From ΛCDM to EDE

Lecture 1: Theory

Laura Herold

CosmoVerse@Corfu May 13 - 18, 2024

Overview

Lecture 1: Theory from ΛCDM to EDE (1:30h)

Hands-on session 1: From theory to predictions (1h)

Lecture 2: Observation — Can EDE solve the Hubble tension? (1h)

Hands-on session 2: Let’s analyse EDE with the cosmic microwave background and supernovae (1:30h)

Outline: From ΛCDM to EDE

1. Introduction: basic equations in a homogeneous & isotropic universe (Friedmann equations, matter content in the universe, defining distances)

2. Overview over the timeline of the universe

3. Hubble tension (How does CMB constrain H_0, how do SNe constrain H_0)

4. Early Dark Energy: How to solve the Hubble tension with new physics? Scalar field in an expanding background

Conventions

- Natural units: $c = 1$

- Only background equations (perturbations are covered in a later lecture)

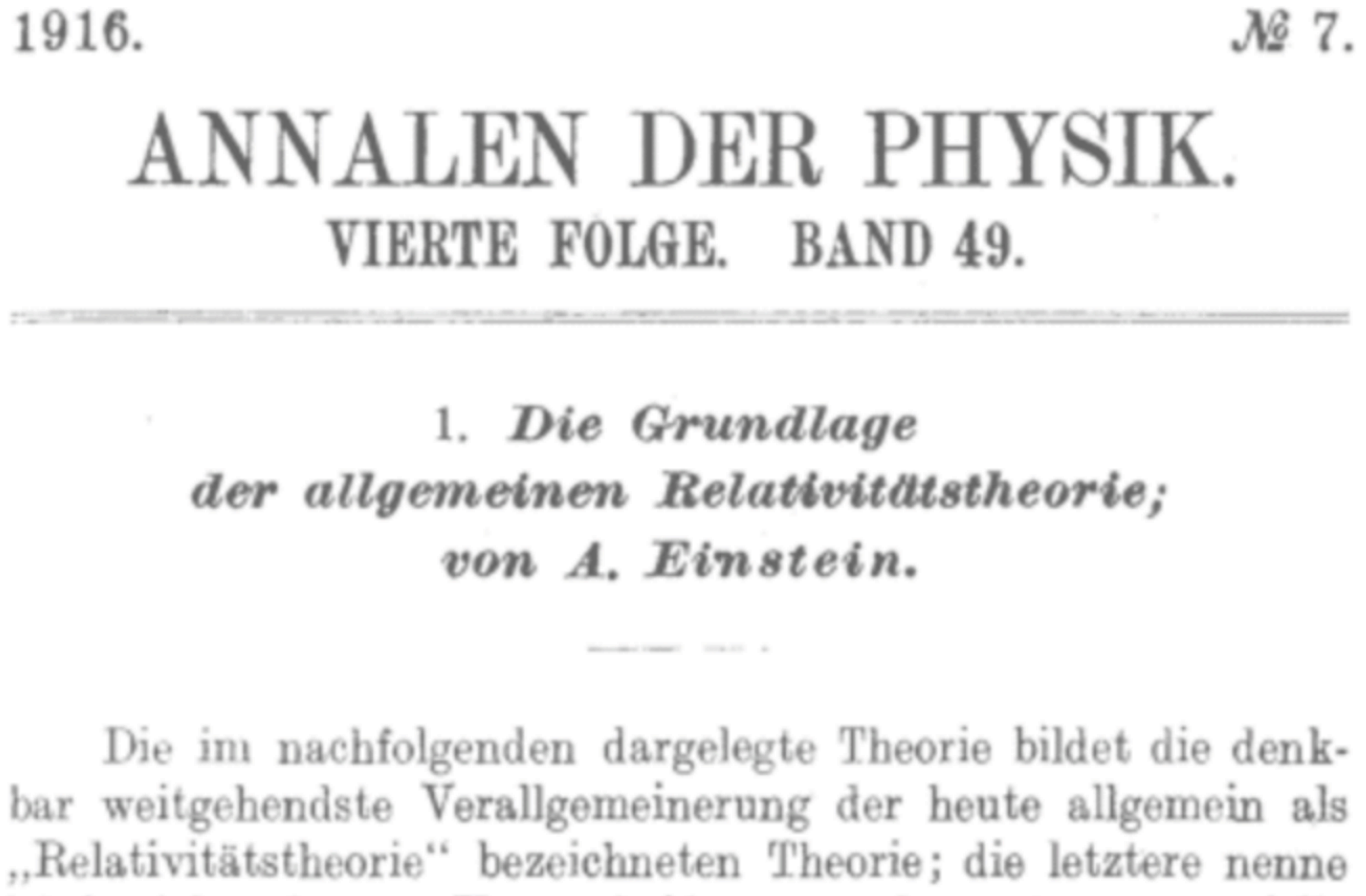
- Add conventions
- Make sure, they apply to all slides

Introduction

Short “crash course” to fix the notation

General Relativity

- Let’s imagine it was 100 years ago:
  - We didn’t know about dark matter (DM)
  - We didn’t know about dark energy (DE)
  - But a few years ago, Albert Einstein had published the theory of General Relativity (GR)



1916. № 7.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 49.

1. *Die Grundlage der allgemeinen Relativitätstheorie;*
von A. Einstein.

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie“ bezeichneten Theorie; die letztere nenne

General Relativity

For more about GR, see Matteo Martinelli's lecture

- We will not go into details here but only sketch the rough idea
- Einstein Equations:

$$R^{\mu\nu} - \frac{R}{2}g^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu},$$

Ricci curvature tensor

metric

cosmological constant

energy-momentum tensor

“Matter tells space how to curve, space tells matter how to move” (Misner++ 1973)

General Relativity

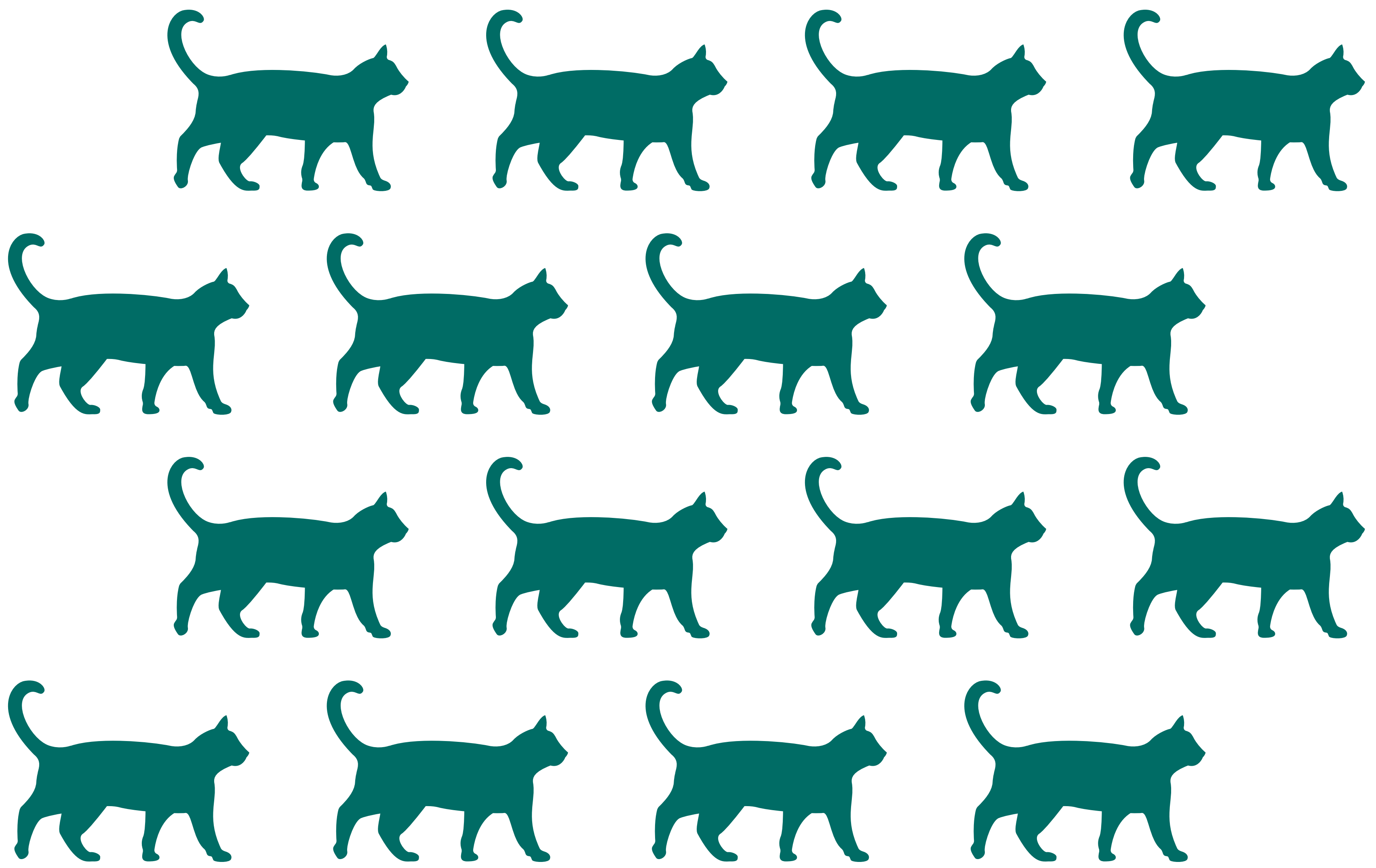
- Solution for Einstein Equations for the universe as a whole?
- Cosmological Principle:

"On sufficiently large scales, the properties of the universe is the same for all observers."

or

"The universe is spatially homogeneous and isotropic on large scales."

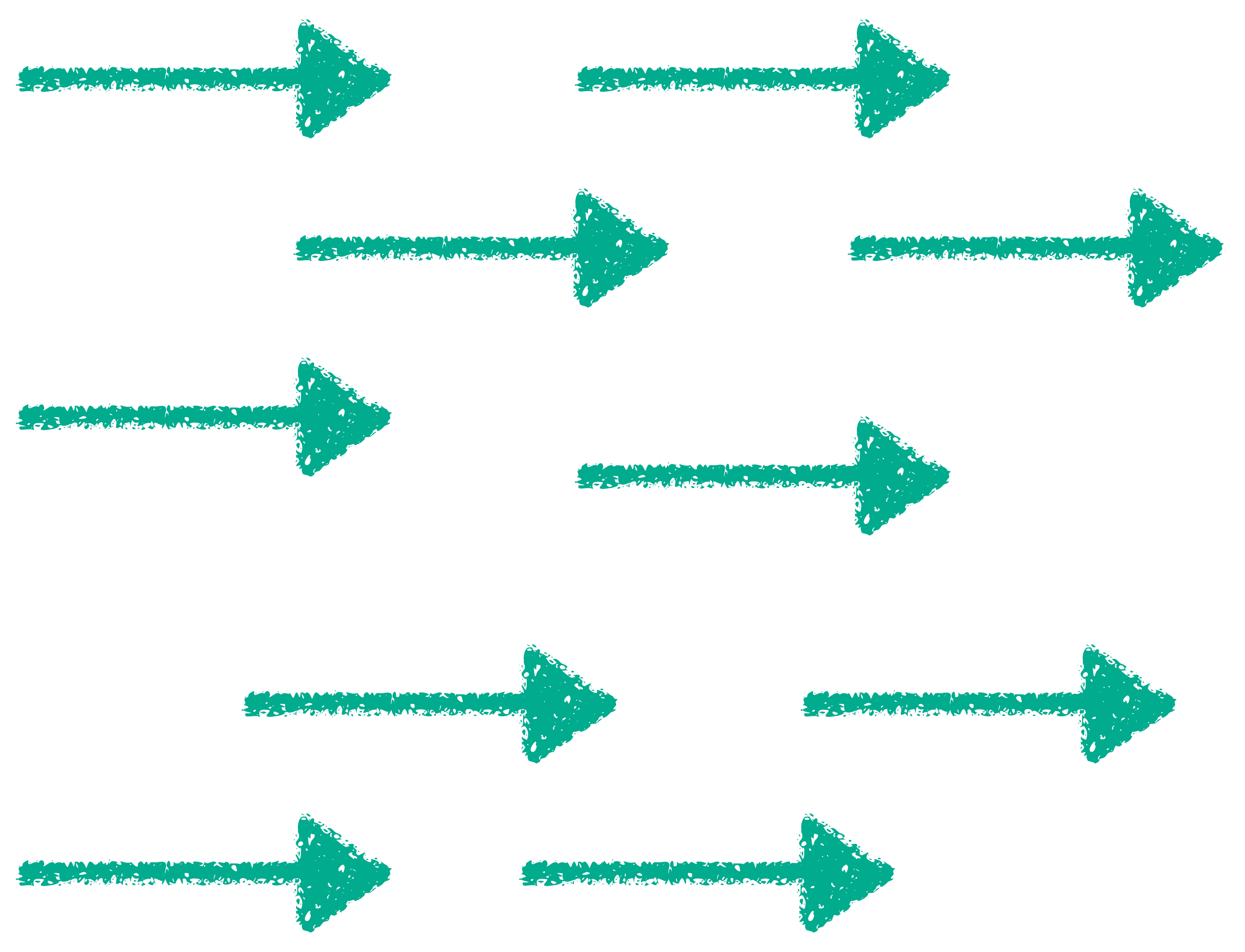
Isotropic or homogeneous?



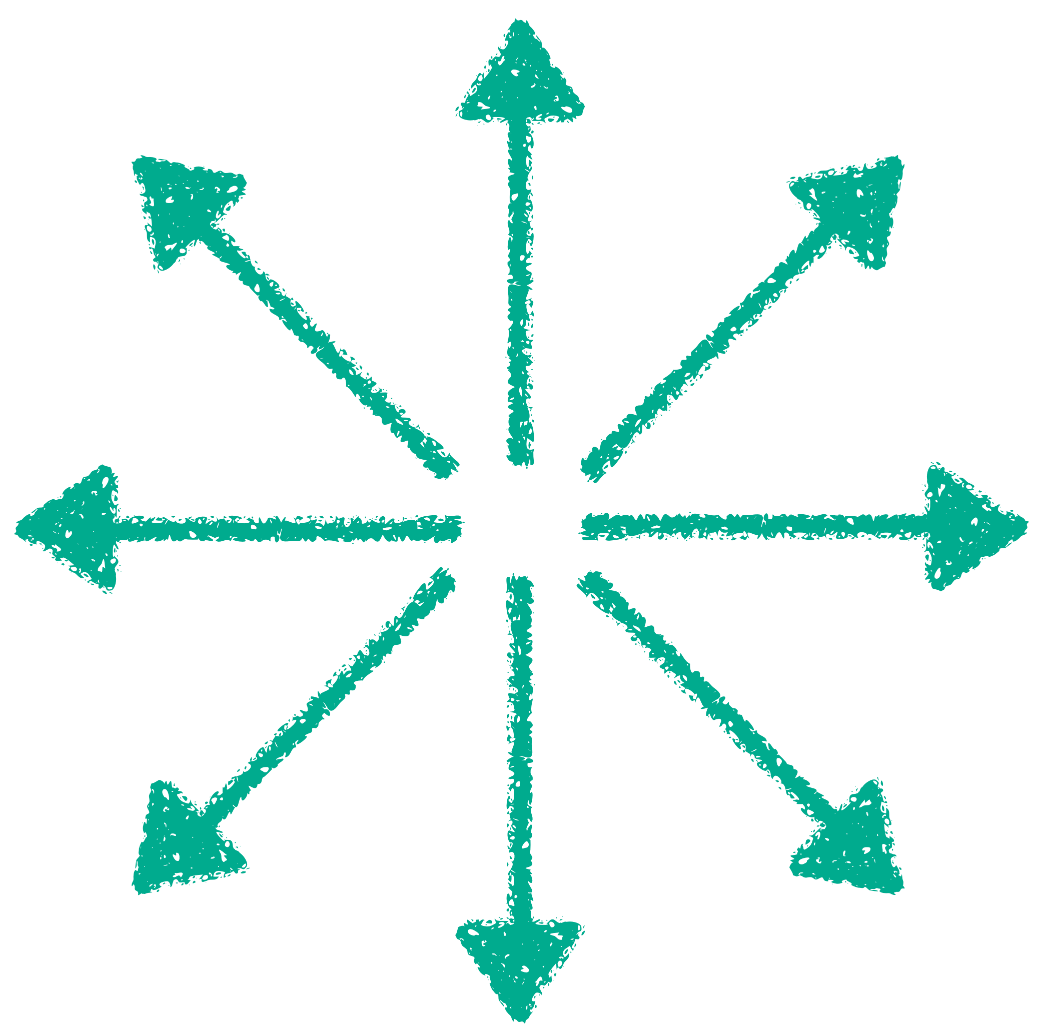
homogenous but not isotropic

isotropic but not homogenous

Isotropic or homogeneous?



homogenous but not isotropic



isotropic but not homogenous

Friedmann Equations

- Friedmann 1922, Robertson 1935, Walker 1937: For a spatially homogeneous and isotropic universe, the metric simplifies to

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\frac{\delta_{ab}}{1+\frac{k}{4}|\boldsymbol{x}|^2}\mathrm{d}x^a\mathrm{d}x^b$$

spacetime line element

Figure credit: Bianchi, Rovelli, Kolb

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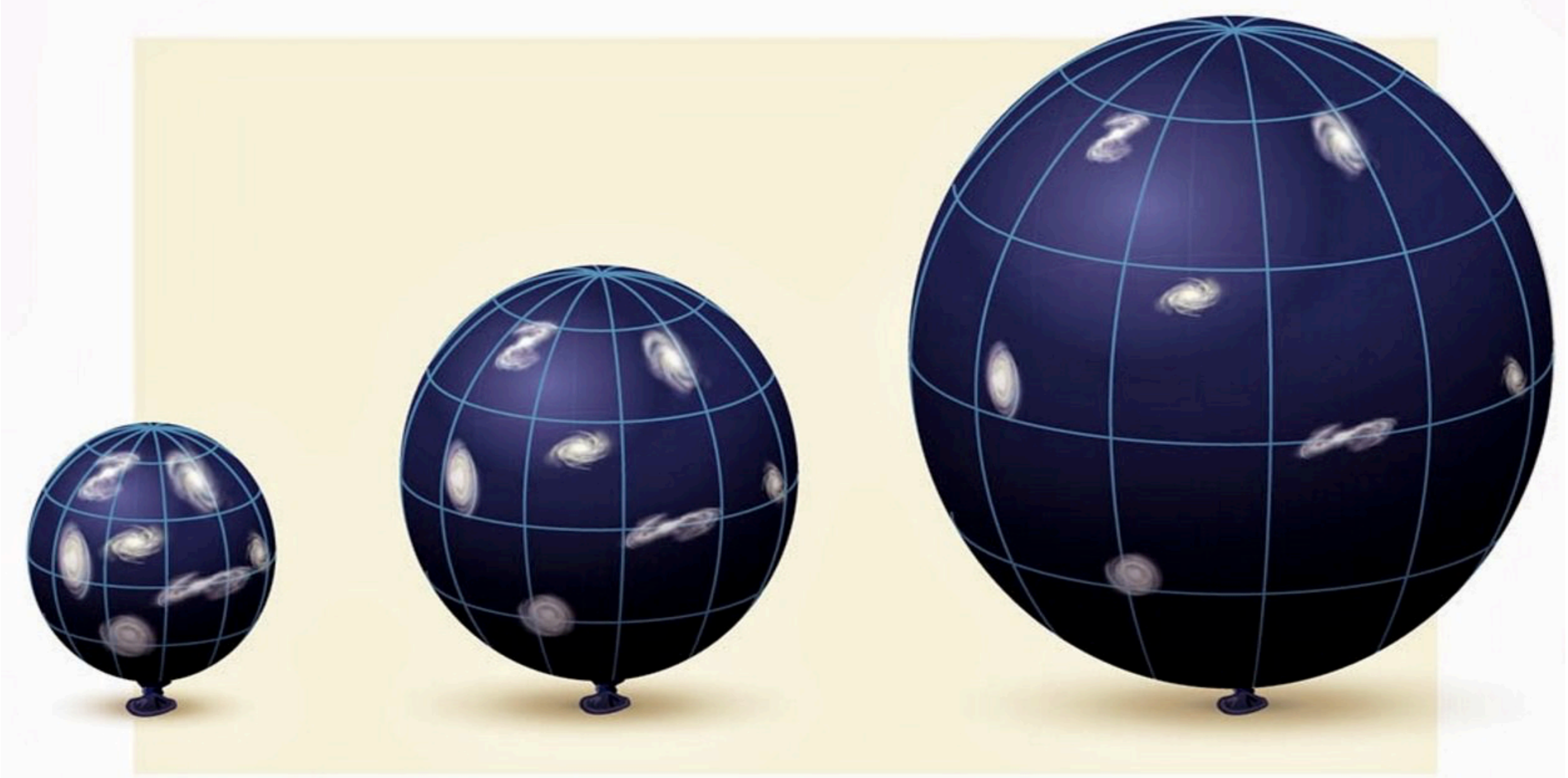
spacetime line element

scale factor



$$a(t_{\text{today}}) = a_0 = 1$$

Figure credit: Bianchi, Rovelli, Kolb

Friedmann Equations

- Friedmann 1922, Robertson 1935, Walker 1937: For a spatially homogeneous and isotropic universe, the metric simplifies to

$$\underbrace{\mathrm{d}s^2}_{\text{spacetime line element}} = -\,\mathrm{d}t^2 + \underbrace{a^2(t)}_{\text{scale factor}} \frac{\delta_{ab}}{1 + \underbrace{\frac{k}{4}}_{\text{curvature parameter}} |\boldsymbol{x}|^2} \mathrm{d}x^a \mathrm{d}x^b$$

$$k = \begin{cases} -1 & \text{hyperbolic} \\ 0 & \text{flat} \\ +1 & \text{spherical} \end{cases}$$

Friedmann Equations

- A perfect fluid is a fluid, which can be completely characterised by its (energy) density and pressure

- The energy momentum tensor of a perfect fluid is

$$T_{\mu\nu}^{\mathrm{pf}} = (\underbrace{\rho}_{\text{energy density}} + \underbrace{p}_{\text{pressure}})\, u_\mu u_\nu + p g_{\mu\nu}$$

- $u_\mu = \dot{x}_\mu$ is the 4-velocity of the observer

- Inserting this in the Einstein Equations, yields the Friedmann Equations

Friedmann Equations

- Inserting the FLRW-metric and the energy momentum tensor of the perfect fluid into the Einstein equations, yields the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \quad \text{(i)}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_{\mathrm{N}}}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad \text{(ii)}$$

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- Inserting (ii) into the temporal derivative of (i), yields the continuity equation:

$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a} \quad \text{(iii)}$$

Friedmann Equations

$$\boxed{\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} &= \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \quad \text{(i)} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad \text{(ii)} \\ \dot{\rho} &= -3(\rho + p)\frac{\dot{a}}{a} \quad \text{(iii)} \end{aligned}}$$

- Where $\rho = \rho_{\text{tot}}$ is the energy density of the universe.
- If we define $\rho_k = -\dfrac{3}{8\pi G_{\text{N}}}\dfrac{k}{a^2}$ and $\rho_\Lambda = \dfrac{\Lambda}{8\pi G_{\text{N}}}$, one can rewrite (i) as

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{tot}} \quad \text{(i)}}$$

$$\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda + \rho_k$$

ΛCDM model $\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda + \rho_k$

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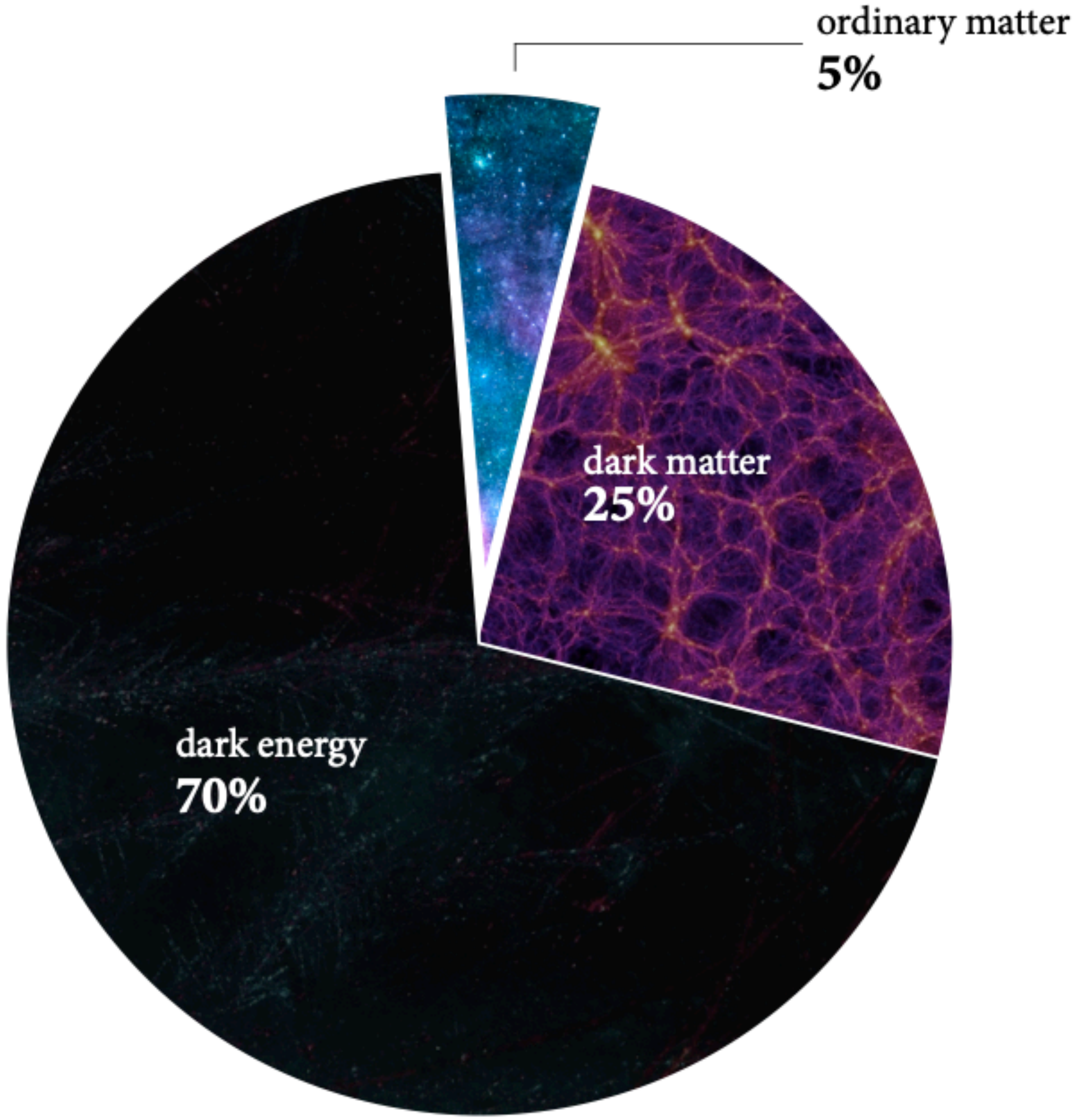


Figure credit: Florian Wolz

ΛCDM model $\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda$

- Equation of state: $p = w \cdot \rho$

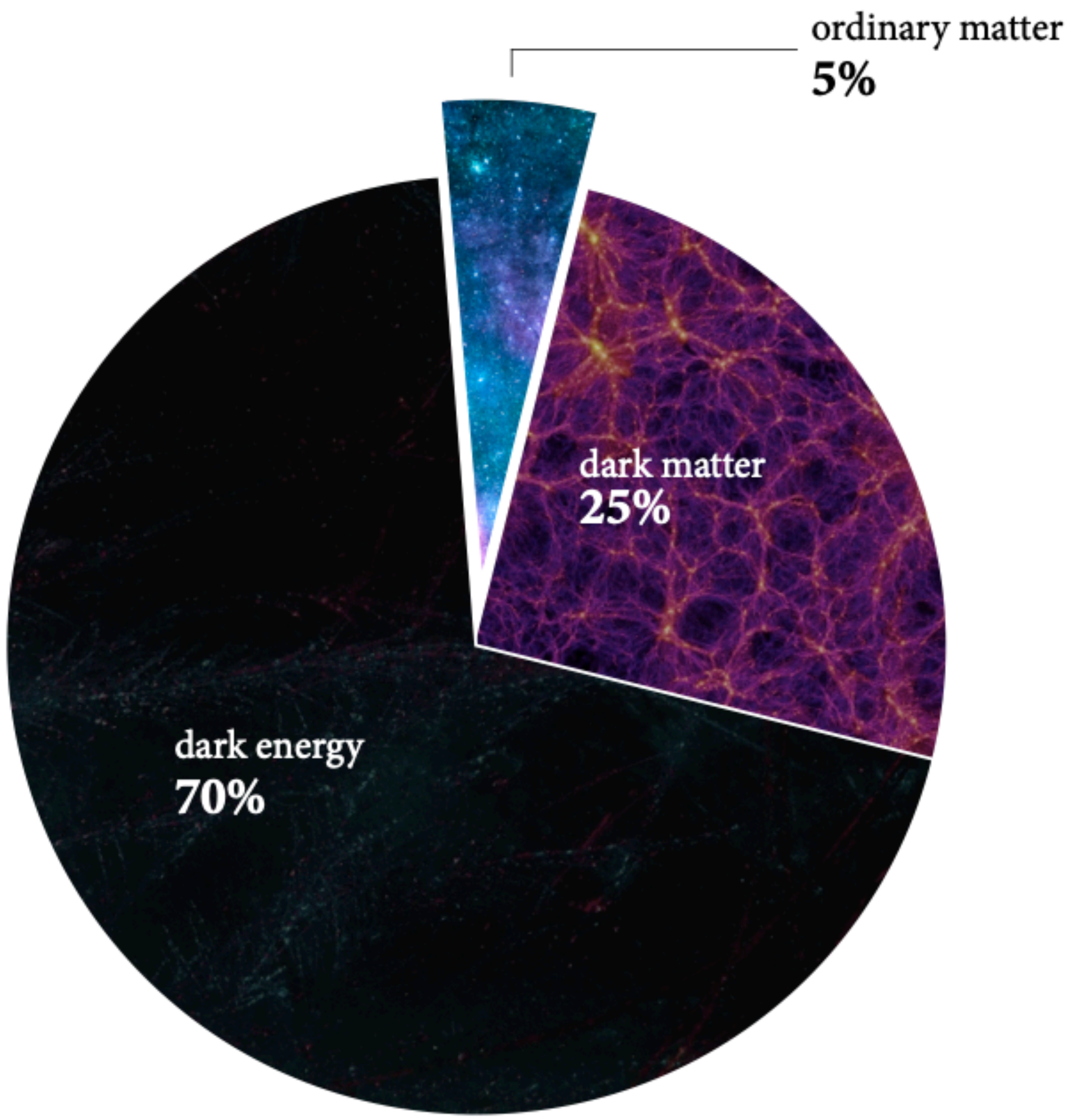
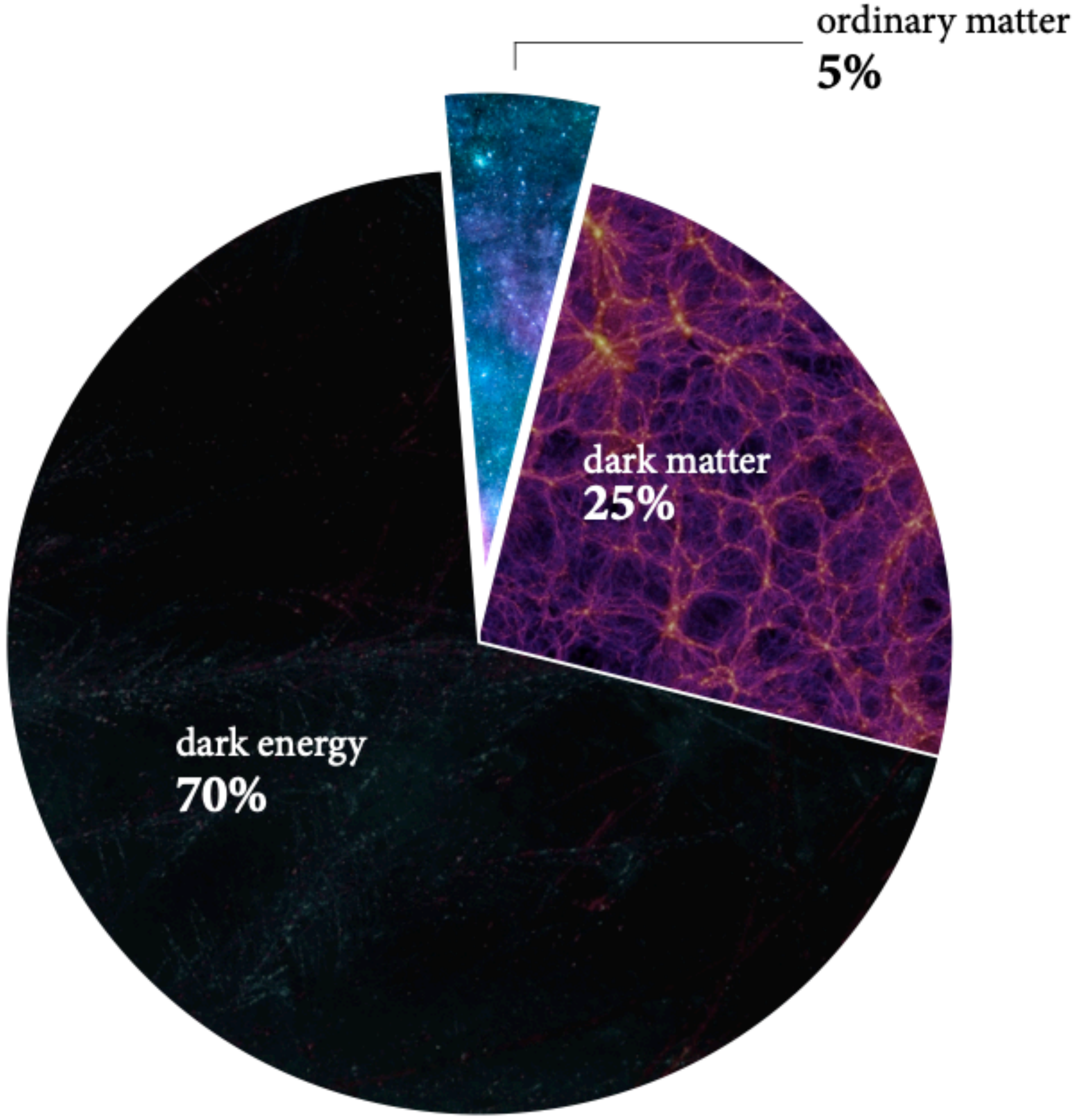


Figure credit: Florian Wolz

ΛCDM model $\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda$

- Equation of state: $p = w \cdot \rho$
- EOS-parameter w for different matter species:
  - Dark matter and baryonic matter: $w = 0$



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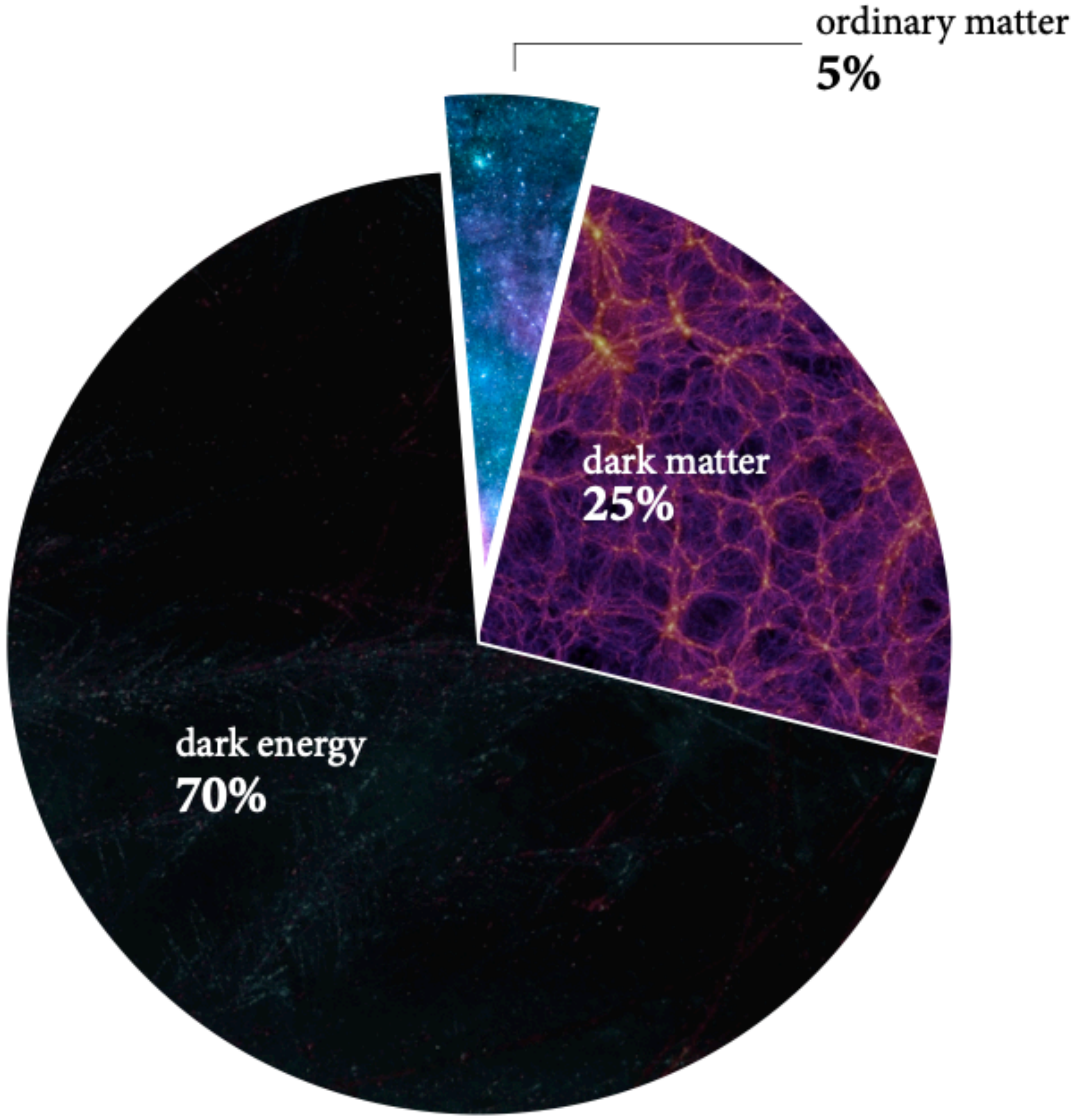

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ΛCDM model $\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda$

- Equation of state: $p = w \cdot \rho$
- EOS-parameter w for different matter species:
  - Dark matter and baryonic matter: $w = 0$
  - Radiation: $w = \dfrac{1}{3}$
  - Dark energy: $w = -1$

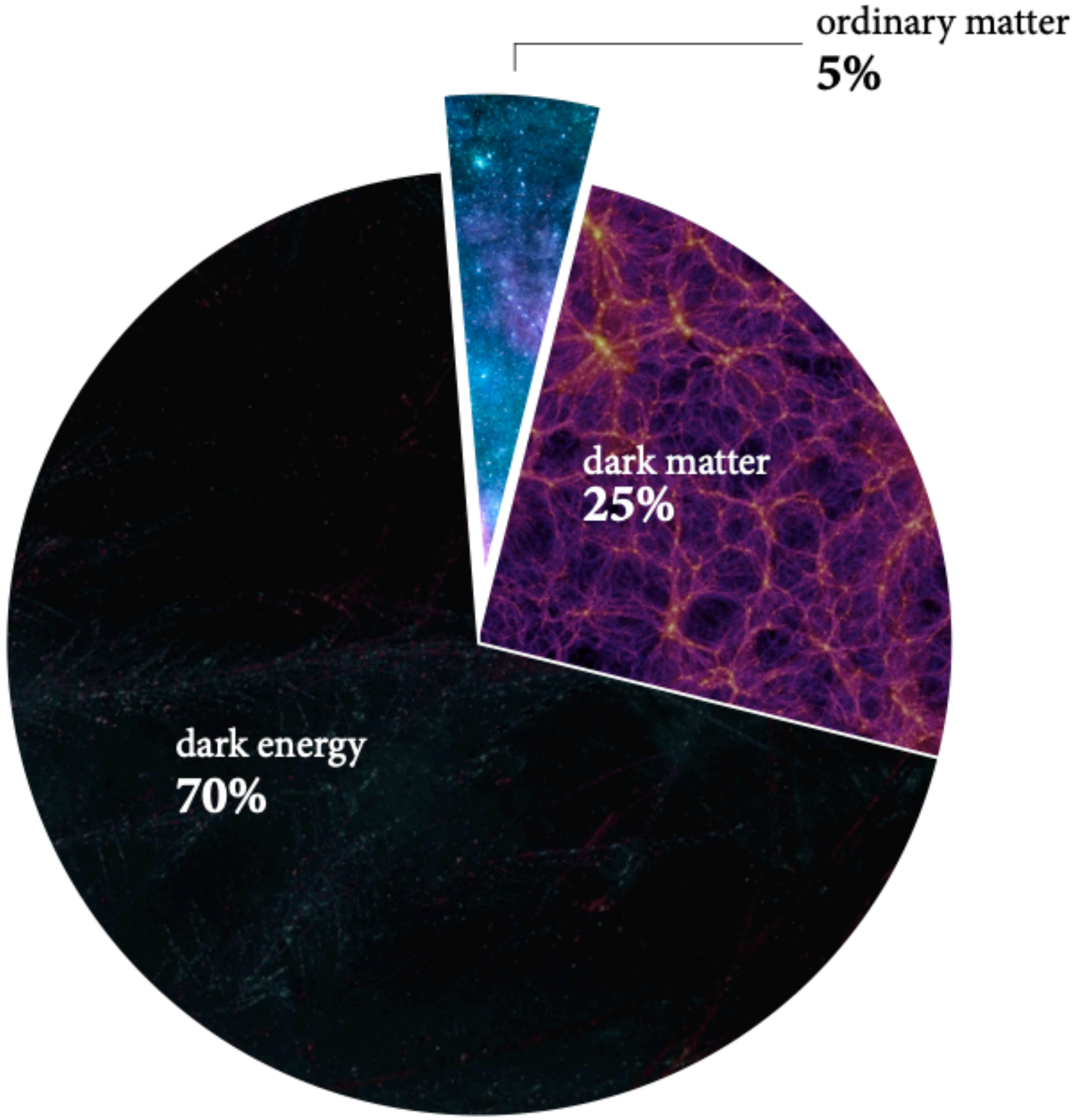

Figure credit: Florian Wolz

Friedmann Equations

- Equation of state: $p = w \cdot \rho$

- Inserting the E.O.S into the continuity equation $\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}$ yields:

$$\dot{\rho} = -3\rho(1 + w)\frac{\dot{a}}{a}$$

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$$\dot{\rho} = -3\rho(1+w)\frac{\dot{a}}{a}$$

… (your exercise later)

$$\boxed{\frac{\rho(a)}{\rho_0} = \left(\frac{a}{a_0}\right)^{-3(1+w)}}$$

The subscript 0 refers to the current time "today"

The Hubble parameter

- The first Friedmann equation describes the rate of expansion as a function of the energy content ρ of the universe:

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\mathrm{tot}} \quad \text{(i)}}$$

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$$H(t) = \frac{\dot{a}}{a}$$

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- The Hubble parameter **today** is called the Hubble constant:

$$H_0 = H(t = t_0)$$

today

The Hubble parameter

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$$\boxed{H^2(t) = \frac{8\pi G}{3}\rho_{\text{tot}} \quad \text{(i)}}$$

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The Hubble parameter

$$H^2(t) = \frac{8\pi G_{\mathrm{N}}}{3}\rho_{\mathrm{tot}}(t)$$

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The Hubble parameter

$$H^2(t) = \frac{8\pi G_{\rm N}}{3}\rho_{\rm tot}(t)$$

- Defining $\rho_{\rm crit} = \dfrac{3H_0^2}{8\pi G_{\rm N}}$

$$\frac{H^2(a)}{H_0^2} = \frac{\rho_{\rm tot}(a)}{\rho_{\rm crit}}$$

The Hubble parameter

$$H^2(t) = \frac{8\pi G_N}{3}\rho_{tot}(t)$$

- Defining $\rho_{crit} = \dfrac{3H_0^2}{8\pi G_N}$ and the fractional energy densities $\Omega_I = \dfrac{\rho_I}{\rho_{crit}}$:

$$\frac{H^2(a)}{H_0^2} = \frac{\rho_{tot}(a)}{\rho_{crit}}$$

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The Hubble parameter

$$H^2(t) = \frac{8\pi G_\mathrm{N}}{3}\rho_\mathrm{tot}(t)$$

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$$\frac{H^2(a)}{H_0^2} = \frac{\rho_\mathrm{tot}(a)}{\rho_\mathrm{crit}} = \Omega_\mathrm{tot}(a) = \Omega_{r,0}\left(\frac{a_0}{a}\right)^4 + \Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{k,0}\left(\frac{a_0}{a}\right)^2 + \Omega_{\Lambda,0}$$

The Hubble parameter

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Defining the redshift $1 + z = \dfrac{a_0}{a}$, one can rewrite this in the famous form:

The Hubble parameter

$$H^2(t) = \frac{8\pi G_N}{3}\rho_{tot}(t)$$

- Defining $\rho_{crit} = \dfrac{3H_0^2}{8\pi G_N}$ and the fractional energy densities $\Omega_I = \dfrac{\rho_I}{\rho_{crit}}$:

$$\frac{H^2(a)}{H_0^2} = \frac{\rho_{tot}(a)}{\rho_{crit}} = \Omega_{tot}(a) = \Omega_{r,0}\left(\frac{a_0}{a}\right)^4 + \Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{k,0}\left(\frac{a_0}{a}\right)^2 + \Omega_{\Lambda,0}$$

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Distances in an expanding universe

Distances in an expanding universe

- In an expanding universe, the "distance" between e.g. two galaxies becomes an ambiguous term; we will discuss common distance definitions in the following

- The comoving distance χ defines a coordinate grid that expands with the universe and is given by

$$\chi(t) = c \int_t^{t_0} \frac{\mathrm{d}t'}{a(t')} = \int_0^{z(t)} \frac{c\,\mathrm{d}z'}{H(z')}$$

- The physical distance r_p is defined as the scale factor times the comoving distance: $r_p(t) = a(t)\,\chi(t)$

Distances in an expanding universe

- In an expanding universe, the “distance” between e.g. two galaxies becomes an ambiguous term; we will discuss common distance definitions in the following

Distances in an expanding universe

- The angular diameter distance D_A is defined such that it holds

$$\theta = \frac{s}{D_A}$$

- To derive an equation for D_A, note that the proper size, s, of the object can also be expressed as $s = \chi(t)\,\theta \cdot a(t)$, where $\chi(t)\,\theta$ corresponds to the comoving size of the object. Hence, the angular diameter distance takes on the form

$$D_A(t) = a(t)\,\chi(t) = \frac{c}{1+z(t)} \int_0^{z(t)} \frac{\mathrm{d}z'}{H(z')}$$

Distances in an expanding universe

- The luminosity distance D_L is defined such that it holds

$$F = \frac{L}{4\pi D_L^2}$$

- Since the Universe is expanding, it holds that $F = \dfrac{L\,a^2}{4\pi\chi^2(a)}$, where the additional factor of a^2 comes from the fact that the expansion of the Universe leads to a dilution of photons ($\times\, a$) and to an increase in wavelength ($\times\, a$). Hence, the luminosity distance is given by

$$D_L(t) = \frac{\chi(t)}{a(t)} = [1 + z(t)] \int_0^{z(t)} \frac{c\,\mathrm{d}z'}{H(z')}$$

- adapt distances

How to solve the Hubble tension?

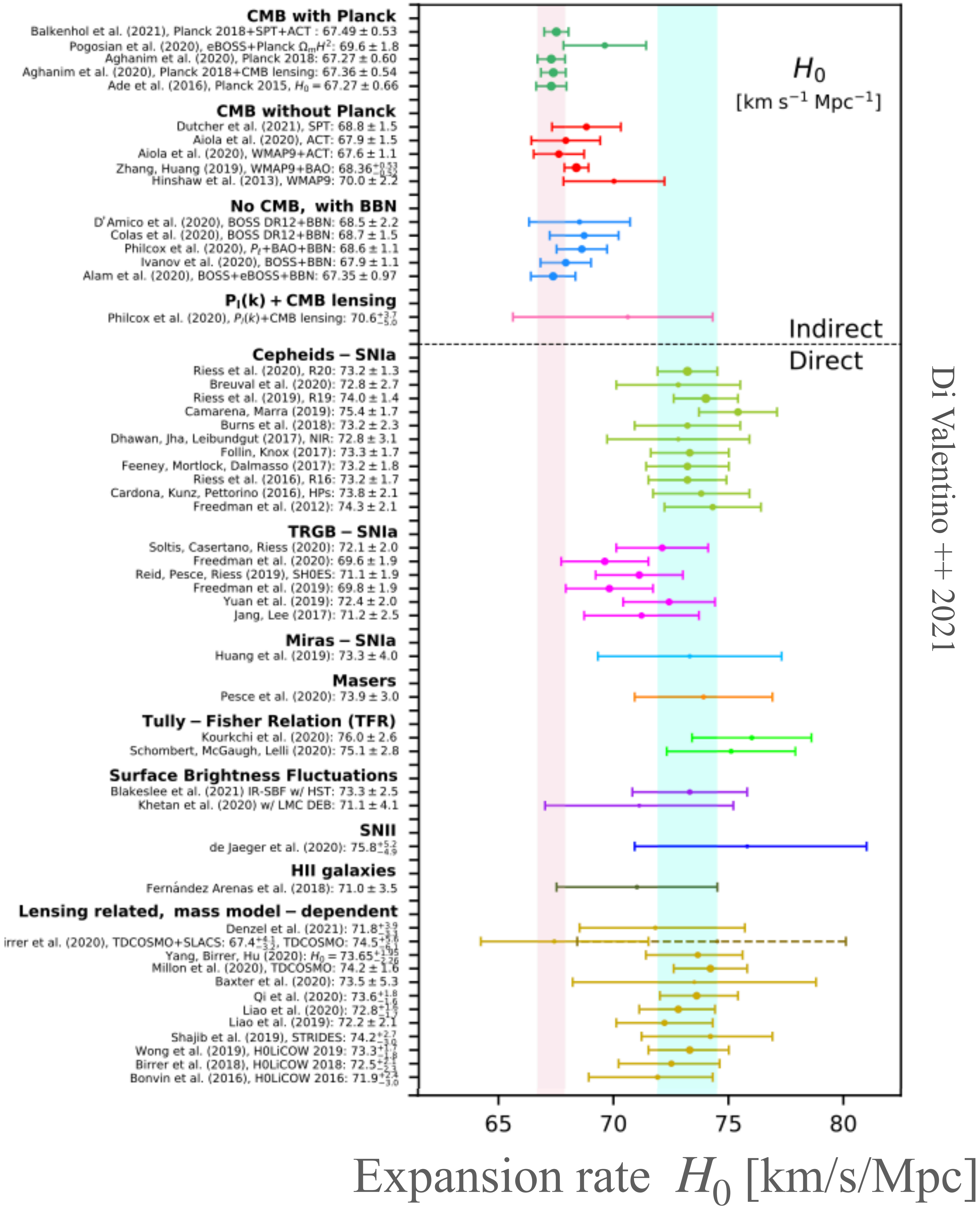
Direct vs. indirect? Early vs. late measurements?

How does the CMB constrain H_0?

The Hubble tension

- Indirect measurements: cosmic microwave background (CMB), baryon acoustic oscillations (BAO), galaxy clustering

- Direct measurements: distance ladder (Cepheids, TRGB, SNe, …), gravitational lensing, …



The Hubble tension

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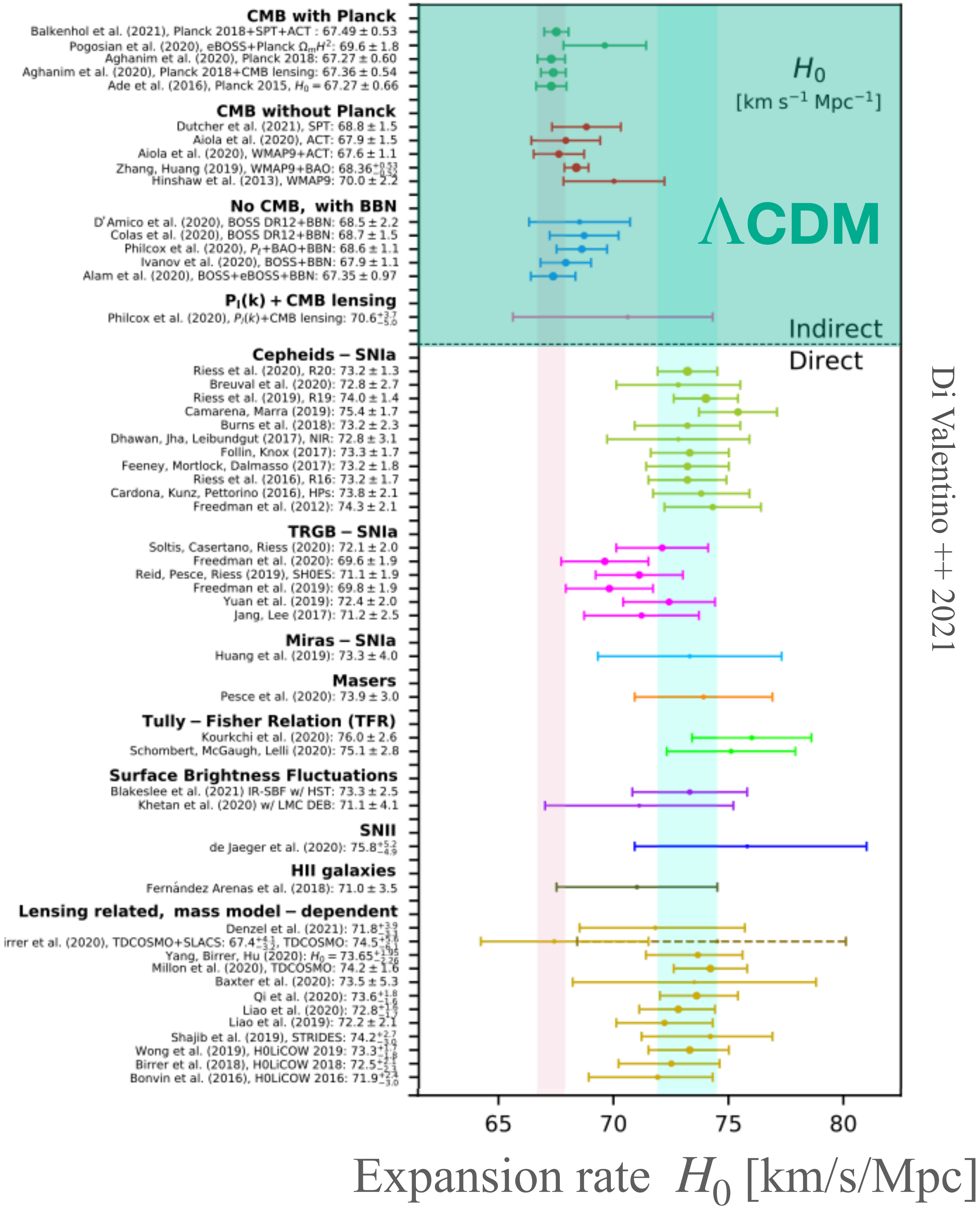
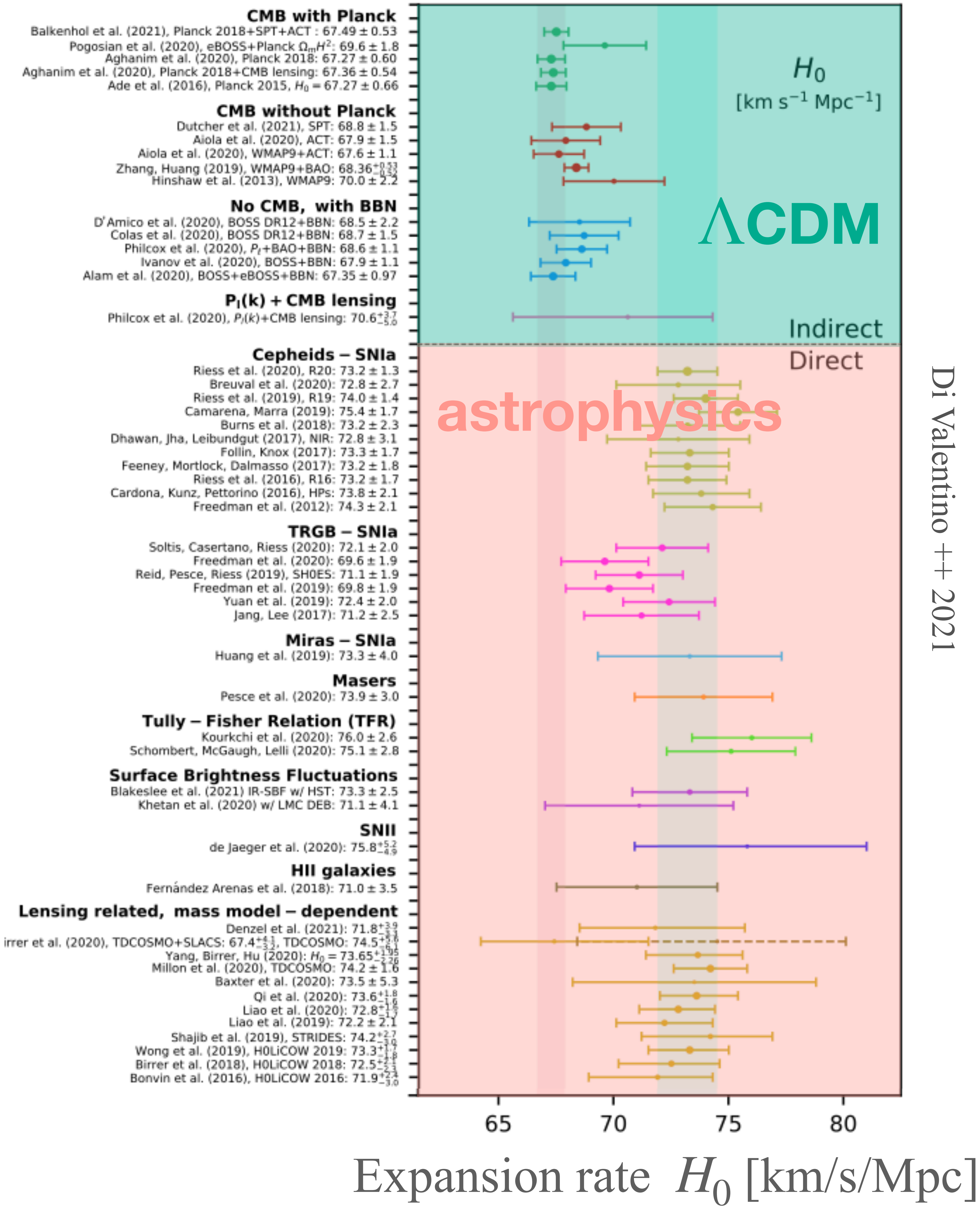


The Hubble tension

- Indirect measurements: cosmic microwave background (CMB), baryon acoustic oscillations (BAO), galaxy clustering → precise but model dependent

- Direct measurements: distance ladder (Cepheids, TRGB, SNe, …), gravitational lensing, … → less precise due to astrophysical modelling but independent of cosmological model

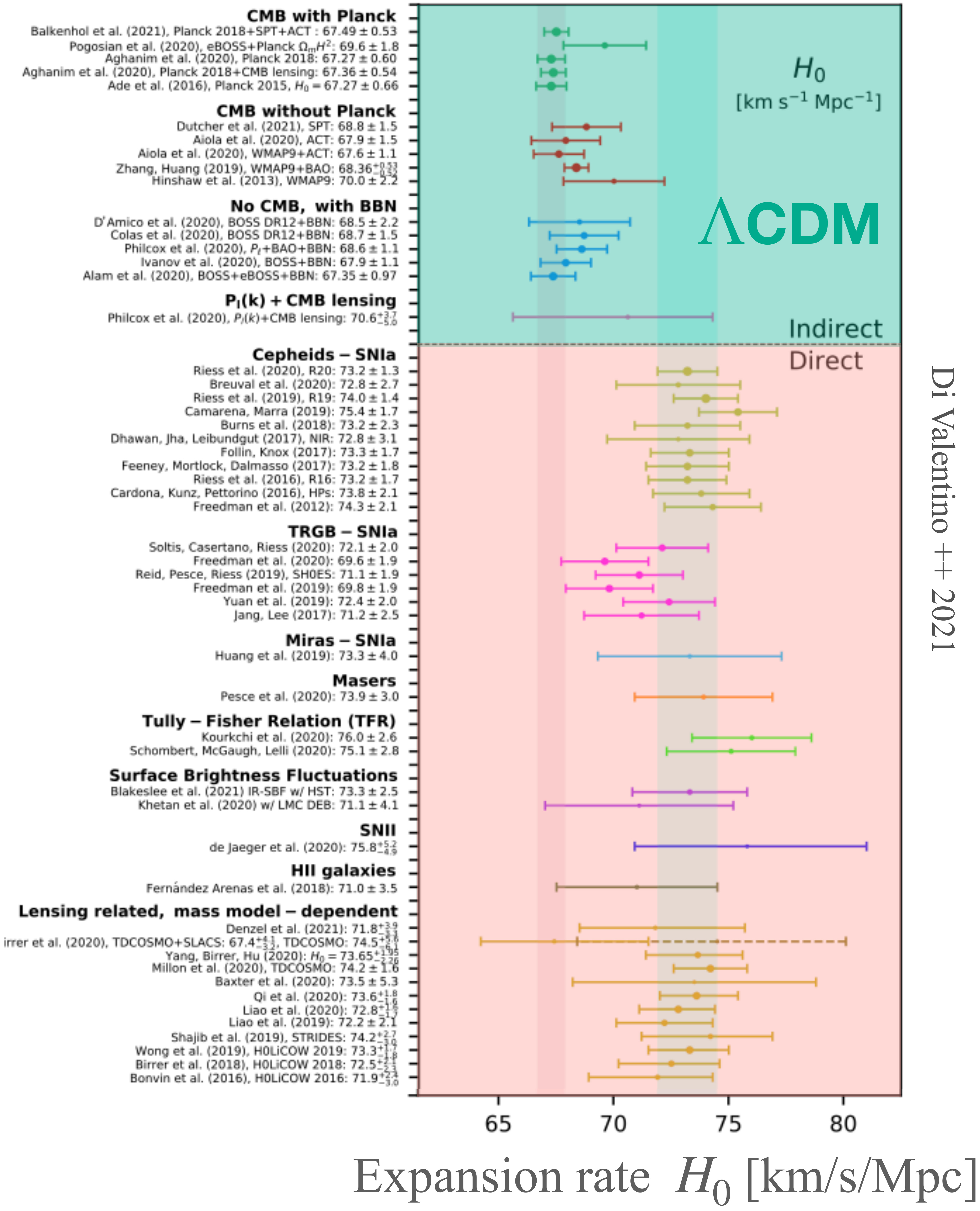


Di Valentino ++ 2021

The Hubble tension

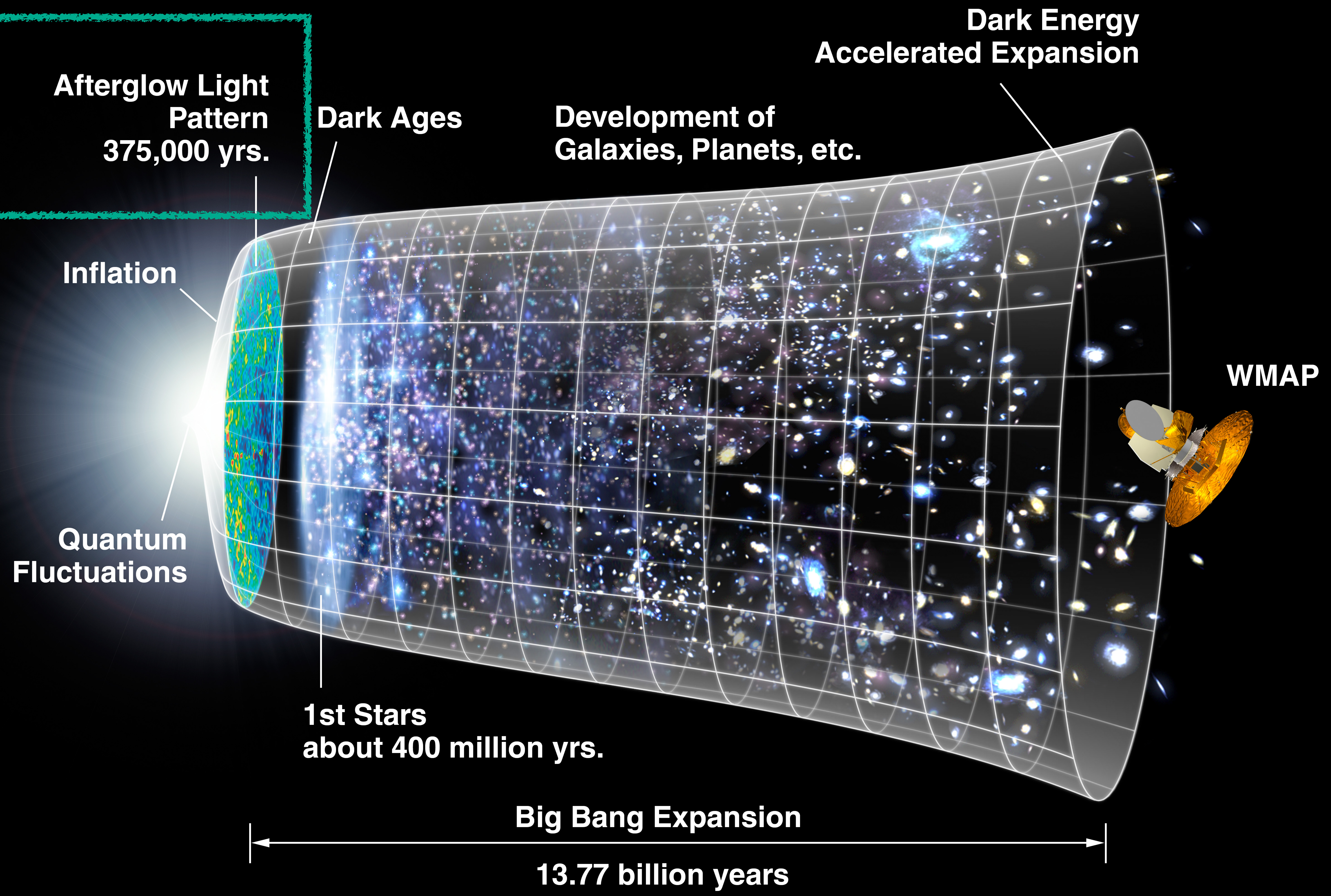
- General strategy to solve the H_0 tension: Assume direct measurements are correct and change cosmological model in order to *infer* a higher H_0

- Goal: “get CMB-H_0 to ~73 km/s/Mpc”



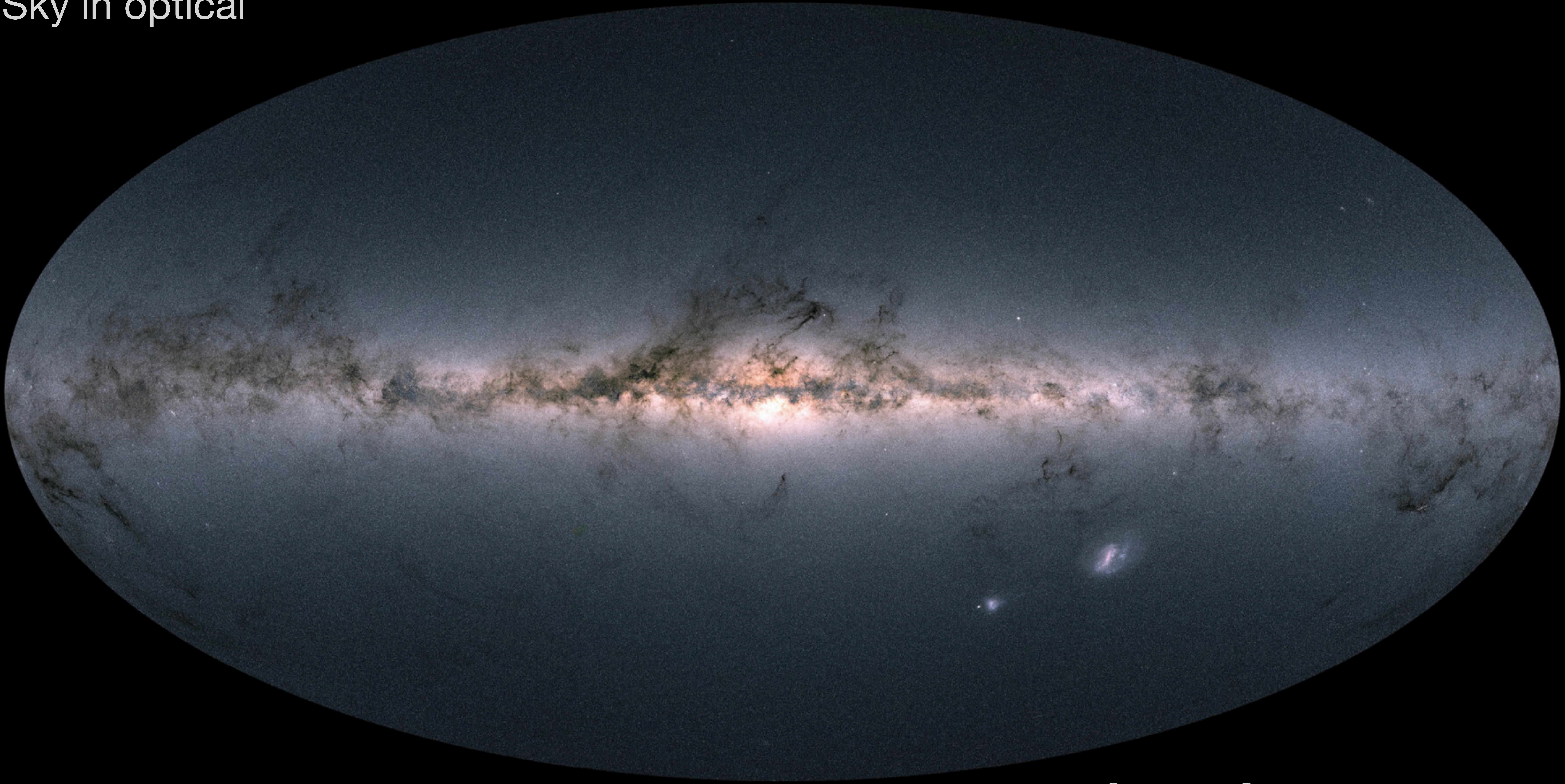
How does the CMB constrain H_0?

- In order to understand how we can solve the Hubble tension, we need to understand how the CMB constrains H_0
- The CMB provides the most important cosmological probe when it comes to constraining the parameters of the cosmological model with high accuracy
- However, it is an indirect probe of H_0: it depends on the cosmological model that is assumed $\rightarrow$ the CMB constrains the universe at early times and predicts H_0 today



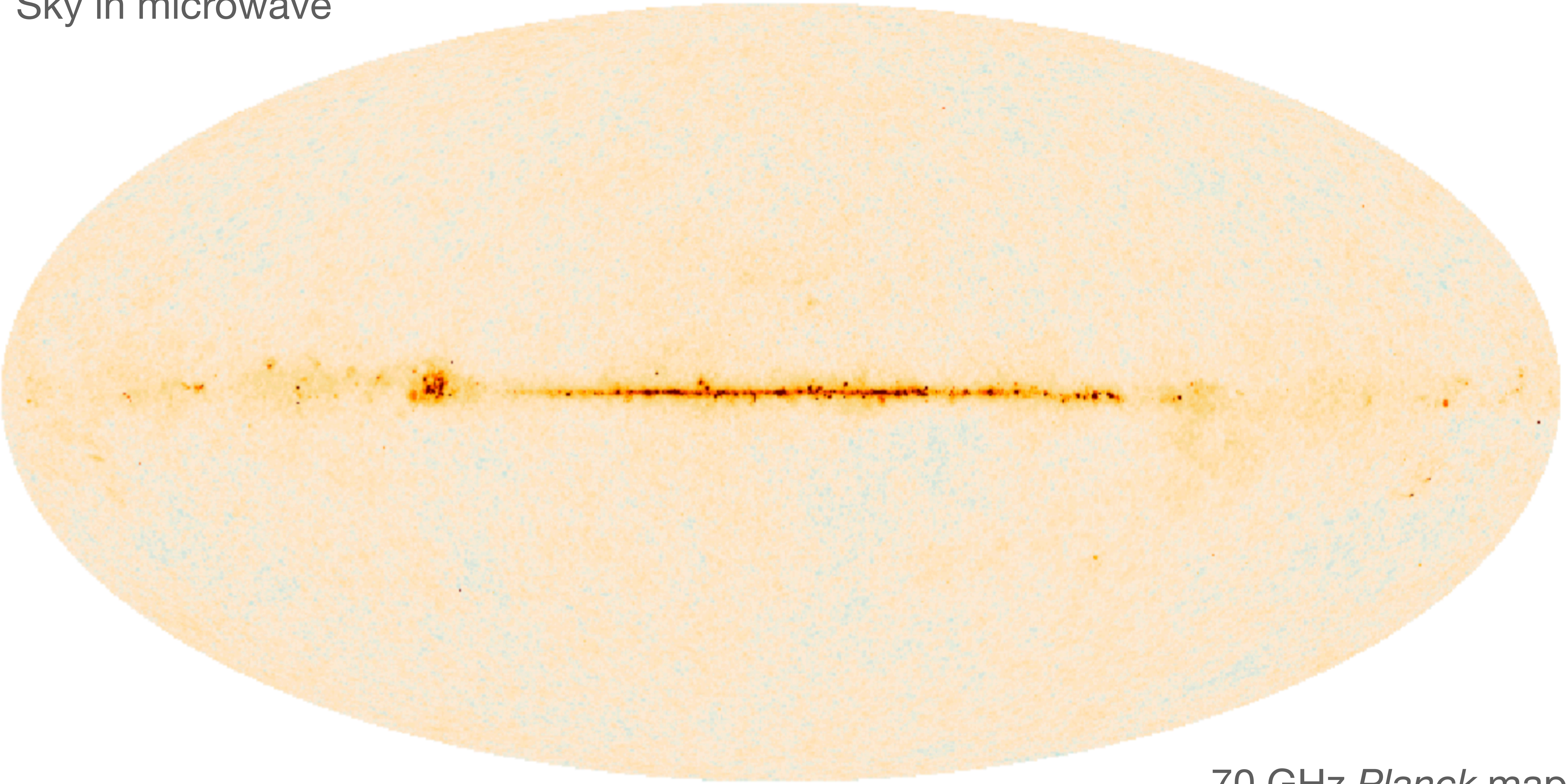
Credit: WMAP Collaboration

Sky in optical



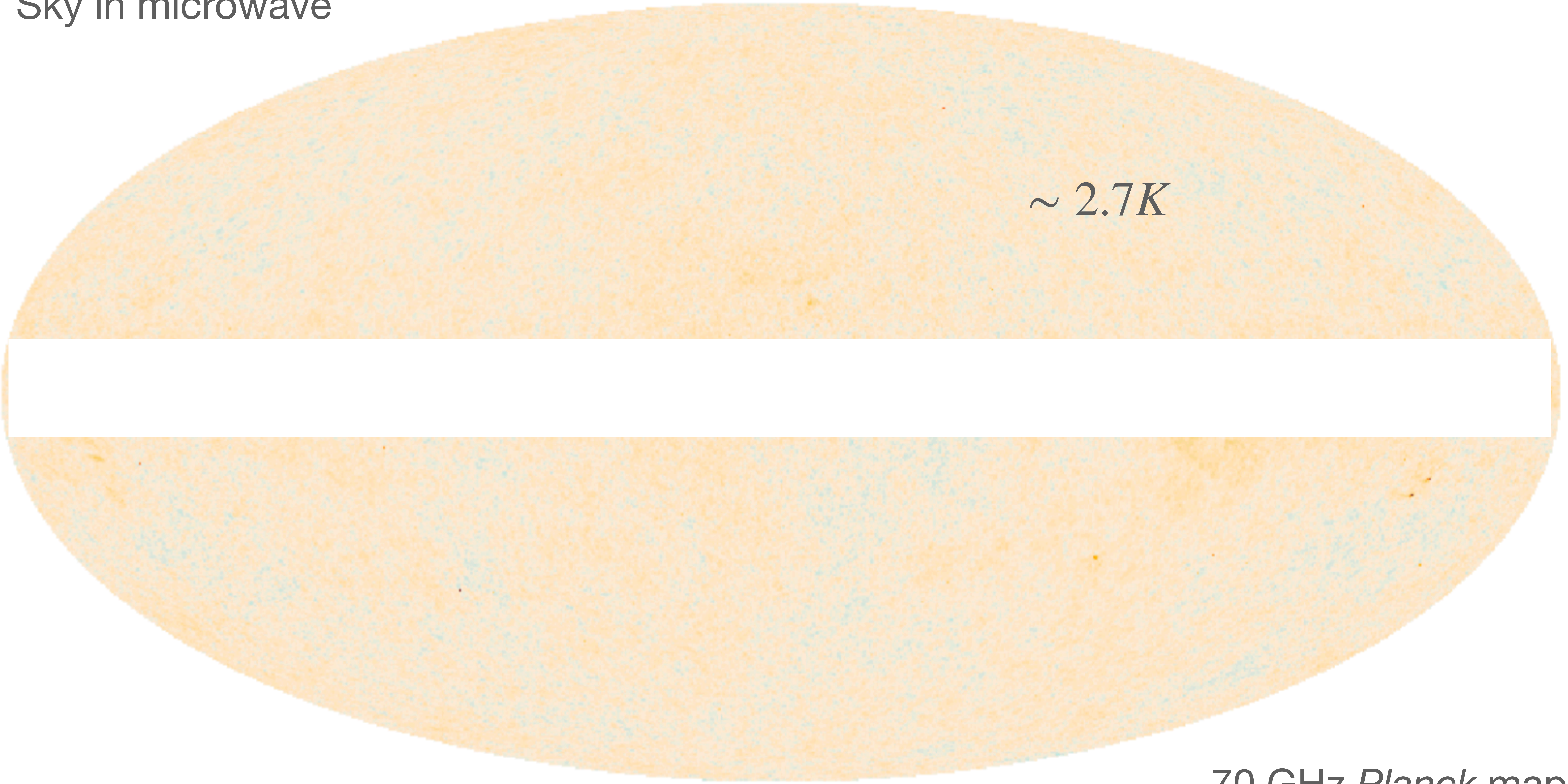
Credit: Gaia collaboration

Sky in microwave



70 GHz *Planck* map

Sky in microwave



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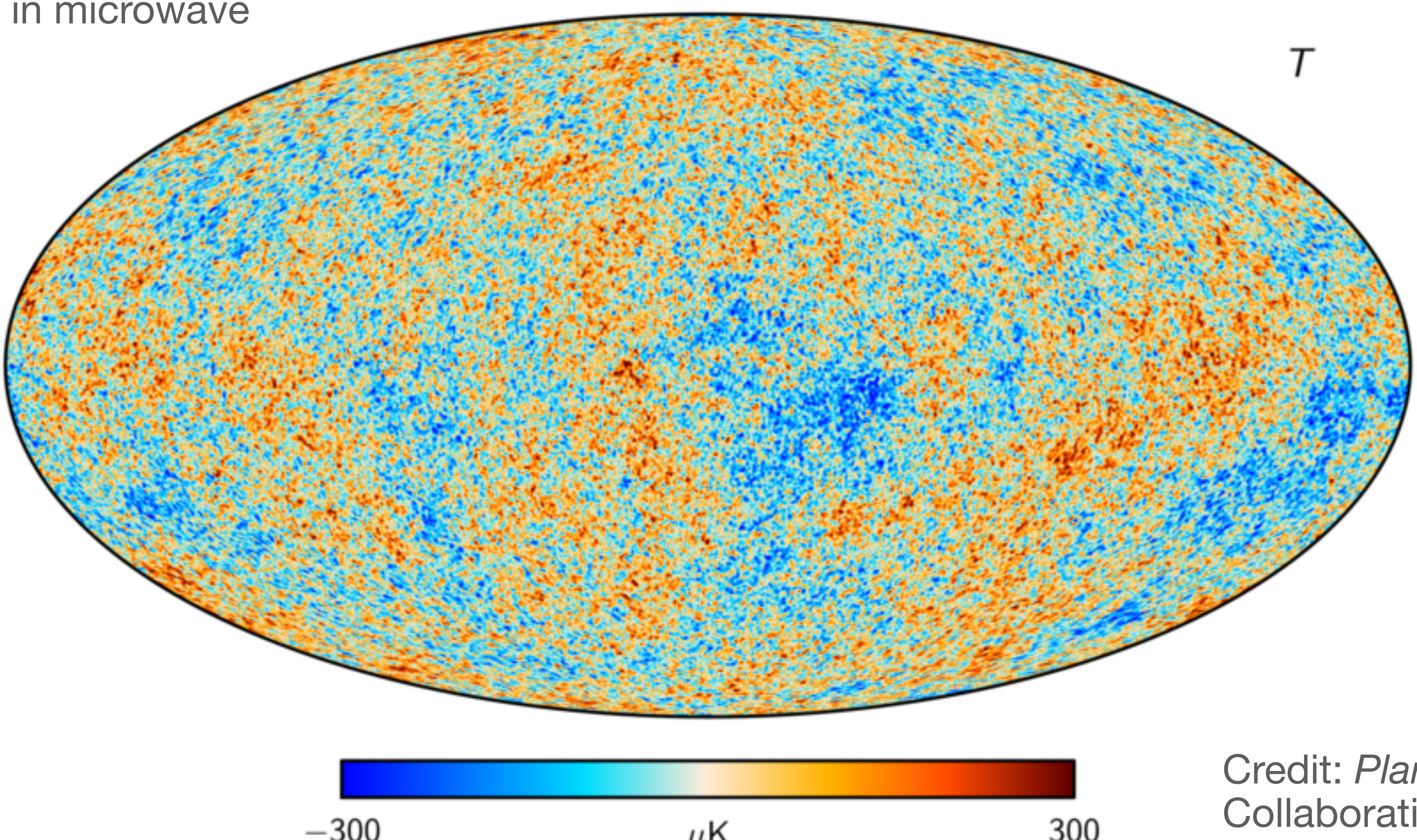


Credit: *Planck* Collaboration

- add LCDM parameters

The CMB

- How to analyse the CMB map with temperature fluctuations?
  - 1. Decompose the temperature fluctuations into a set of waves with various wavelength
  - 2. Plot the strength of each wavelength: Power spectrum
- In 2 dimensions: decomposing into waves = Fourier transform
- On the unit sphere: decomposing into waves = Spherical harmonics decomposition

The CMB

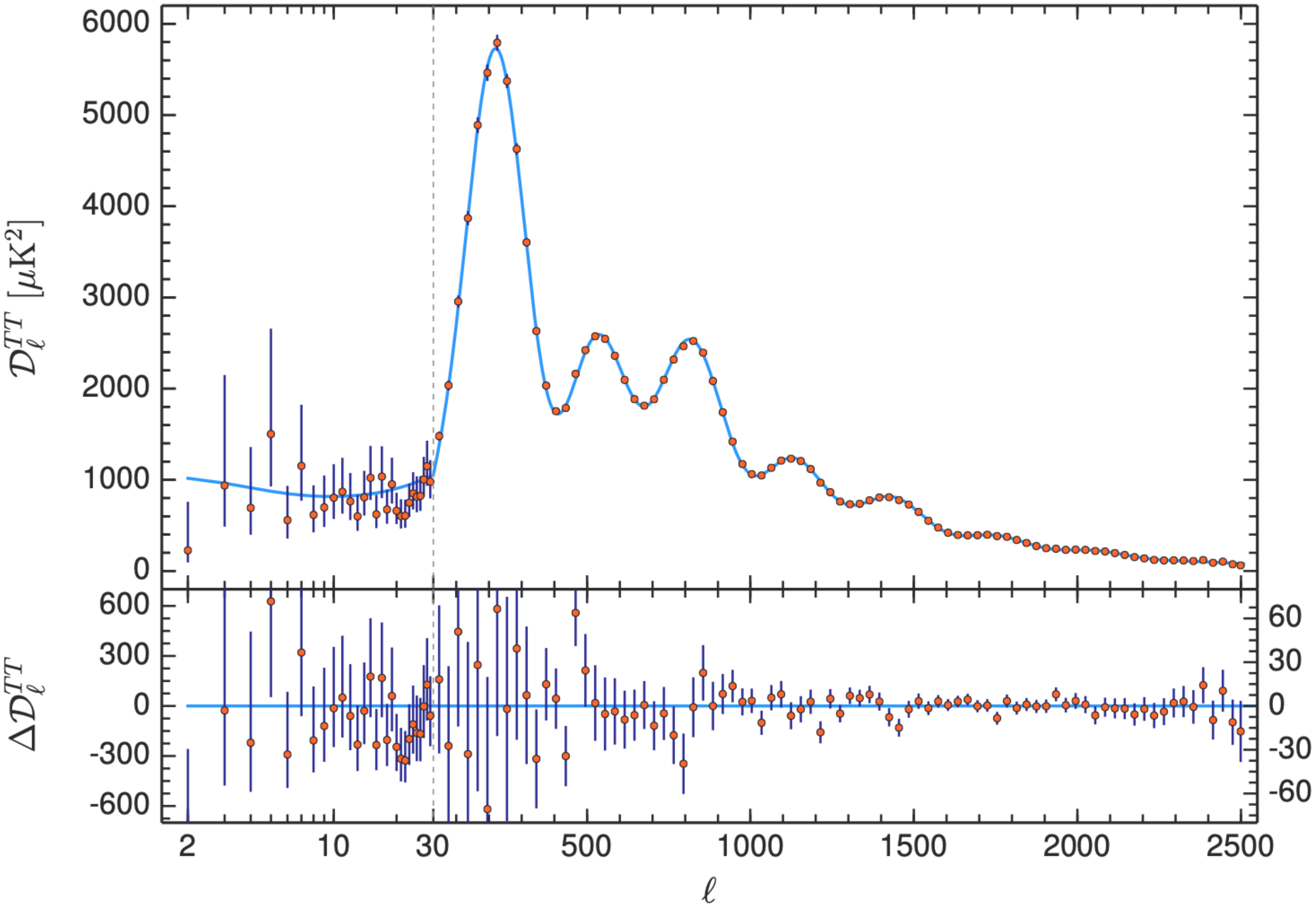
- Decompose temperature fluctuations: $\Delta T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} \, Y_{\ell m}(\theta, \phi)$

spherical harmonics

- The $a_{\ell m}$ contain the same information as the map itself
- ℓ defines the multipole, the higher ℓ the smaller the scale
- The power spectrum is then given by:

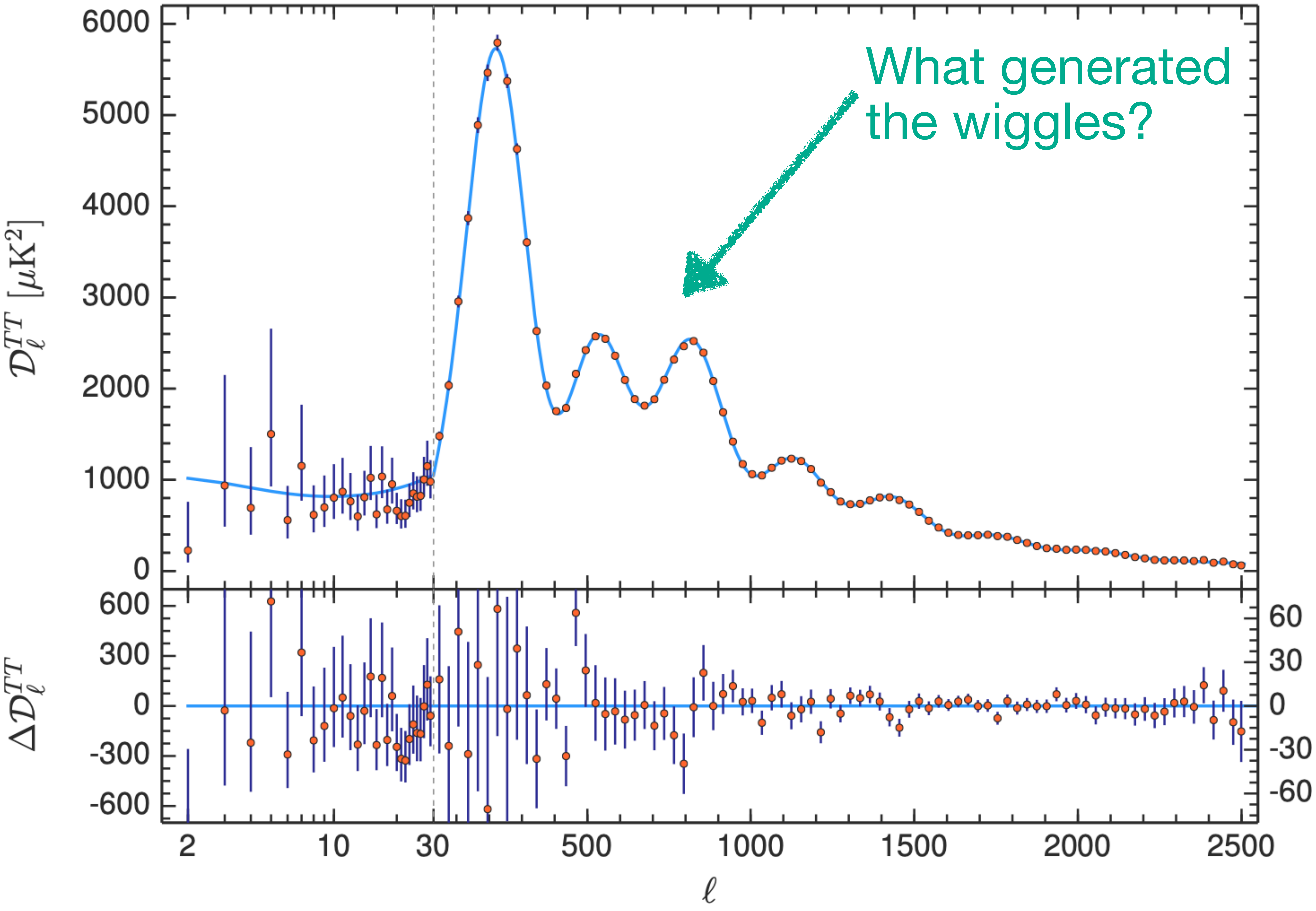
$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell,m} \, a^*_{\ell,m}$$

The CMB



$$D_\ell = \frac{\ell(\ell+1)}{2\pi} C_\ell$$

The CMB



$$D_\ell = \frac{\ell(\ell+1)}{2\pi} C_\ell$$

The baryon acoustic oscillations (BAO)

- The early universe was a hot plasma of baryons and photons
- Small density fluctuations sourced pressure-density waves
  - Gravity as driving force
  - Photon pressure as restoring force
- Like rain drops on the water surface many waves overlap
- Once electrons and protons recombine, photons can travel freely → the density waves freeze

Credit: Mabel Amber

The baryon acoustic oscillations (BAO)

- Let's compute the (comoving) distance that the sound waves travelled: the sound horizon

- The sound waves travel from the big bang to the time of recombination t^* (when protons and electrons combine into atoms): $r_s = \int_{BB}^{t^*} \cancel{c_s(t)}\, \mathrm{d}t$

$$\frac{c_s(t)}{a(t)}$$

- While the sound waves travel, the universe expands. This slows down the sound waves in the comoving coordinate frame

The baryon acoustic oscillations (BAO)

- Comoving sound horizon:

$$r_s = \int_{BB}^{t*} \frac{c_s(t)}{a(t)}\,\mathrm{d}t = \int_0^{a*} \frac{c_s(a)}{\dot{a}a}\,\mathrm{d}a = -\int_{\infty}^{z*} \frac{c_s(z)a^2}{\dot{a}a}\,\mathrm{d}z = \int_{z*}^{\infty} \frac{c_s(z)}{H(z)}\,\mathrm{d}z$$

The baryon acoustic oscillations (BAO)

- Comoving sound horizon:

$$r_s = \int_{BB}^{t*} \frac{c_s(t)}{a(t)}\,\mathrm{d}t = \int_0^{a*} \frac{c_s(a)}{\dot{a}a}\,\mathrm{d}a = -\int_{\infty}^{z*} \frac{c_s(z)a^2}{\dot{a}a}\,\mathrm{d}z = \int_{z*}^{\infty} \frac{c_s(z)}{H(z)}\,\mathrm{d}z$$

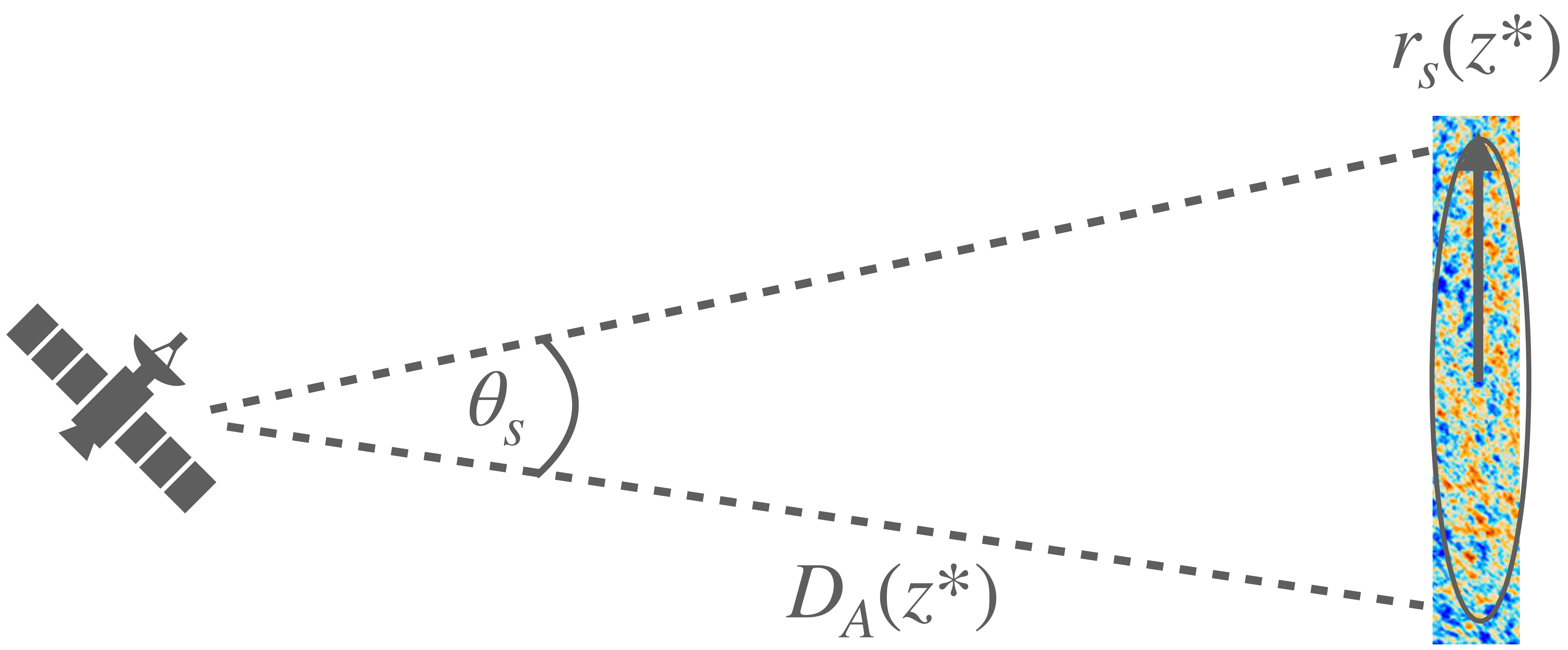
- The sound speed can be expressed as (e.g. Dodelson&Schmidt, 2020):

$$c_s(z) = \frac{1}{\sqrt{3(1+R(z))}}$$

- With R being the baryon-photon ratio:

$$R(z) = \frac{3\omega_b}{4\omega_\gamma}\frac{1}{1+z}$$

The baryon acoustic oscillations (BAO)



- What we observe is the *angular size of the sound horizon θ_s*

- In the small-angle approximation:

$$\theta_s = \frac{r_s(z^*)}{D_A(z^*)}$$

- where the angular diameter distance is (see slide earlier):

$$D_A(t) = \int_0^{z(t)} \frac{\mathrm{d}z'}{H(z')}$$

How does the CMB constrain H_0?

- We know that the CMB directly constrains

$$\theta_s = \frac{a(z^*)\ r_s(z^*)}{D_A(z^*)} = \frac{\int_{z^*}^{\infty} c_s(z)\mathrm{d}z/H(z)}{\int_0^{z^*} \mathrm{d}z/H(z)}$$

- Cosmology enters mainly over the Hubble parameter here:

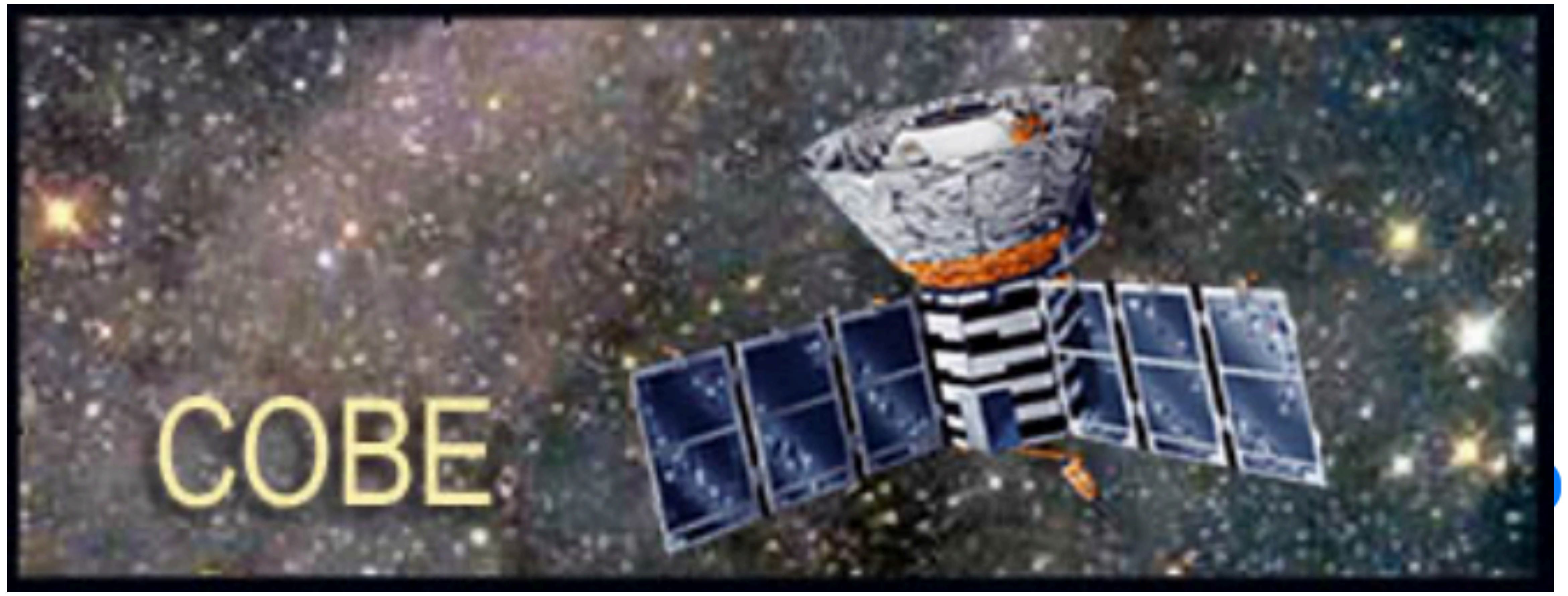
$$H(z) = H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$

- If we get Ω_r, Ω_m, Ω_Λ from somewhere else, this becomes an implicit equation for H_0 — How does the CMB constrain Ω_r, Ω_m, Ω_Λ?

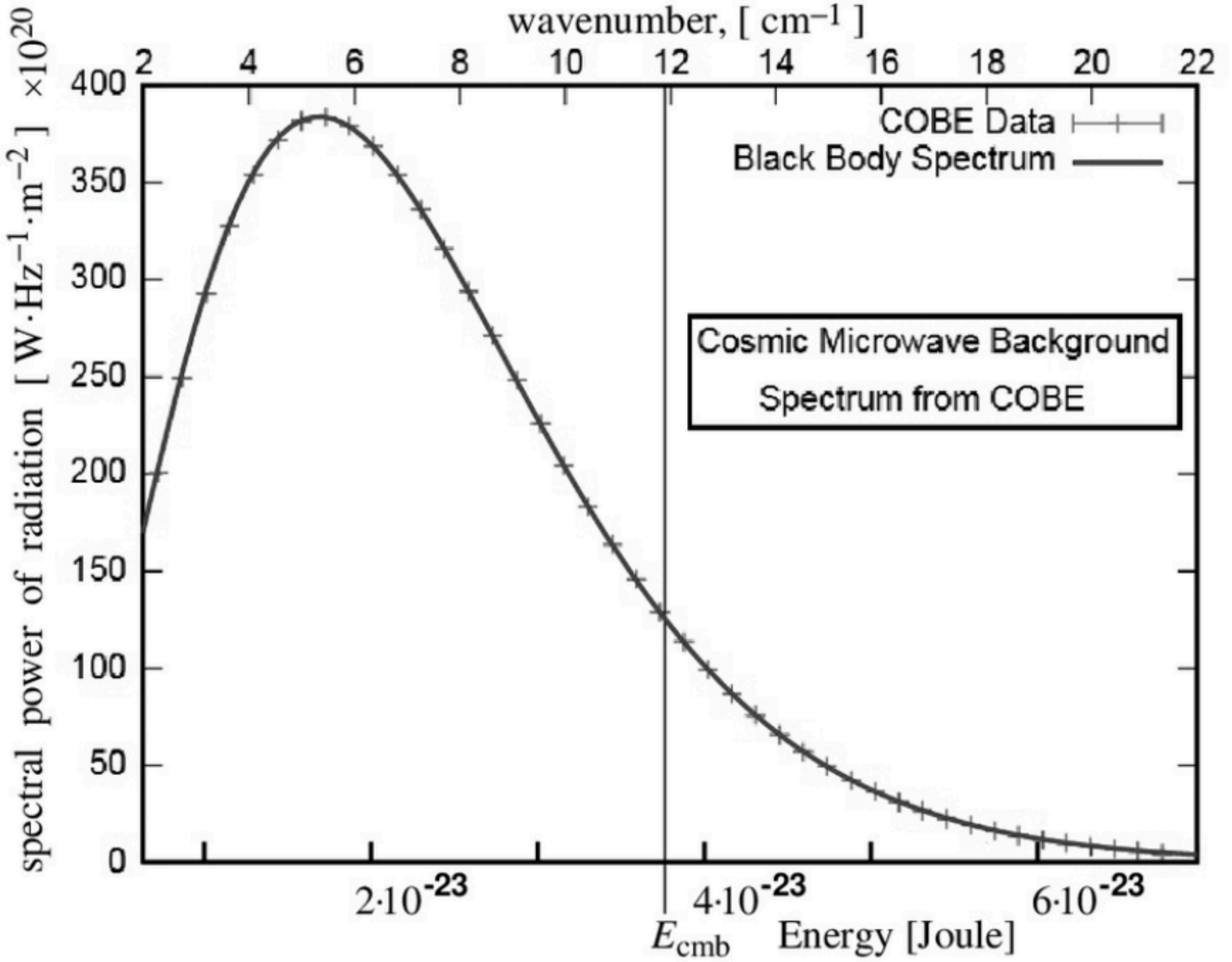
How does the CMB constrain Ω_r?

- The radiation density Ω_r is precisely measured by the CMB temperature

$$T = 2.725 \pm 0.002\,\mathrm{K}$$ (COBE - FIRAS measurement)



Credit: Berkeley Center for Cosmological Physics



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- For a black-body spectrum, the energy density in radiation is then given by (e.g. Weinberg 2008, Ch. 2.1):

$$\rho_{0,\mathrm{CMB}} = \int_0^\infty h\nu \cdot n(\nu)\mathrm{d}\nu = \frac{8\pi^5 k_B^4}{15h^3c^3}T^4 = 4.64 \cdot 10^{-34}\,\mathrm{g/cm^3}$$

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For more about neutrinos, see Olga Mena's lecture

- Taking into account the earlier time of decoupling of neutrinos, one finds: $\rho_\nu = 0.4\,\rho_{0,\mathrm{CMB}}$

- Since the CMB and neutrinos are by far the dominant contribution:

$$\Omega_r = 1.4\,\rho_{0,\mathrm{CMB}}/\rho_{\mathrm{crit}} = 4.15 \cdot 10^{-5} h^{-2}$$

How does the CMB constrain Ω_m?

- It does not constrain Ω_m, but $\omega_m = h^2\Omega_m$, where $\omega_m = \omega_{\rm cdm} + \omega_b$
- ω_m is constrained by the height of the acoustic peaks:
  - a smaller ω_m leads to a later time of matter-radiation equality
  - this leads to a stronger decay of the gravitational potentials at recombination $\rightarrow$ early integrated Sachs-Wolfe effect (eISW)
  - Since the eISW adds in phase with the BAO, this leads to a boost of all peaks, particularly the first peak.

How does the CMB constrain Ω_Λ?

- The sum of the energy densities has to satisfy:

$$\Omega_r + \Omega_m + \Omega_\Lambda = \Omega_k$$

- For a flat universe:

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1$$

- Hence, if we know Ω_r and $\omega_m = h^2\Omega_m$, we can compute

$$\Omega_\Lambda = 1 - \Omega_r - \omega_m/h^2$$

How does the CMB constrain H_0?

- We know that the CMB directly constrains

$$\theta_s = \frac{a(z^*)\ r_s(z^*)}{D_A(z^*)} = \frac{\int_{z^*}^{\infty} c_s(z)\mathrm{d}z/H(z)}{\int_0^{z^*} \mathrm{d}z/H(z)}$$

- Cosmology enters mainly over the Hubble parameter here:

$$H(z) = H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$

- Since we can constrain Ω_r and ω_m independently, this becomes an implicit equation for h

$$\frac{H^2(z)}{(100\,\mathrm{km/s/Mpc})^2} = h^2\Omega_r(1+z)^4 + \omega_m(1+z)^3 + (h^2 - \omega_m - h^2\Omega_r)$$

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* $c_s(z)$ and z^* also depend on cosmology but we neglect that here

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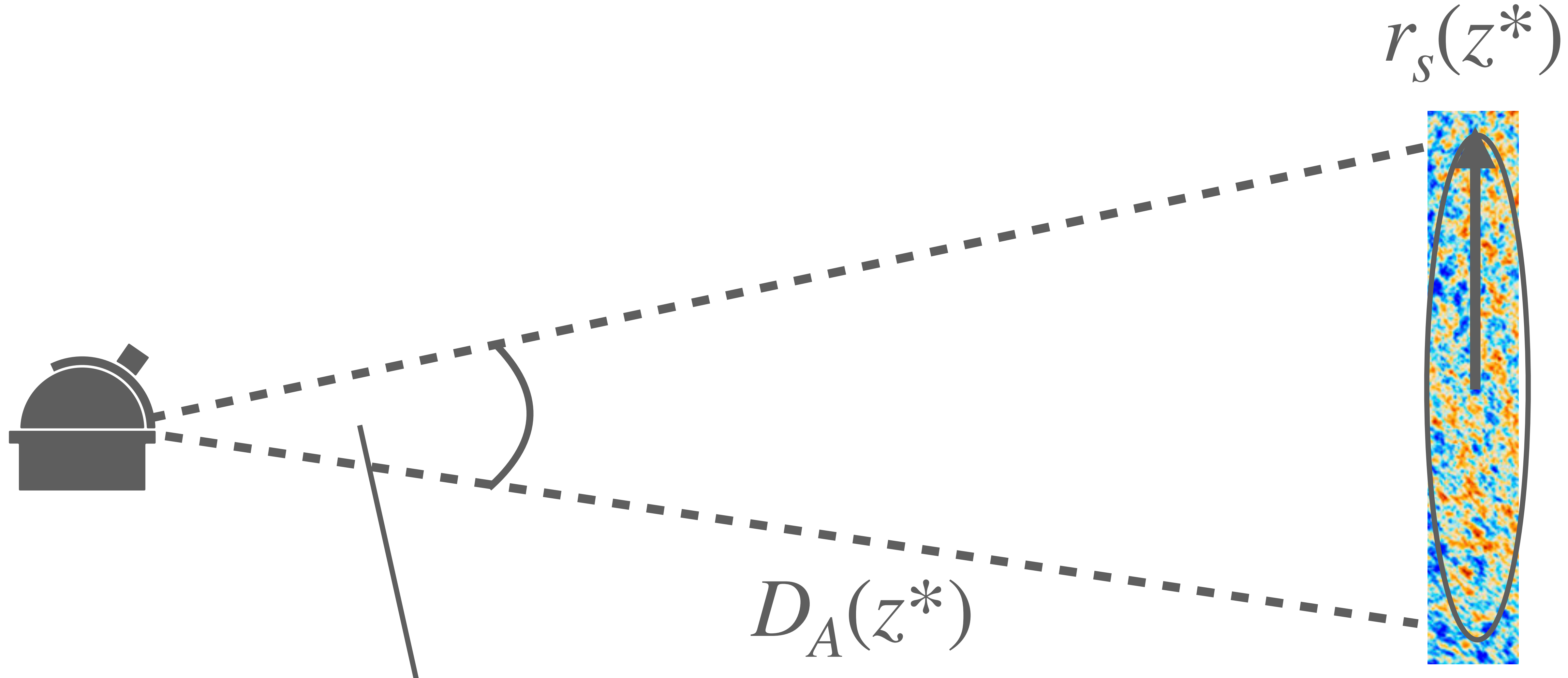
Solutions to the Hubble tension

Solutions to the Hubble tension



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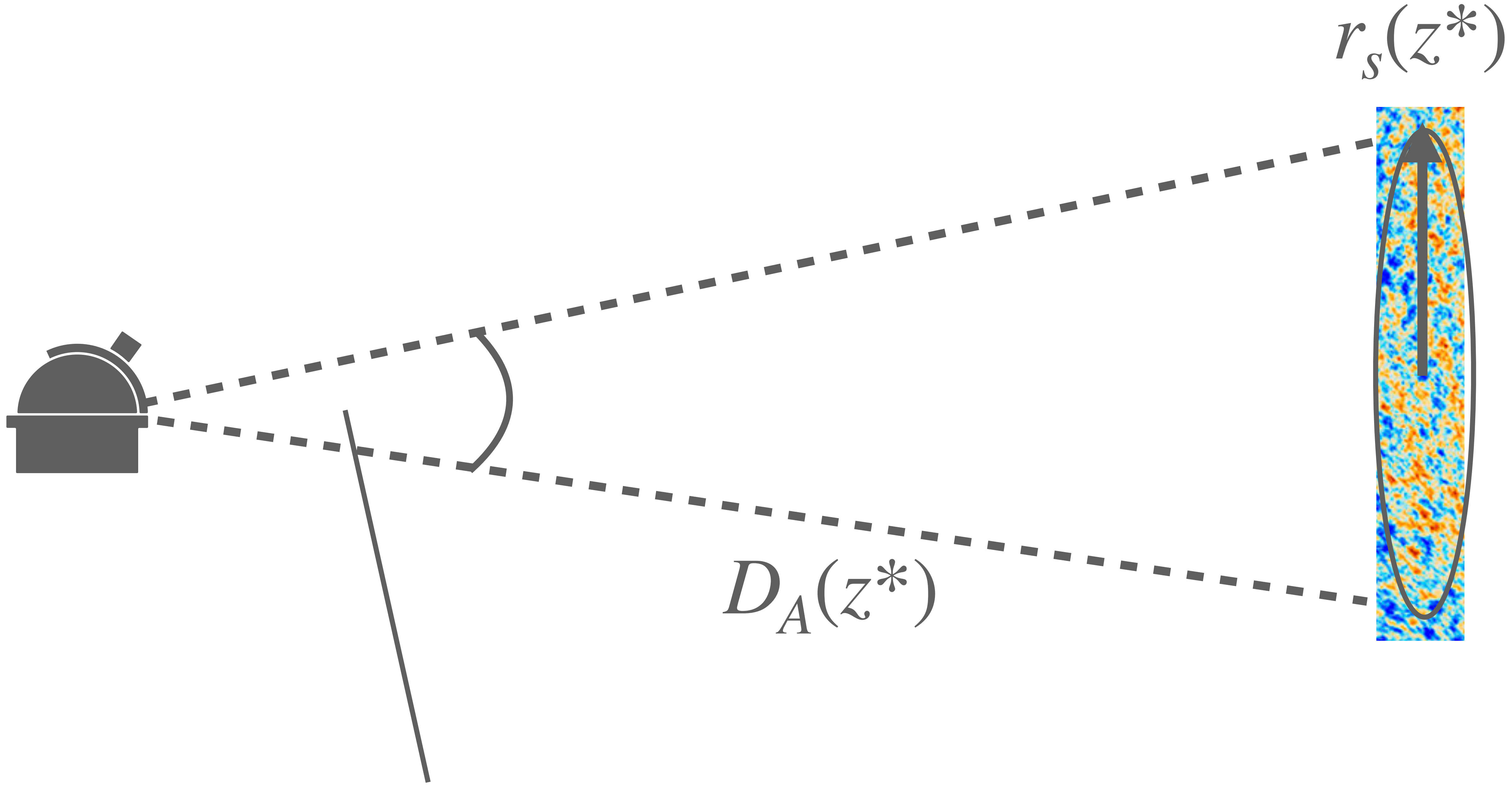
- θ_s is measured precisely by CMB $\rightarrow \theta_s$ fixed
- Two options to solve the Hubble tension:

Solutions to the Hubble tension



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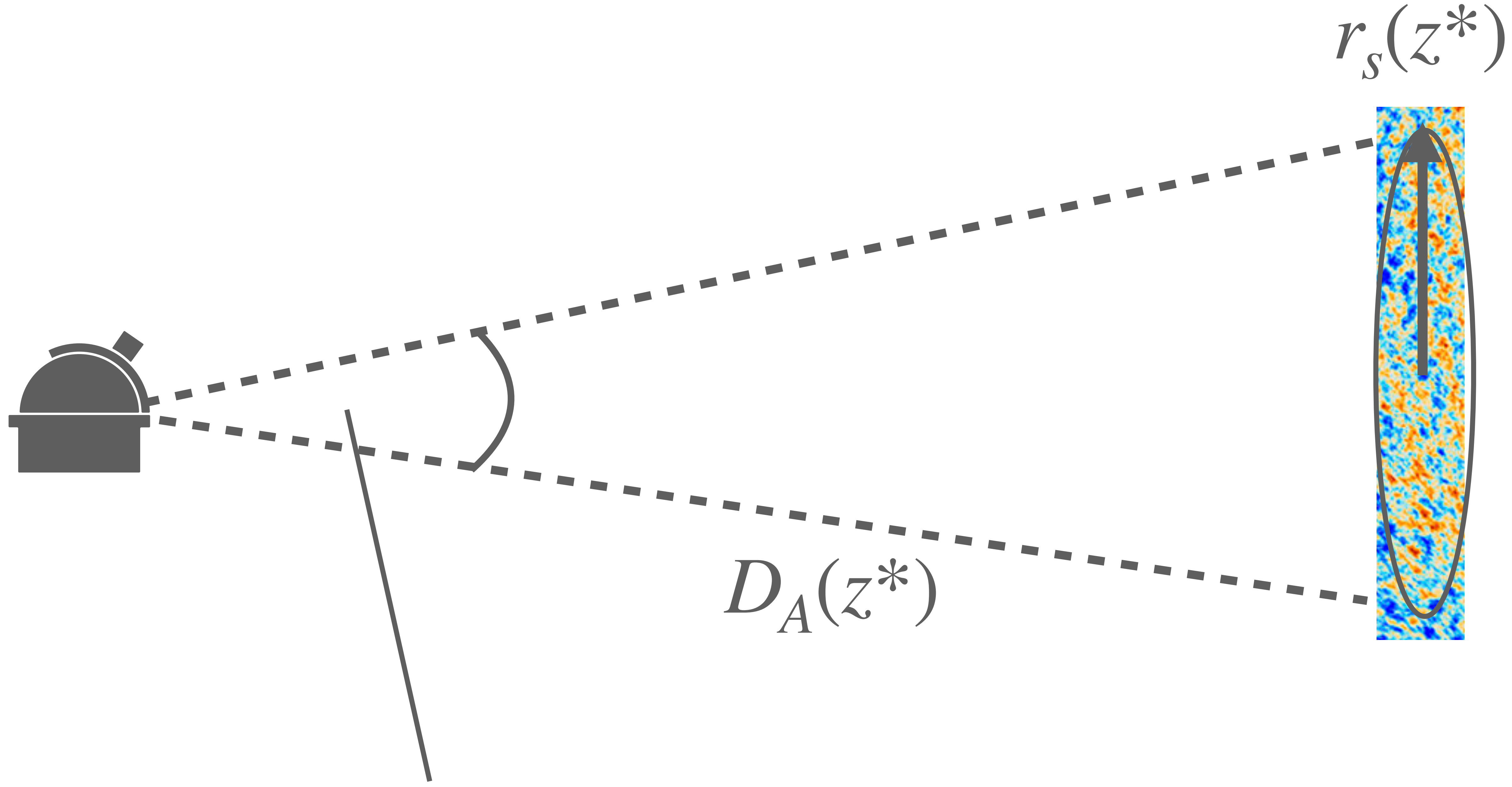
Early-time solutions modify r_s

Solutions to the Hubble tension



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- θ_s is measured precisely by CMB $\rightarrow \theta_s$ fixed
- Two options to solve the Hubble tension:

Early-time solutions modify r_s

Late-time solutions modify D_A

Late-time solutions

- Modify $D_A(z^*) = \int_0^{z^*} \frac{\mathrm{d}z}{H(z)}$ by modifying the expansion rate between today (0) and recombination z^* (since $\Omega_r \approx 0$ at times after z^*):

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}$$

- However, $H(z)$ is well constrained by galaxy BAO data and supernova data
  $\rightarrow$ There is not enough wiggle-room to increase H_0 enough

- Late-time solutions are somewhat disfavoured

Early-time solutions

- Modify $r_s = \int_{z^*}^{\infty} \frac{c_s(z)\ \mathrm{d}z}{H(z)}$ → 3 options:
    - modify z^*
    - modify $c_s(z)$
    - modify $H(z)$

Early-time solutions

- Modify $r_s = \int_{z^*}^{\infty} \frac{c_s(z)\ \mathrm{d}z}{H(z)}$ → 3 options:
  - **modify z^*** →
  - modify $c_s(z)$
  - modify $H(z)$

E.g. by modifying the mass of the electron m_e:

→ This shifts the energy levels of the atoms,

→ and changes the ionization energy,

→ which changes the time of recombination z^*

- modify cs

Early-time solutions

- Modify $r_s = \int_{z^*}^{\infty} \frac{c_s(z)\ \mathrm{d}z}{H(z)}$ → 3 options:
  - modify z^*
  - modify $c_s(z)$
  - **modify $H(z)$** →

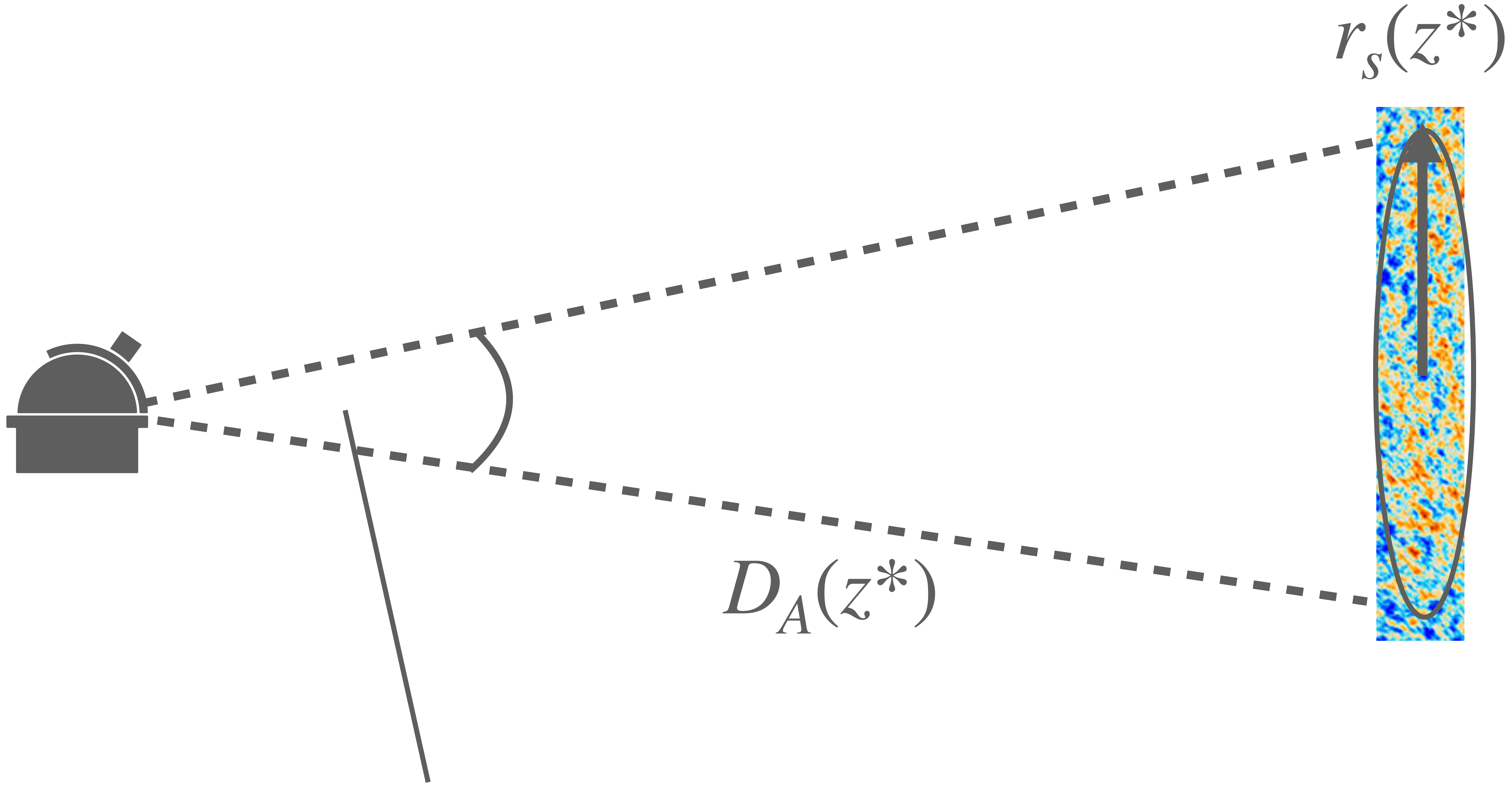
E.g. by introducing an energy density, which boosts the expansion rate before recombination z^* (at early times $\Omega_\Lambda \approx 0$):

$$H(z) = H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_?}$$

This leads to a smaller r_s.

The most successful of such ideas is EDE

Idea behind Early Dark Energy



$$\theta_s = \frac{r_s(z^*)}{D_A(z^*)} = \frac{\int_{z^*}^{\infty} c_s(z)\mathrm{d}z/H(z)}{\int_0^{z^*} \mathrm{d}z/H(z)}$$

Angular scale of sound horizon θ_s measured with 0.03% precision by *Planck*.

$\downarrow$

Introducing an additional component before recombination reduces r_s.

$\downarrow$ θ_s fixed

Angular diameter distance D_A decreases.

$\downarrow$ $H(z) = H_0\sqrt{\Omega_m(z) + \Omega_r(z) + \Omega_\Lambda}$

H_0 increases.

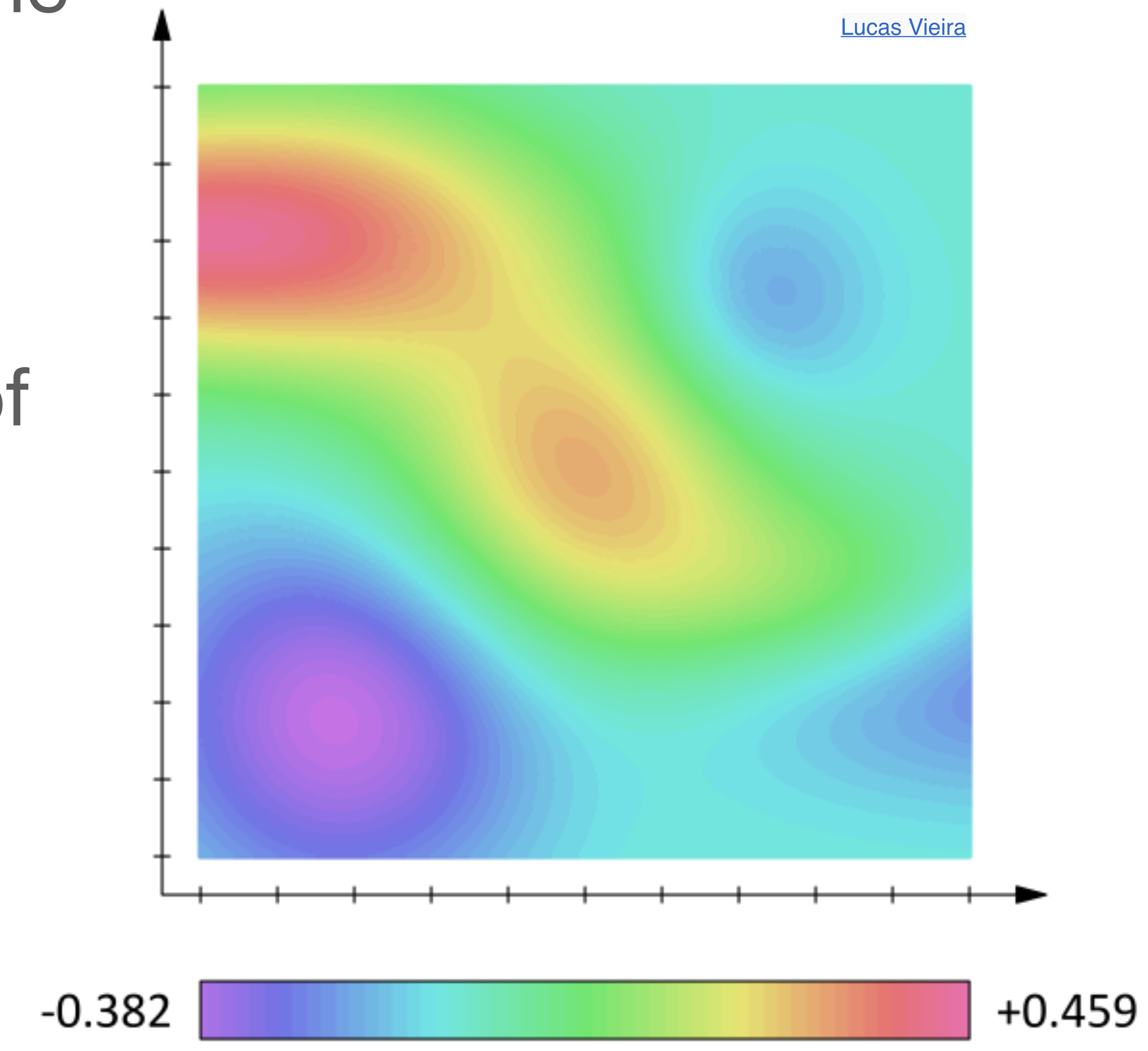
Short history of Early Dark Energy

- EDE has already been studied before the Hubble tension emerged in the context of quintessence models (Doran++ 2001, Wetterich++ 2004, Doran&Robbers 2006, Kamionkowski++ 2014)
- The axion-like EDE was proposed as a solution to the Hubble tension (Karwal&Kamionkowski 2016, Poulin++ 2018, 2019)
- There are many versions of the EDE model, but we will focus on the (most commonly studied) axion-like EDE model
- The axion-like EDE model is modelled as a scalar field

Scalar fields in an expanding spacetime

Intuition

- A scalar field $\phi(t, \vec{x})$ assigns each point in spacetime a single number
- Examples
    - Temperature, density, pressure,… as a function of position and time
    - Potential fields like the gravitational potential, electric potential
    - In quantum field theory, the Higgs particle is described as a scalar field, and the pions
    - The inflaton driving inflation is most commonly a scalar field



temperature

Lagrangian

- The Lagrangian of a scalar field minimally coupled to the metric is:

$$\mathcal{S}_\phi = \int \mathrm{d}^4x\sqrt{|\det(g)|}\;\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]$$

integral over whole spacetime

determinant of metric

kinetic term

potential term

- One can compute the energy momentum tensor of the scalar field via:

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\right)$$

Lagrangian

- This gives for the energy-momentum tensor:

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\right) = \frac{\dot{\phi}^2}{2}\delta_\mu^0\delta_\nu^0 + \left[\frac{\dot{\phi}^2}{2} - V(\phi)\right]g_{\mu\nu}$$

- By comparing $T_{\mu\nu}$ to the energy momentum tensor of the perfect fluid: $T_{\mu\nu} = (\rho + p)\delta_\mu^0\delta_\nu^0 + pg_{\mu\nu}$, one can read off the energy density and pressure of the scalar field:

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)\,.$$

Lagrangian

- Inserting $\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$ into the first Friedmann equation and the continuity equation yields:

$$\boxed{\begin{gathered} H^2 = \frac{8\pi G_\mathrm{N}}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right) \\ \ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0 \end{gathered}}$$

"Hubble friction"

Potential term

Lagrangian

- Inserting $\rho_\phi = \dfrac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \dfrac{\dot{\phi}^2}{2} - V(\phi)$ into the first Friedmann equation and the continuity equation yields:

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- The second equation is the Klein-Gordon equation for a scalar field in an expanding space, where the second term ($3H\dot{\phi}$) is the Hubble-drag term and the third term ($\frac{1}{2}\mathrm{d}V/\mathrm{d}\phi$) is the potential-gradient term

Early Dark Energy

Early Dark Energy

- EDE is a scalar field ϕ in a potential $V(\phi)$, which boosts the expansion rate before recombination

- Potential motivated by “axion”:

$$V(\phi) = V_0 \, [1 - \cos(\phi/f)]^n$$

index

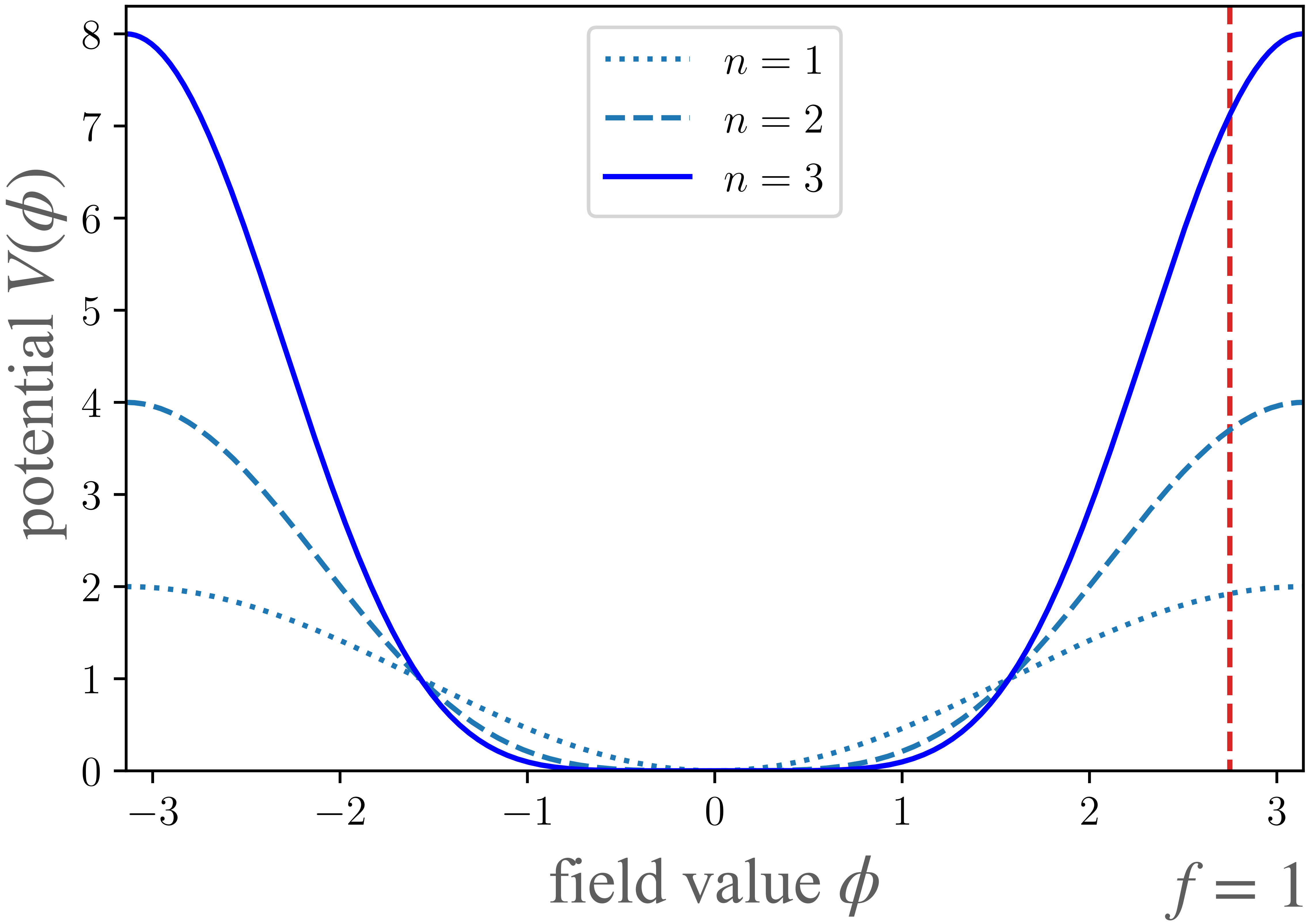
normalization of
the potential

decay constant

- Explain axion, f, m, etc

Early Dark Energy

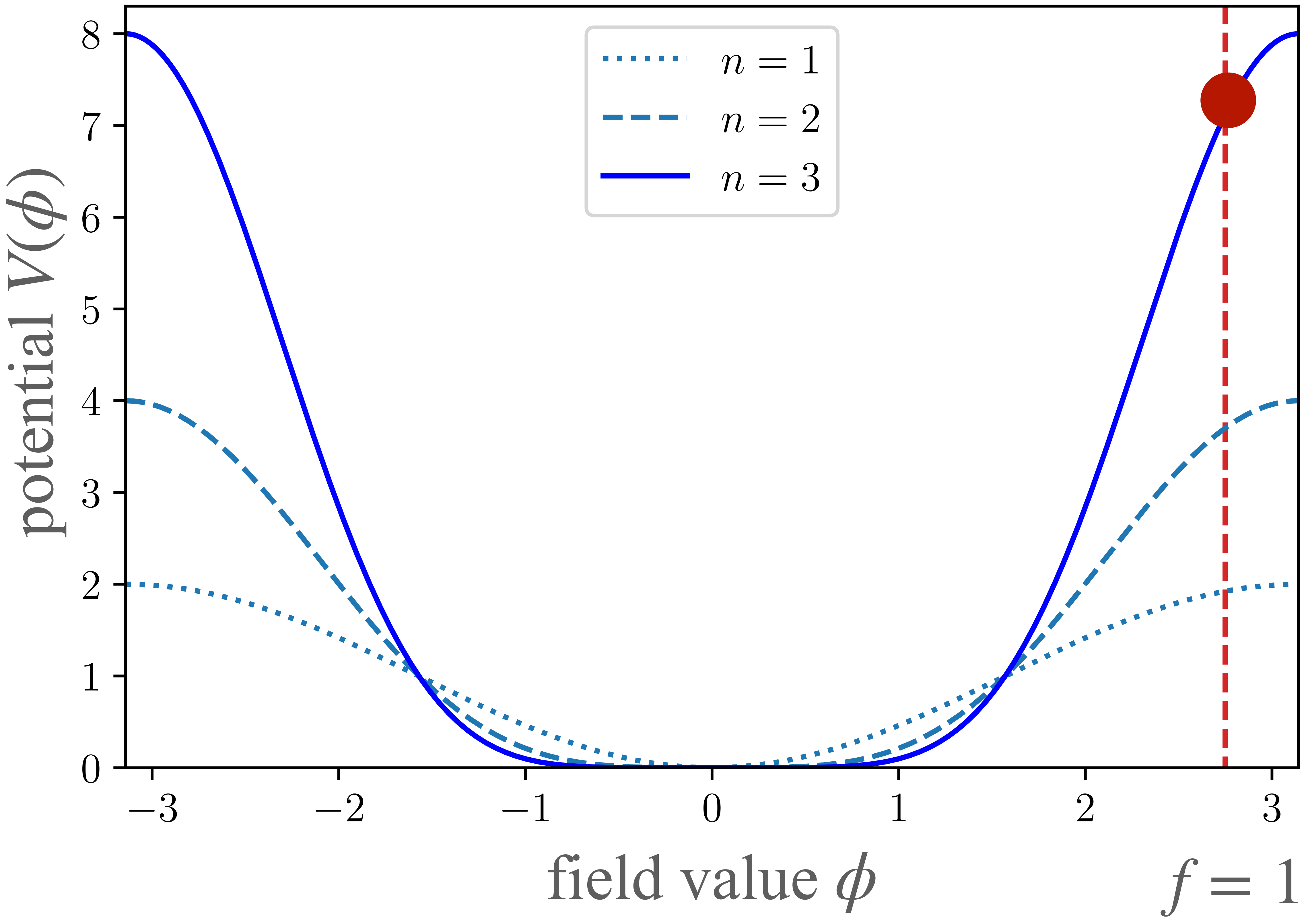
$$V(\phi) = V_0\,[1 - \cos(\phi/f)]^n$$



$f = 1$

Early Dark Energy

$$V(\phi) = V_0\,[1 - \cos(\phi/f)]^n$$



- Dynamics governed by Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} + \cancel{\frac{\mathrm{d}V}{\mathrm{d}\phi}} = 0$$

- ϕ starts high up in the potential, where the potential is very flat:

$$\frac{\mathrm{d}V}{\mathrm{d}\phi} \approx 0$$

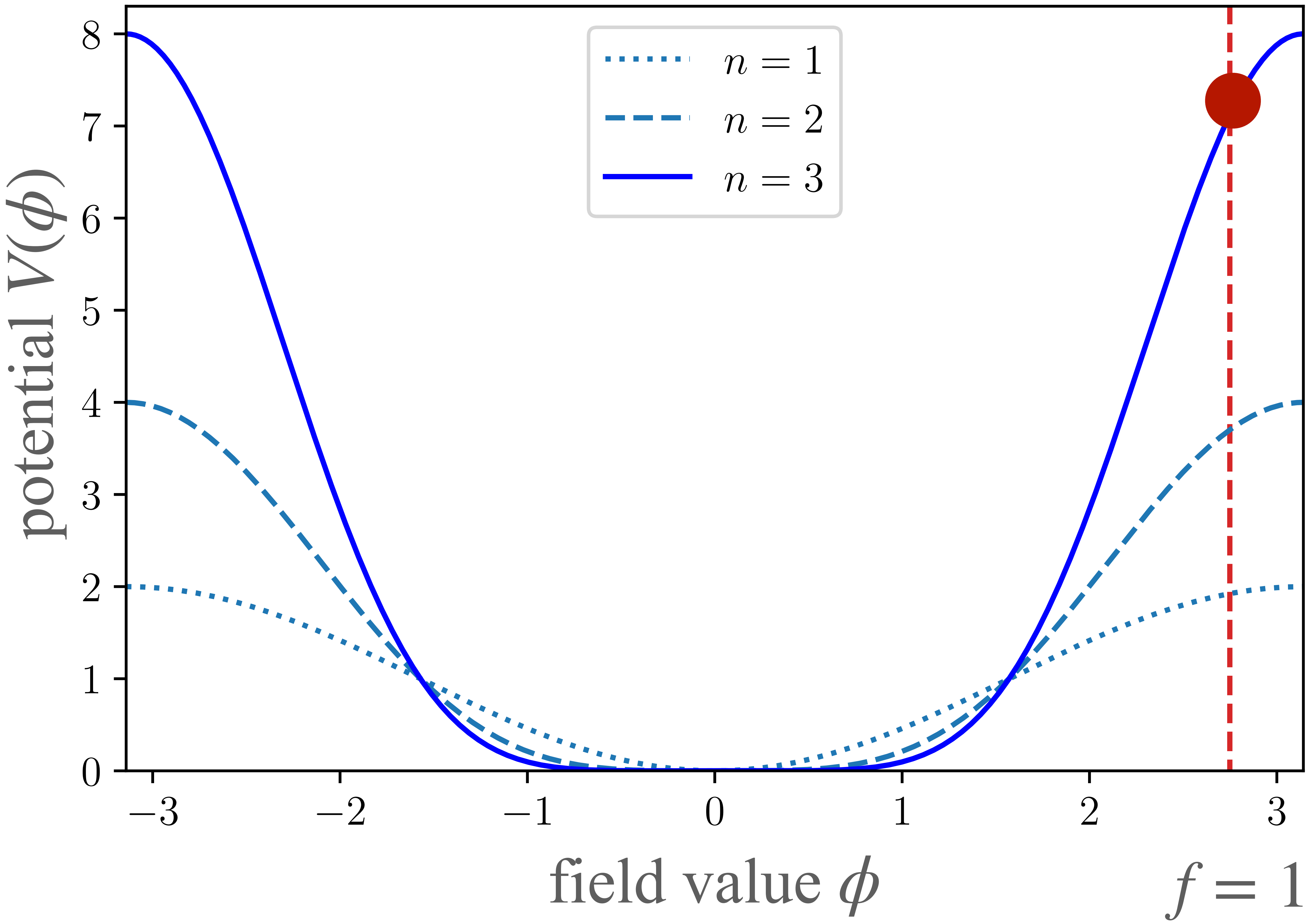
- Moreover, at early times the expansion rate H is large; hence "Hubble friction" dominates:

$$3H\dot{\phi} \gg \frac{\mathrm{d}V}{\mathrm{d}\phi}$$

- Both of these factors "freeze" the scalar field in place: $\dot{\phi} \approx 0$

Early Dark Energy

$$V(\phi) = V_0\,[1 - \cos(\phi/f)]^n$$



$f = 1$

- At early times: $\dot{\phi} \approx 0$
- This means for the energy density& pressure

$$\rho_\phi = \cancel{\frac{\dot{\phi}^2}{2}} + V(\phi), \quad p_\phi = \cancel{\frac{\dot{\phi}^2}{2}} - V(\phi)\,.$$

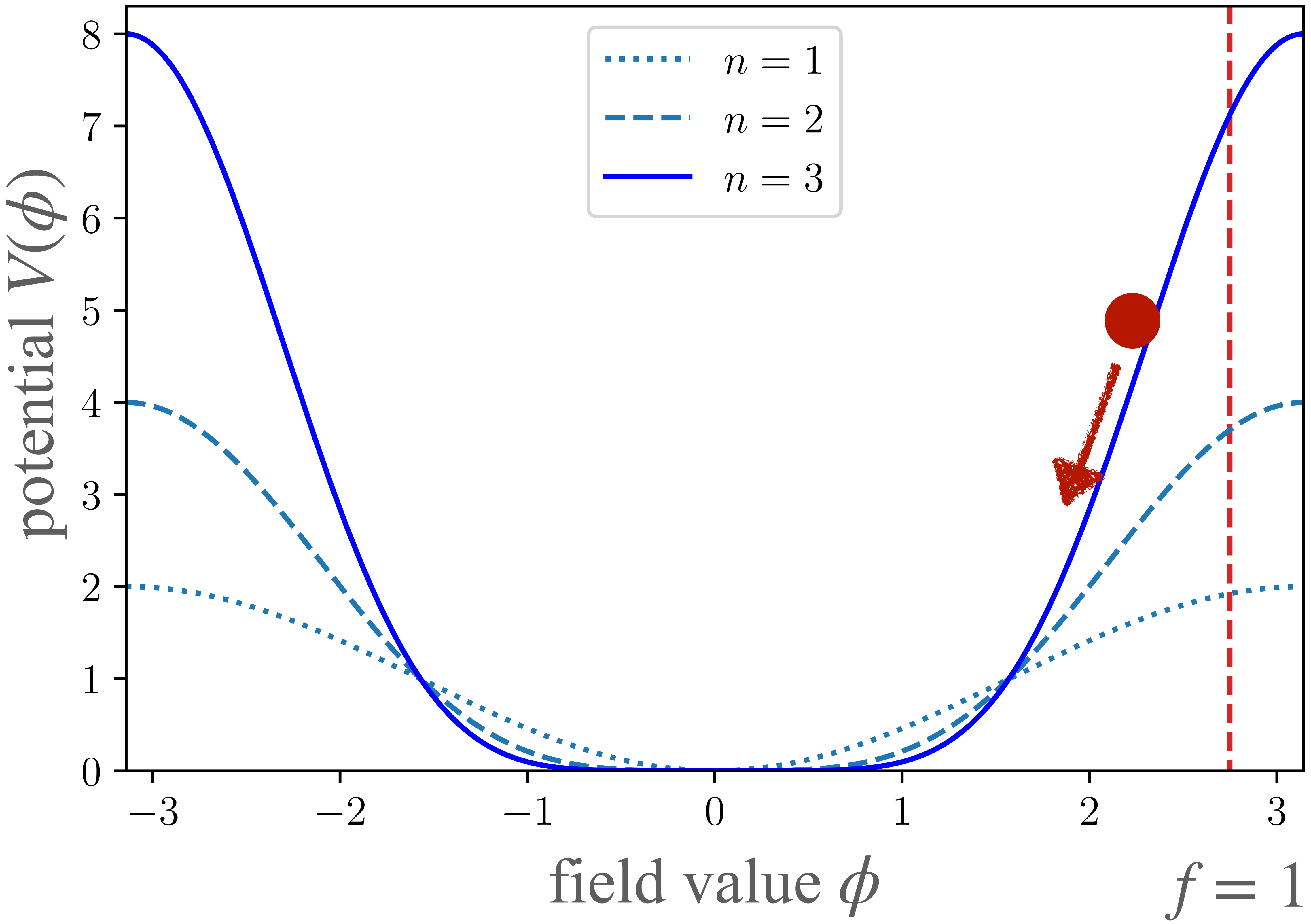
- The equation of state becomes:

$$w = \frac{p_\phi}{\rho_\phi} \approx -1$$

- Hence, at early times the field behaves like dark energy → EDE

Early Dark Energy

$$V(\phi) = V_0\,[1 - \cos(\phi/f)]^n$$

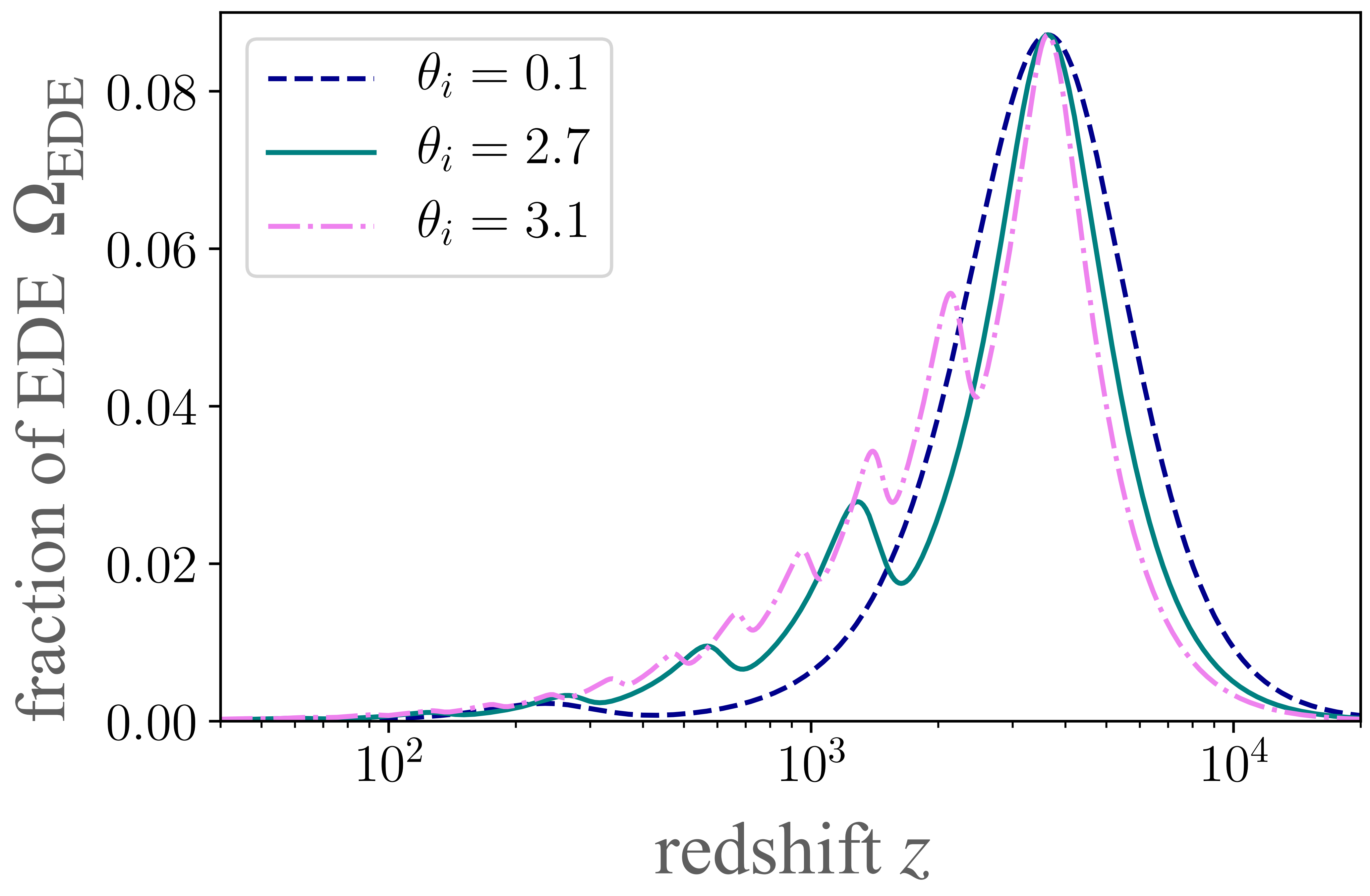


$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0$$

- As time passes, $H(z)$ decreases, untial at some critical redshift z_c:
- $3H\dot{\phi} \ll \dfrac{\mathrm{d}V}{\mathrm{d}\phi}$
- In this phase, the field oscillates around the minimum with equation of state

$$\langle w \rangle = \frac{n-1}{n+1} = \frac{1}{2}$$

Early Dark Energy



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- Think about slow-roll phase more
- How to get equation of state in oscillating phase
- Add description about m, f

Recap Lecture 1

- Add recap