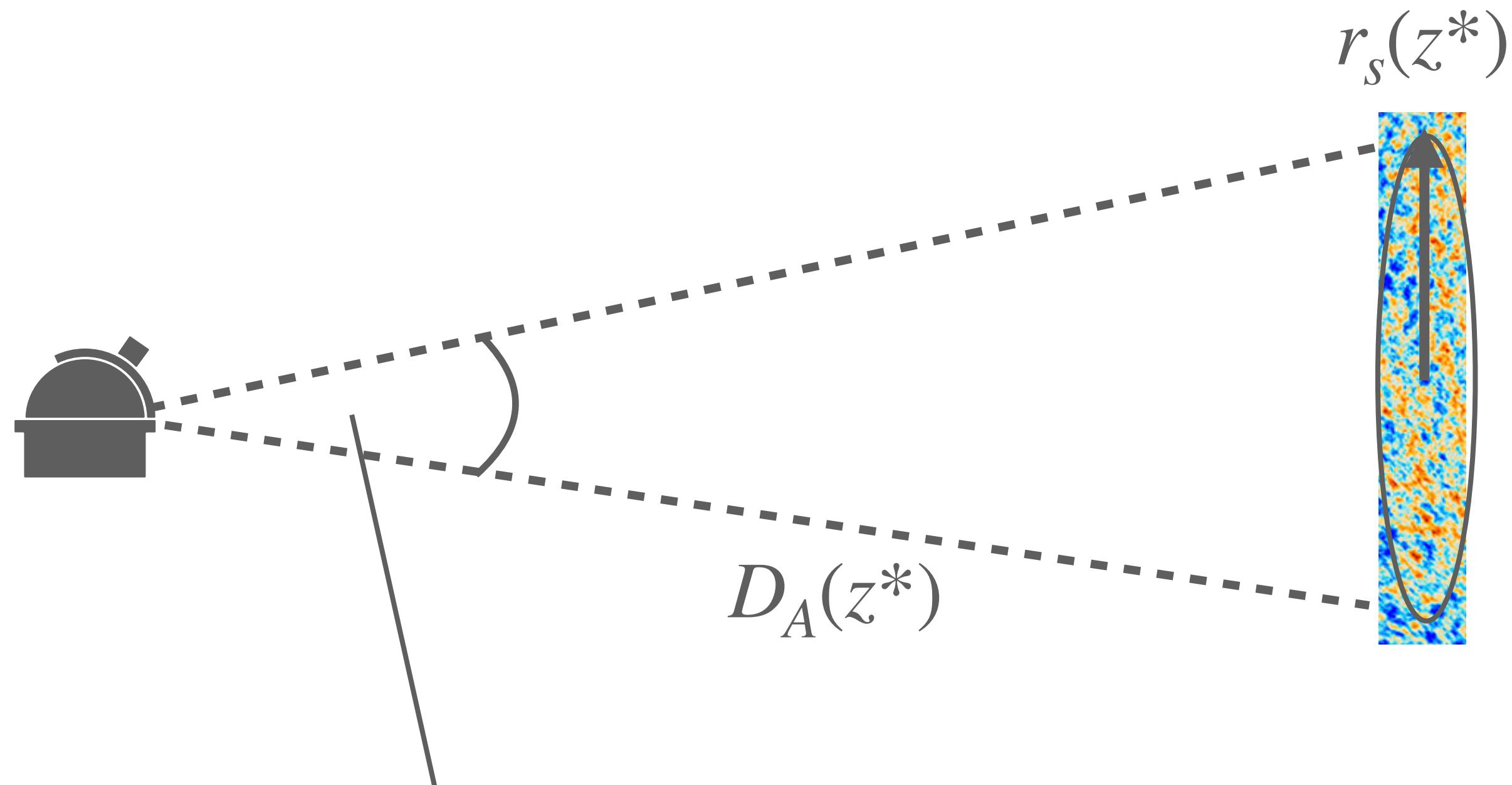
E.g. by introducing an energy density, which boosts the expansion rate before recombination  $z^*$  (at early times  $\Omega_\Lambda \approx 0$ ):

$$H(z) = H_0 \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_q}$$

This leads to a smaller  $r_s$ .

The most successful of such ideas is **EDE**

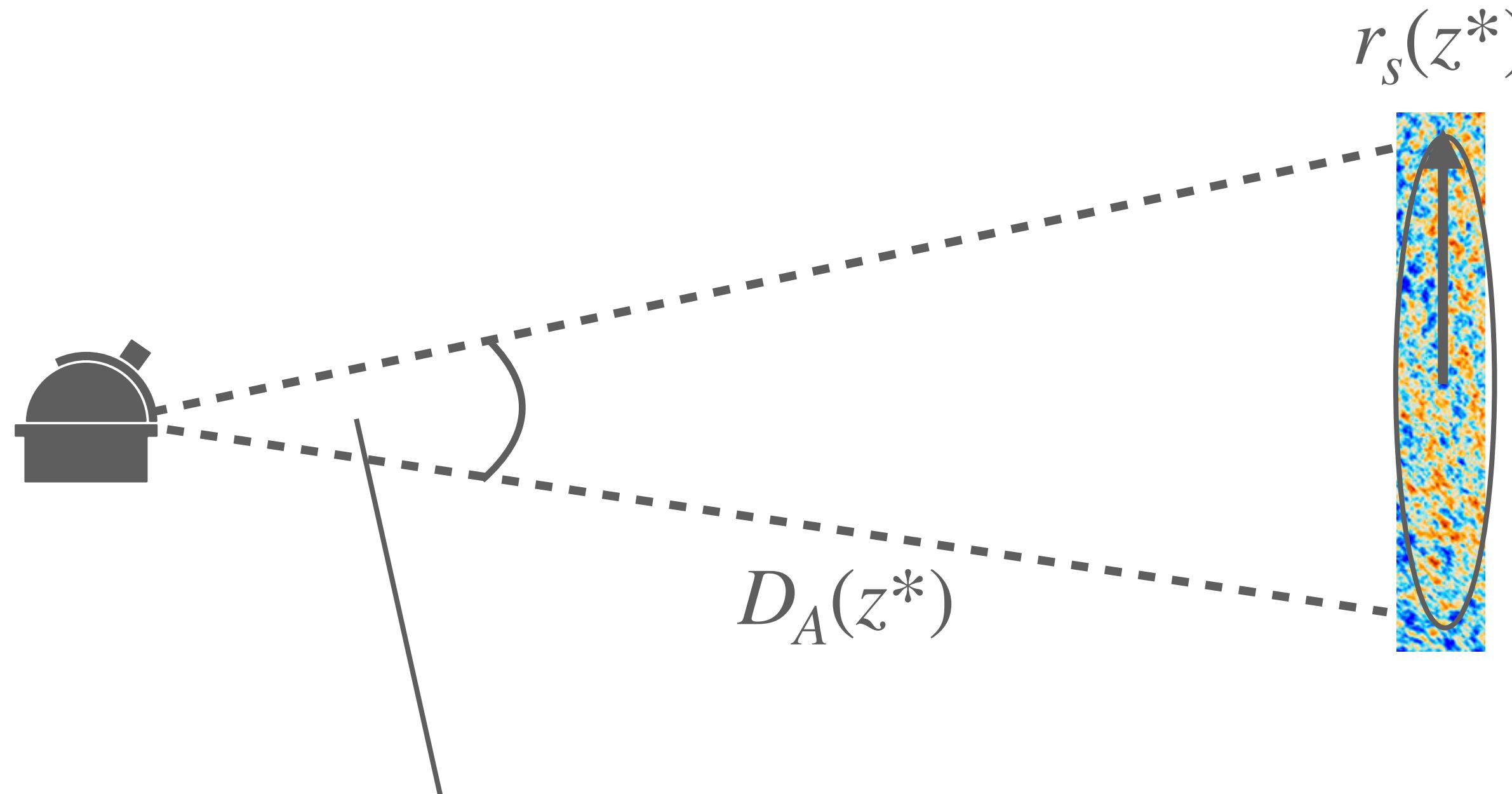
# Idea behind Early Dark Energy



Angular scale of sound horizon  $\theta_s$   
measured with 0.03% precision by *Planck*.

$$\theta_s = \frac{r_s(z^*)}{D_A(z^*)} = \frac{\int_{z^*}^{\infty} c_s(z) dz / H(z)}{\int_0^{z^*} dz / H(z)}$$

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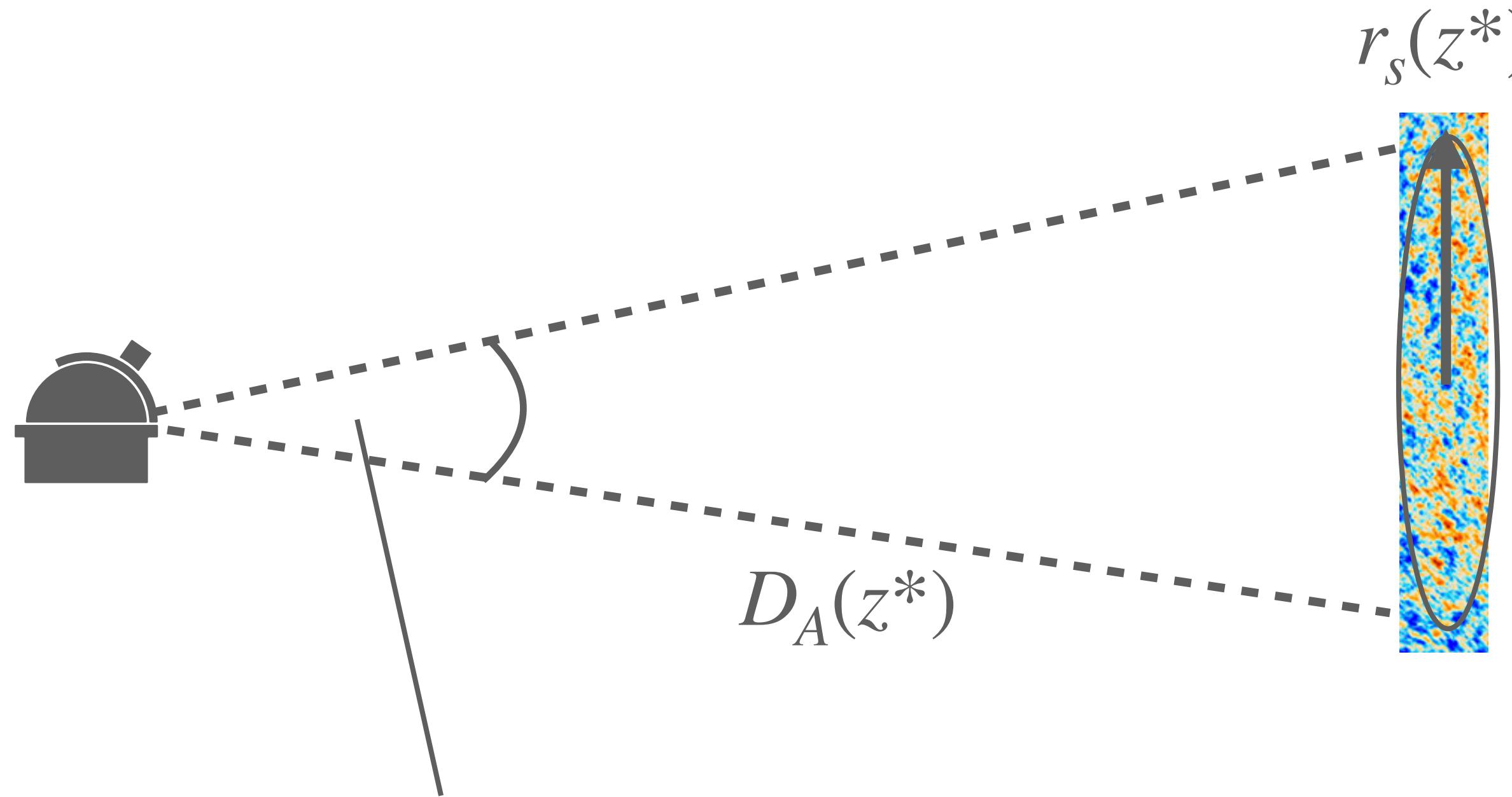


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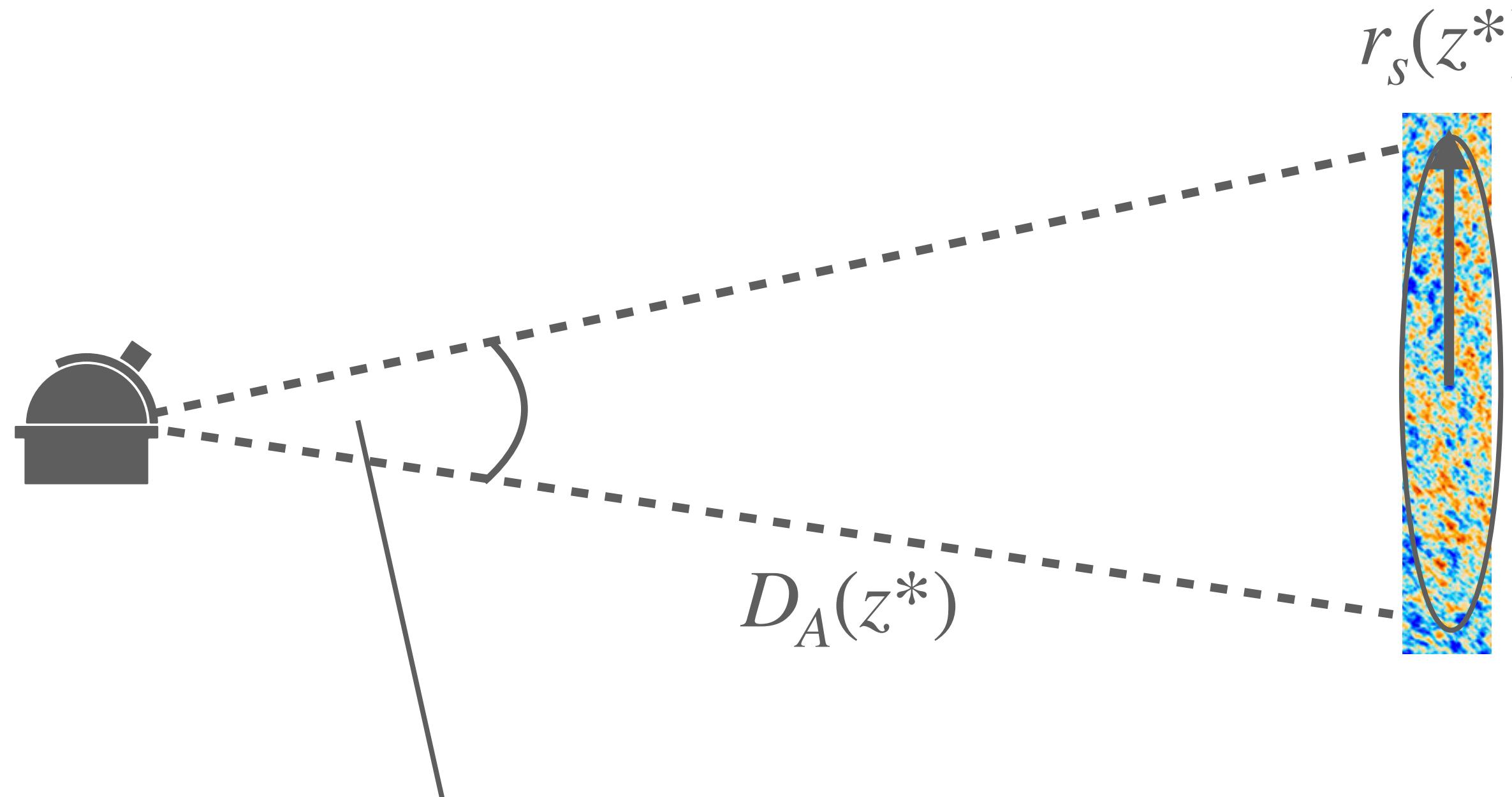
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Angular diameter distance  
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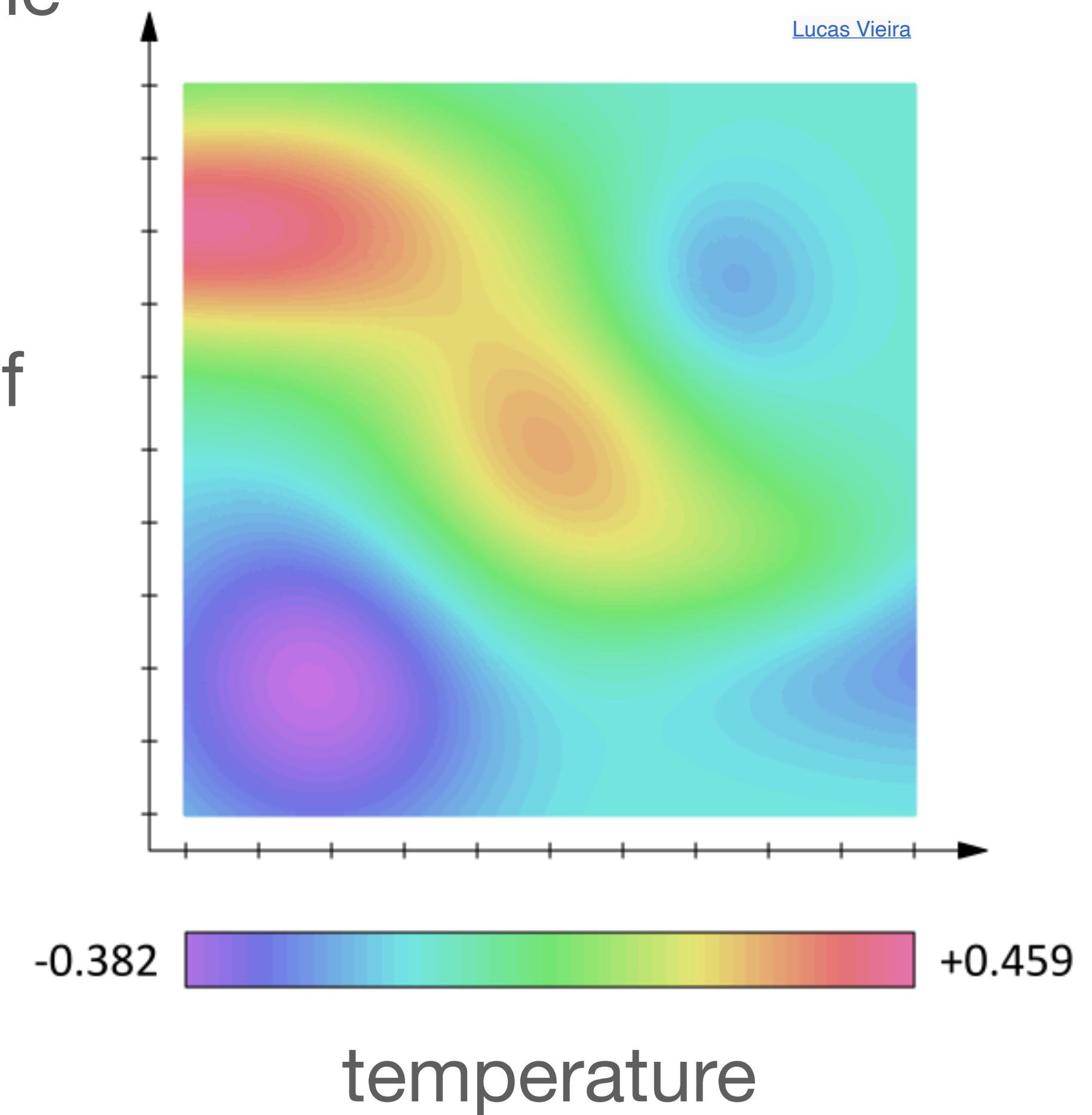
# Short history of Early Dark Energy

- EDE has already been studied before the Hubble tension emerged in the context of quintessence models (Doran++ 2001, Wetterich++ 2004, Doran&Robbers 2006, Kamionkowski++ 2014)
- The axion-like EDE was proposed as a solution to the Hubble tension (Karwal&Kamionkowski 2016, Poulin++ 2018, 2019)
- There are many versions of the EDE model, but we will focus on the (most commonly studied) axion-like EDE model
- The axion-like EDE model is modelled as a **scalar field**
- Here: Only background equations

# Scalar fields in an expanding spacetime

# Intuition

- A scalar field  $\phi(t, \vec{x})$  assigns each point in spacetime a single number
- Examples
  - Temperature, density, pressure,... as a function of position and time
  - Potential fields like the gravitational potential, electric potential
  - The inflaton driving inflation is most commonly a scalar field

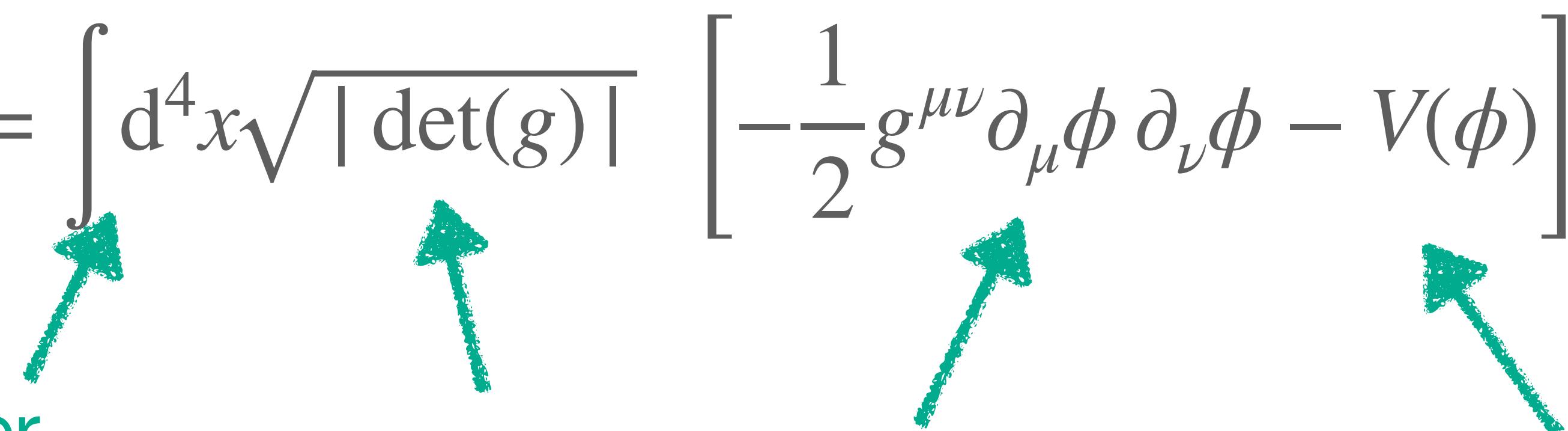


# Scalar field

- The action of a scalar field minimally coupled to the metric is:

$$\mathcal{S}_\phi = \int d^4x \sqrt{|\det(g)|} \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

integral over whole spacetime      determinant of metric      kinetic term      potential term



Very similar to inflation: see William Giare's lecture

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integral over whole spacetime      determinant of metric      kinetic term      potential term

- One can compute the energy momentum tensor of the scalar field via:

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left( \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - V(\phi) \right)$$

# Scalar field

- This gives for the energy-momentum tensor:

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- By comparing  $T_{\mu\nu}$  to the energy momentum tensor of the perfect fluid:  
 $T_{\mu\nu} = (\rho + p)\delta_\mu^0 \delta_\nu^0 + pg_{\mu\nu}$ , one can read off the energy density and pressure of the scalar field:

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi).$$

# Scalar field

- Inserting  $\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$ ,  $p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$  into the first Friedmann equation and the continuity equation yields:

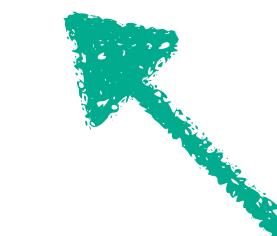
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$$H^2 = \frac{8\pi G_N}{3} \left( \frac{\dot{\phi}^2}{2} + V(\phi) \right)$$
$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$



“Hubble  
friction”



Potential  
term

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- The second equation is the **Klein-Gordon equation** for a scalar field in an expanding space, where the second term ( $3H\dot{\phi}$ ) is the Hubble-drag term and the third term ( $\frac{1}{2}dV/d\phi$ ) is the potential-gradient term

# Early Dark Energy

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$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$

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index  $n$ :

$n = 1$  “standard” axion; however: doesn’t decay quickly enough

$n = 3$ : is preferred value by the data (Poulin++ 2020)  
→ decays fast enough (commonly fixed in EDE data analysis)

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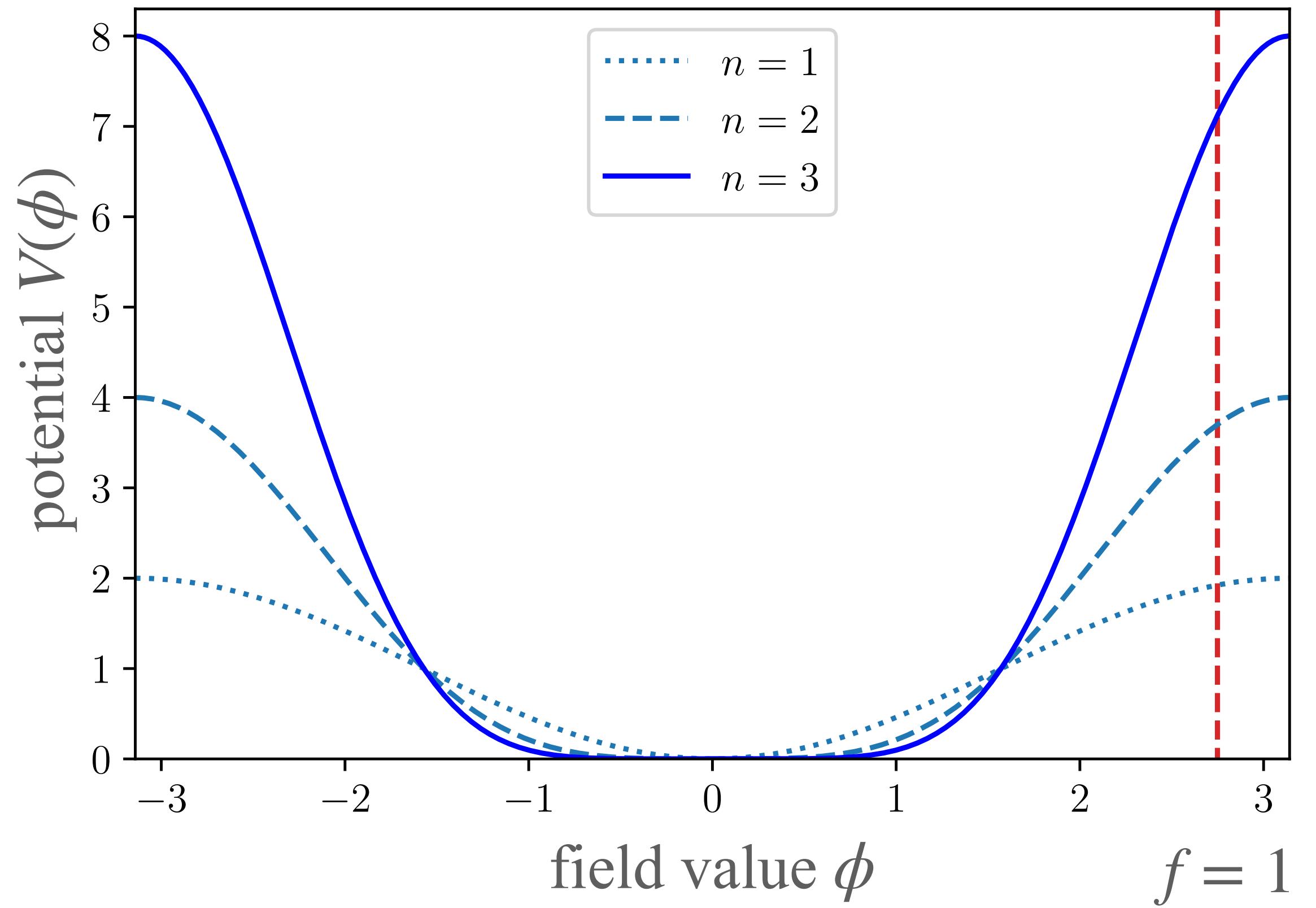
$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$

Free parameters of the model:

- $m$  mass ( $V_0 = m^2 f^2$ )
- $f$  “decay constant”
- $\theta_i = \phi_i/f$  initial value of the field
- ( $n = 3$ )

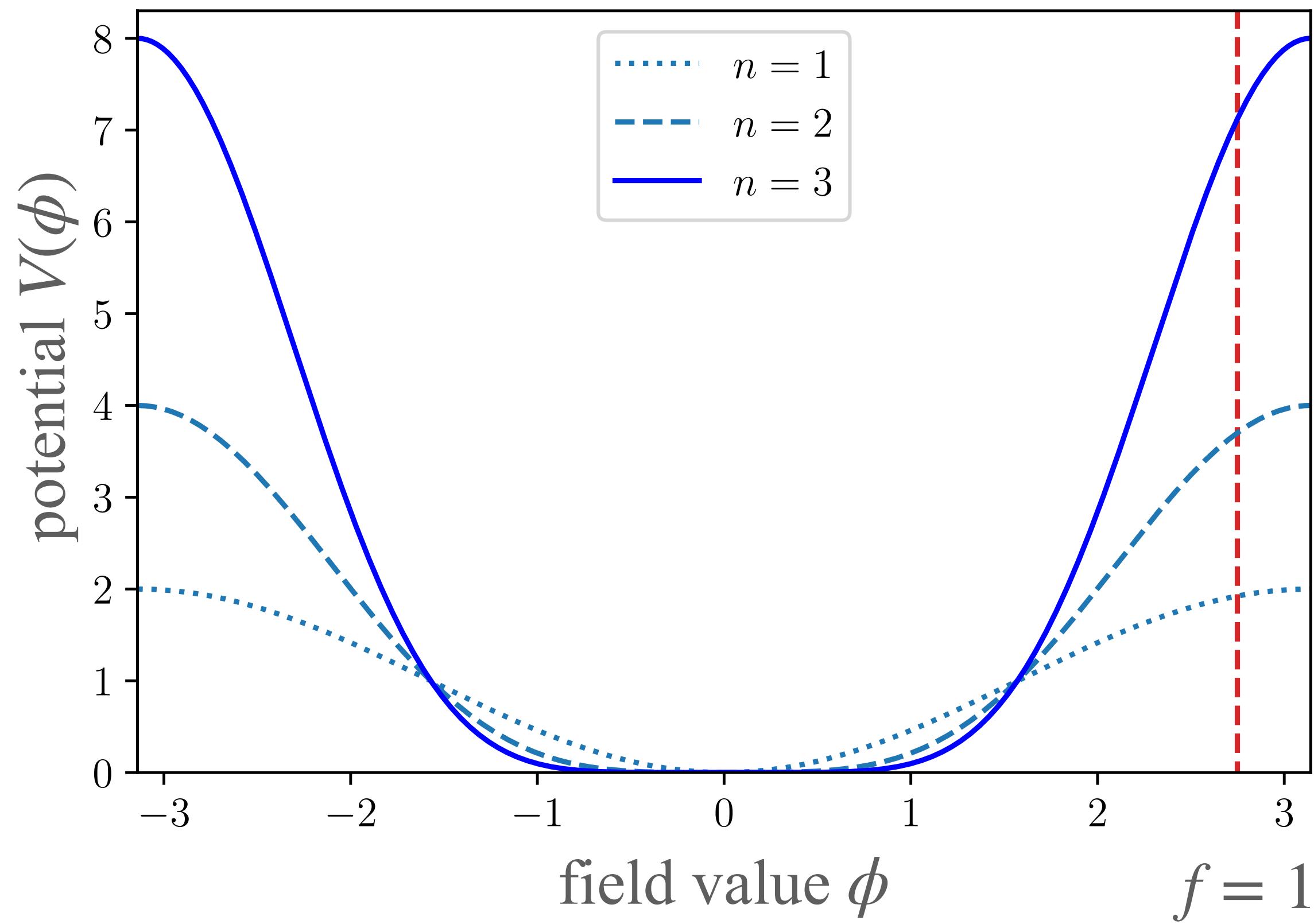
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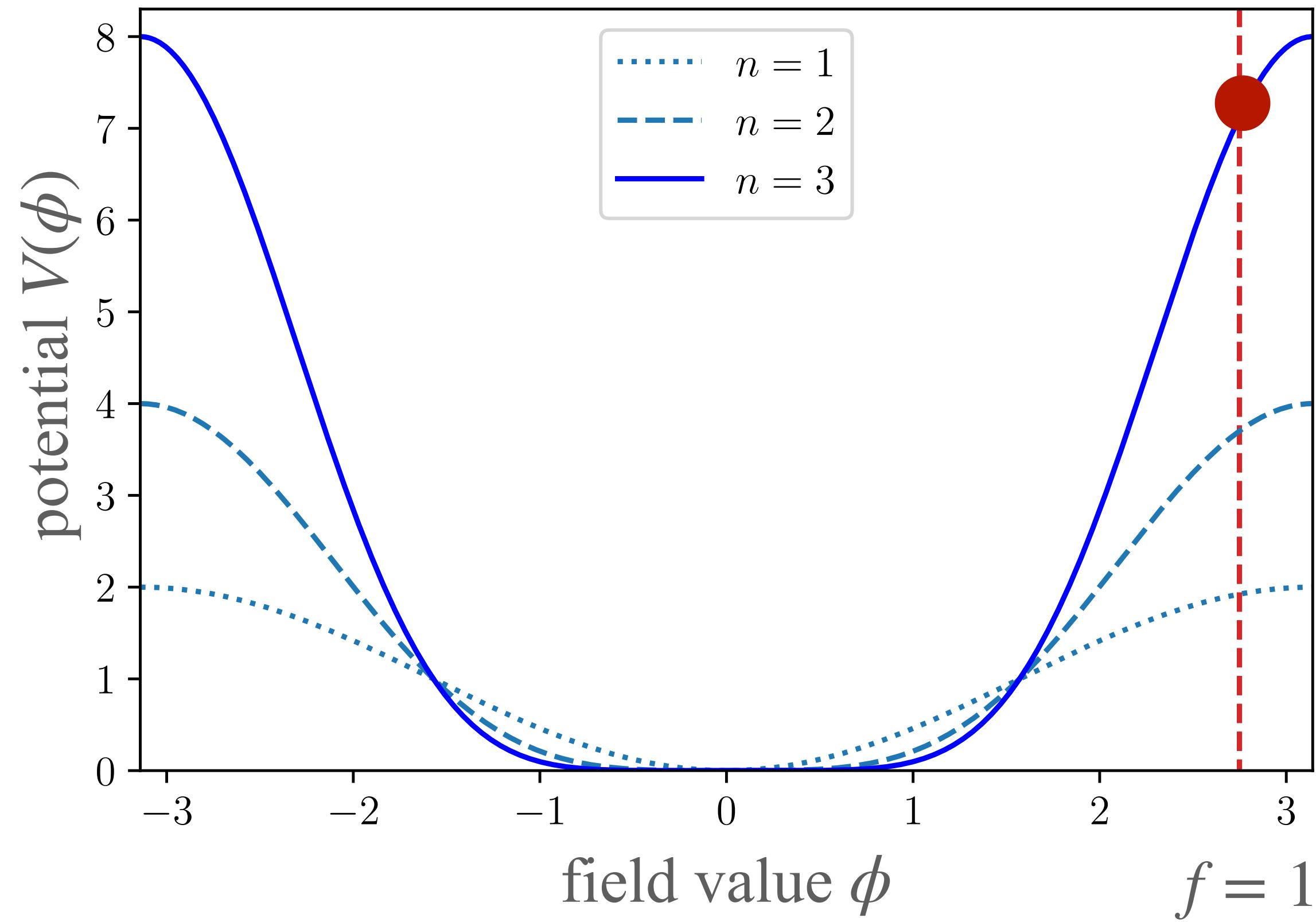


Disclaimer: the following will include many “hand-wavy” approximations.

Analytical computations are too complicated otherwise and numerics is needed.

# Early Dark Energy

$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$



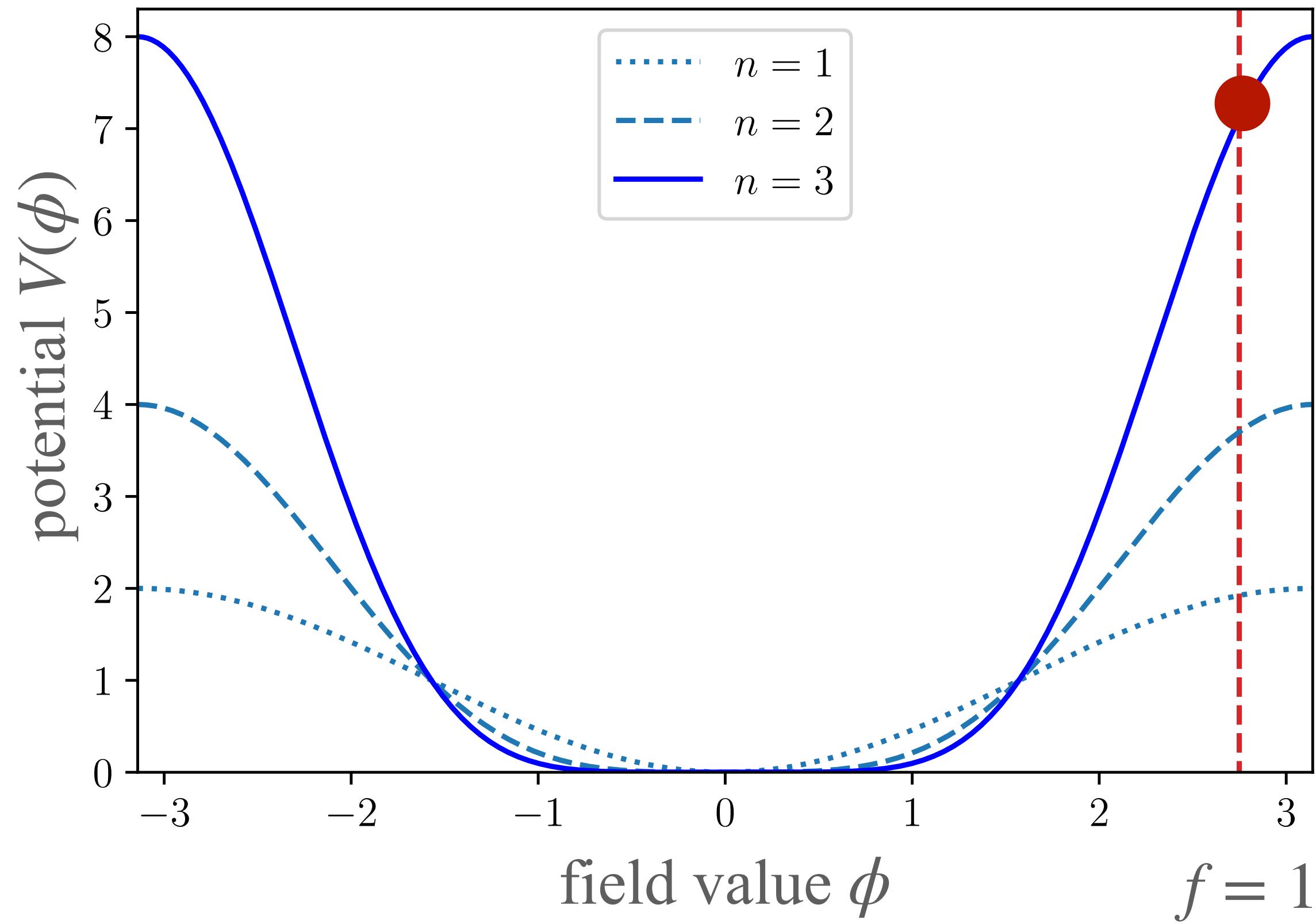
At early times:

- Dynamics governed by Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} + \cancel{\frac{dV}{d\phi}} = 0$$

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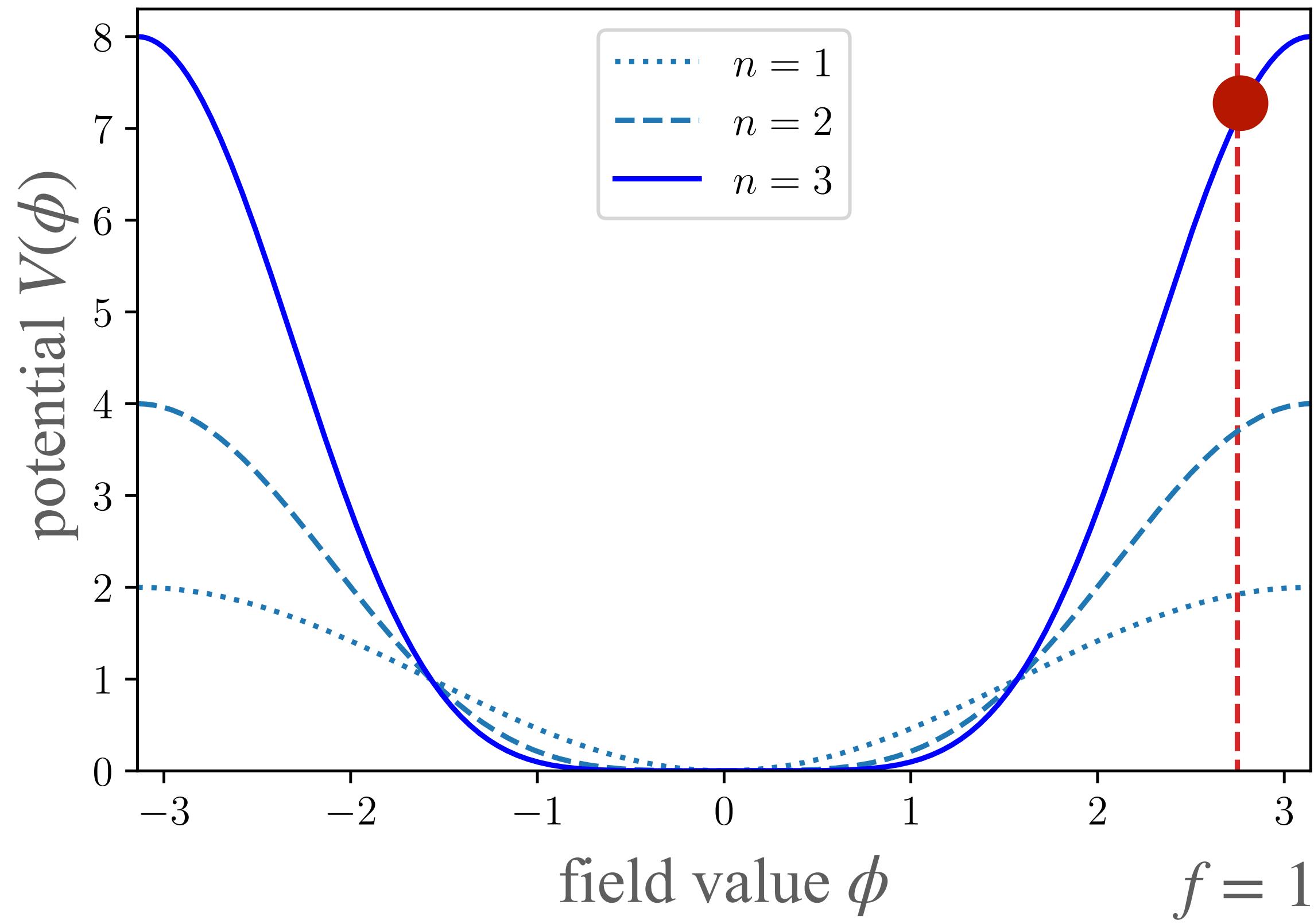
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$$\frac{dV}{d\phi} \approx 0$$

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- Moreover, at early times the expansion rate  $H$  is large; hence “Hubble friction” dominates:

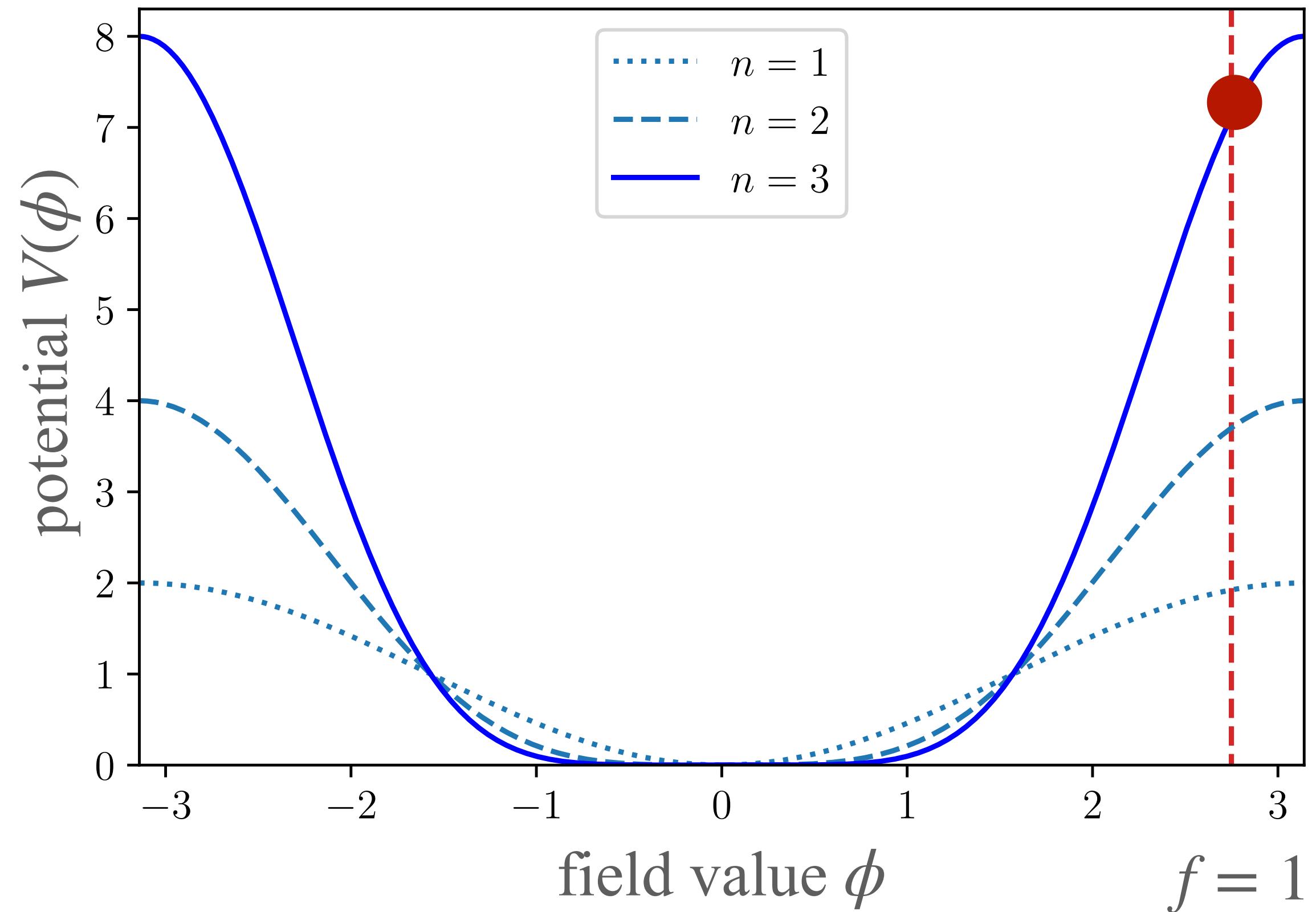
$$3H\dot{\phi} \gg \frac{dV}{d\phi}$$

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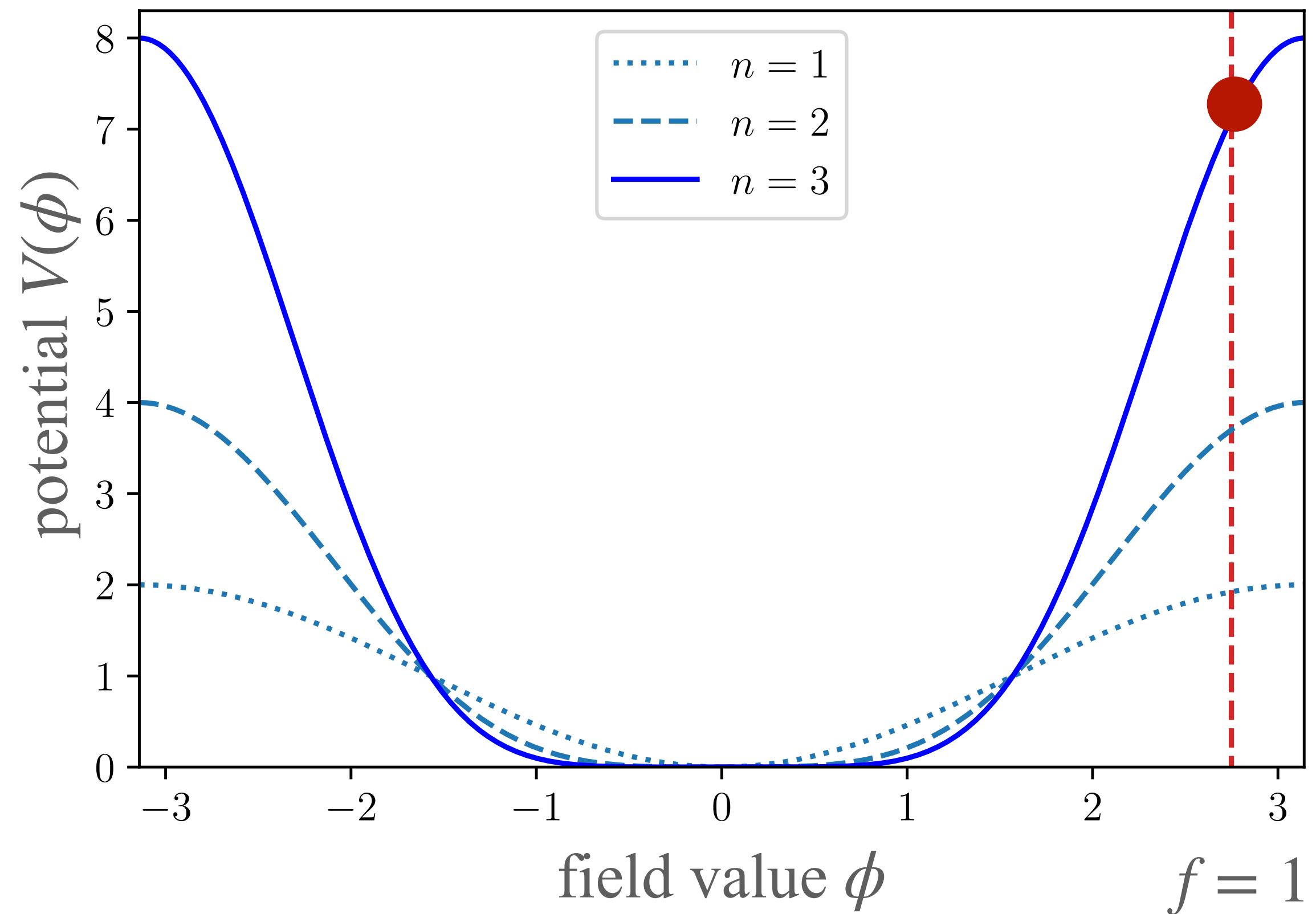


# Early Dark Energy

At early times:

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$$\ddot{\phi} + 3H\dot{\phi} = 0$$



- Define  $\psi = \dot{\phi}$ :

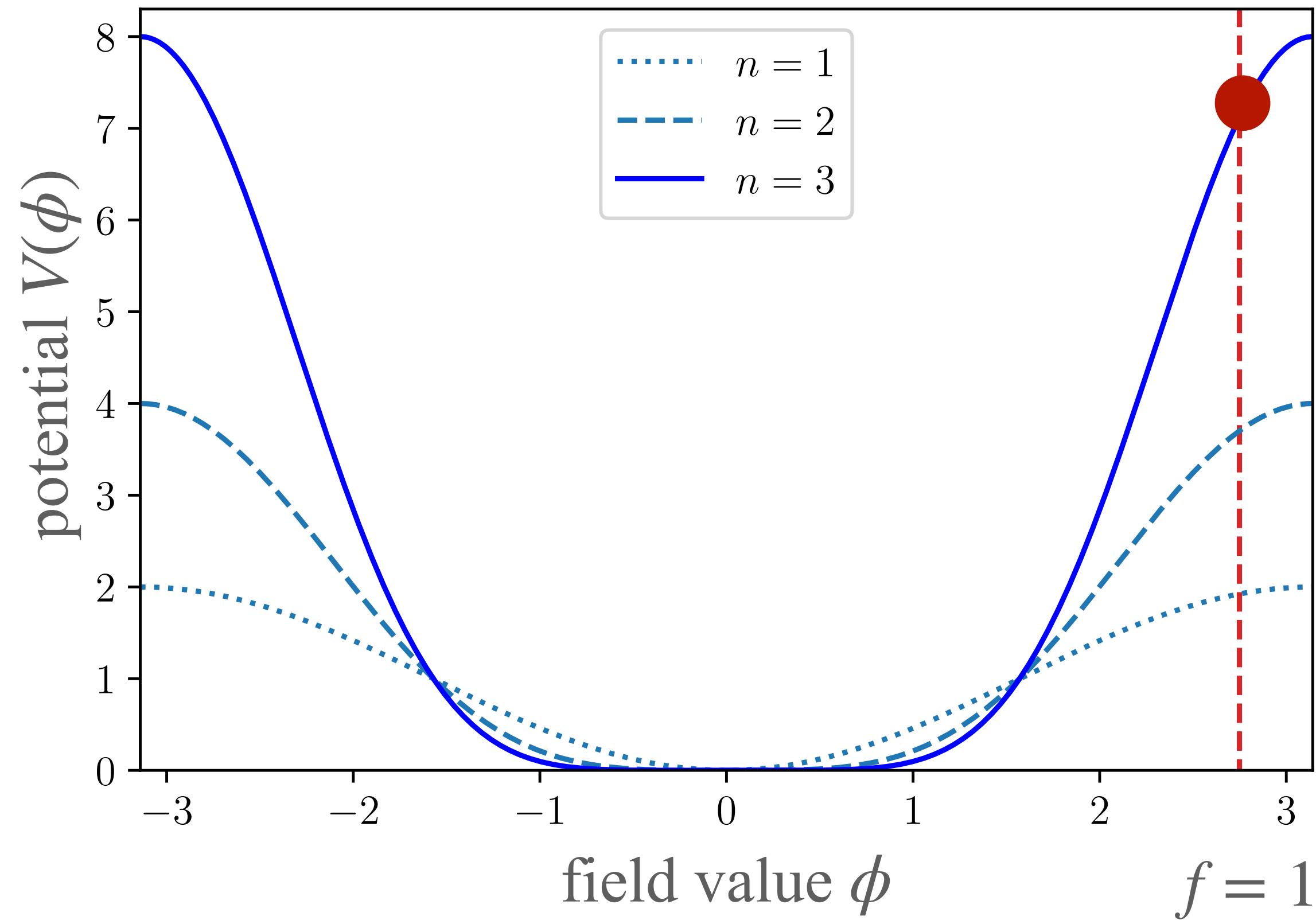
$$\dot{\psi} = -3H\psi$$

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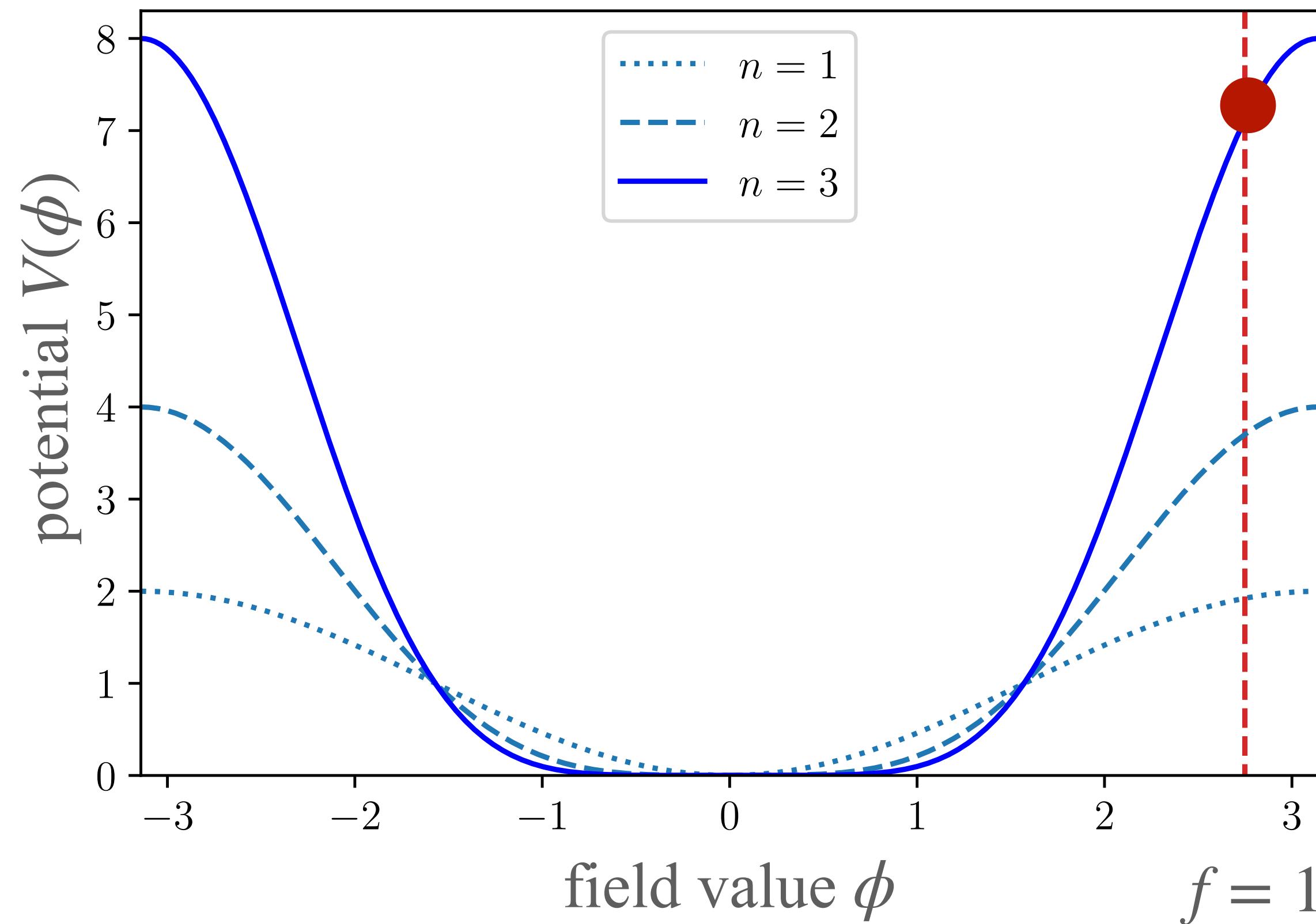
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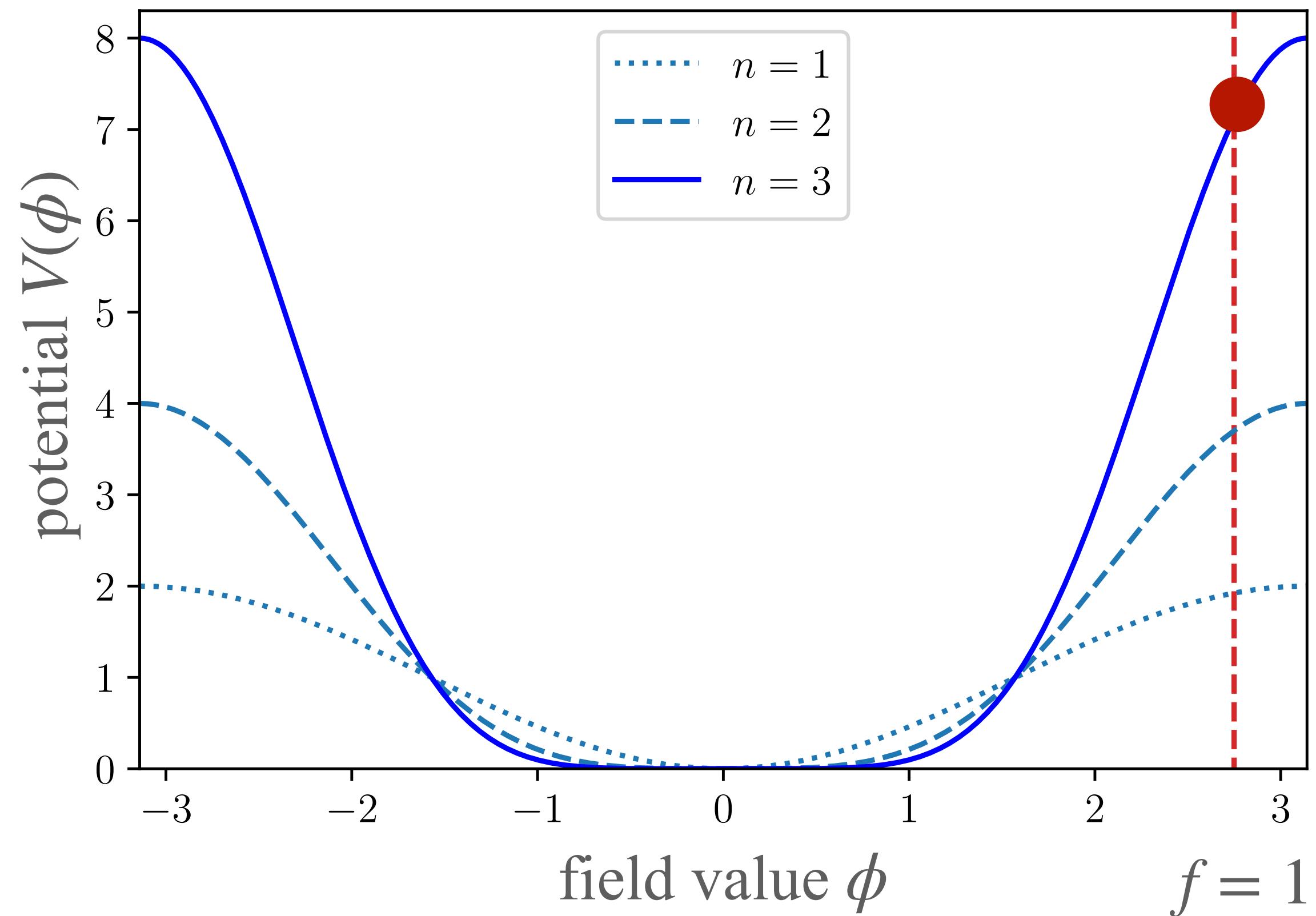
$H$  large at early times

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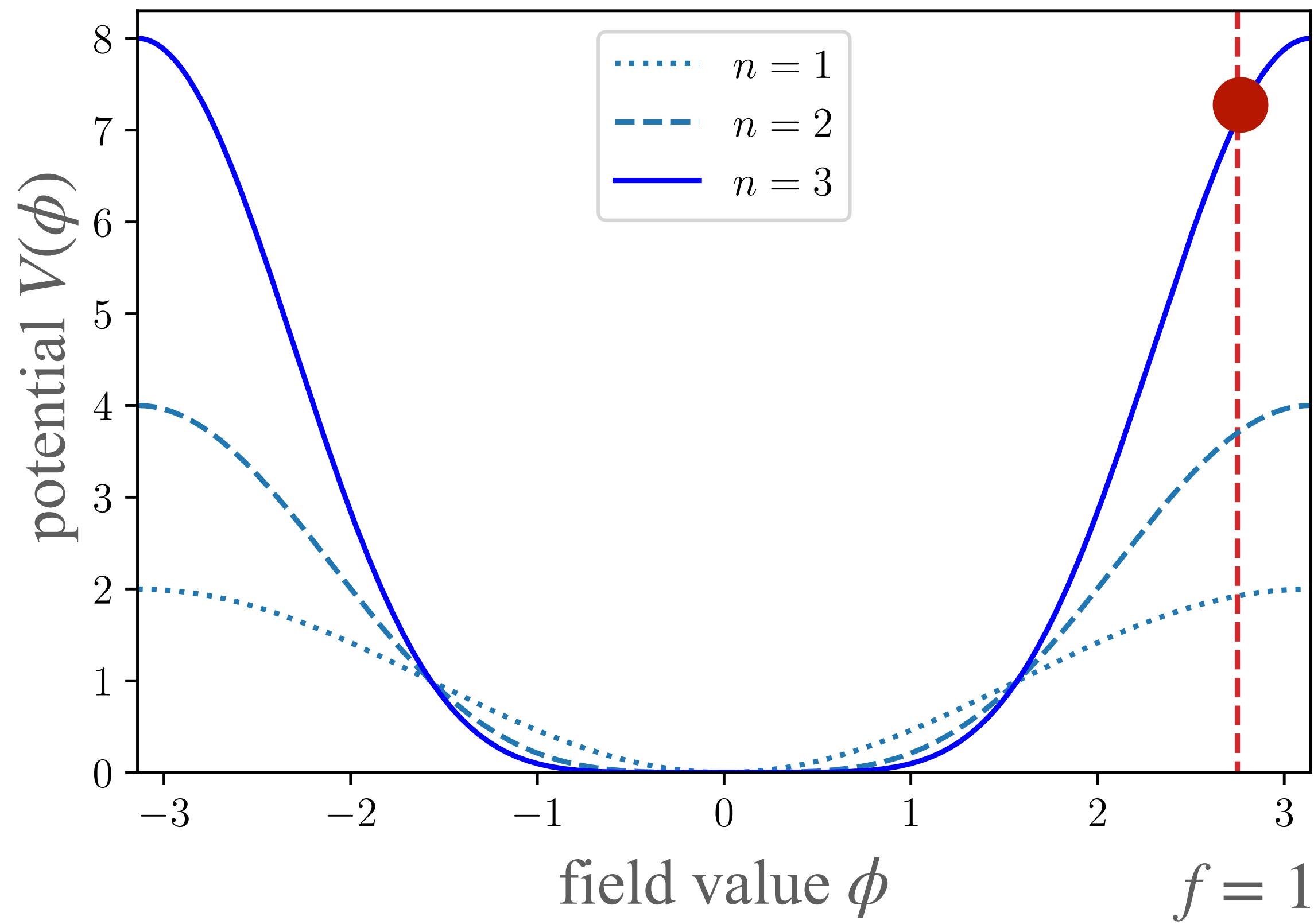
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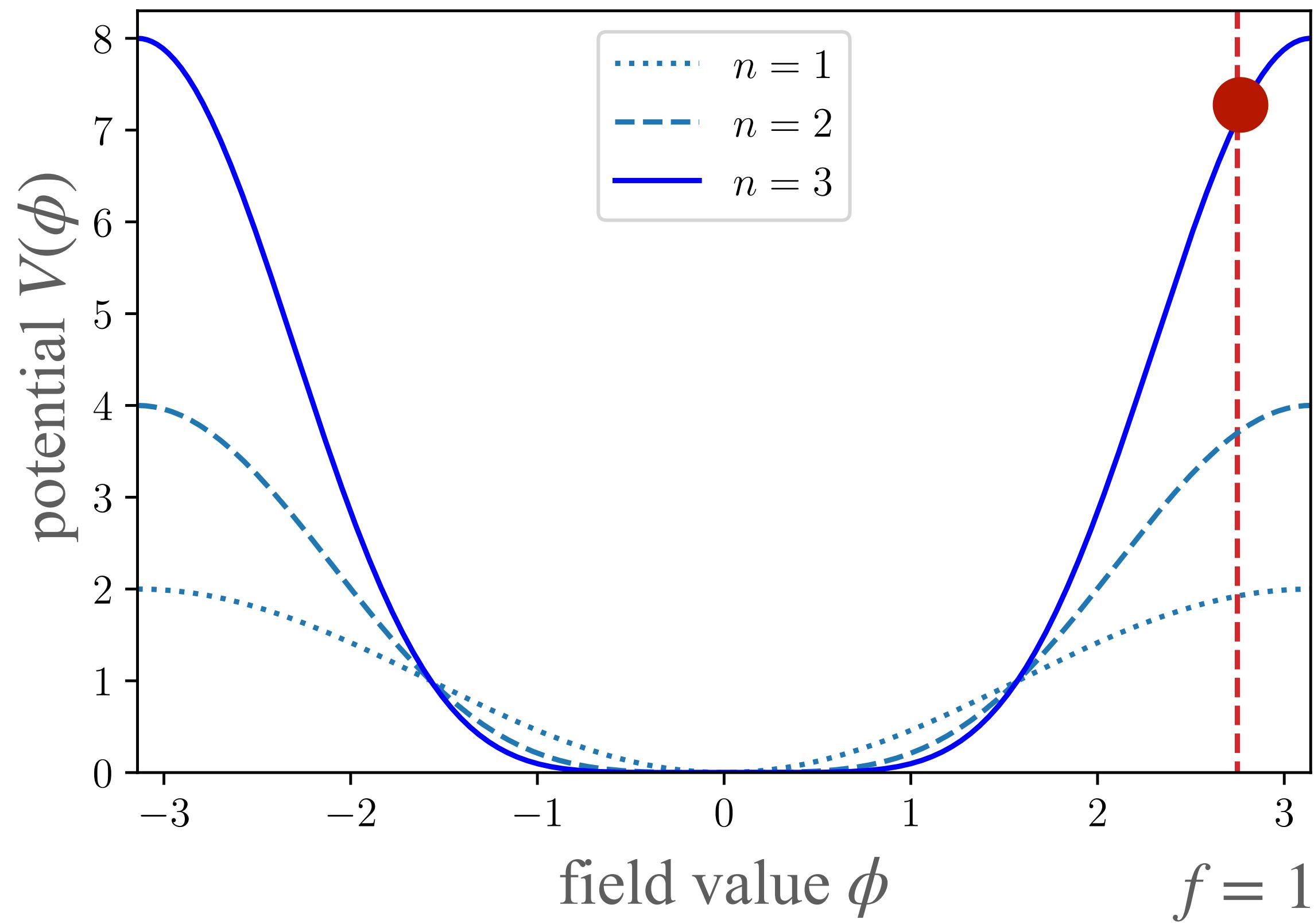
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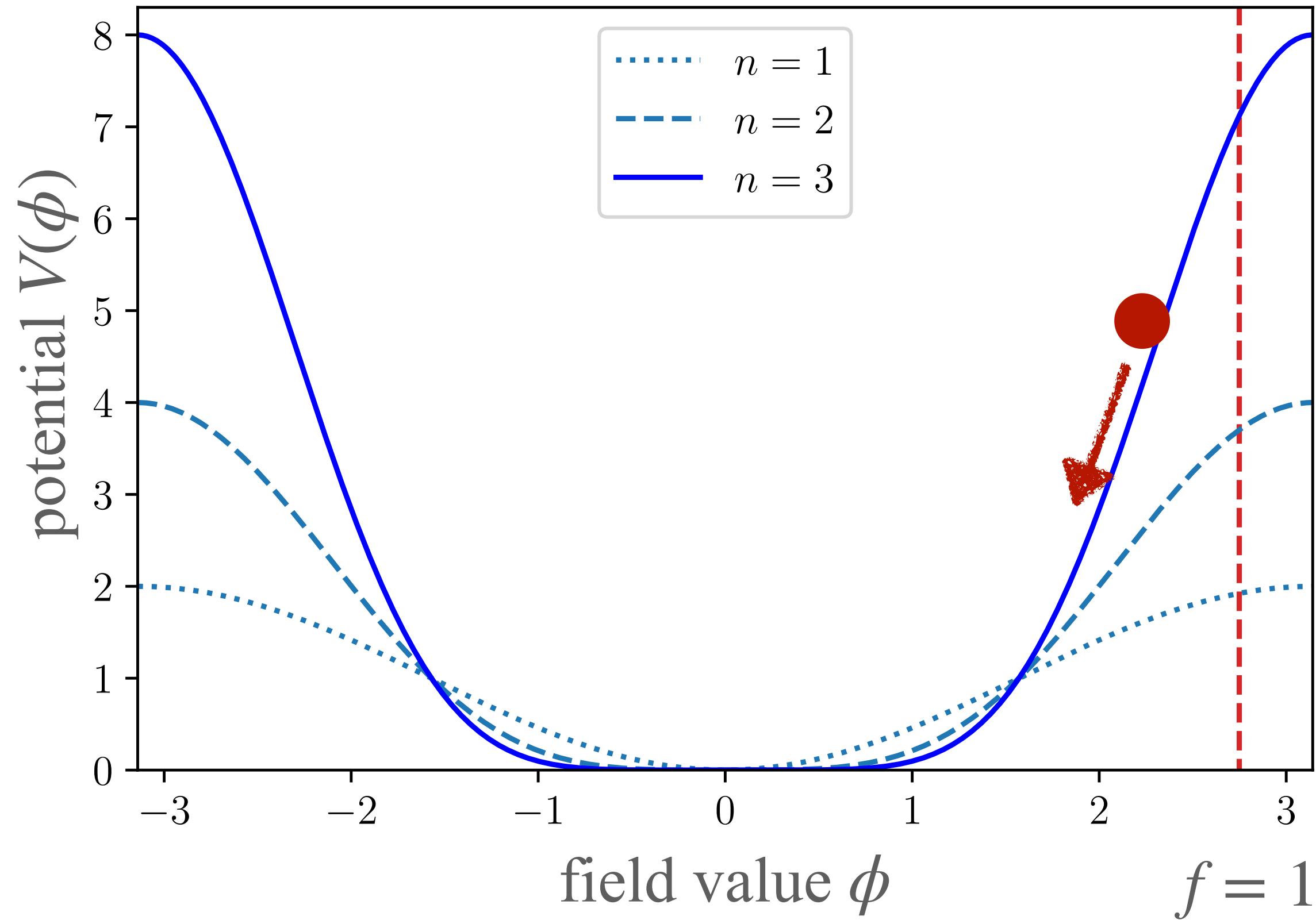
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$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi).$$

- The equation of state becomes:
$$w = \frac{p_\phi}{\rho_\phi} \approx -1$$
- Hence, at early times the field behaves like dark energy  $\rightarrow$  “EDE”

# Early Dark Energy

$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$



At the critical redshift:

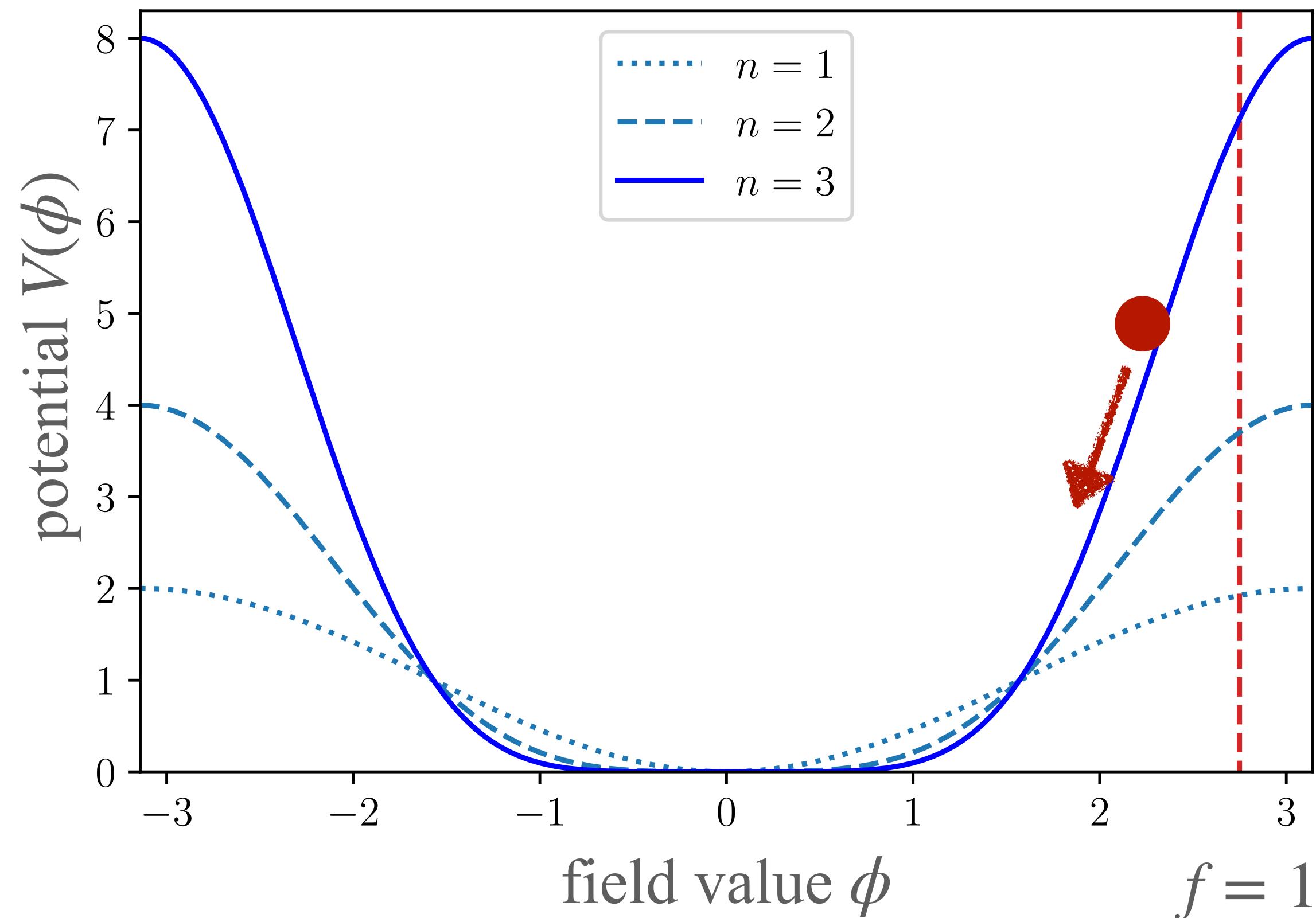
$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

- As time passes,  $H(z)$  decreases, until at some critical redshift  $z_c$ :

$$3H\dot{\phi} \approx \frac{dV}{d\phi}$$

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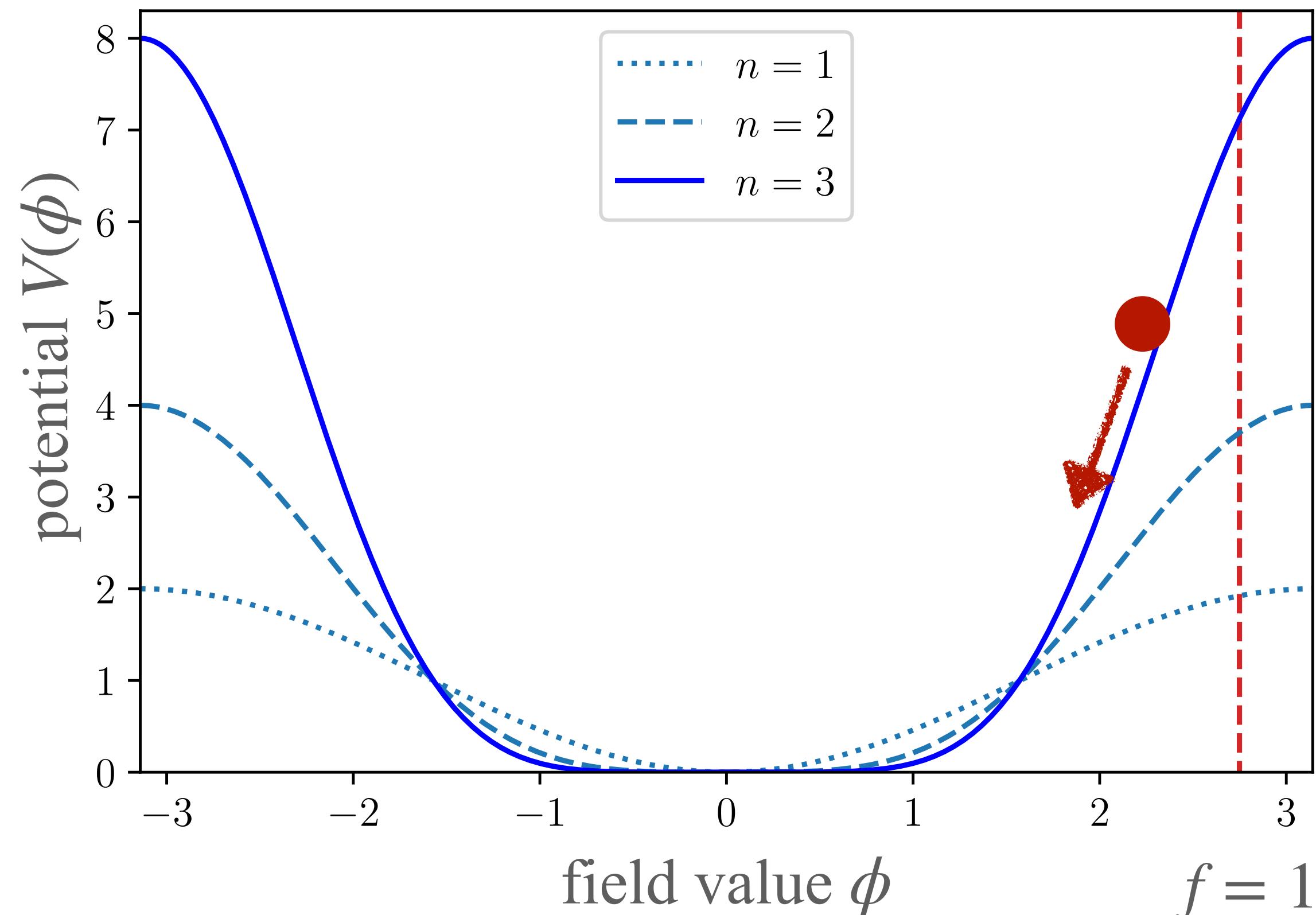
- Let's approximate:

- $V(\phi) = m^2 f^2 (1 - \cos(\phi/f))^n \approx$

For  $n = 1$  and small  $\phi$ :  $1 - \cos(\phi/f) \approx \frac{\phi^2}{2f^2}$

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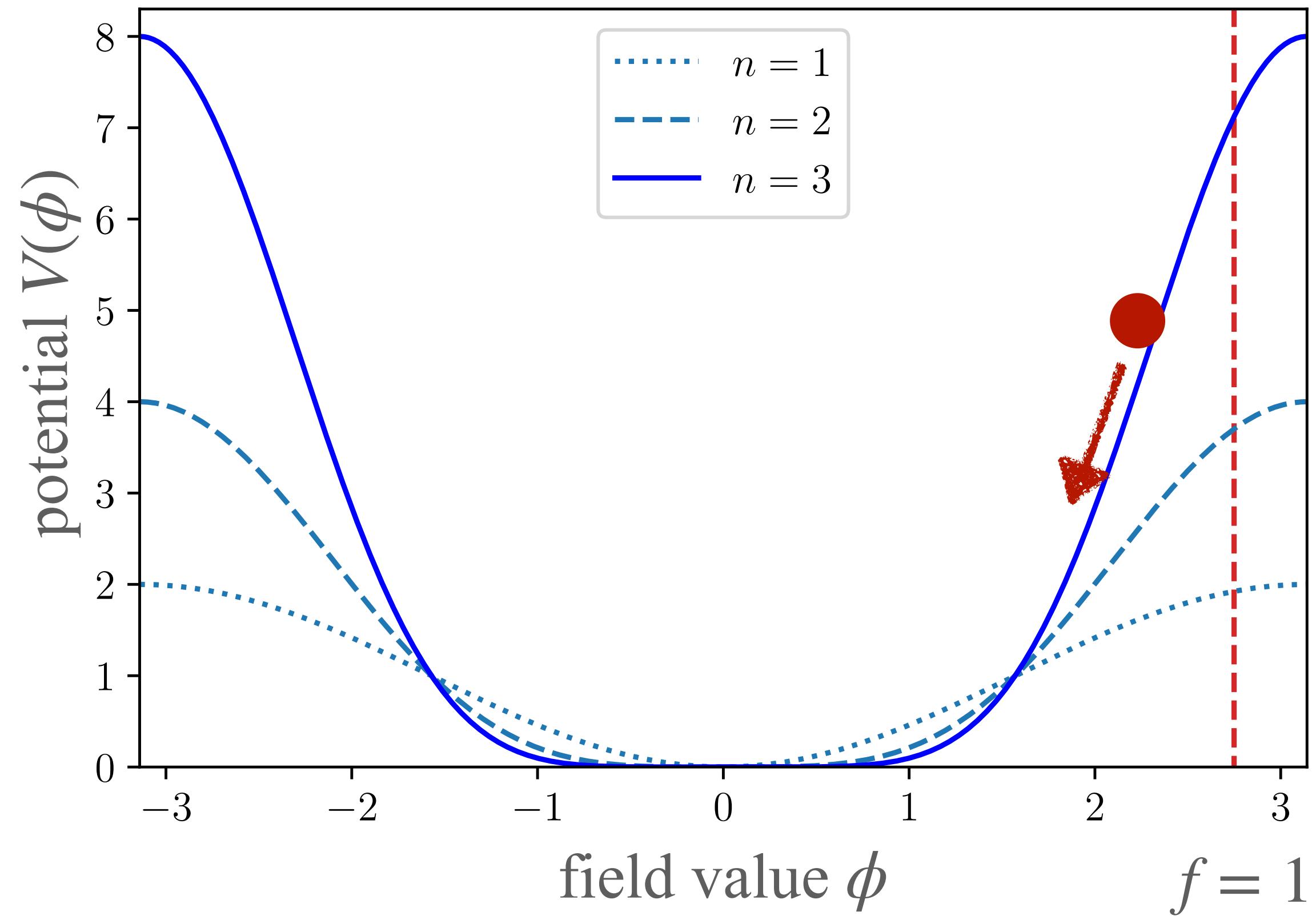
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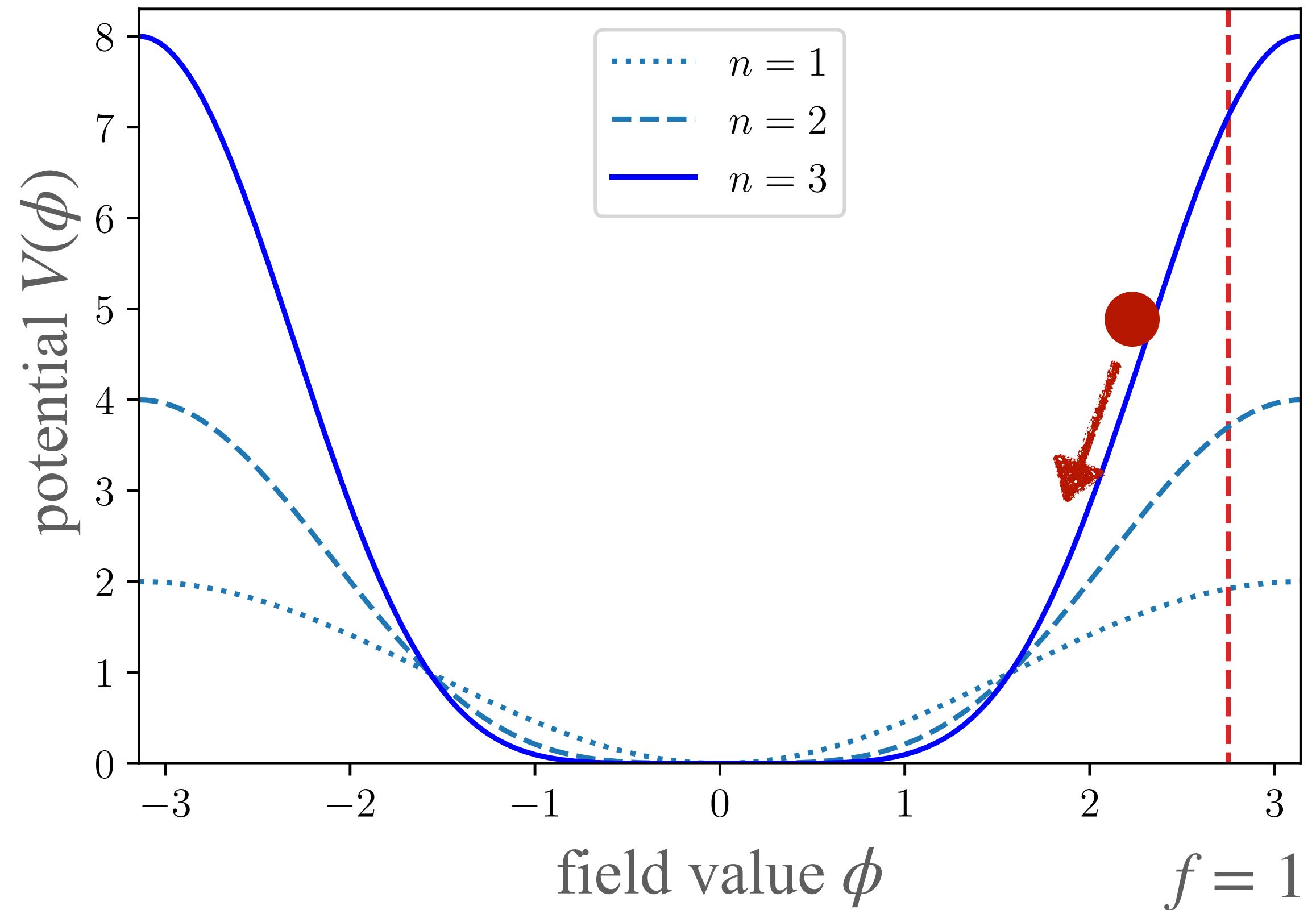


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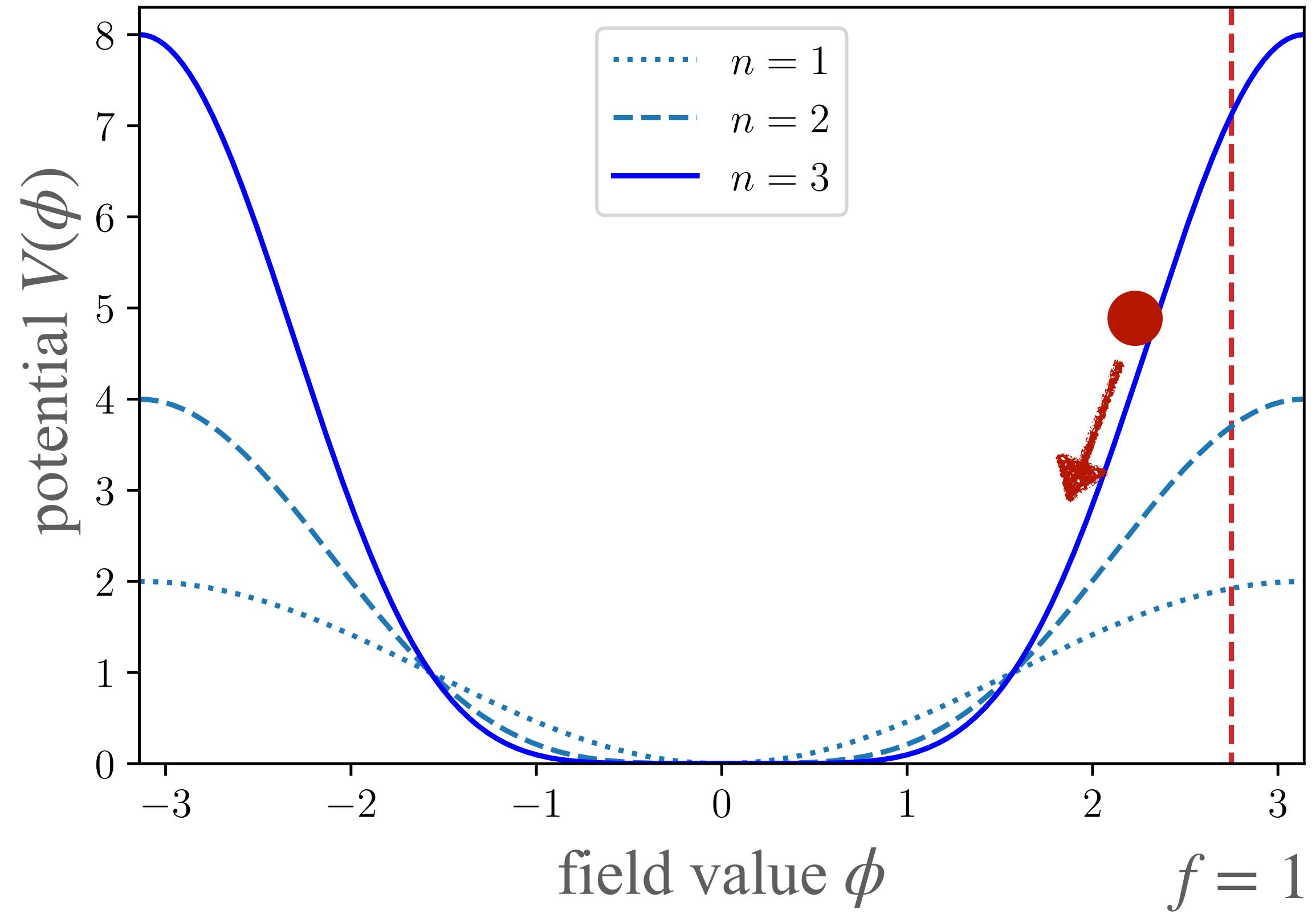
At the critical redshift:

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- $V(\phi) \approx \frac{m^2}{2}\phi^2 \rightarrow \frac{dV}{d\phi} \approx m^2\phi$ :

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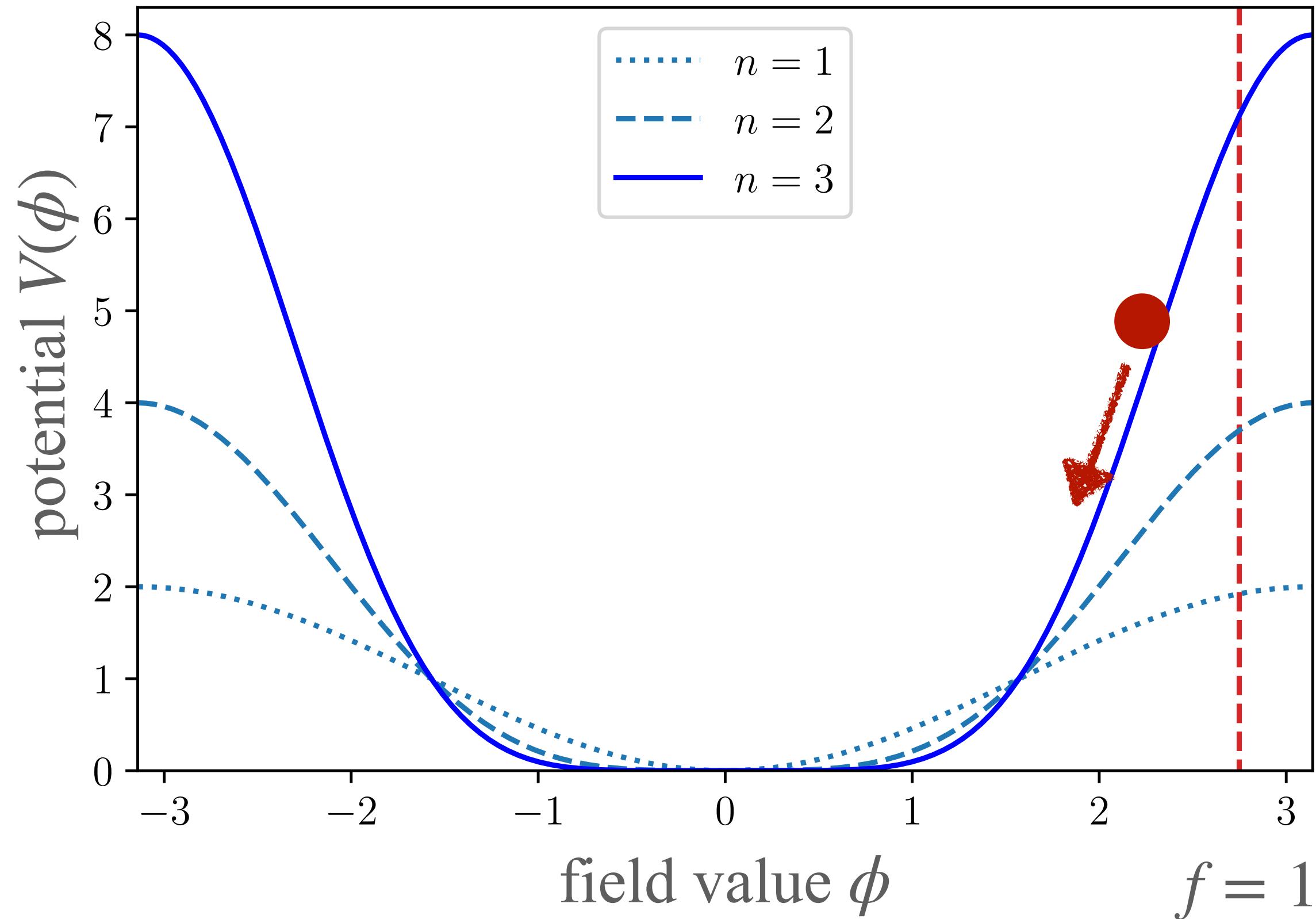
$$3H\phi \approx \frac{dV}{d\phi}$$

- $V(\phi) \approx \frac{m^2}{2}\phi^2 \rightarrow \frac{dV}{d\phi} \approx m^2\phi$ :

$$3H \approx m^2$$

# Early Dark Energy

$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$



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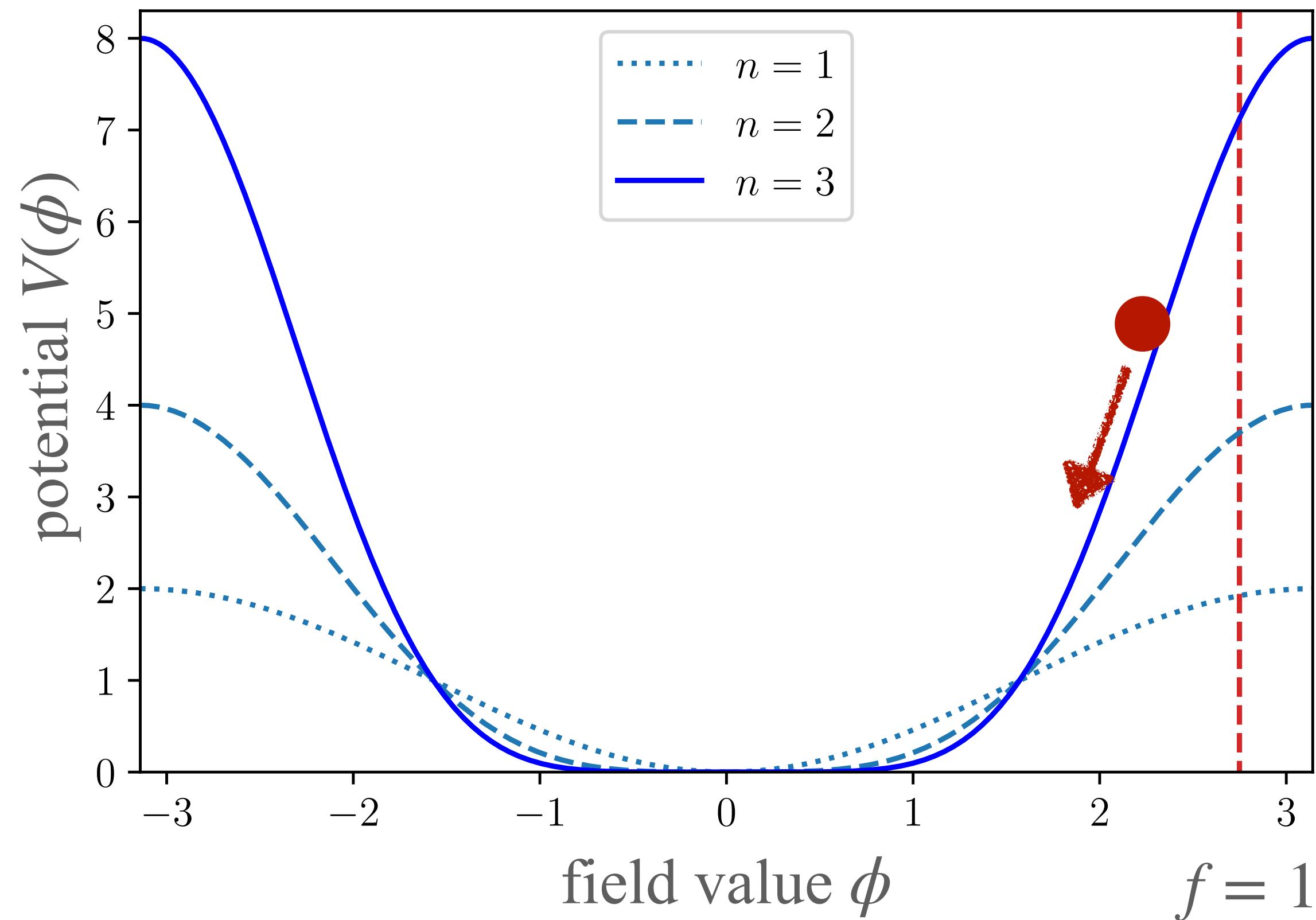
$$3H \approx m^2$$

- Using this, one can determine the “mass” of the EDE field:

$$m \sim 10^{-28} \text{ eV}$$

# Early Dark Energy

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At times after  $z_c$ :

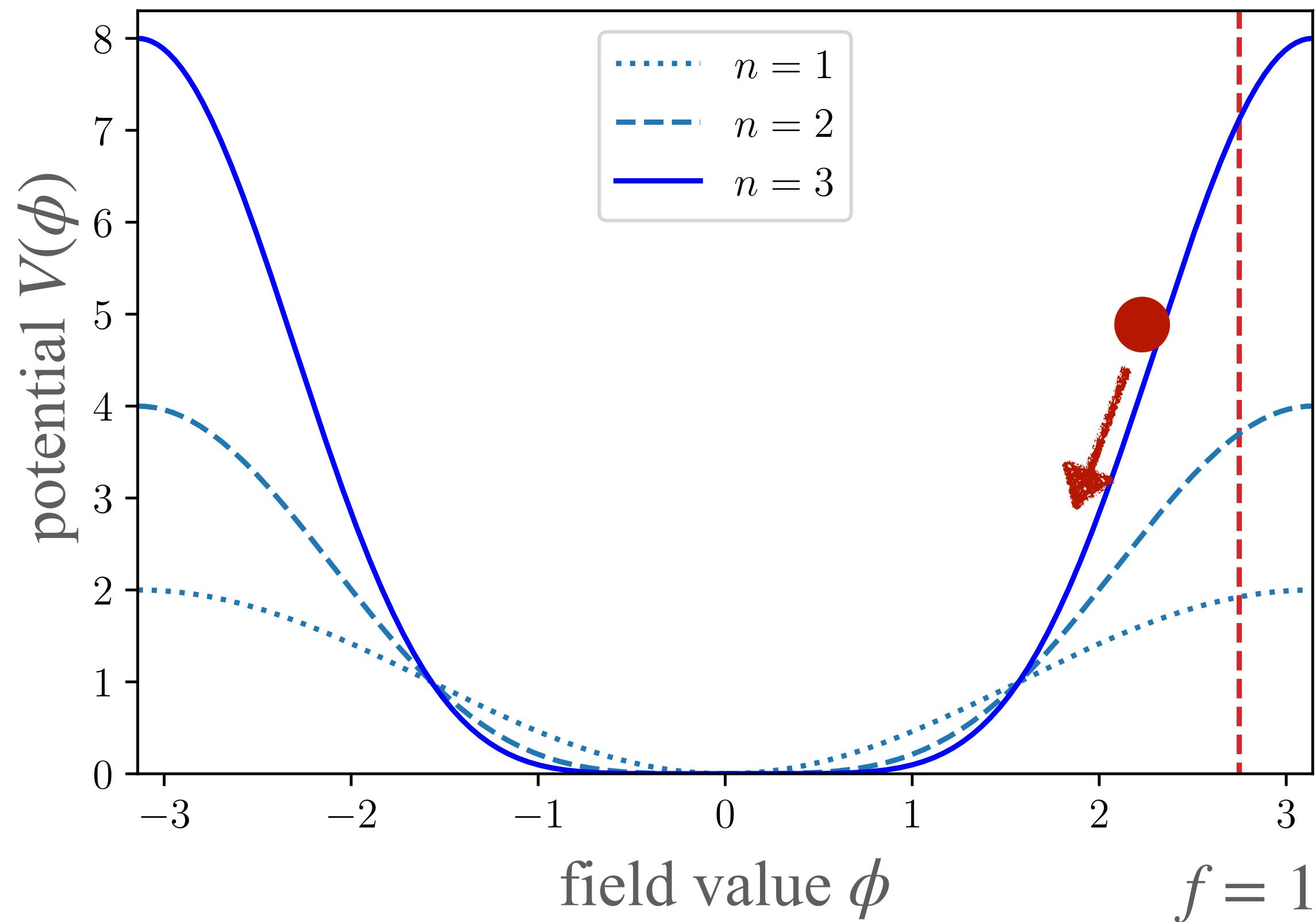
$$3H\dot{\phi} \ll \frac{dV}{d\phi}$$

- Hence the Klein-Gordon eq. is:

$$\ddot{\phi} + \frac{dV}{d\phi} \approx 0$$

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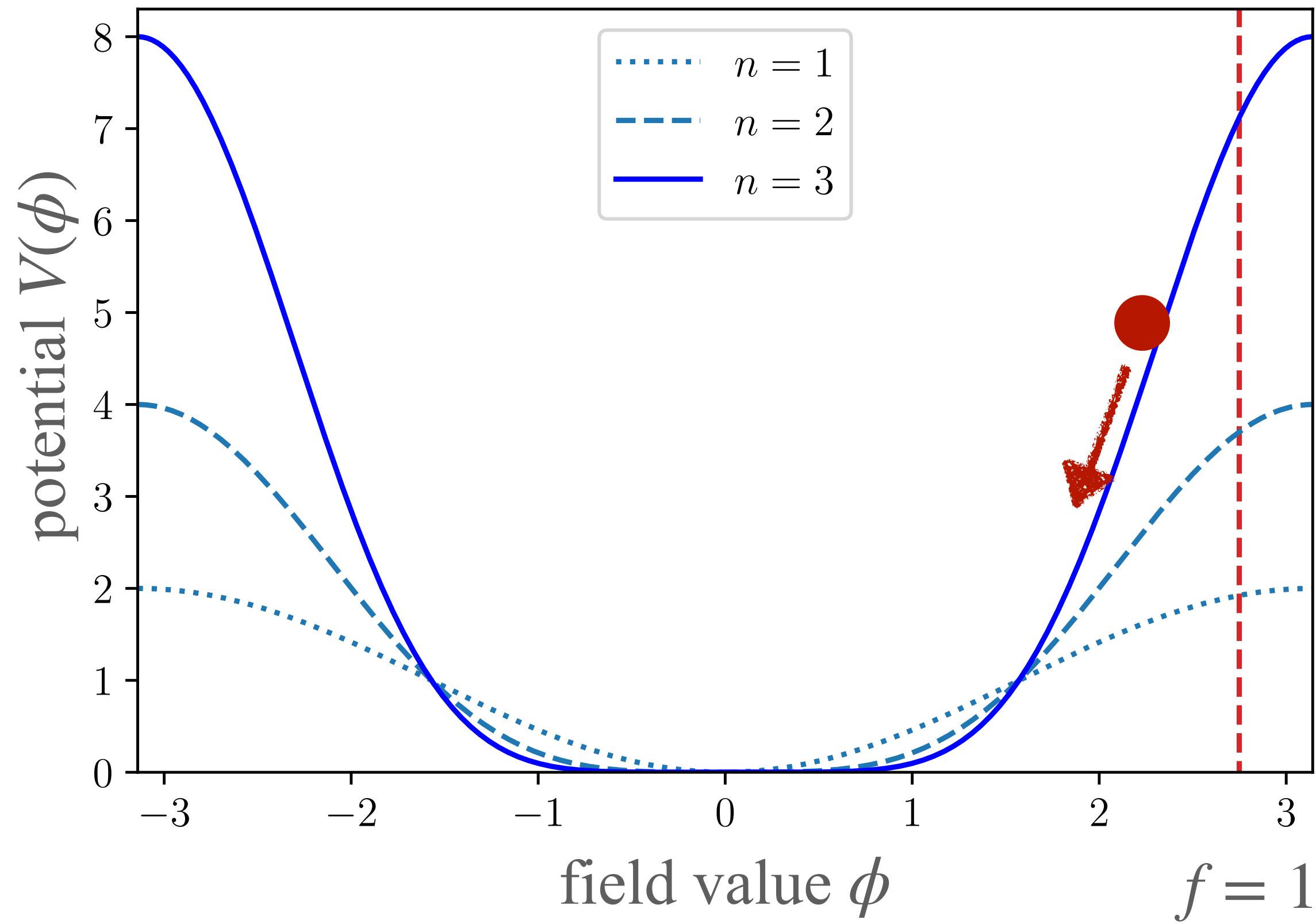
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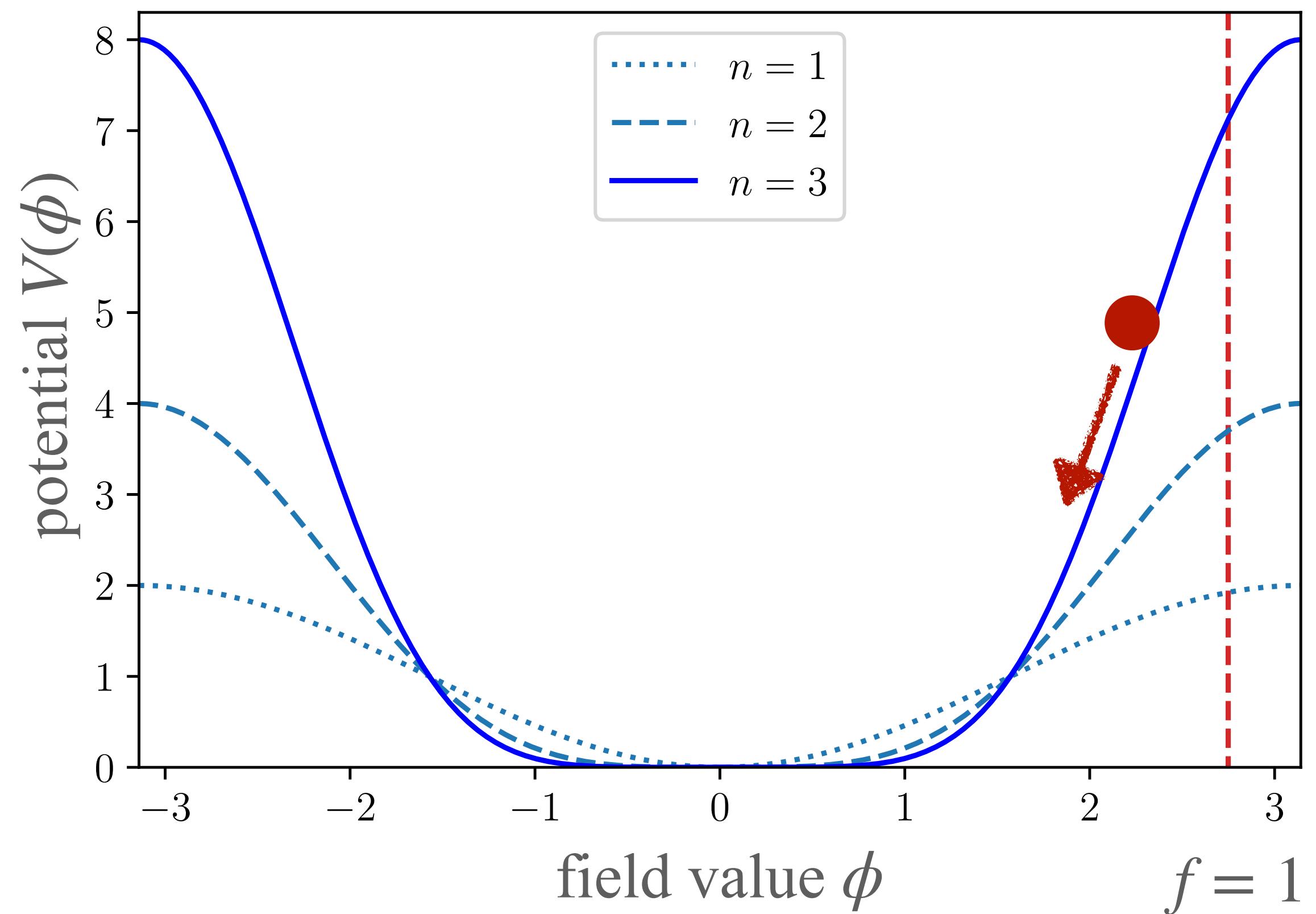
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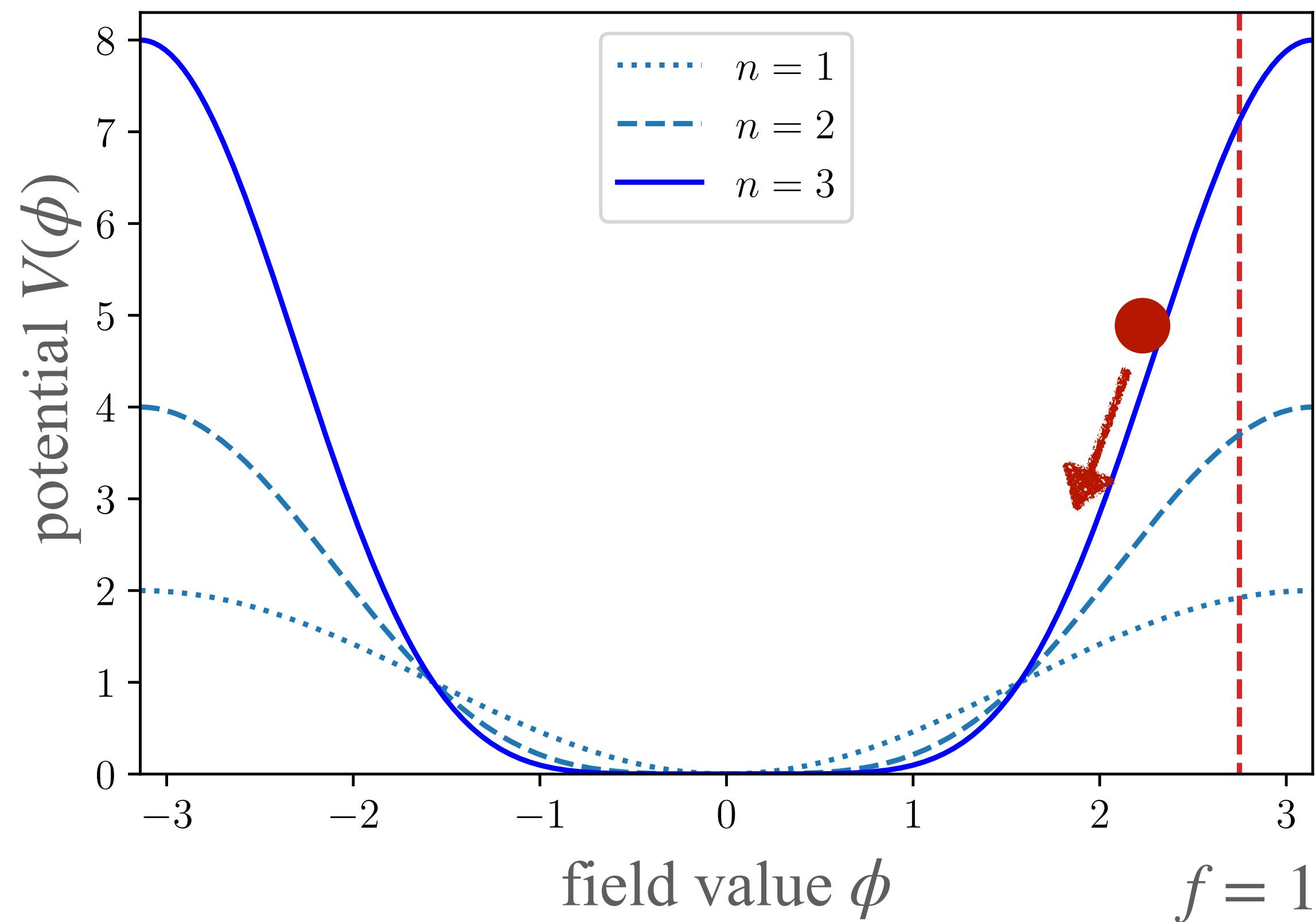
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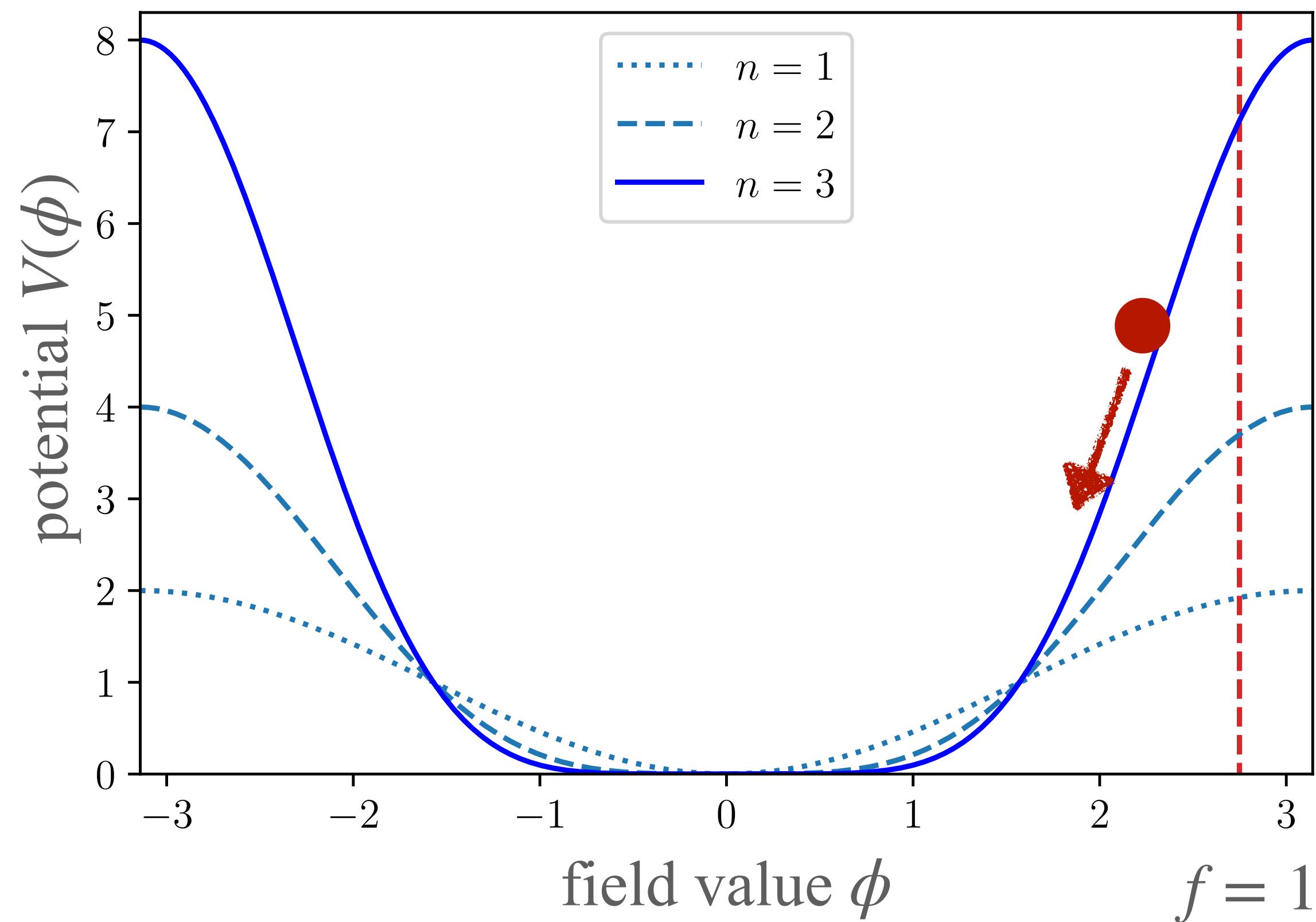
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- This is an oscillation!

$$\phi(t) = \phi_0 \cos(mt)$$

# Early Dark Energy

$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$



At times after  $z_c$ :

$$\ddot{\phi} + m^2\phi \approx 0$$

- This is an oscillation!

$$\phi(t) = \phi_0 \cos(mt)$$

- One can now numerically show that the EOS is:

$$\langle w \rangle = \frac{n-1}{n+1}$$

For  $n = 3$ :  $\langle w \rangle = \frac{1}{2}$

# Early Dark Energy

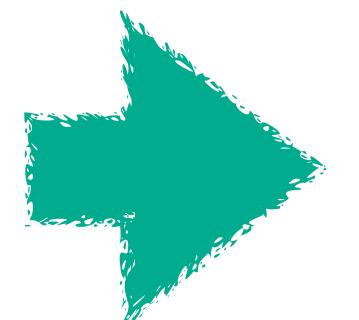
Free “particle-physics” parameters:

- $m$  mass ( $V_0 = m^2 f^2$ )
- $f$  “decay constant”
- $\theta_i = \phi_i/f$  initial value of the field
- $(n = 3)$

# Early Dark Energy

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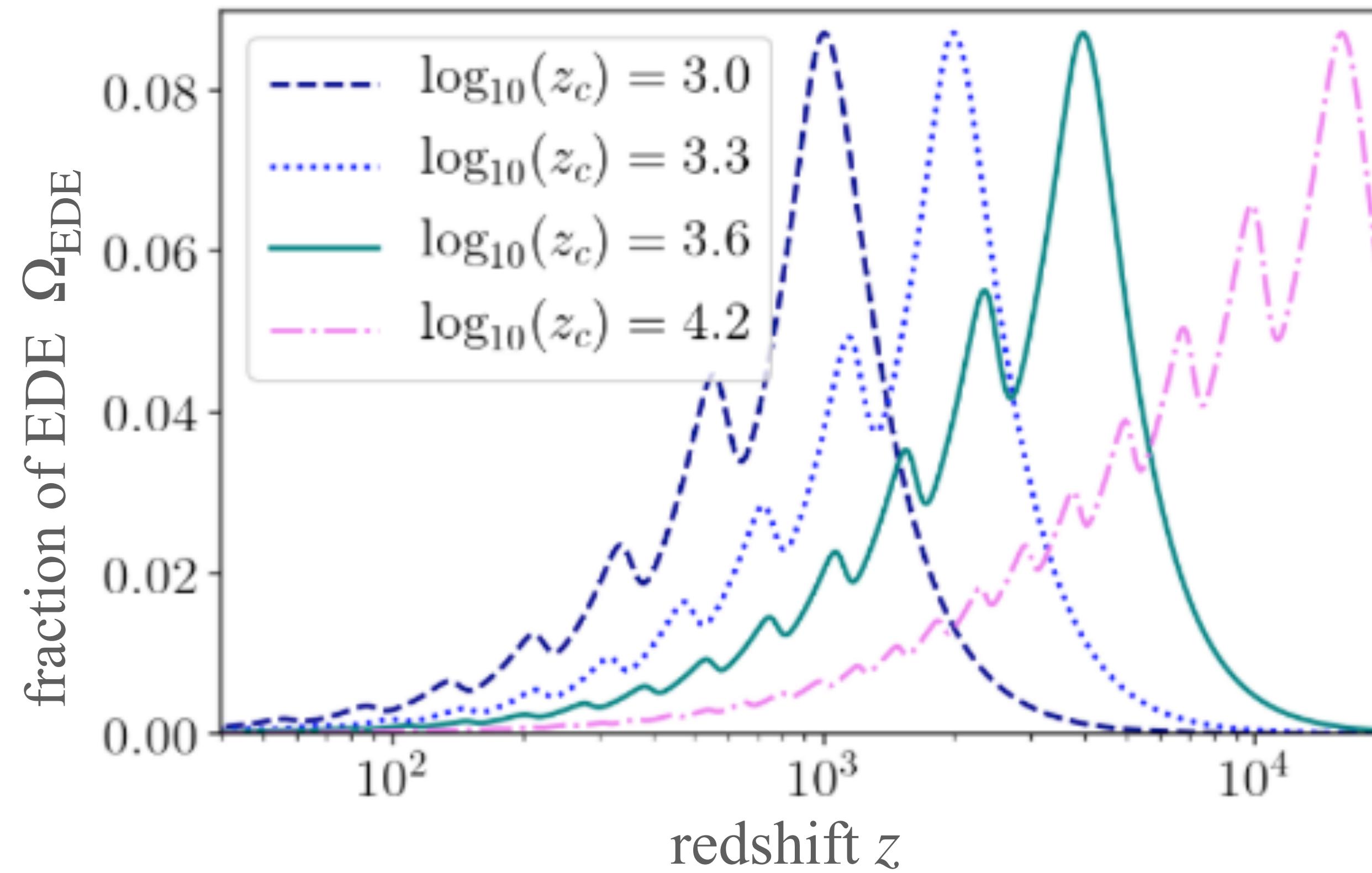
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Free “phenomenological” parameters:

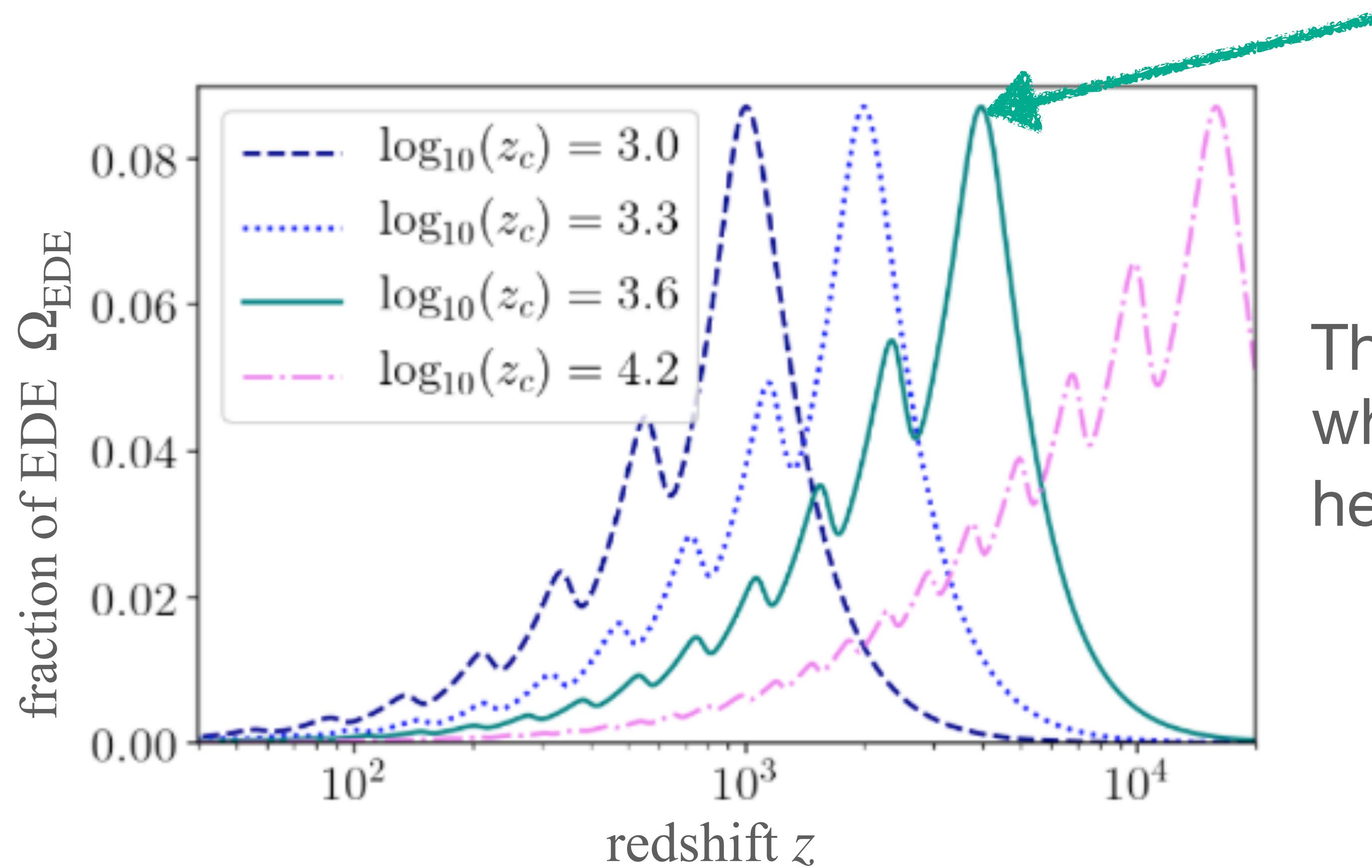
- $f_{\text{EDE}}$  fraction of EDE at  $z_c$
- $z_c$  the critical redshift
- $\theta_i = \phi_i/f$  initial value of the field
- $(n = 3)$

# Early Dark Energy



The critical redshift  $z_c$  determines when the field starts oscillating, hence when  $\Omega_{\text{EDE}}$  peaks

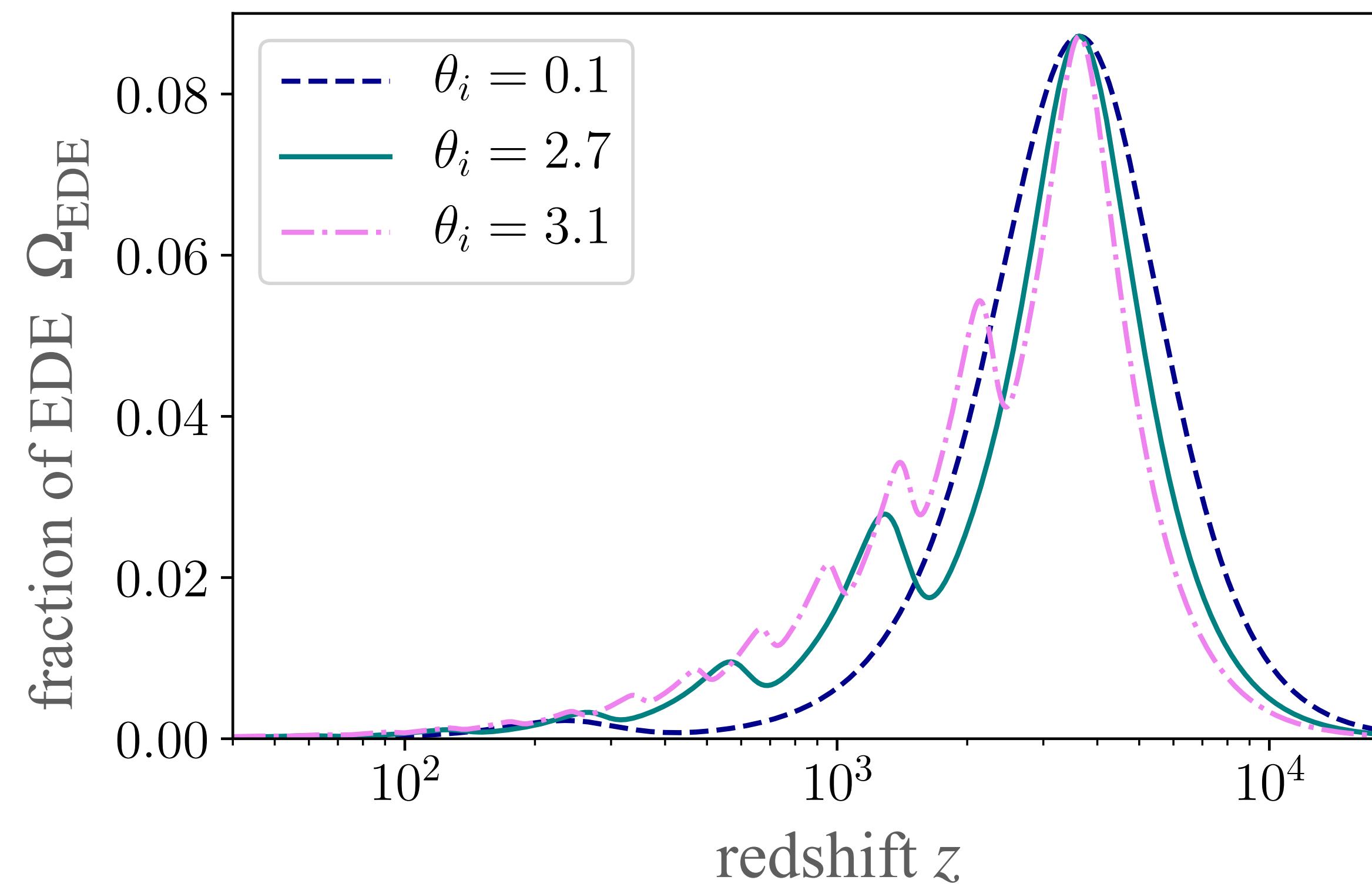
# Early Dark Energy



$f_{\text{EDE}} = \Omega_{\text{EDE}}(z_c)$ : determines the height of the peak

The critical redshift  $z_c$  determines when the field starts oscillating, hence when  $\Omega_{\text{EDE}}$  peaks

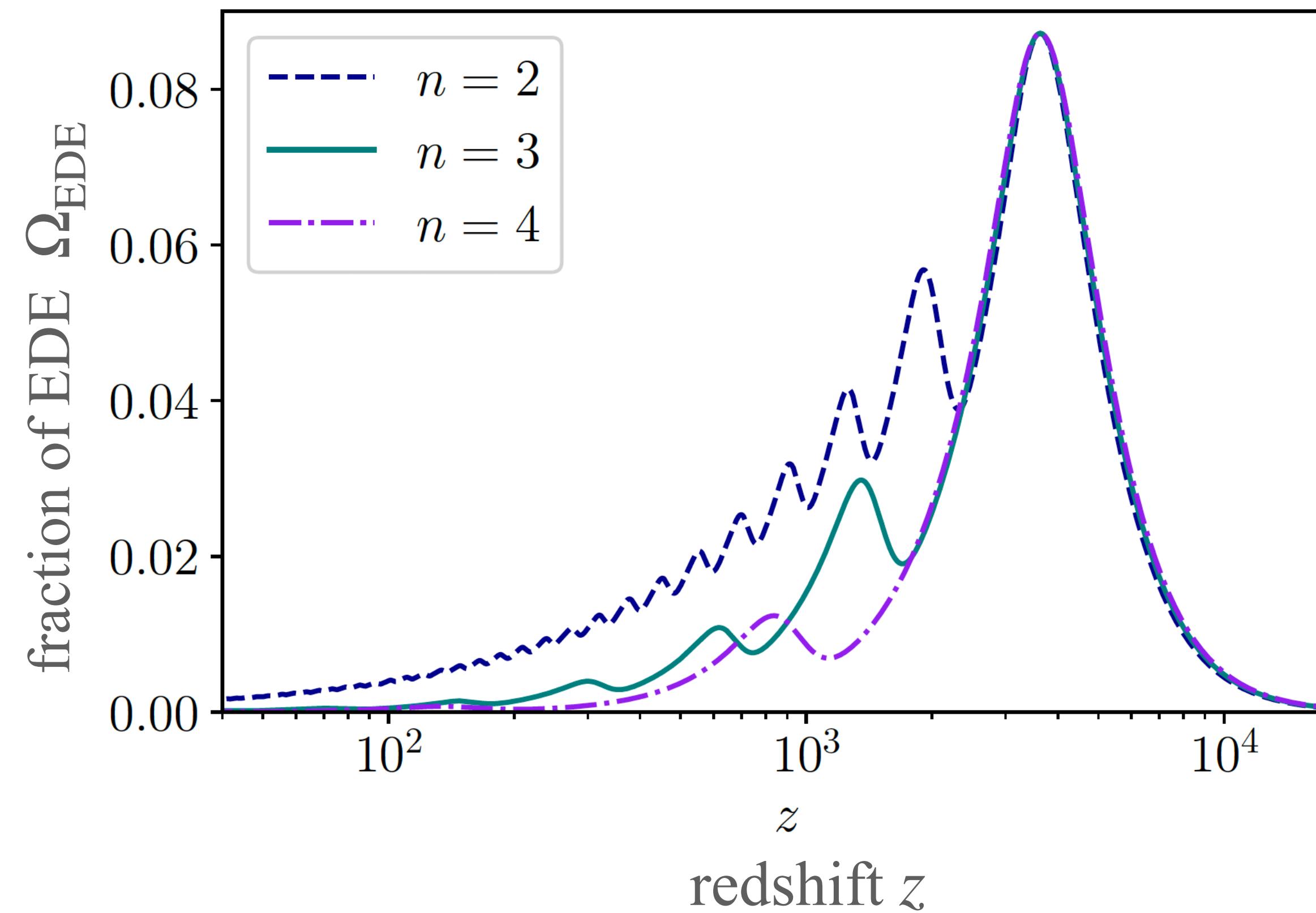
# Early Dark Energy



The initial value  $\theta_i \in [0, \pi]$  determines how fast the field oscillates when it decays:

The closer to  $\pi$  the faster the oscillations

# Early Dark Energy



The index  $n = 1, 2, 3, \dots$  determines how fast the field decays:

The higher  $n$ , the faster the decay

$n = 3$  was shown to fit the data best

# Recap Lecture 1

# Introduction

- Friedmann equations describe (background) evolution of the components
- Hubble parameter
- distances
- CMB basics
- BAO
- How the CMB constrains  $H_0$

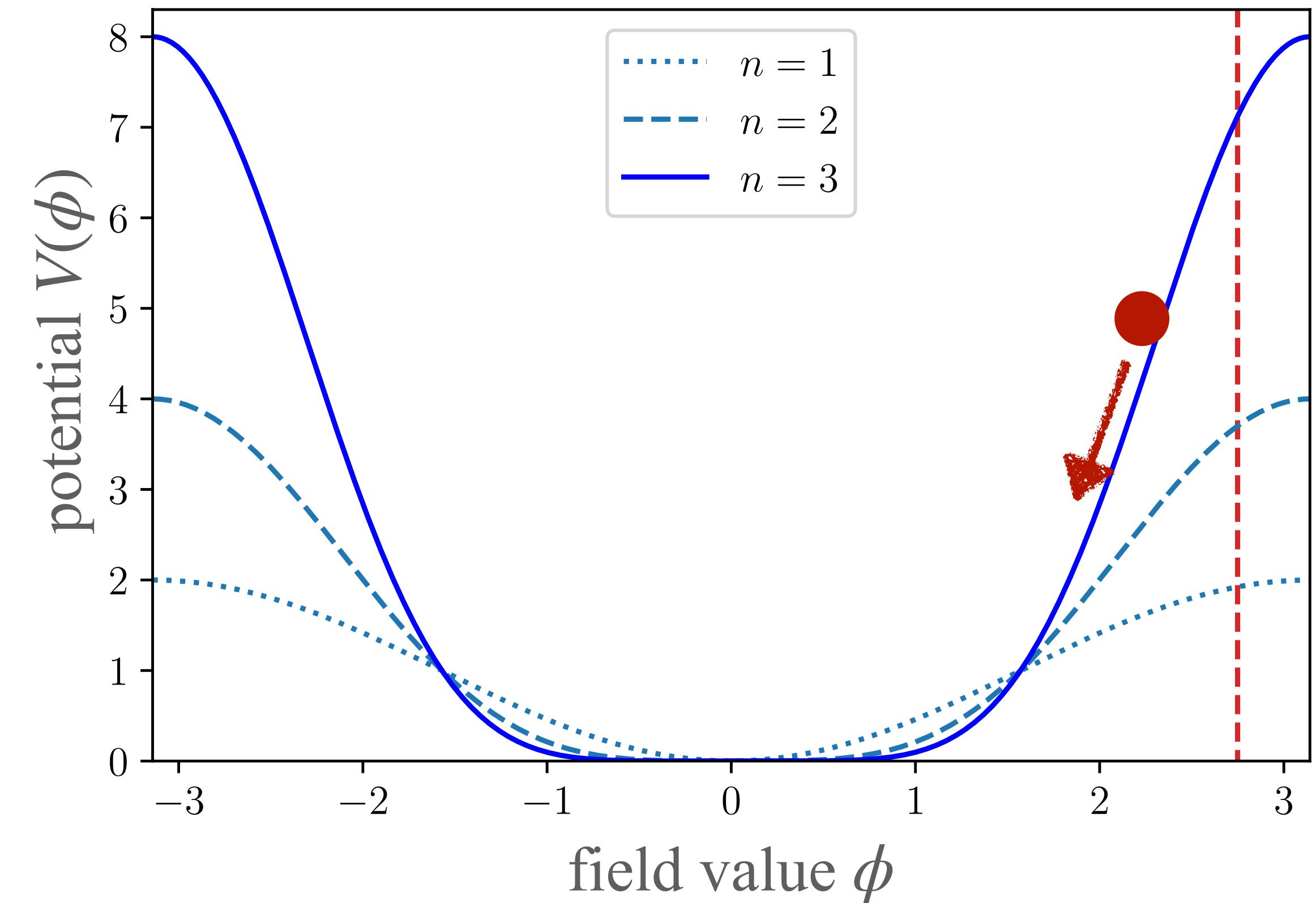
$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \quad (\text{i})$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad (\text{ii})$$

$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a} \quad (\text{iii})$$

# Hubble tension and EDE

- Early- and late-universe solutions
$$\theta_s = \frac{r_s}{D_A(z^*)}$$
- EDE: scalar field in an expanding universe with a “ $1 - \cos$ ” potential
- EDE increases  $H(z)$  before recombination and decays quickly after
- More about EDE: tomorrow



# References

- Books: Dodelson&Schmidt “Modern Cosmology”, Weinberg: “Cosmology”, Huterer: “A course in cosmology”
- Komatsu “Physics of the Cosmic Microwave Background” (recorded lecture)
- Reviews about EDE:
  - Poulin et al. “The Ups and Downs of Early Dark Energy solutions to the Hubble tension: A review of models, hints and constraints circa 2023”
  - Kamionkowski&Riess: “The Hubble Tension and Early Dark Energy”