

From Λ CDM to EDE

Lecture 1: Theory

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CosmoVerse@Corfu May 13 - 18, 2024

Overview

Lecture 1: Theory from Λ CDM to EDE (1:30h)

Hands-on session 1: From theory to predictions (1h)

Lecture 2: Observation – Can EDE solve the Hubble tension? (1h)

Hands-on session 2: Let's analyse EDE with the cosmic microwave background and supernovae (1:30h)

Outline: From Λ CDM to EDE

1. Introduction: basic equations in a homogeneous & isotropic universe
(Friedmann equations, matter content in the universe, defining distances)
2. Overview over the timeline of the universe
3. Hubble tension (How does CMB constrain H_0 , how do SNe constrain H_0)
4. Early Dark Energy: How to solve the Hubble tension with new physics?
Scalar field in an expanding background

Introduction

Short “crash course” to fix the notation

Natural units: $c = 1$

General Relativity

- Let's imagine it was 100 years ago:
 - We didn't know about dark matter (DM)
 - We didn't know about dark energy (DE)
 - But a few years ago, Albert Einstein had published the theory of General Relativity (GR)

1916.

Nº 7.

ANNALEN DER PHYSIK.
VIERTE FOLGE. BAND 49.

1. *Die Grundlage
der allgemeinen Relativitätstheorie;
von A. Einstein.*

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie“ bezeichneten Theorie; die letztere nennt

General Relativity

For more about GR, see Matteo Martinelli's lecture

- We will not go into details here but only sketch the rough idea
- Einstein Equations:

$$R^{\mu\nu} - \frac{R}{2}g^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu},$$

Ricci curvature tensor metric cosmological constant energy-momentum tensor

The diagram shows the Einstein field equations: $R^{\mu\nu} - \frac{R}{2}g^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu}$. Four green arrows point to the terms from left to right: 1) Ricci curvature tensor $R^{\mu\nu}$, 2) metric $g^{\mu\nu}$, 3) cosmological constant $\Lambda g^{\mu\nu}$, and 4) energy-momentum tensor $8\pi G T^{\mu\nu}$.

“Matter tells space how to curve, space tells matter how to move”
(Misner++ 1973)

General Relativity

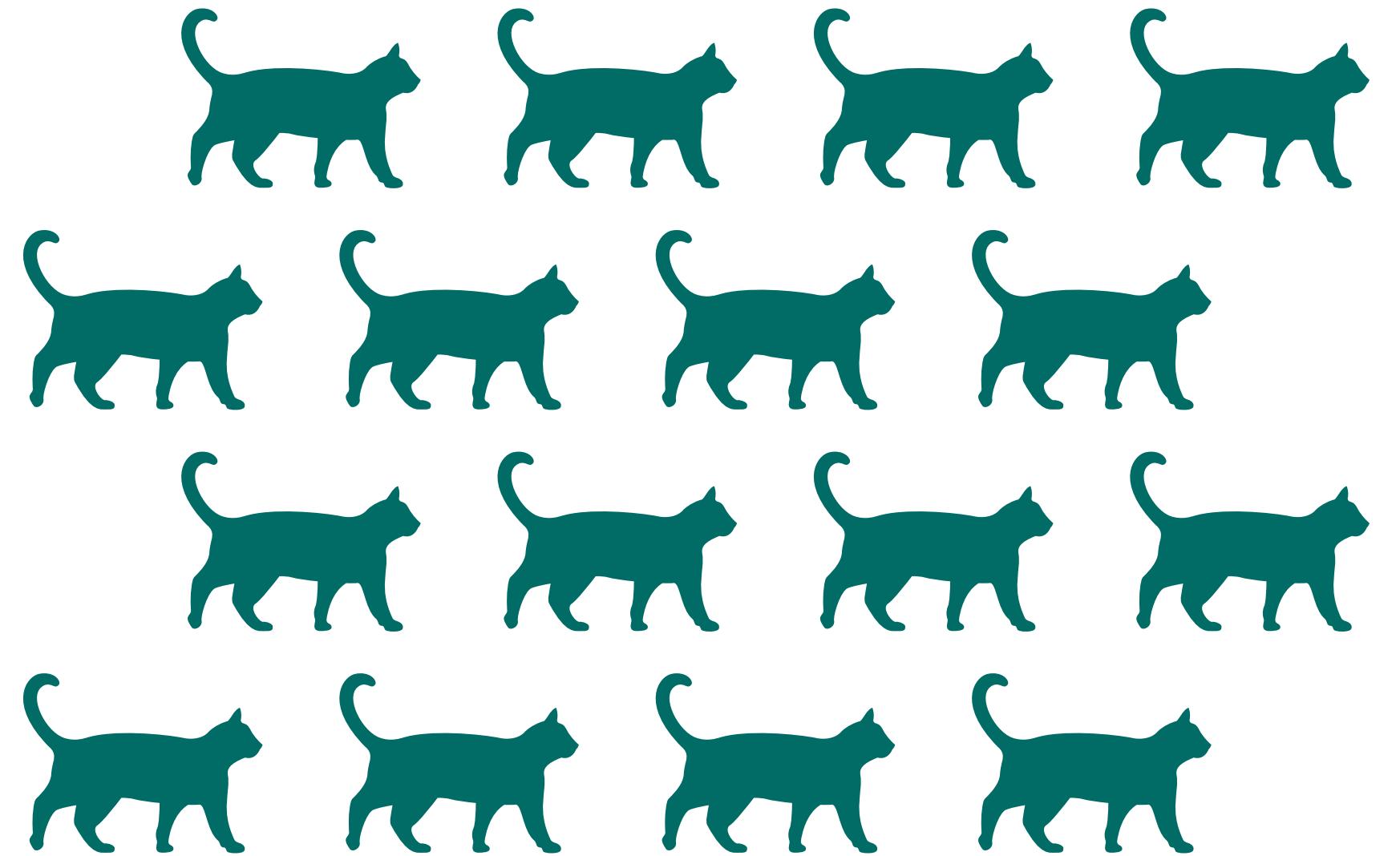
- Solution for Einstein Equations for the universe as a whole?
- Cosmological Principle:

“On sufficiently large scales, the properties of the universe is the same for all observers.”

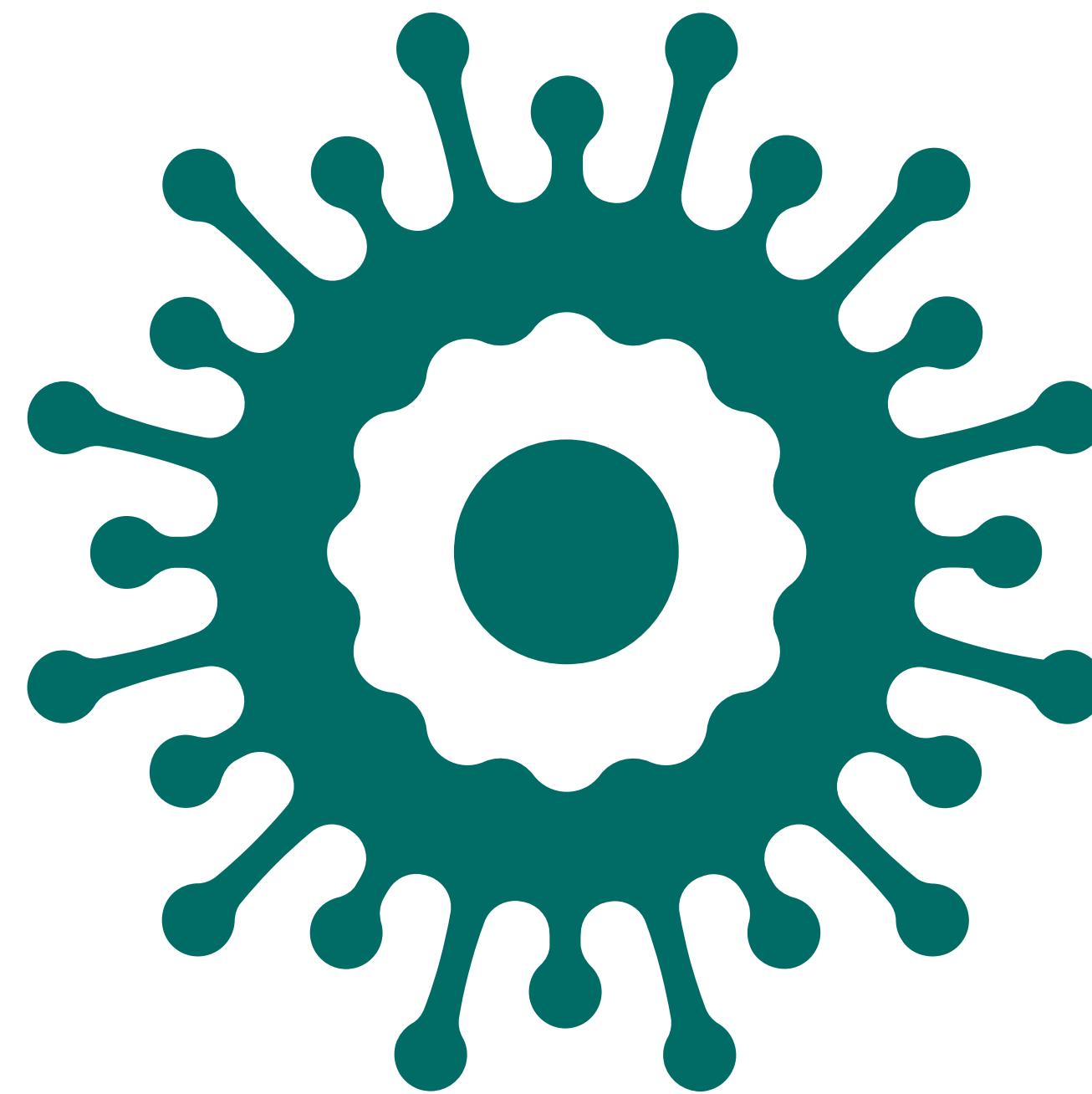
or

“The universe is spatially homogeneous and isotropic on large scales.”

Isotropic or homogeneous?

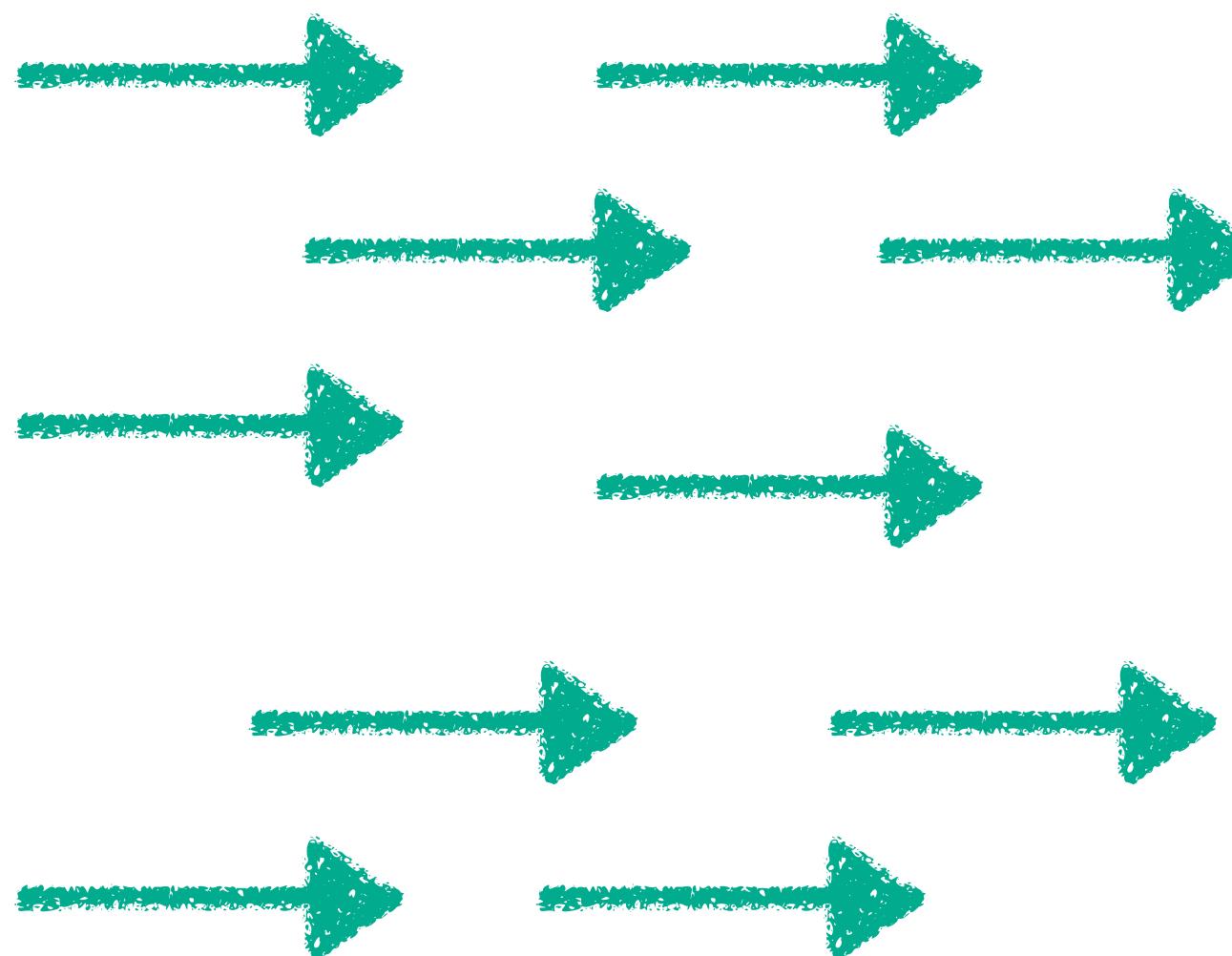


homogenous but not isotropic

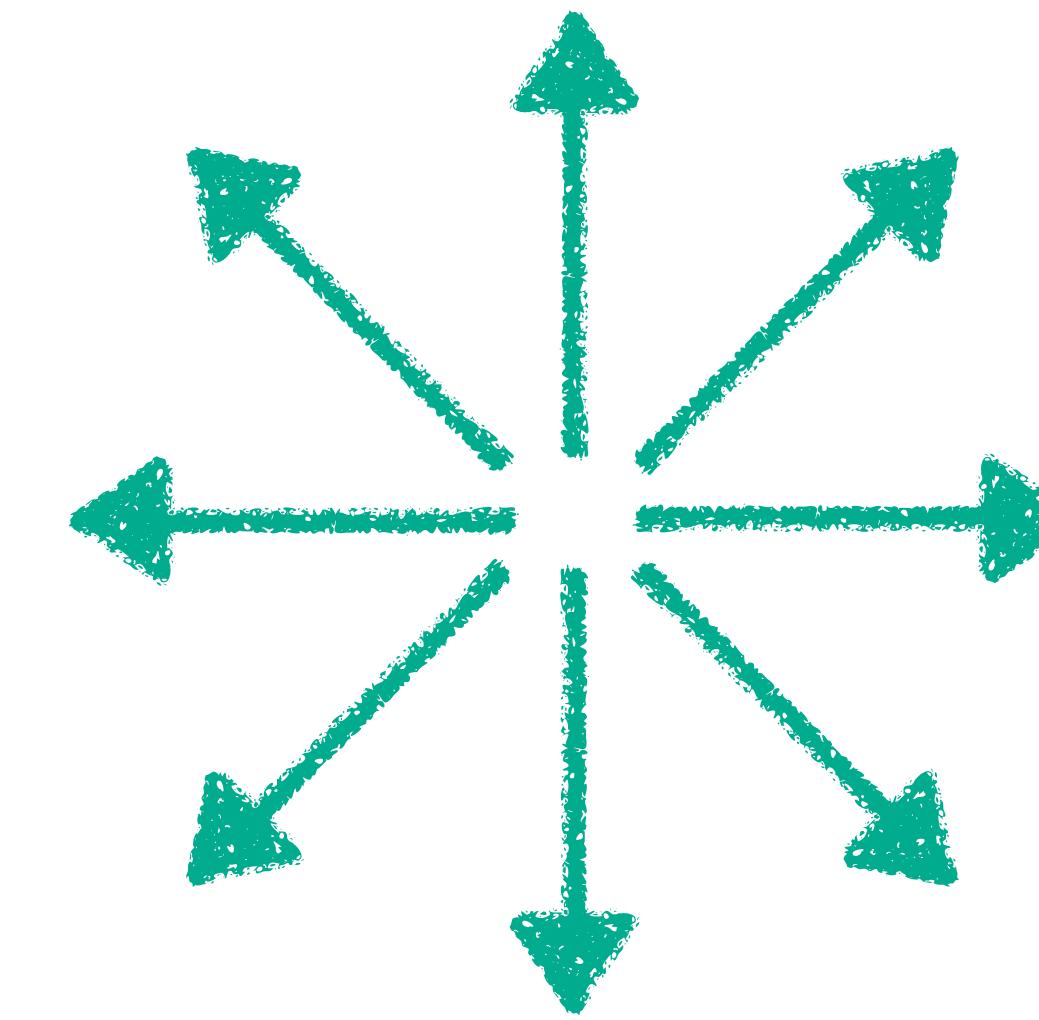


isotropic but not homogenous

Isotropic or homogeneous?



homogenous but not isotropic

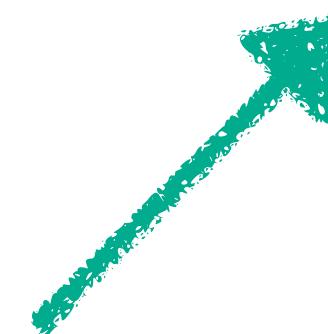


isotropic but not homogenous

Friedmann Equations

- Friedmann 1922, Robertson 1935, Walker 1937: For a spatially homogeneous and isotropic universe, the metric simplifies to

$$ds^2 = -dt^2 + a^2(t) \frac{\delta_{ab}}{1 + \frac{k}{4} |x|^2} dx^a dx^b$$



spacetime line element

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spacetime line element scale factor



$$a(t_{\text{today}}) = a_0 = 1$$

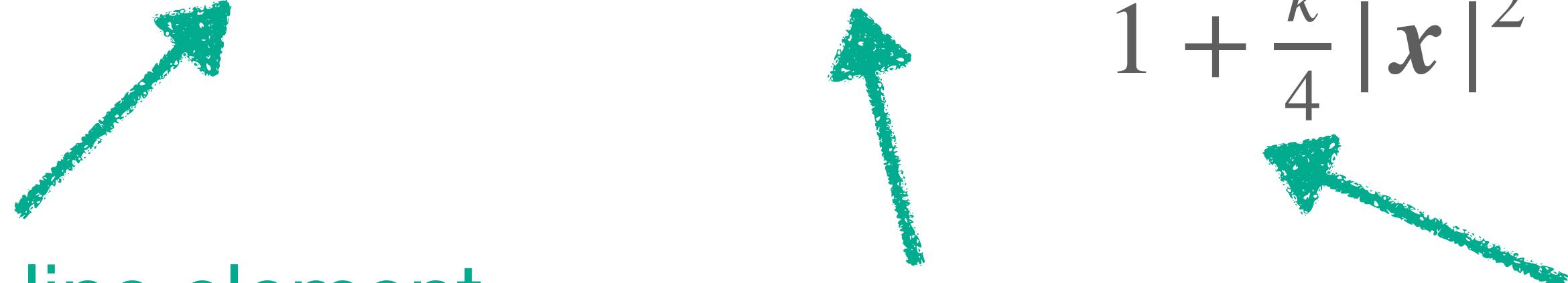
Figure credit: Bianchi, Rovelli, Kolb

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spacetime line element scale factor curvature parameter



$$k = \begin{cases} -1 & \text{hyperbolic} \\ 0 & \text{flat} \\ +1 & \text{spherical} \end{cases}$$

Friedmann Equations

- A perfect fluid is a fluid, which can be completely characterised by its (energy) density and pressure
- The energy momentum tensor of a perfect fluid is

$$T_{\mu\nu}^{\text{pf}} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$$


energy density pressure

- $u_\mu = \dot{x}_\mu$ is the 4-velocity of the observer
- Inserting this in the Einstein Equations, yields the Friedmann Equations

Friedmann Equations

- Inserting the FLRW-metric and the energy momentum tensor of the perfect fluid into the Einstein equations, yields the **Friedmann equations**:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \quad (\text{i})$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad (\text{ii})$$

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- Inserting (ii) into the temporal derivative of (i), yields the **continuity equation**:

$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a} \quad (\text{iii})$$

Friedmann Equations

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$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a} \quad (\text{iii})$$

- Where $\rho = \rho_{\text{tot}}$ is the energy density of the universe.

- If we define $\rho_k = -\frac{3}{8\pi G_N}\frac{k}{a^2}$ and $\rho_\Lambda = \frac{\Lambda}{8\pi G_N}$, one can rewrite (i) as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{tot}} \quad (\text{i})$$



$$\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda + \rho_k$$

$$\Lambda\text{CDM model} \quad \rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda + \rho_k$$

Λ CDM model

$$\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda + \cancel{\rho_k}$$

Λ CDM model

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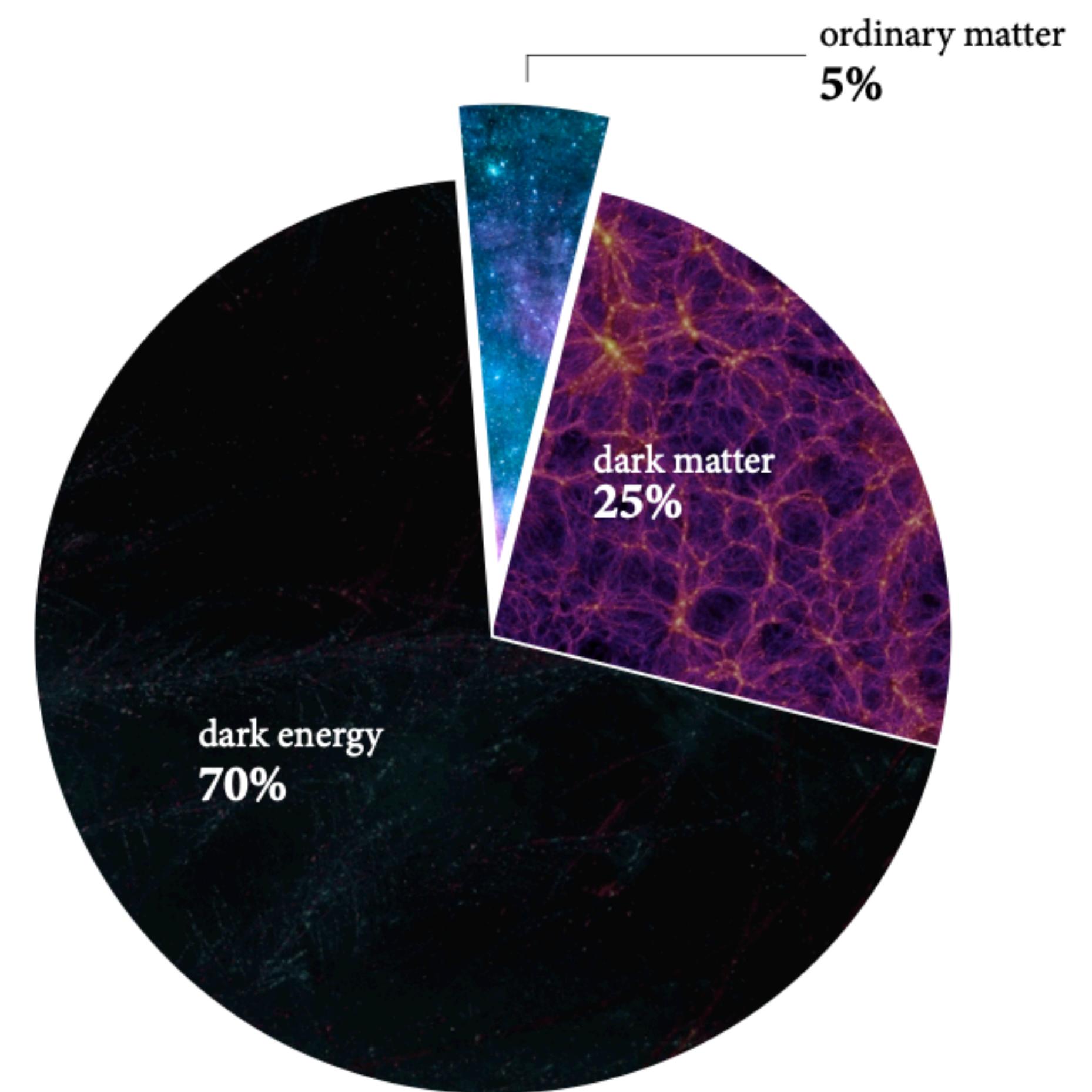


Figure credit: Florian Wolz

Λ CDM model

- Equation of state:

$$\rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda$$

$$p = w \cdot \rho$$

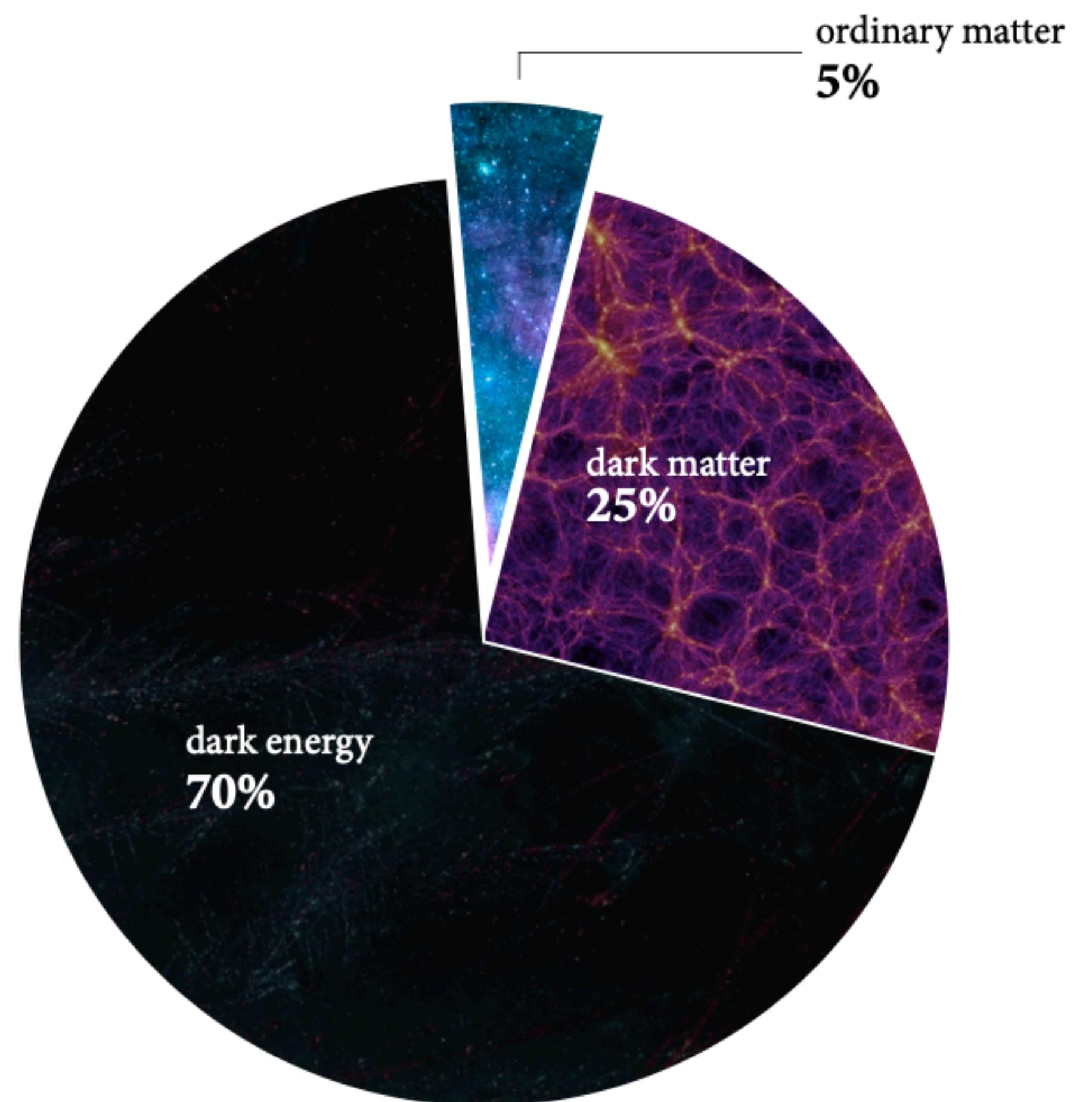


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Λ CDM model

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- EOS-parameter w for different matter species:
 - Dark matter and baryonic matter: $w = 0$

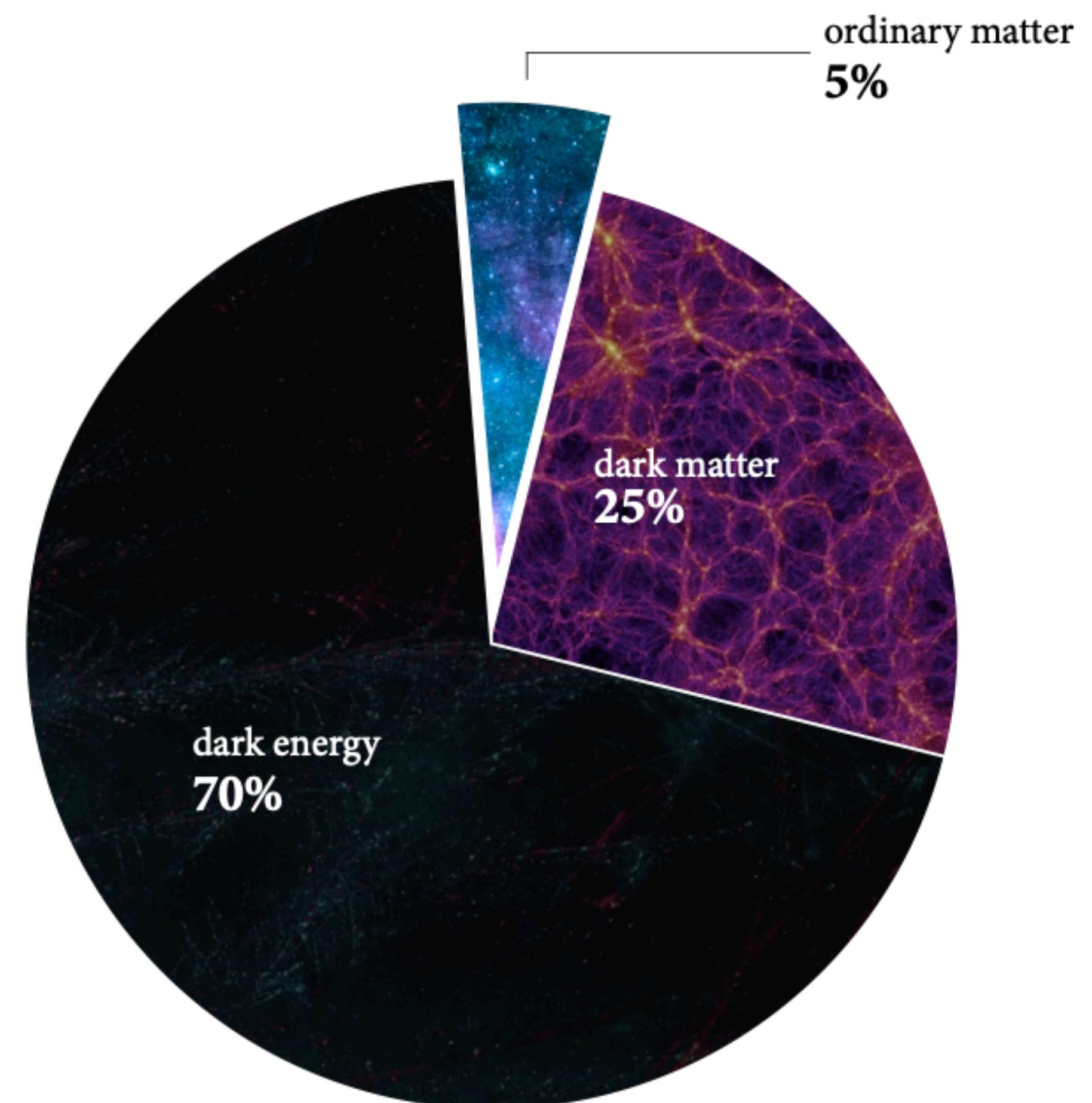


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 - Radiation: $w = \frac{1}{3}$

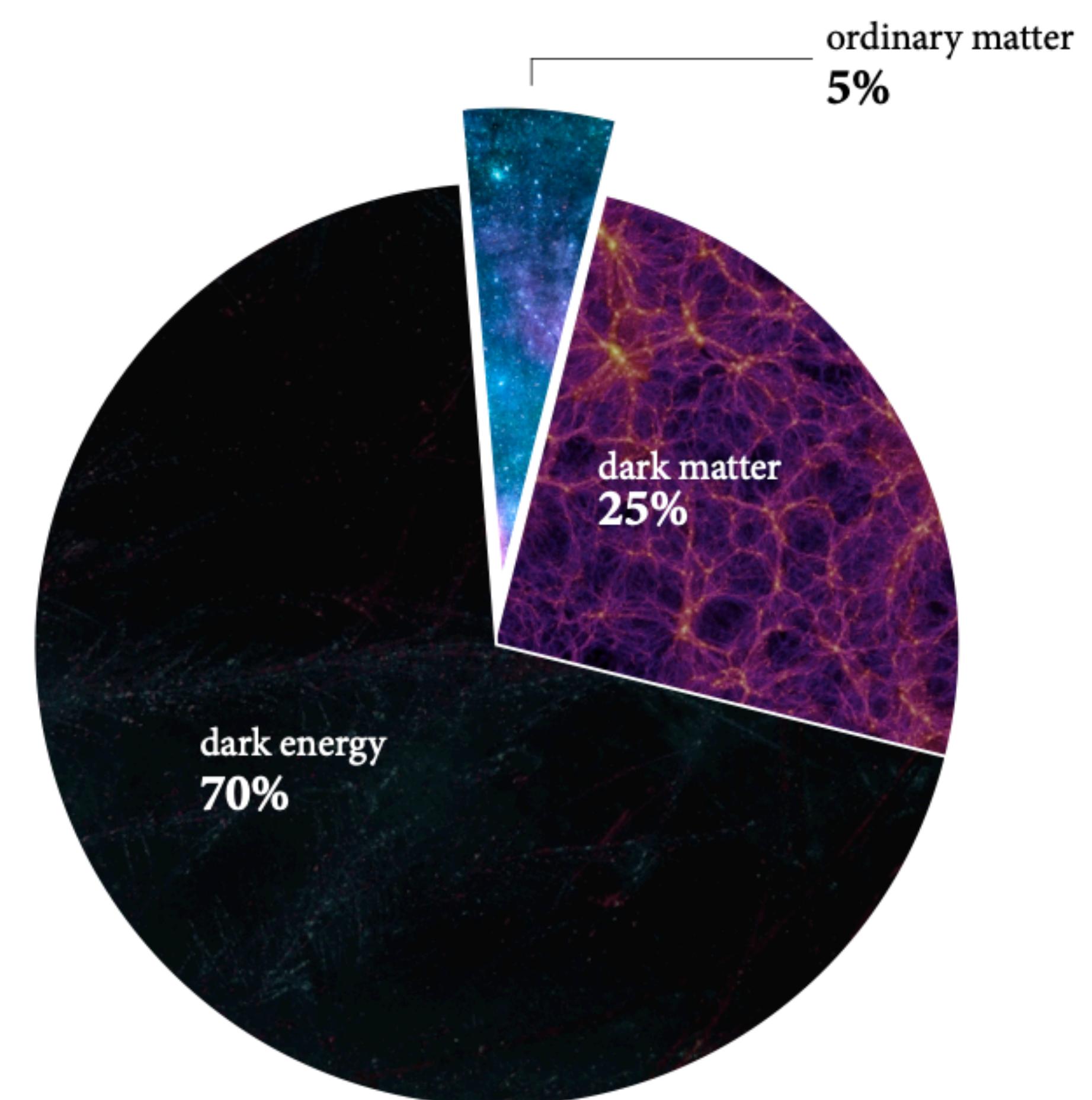


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- Equation of state: $p = w \cdot \rho$
- EOS-parameter w for different matter species:
 - Dark matter and baryonic matter: $w = 0$
 - Radiation: $w = \frac{1}{3}$
 - Dark energy: $w = -1$

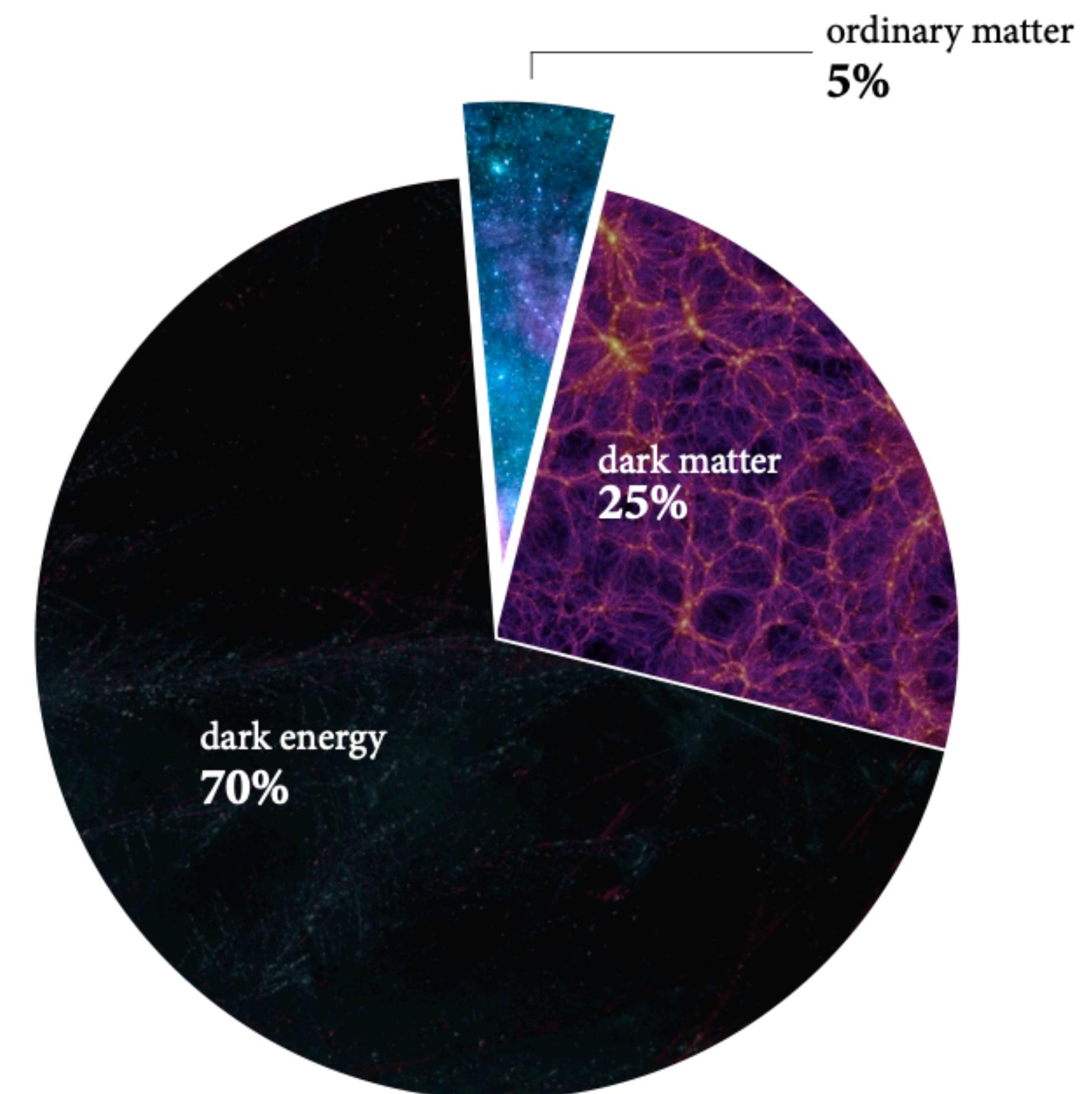


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Friedmann Equations

- Equation of state: $p = w \cdot \rho$
- Inserting the E.O.S into the continuity equation $\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}$ yields:
$$\dot{\rho} = -3\rho(1 + w)\frac{\dot{a}}{a}$$

Friedmann Equations

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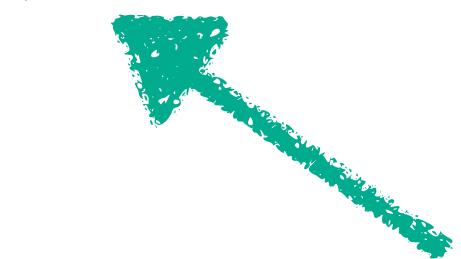
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... (your exercise later)

$$\frac{\rho(a)}{\rho_0} = \left(\frac{a}{a_0}\right)^{-3(1+w)}$$



The subscript 0 refers to the current time “today”

The Hubble parameter

- The first Friedmann equation describes the rate of expansion as a function of the energy content ρ of the universe:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho_{\text{tot}} \quad (\text{i})$$

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$$H_0 = H(t = t_0)$$


today

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- Defining $\rho_{\text{crit}} = \frac{3H_0^2}{8\pi G_N}$

$$\frac{H^2(a)}{H_0^2} = \frac{\rho_{\text{tot}}(a)}{\rho_{\text{crit}}}$$

The Hubble parameter

$$H^2(t) = \frac{8\pi G_N}{3} \rho_{\text{tot}}(t)$$

- Defining $\rho_{\text{crit}} = \frac{3H_0^2}{8\pi G_N}$ and the fractional energy densities $\Omega_I = \frac{\rho_I}{\rho_{\text{crit}}}$:

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Defining the redshift $1 + z = \frac{a_0}{a}$, one can rewrite this in the famous form:

The Hubble parameter

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Distances in an expanding universe

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- The **comoving distance** χ defines a coordinate grid that expands with the universe and is given by

$$\chi(t) = c \int_t^{t_0} \frac{dt'}{a(t')} = \int_0^{z(t)} \frac{c dz'}{H(z')}$$

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- The **physical distance** r_p is defined as the scale factor times the comoving distance: $r_p(t) = a(t)\chi(t)$

Distances in an expanding universe

- The angular diameter distance D_A is defined such that it holds

$$\theta = \frac{s}{D_A}$$

angle ↗ θ ↗ physical size of the object
ang. diam. distance ↗ ↗

Distances in an expanding universe

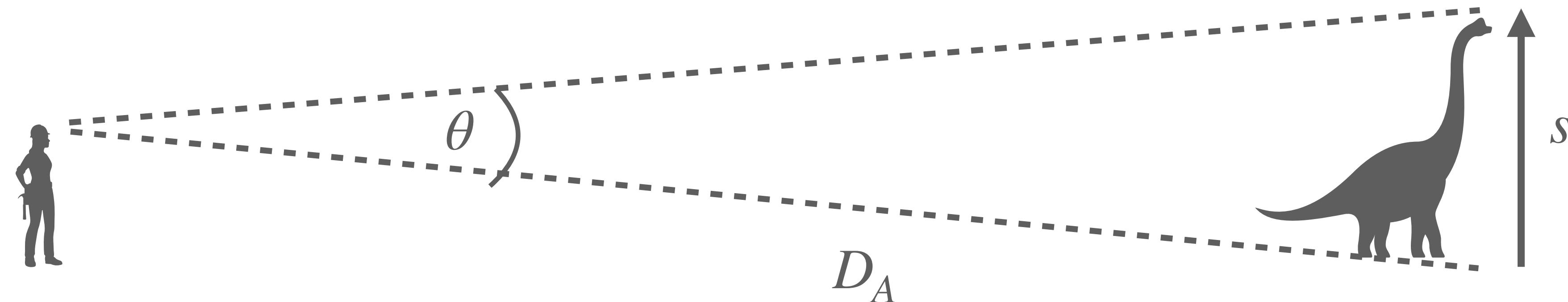
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angle

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ang. diam. distance



Distances in an expanding universe

- The angular diameter distance D_A is defined such that it holds

$$\theta = \frac{s}{D_A}$$

$$D_A(t) = a(t) \chi(t) = \frac{1}{1 + z(t)} \int_0^{z(t)} \frac{dz'}{H(z')}$$

*To derive an equation for D_A , note that the proper size, s , of the object can also be expressed as $s = \chi(t) \theta \cdot a(t)$, where $\chi(t) \theta$ corresponds to the comoving size of the object.

Distances in an expanding universe

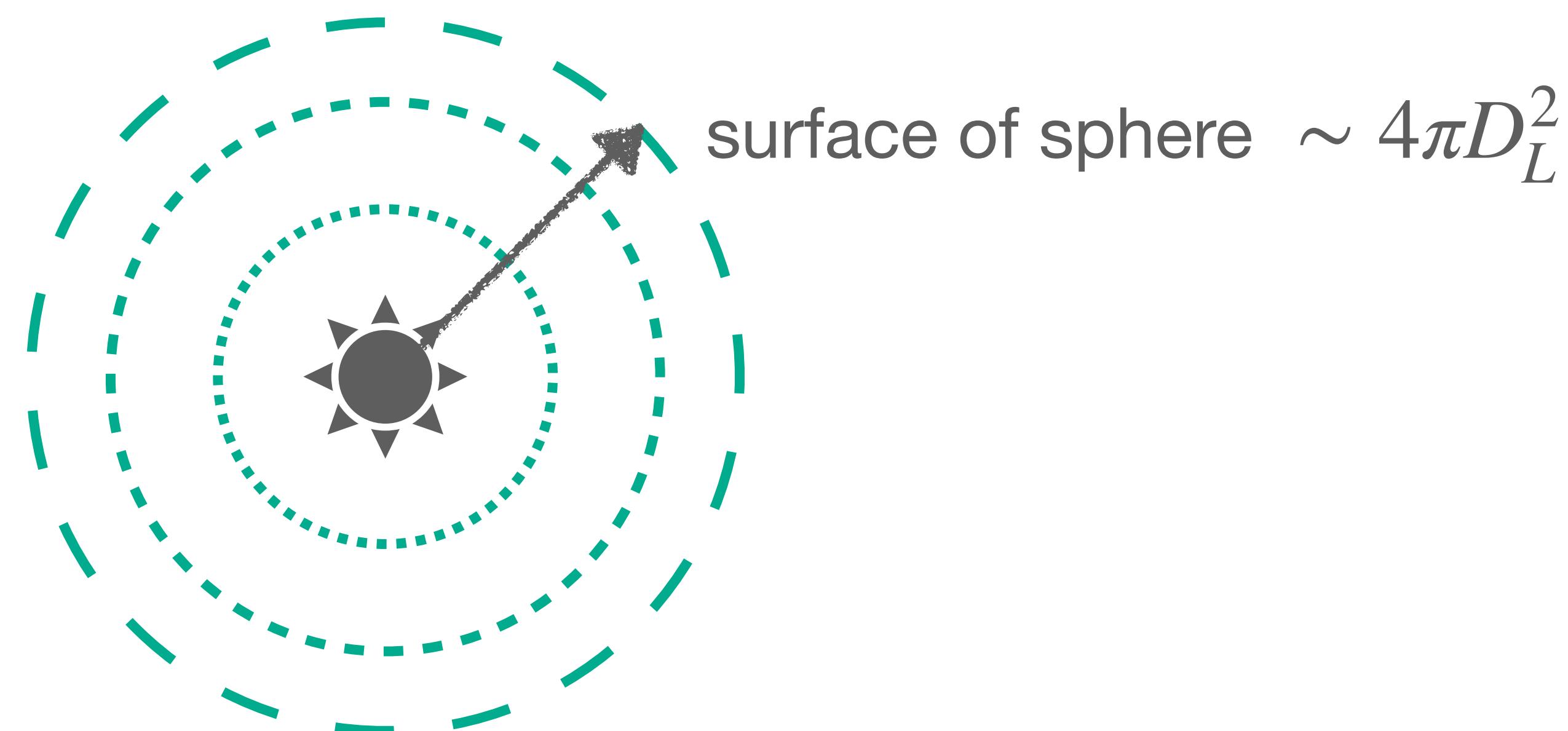
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Distances in an expanding universe

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$$F = \frac{L}{4\pi D_L^2}$$

$$D_L(t) = \frac{\chi(t)}{a(t)} = [1 + z(t)] \int_0^{z(t)} \frac{c \, dz'}{H(z')}$$

*Since the Universe is expanding, it holds that $F = \frac{La^2}{4\pi\chi^2(a)}$, where the additional factor of a^2 comes from the fact that the expansion of the Universe leads to a dilution of photons ($\propto a$) and to an increase in wavelength ($\propto a$).

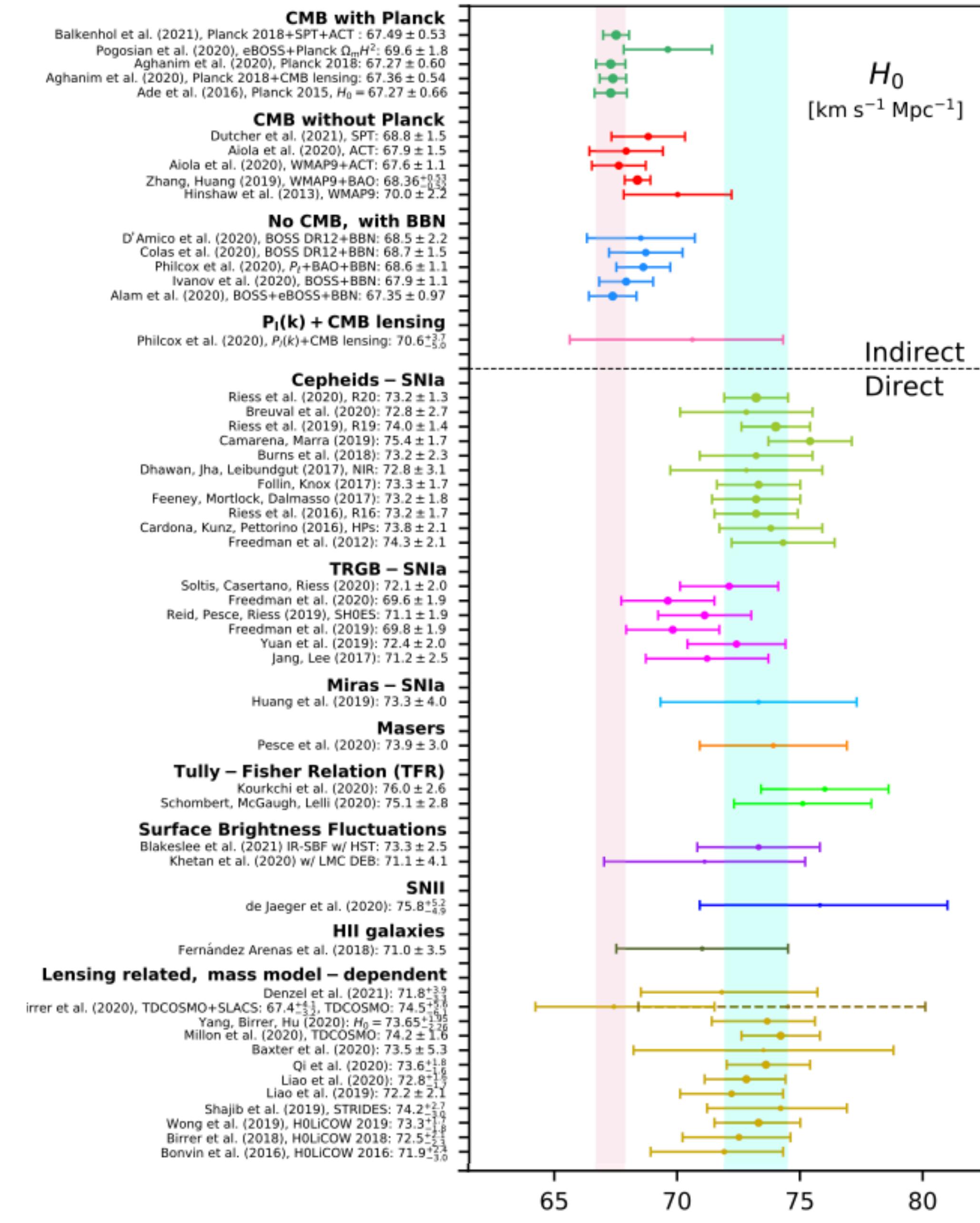
How to solve the Hubble tension?

Direct vs. indirect? Early vs. late measurements?

How does the CMB constrain H_0 ?

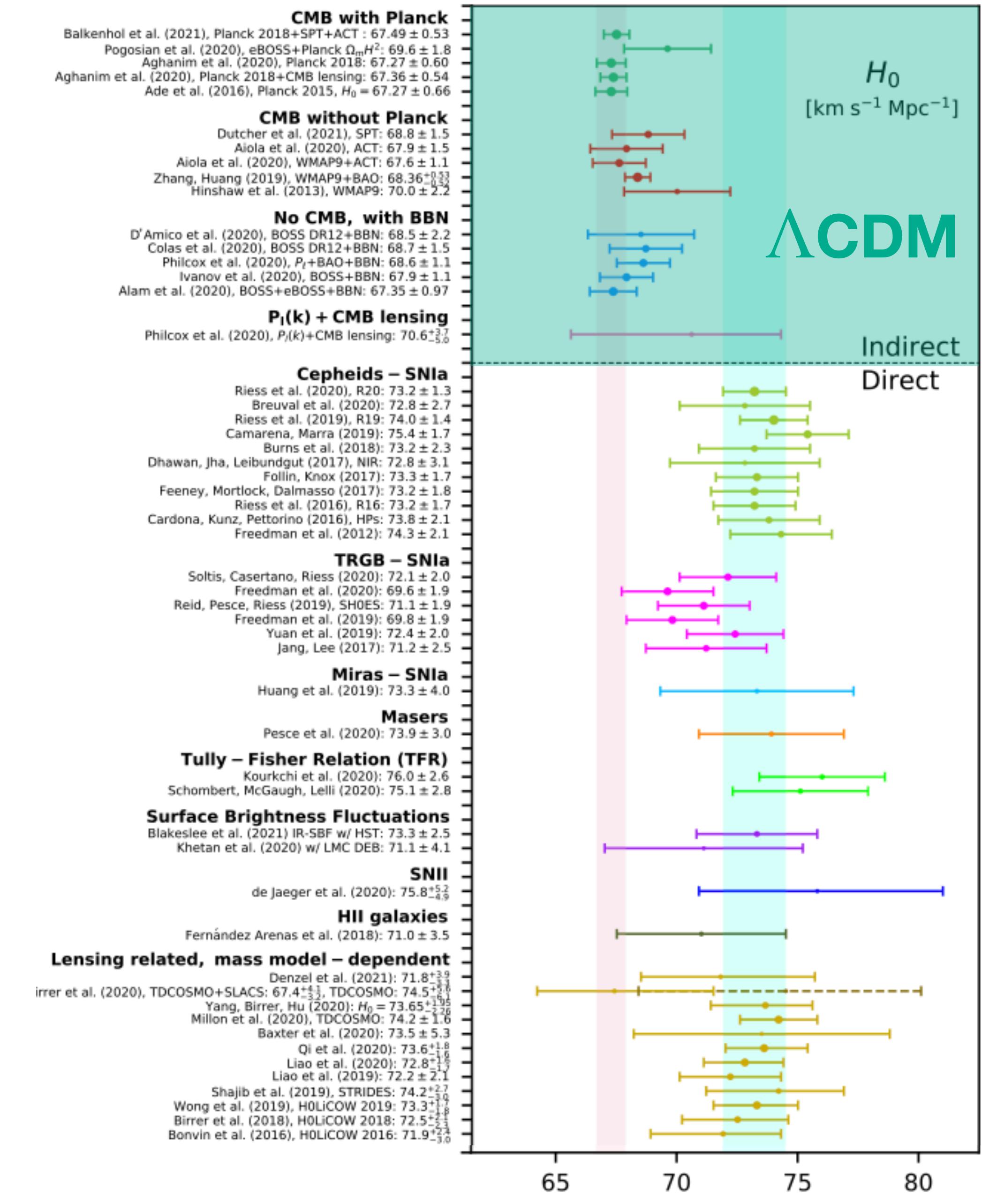
The Hubble tension

- **Indirect measurements:** cosmic microwave background (CMB), baryon acoustic oscillations (BAO), galaxy clustering
- **Direct measurements:** distance ladder (Cepheids, TRGB, SNe, ...), gravitational lensing, ...



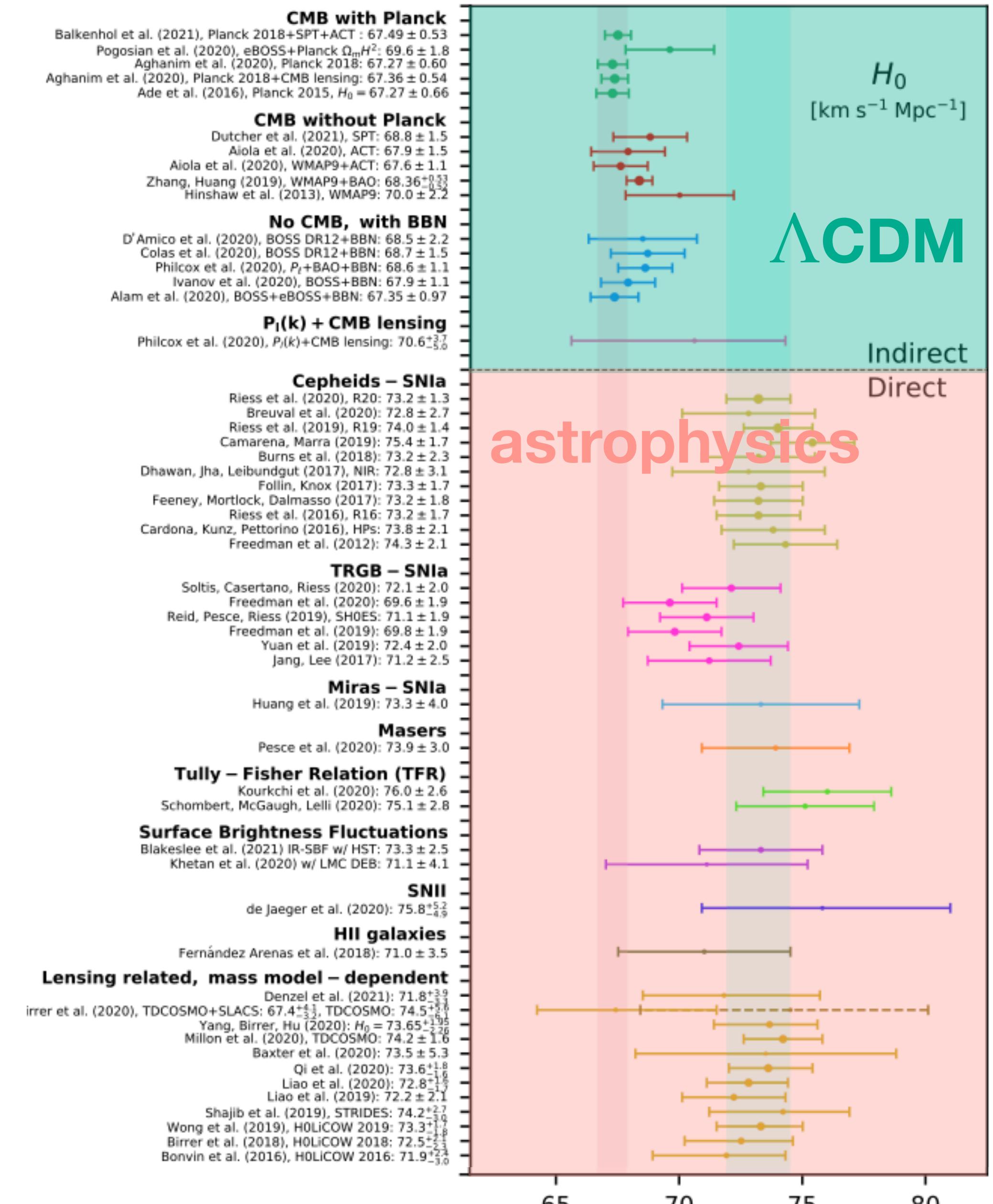
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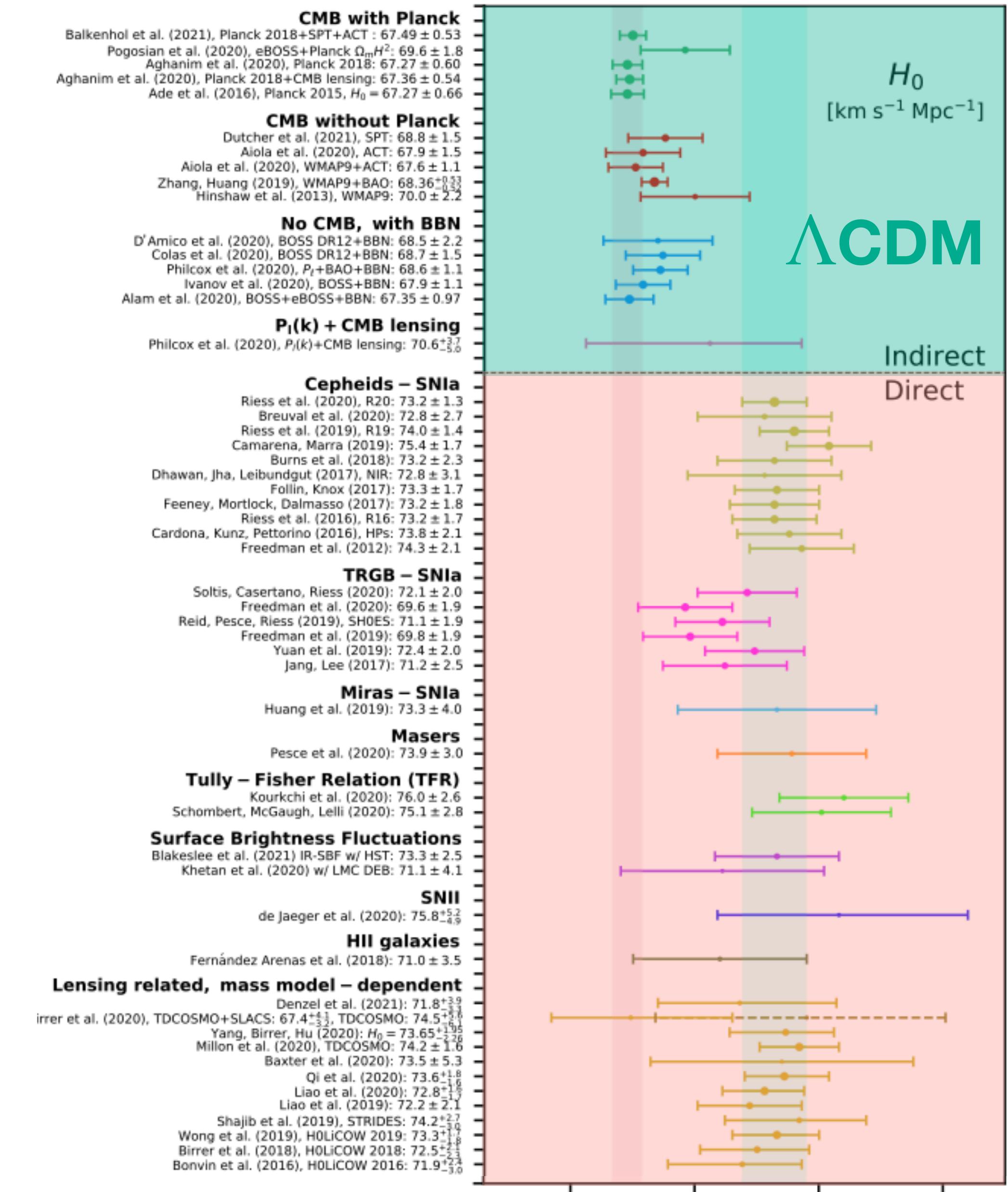
The Hubble tension

- **Indirect measurements:** cosmic microwave background (CMB), baryon acoustic oscillations (BAO), galaxy clustering → precise but model dependent
- **Direct measurements:** distance ladder (Cepheids, TRGB, SNe, ...), gravitational lensing, ... → less precise due to astrophysical modelling but independent of cosmological model



The Hubble tension

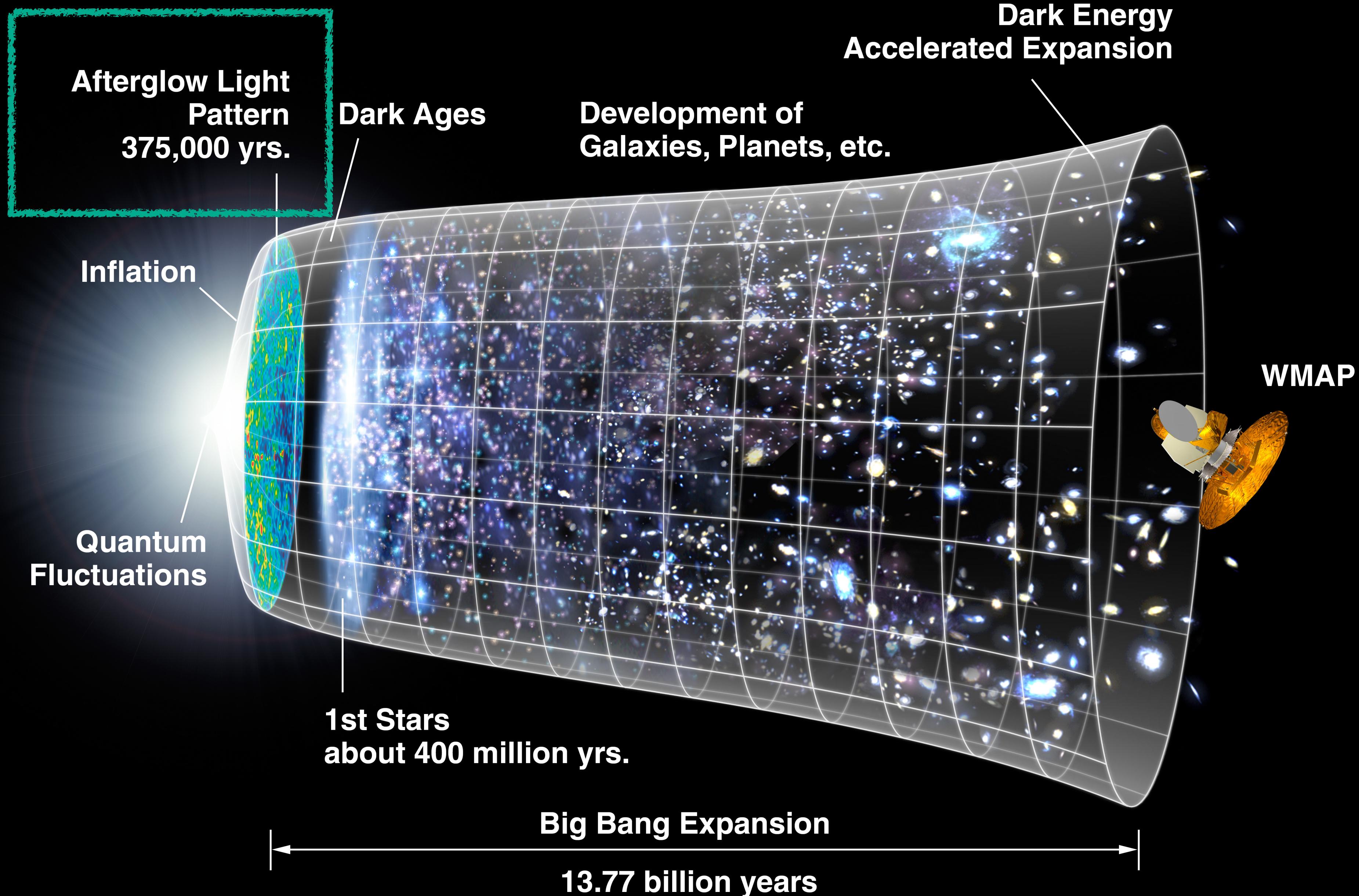
- General strategy to solve the H_0 tension:
Assume direct measurements are correct
and **change cosmological model** in order
to *infer* a higher H_0
- Goal: “get CMB- H_0 to ~ 73 km/s/Mpc”



Expansion rate H_0 [km/s/Mpc]

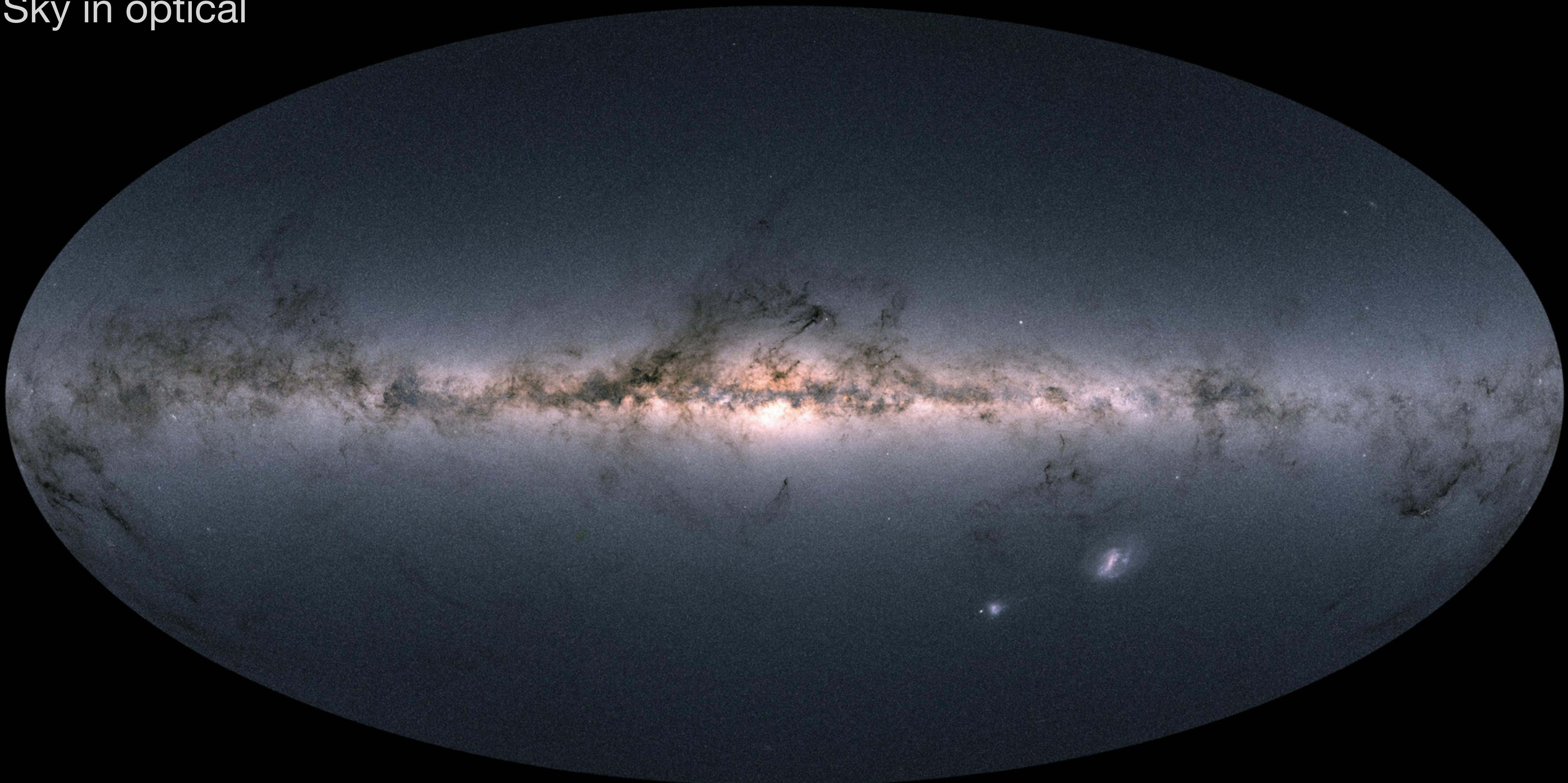
How does the CMB constrain H_0 ?

- In order to understand how we can solve the Hubble tension, we need to understand how the CMB constrains H_0
- The CMB provides the most important cosmological probe when it comes to constraining the parameters of the cosmological model with high accuracy
- However, it is an indirect probe of H_0 : **it depends on the cosmological model** that is assumed → the CMB constrains the universe **at early times** and **predicts H_0 today**



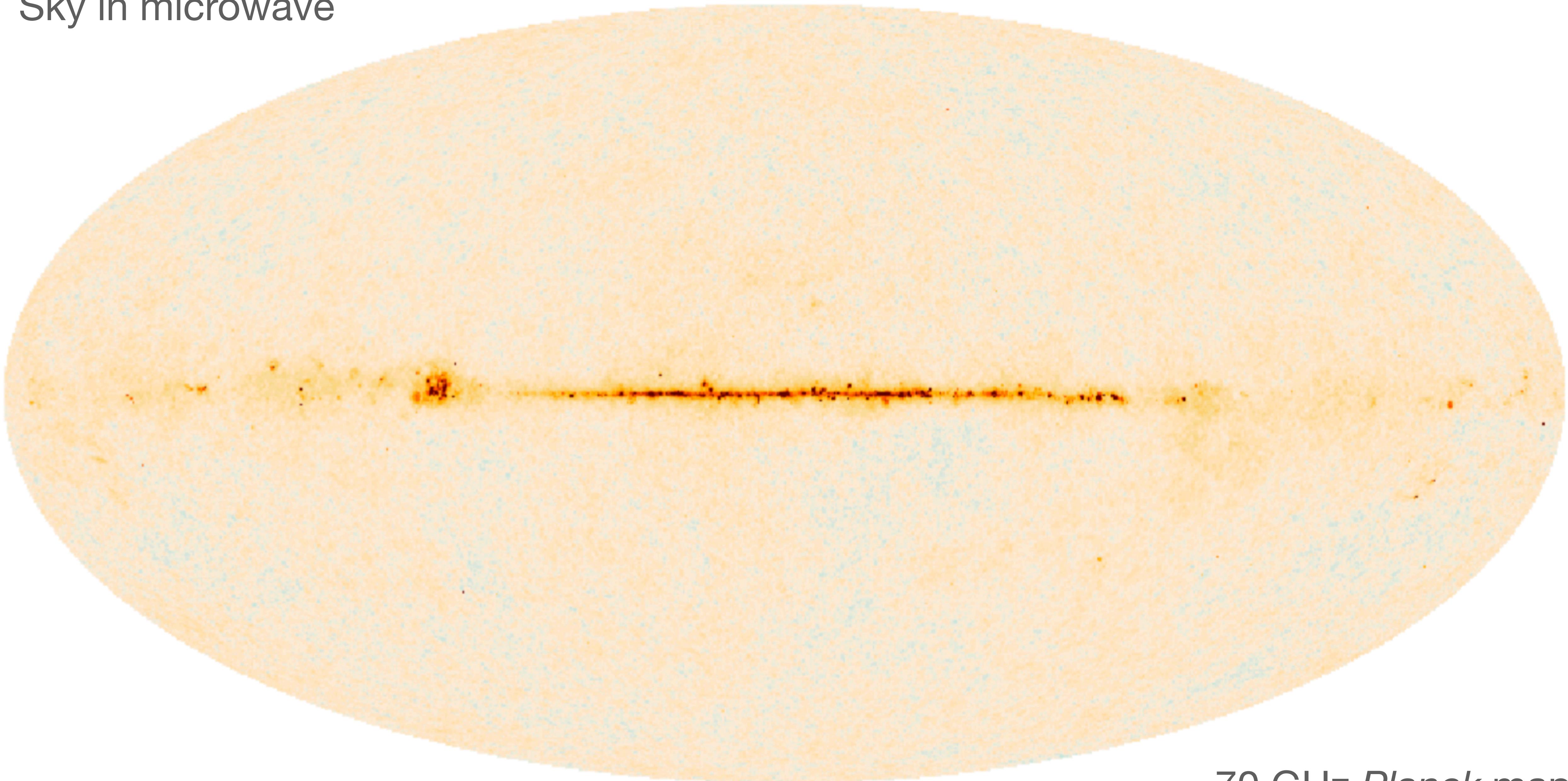
Credit: WMAP Collaboration

Sky in optical



Credit: Gaia collaboration

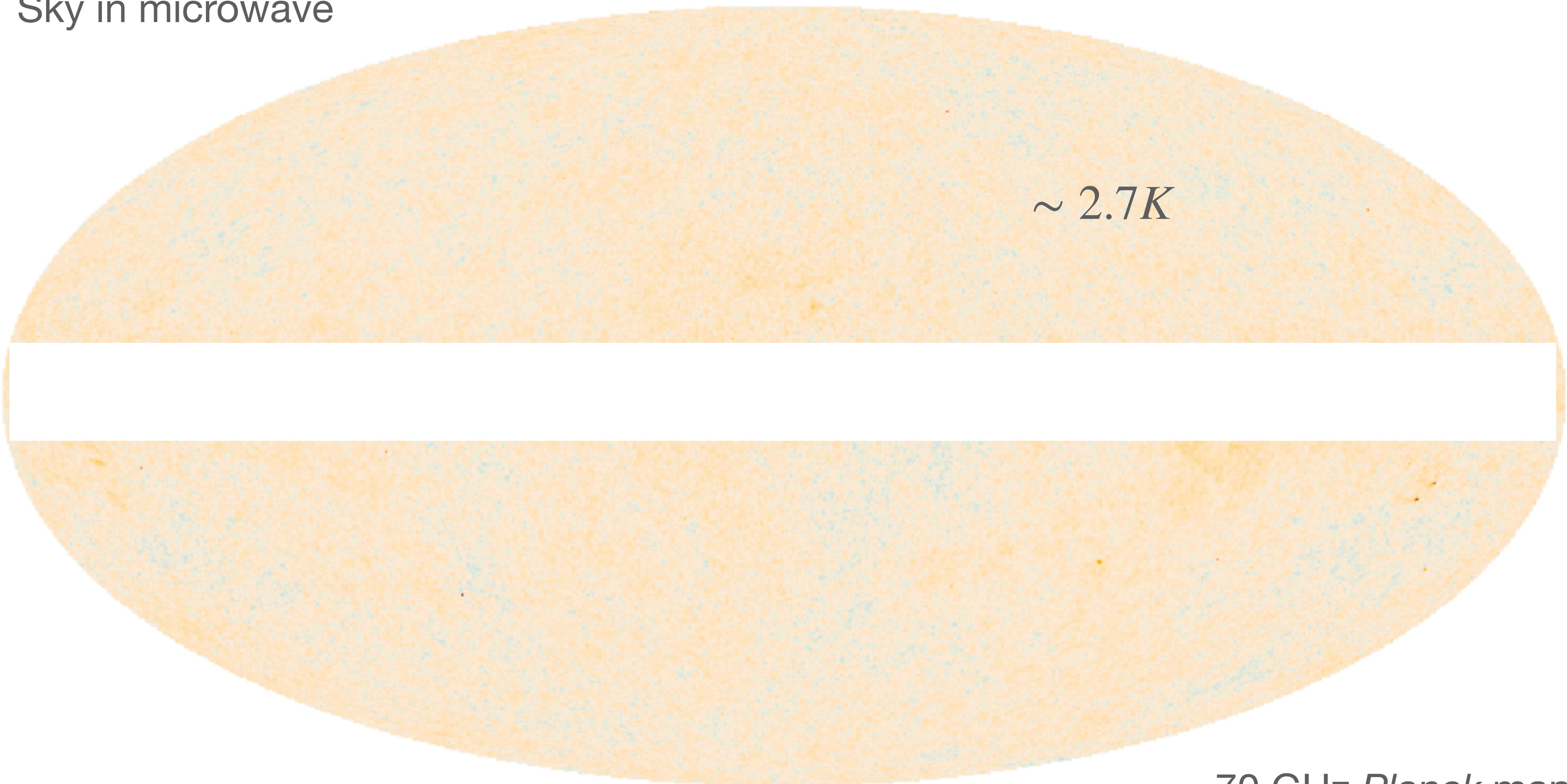
Sky in microwave



70 GHz *Planck* map

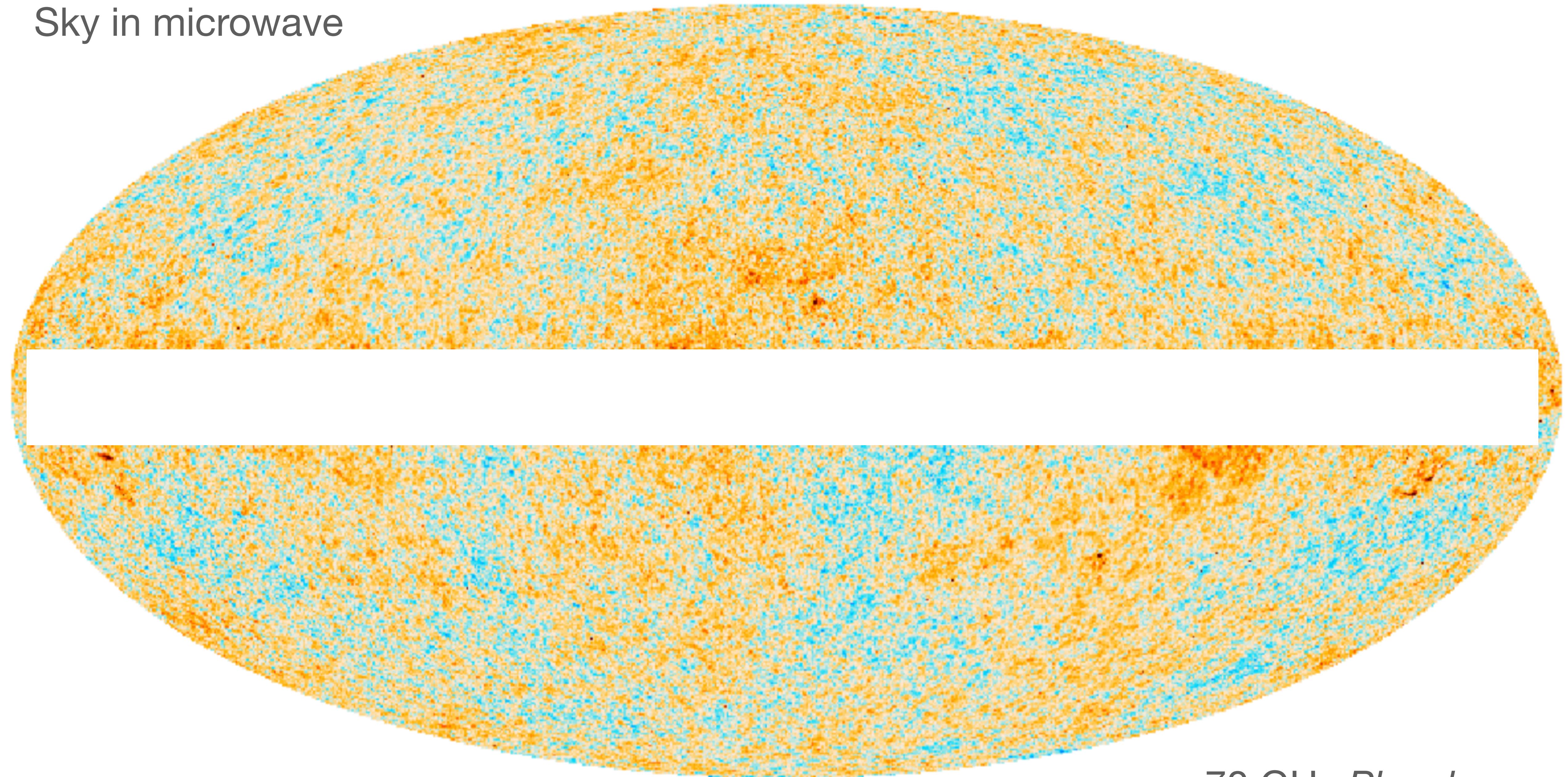
Sky in microwave

$\sim 2.7K$



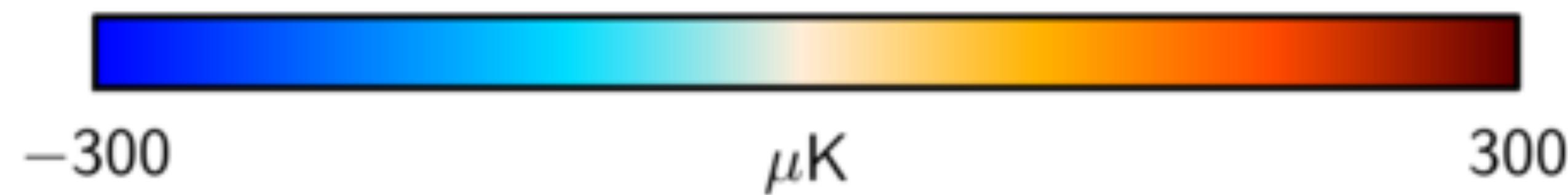
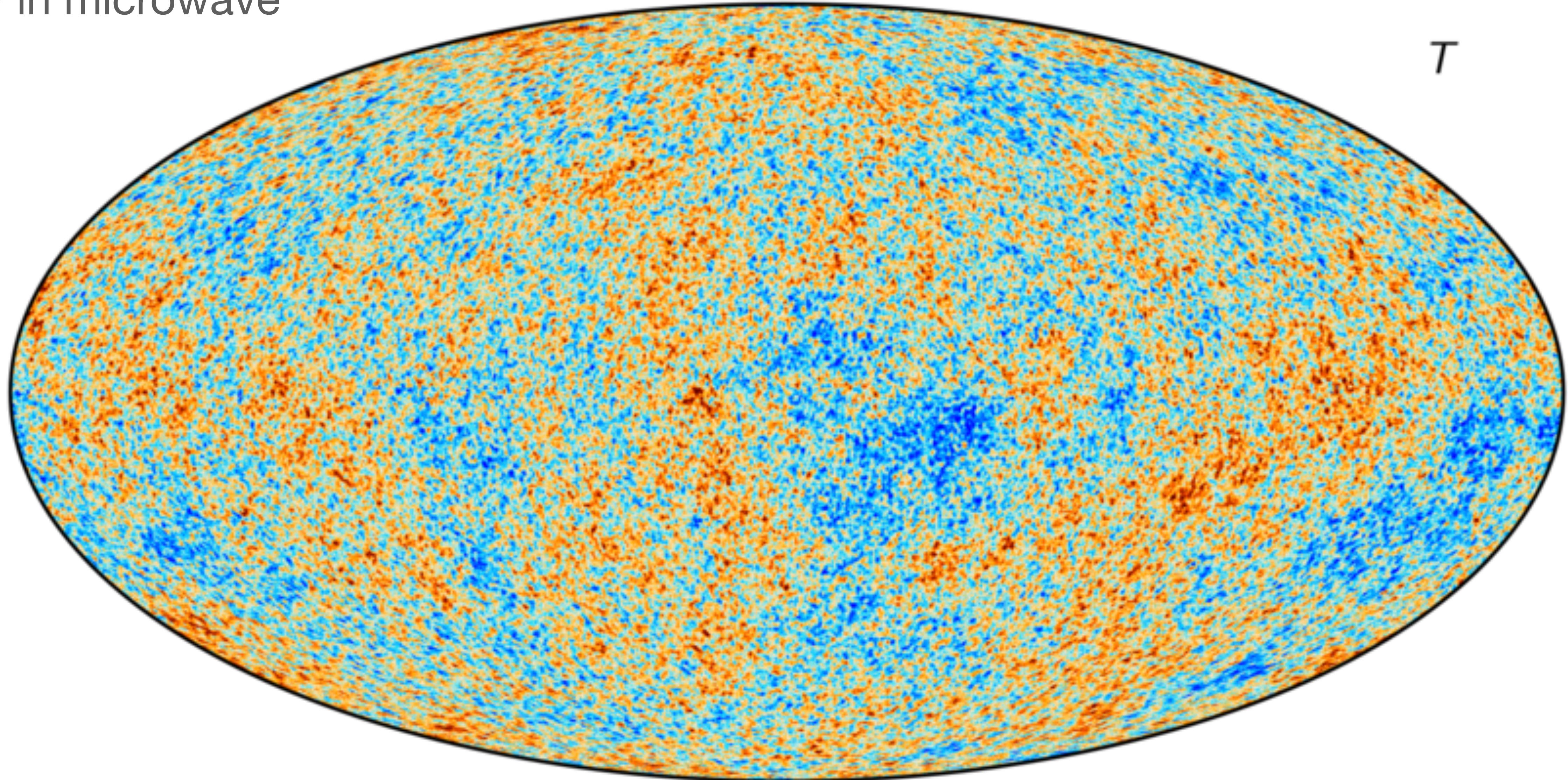
70 GHz *Planck* map

Sky in microwave



70 GHz Planck map

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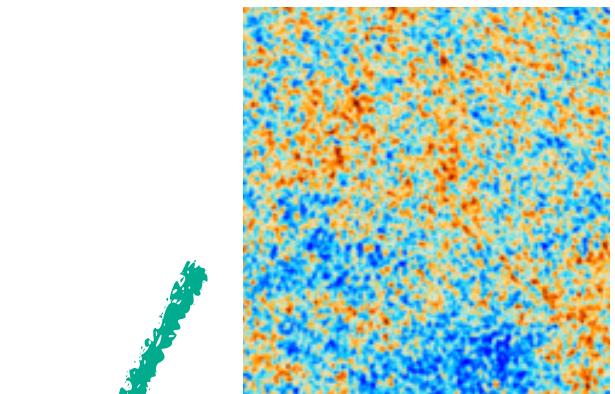
Credit: Planck
Collaboration

The CMB

- How to analyse the CMB map with temperature fluctuations?
 - 1. Decompose the temperature fluctuations into a set of waves with various wavelength
 - 2. Plot the strength of each wavelength: **Power spectrum**
- In 2 dimensions: decomposing into waves = Fourier transform
- On the unit sphere: decomposing into waves = **Spherical harmonics** decomposition

The CMB

- Decompose temperature fluctuations: $\Delta T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$

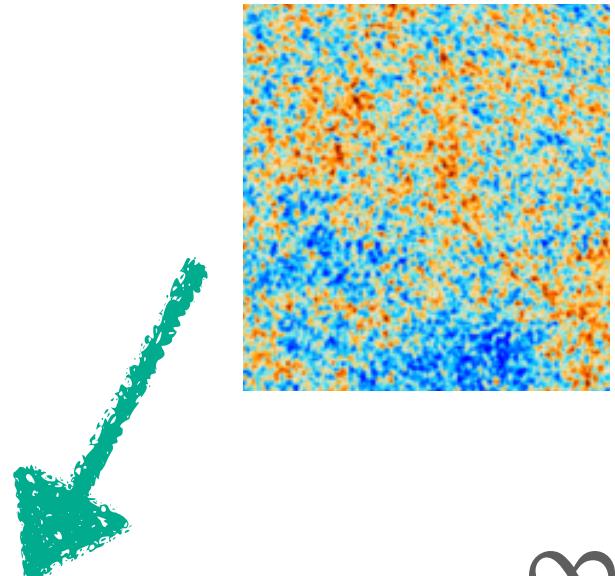


coefficients

spherical harmonics

The CMB

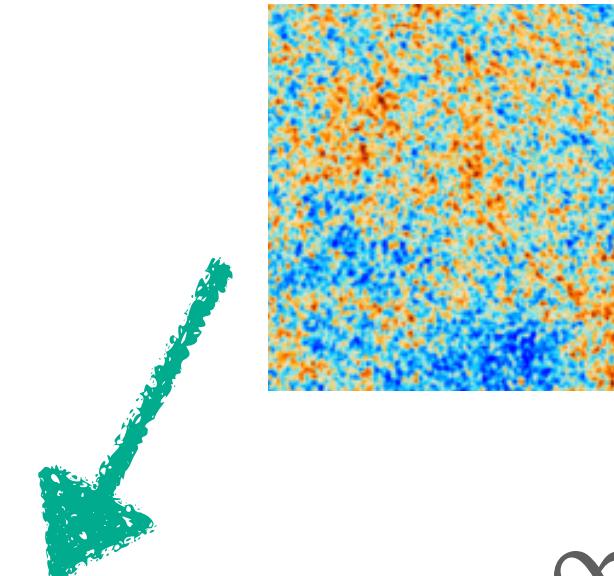
- Decompose temperature fluctuations: $\Delta T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$
- The $a_{\ell m}$ contain the same information as the map itself



spherical harmonics

The CMB

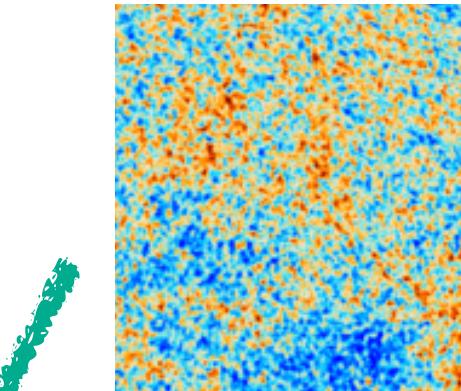
- Decompose temperature fluctuations: $\Delta T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$



- The $a_{\ell m}$ contain the same information as the map itself
- ℓ defines the multipole, the higher ℓ the smaller the scale

spherical harmonics

The CMB

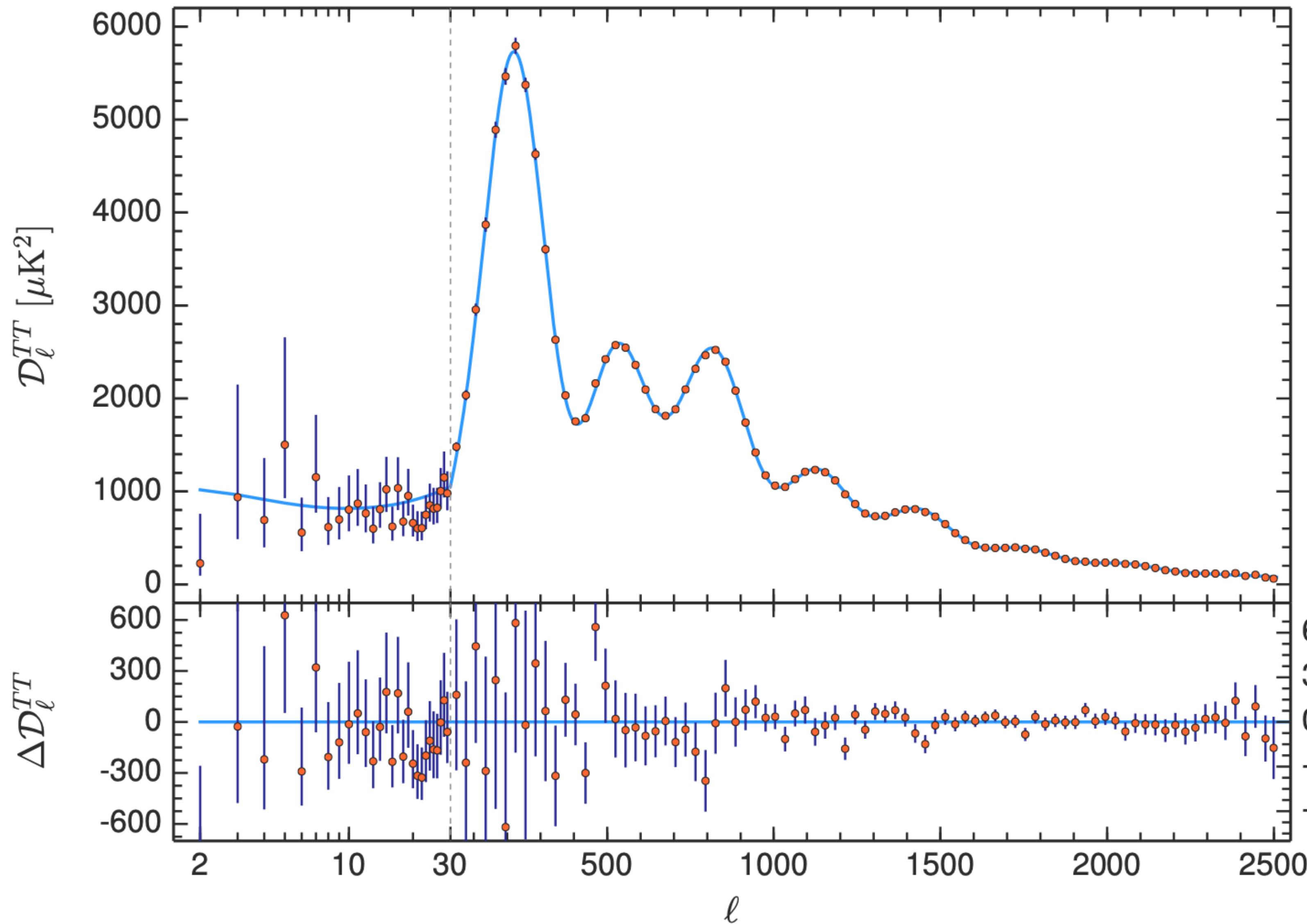


- Decompose temperature fluctuations: $\Delta T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$
- The $a_{\ell m}$ contain the same information as the map itself
- ℓ defines the multipole, the higher ℓ the smaller the scale
- The power spectrum is then given by:

$$C_{\ell} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell,m} a_{\ell,m}^*$$

spherical harmonics

The CMB

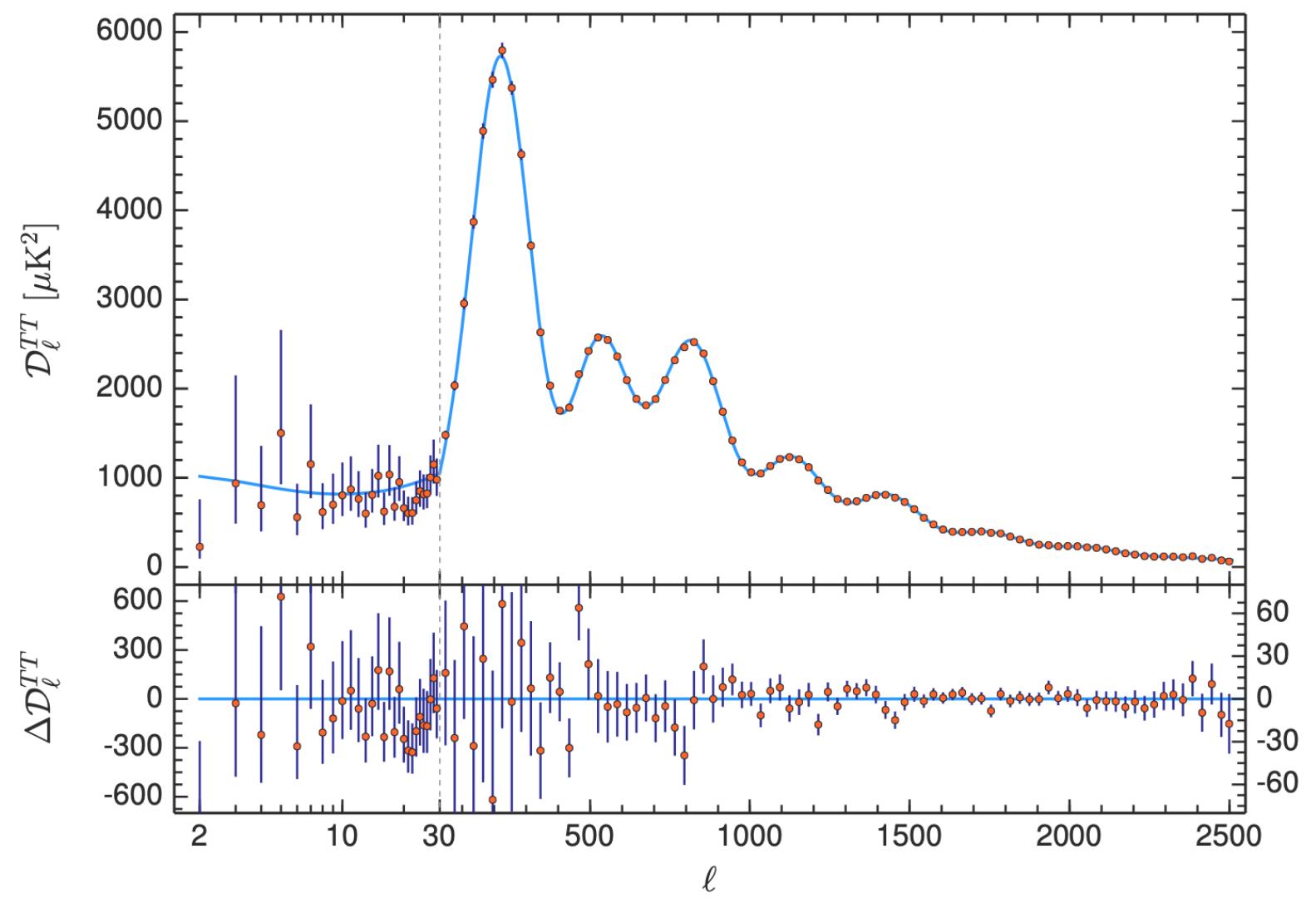


$$D_\ell = \frac{\ell(\ell + 1)}{2\pi} C_\ell$$

*You will need this in the hands-on session later

The CMB

How to analyse data like this?



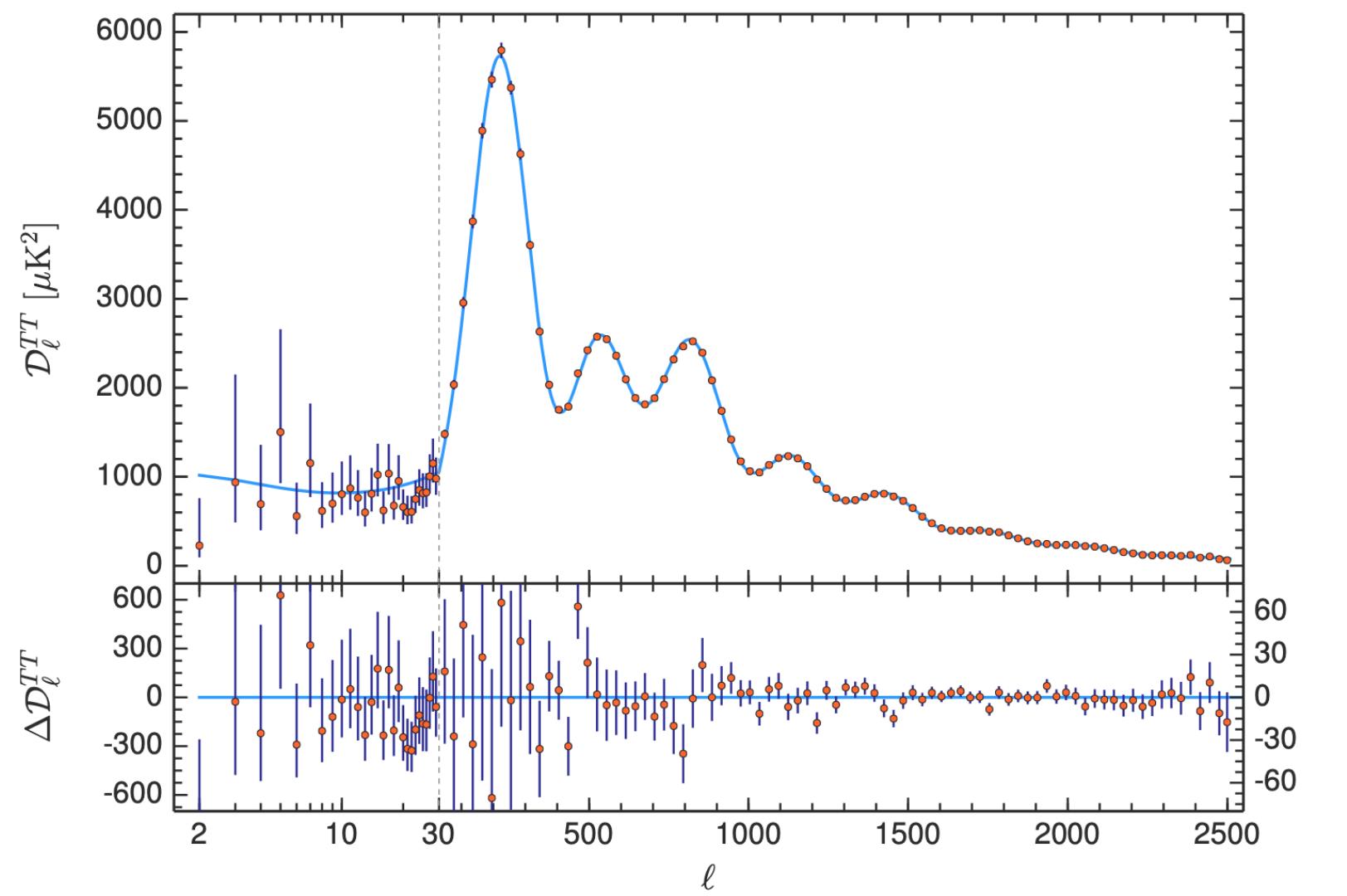
- In practice one needs to solve full fluid-equations (the Boltzmann equations) for all components of the universe to predict the CMB power spectrum
- This can be done with cosmological Boltzmann solvers like **CAMB** (Lewis&Bridle 2002) and **CLASS** (Blas, Lesgourgues, Tram 2011)
- These Boltzmann solvers take as input the ΛCDM parameters and can be sampled within MCMC samplers

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The CMB

Λ CDM model

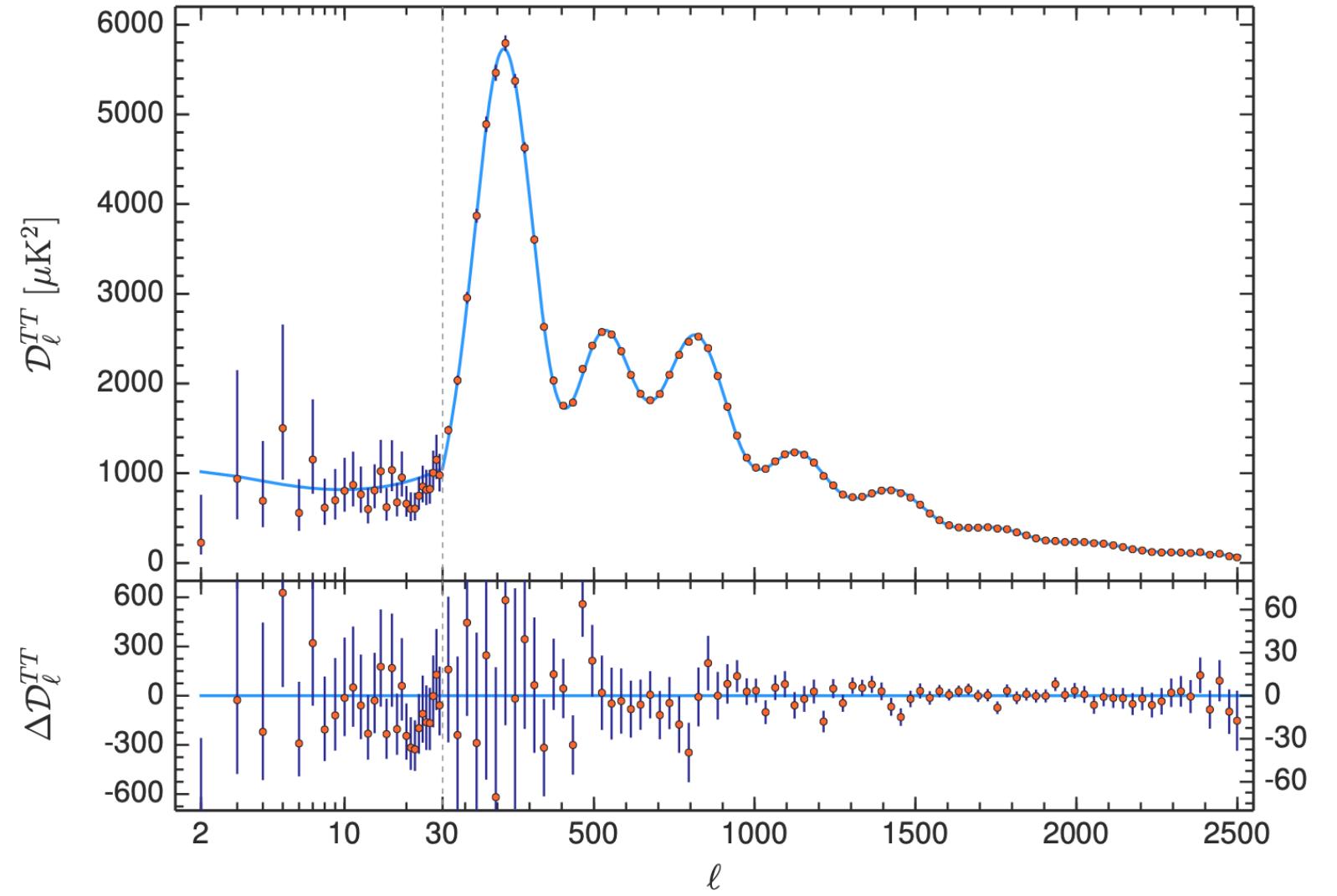
$$\rho_{\text{tot}} = \rho_r + \rho_m + \rho_{\Lambda}$$



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The CMB

$$\Lambda\text{CDM model} \quad \rho_{\text{tot}} = \rho_r + \rho_m + \rho_\Lambda$$

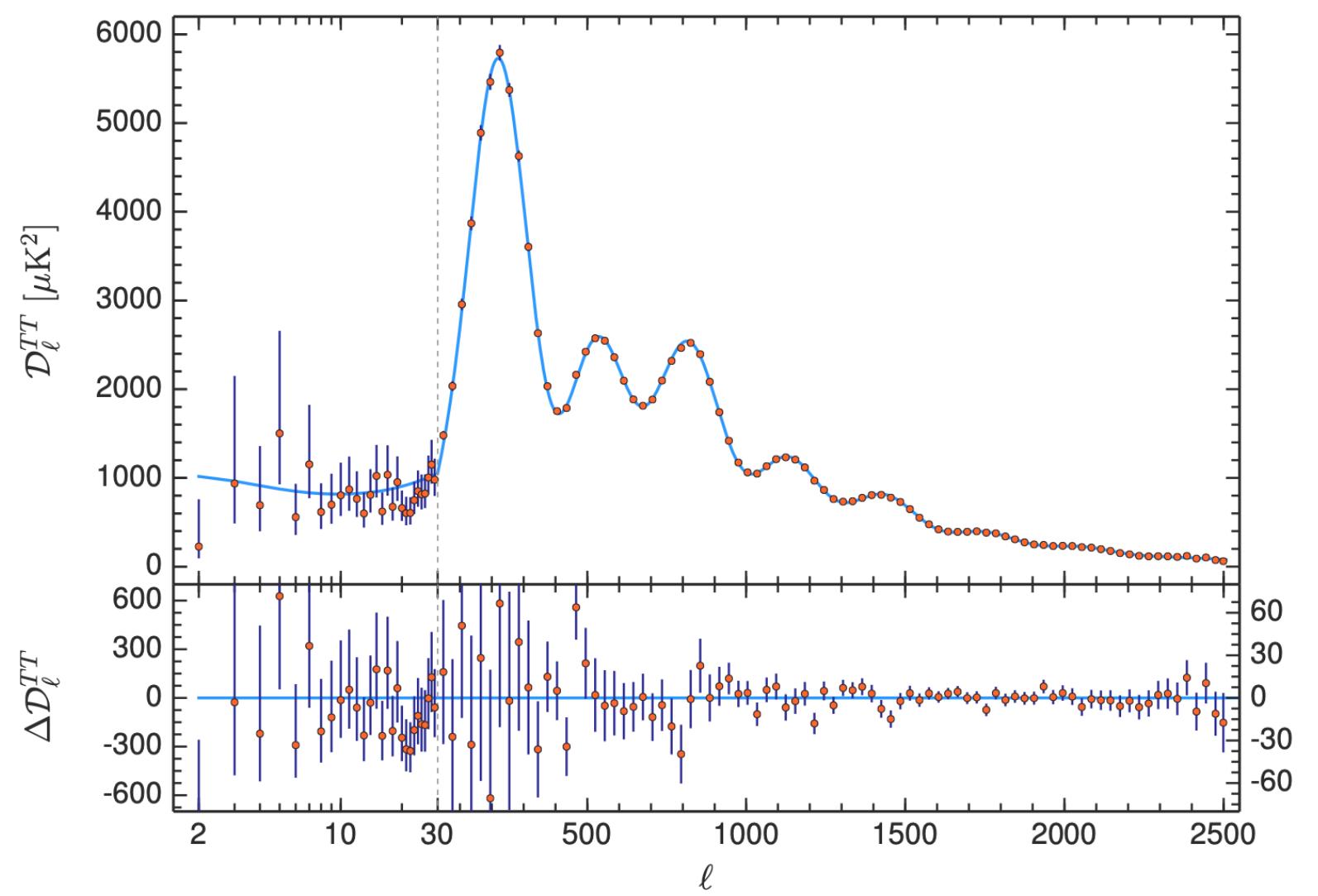


- $\omega_{\text{cdm}} = h^2 \Omega_{\text{cdm}}$:
- $\omega_b = h^2 \Omega_b$:
- $h = H_0 / (100 \text{ km/s/Mpc})$:
- τ_{reio} :
- A_s :
- n_s :

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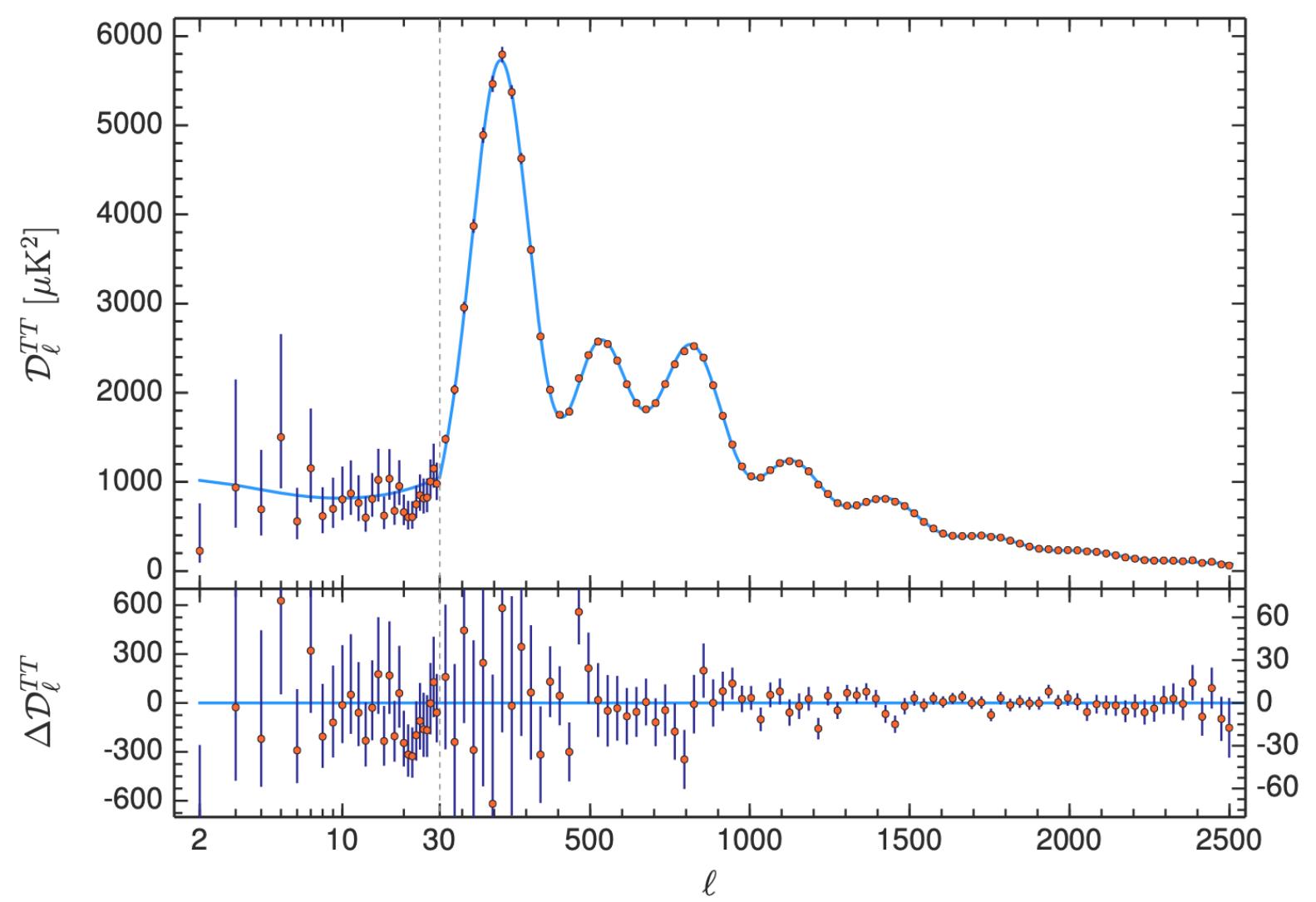


- $\omega_{\text{cdm}} = h^2\Omega_{\text{cdm}}$: physical energy density in CDM
- $\omega_b = h^2\Omega_b$:
- $h = H_0/(100 \text{ km/s/Mpc})$:
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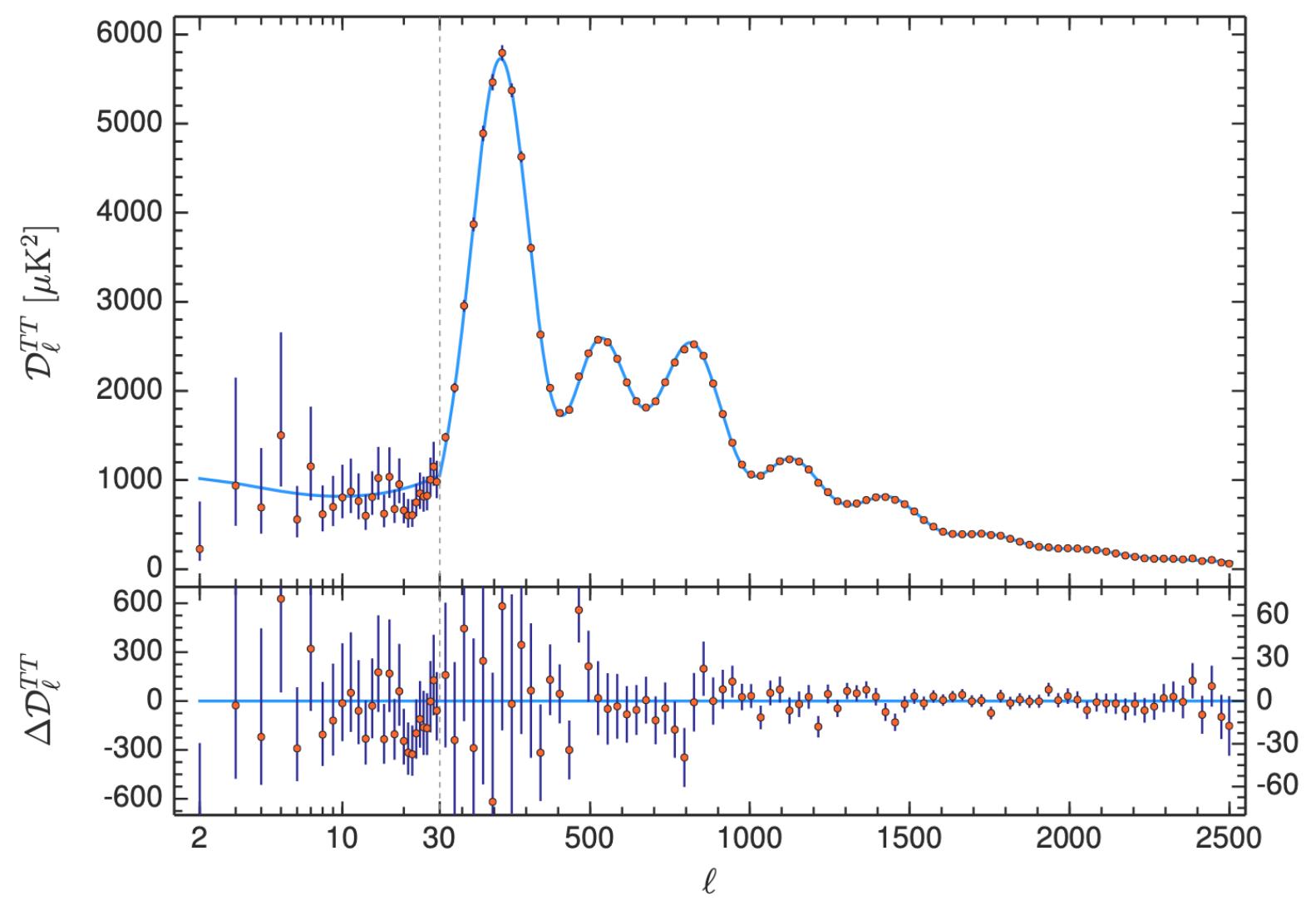


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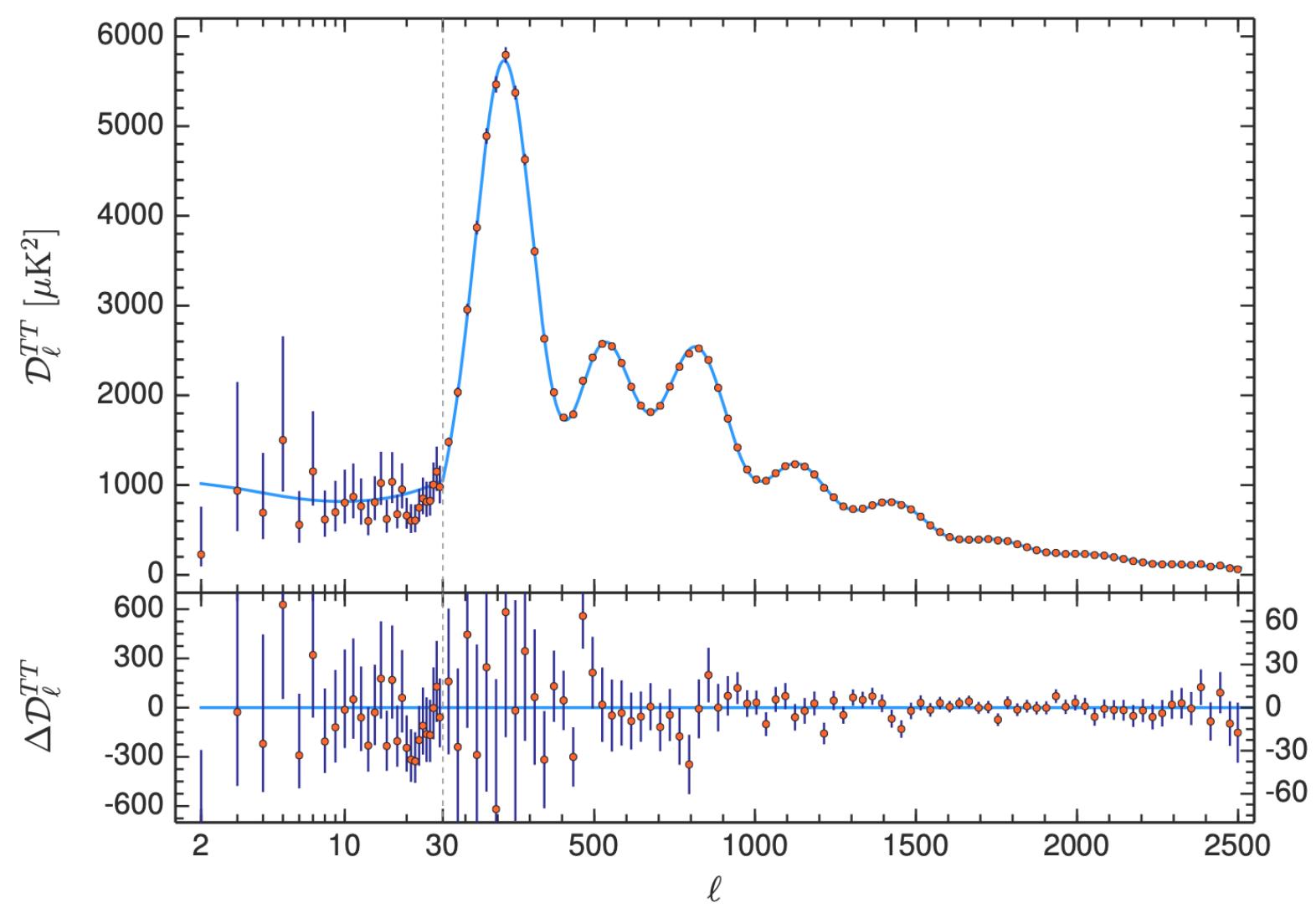


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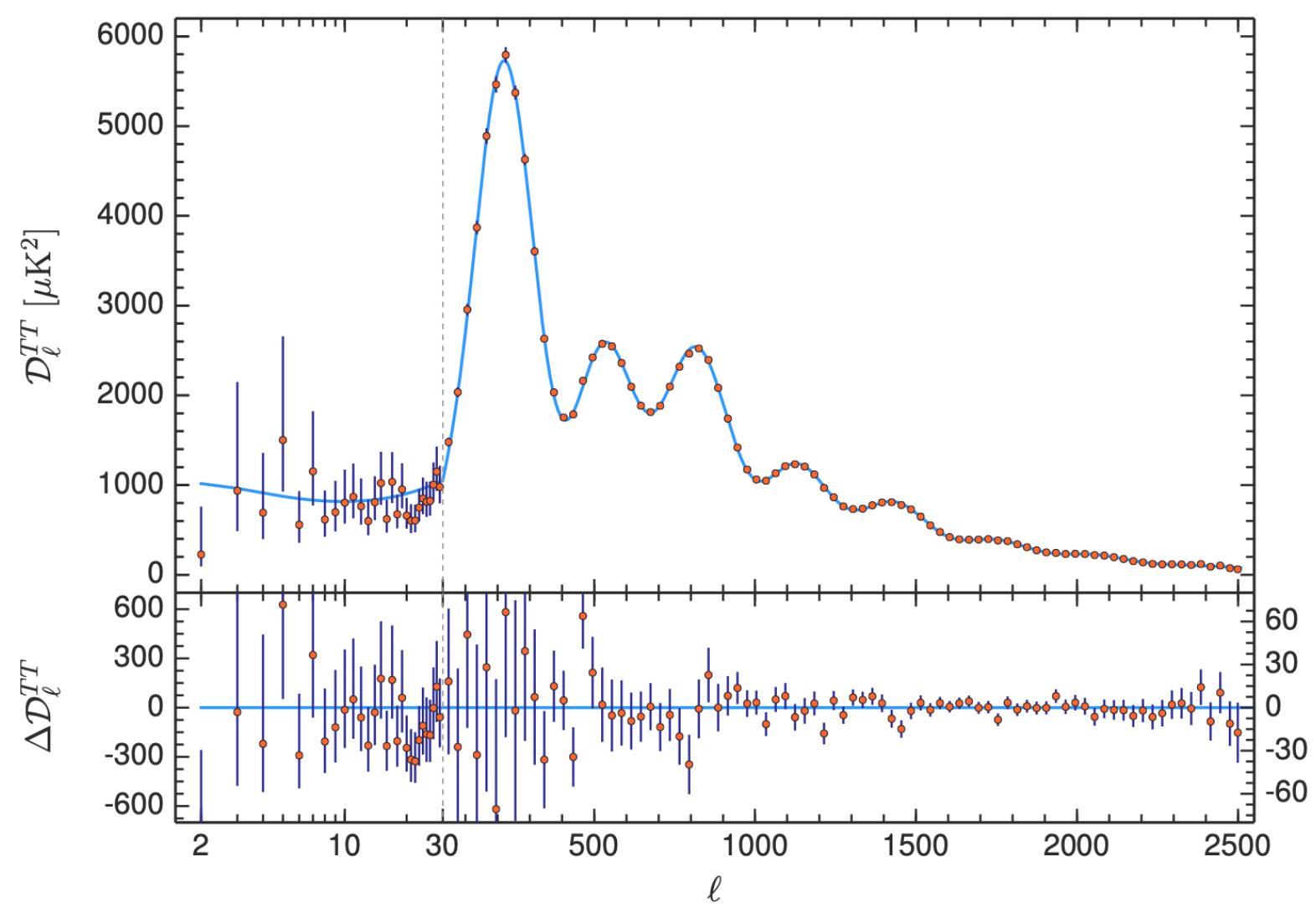


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- τ_{reio} : optical depth to reionization
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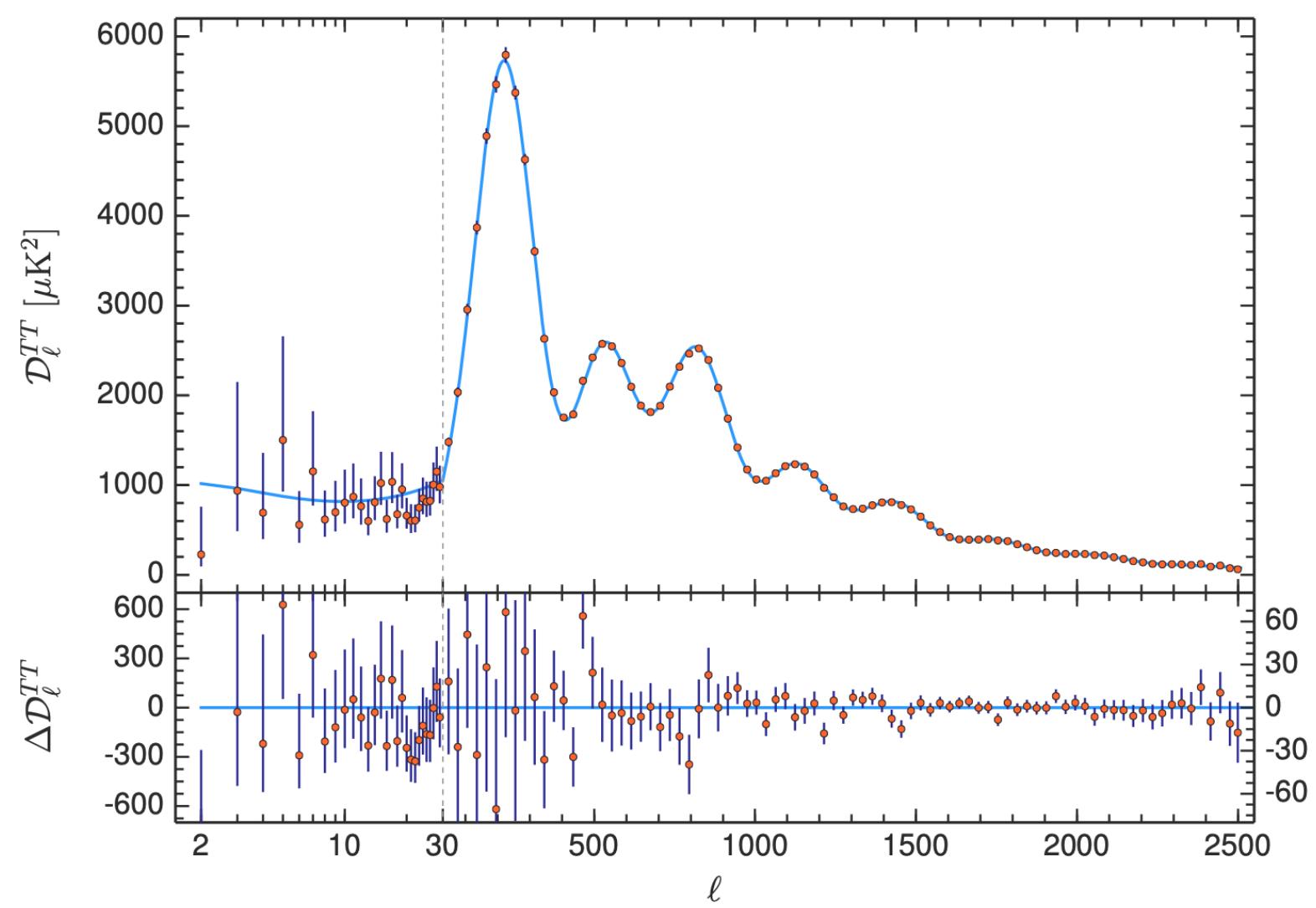


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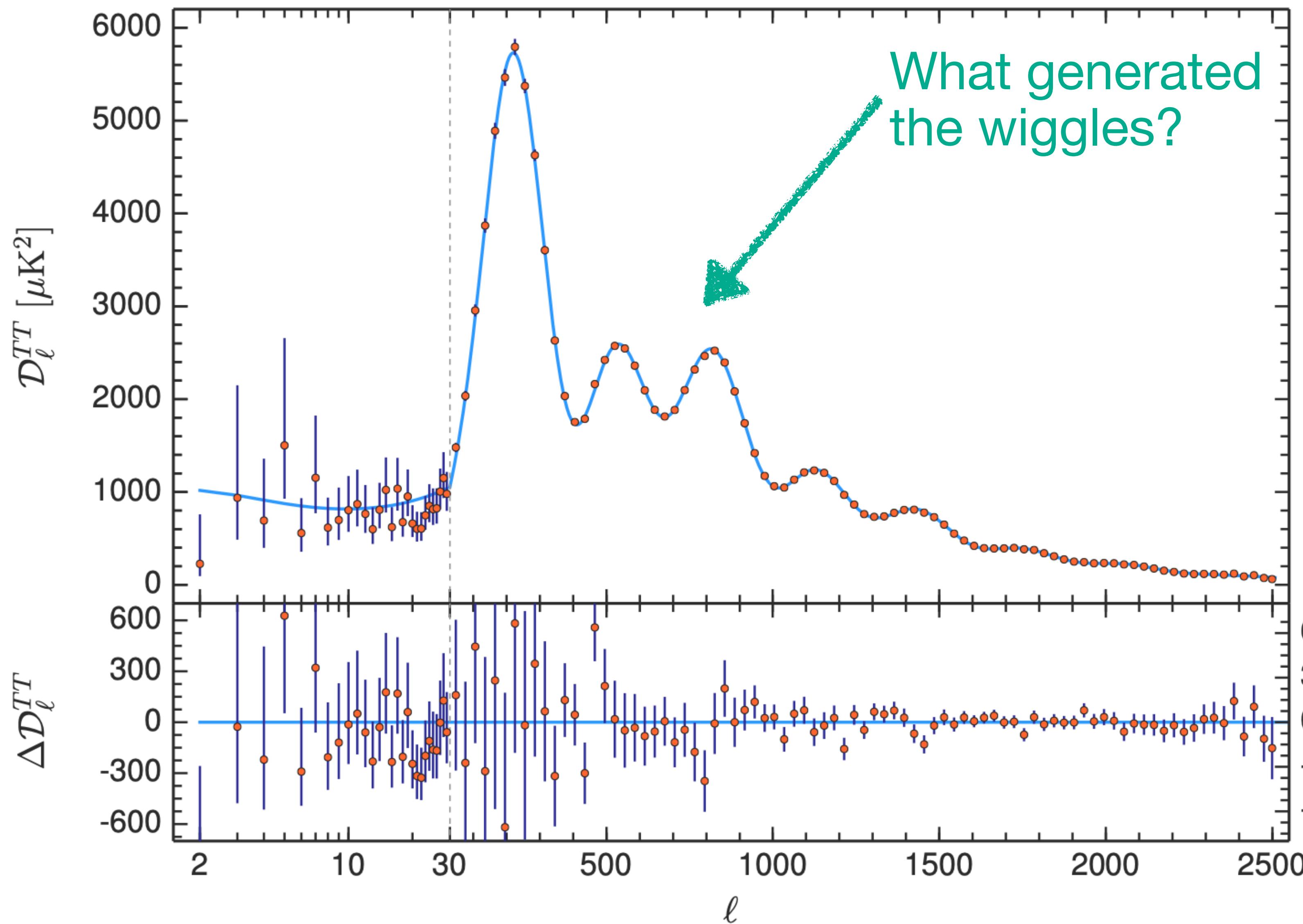
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- τ_{reio} : optical depth to reionization
- A_s : amplitude of the primordial power spectrum
- n_s : tilt of the primordial power spectrum

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The CMB



$$D_\ell = \frac{\ell(\ell + 1)}{2\pi} C_\ell$$

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The baryon acoustic oscillations (BAO)

- The early universe was a hot plasma of baryons and photons

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Credit: Mabel Amber

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- Small density fluctuations sourced pressure-density waves
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 - Photon pressure as restoring force
- Like rain drops on the water surface many waves overlap
- Once electrons and protons recombine, photons can travel freely → the density waves freeze



Credit: Mabel Amber

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- Let's compute the (comoving) distance that the sound waves travelled: the sound horizon
- The sound waves travel from the big bang to the time of recombination t^* (when protons and electrons combine into atoms):
$$r_s = \frac{\int_{BB}^{t^*} c_s(t) dt}{\frac{c_s(t)}{a(t)}}$$
- While the sound waves travel, the universe expands. This slows down the sound waves in the comoving coordinate frame

The baryon acoustic oscillations (BAO)

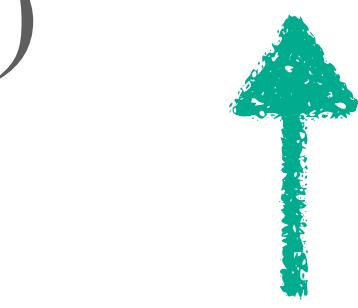
- Comoving sound horizon:

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$$r_s = \int_{BB}^{t^*} \frac{c_s(t)}{a(t)} dt = \int_0^{a^*} \frac{c_s(a)}{\dot{a}} da$$



$$\frac{da}{dt} = \dot{a}$$

The baryon acoustic oscillations (BAO)

- Comoving sound horizon:

$$r_s = \int_{BB}^{t^*} \frac{c_s(t)}{a(t)} dt = \int_0^{a^*} \frac{c_s(a)}{\dot{a}a} da = - \int_{\infty}^{z^*} \frac{c_s(z)a^2}{\dot{a}a} dz$$



$$\frac{dz}{da} = \frac{d(1/a)}{da} = -\frac{1}{a^2}$$

The baryon acoustic oscillations (BAO)

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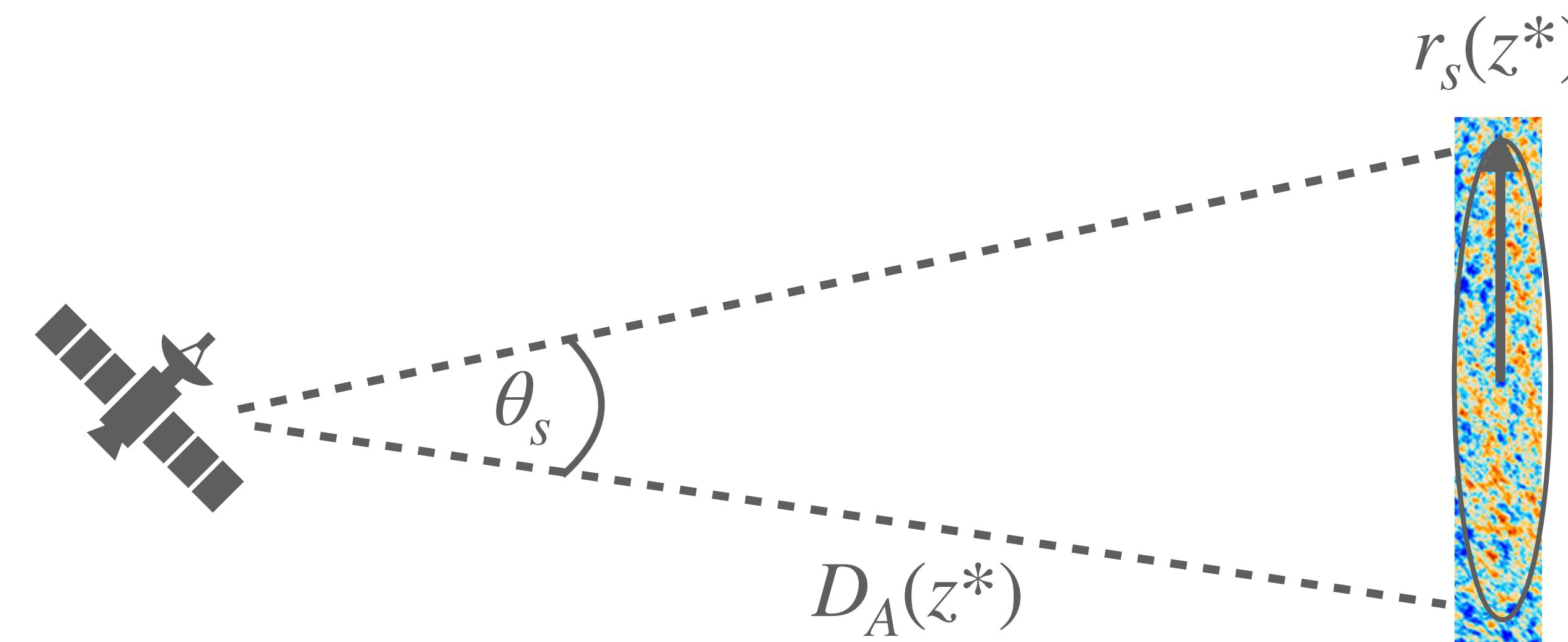
- The sound speed can be expressed as (e.g. Dodelson&Schmidt, 2020):

$$c_s(z) = \frac{1}{\sqrt{3(1 + R(z))}}$$

- With R being the baryon-photon ratio:

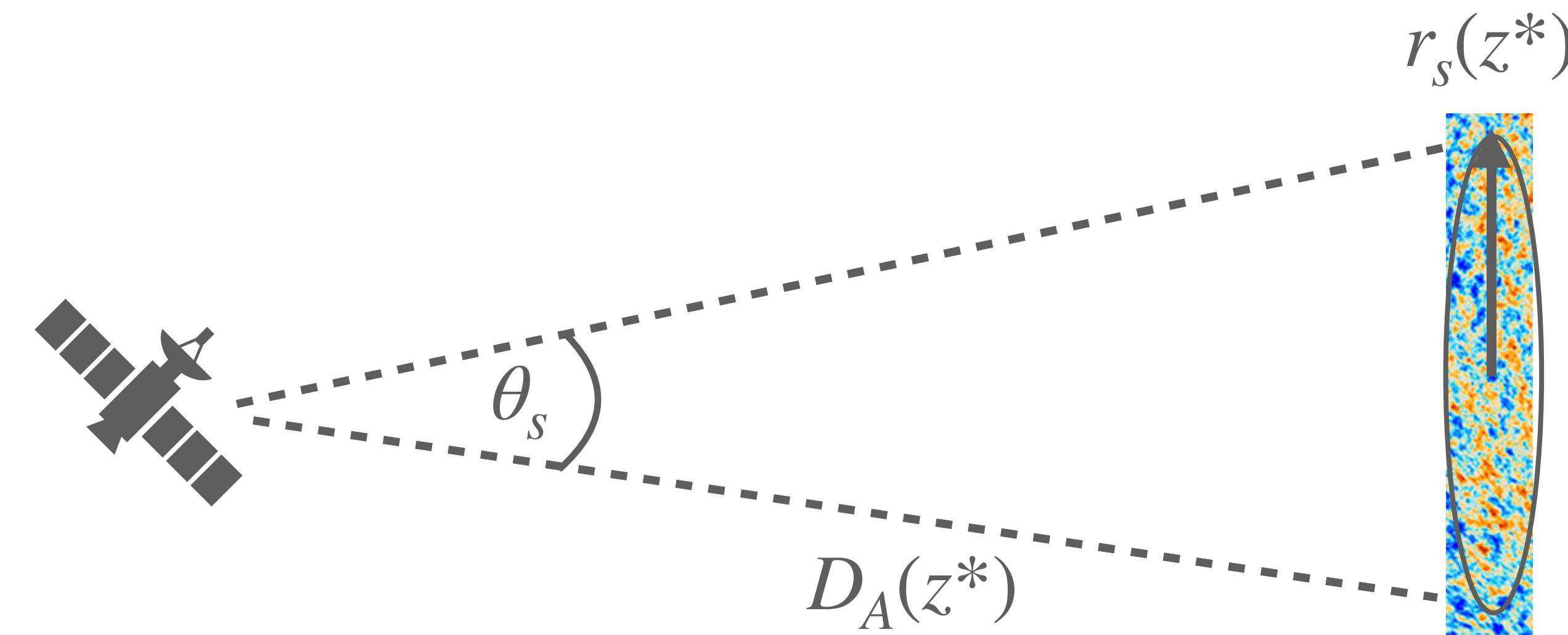
$$R(z) = \frac{3\omega_b}{4\omega_\gamma} \frac{1}{1+z}$$

The baryon acoustic oscillations (BAO)



- What we observe is the *angular size of the sound horizon* θ_s

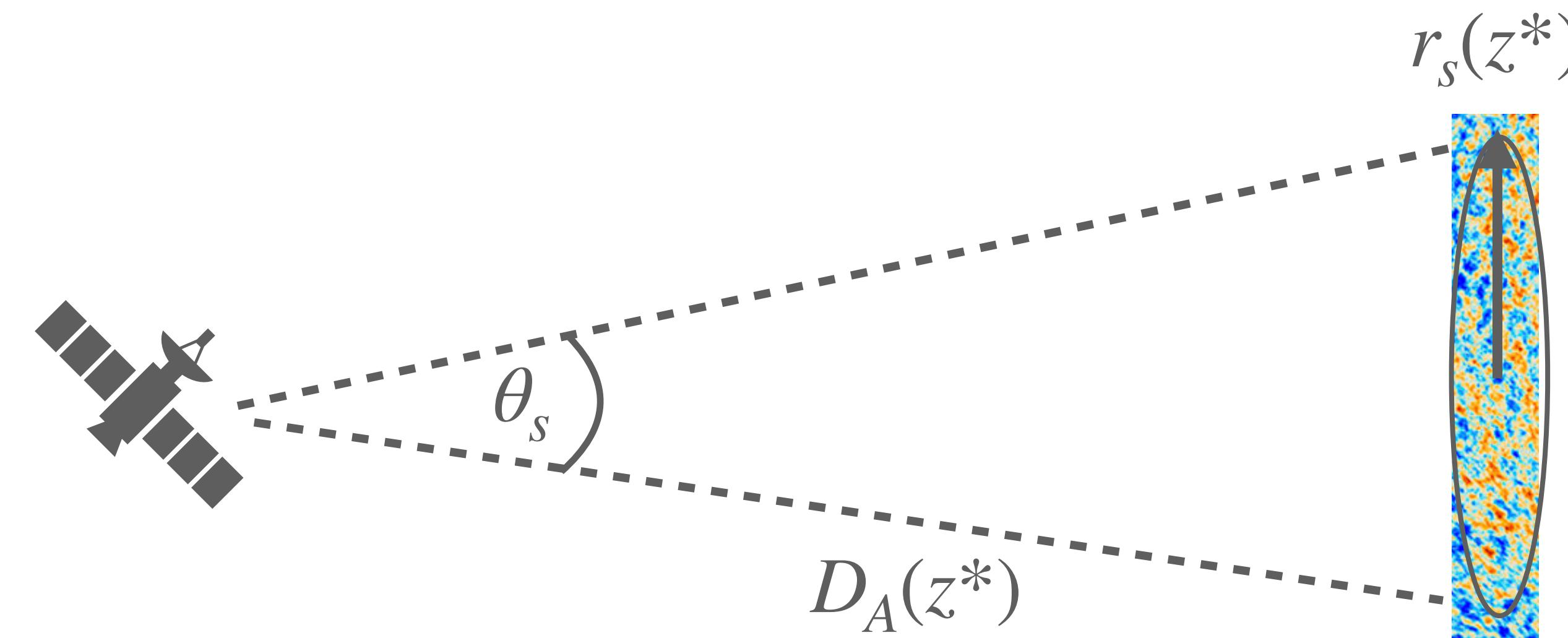
The baryon acoustic oscillations (BAO)



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- In the small-angle approximation:

$$\theta_s = \frac{r_s(z^*)}{D_A(z^*)}$$

The baryon acoustic oscillations (BAO)



- What we observe is the *angular size of the sound horizon* θ_s
- In the small-angle approximation:
- where the angular diameter distance is (see slide earlier):

$$D_A(t) = \int_0^{z(t)} \frac{dz'}{H(z')}$$

How does the CMB constrain H_0 ?

- We know that the CMB directly constrains

$$\theta_s = \frac{a(z^*) r_s(z^*)}{D_A(z^*)} = \frac{\int_{z^*}^{\infty} c_s(z) dz / H(z)}{\int_0^{z^*} dz / H(z)}$$

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$$H(z) = H_0 \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$

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- If we get Ω_r , Ω_m , Ω_Λ from somewhere else, this becomes an implicit equation for H_0 – How does the CMB constrain Ω_r , Ω_m , Ω_Λ ?

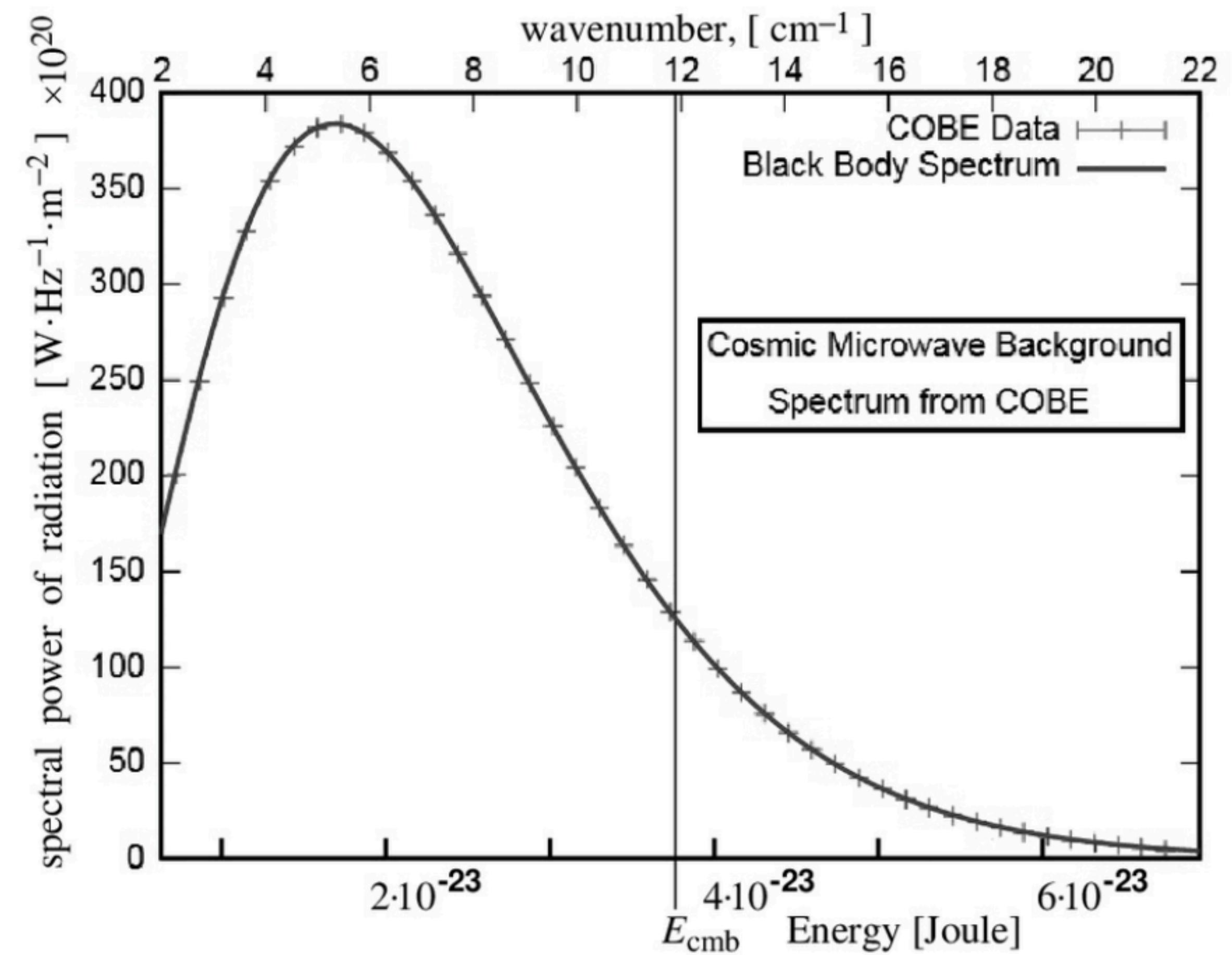
How does the CMB constrain Ω_r ?

- The radiation density Ω_r is precisely measured by the CMB temperature

$$T = 2.725 \pm 0.002 \text{ K} \text{ (COBE - FIRAS measurement)}$$



Credit: Berkeley Center for Cosmological Physics



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- For a black-body spectrum, the energy density in radiation is then given by (e.g. Weinberg 2008, Ch. 2.1):

$$\rho_{0,\text{CMB}} = \int_0^{\infty} h\nu \cdot n(\nu) d\nu = \frac{8\pi^5 k_B^4}{15h^3 c^3} T^4 = 4.64 \cdot 10^{-34} \text{ g/cm}^3$$

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- Taking into account the earlier time of decoupling of neutrinos, one finds:

$$\rho_\nu = 0.4 \rho_{0,\text{CMB}}$$

- Since the CMB and neutrinos are by far the dominant contribution:

$$\Omega_r = 1.4 \rho_{0,\text{CMB}} / \rho_{\text{crit}} = 4.15 \cdot 10^{-5} h^{-2}$$

For more about neutrinos, see Olga Mena's lecture

How does the CMB constrain Ω_m ?

- It does not constrain Ω_m , but $\omega_m = h^2\Omega_m$, where $\omega_m = \omega_{\text{cdm}} + \omega_b$

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- It does not constrain Ω_m , but $\omega_m = h^2 \Omega_m$, where $\omega_m = \omega_{\text{cdm}} + \omega_b$
- ω_m is constrained by the height of the acoustic peaks:
 - a smaller ω_m leads to a later time of matter-radiation equality
 - this leads to a stronger decay of the gravitational potentials at recombination → **early integrated Sachs-Wolfe effect (eISW)**
 - Since the eISW adds in phase with the BAO, this leads to a boost of all peaks, particularly the first peak.

How does the CMB constrain Ω_Λ ?

- The sum of the energy densities has to satisfy:

$$\Omega_r + \Omega_m + \Omega_\Lambda = \Omega_k$$

- For a flat universe:

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1$$

- Hence, if we know Ω_r and $\omega_m = h^2\Omega_m$, we can compute

$$\Omega_\Lambda = 1 - \Omega_r - \omega_m/h^2$$

How does the CMB constrain H_0 ?

- We know that the CMB directly constrains

$$\theta_s = \frac{a(z^*) r_s(z^*)}{D_A(z^*)} = \frac{\int_{z^*}^{\infty} c_s(z) dz / H(z)}{\int_0^{z^*} dz / H(z)}$$

- Cosmology enters mainly over the Hubble parameter here:

$$H(z) = H_0 \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$

- Since we can constrain Ω_r and ω_m independently, this becomes an implicit equation for h

$$\frac{H^2(z)}{(100 \text{ km/s/Mpc})^2} = h^2 \Omega_r (1+z)^4 + \omega_m (1+z)^3 + (h^2 - \omega_m - h^2 \Omega_r)$$

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* $c_s(z)$ and z^* also depend on cosmology but we neglect that here

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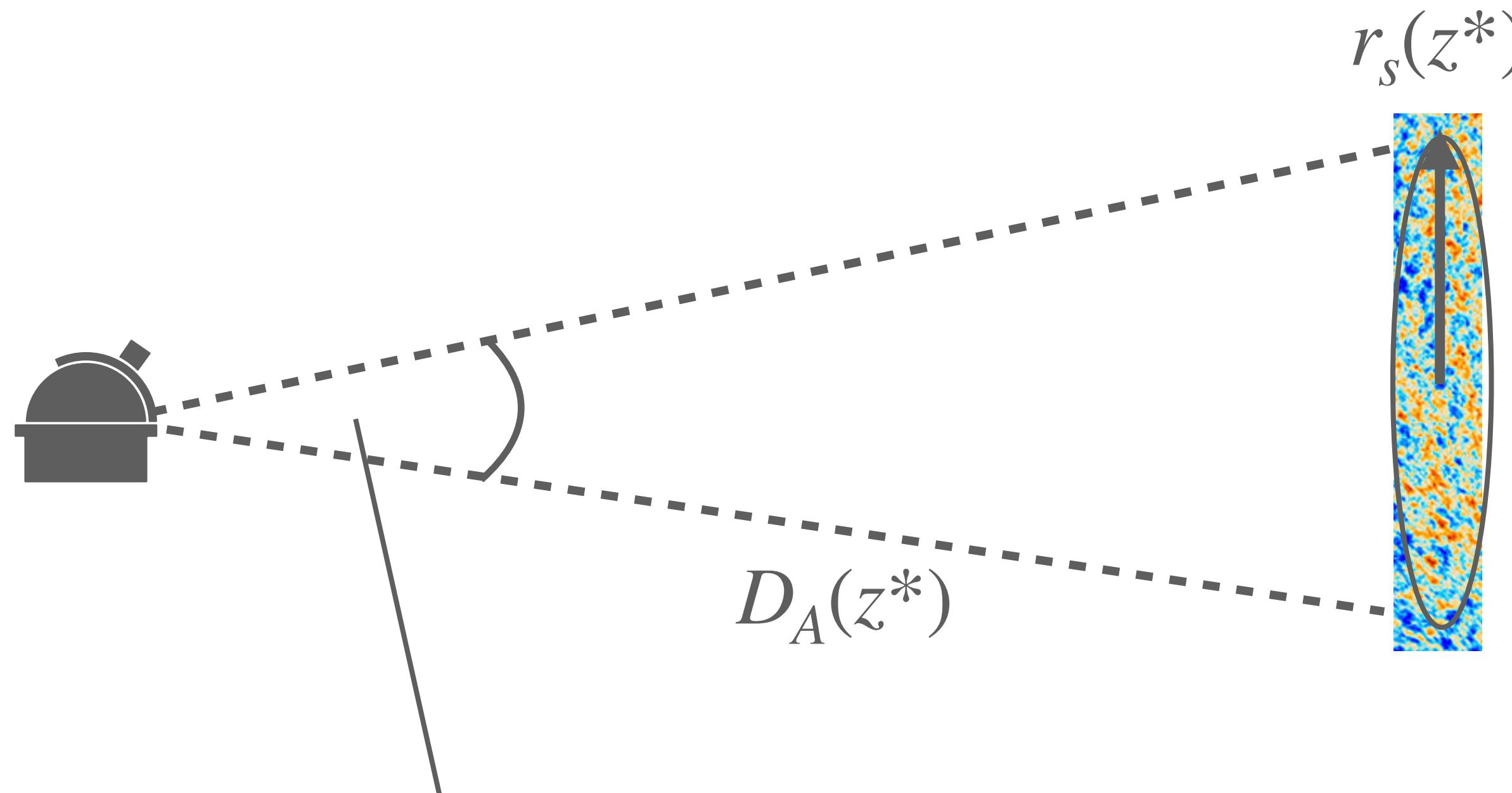
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Solutions to the Hubble tension

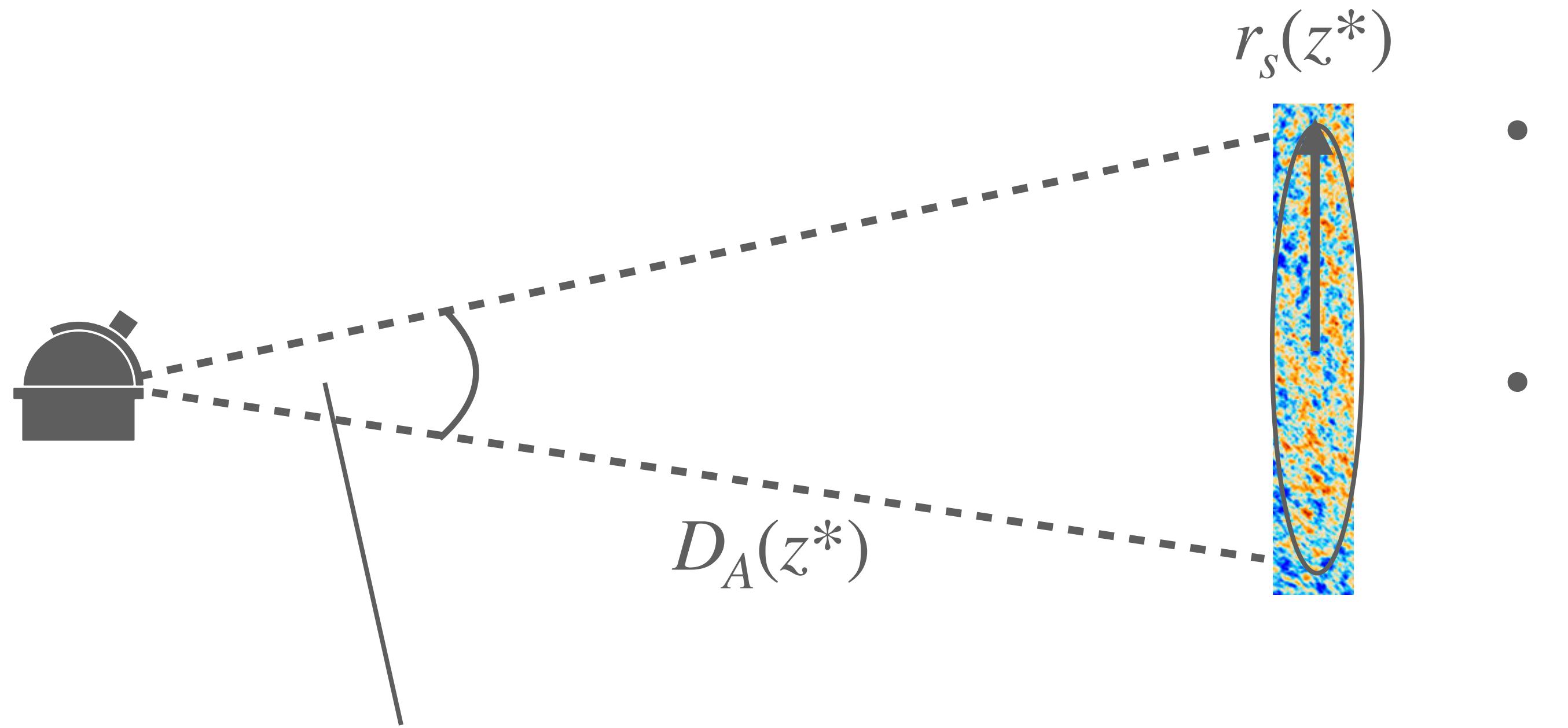
Solutions to the Hubble tension



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- θ_s is measured precisely by CMB
 $\rightarrow \theta_s$ fixed
- Two options to solve the Hubble tension:

Solutions to the Hubble tension

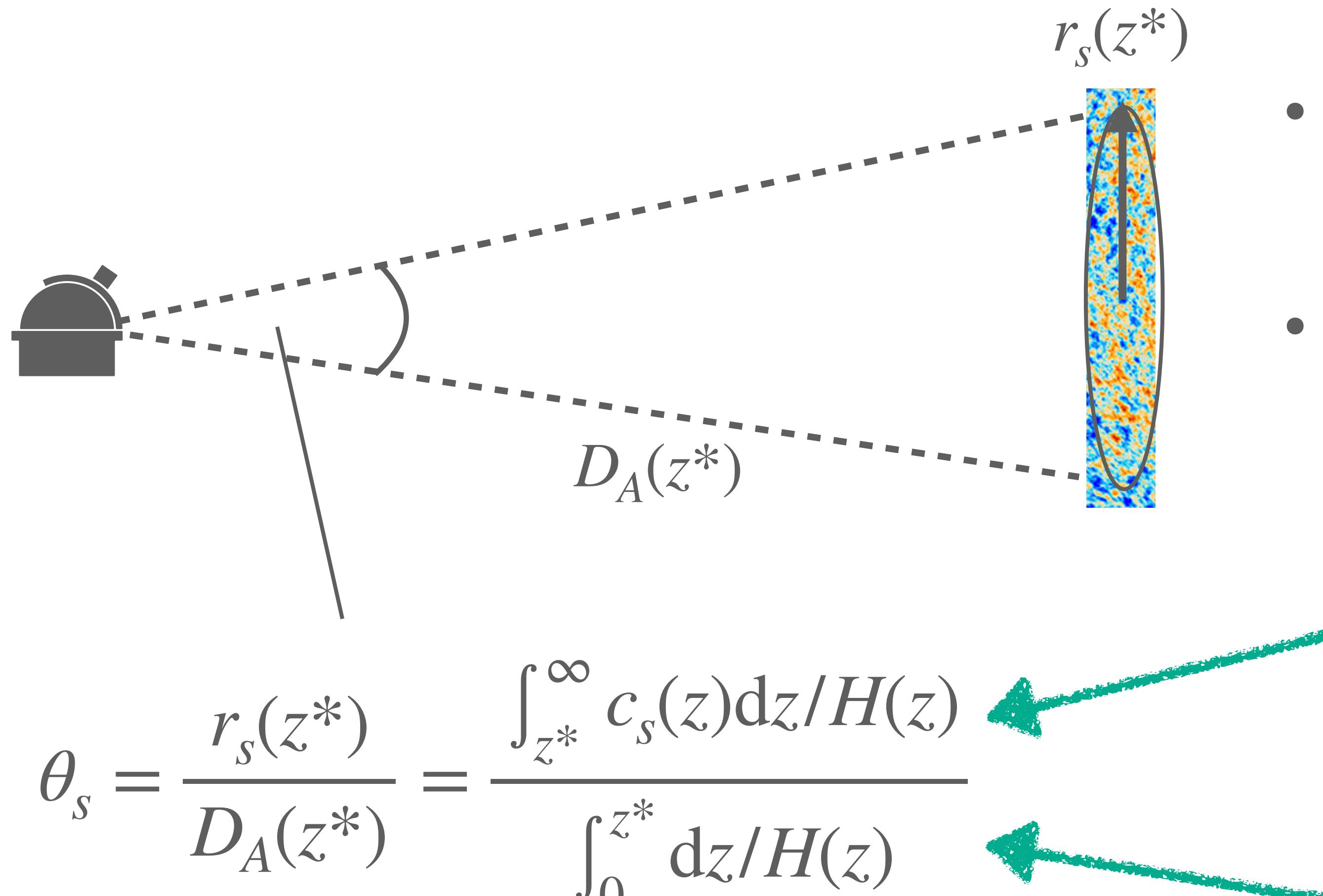


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Early-time solutions
modify r_s

Solutions to the Hubble tension



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Early-time solutions
modify r_s

Late-time solutions
modify D_A

Late-time solutions

- Modify $D_A(z^*) = \int_0^{z^*} \frac{dz}{H(z)}$ by modifying the expansion rate between today (0) and recombination z^* (since $\Omega_r \approx 0$ at times after z^*):

$$H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}$$

Late-time solutions

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$$H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}$$

- However, $H(z)$ is well constrained by galaxy BAO data and supernova data
→ There is not enough wiggle-room to increase H_0 enough
- Late-time solutions are somewhat disfavoured

Early-time solutions

- Modify $r_s = \int_{z^*}^{\infty} \frac{c_s(z) dz}{H(z)} \rightarrow$ 3 options:
 - ▶ modify z^*
 - ▶ modify $c_s(z)$
 - ▶ modify $H(z)$

Early-time solutions

- Modify $r_s = \int_{z^*}^{\infty} \frac{c_s(z) dz}{H(z)} \rightarrow 3 \text{ options:}$

► **modify z^***



- modify $c_s(z)$
- modify $H(z)$

E.g. by modifying the mass of the electron m_e :

- This shifts the energy levels of the atoms,
- and changes the ionization energy,
- which changes the time of recombination z^*

Early-time solutions

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 - ▶ **modify $H(z)$** →

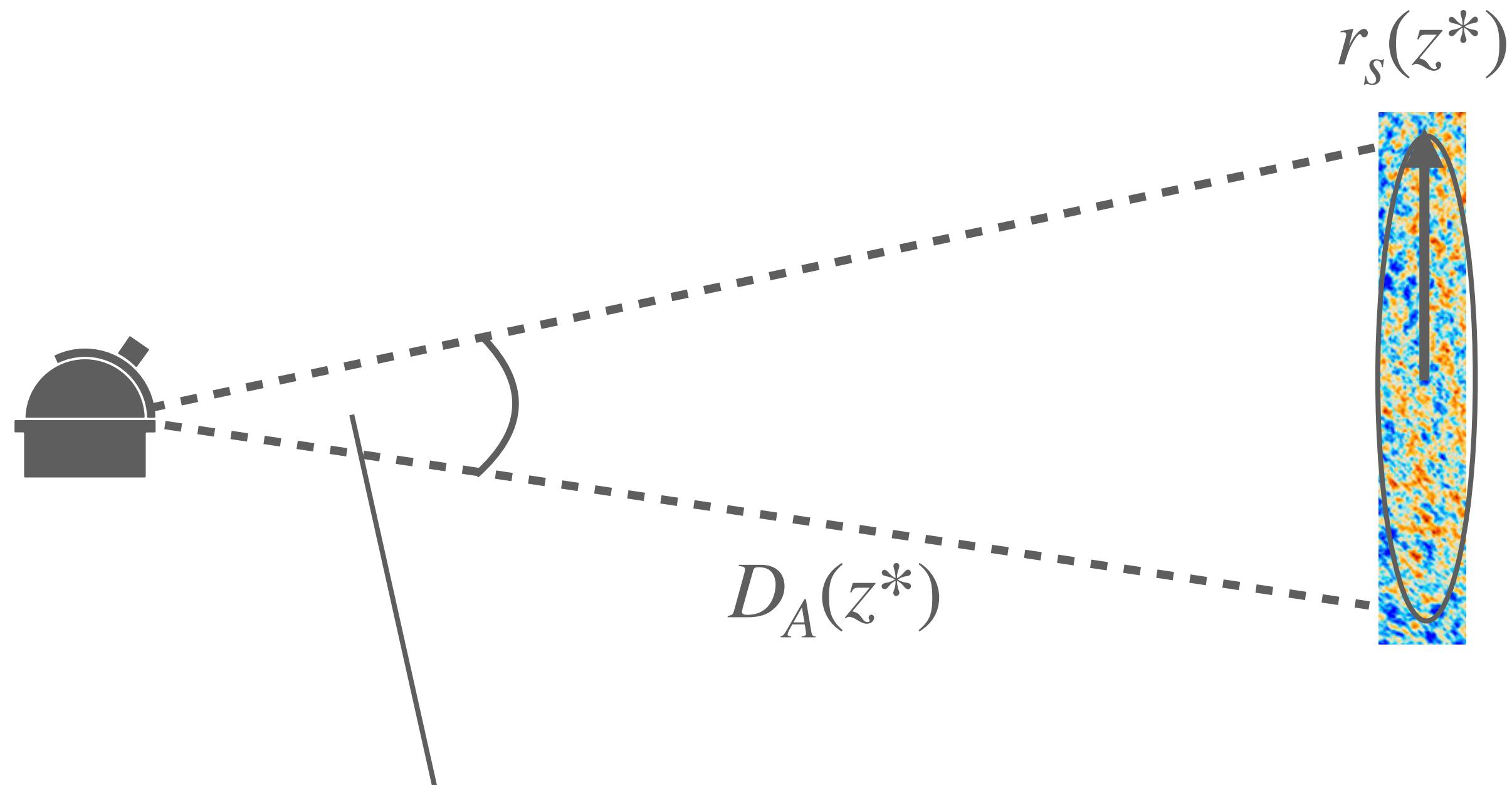
E.g. by introducing an energy density, which boosts the expansion rate before recombination z^* (at early times $\Omega_\Lambda \approx 0$):

$$H(z) = H_0 \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_q}$$

This leads to a smaller r_s .

The most successful of such ideas is **EDE**

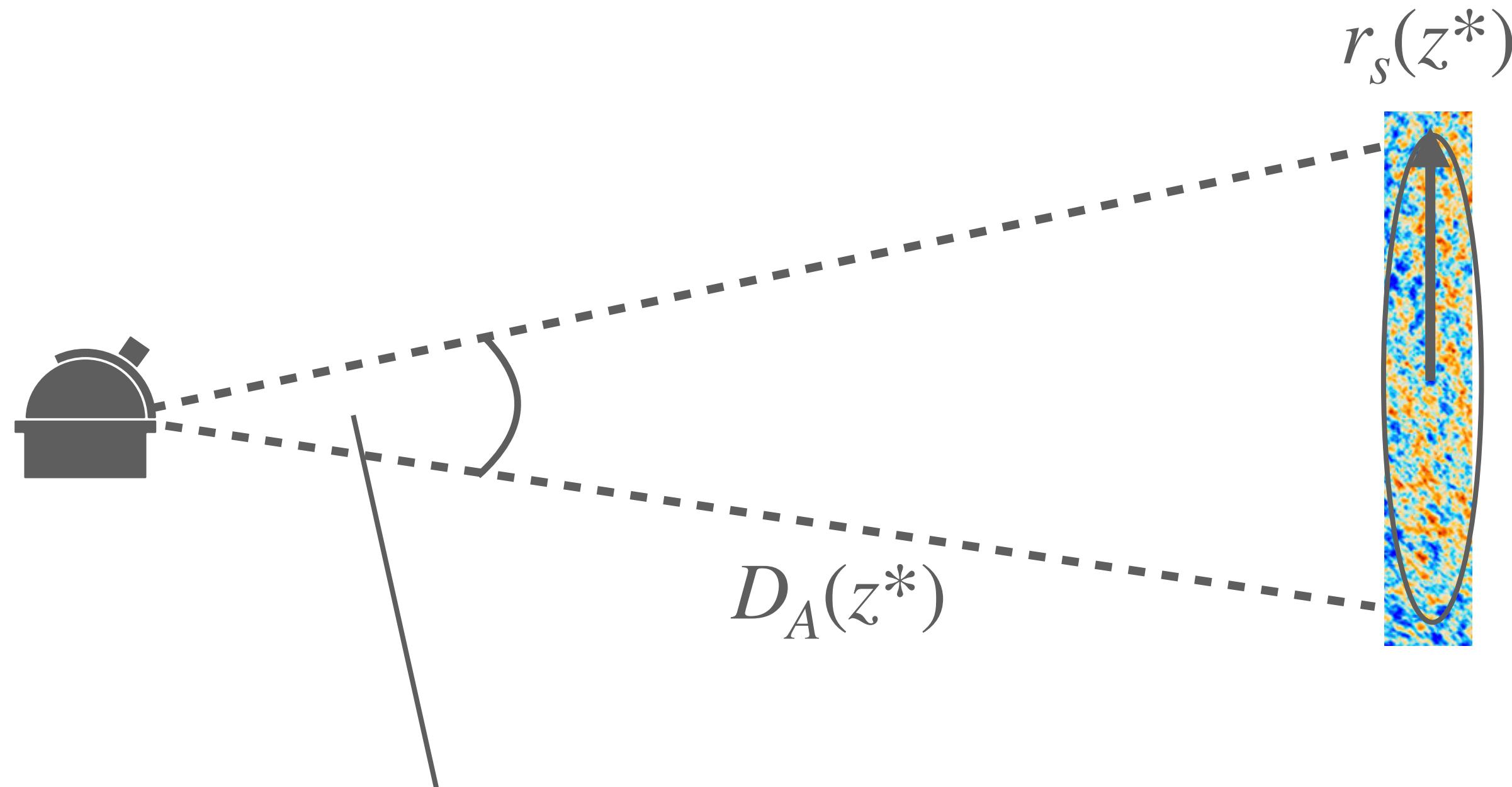
Idea behind Early Dark Energy



Angular scale of sound horizon θ_s
measured with 0.03% precision by *Planck*.

$$\theta_s = \frac{r_s(z^*)}{D_A(z^*)} = \frac{\int_{z^*}^{\infty} c_s(z) dz / H(z)}{\int_0^{z^*} dz / H(z)}$$

Idea behind Early Dark Energy

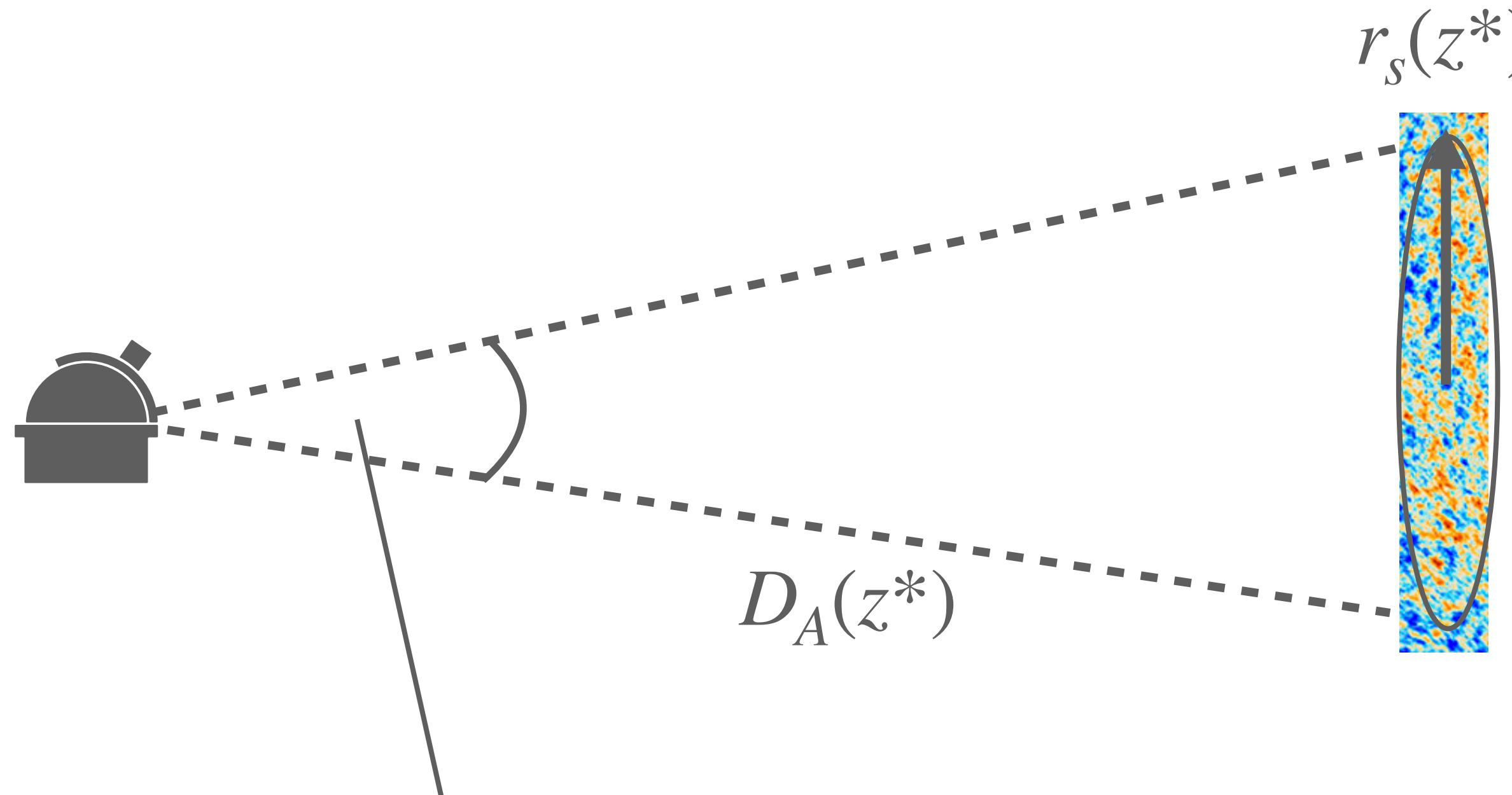


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Introducing an additional component
before recombination reduces r_s .

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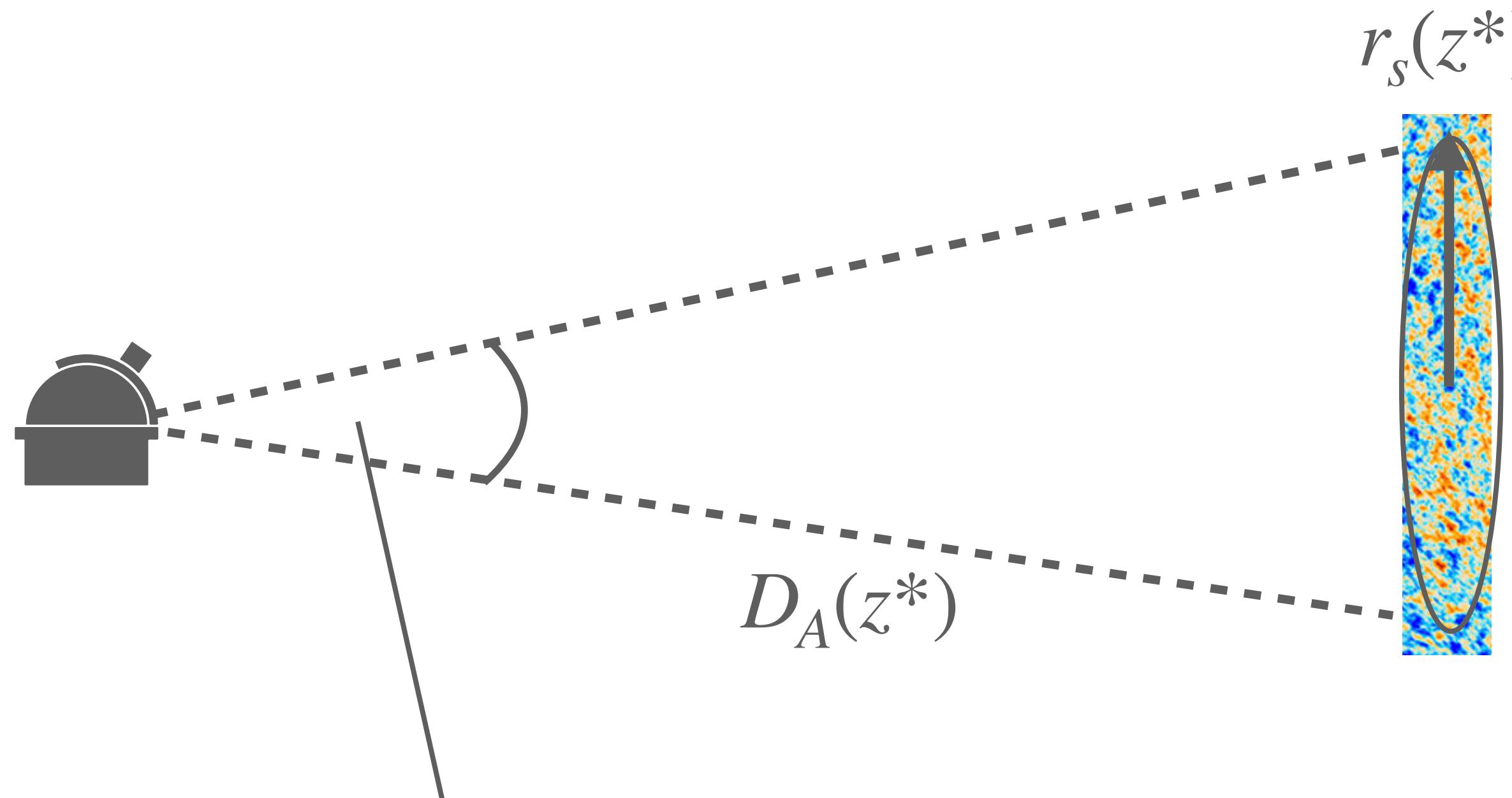
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θ_s fixed

Angular diameter distance
 D_A decreases.

Idea behind Early Dark Energy



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Introducing an additional component before recombination reduces r_s .

θ_s fixed

Angular diameter distance D_A decreases.

$H(z) = H_0 \sqrt{\Omega_m(z) + \Omega_r(z) + \Omega_\Lambda}$

H_0 increases.

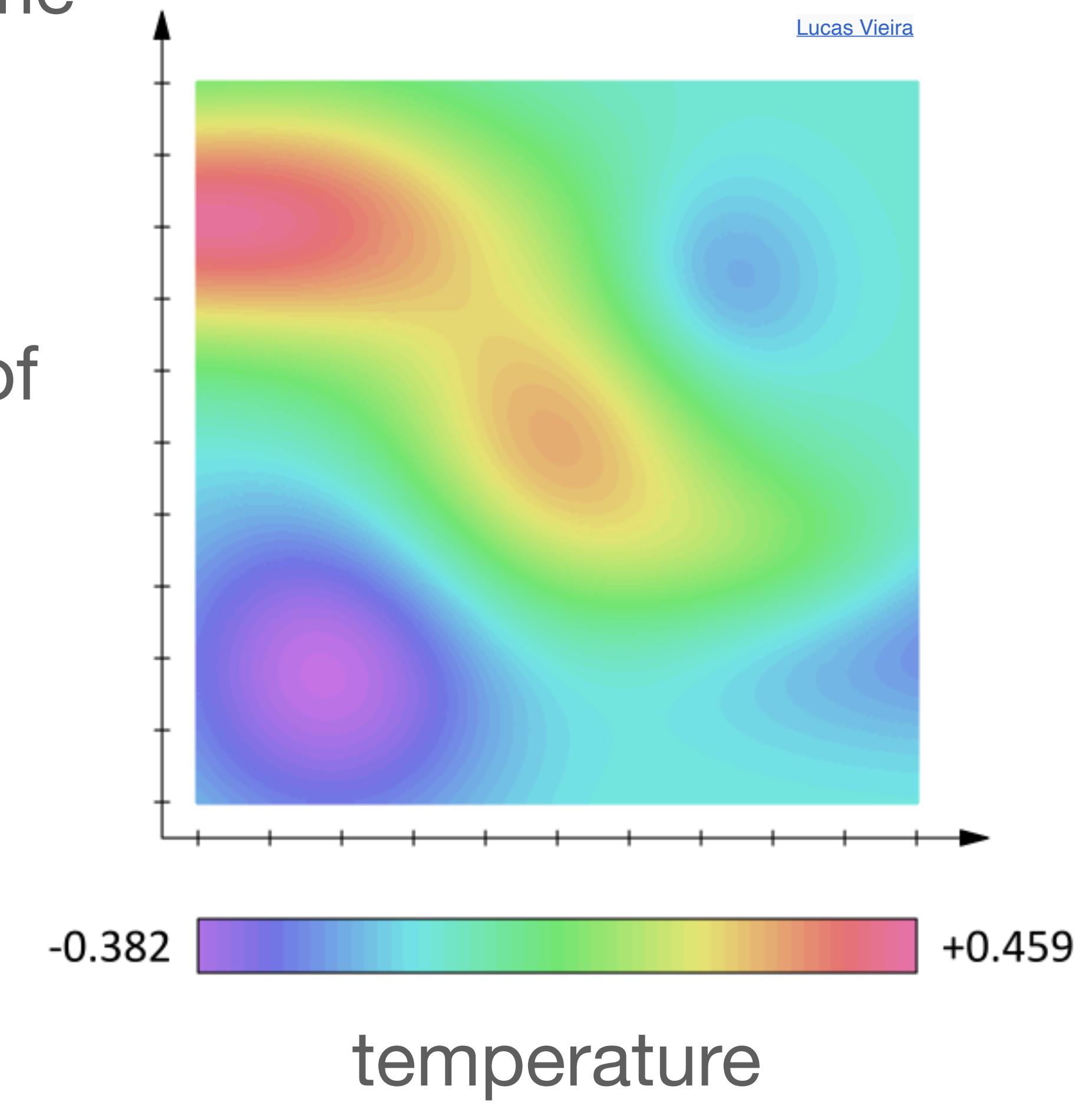
Short history of Early Dark Energy

- EDE has already been studied before the Hubble tension emerged in the context of quintessence models (Doran++ 2001, Wetterich++ 2004, Doran&Robbers 2006, Kamionkowski++ 2014)
- The axion-like EDE was proposed as a solution to the Hubble tension (Karwal&Kamionkowski 2016, Poulin++ 2018, 2019)
- There are many versions of the EDE model, but we will focus on the (most commonly studied) axion-like EDE model
- The axion-like EDE model is modelled as a **scalar field**
- Here: Only background equations

Scalar fields in an expanding spacetime

Intuition

- A scalar field $\phi(t, \vec{x})$ assigns each point in spacetime a single number
- Examples
 - Temperature, density, pressure,... as a function of position and time
 - Potential fields like the gravitational potential, electric potential
 - In quantum field theory, the Higgs particle is described as a scalar field, and the pions
 - The inflaton driving inflation is most commonly a scalar field



Lagrangian

- The Lagrangian of a scalar field minimally coupled to the metric is:

$$\mathcal{S}_\phi = \int d^4x \sqrt{|\det(g)|} \left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \right]$$

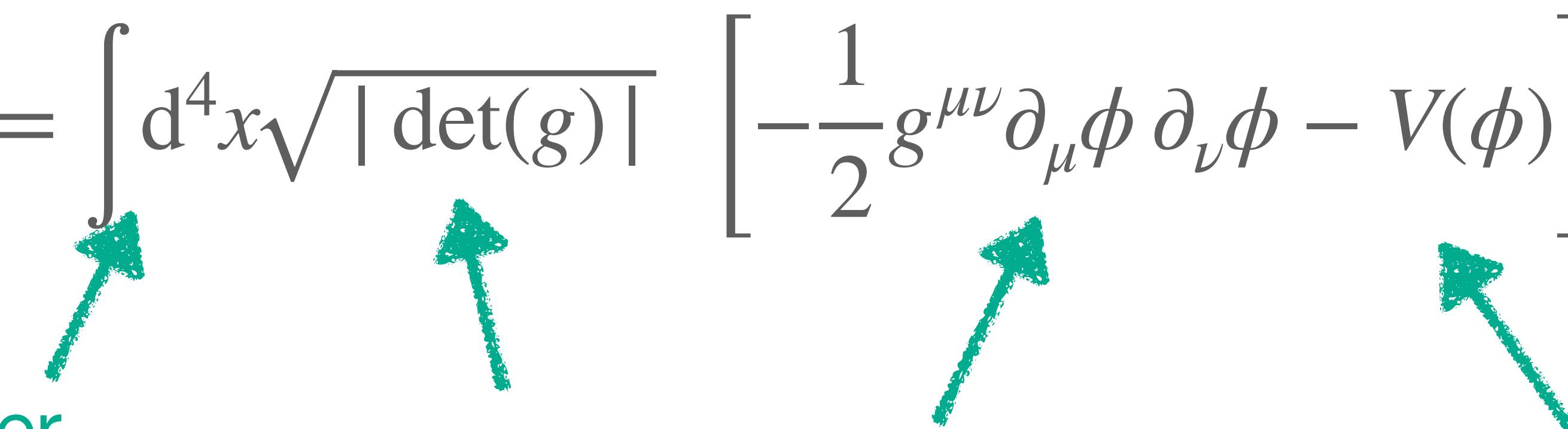
integral over whole spacetime determinant of metric kinetic term potential term

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integral over whole spacetime determinant of metric kinetic term potential term



- One can compute the energy momentum tensor of the scalar field via:

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu} \left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi) \right)$$

Lagrangian

- This gives for the energy-momentum tensor:

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - V(\phi) \right) = \frac{\dot{\phi}^2}{2} \delta_\mu^0 \delta_\nu^0 + \left[\frac{\dot{\phi}^2}{2} - V(\phi) \right] g_{\mu\nu}$$

Lagrangian

- This gives for the energy-momentum tensor:

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- By comparing $T_{\mu\nu}$ to the energy momentum tensor of the perfect fluid:
 $T_{\mu\nu} = (\rho + p)\delta_\mu^0 \delta_\nu^0 + pg_{\mu\nu}$, one can read off the energy density and pressure of the scalar field:

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi).$$

Lagrangian

- Inserting $\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$, $p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$ into the first Friedmann equation and the continuity equation yields:

Lagrangian

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$$H^2 = \frac{8\pi G_N}{3} \left(\frac{\dot{\phi}^2}{2} + V(\phi) \right)$$
$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

“Hubble friction” Potential term

Lagrangian

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$$\boxed{H^2 = \frac{8\pi G_N}{3} \left(\frac{\dot{\phi}^2}{2} + V(\phi) \right)}$$
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- The second equation is the **Klein-Gordon equation** for a scalar field in an expanding space, where the second term ($3H\dot{\phi}$) is the Hubble-drag term and the third term ($\frac{1}{2}dV/d\phi$) is the potential-gradient term

Early Dark Energy

Early Dark Energy

- EDE is a scalar field ϕ in a potential $V(\phi)$, which boosts the expansion rate before recombination

Early Dark Energy

- EDE is a scalar field ϕ in a potential $V(\phi)$, which boosts the expansion rate before recombination
- Potential motivated by “axion”:

$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$

normalization of the potential

decay constant

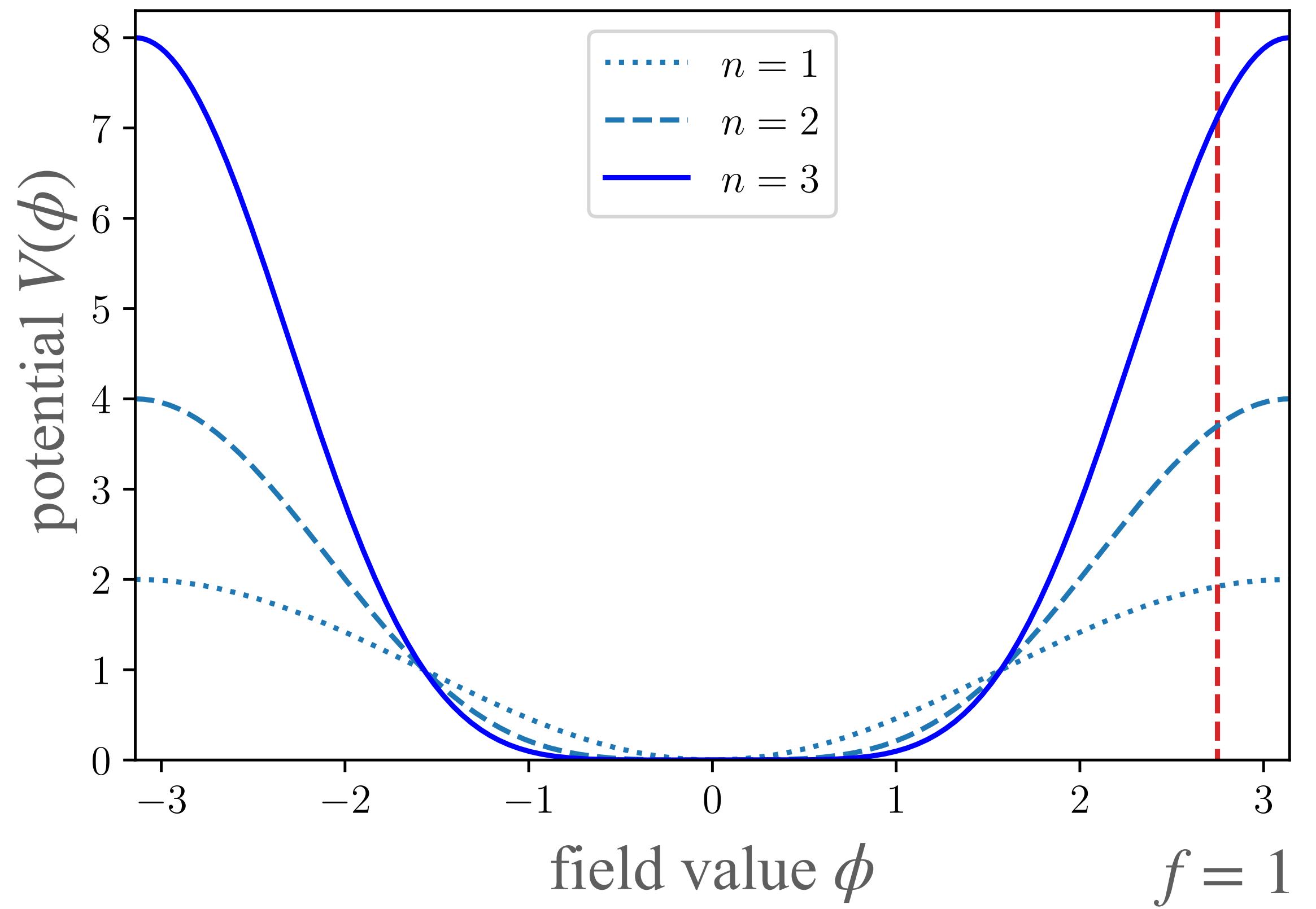
index

The diagram shows the potential function $V(\phi) = V_0 [1 - \cos(\phi/f)]^n$. Three green arrows originate from the text labels below and point to specific parts of the equation: one arrow points to V_0 with the label "normalization of the potential", another points to f with the label "decay constant", and a third points to n with the label "index".

- Explain axion, f, m, etc

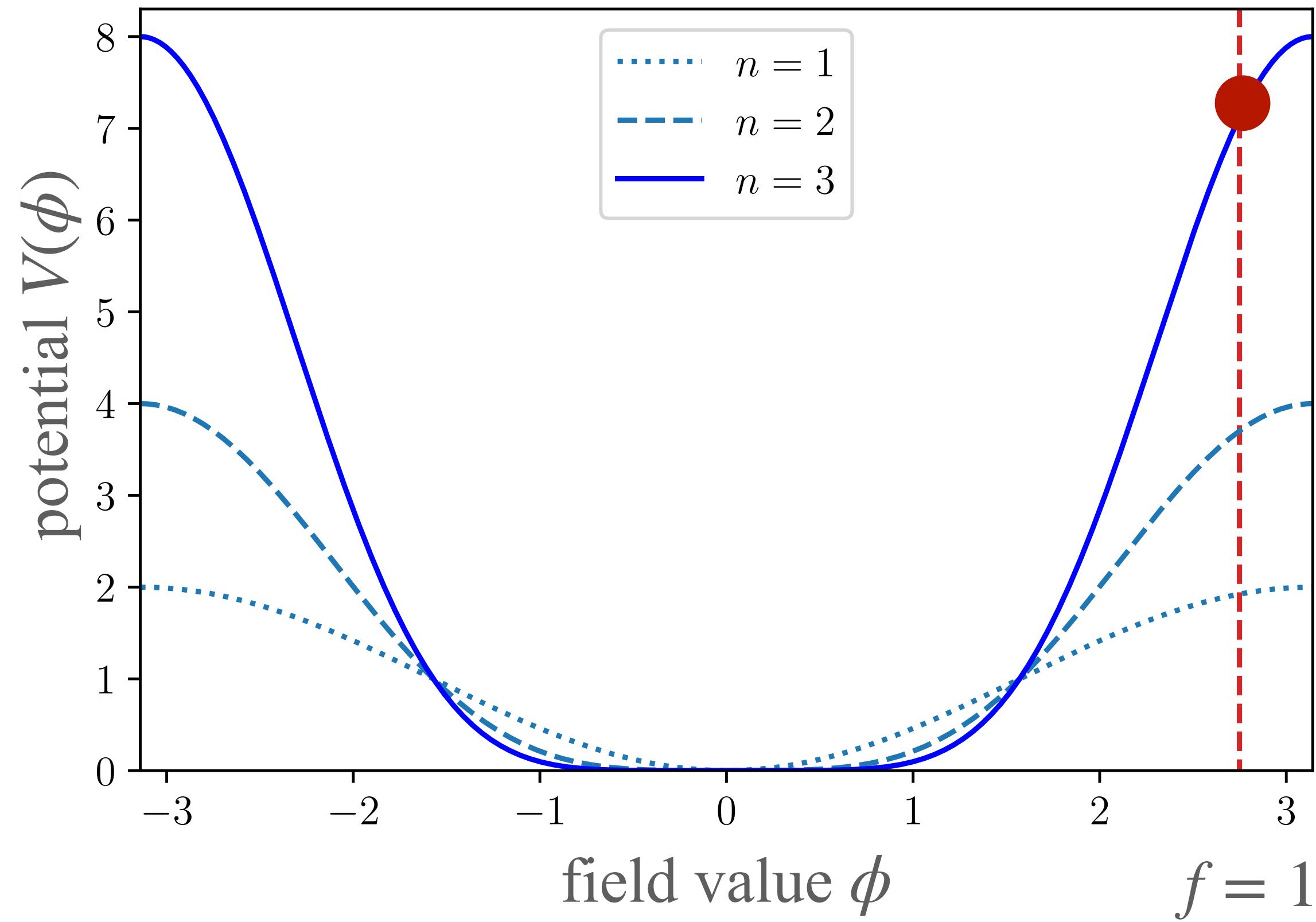
Early Dark Energy

$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$



Early Dark Energy

$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$



- Dynamics governed by Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} + \cancel{\frac{dV}{d\phi}} = 0$$

- ϕ starts high up in the potential, where the potential is very flat:

$$\frac{dV}{d\phi} \approx 0$$

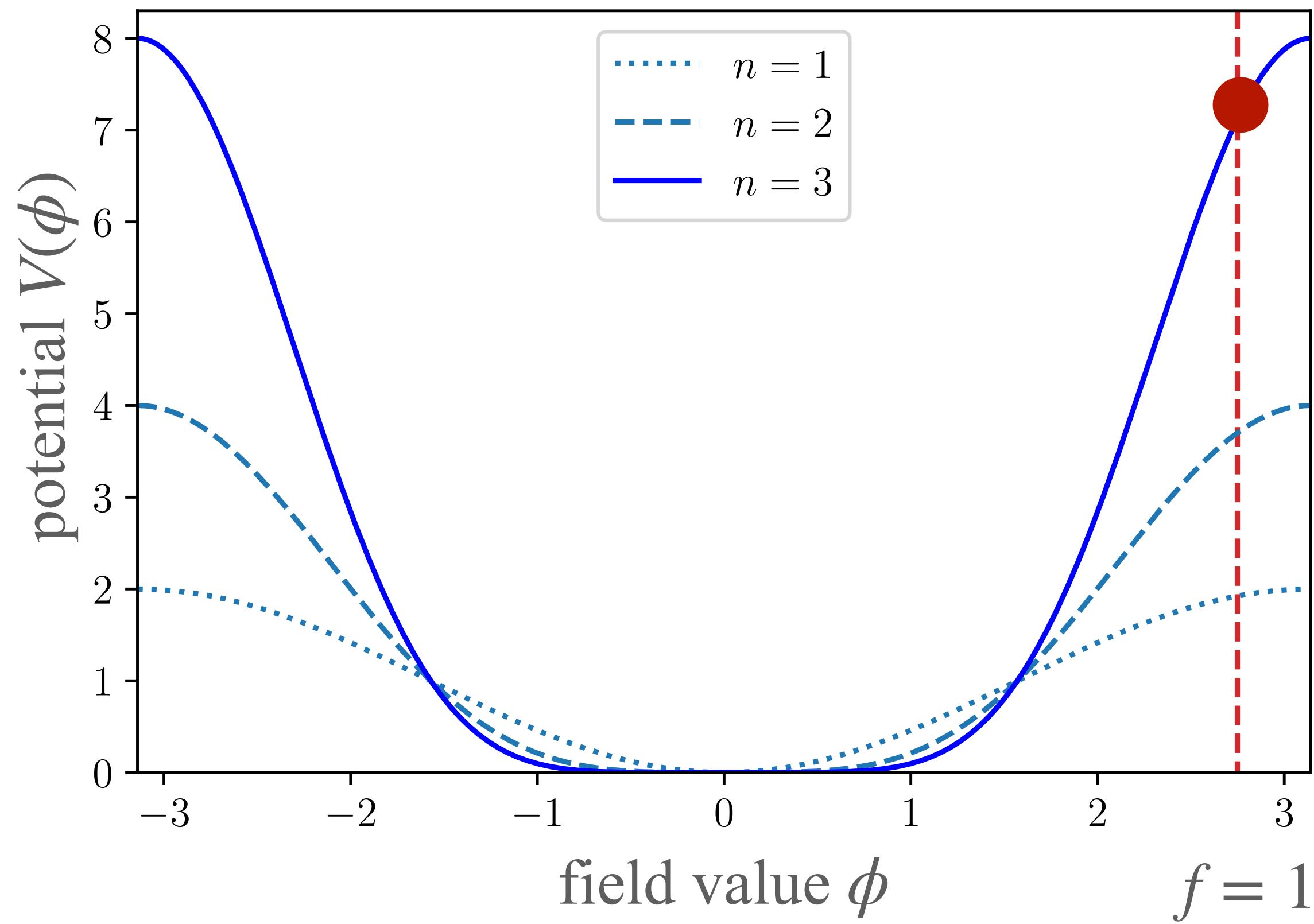
- Moreover, at early times the expansion rate H is large; hence “Hubble friction” dominates:

$$3H\dot{\phi} \gg \frac{dV}{d\phi}$$

- Both of these factors “freeze” the scalar field in place: $\dot{\phi} \approx 0$

Early Dark Energy

$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$



- At early times: $\dot{\phi} \approx 0$
- This means for the energy density & pressure

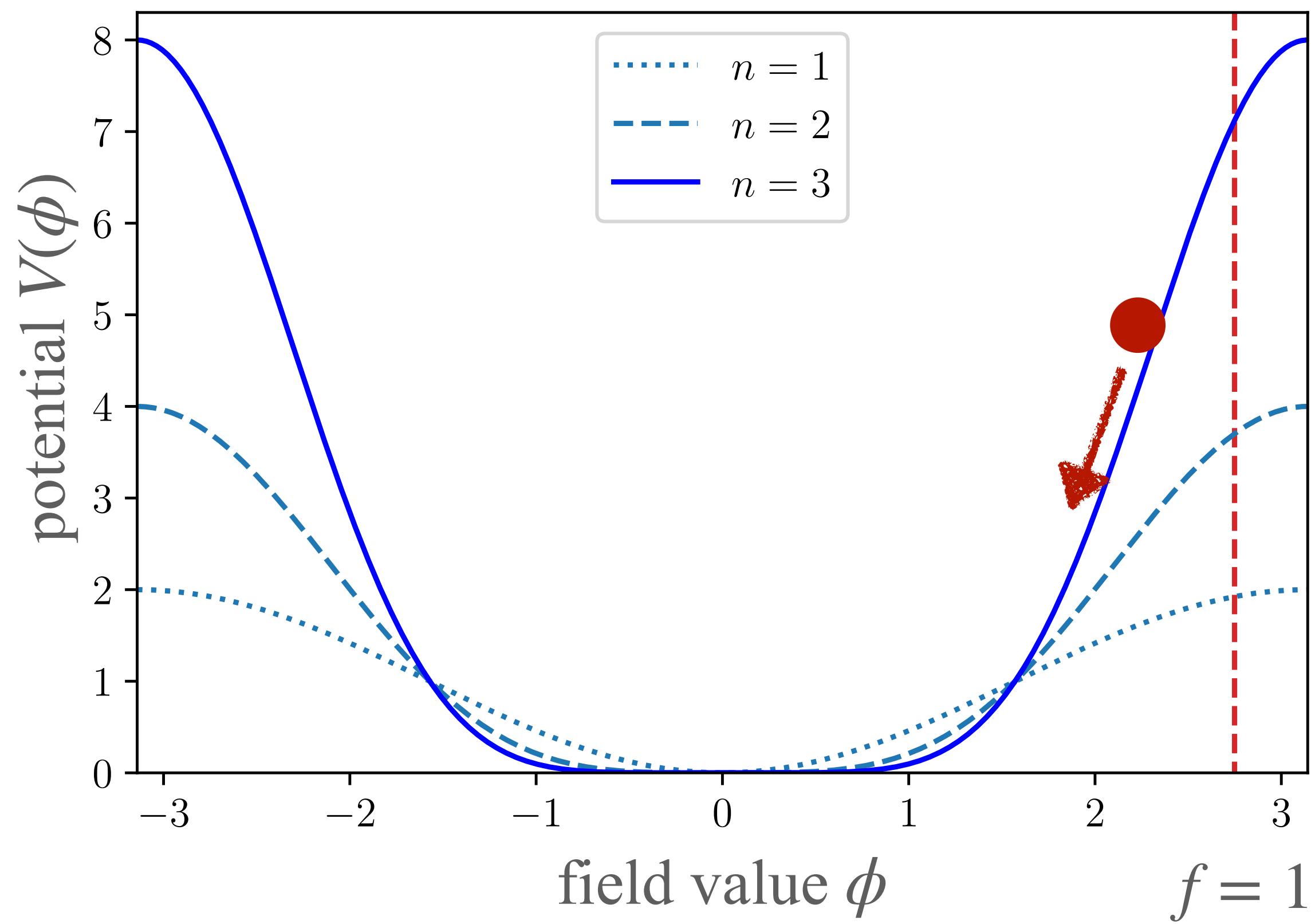
$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi).$$

- The equation of state becomes:
- Hence, at early times the field behaves like dark energy \rightarrow EDE

$$w = \frac{p_\phi}{\rho_\phi} \approx -1$$

Early Dark Energy

$$V(\phi) = V_0 [1 - \cos(\phi/f)]^n$$



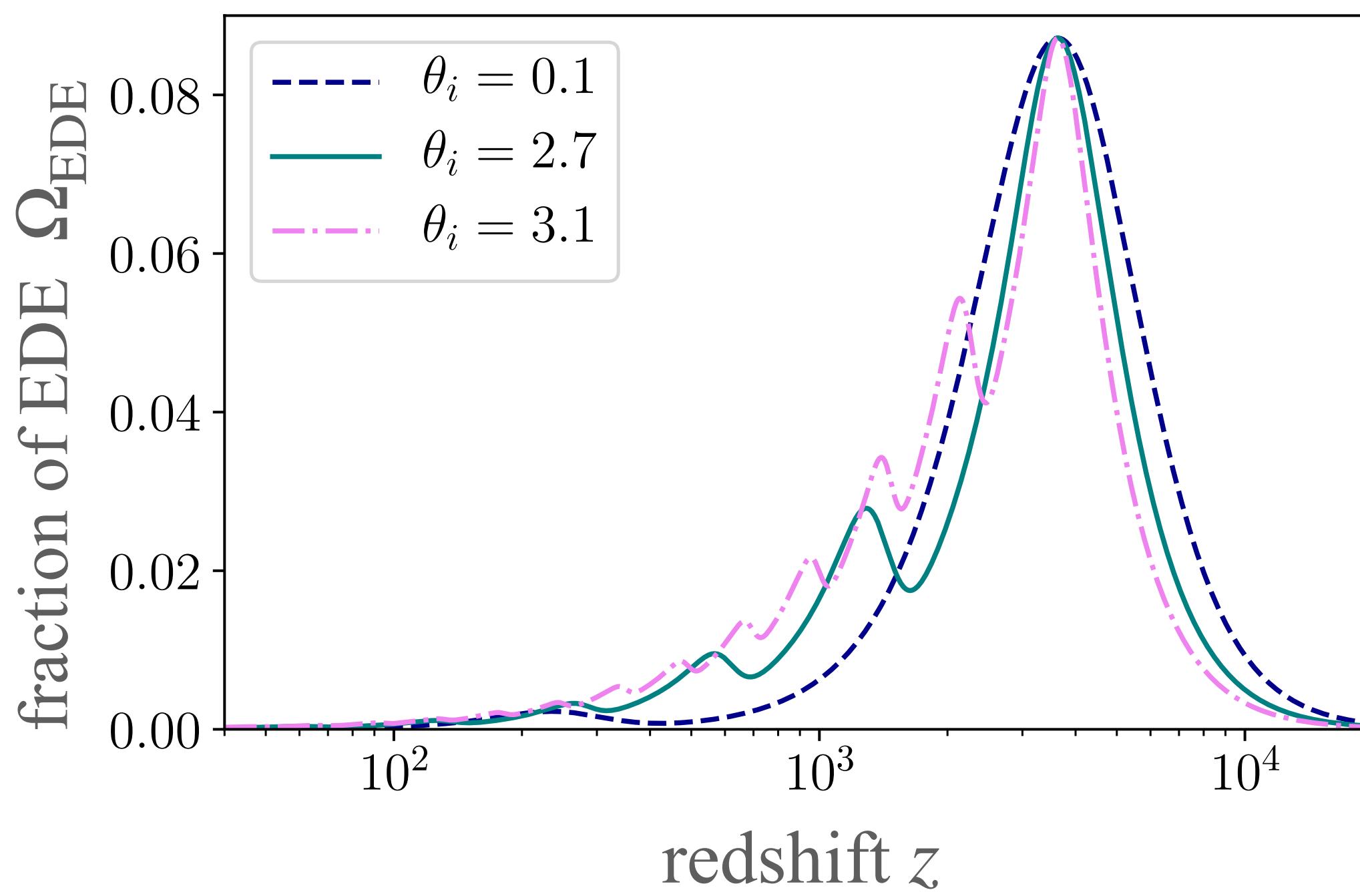
$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

- As time passes, $H(z)$ decreases, until at some critical redshift z_c :
- $3H\dot{\phi} \ll \frac{dV}{d\phi}$
- In this phase, the field oscillates around the minimum with equation of state

$$\langle w \rangle = \frac{n-1}{n+1} = \frac{1}{2}$$

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- Think about slow-roll phase more
- How to get equation of state in oscillating phase
- Add description about m , f

Recap Lecture 1

- Add recap
- Add sources