From ΛCDM to EDE

Lecture 2: Observation

Laura Herold

CosmoVerse@Corfu May 13 - 18, 2024

Overview

Lecture 1: Theory from ΛCDM to EDE (1:30h)

Hands-on session 1: From theory to predictions (1h)

Lecture 2: Observation — Can EDE solve the Hubble tension? (1h)

Hands-on session 2: Compare EDE predictions with CMB and SN data (1:30h)

Outline: Can EDE solve the Hubble tension?

1. Short overview over statistical tools
2. Recap EDE
3. Can EDE resolve the H_0 tension? — Review of constraints from the literature
4. Conclusions

Statistical tools

Very short overview Bayesian & frequentist statistics

Statistical tools

- I will only give a very short overview here about:
  - Point estimates
  - Bayesian and frequentist parameter intervals
- Matteo Martinelli will go into details about Bayesian statistics and MCMC in the afternoon lecture!

The likelihood

- The central quantity in both Bayesian & frequentist statistics is the likelihood:

$$\boxed{\mathscr{L}(\boldsymbol{x}\,|\,\boldsymbol{\mu}(\boldsymbol{\theta})) \equiv \mathscr{L}_{\boldsymbol{x}}(\boldsymbol{\theta})}$$

- "Probability of the data $\boldsymbol{x}$ given the model $\boldsymbol{\mu}$ with parameter(s) $\boldsymbol{\theta}$"
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- The likelihood is typically computed for a fixed (measured or simulated) data set, hence $\boldsymbol{x}$ is often written as a subscript or dropped
- The larger $\mathscr{L}_{\boldsymbol{x}}(\boldsymbol{\theta})$ the better the agreement of the model with the data

Gaussian likelihood

- One very common likelihood is the Gaussian likelihood (in n dim): $\boldsymbol{x} = \vec{x} = (x_1, \ldots, x_n)$

$$\mathcal{L}_{\boldsymbol{x}}(\boldsymbol{\mu}(\boldsymbol{\theta})) = \frac{1}{\sqrt{(2\pi)^n |C|}} \exp\left[-\frac{1}{2}\sum_{i,j}[x^i - \mu^i(\boldsymbol{\theta})]^T C_{ij}^{-1}[x^j - \mu^j(\boldsymbol{\theta})]\right]$$

"data - model" covariance matrix

- The covariance matrix C_{ij} gives the correlation between different data points; it typically needs to be estimated from simulations or analytically

Gaussian likelihood

- In one dimension, the likelihood simplifies:

$$\mathscr{L}_x(\mu(\theta)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{[x-\mu(\theta)]^2}{2\sigma^2}\right]$$

variance

- The Gaussian likelihood is commonly encountered in cosmological data analysis and many other areas

Point Estimates

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1. Maximum likelihood estimator (MLE) $\hat{\boldsymbol{\theta}}$:

$$\boxed{\left.\frac{\partial \ln(\mathscr{L})}{\partial \theta_i}\right|_{\theta=\hat{\theta}} = 0}$$

- I.e. $\hat{\boldsymbol{\theta}}$ is the parameter (set), which maximizes the likelihood
- Since $\mathscr{L}$ typically has one extremum, the above condition is enough
- This is also called the “bestfit”

Point Estimates

2. Least squares estimator:

$$\left.\frac{\partial\chi^2}{\partial\theta_i}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = 0 \text{ with}$$

$$\boxed{\chi^2(\boldsymbol{\theta}) = \sum_{i,j}[x^i - \mu^i(\boldsymbol{\theta})]^T C_{ij}^{-1}[x^j - \mu^j(\boldsymbol{\theta})]}$$

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$$\mathscr{L}_{\boldsymbol{x}}(\boldsymbol{\mu}(\boldsymbol{\theta})) \sim \exp\left[-\frac{1}{2}\sum_{i,j}[x^i - \mu^i(\boldsymbol{\theta})]^T C_{ij}^{-1}[x^j - \mu^j(\boldsymbol{\theta})]\right]$$

- The least-squares estimator coincides with the MLE for a Gaussian likelihood $\chi^2 = -2\log\mathscr{L}$

Bayes theorem

likelihood

$$P(\boldsymbol{\theta}\,|\,\boldsymbol{x}) = \frac{\mathscr{L}(\boldsymbol{x}\,|\,\boldsymbol{\theta}) \cdot \pi(\boldsymbol{\theta})}{P(\boldsymbol{x})}$$

prior

posterior

"evidence"

- The posterior gives the "probability of the model parameters $\boldsymbol{\theta}$ given the data $\boldsymbol{x}$"
- Since the evidence depends only on the data, it contributes only a constant factor and is not important

Parameter intervals

- Apart from finding the “bestfit” or “mean” parameter, it is important to give an errorbar or interval of a parameter, e.g. $H_0 = 70.0 \pm 1.0$ km/s/Mpc
- There are two main philosophies of statistics:
  - Bayesian statistics
  - Frequentist statistics
- Both construct parameter intervals in different ways

Bayesian credible intervals

- Uses Bayes theorem: based on posterior (data & prior):

$$P(\theta\,|\,x) \sim \mathscr{L}(x\,|\,\theta) \cdot \pi(\theta)$$

- Credible interval $[\theta^-, \theta^+]$ of C.L. α:

$$\int_{\theta^-}^{\theta^+} P(\theta\,|\,x)\, d\theta = \alpha$$

- Method: Obtain posterior from Markov-Chain Monte-Carlo (MCMC)

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- Method: Obtain posterior from Markov-Chain Monte-Carlo (MCMC)
- 1D interval — marginalization: $$P(\theta\,|\,\boldsymbol{x}) = \int d\boldsymbol{\nu}\, P(\theta, \boldsymbol{\nu}\,|\,\boldsymbol{x})$$
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Bayesian credible intervals

- How to choose the prior?

$$P(\theta \mid x) \sim \mathscr{L}(x \mid \theta) \cdot \pi(\theta)$$

- The prior can encode prior information and believes
- It can be flat, log, Gaussian,…

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- If the data is not very constraining, i.e. the likelihood $\mathscr{L}$ is very flat, then the parameter constraints can have a strong dependence on the prior π
- Hence, it is important to check the sensitivity of the results on the choice of π

Frequentist confidence intervals

- Based on the goodness of fit to the data, i.e. on the likelihood (only data):

$$\mathscr{L}(x \,|\, \theta)$$

- No posterior, no prior

- Interpretation: "The true value θ_0 is contained in $[\theta^-, \theta^+]$ a fraction α of the experiments" — statement about the repetition of the experiment

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- Method: Neyman construction

  - Profile likelihood: in the limit of a large data set (the likelihood ratio follows a χ^2 -distribution)

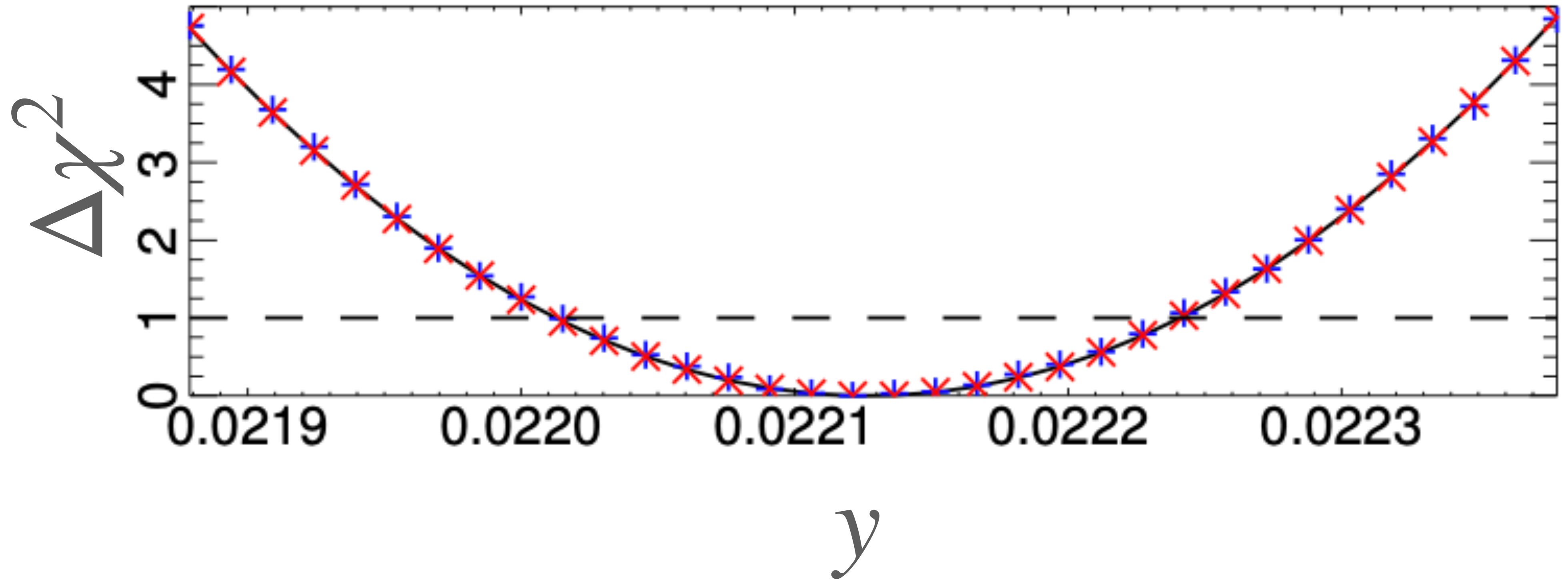
Frequentist confidence intervals

Profile likelihood: Fix parameter y of interest to different values, minimize $\chi^2 = -2\log(\mathscr{L})$ w.r.t. all other parameters

Confidence interval (graphical method):

In the asymptotic limit (Wilks theorem):

- Read off 1σ at the intersection with $\Delta\chi^2 = 1$
- Read off 2σ at the intersection with $\Delta\chi^2 = 4$
- etc.



Example: Planck col. XVI, 2013

Bayesian and frequentist intervals

Bayesian intervals (MCMC):

- Includes prior knowledge as priors: $P(M \mid D) \sim \mathscr{L}(D \mid M) \cdot P(M)$.
- Identifies bulk volumes that fit data well.

- Problem: If data is not constraining enough, can be subject to prior effects.

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Both statistical approaches should agree in the limit of a large data set.

Recap EDE

Early Dark Energy (EDE)

Kamionkowski et al. 2014, Karwal & Kamionkowski 2016, Caldwell & Devulder 2018, Poulin et al. 2019

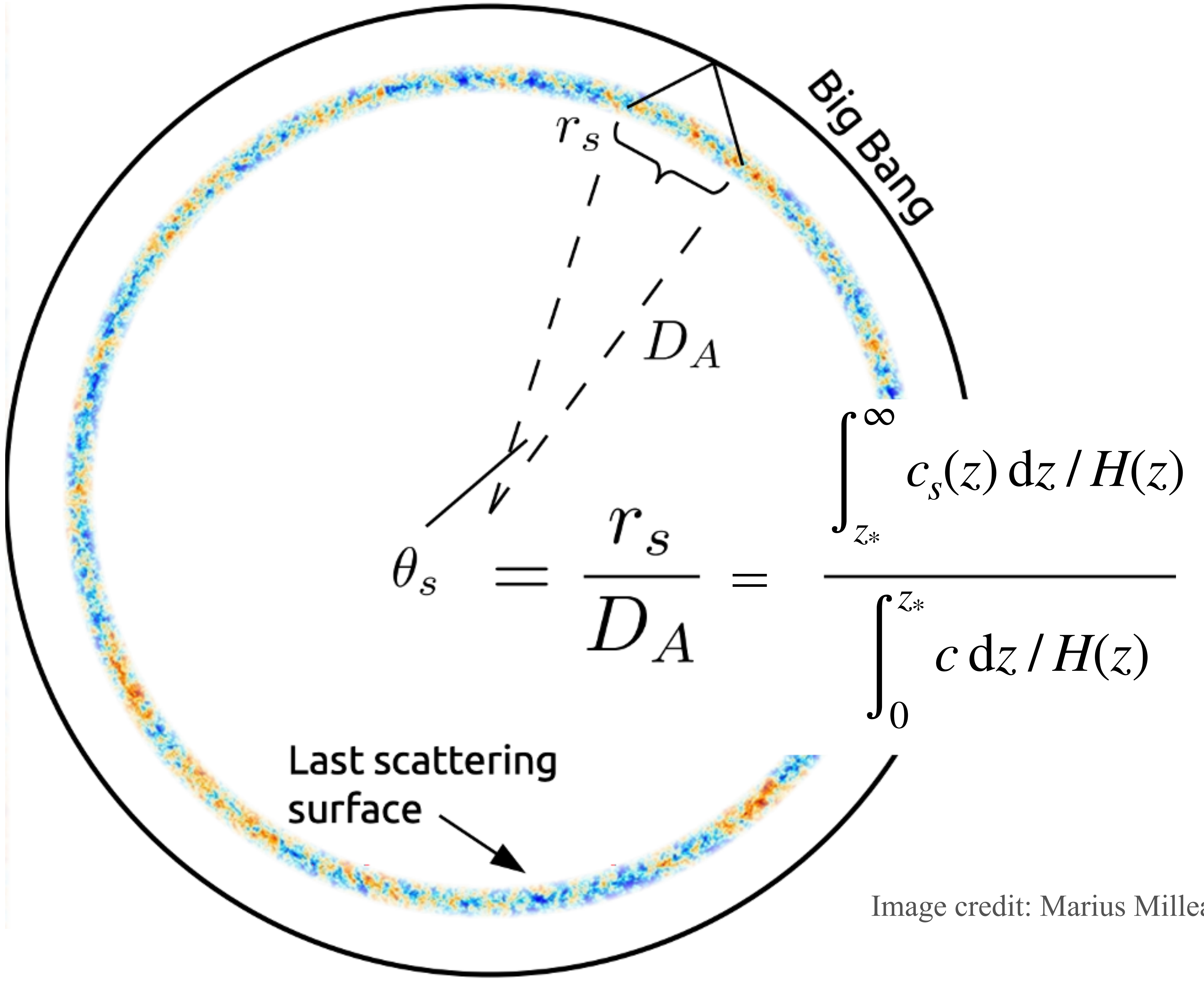

Image credit: Marius Millea

Idea of EDE:

Introducing an additional component before recombination reduces the sound horizon r_s.

$\downarrow$ θ_s fixed

Angular diameter distance D_A decreases.

$\downarrow$ $H(z) = H_0\sqrt{\Omega_m(z) + \Omega_r(z) + \Omega_\Lambda}$

H_0 increases.

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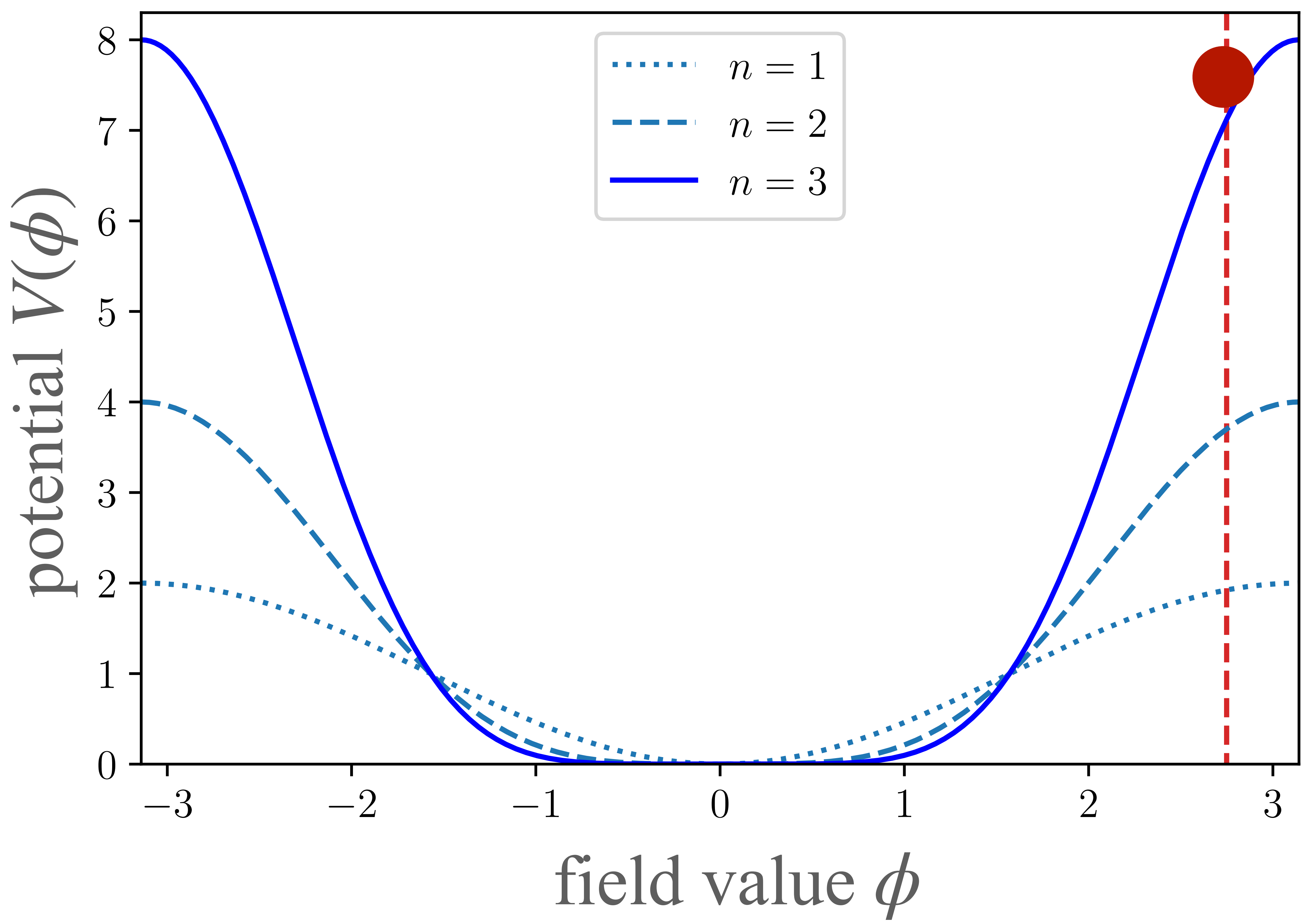
Axion-like EDE model: scalar field ϕ with potential

$$V(\phi) = V_0\,[1 - \cos(\phi/f)]^n$$



Free parameters:

m: 'mass' of ϕ $\rightarrow V_0 = m^2 f^2$,

f: decay constant,

$\theta_i \equiv \phi_i/f$: initial value of the field,

$n = 3$: EDE decays quickly enough.

$\rightarrow$ Field inspired by axion ($n = 1$) with extremely small mass $\sim 10^{-27}$eV

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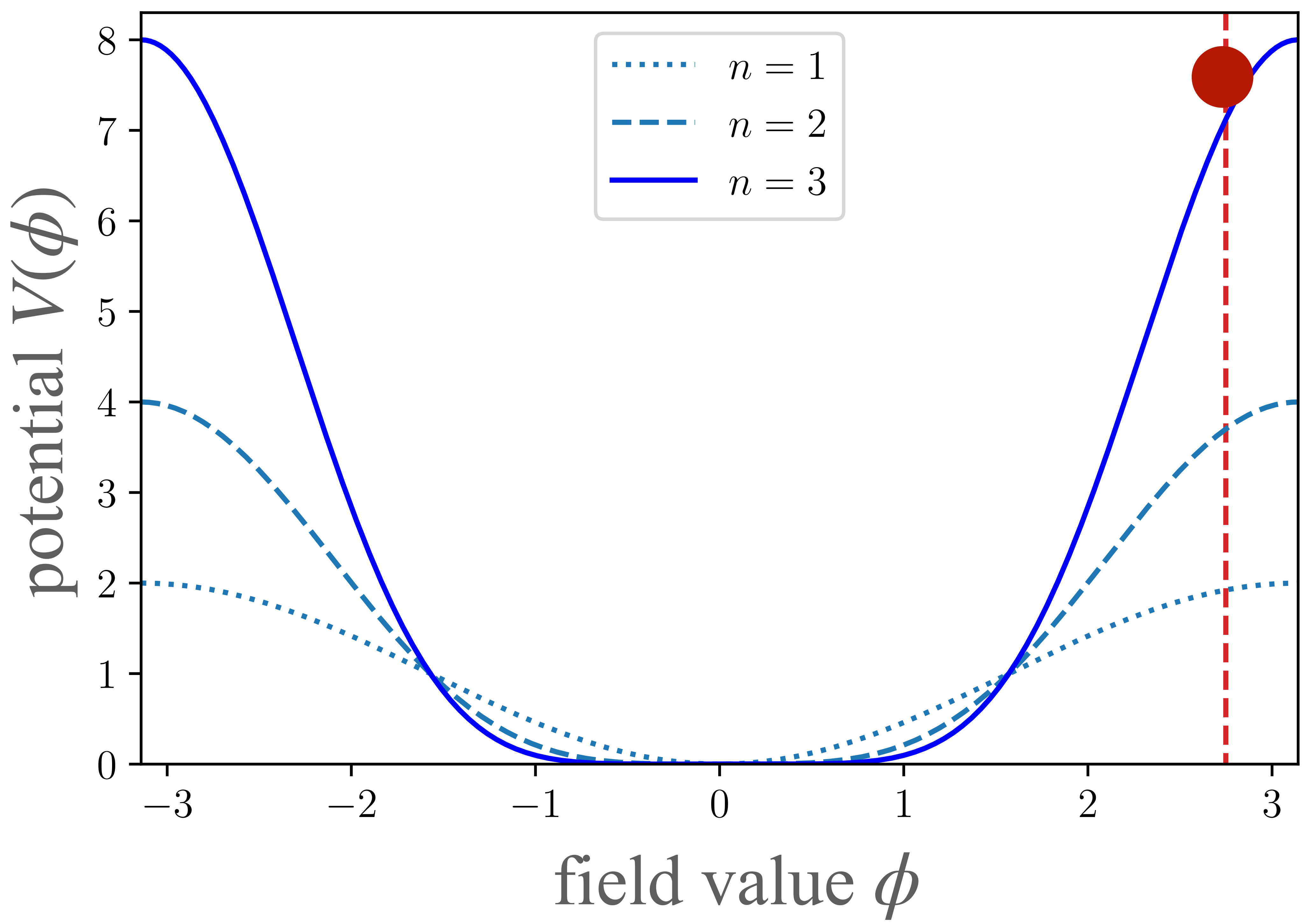
Dynamics: $\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = 0$

- Initially: Hubble friction dominates: ϕ frozen —> behaves as DE

- At z_c: Hubble friction < potential term: ϕ starts oscillating and decays faster than matter



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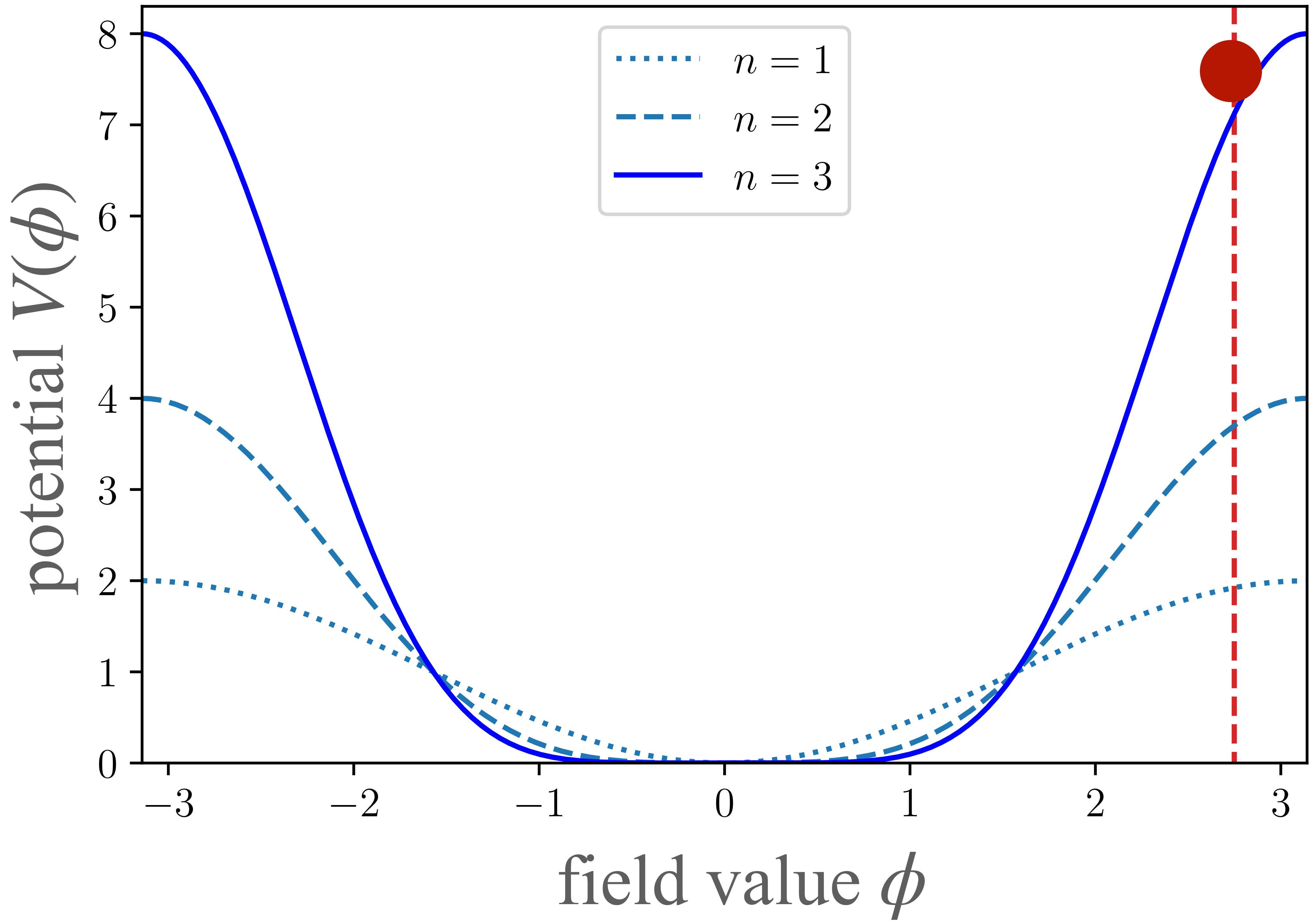
Eq. of state: $$\begin{cases} w = -1 \text{ for } z > z_c \\ \langle w \rangle = \dfrac{n-1}{n+1} = \dfrac{1}{2} \text{ for } z < z_c \end{cases}$$



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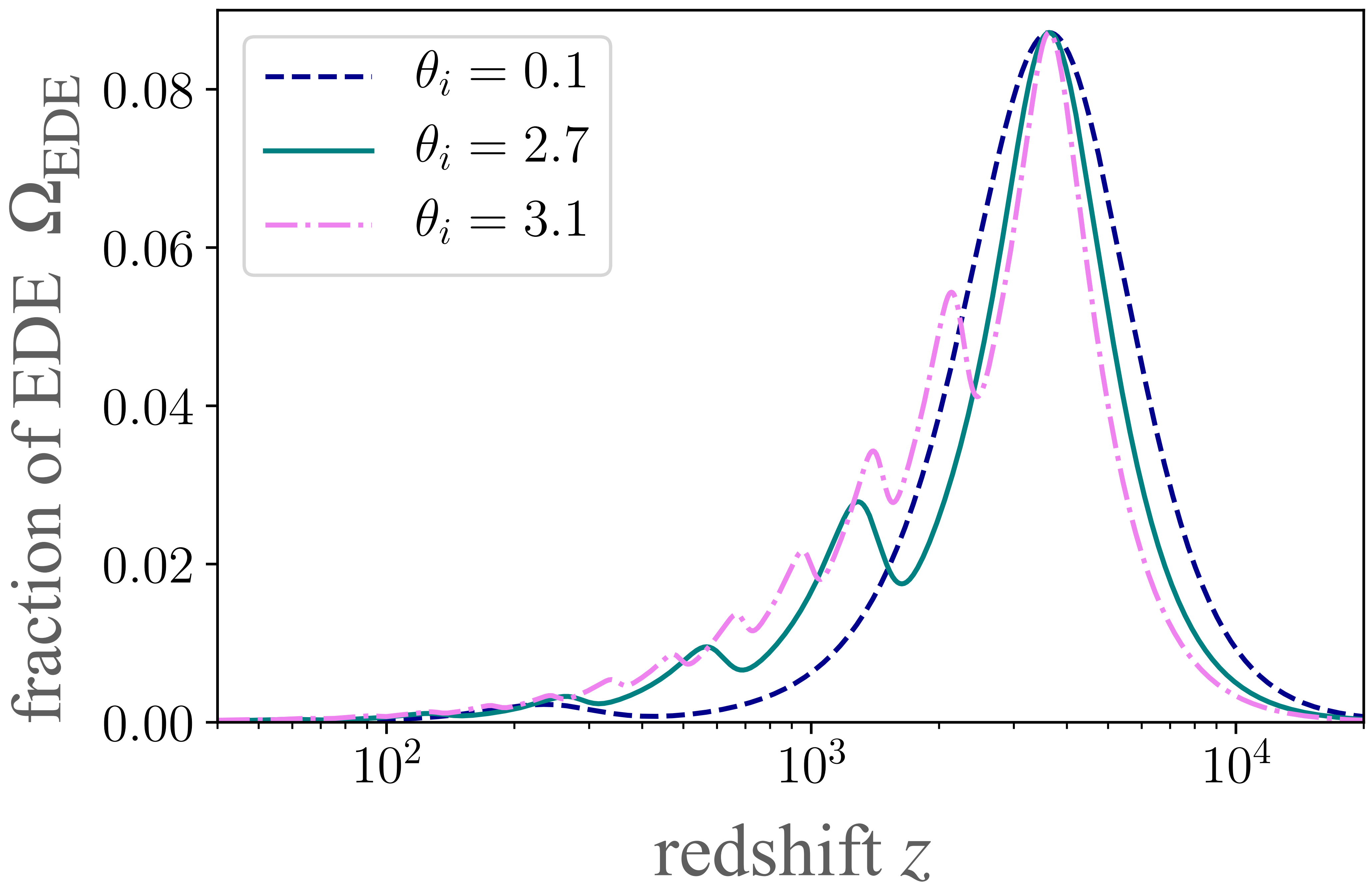

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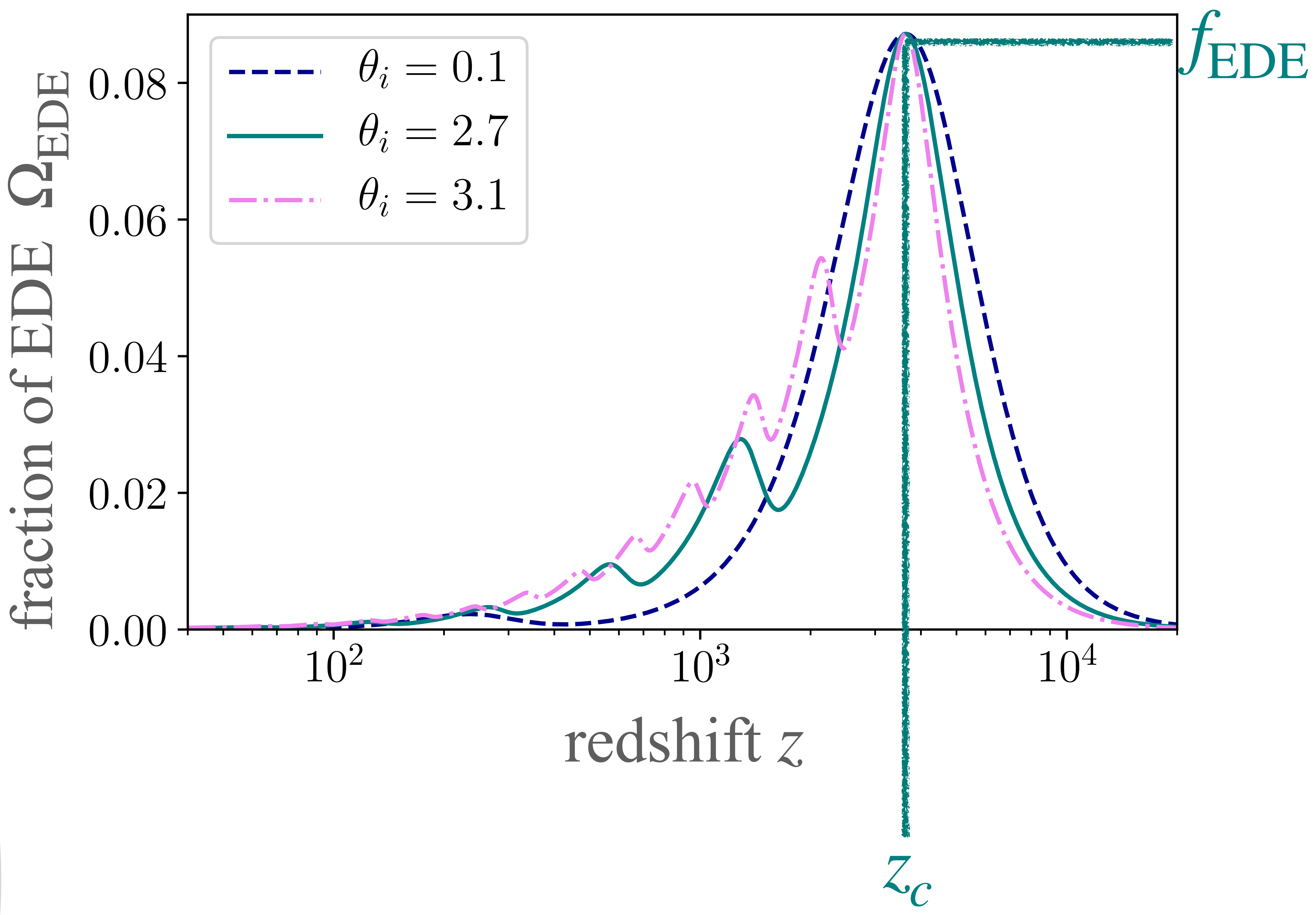

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$f_{\rm EDE}$, z_c, θ_i (3 additional parameters to ΛCDM)

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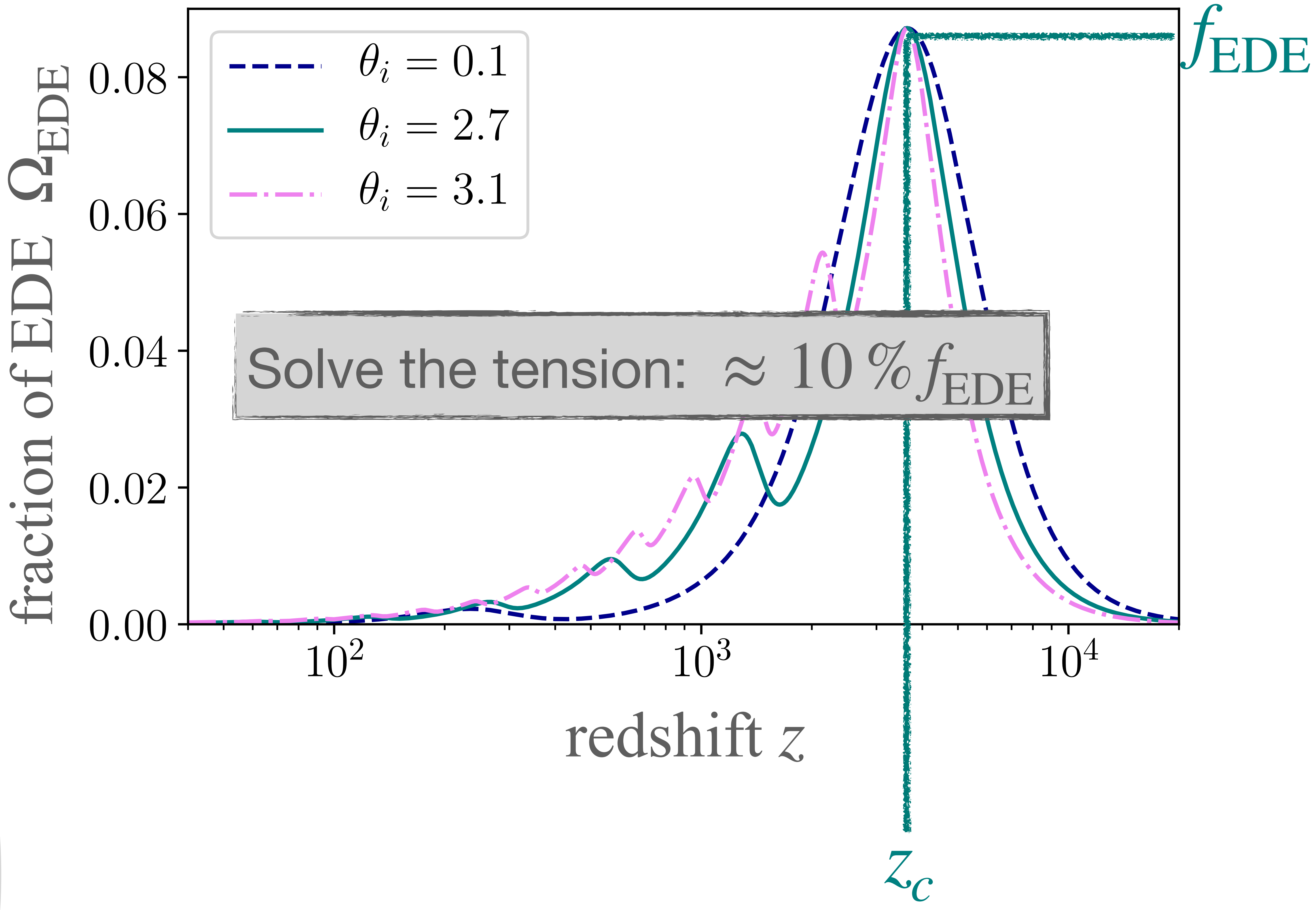

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Can EDE resolve the H_0 tension

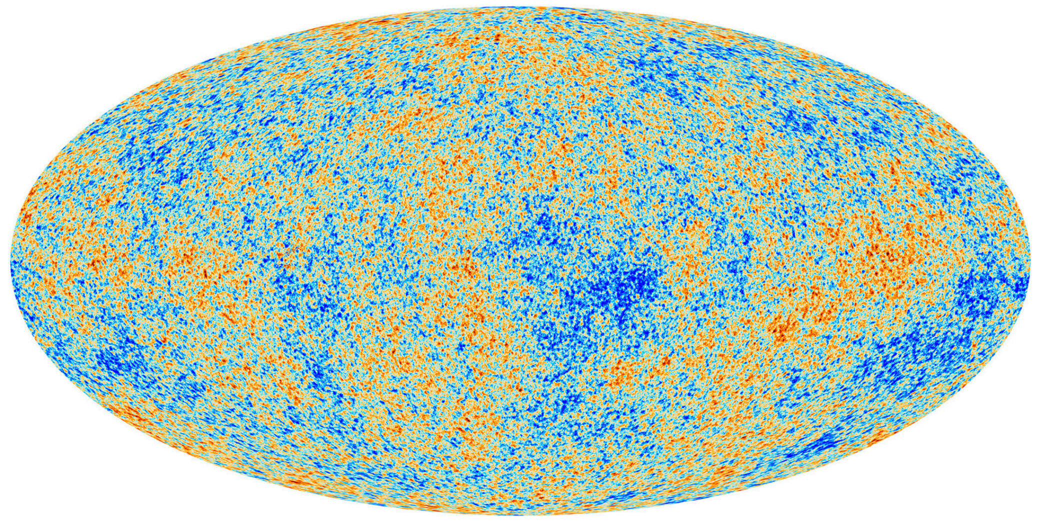
… and fit all available data sets?

Disclaimer: this review about constraints on EDE is **not** complete

Can EDE solve the H_0 tension and fit these data sets?

Data sets: Planck + 6dFGS + BOSS DR12 BAO/ RSD + Pantheon + SH0ES 2016

CMB (TT, TE, EE, lensing)



ESA

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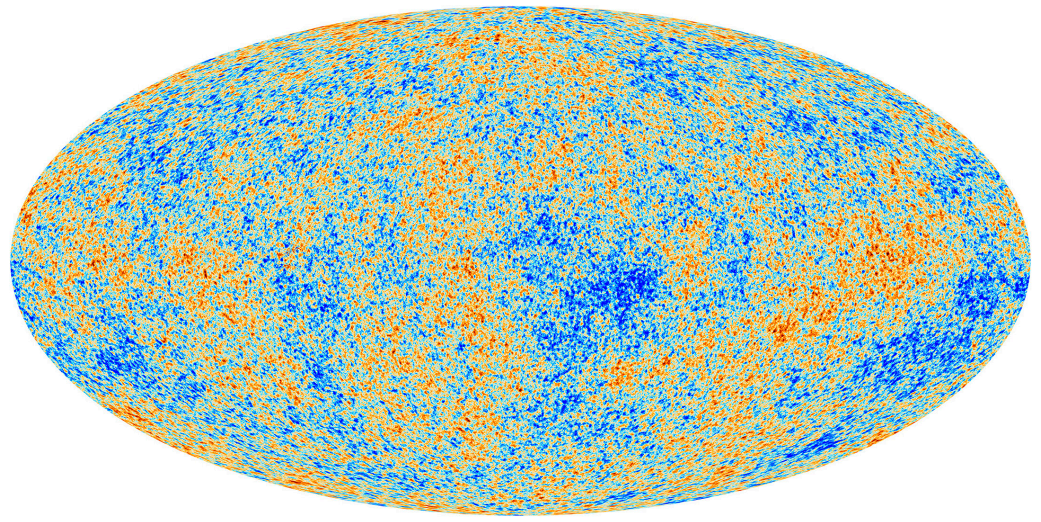
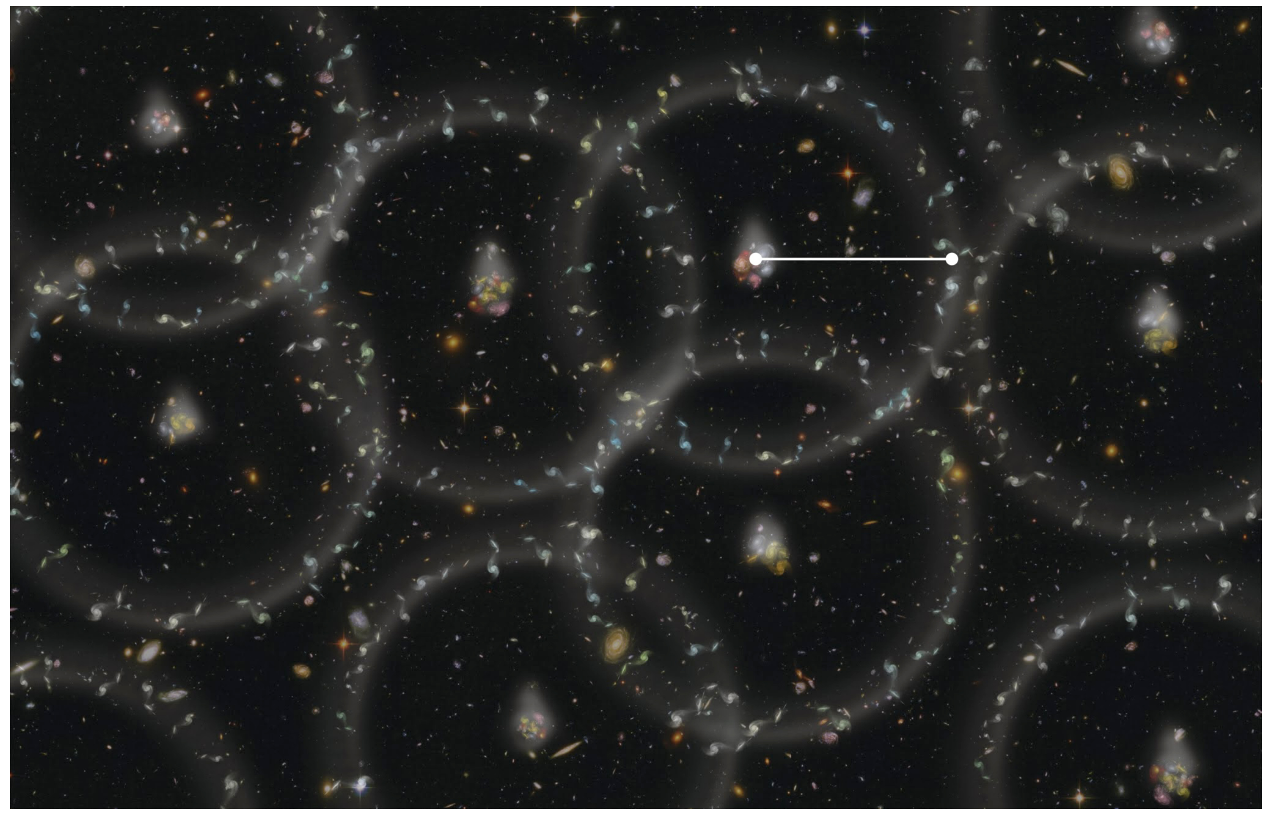
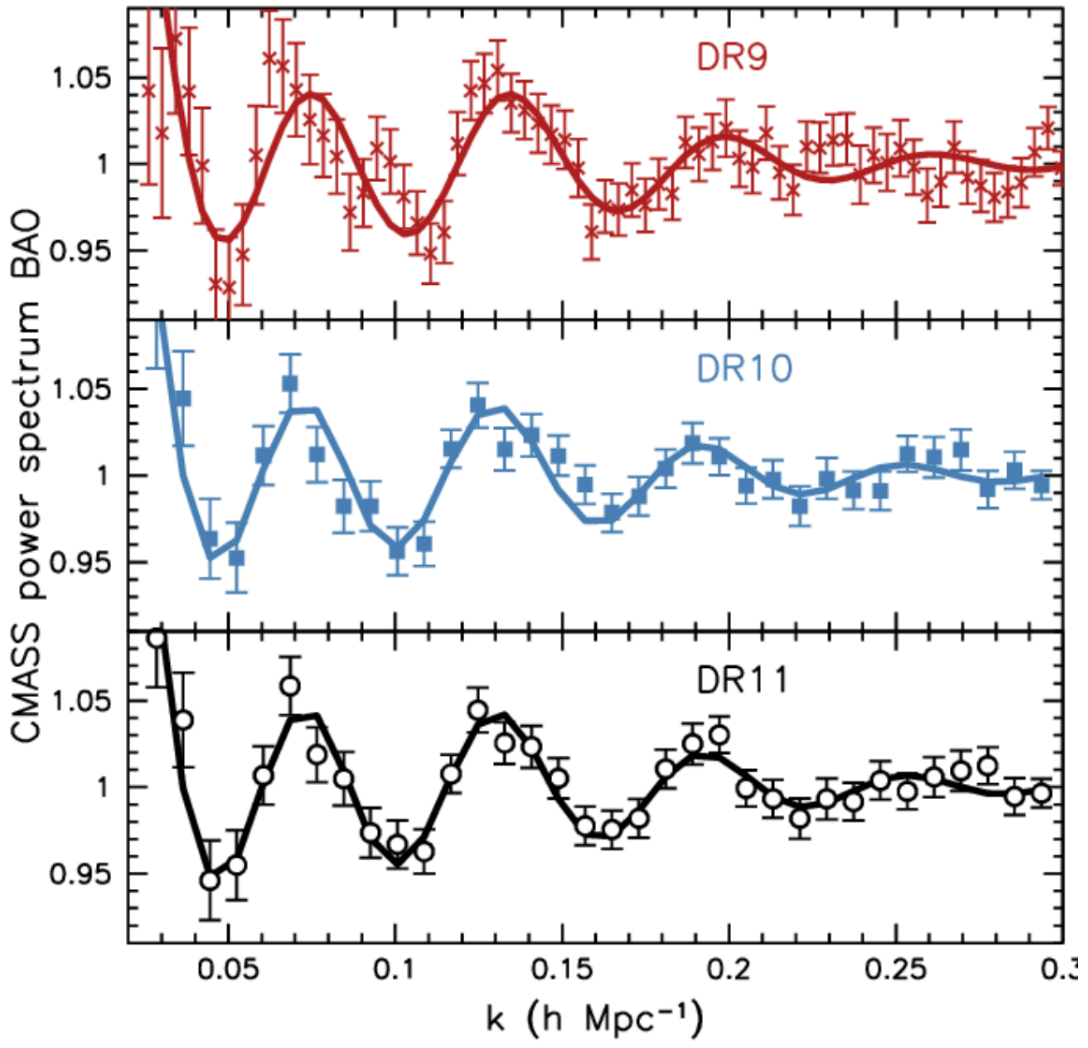


ESA

Baryon acoustic oscillations



BOSS collaboration



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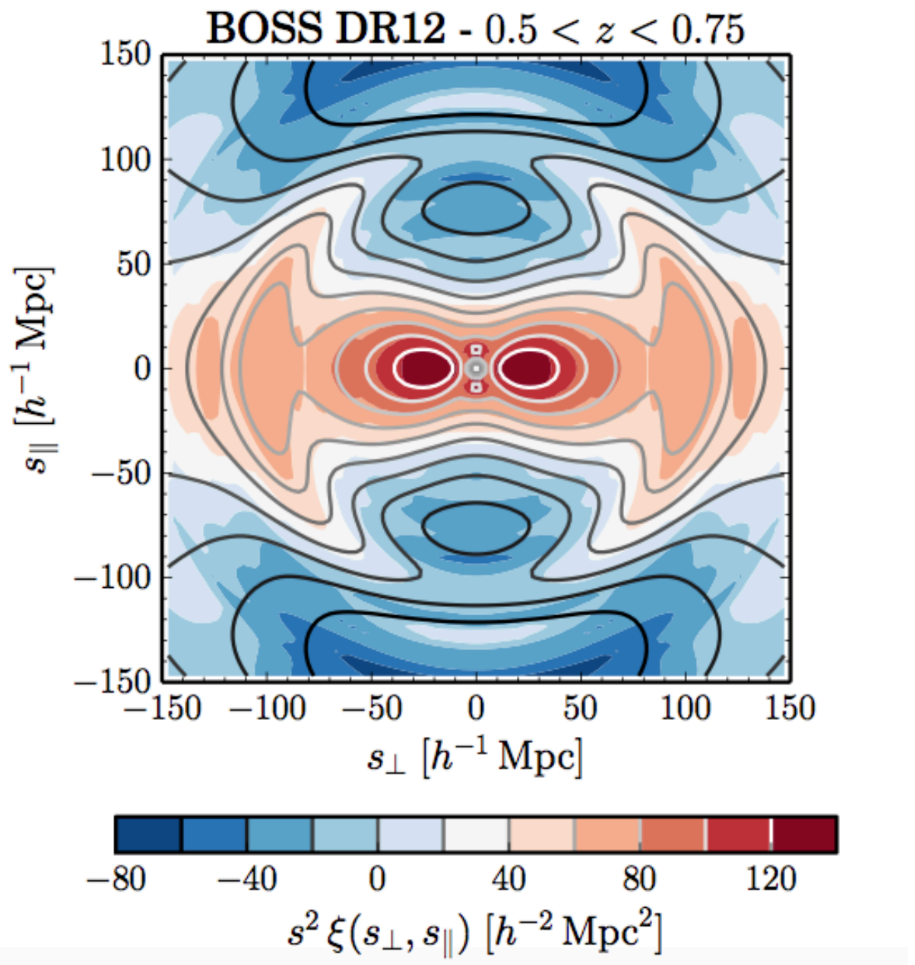
Redshift space distortions



BOSS collaboration (Sanchez++ 2016)

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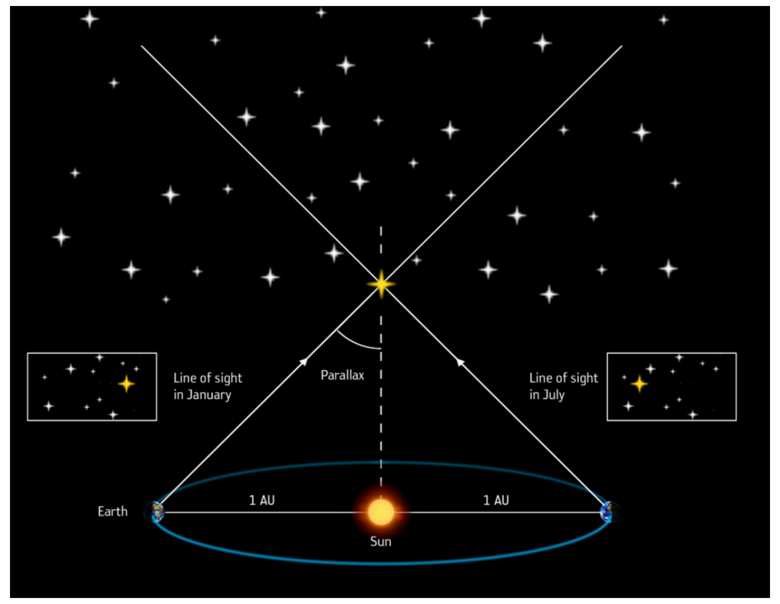
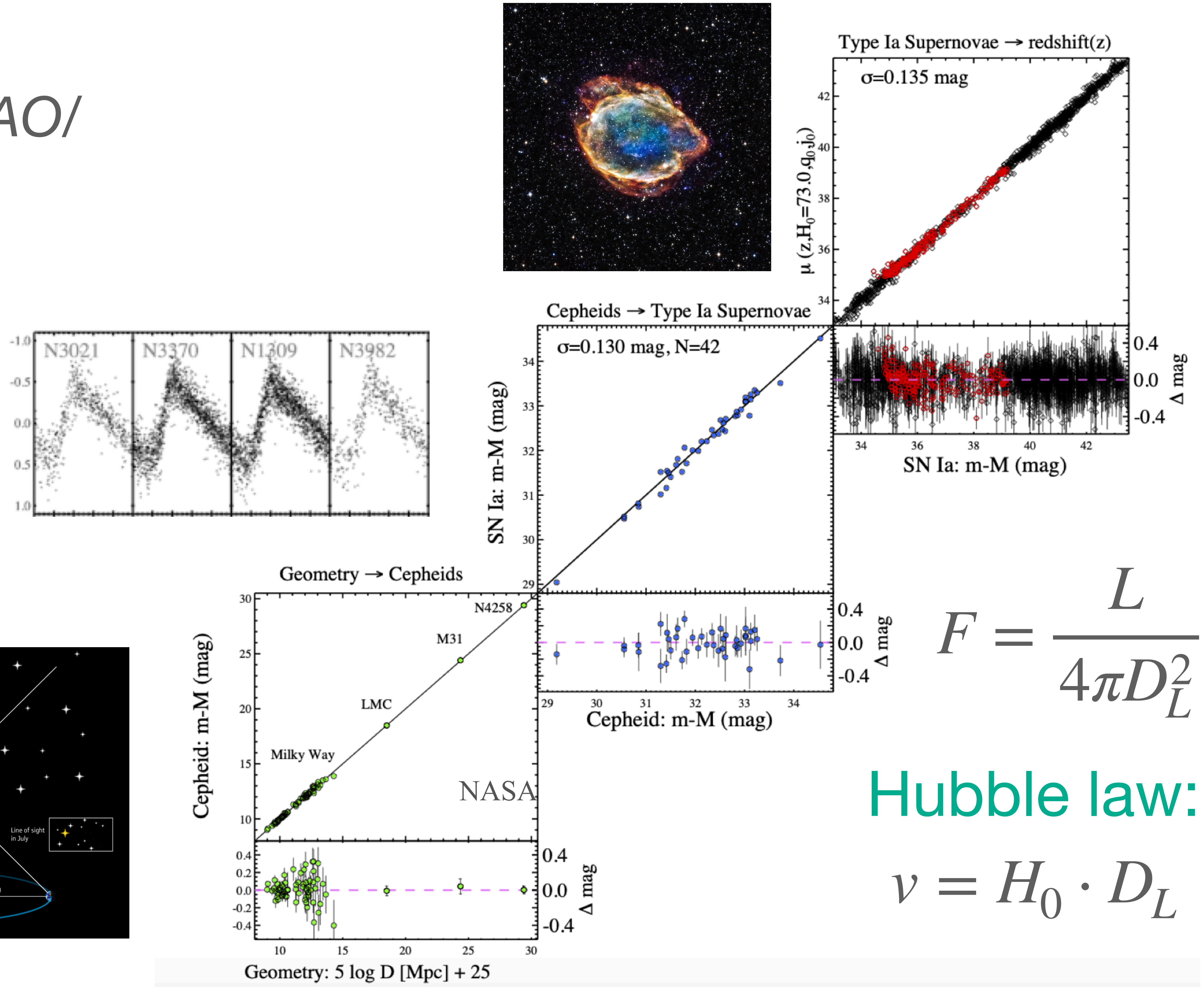
Cepheid-calibrated SNIa
(distance ladder)

Gaia col.





NASA

$$F = \frac{L}{4\pi D_L^2}$$

Hubble law:

$$v = H_0 \cdot D_L$$

SH0ES collaboration 2022 (Riess et al.)

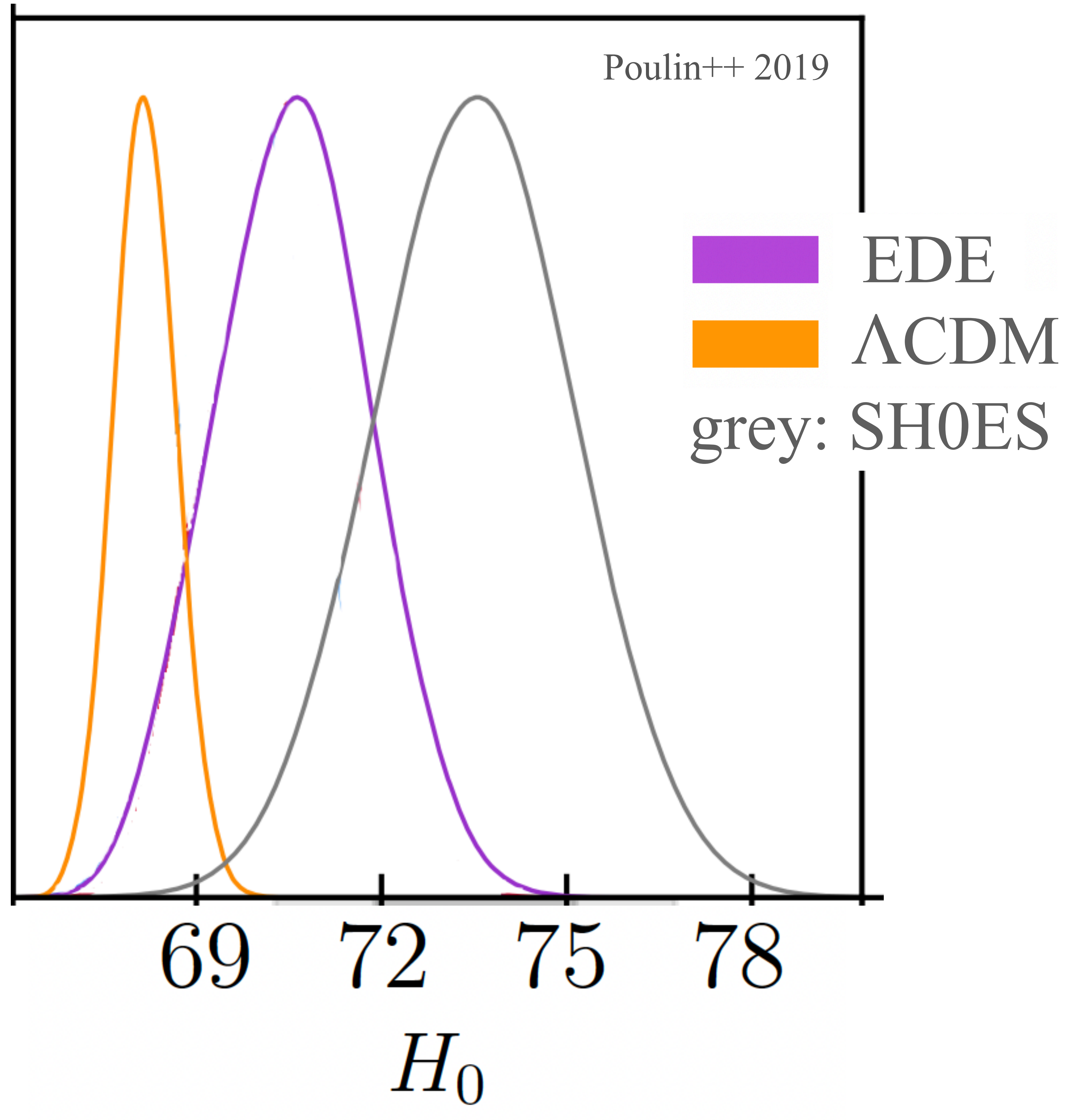
2019: EDE can solve the H_0 tension!

Poulin, Smith, Karwal, Kamionkowski, 2019

Data sets: Planck + 6dFGS + BOSS DR12 BAO/RSD + Pantheon + SH0ES 2016

- $f_{\mathrm{EDE}} = 0.107^{+0.035}_{-0.030}$ (mean $\pm 1\sigma$)
- $H_0 = 71.49 \pm 1.20$ km/s/Mpc

MCMC posterior



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- Also other parameters shift:
  EDE suppresses growth of perturbations at early times (eISW)
  $\rightarrow$ $\omega_{\rm CDM}$ and n_s increase

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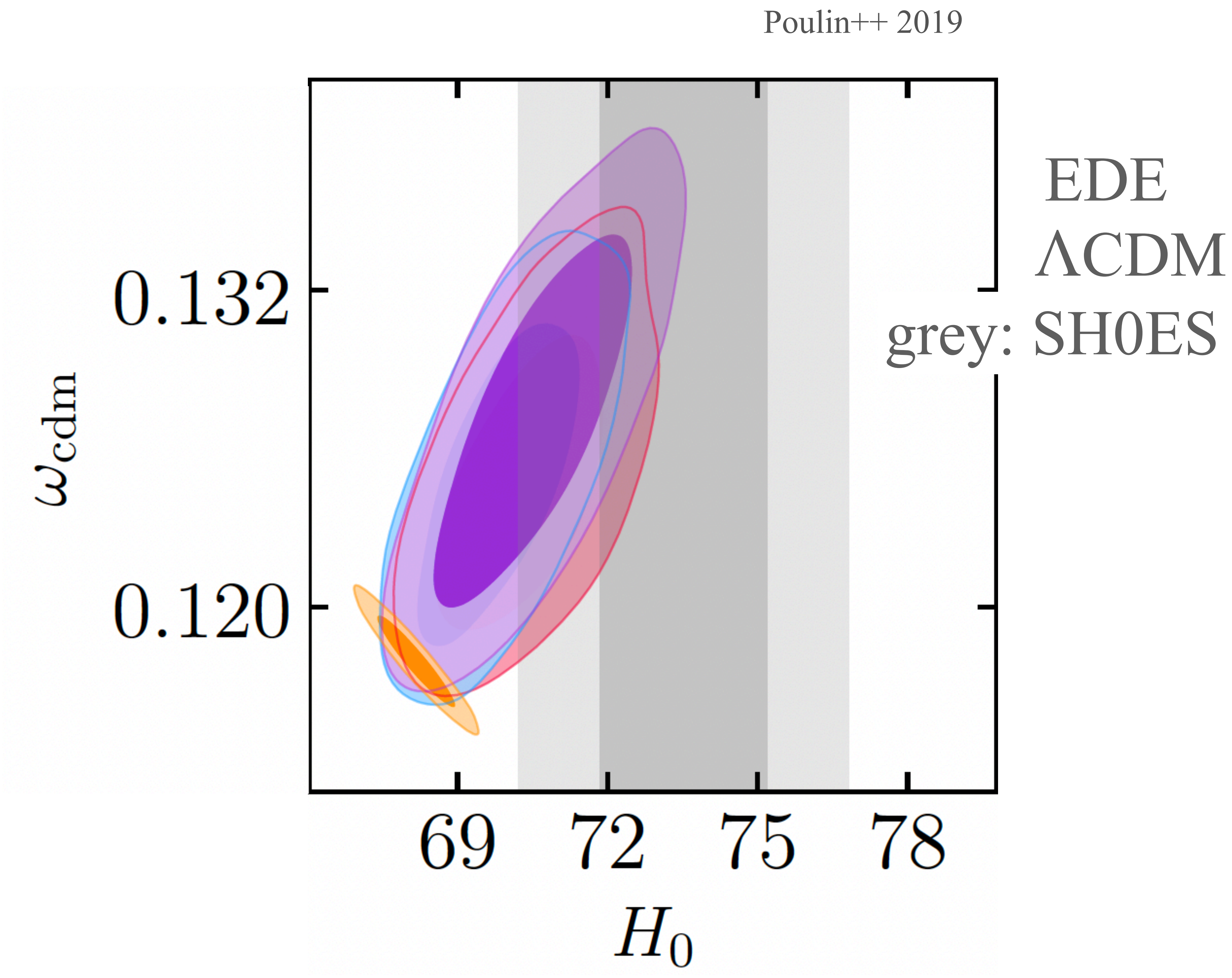
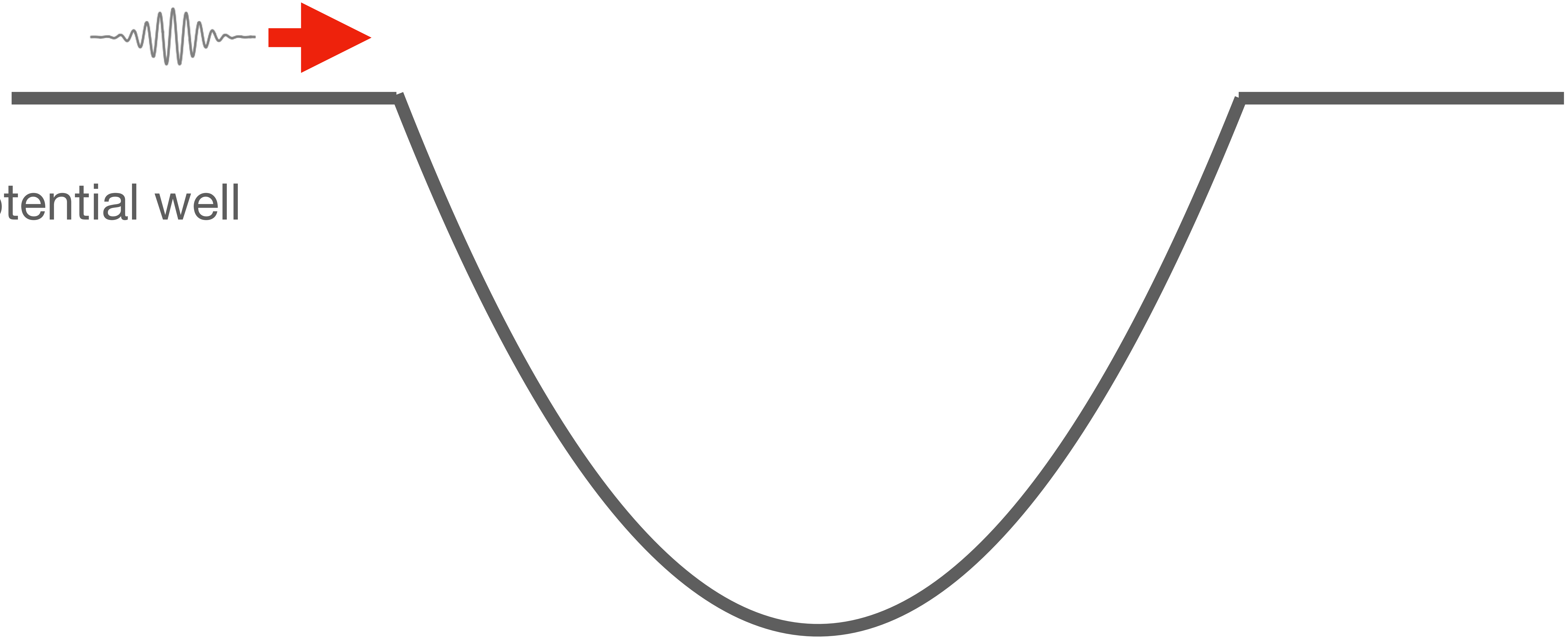


Aside: early Integrated Sachs-Wolfe effect



- potential well

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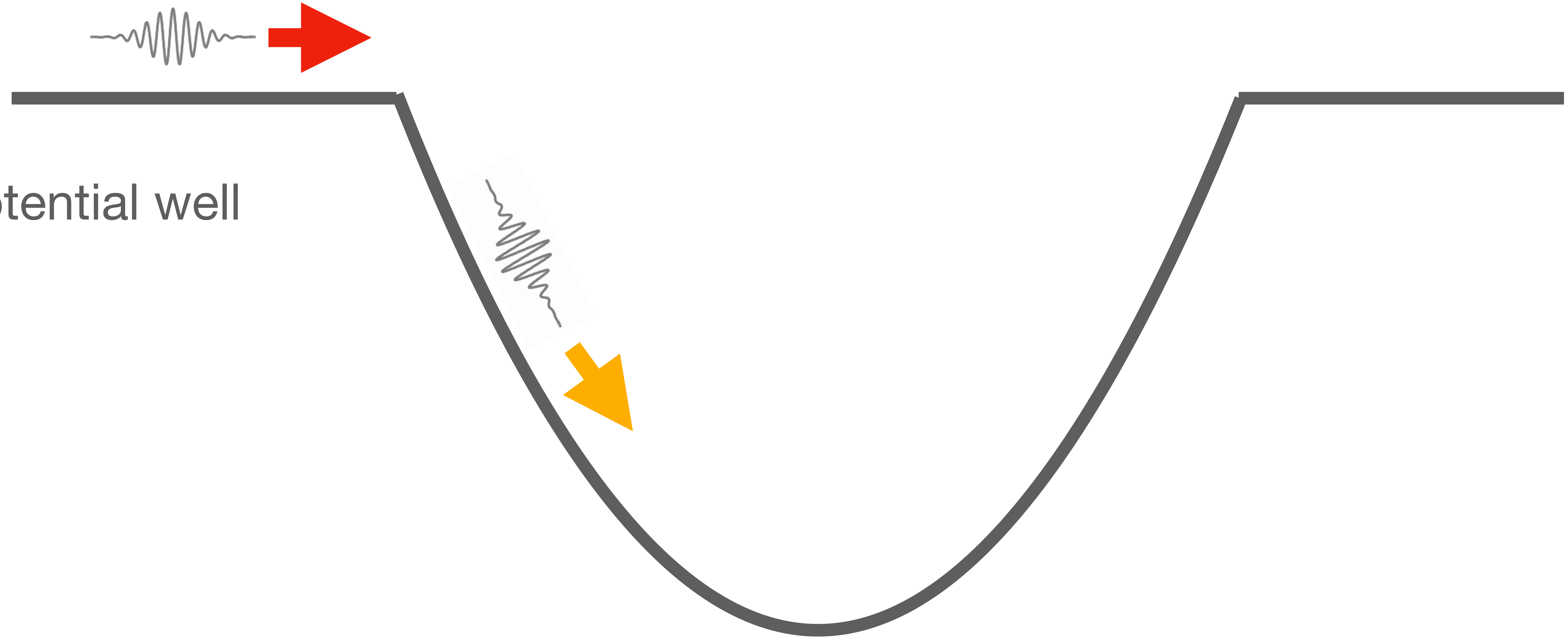

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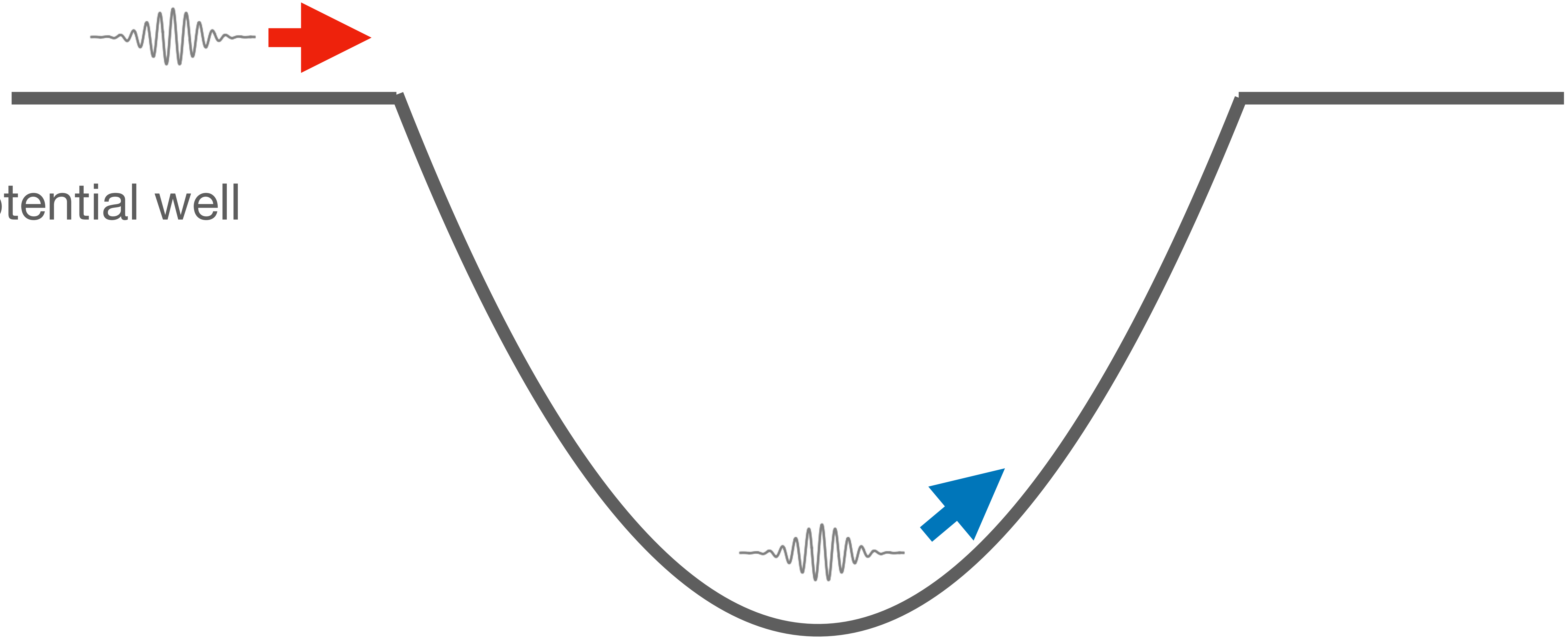

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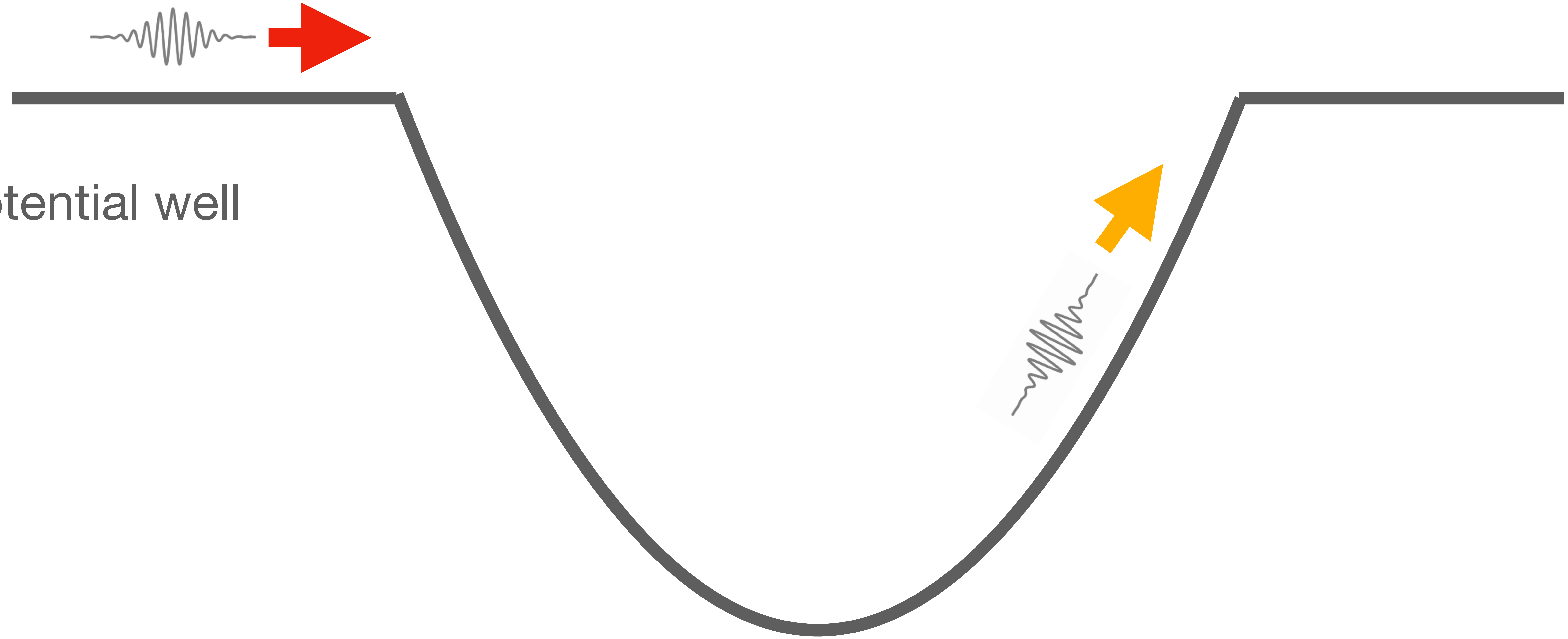

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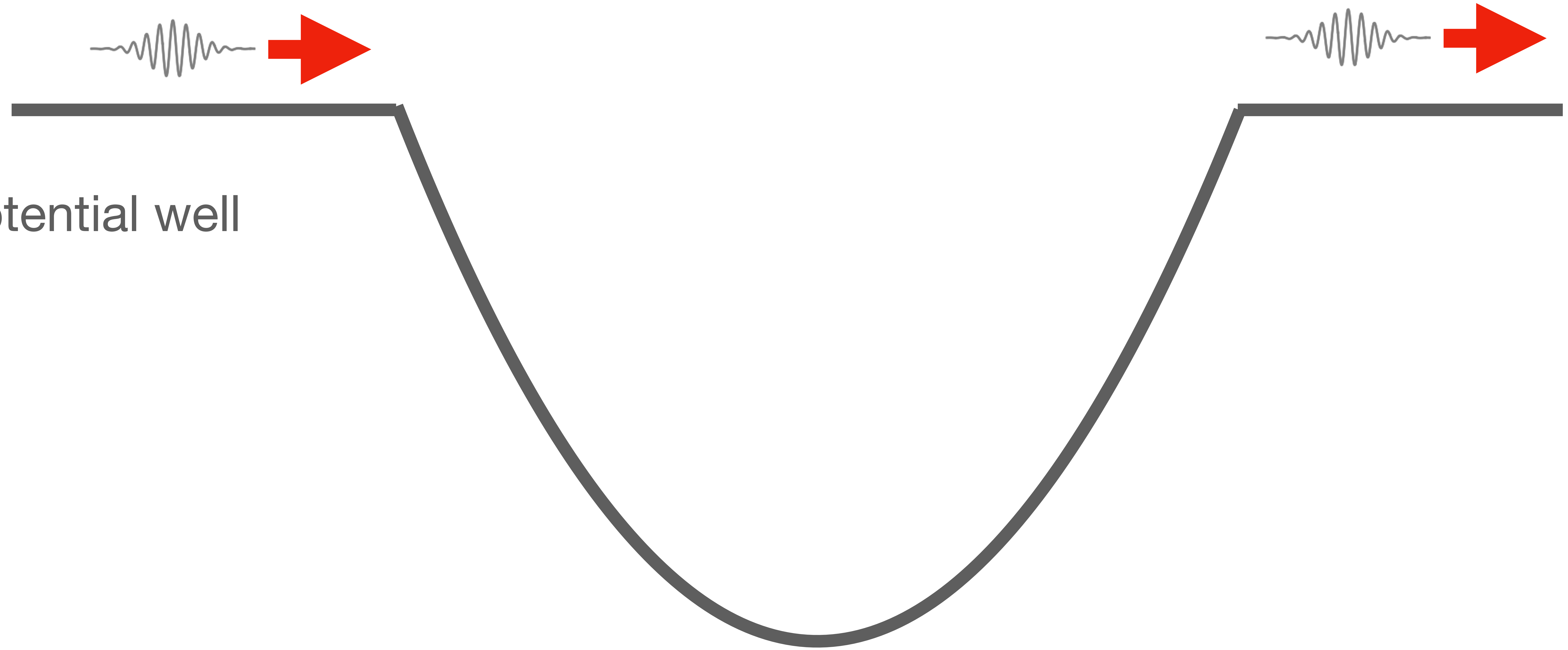

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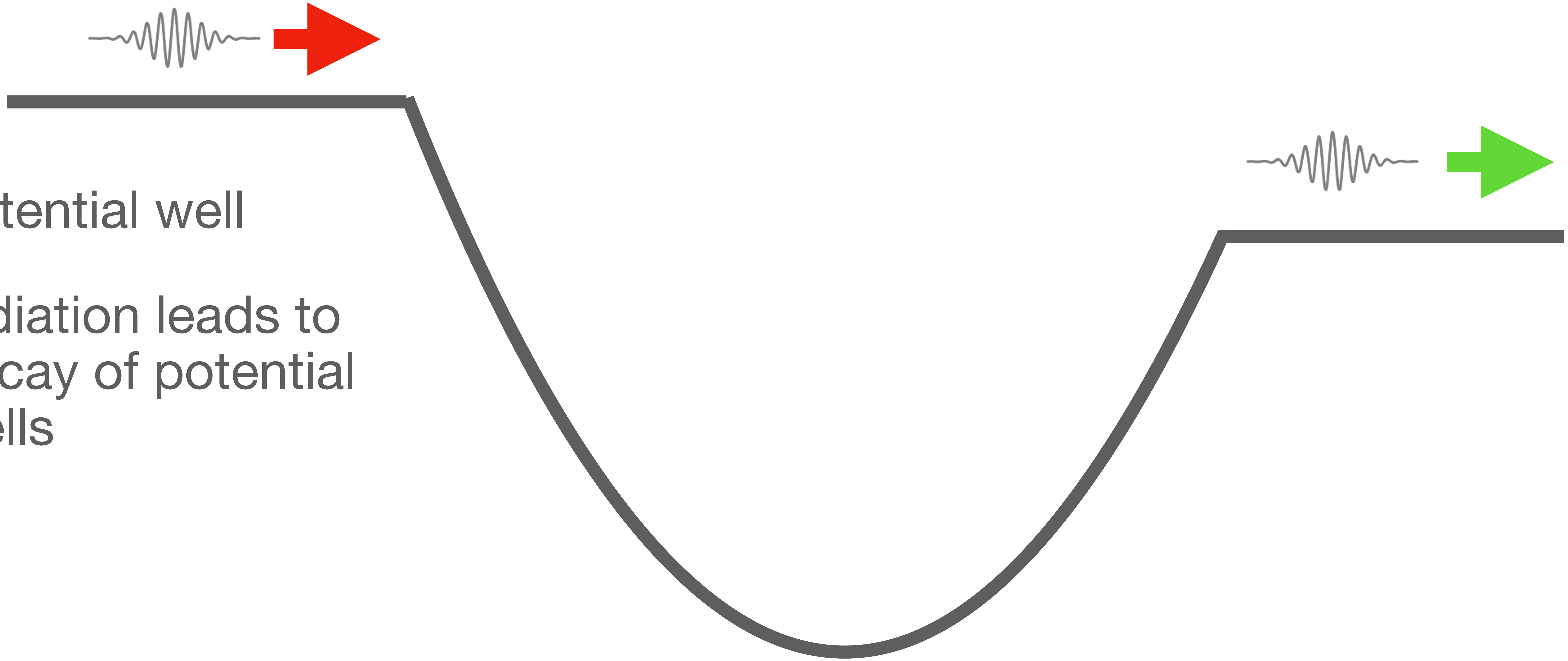

- potential well
- radiation leads to decay of potential wells

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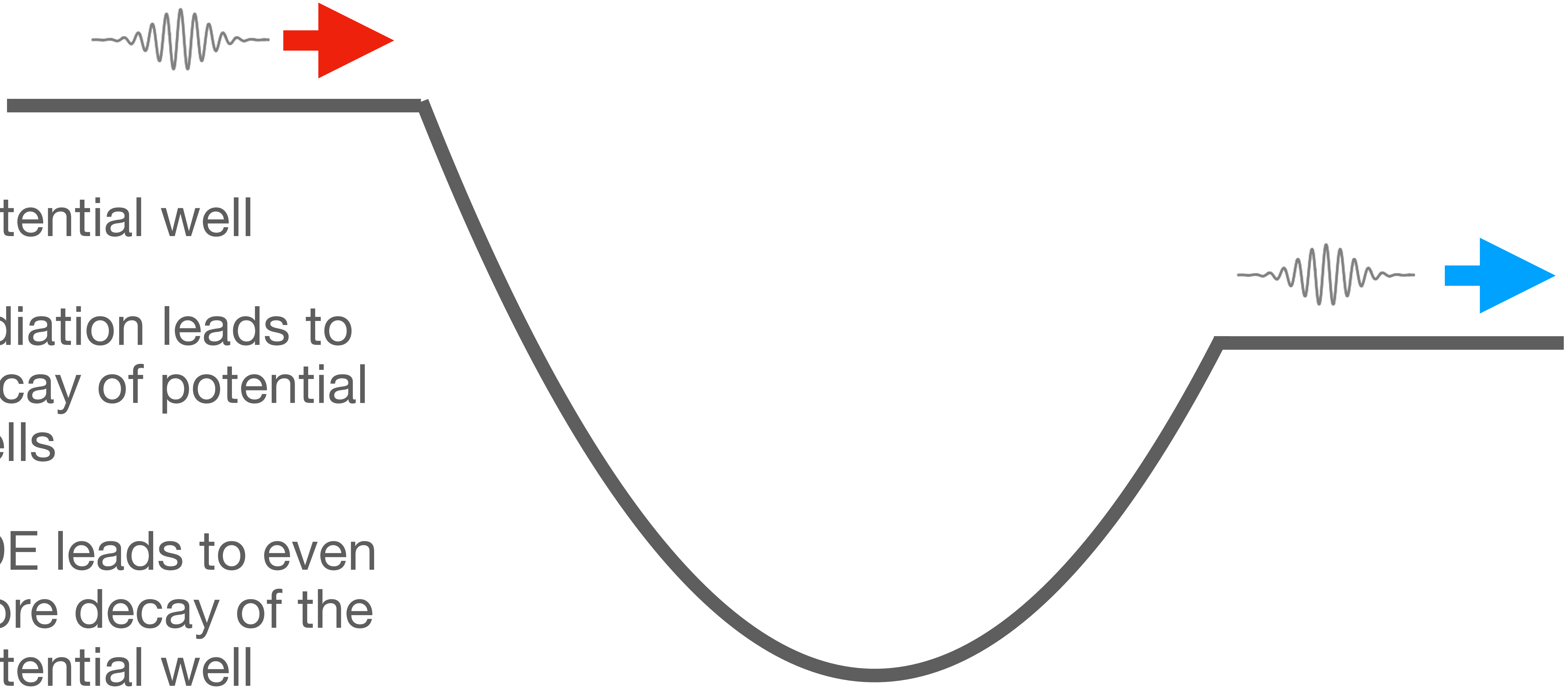

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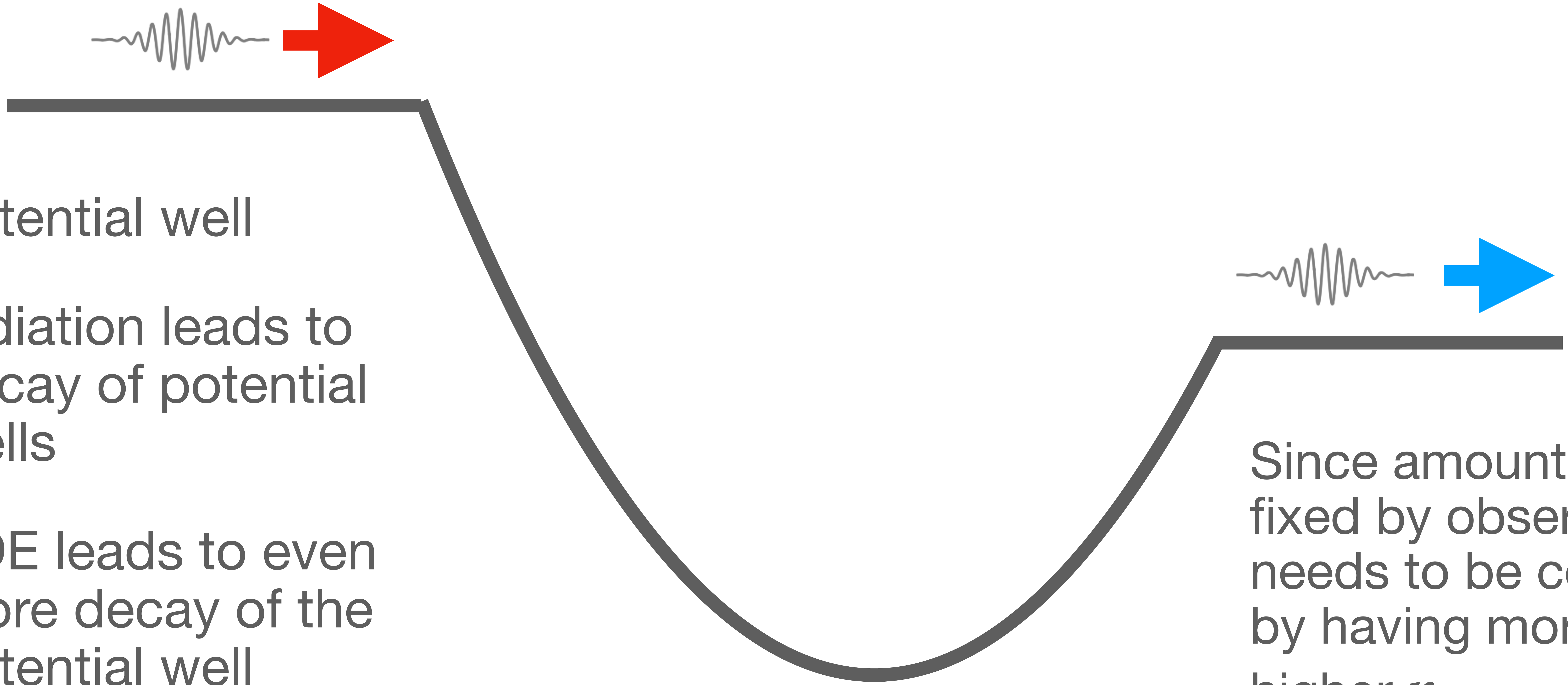

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Since amount of eISW is fixed by observation, this needs to be compensated by having more DM and higher n_s

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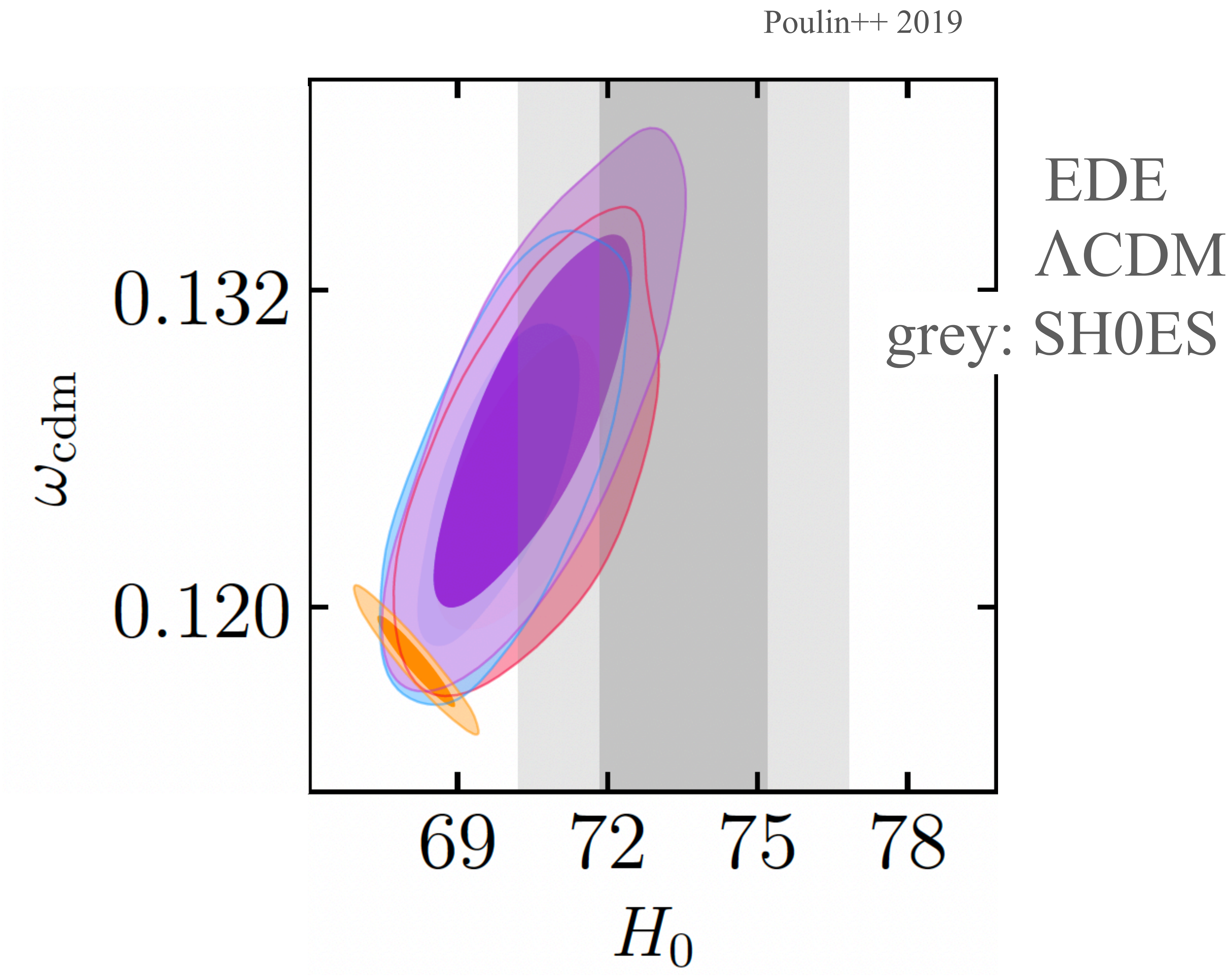
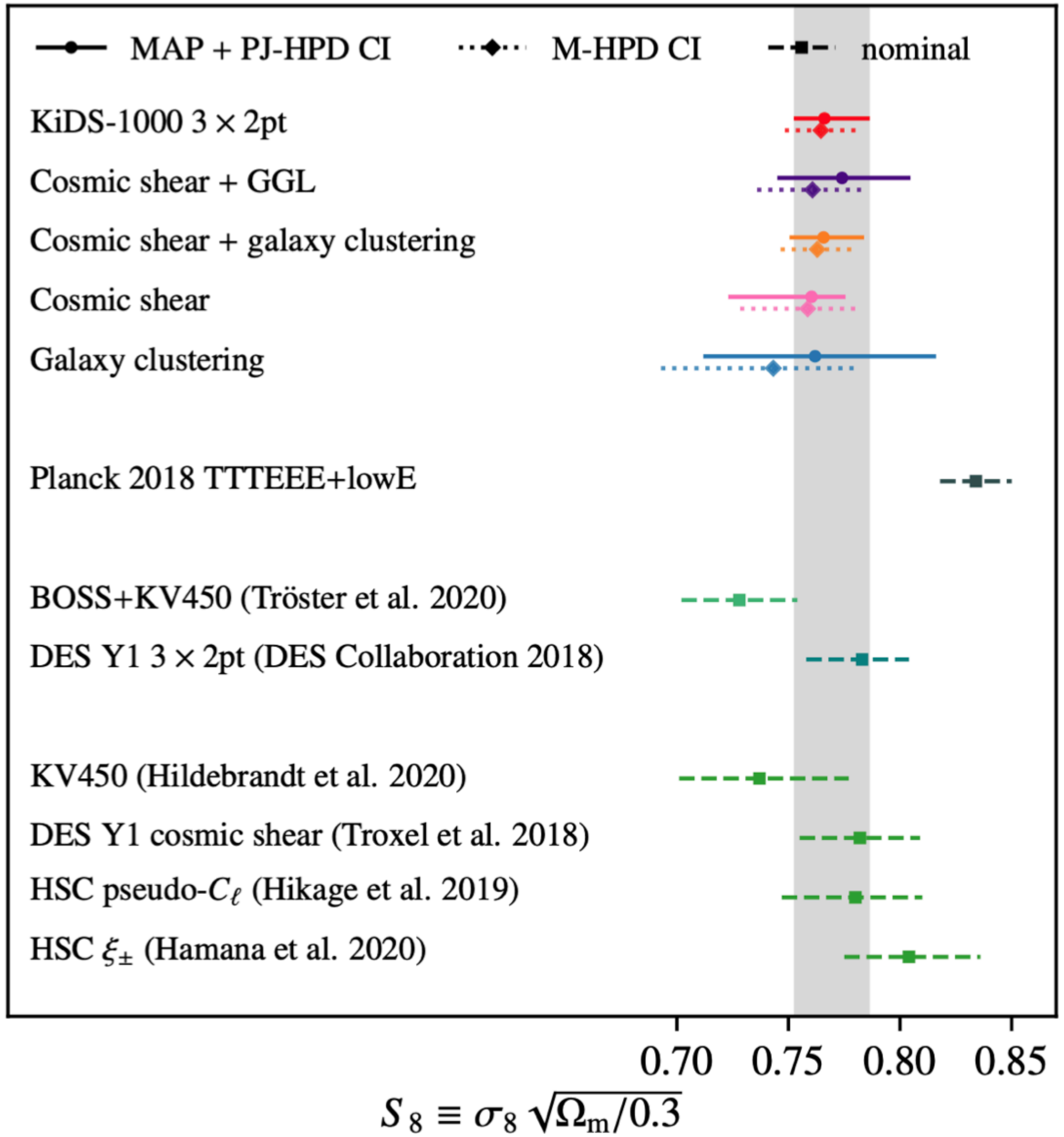
  $\rightarrow$ σ_8 increases, worsening the so-called σ_8 discrepancy

MCMC posterior



Aside: σ_8 discrepancy

Cosmology Intertwined 2021



$$\sigma_8^2 = \int dk^3 \, | W(k) |^2 P_{\rm lin}(k), \qquad S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$$

Fourier transform of top-hat filter
with radius 8 Mpc/h

linear matter power spectrum

Planck prefers 2-3 σ higher S_8 than
weak lensing experiments (DES, KiDS, HSC)

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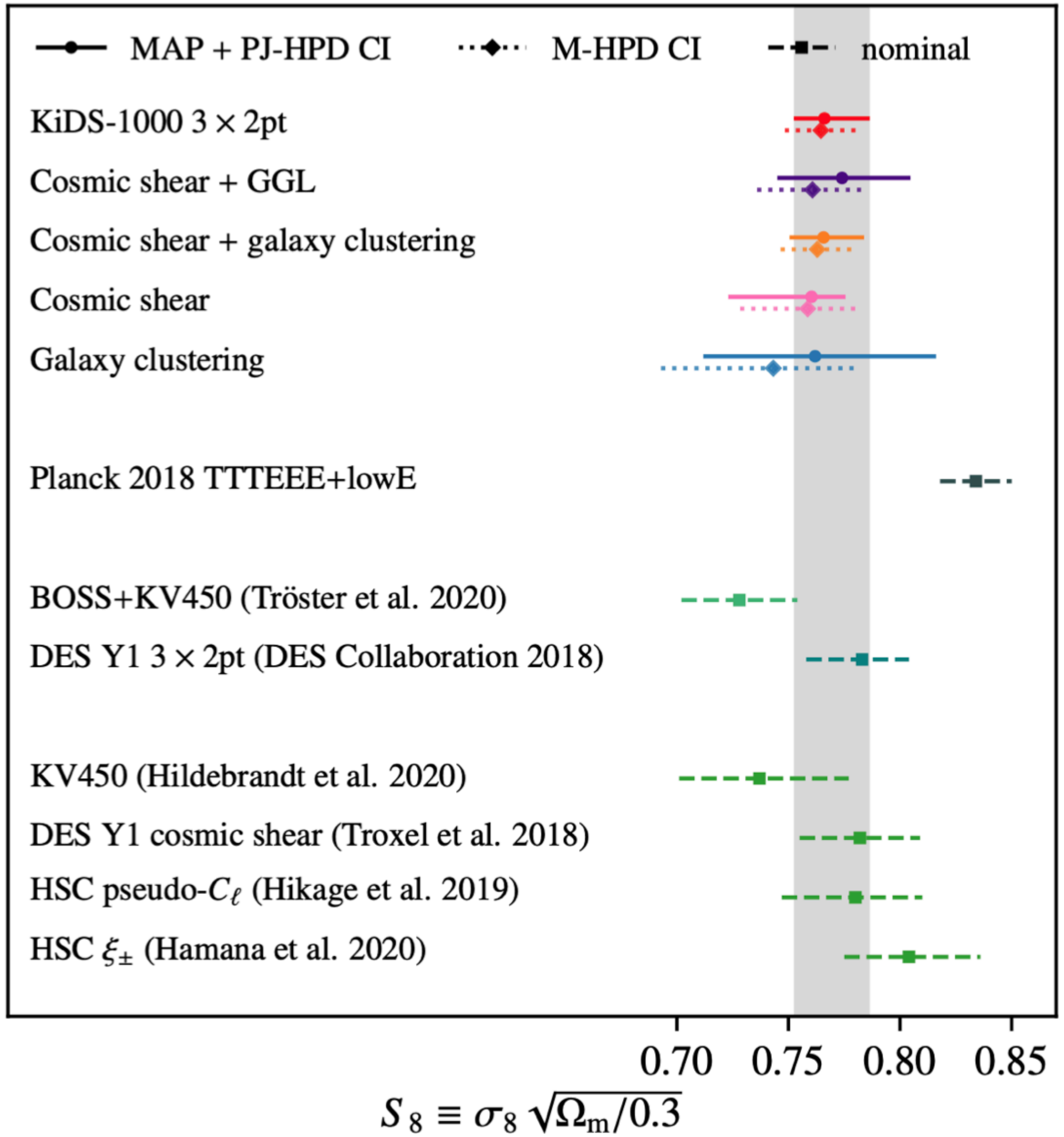

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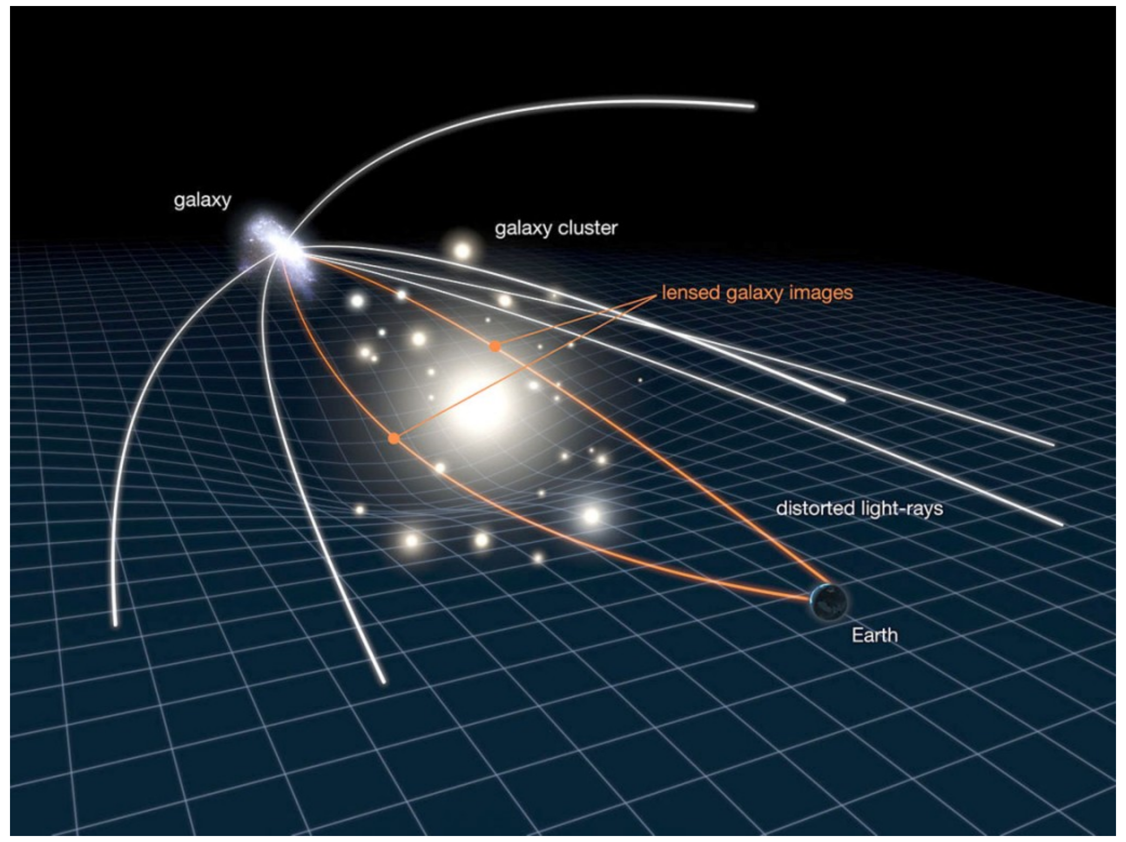


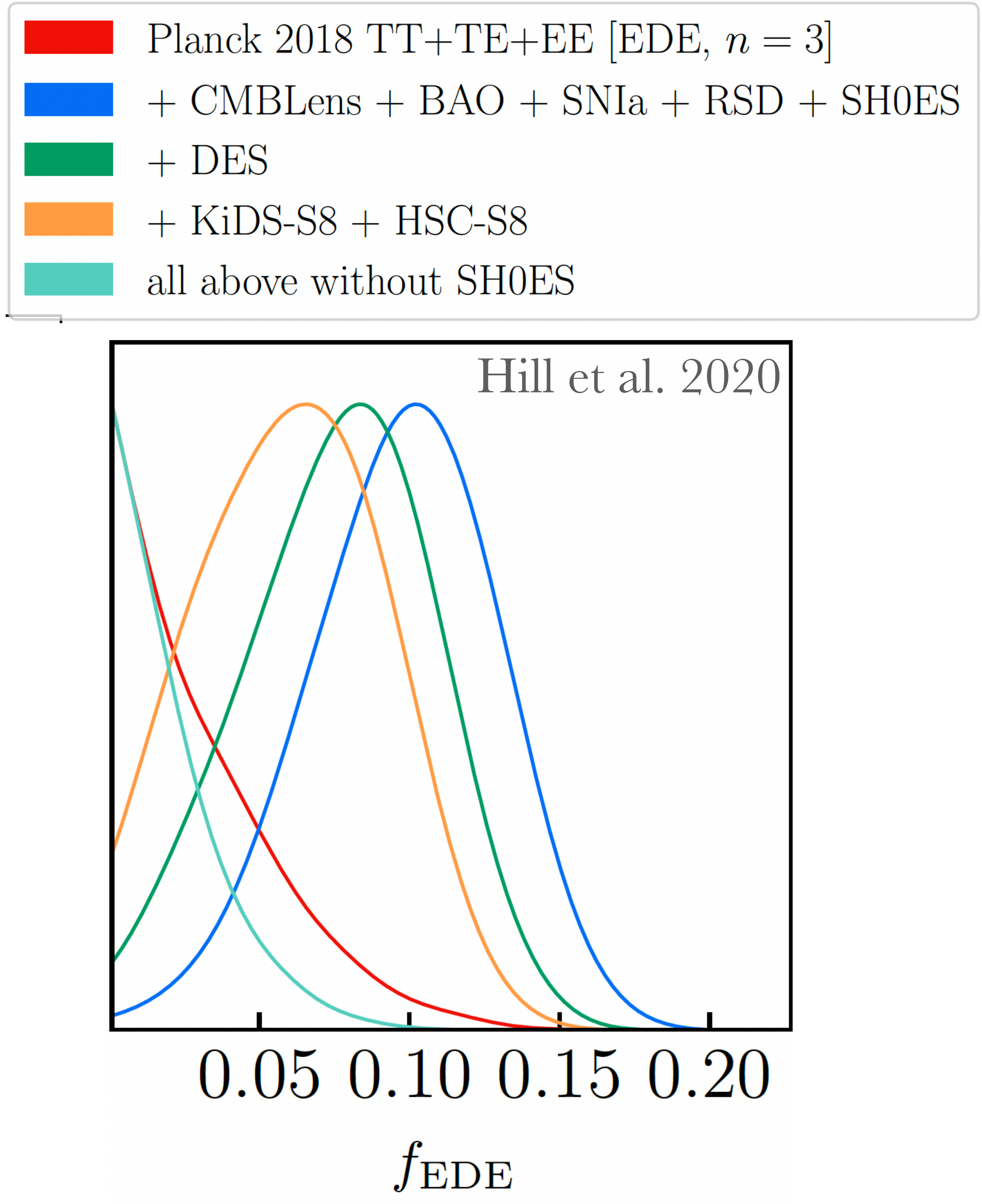
NASA

Adding LSS data: EDE is ruled out?

Hill et al. 2020; Murgia et al. 2021

Data sets: Planck + 6dFGS + BOSS DR12 BAO/RSD + Pantheon ~~+ SH0ES 2016~~ + DES + KiDS + HSC

- $f_{\mathrm{EDE}} < 0.06$ (95% C.L.)
- $H_0 = 68.92^{+0.57}_{-0.59}$ km/s/Mpc
- No SH0ES → no preference for EDE



Adding LSS data: EDE is ruled out?

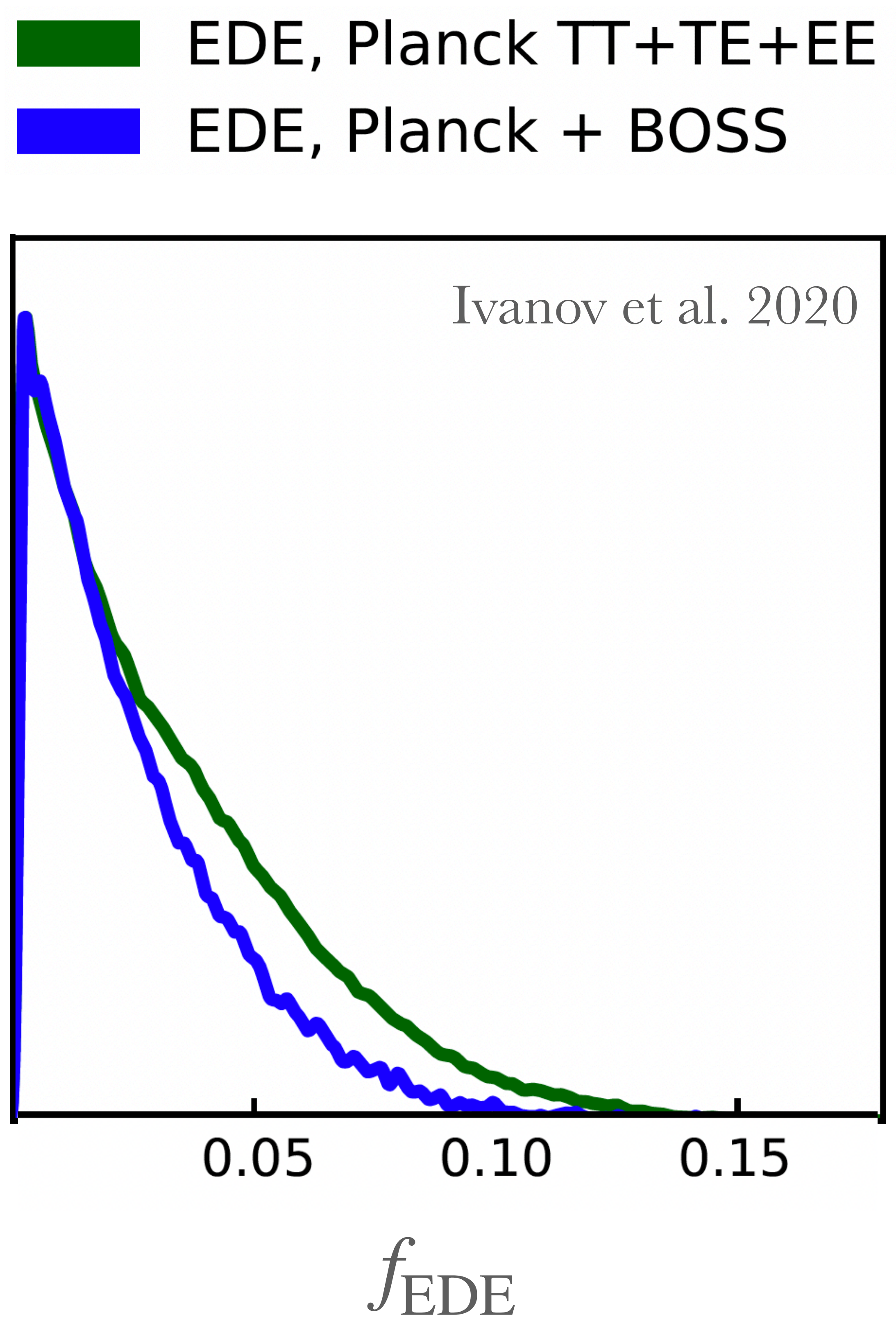
Ivanov et al. 2020; D'Amico et al. 2020

MCMC posterior

Data sets: *Planck + BOSS DR12 BAO*
+ full-shape analysis based on EFT of LSS

- $f_{\mathrm{EDE}} < 0.072$ (95% CL)
- $H_0 = 68.54^{+0.52}_{-0.95}$ km/s/Mpc

→ EDE cannot restore cosmological concordance



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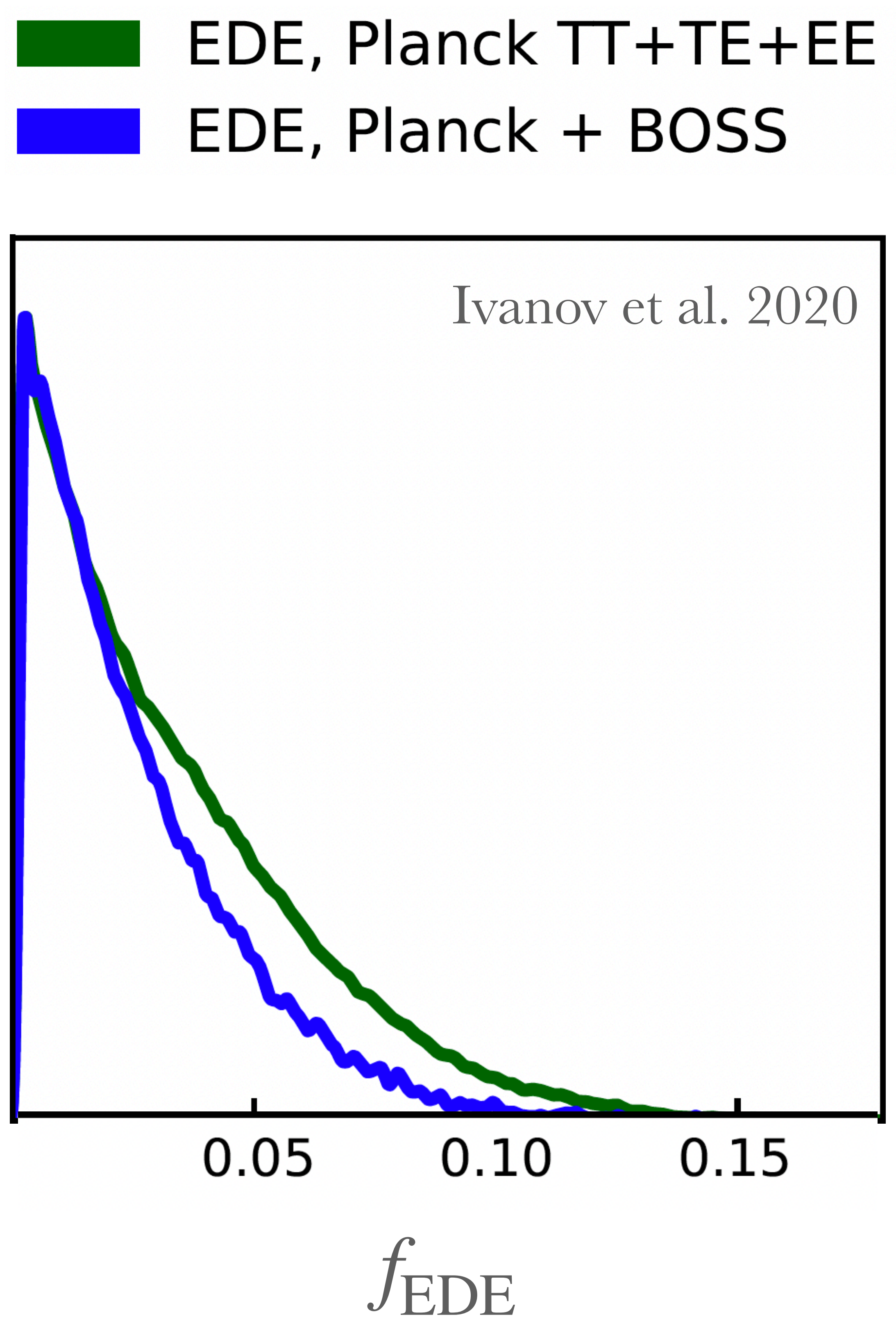


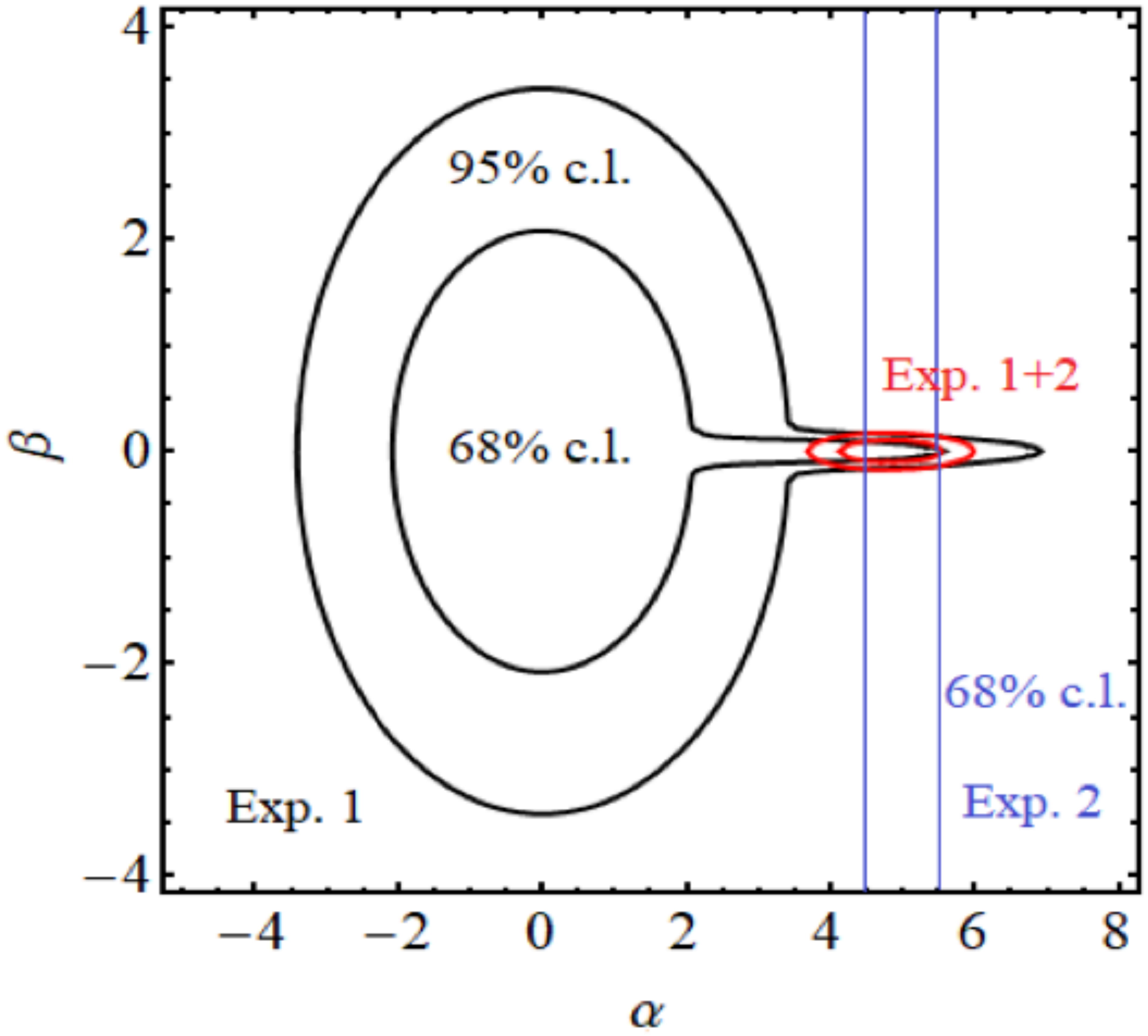
Could **volume effects** affect the results?

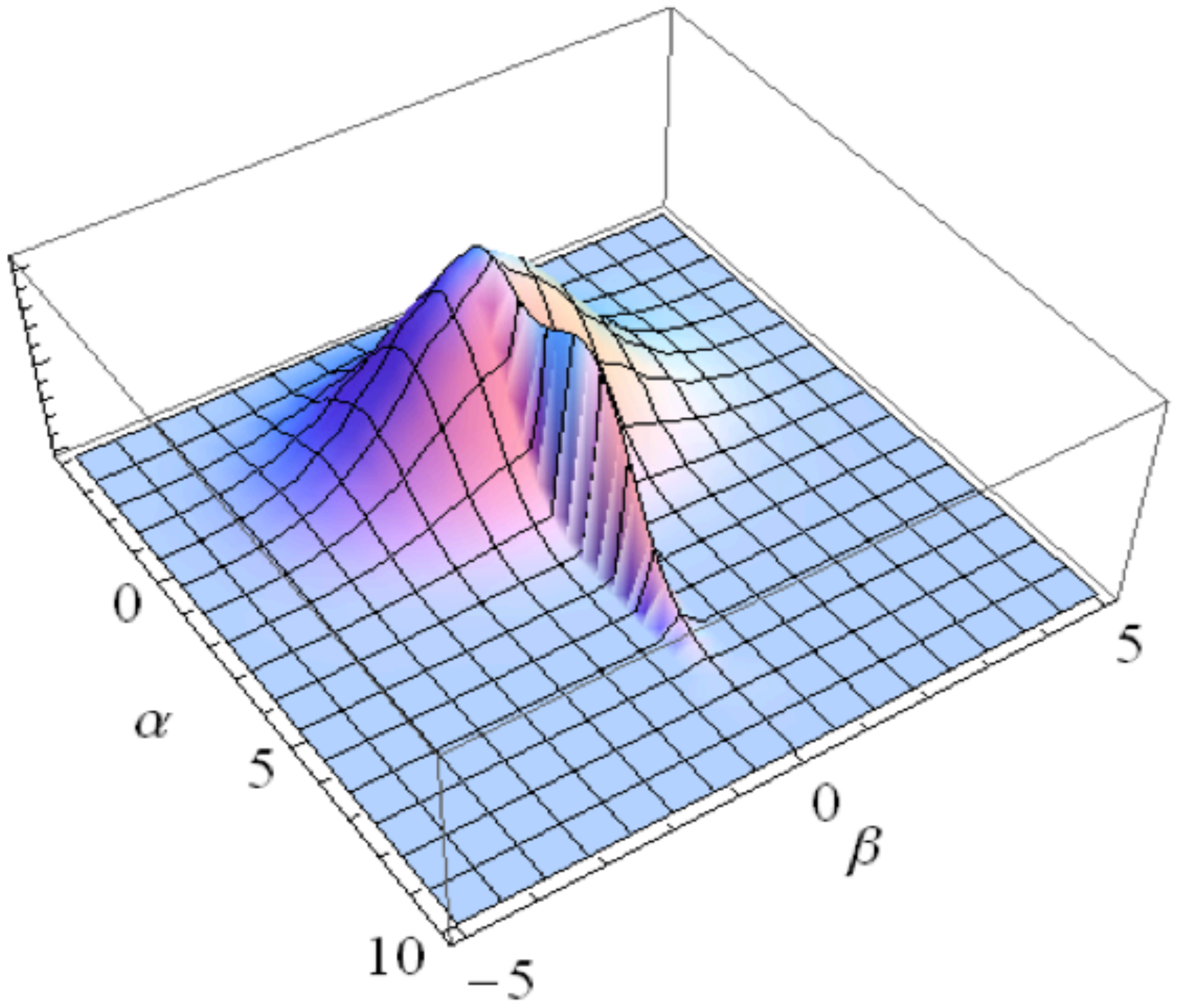
Smith++ 2020, Niedermann++ 2020, Smith++ 2021

Toy example: Prior volume / projection effect

Gomez-Valent 2022

2D posterior





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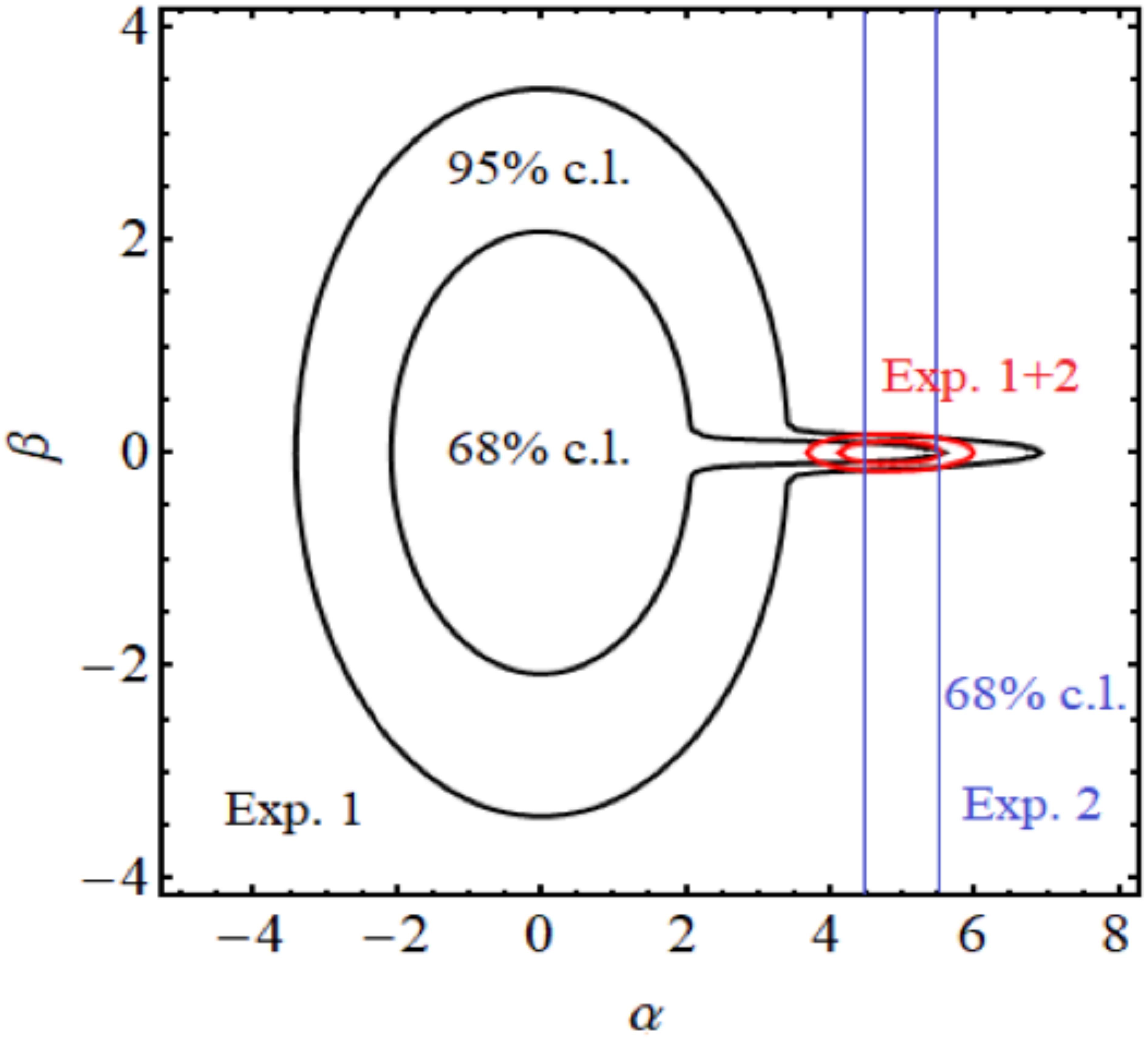
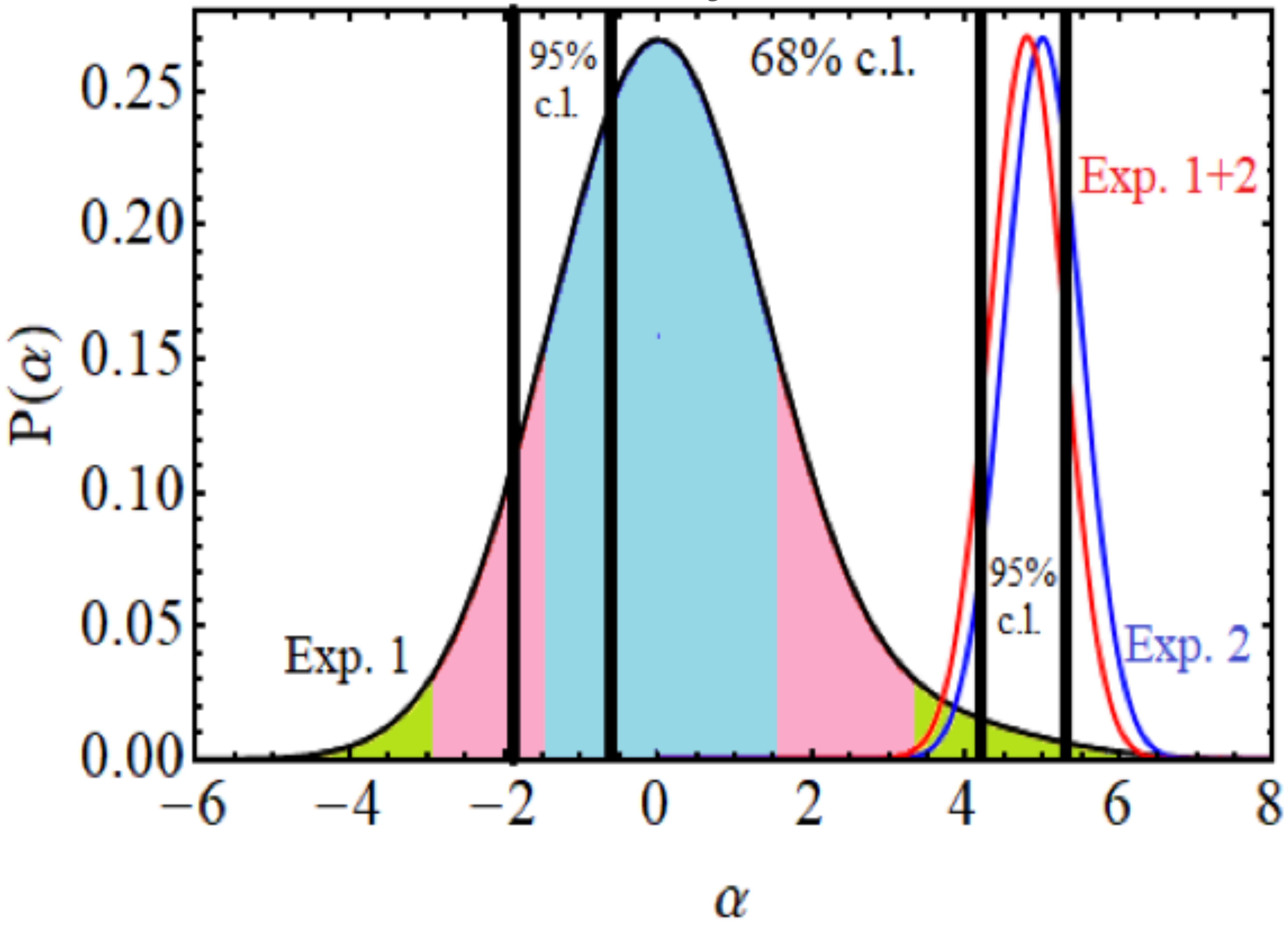


1D marginalized posterior of α

$$P(\alpha\,|\,\boldsymbol{x}) = \int d\beta\, P(\alpha, \beta\,|\,\boldsymbol{x})$$



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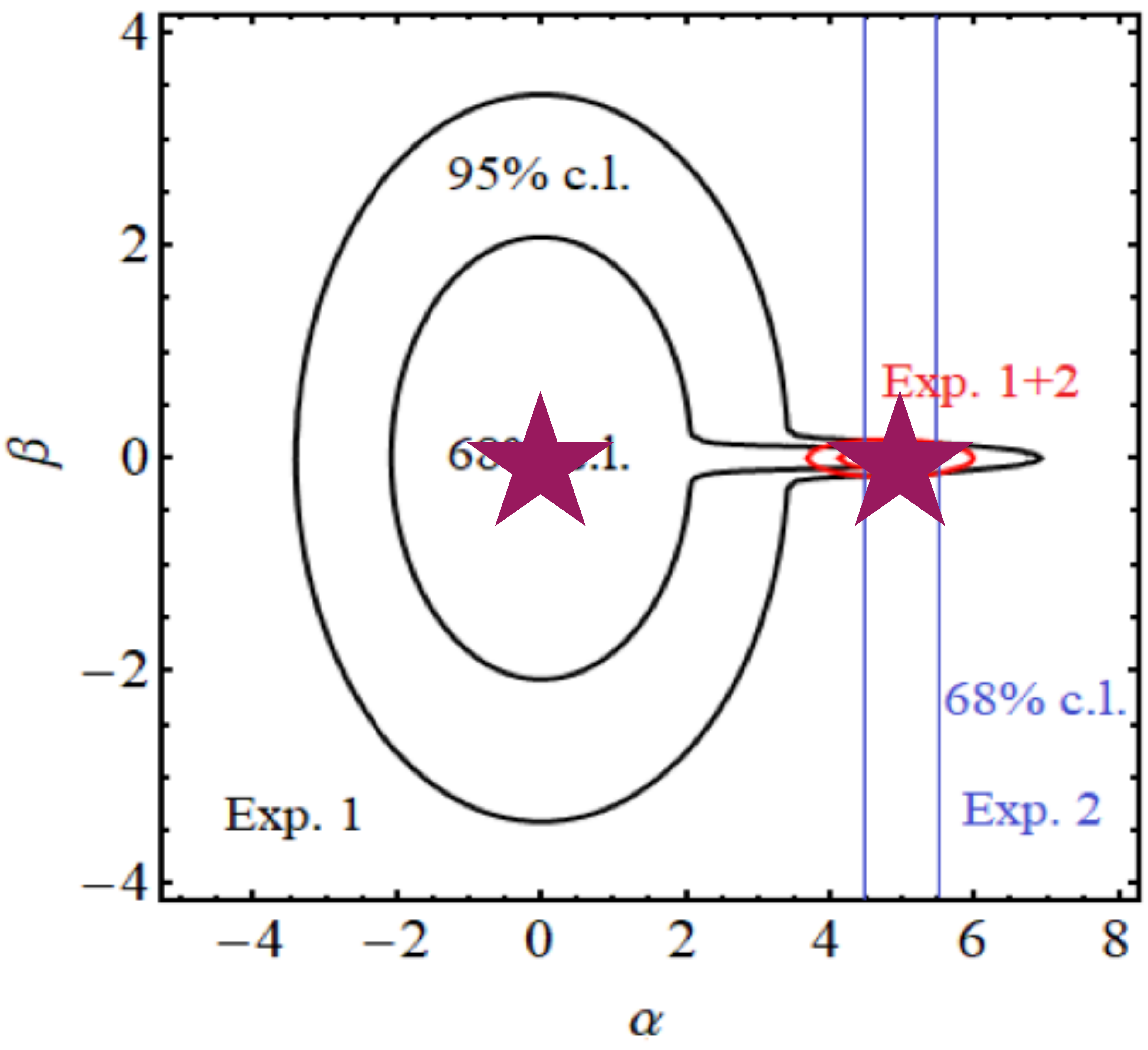

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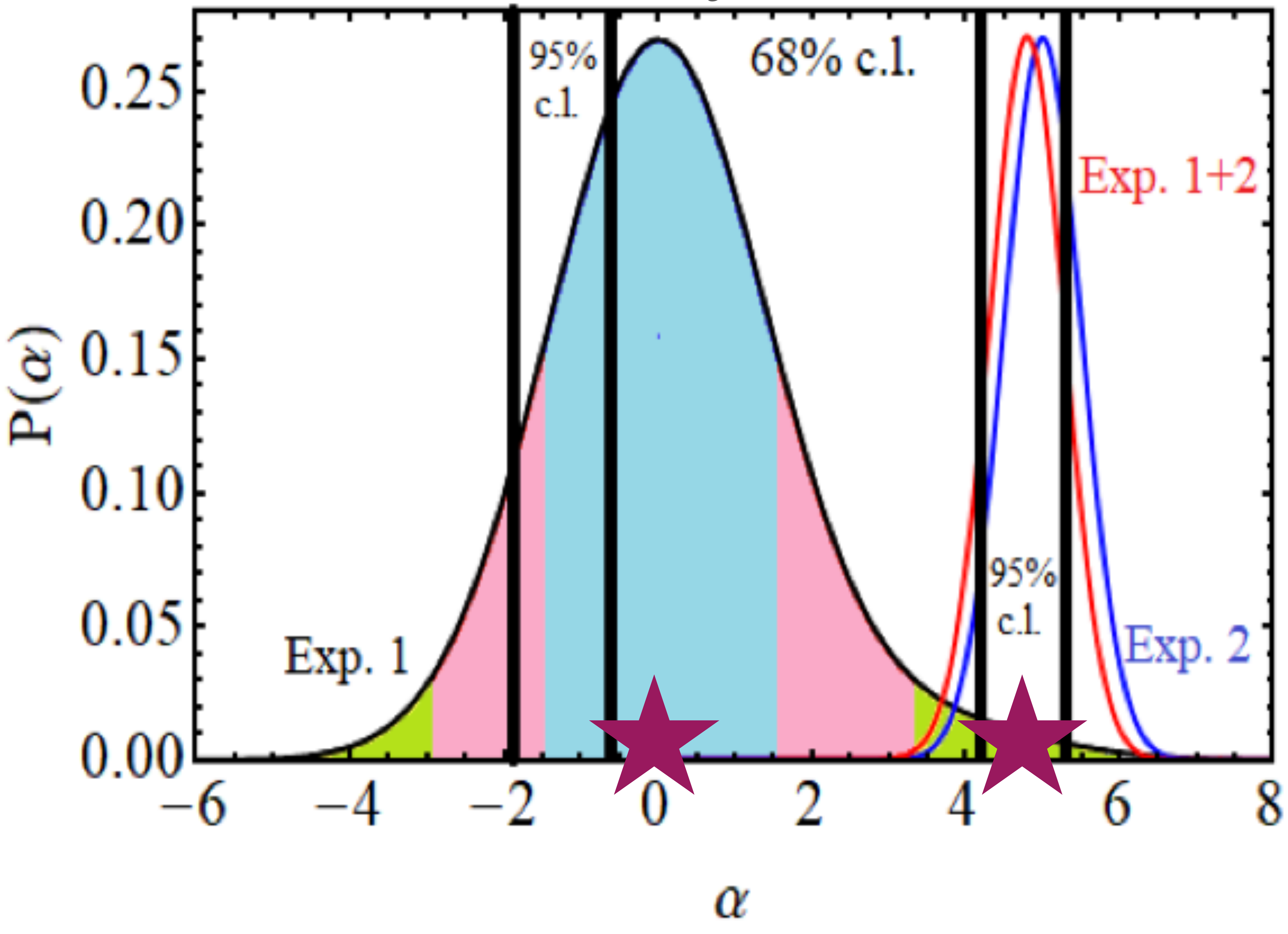


Prior volume / projection / marginalisation effects

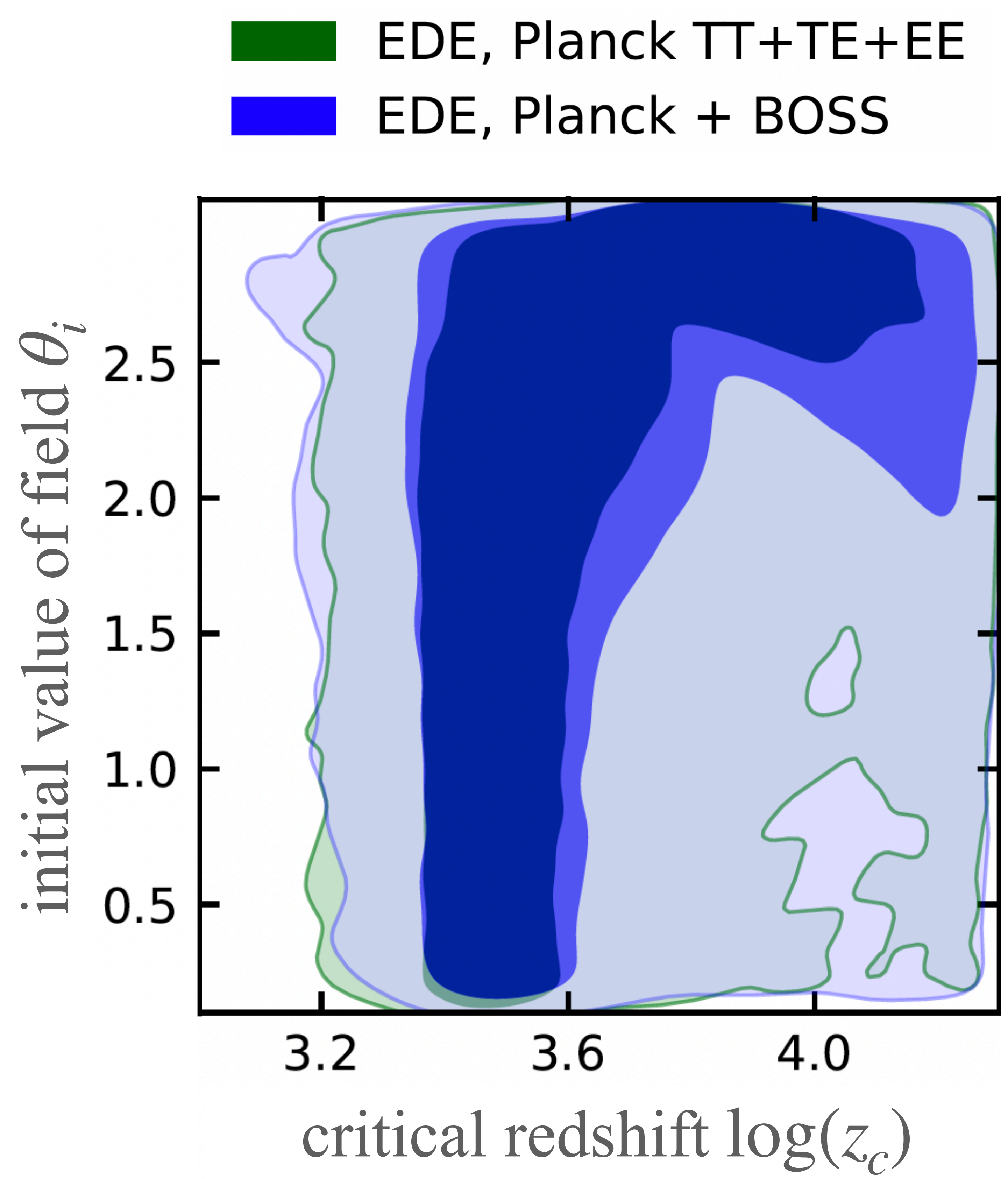
…appear if the posterior is influenced by the prior volume.
Reasons:

- Model has too many parameters / data is not constraining.

- Posterior is very non-Gaussian.

- Parameter structure of the model generates large volume differences.

Ivanov et al. 2020



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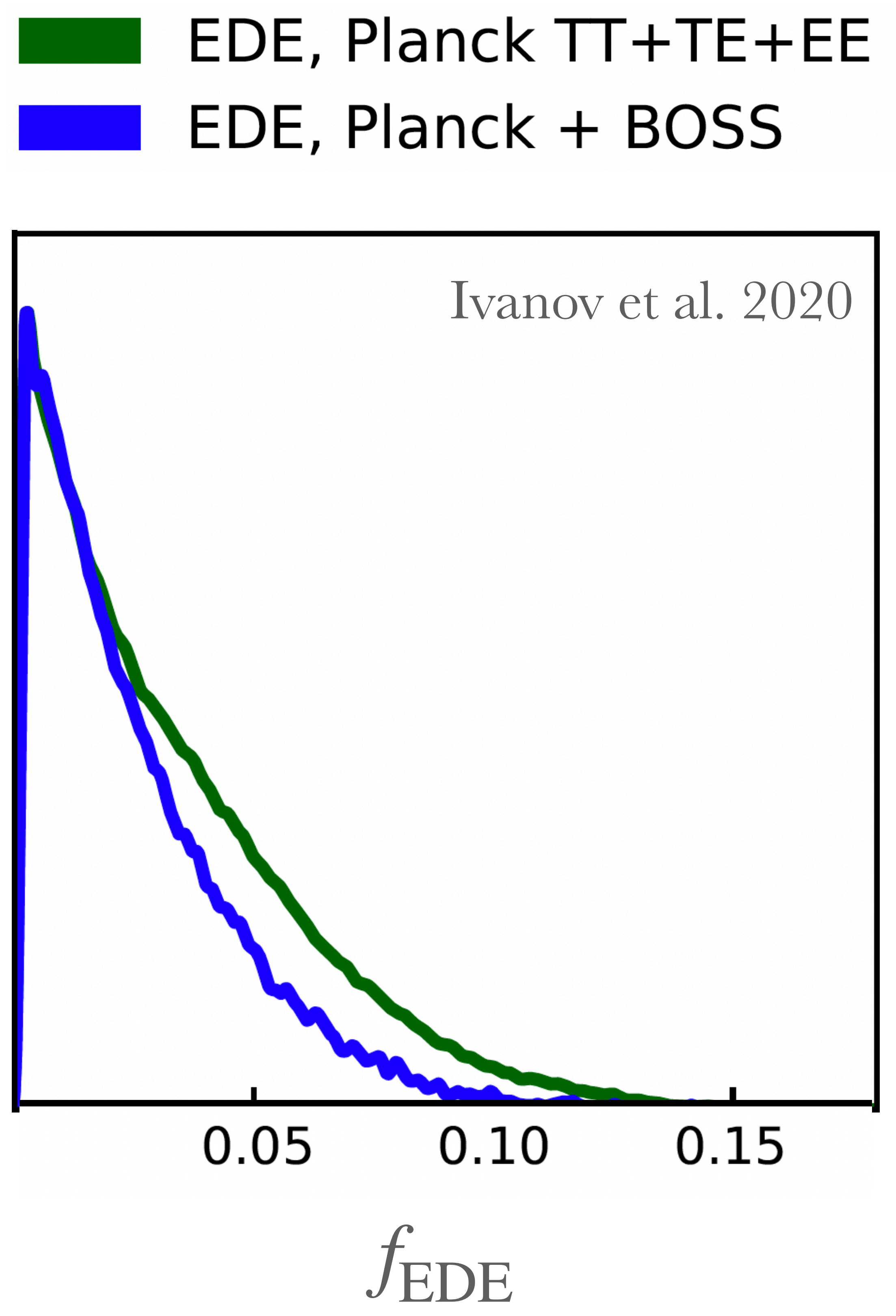

$f_{\rm EDE} \approx 0$: all values of z_c, θ_i unconstrained (ΛCDM limit)

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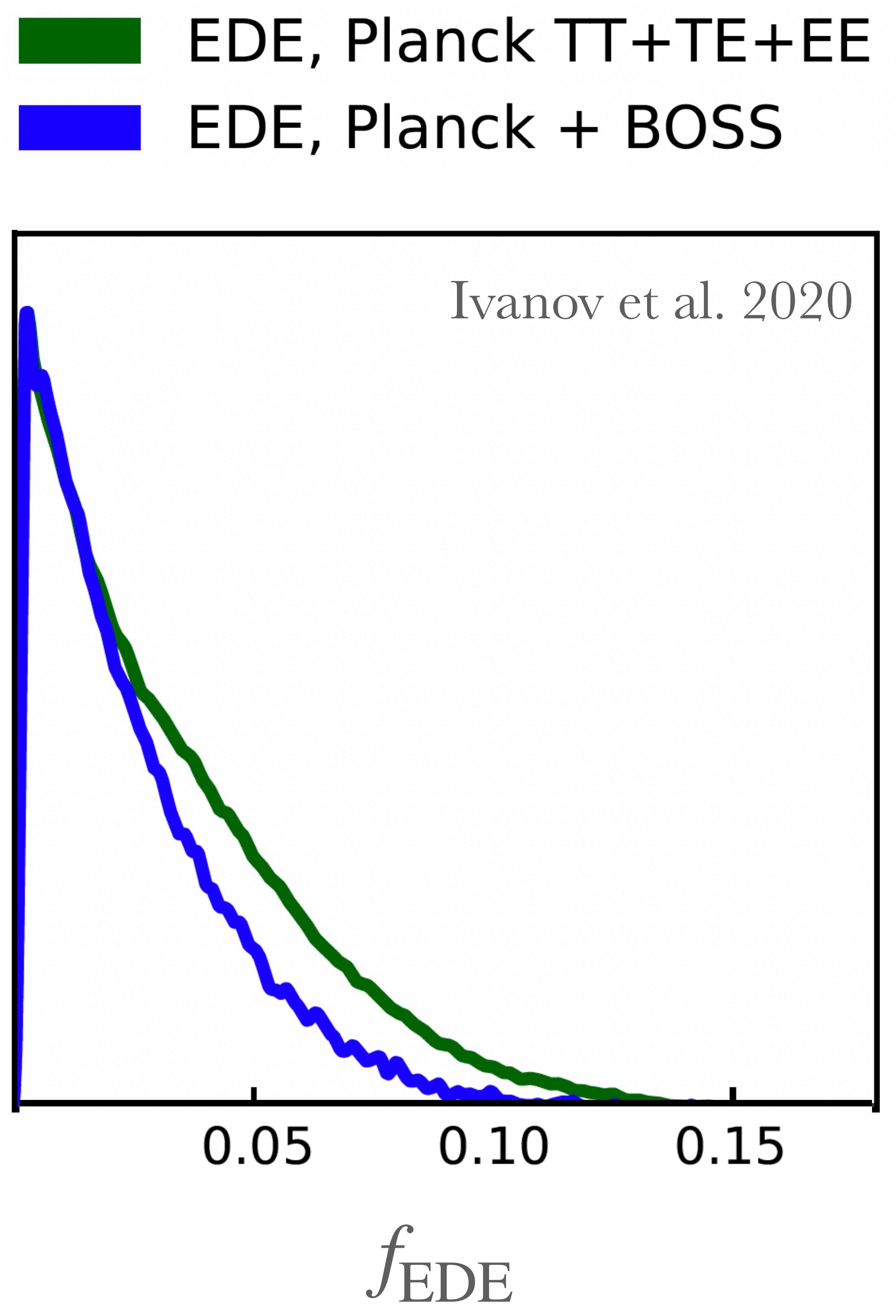
$\rightarrow$ Bias in the marginalised posterior.



$f_{\mathrm{EDE}} \approx 0$: all values of z_c, θ_i unconstrained (ΛCDM limit)

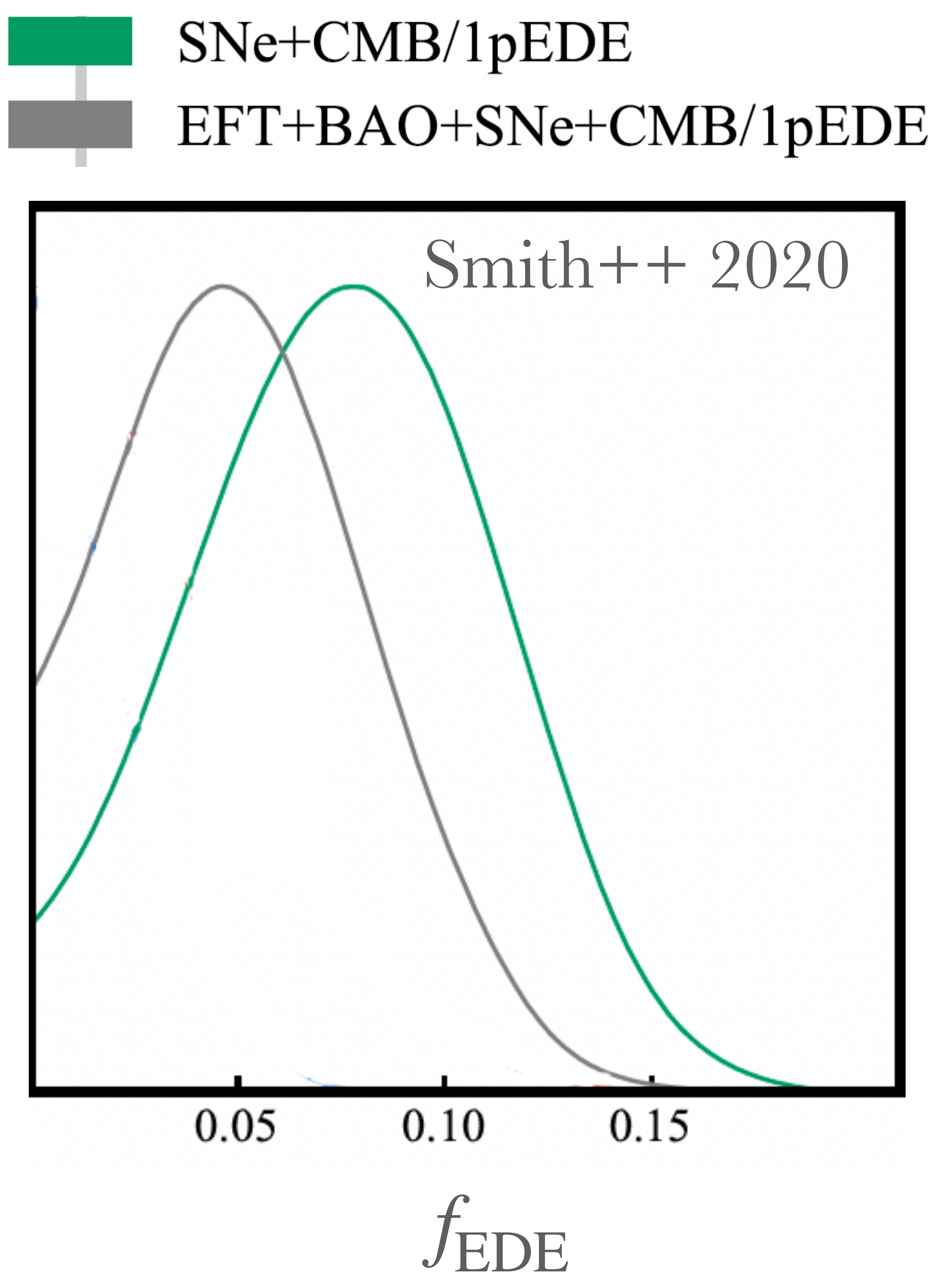
EDE is not ruled out by LSS?

Smith, Poulin, Bernal, Boddy, Kamionkowski, Murgia, 2020; Niedermann, Sloth, 2019 (for NEDE)

Data sets: Planck + BOSS DR12 BAO + full-shape analysis + Pantheon

- fixing z_c, θ_i to bestfit to *Planck* — "1-parameter model"
- $f_{\rm EDE} = 0.072 \pm 0.034$ (mean $\pm 1\sigma$)
- Same data set, but different conclusion than Ivanov et al., D'Amico et al., suspect volume effects

MCMC posterior



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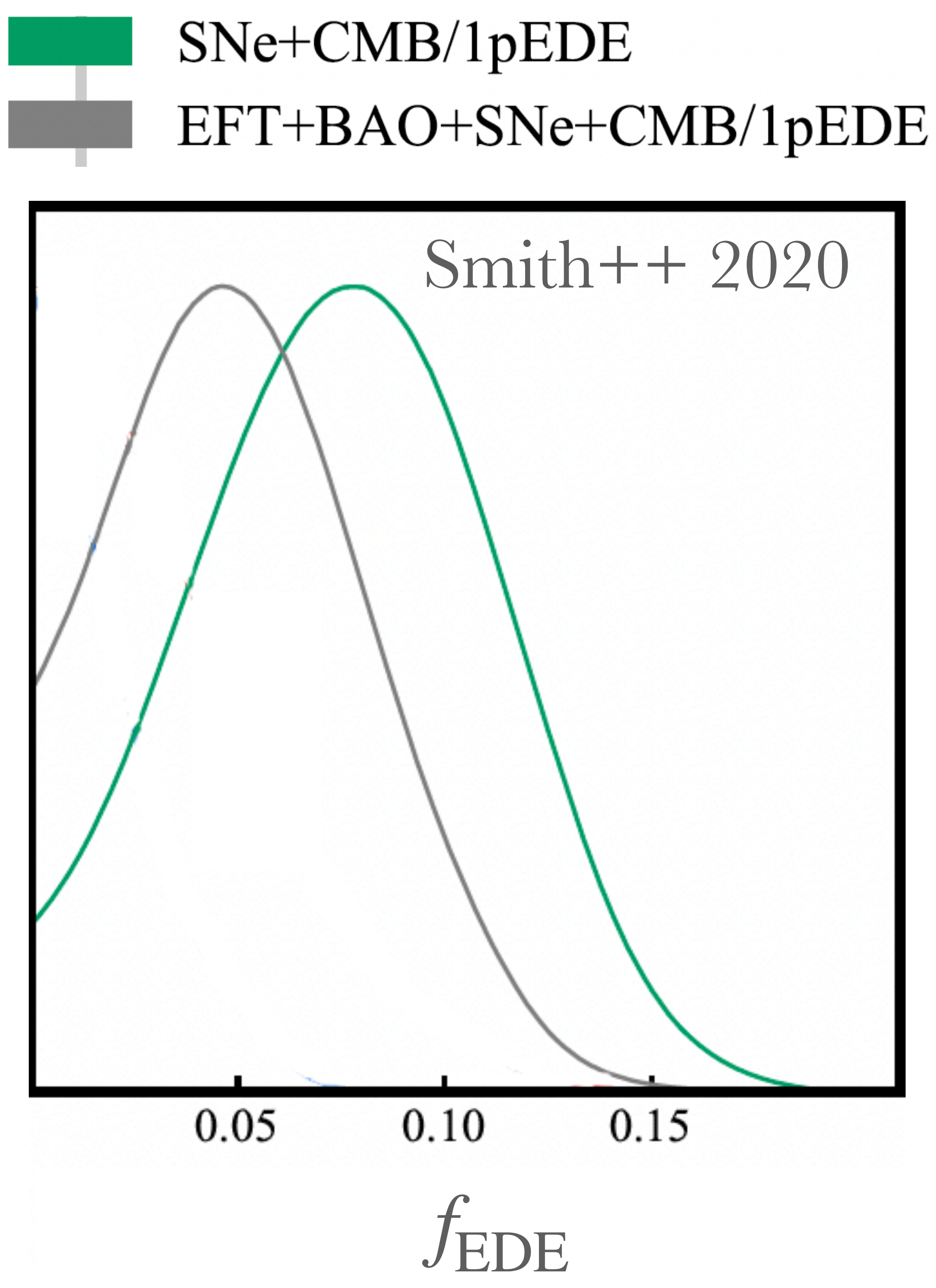
However: Not a full Bayesian analysis

MCMC posterior



Profile likelihood: Planck + BOSS

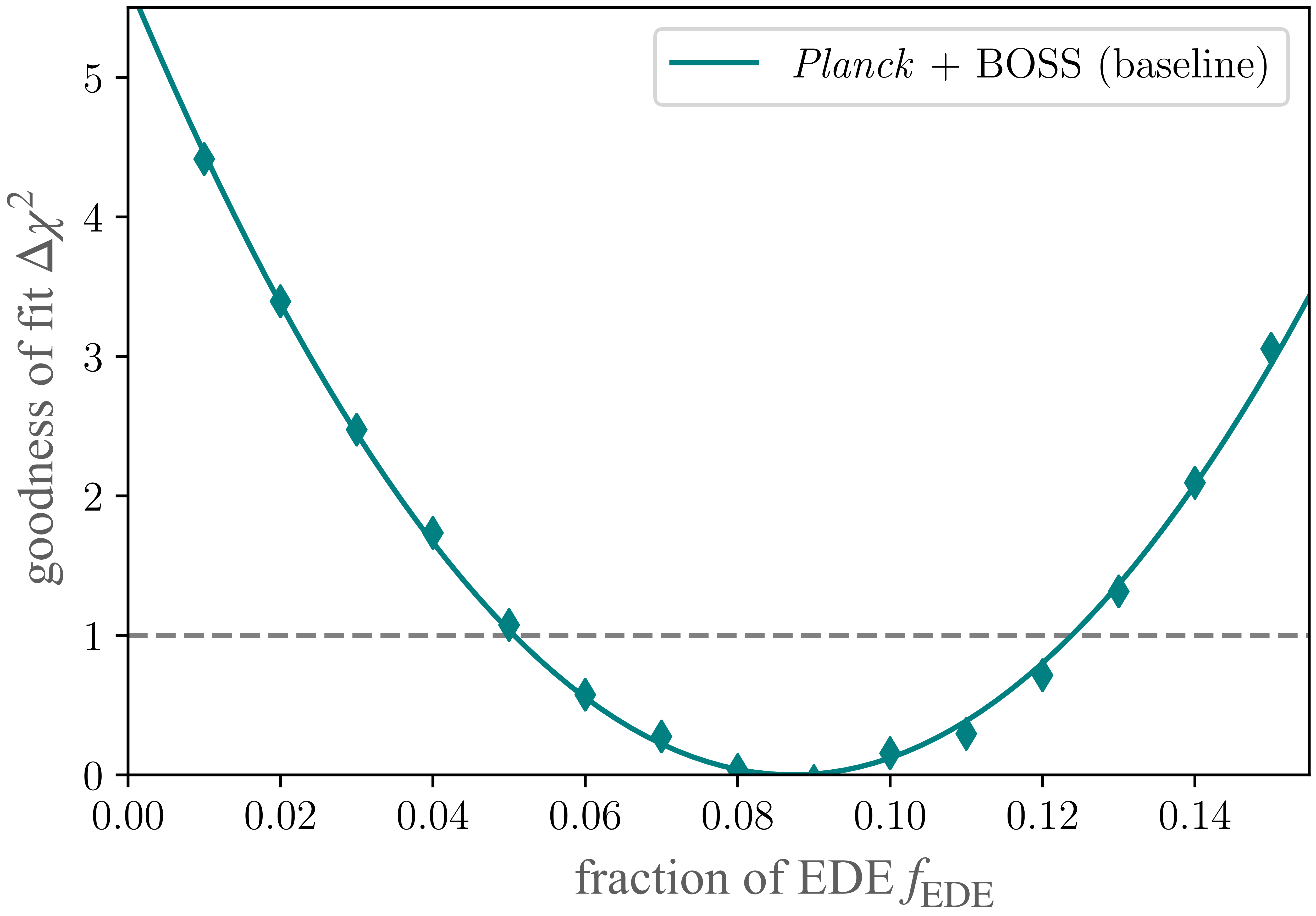
LH, Ferreira, Komatsu 2021; LH, Ferreira 2022

MCMC results:

- $f_{\mathrm{EDE}} < 0.072$ (95% C.L.), $H_0 = 68.55^{+0.62}_{-1.06}$ km/s/Mpc

Profile likelihood results:

- $f_{\mathrm{EDE}} = 0.087 \pm 0.037$,



Profile likelihood

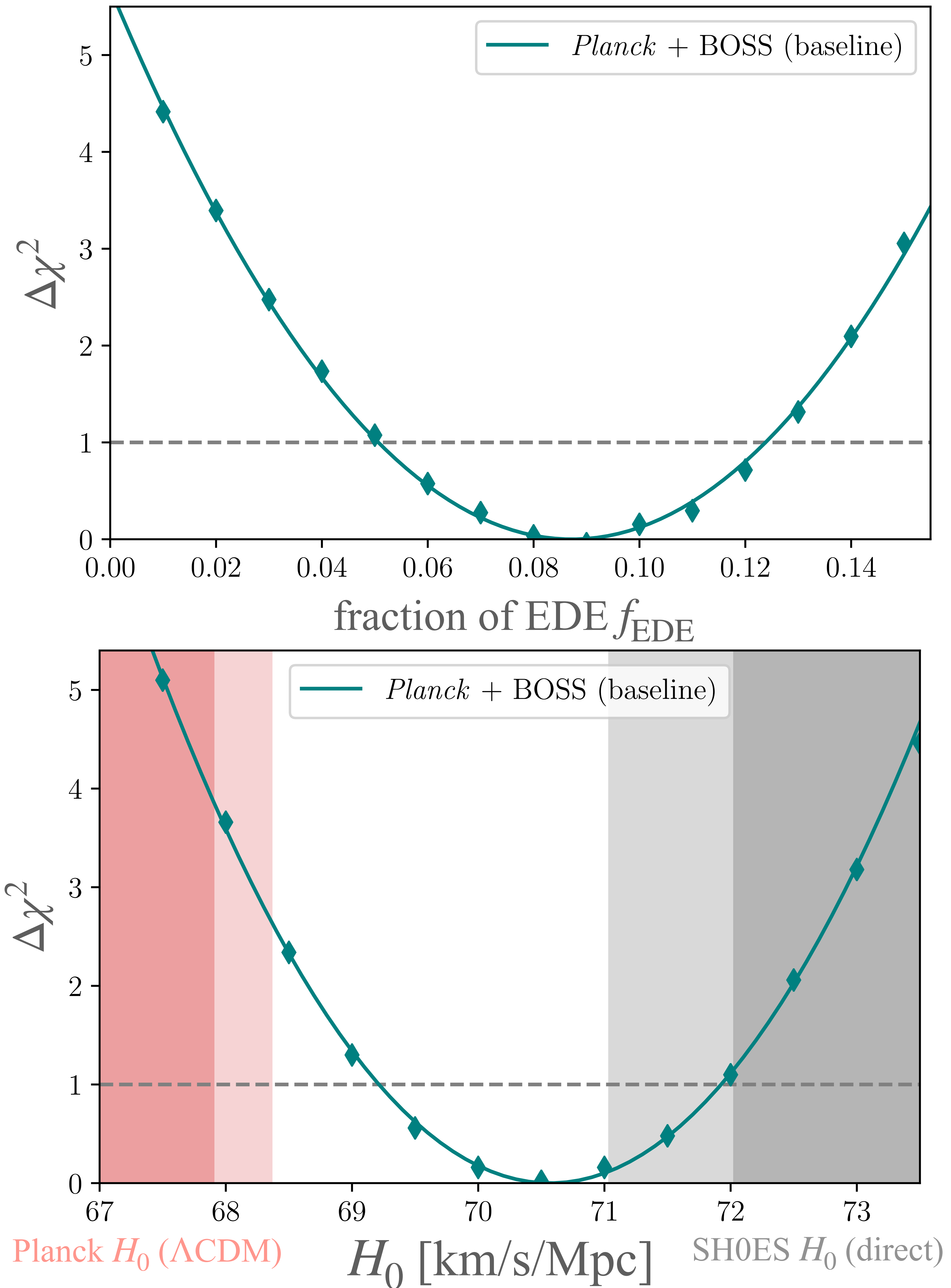
LH, Ferreira 2022

MCMC results:

- $f_{\rm EDE} < 0.072$ (95% C.L.),
  $H_0 = 68.55^{+0.62}_{-1.06}$ km/s/Mpc

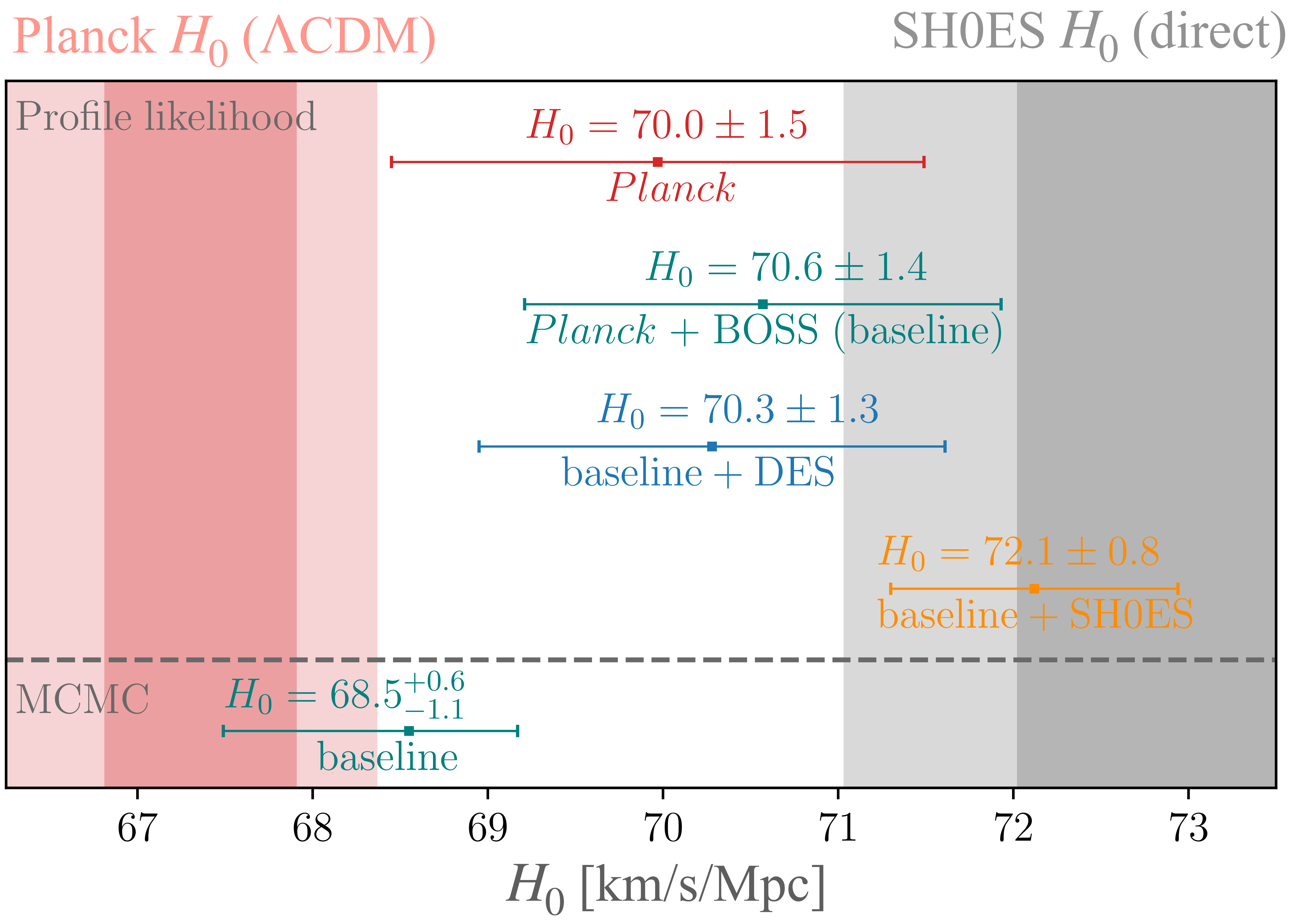
Profile likelihood results:

- $f_{\rm EDE} = 0.087 \pm 0.037$,
  $H_0 = 70.57 \pm 1.36$ km/s/Mpc
- Consistency with SH0ES at $1.4\,\sigma$
- However: S_8 tension worsens slightly
  (ΛCDM: 0.828, EDE: 0.840, DES: 0.776)



Profile likelihood — results

LH, Ferreira 2022

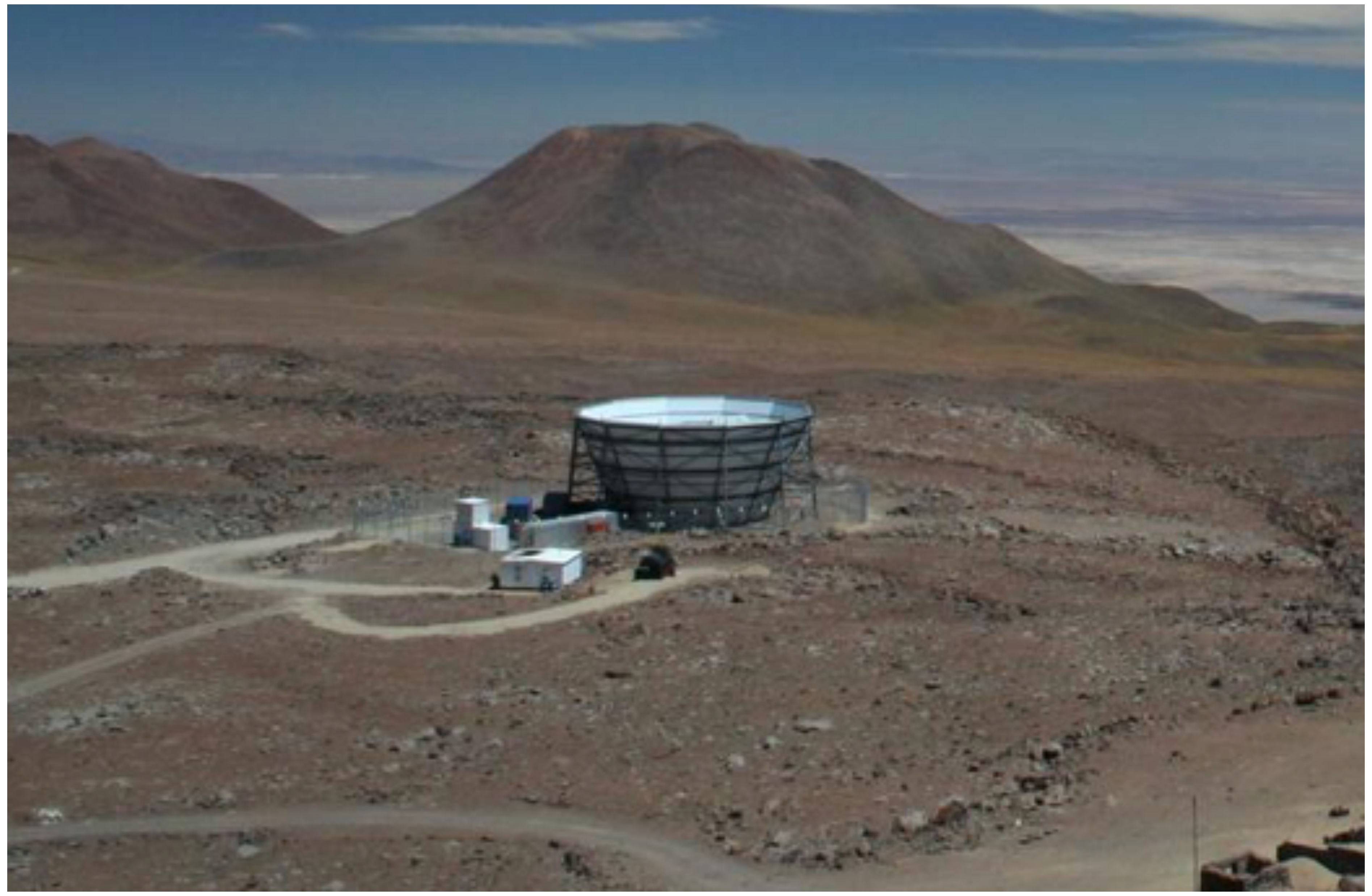


- Evidence for prior volume effects.
- H_0 in EDE model within $1.7\,\sigma$ of SH0ES measurement for all data sets (incl. galaxy clustering, weak lensing).
- EDE viable solution to Hubble tension.

Results from Atacama Cosmology Telescope

Hill et al. 2021, Poulin et al. 2021, Smith et al. 2022

Data sets: ACT DR4 + large-scale *Planck* TT+lensing + BOSS BAO (yellow line)



ACT collaboration

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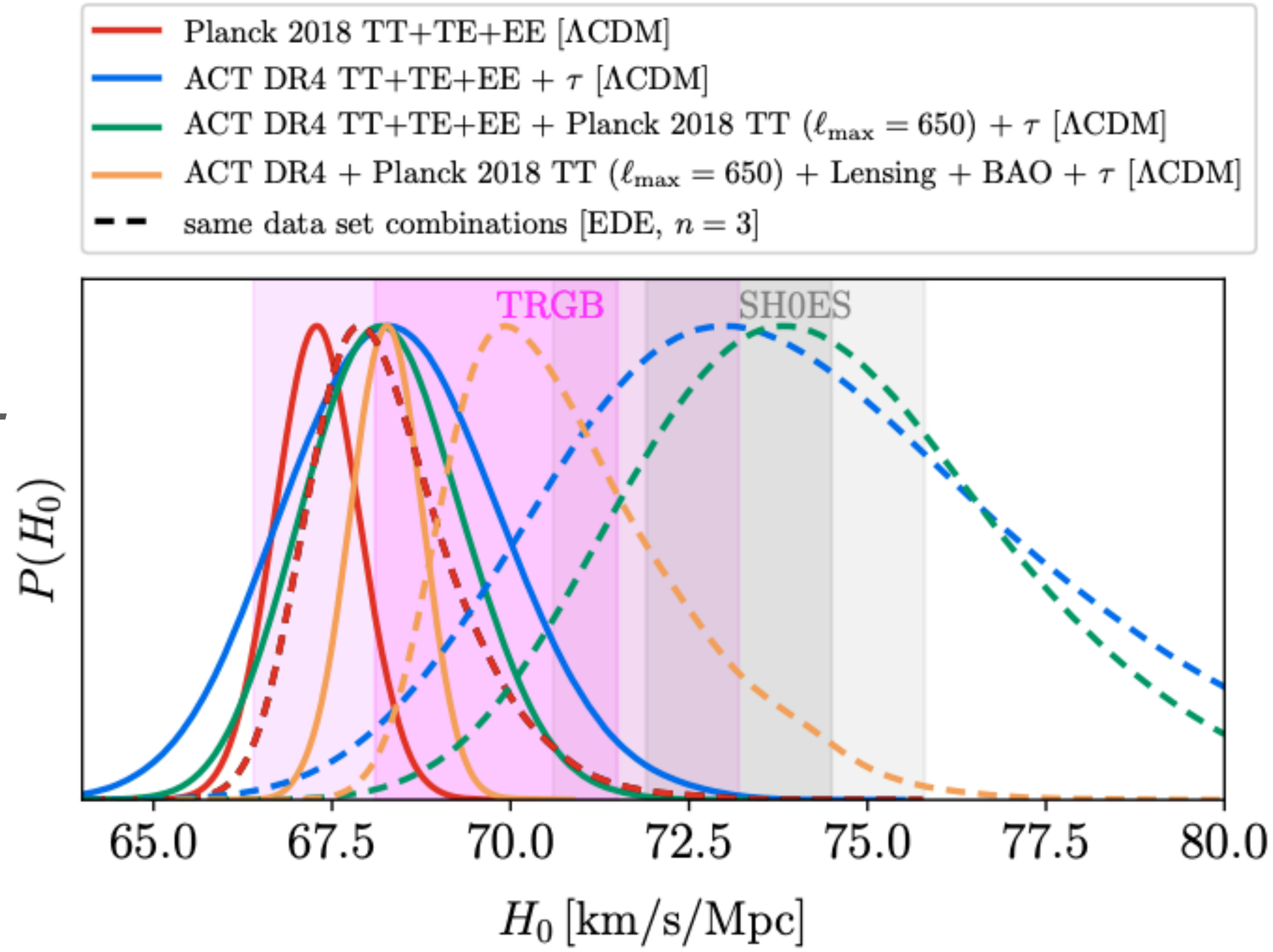
MCMC posterior

Hill et al. 2021



Data sets: ACT DR4 + large-scale *Planck* TT+lensing + BOSS BAO (yellow line)

- prefers the EDE model over ΛCDM by 2-3 σ

- $H_0 = 70.9^{+1.0}_{-2.0}$ km/s/Mpc,
  $f_{\rm EDE} = 0.091^{+0.020}_{-0.036}$

- Driven by ACT *TE+EE* power spectra

EDE constraints from South Pole Telescope

La Posta++ 2021, Smith++ 2022

Credit Luong-Van

EDE constraints from South Pole Telescope

La Posta++ 2021, Smith++ 2022

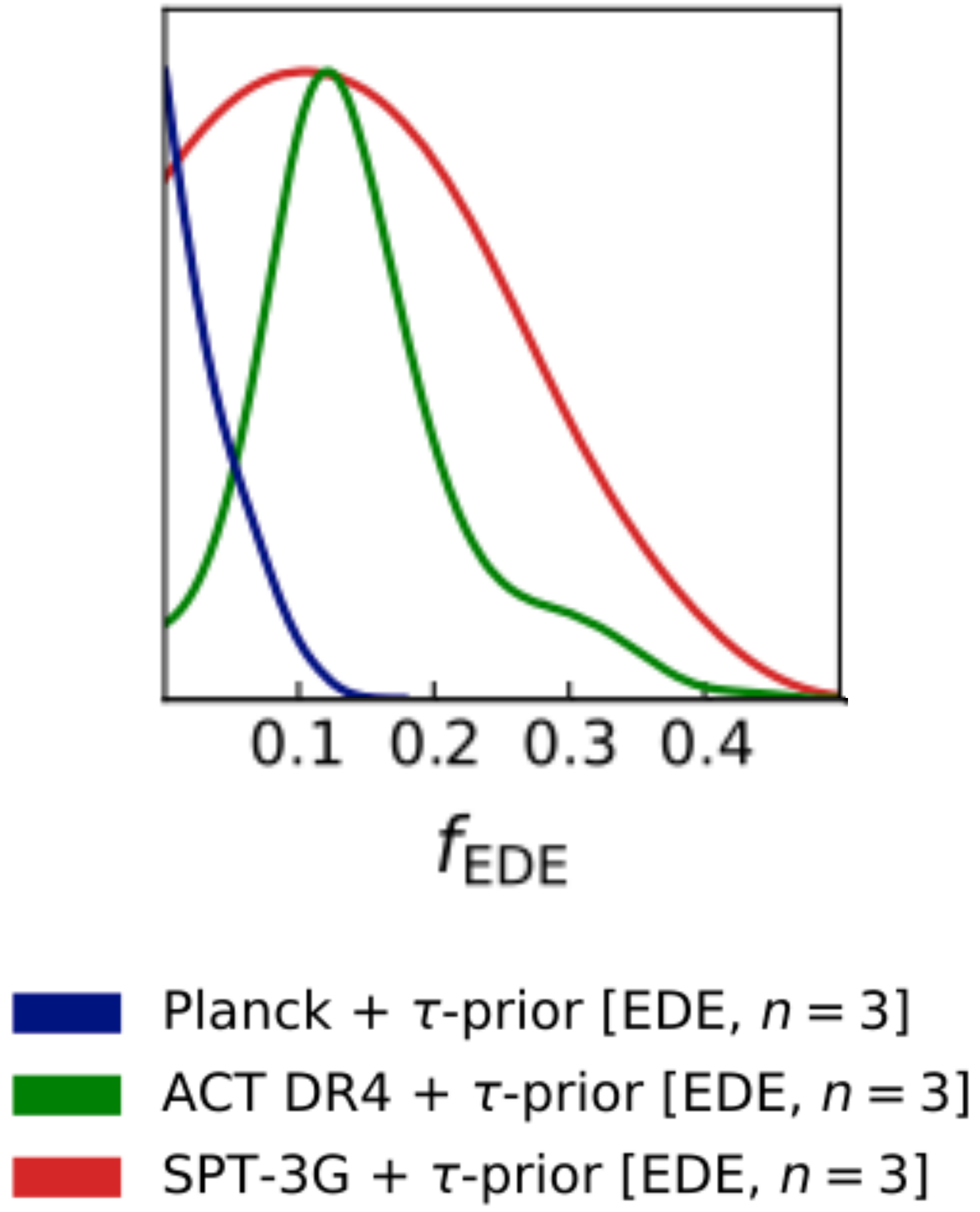
Credit Luong-Van

- SPT is consistent with ACT but with larger errorbar

- It is also consistent with $f_{\rm EDE} = 0$

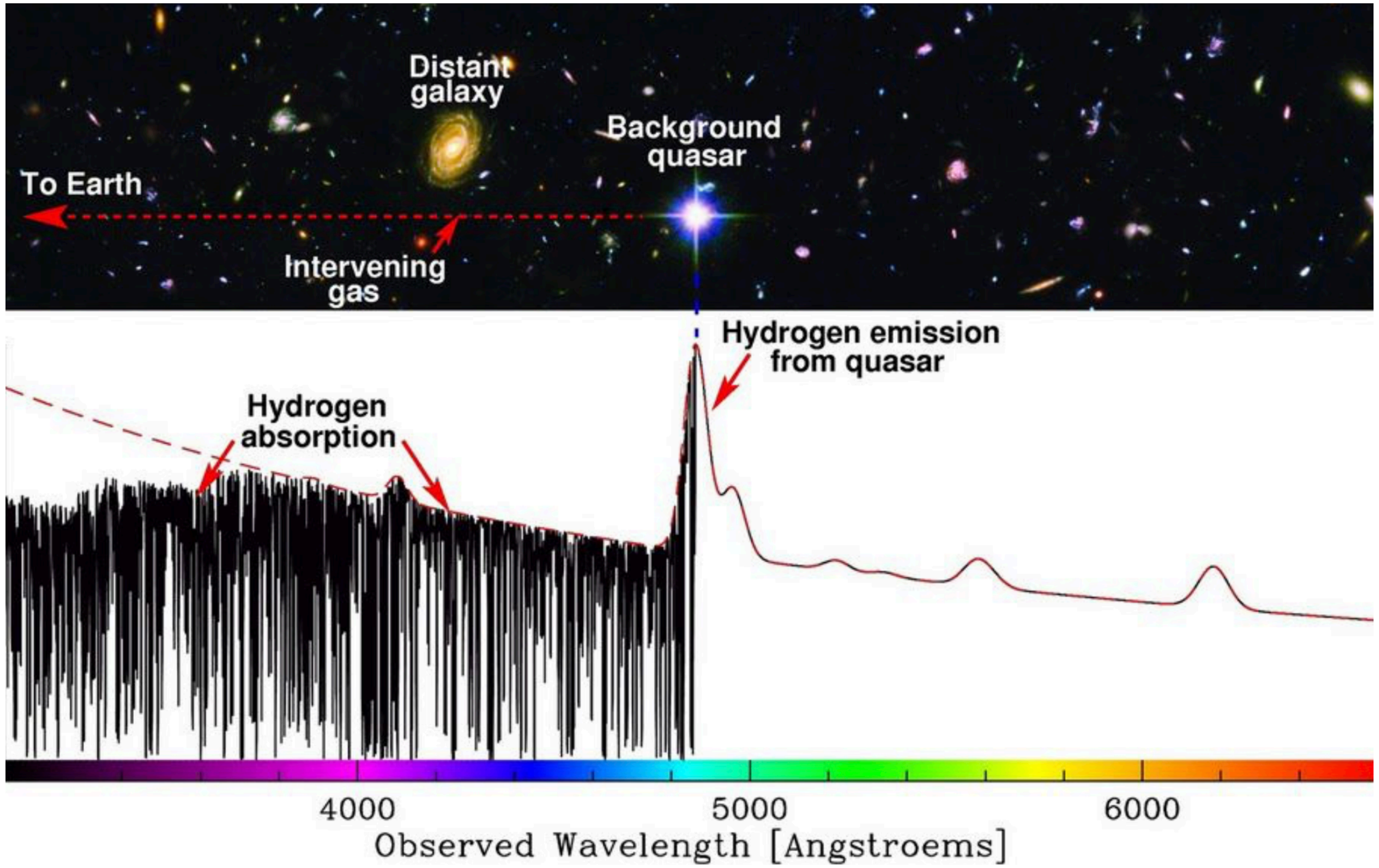
MCMC posterior

La Posta++ 2021



EDE constraints from Ly-α forest

- Lyman-α forest constraints the matter power spectrum at small scales

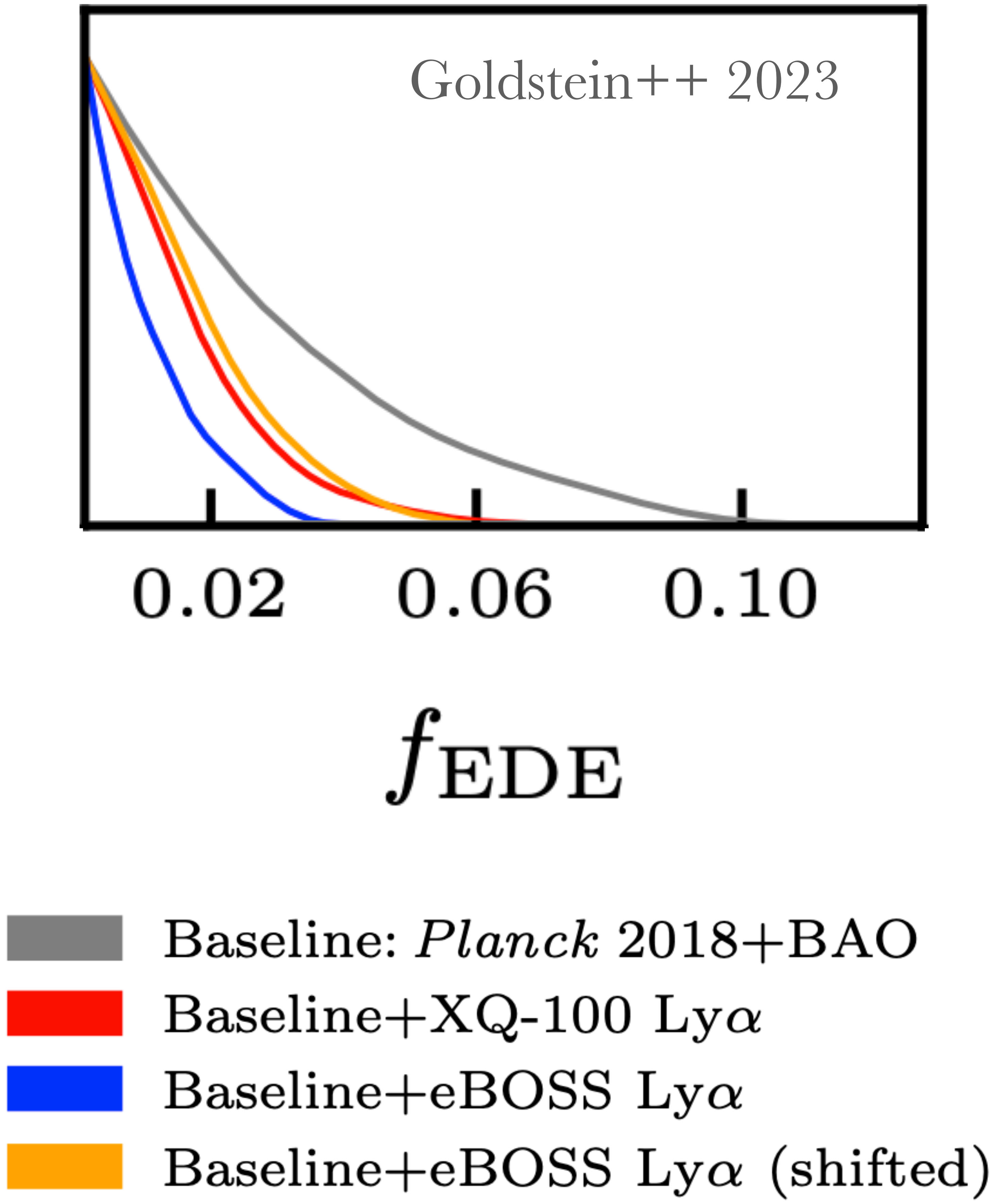


EDE constraints from Ly-α forest

MCMC posterior

Goldstein et al. 2023

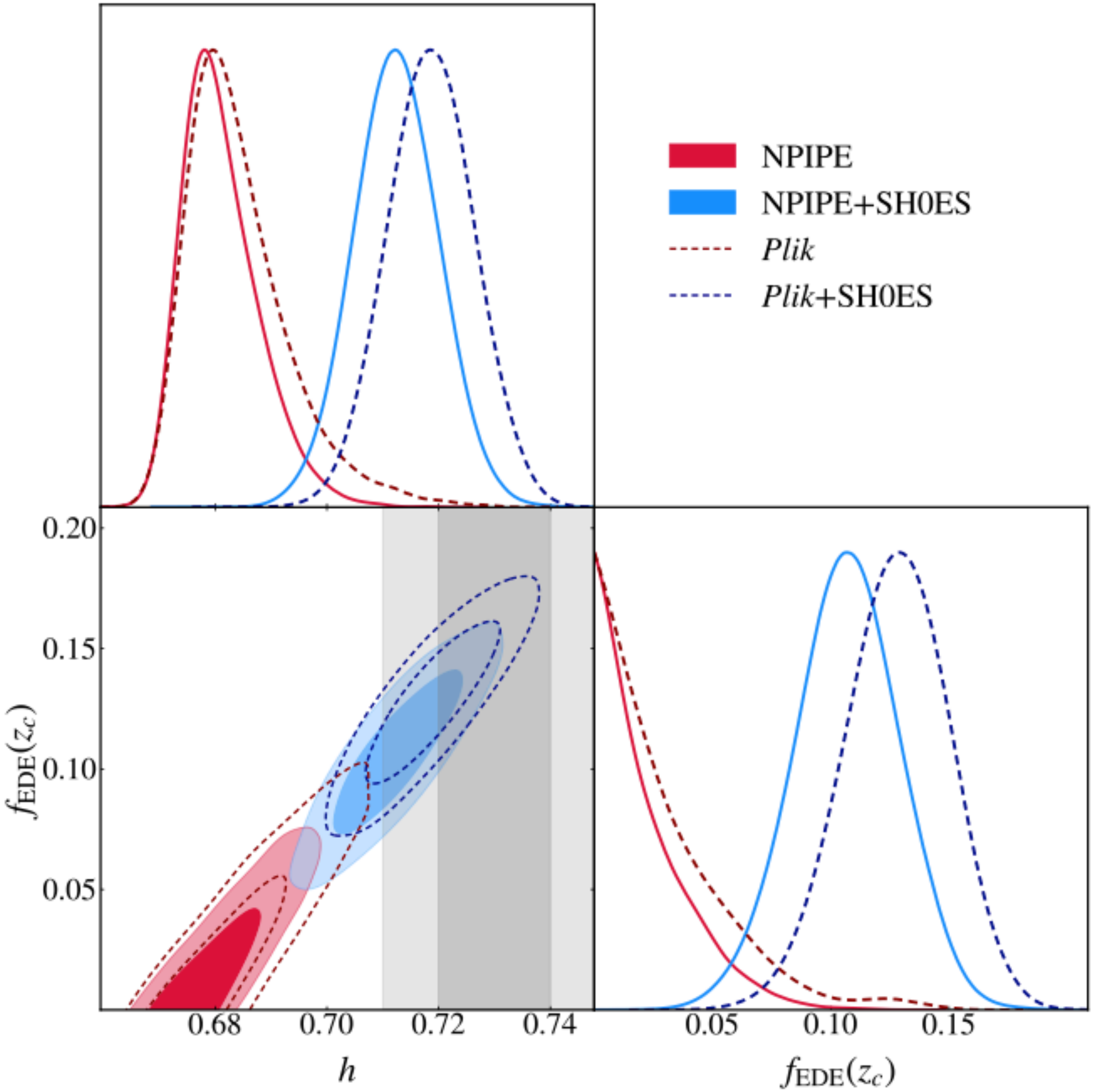
- Lyman-α forest constraints the matter power spectrum at small scales
- Combined with Planck data, it gives very tight upper limits on f_{EDE}
- However, there seem to be some internal discrepancies in Ly-α data, so EDE-constraints need to be taken with a grain of salt



Alternative NPIPE *Planck* pipeline

Efstathiou et al. 2023

- An alternative Planck pipeline NPIPE
  - using more sky area at high frequencies
  - different analysis choices
  - reducing the ΛCDM residuals
- NPIPE gives tighter constraints on f_{EDE} in a Bayesian MCMC…



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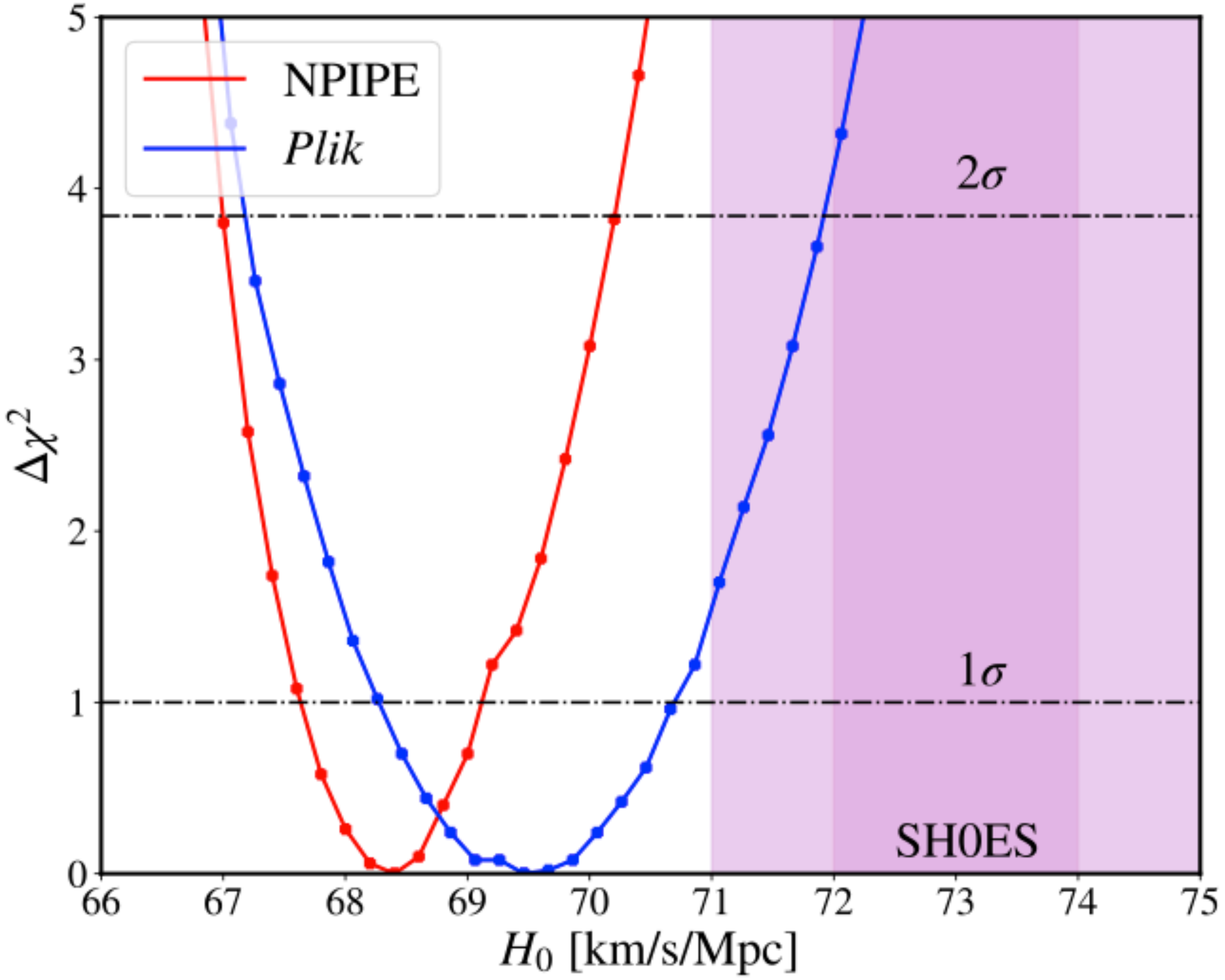

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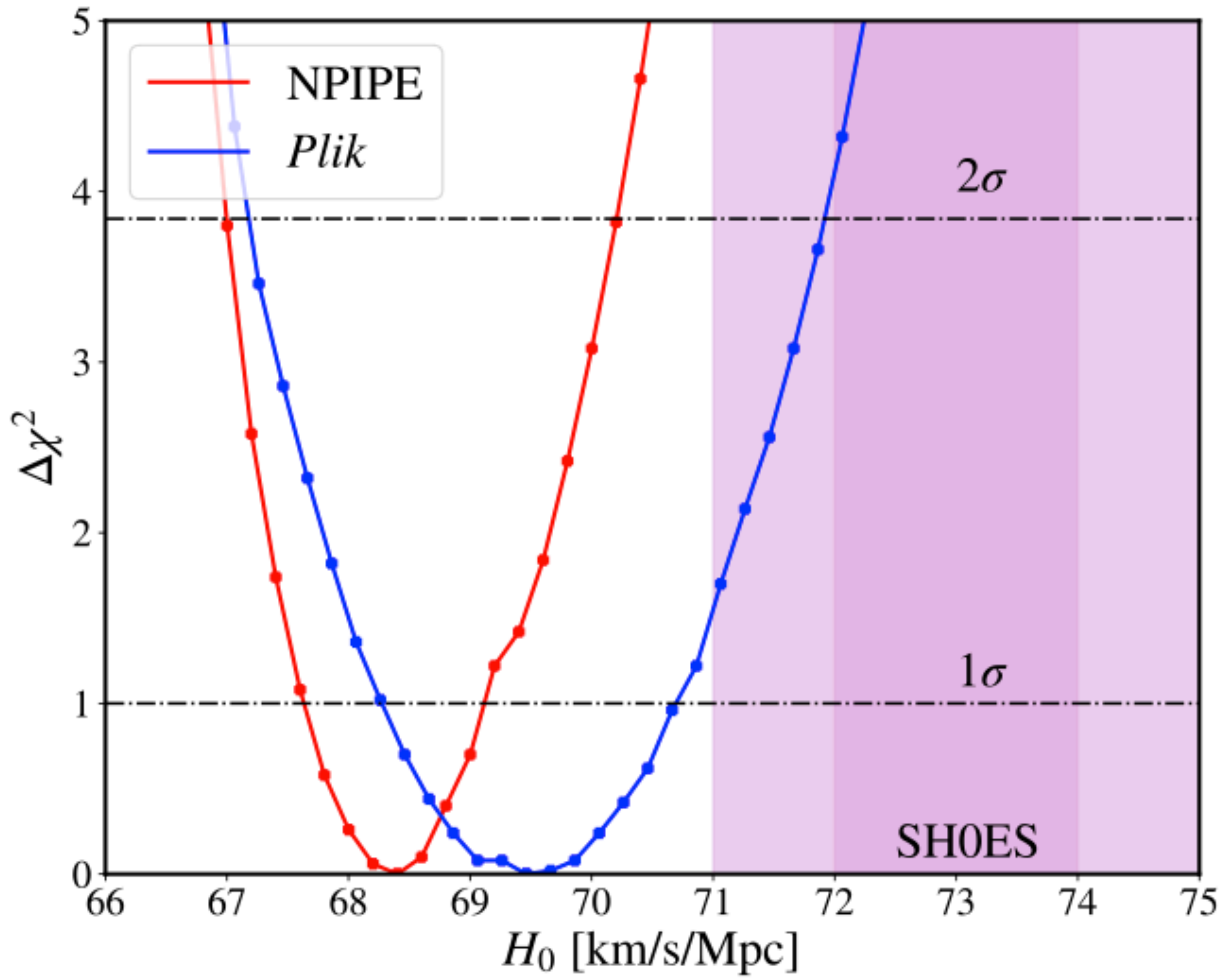

In the limit of a large data set: Both statistical approaches agree

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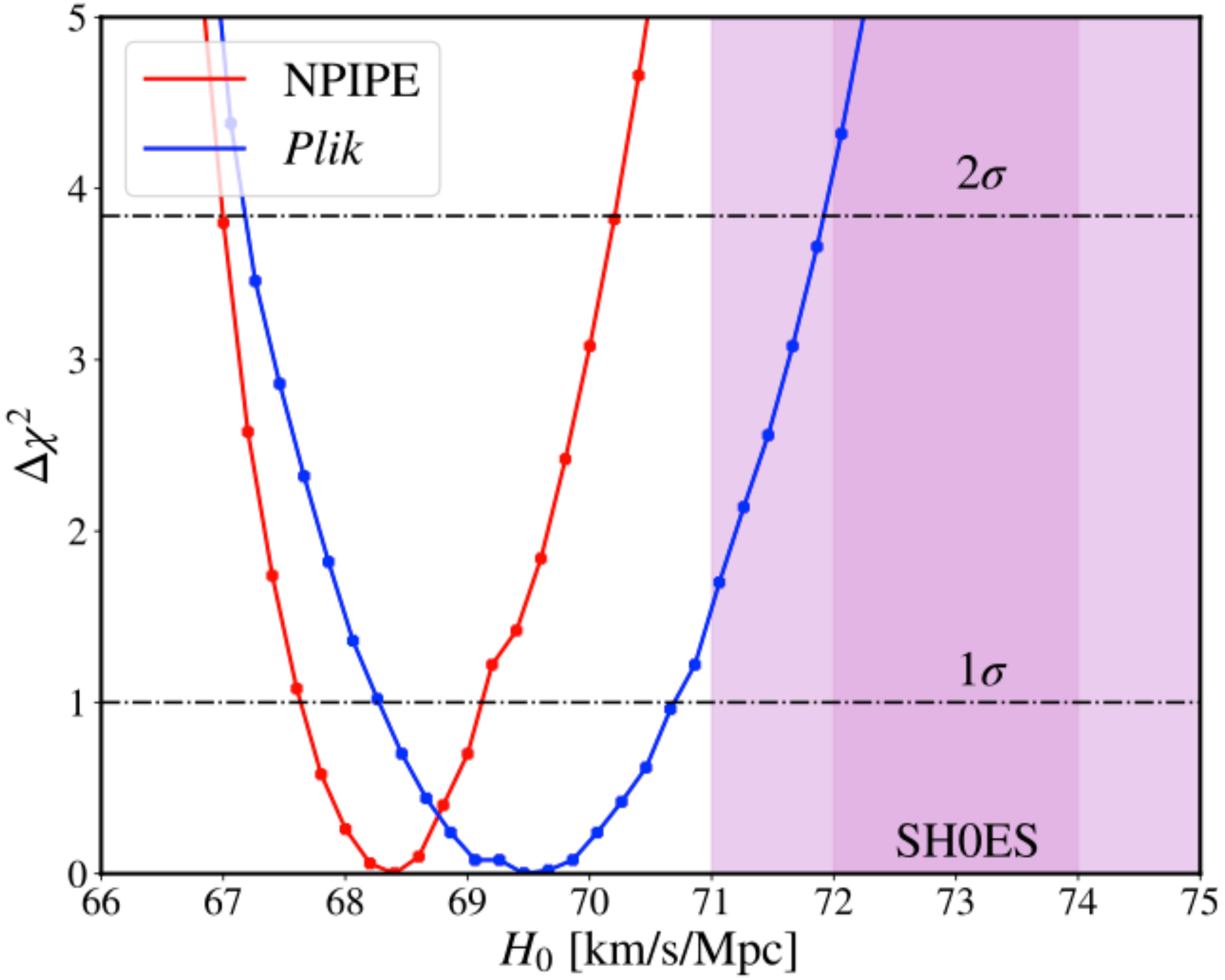

Caveat: not clear whether NPIPE “more correct” than Plik.

Recap Lecture 2

Two statistical philosophies

- Bayesian and frequentist approaches have different interpretations and can differ when there is insufficient data
- If both approaches differ: more data is necessary to give a final answer

The Hubble Tension and Early Dark Energy

- The reason behind the Hubble tension is still unknown
- EDE is a promising solution to the Hubble tension
- EDE has a complicated parameter structure ($f_{\rm EDE}$, z_c, θ_i) and one needs to take care in the analysis
- However, there seem to be more indications that the simple canonical axion-like EDE model is disfavoured (some are still preliminary and might need more analysis/data?)
- EDE-type solutions are still the “least unlikely” ones — Need to be more clever in constructing these models?

Thanks to:

Elisa Ferreira, **Eiichiro Komatsu** and, **Graeme Addison**, who inspired lectures & notebooks!

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Thanks to you for listening!