From ΛCDM to EDE

Lecture 2: Observation

Laura Herold

CosmoVerse@Corfu May 13 - 18, 2024

Overview

Lecture 1: Theory from ΛCDM to EDE (1:30h)

Hands-on session 1: From theory to predictions (1h)

Lecture 2: Observation — Can EDE solve the Hubble tension? (1h)

Hands-on session 2: Let's analyse EDE with the cosmic microwave background and supernovae (1:30h)

Outline: Can EDE solve the Hubble tension?

1. Overview over statistical tools
2. Recap EDE
3. Can EDE resolve the H_0 tension? — Review of constraints from the literature
4. Conclusions

Statistical tools

Overview Bayesian & frequentist statistics

Statistical tools

- I will only give a very short overview here about:
  - Point estimates
  - Bayesian and frequentist parameter intervals
- Matteo Martinelli will go into details about Bayesian statistics and MCMC, so stay tuned for the afternoon lecture!

The likelihood

- The central quantity in both Bayesian & frequentist statistics is the likelihood:

$$\mathscr{L}(\boldsymbol{x}\,|\,\boldsymbol{\mu}(\boldsymbol{\theta})) \equiv \mathscr{L}_{\boldsymbol{x}}(\boldsymbol{\theta})$$

- "Probability of the data $\boldsymbol{x}$ given the model $\boldsymbol{\mu}$ with parameter(s) $\boldsymbol{\theta}$"
- The data $\boldsymbol{x}$, model $\boldsymbol{\mu}$, parameter(s) $\boldsymbol{\theta}$ can be a single number or a vector
- The likelihood is typically computed for a fixed (measured or simulated) data set, hence $\boldsymbol{x}$ is written as a subscript or dropped
- The larger $\mathscr{L}_{\boldsymbol{x}}(\boldsymbol{\theta})$ the better the agreement of the model with the data

Gaussian likelihood

- One very common likelihood is the Gaussian likelihood

$$\mathscr{L}_{\boldsymbol{x}}(\boldsymbol{\mu}(\boldsymbol{\theta})) = \frac{1}{\sqrt{2\pi\,|C|}} \exp\left[\sum_{i,j} [x^i - \mu^i(\boldsymbol{\theta})]^T C_{ij}^{-1} [x^j - \mu^j(\boldsymbol{\theta})]\right]$$

"data - model"

covariance matrix

- The covariance matrix C_{ij} gives the correlation between different data points; it typically needs to be estimated from simulations or analytically

Gaussian likelihood

- If there is no correlation, the likelihood simplifies:

$$\mathscr{L}_{\boldsymbol{x}}(\boldsymbol{\mu}(\boldsymbol{\theta})) = \frac{1}{\sqrt{2\pi|\boldsymbol{\sigma}|^2}} \exp\left[\sum_i \frac{[x^i - \mu^i(\boldsymbol{\theta})]^2}{2\sigma_i^2}\right]$$

variance

- The Gaussian likelihood is commonly encountered in cosmological data analysis

Point Estimates

- One central question of data analysis is to find the most likely value of a model parameter $\boldsymbol{\theta}$

1. Maximum likelihood estimator (MLE) $\hat{\boldsymbol{\theta}}$:

$$\left.\frac{\partial \ln(\mathscr{L})}{\partial \theta_i}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = 0$$

- I.e. $\hat{\boldsymbol{\theta}}$ is the parameter (set), which maximizes the likelihood
- Since $\mathscr{L}$ typically has one extremum, the above condition is enough
- This is also called the “bestfit”

Point Estimates

2. Least squares estimator:

$$\left.\frac{\partial \chi^2}{\partial \theta_i}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = 0 \text{ with}$$

$$\chi^2(\boldsymbol{\theta}) = \sum_{i,j} [x^i - \mu^i(\boldsymbol{\theta})]^T C_{ij}^{-1} [x^j - \mu^j(\boldsymbol{\theta})]$$

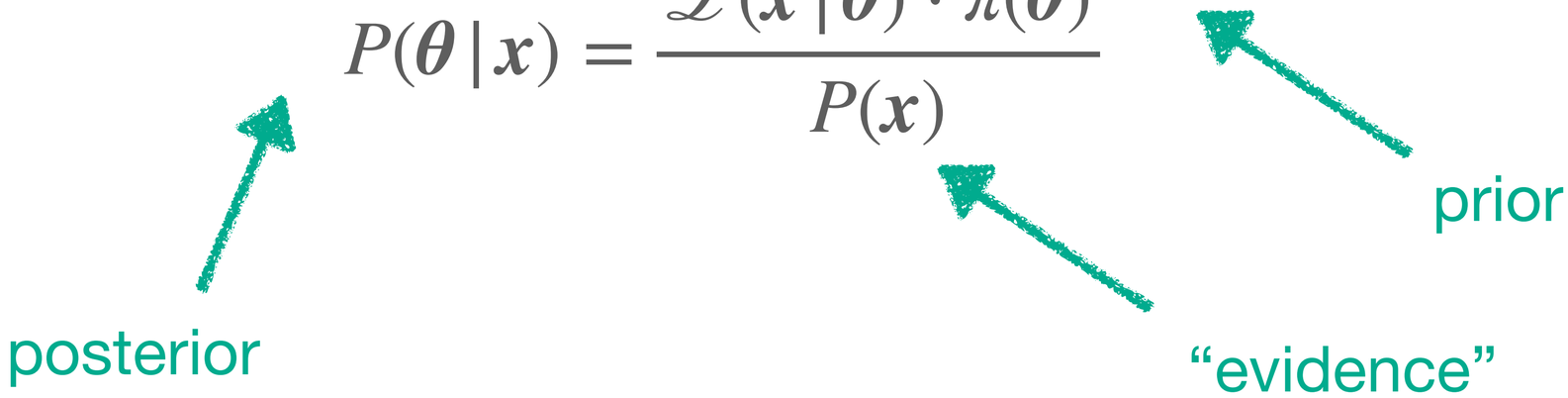
“data - model”

covariance matrix

- This is also called the “minimum chi^2”
- The least-squares estimator coincides with the MLE for a Gaussian likelihood

Bayes theorem

$$P(\boldsymbol{\theta} \,|\, \boldsymbol{x}) = \frac{\mathscr{L}(\boldsymbol{x} \,|\, \boldsymbol{\theta}) \cdot \pi(\boldsymbol{\theta})}{P(\boldsymbol{x})}$$



- The posterior gives the “probability of the model parameters $\boldsymbol{\theta}$ given the data $\boldsymbol{x}$”
- Since the evidence depends only on the data, it contributes only a constant factor and is not important

- Add overview Bayesian statistics (posterior/marginalization)
- Add overview frequentist statistics (profile likelihood)

Recap EDE

- Add recap

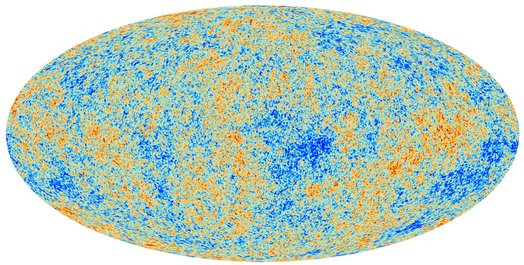
Can EDE resolve the H_0 tension

… and fit all available data sets?

Can EDE solve the H_0 tension and fit these data sets?

Data sets: Planck + 6dFGS + BOSS DR12 BAO/ RSD + Pantheon + SH0ES 2016

CMB (TT, TE, EE, lensing)



ESA

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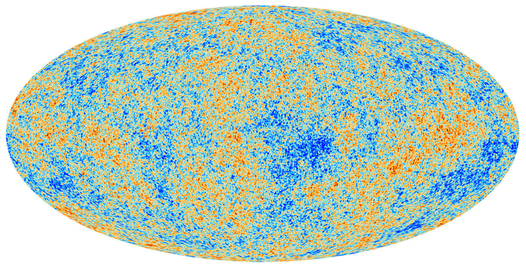
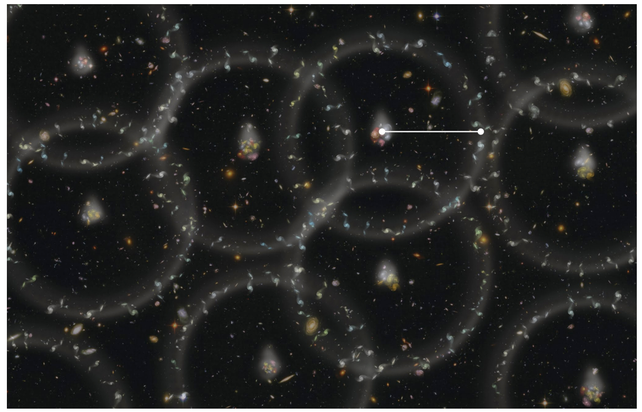


ESA

Baryon acoustic oscillations



BOSS collaboration

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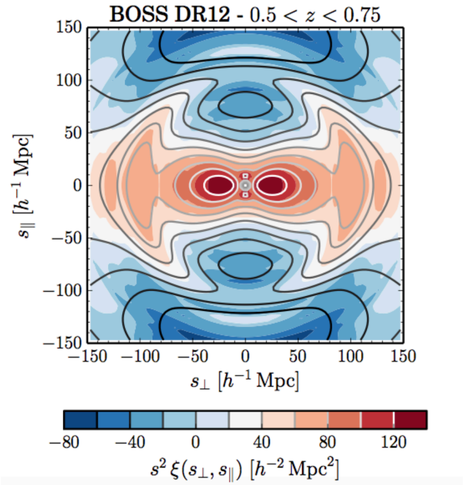
Redshift space distortions



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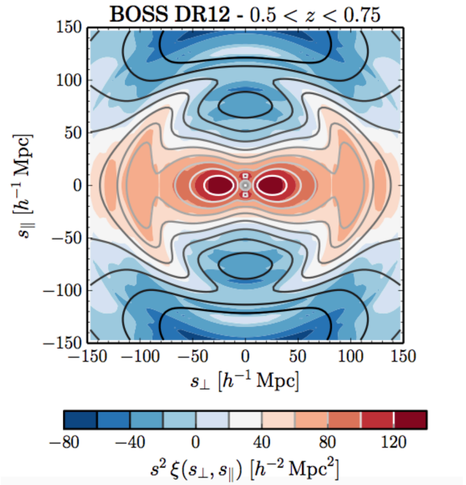
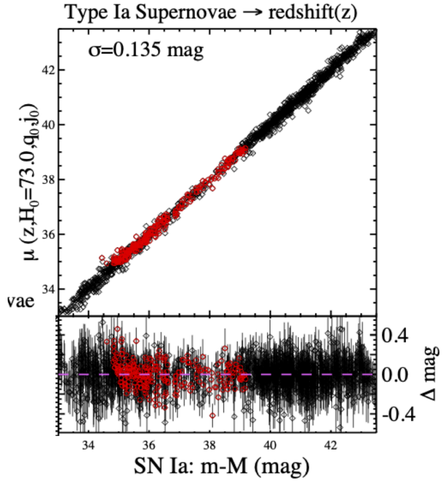


BOSS collaboration (Sanchez++ 2016)

Cepheid-calibrated SNIa

NASA



SH0ES col.

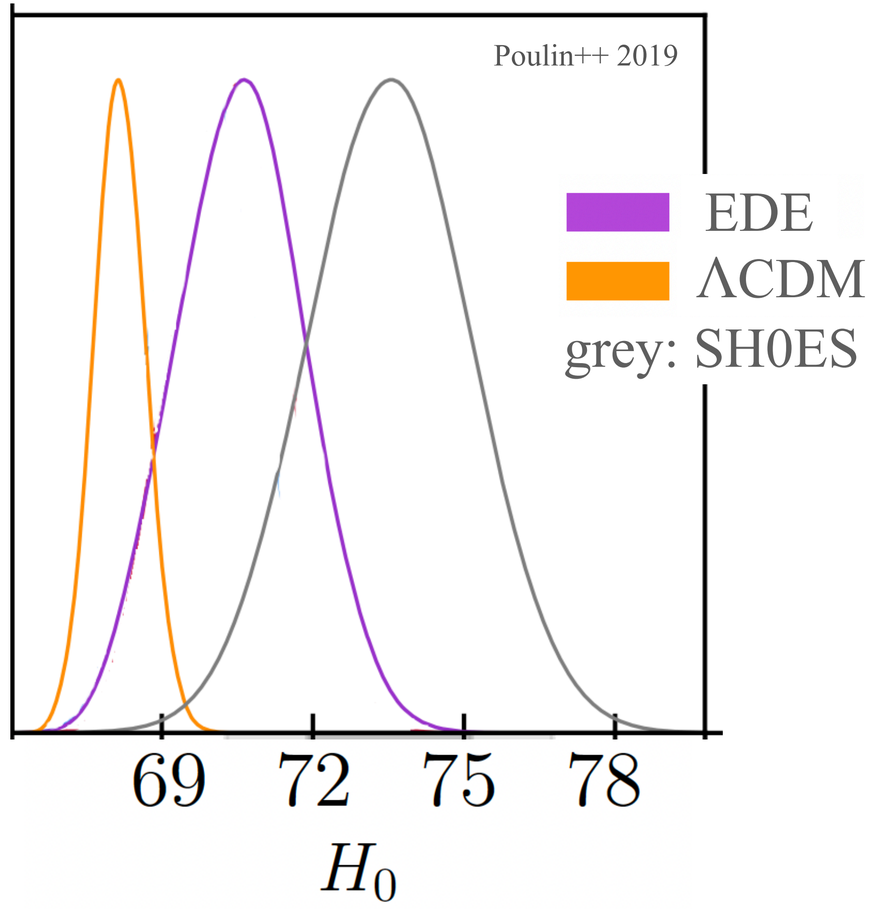
2019: EDE can solve the H_0 tension!

Poulin, Smith, Karwal, Kamionkowski, 2019

Data sets: Planck + 6dFGS + BOSS DR12 BAO/ RSD + Pantheon + SH0ES 2016

- $f_{\rm EDE} = 0.107^{+0.035}_{-0.030}$ (mean $\pm 1\sigma$)
- $H_0 = 71.49 \pm 1.20$ km/s/Mpc

Markov Chain Monte Carlo (MCMC)



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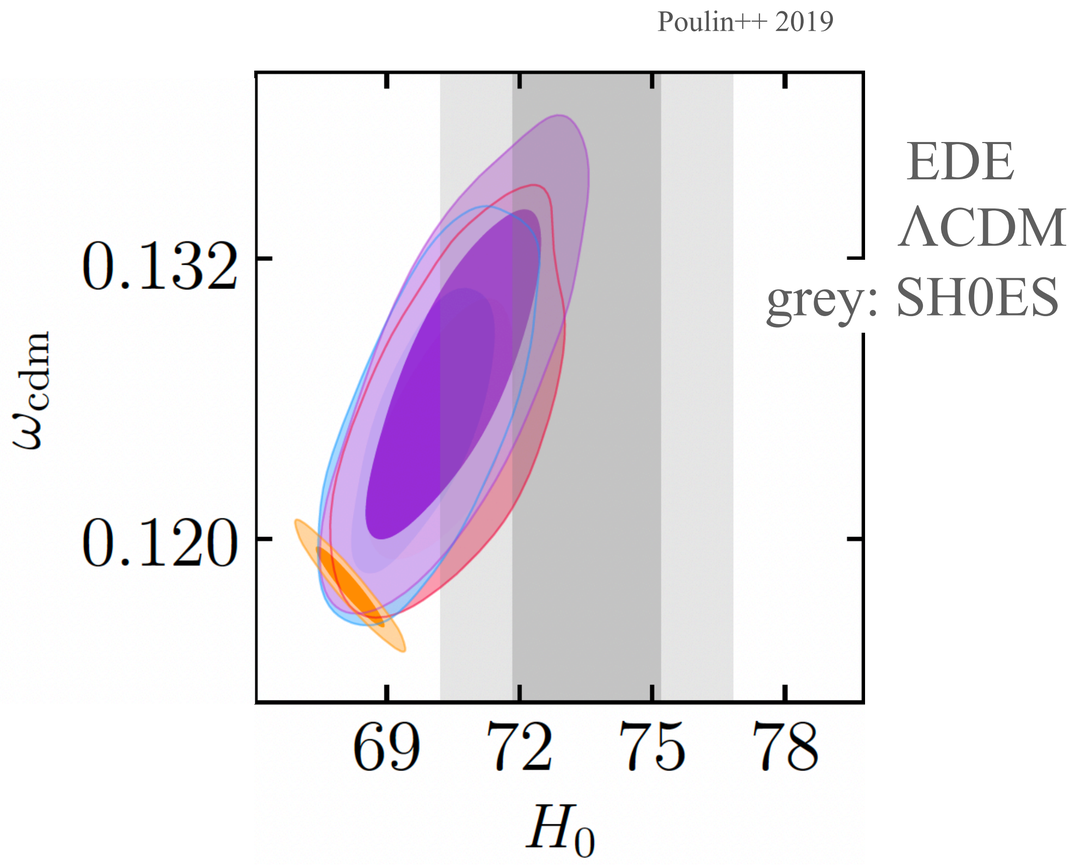
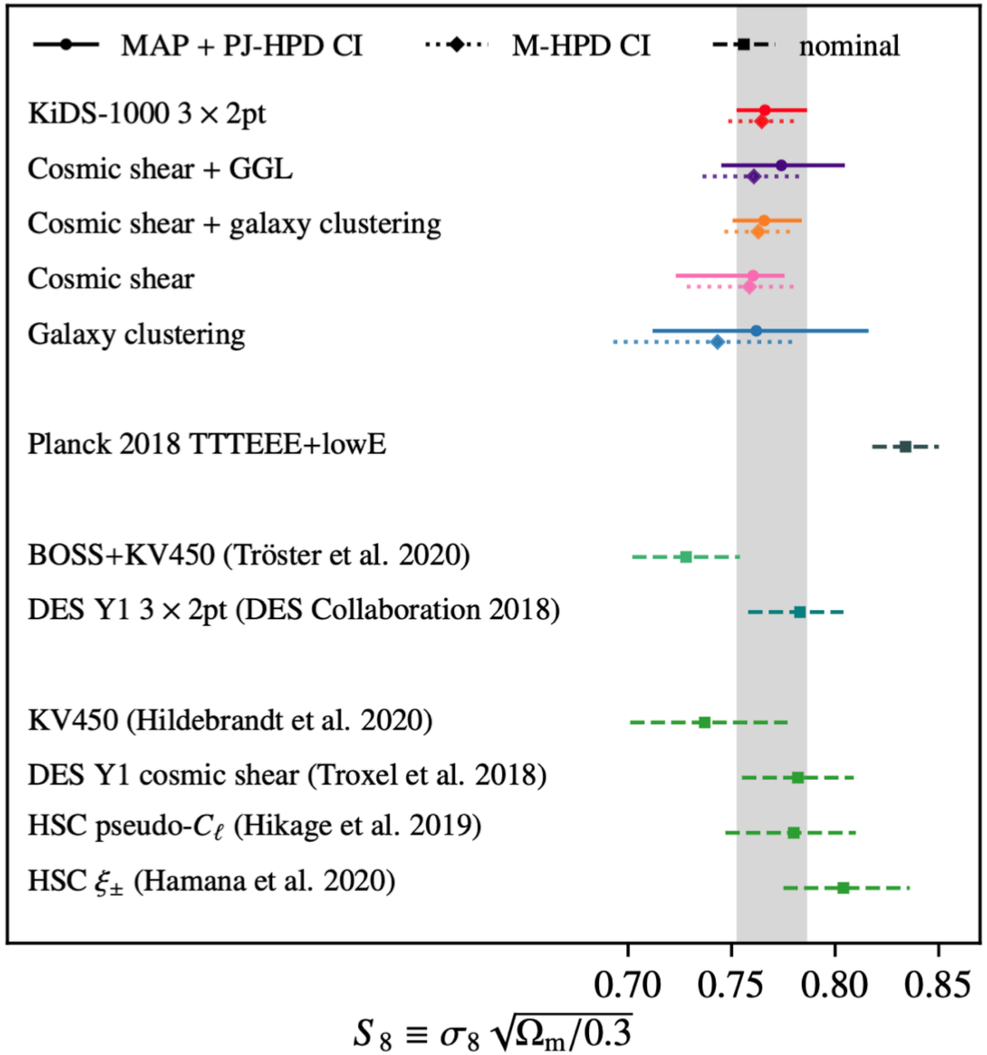
- $f_{\rm EDE} = 0.107^{+0.035}_{-0.030}$ (mean $\pm 1\sigma$)
- $H_0 = 71.49 \pm 1.20$ km/s/Mpc
- Also other parameters shift:
  EDE suppresses growth of perturbations at early times
  $\rightarrow$ $\omega_{\rm CDM}$ and n_s increase
  $\rightarrow$ σ_8 increases, worsening the so-called σ_8 discrepancy



Aside: σ_8 discrepancy

Cosmology Intertwined 2021



$$\sigma_8^2 = \int dk^3 \, |\, W(k) \,|^2 P_{\rm lin}(k), \qquad S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$$

Fourier transform of top-hat filter with radius 8 Mpc/h

linear matter power spectrum

Planck prefers 2-3 σ higher S_8 than weak lensing experiments (DES, KiDS, HSC)

Aside: σ_8 discrepancy

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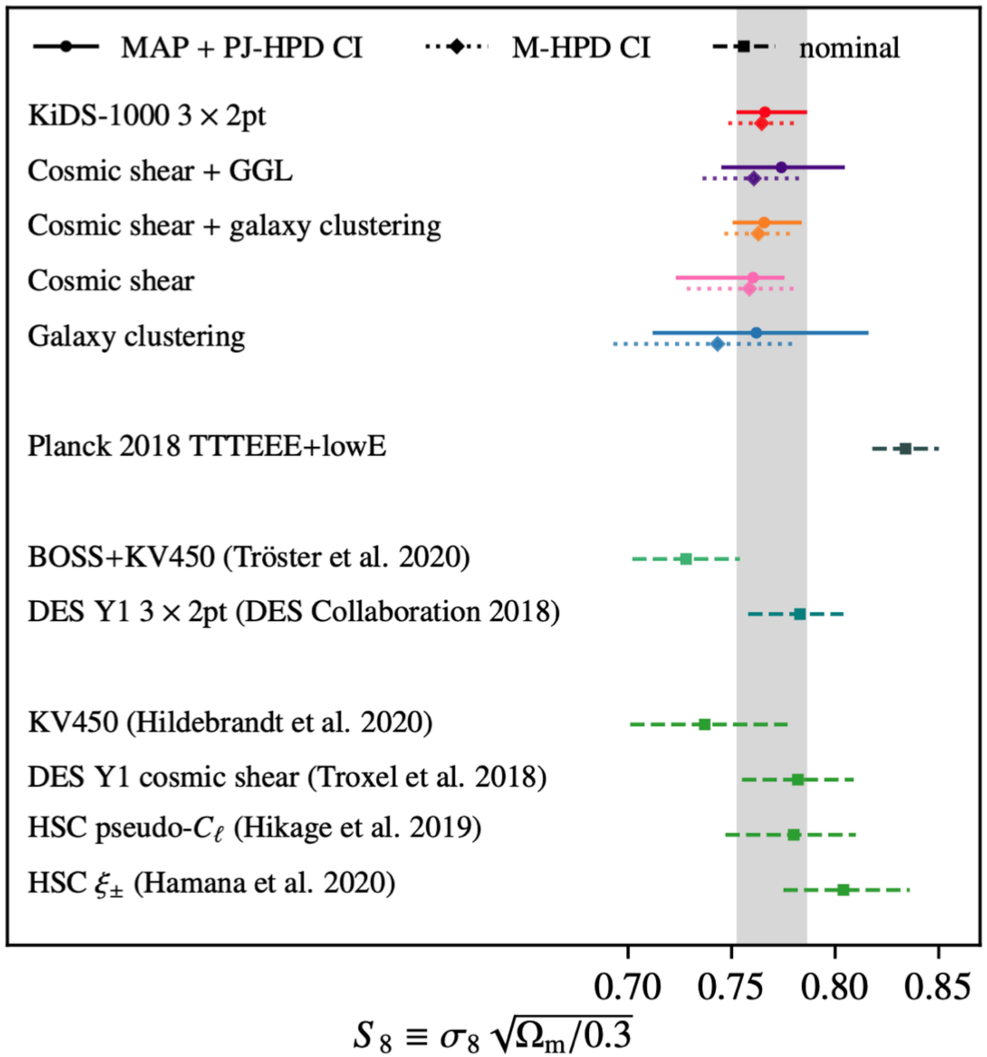

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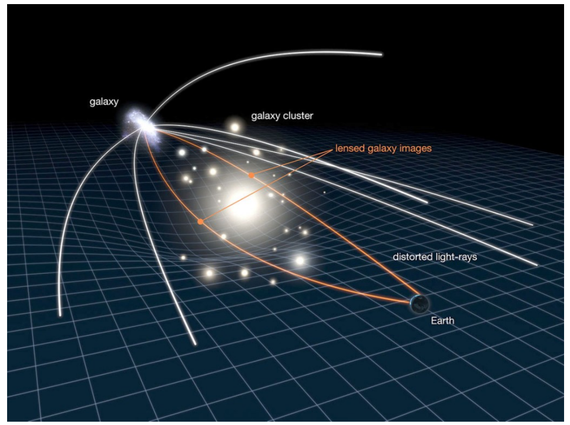


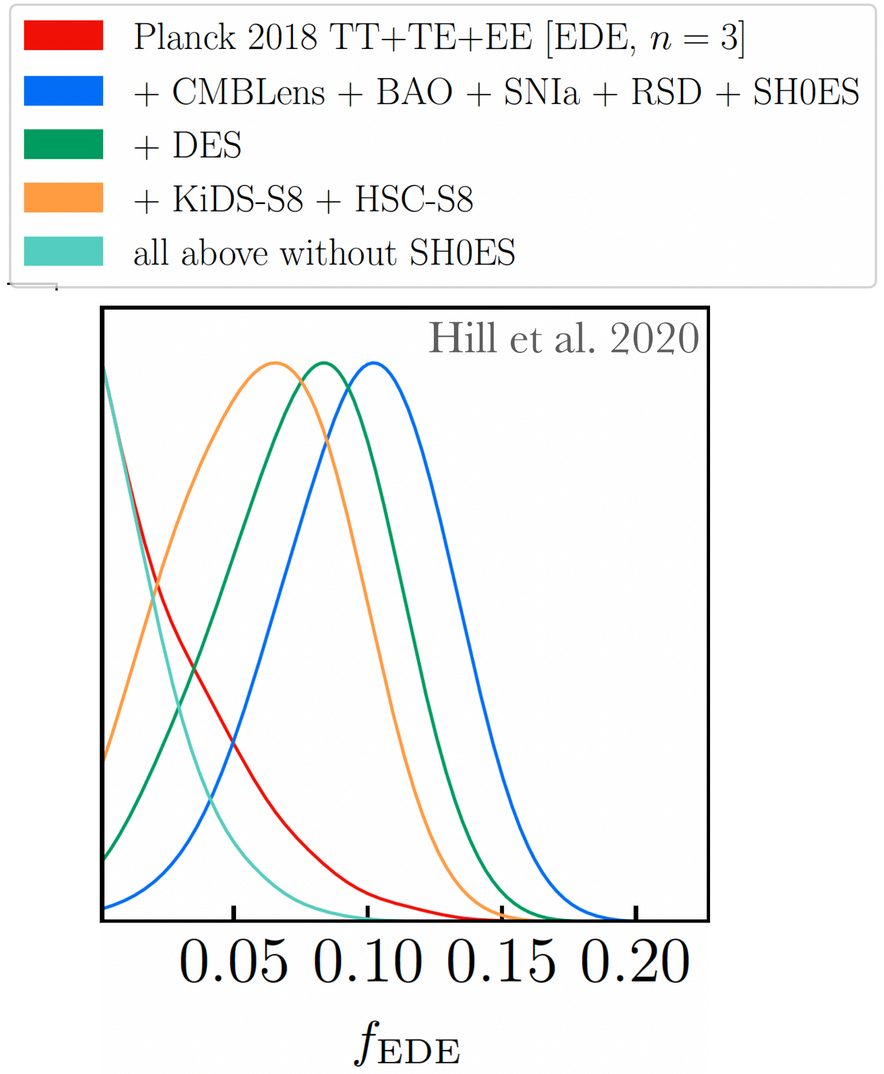
NASA

Adding LSS data: EDE is ruled out?

Hill et al. 2020; Murgia et al. 2021

Data sets: Planck + 6dFGS + BOSS DR12 BAO/RSD + Pantheon ~~+ SH0ES 2016~~ + DES + KiDS + HSC

- $f_{\rm EDE} < 0.06$ (95% C.L.)
- $H_0 = 68.92^{+0.57}_{-0.59}$ km/s/Mpc
- No SH0ES → no preference for EDE



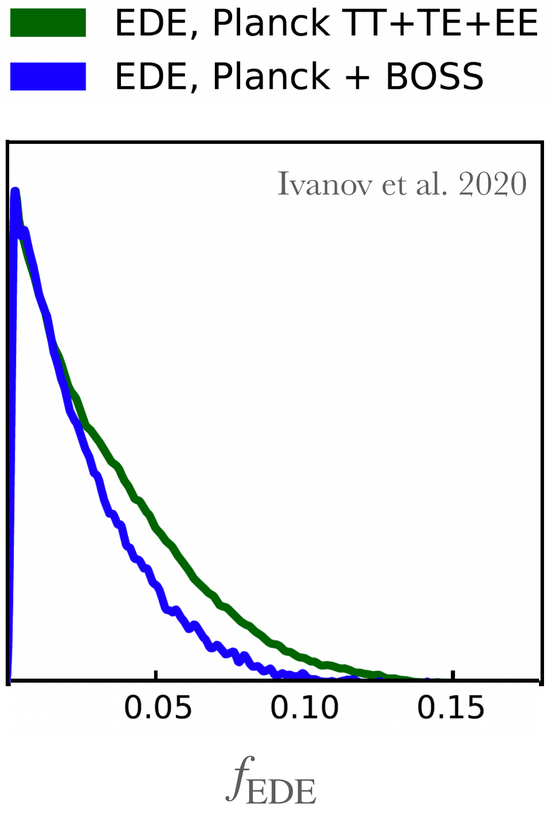
Adding LSS data: EDE is ruled out?

Ivanov et al. 2020; D'Amico et al. 2020

Data sets: Planck + BOSS DR12 BAO
+ full-shape analysis based on EFT of LSS

- $f_{\rm EDE} < 0.072$ (95% CL)
- $H_0 = 68.54^{+0.52}_{-0.95}$ km/s/Mpc

→ EDE cannot restore cosmological concordance



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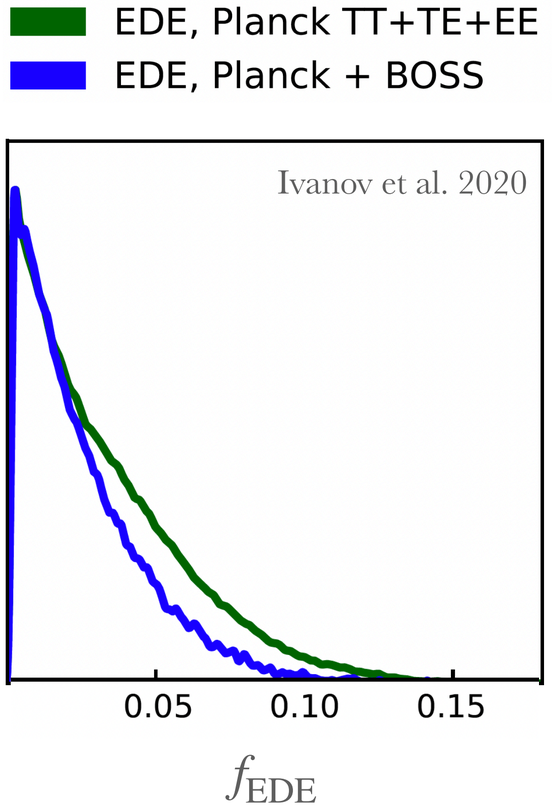
Could **volume effects** affect the results?

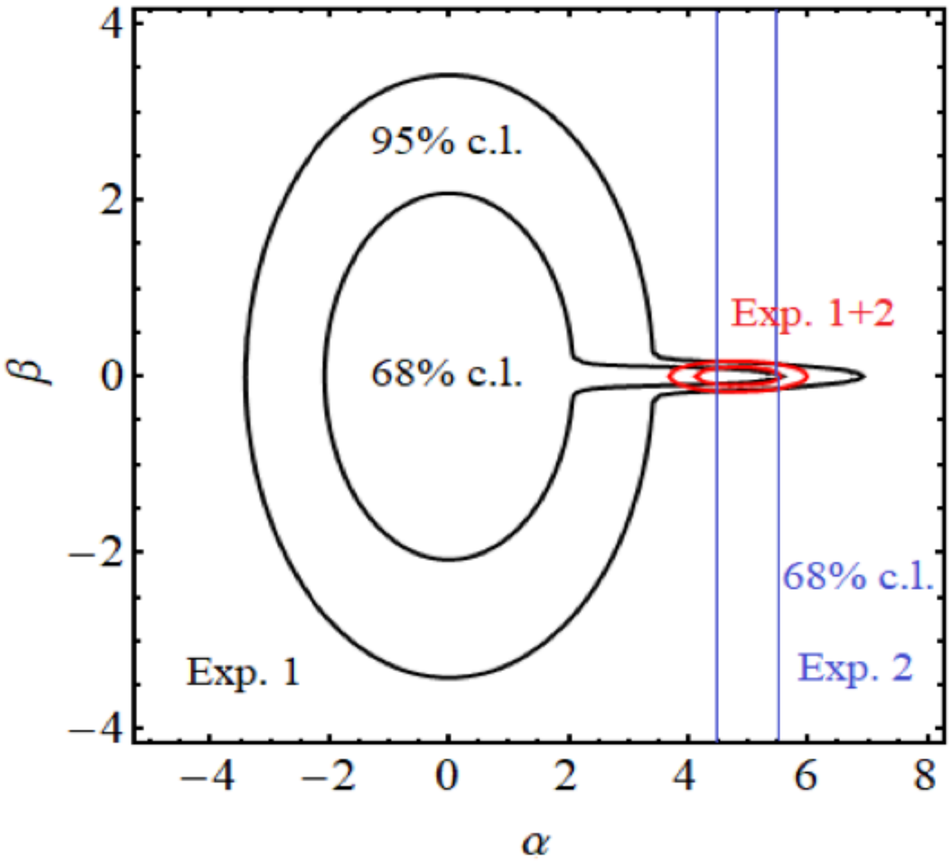
Smith++ 2020, Niedermann++ 2020, Smith++ 2021

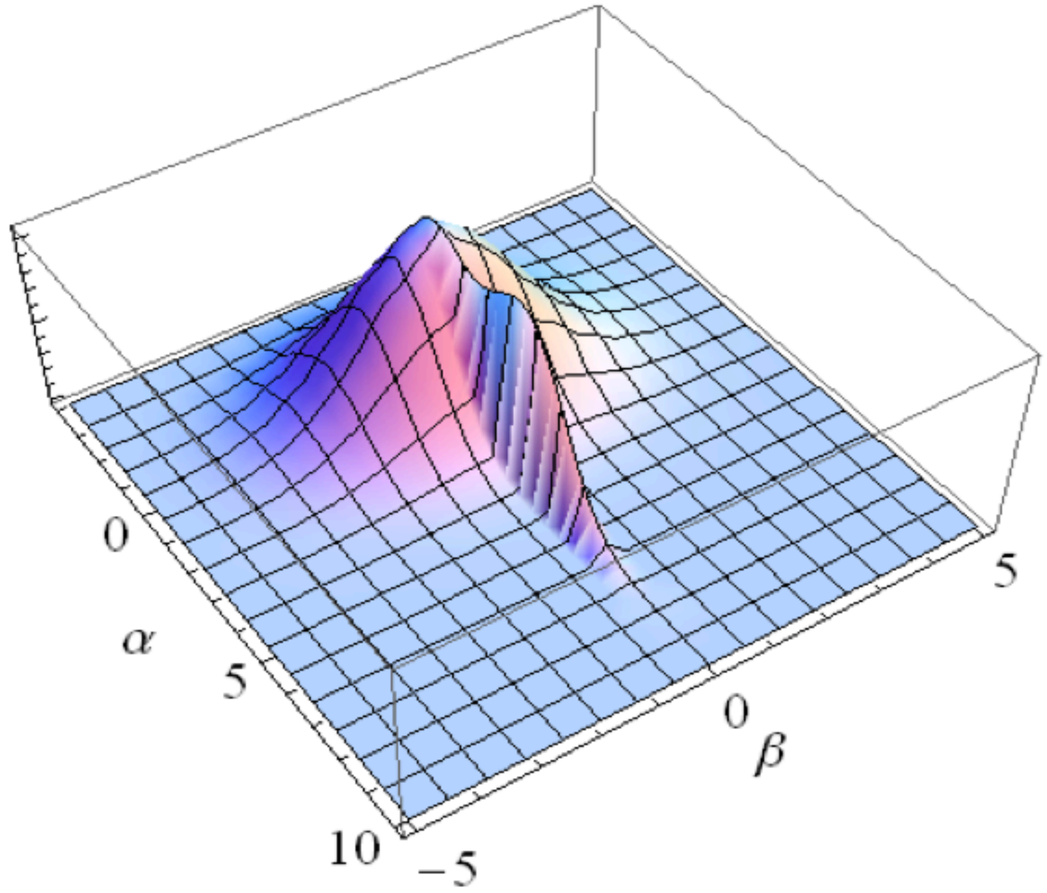


Toy example: Prior volume / projection effect

Gomez-Valent 2022

2D posterior





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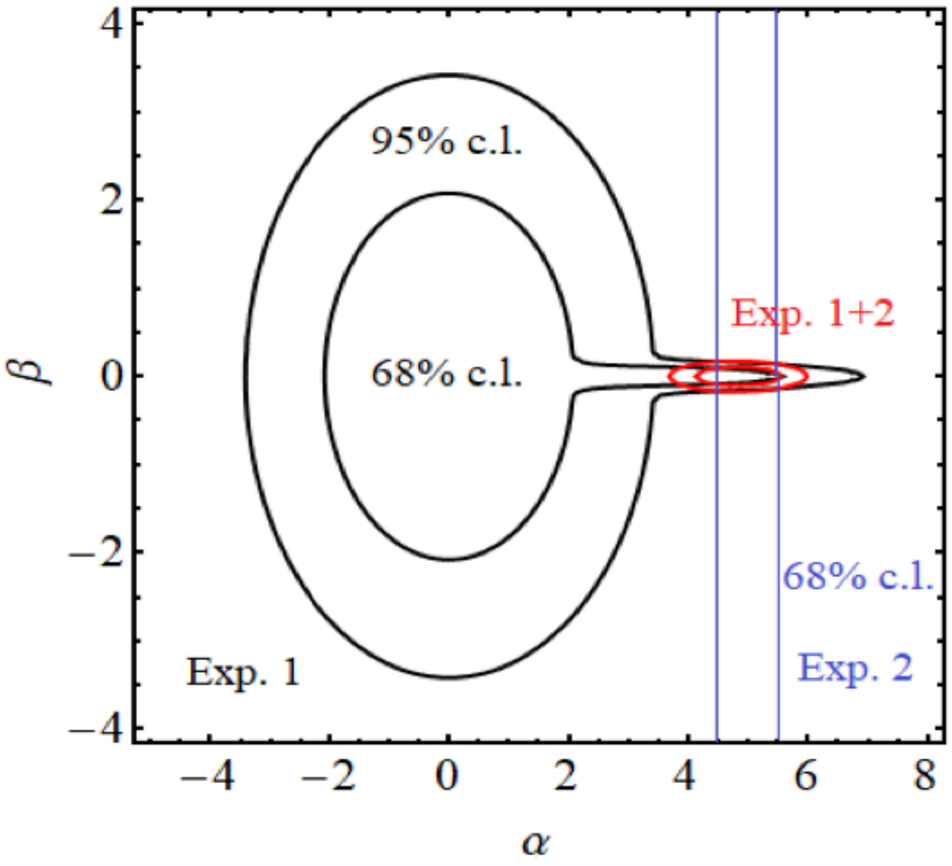
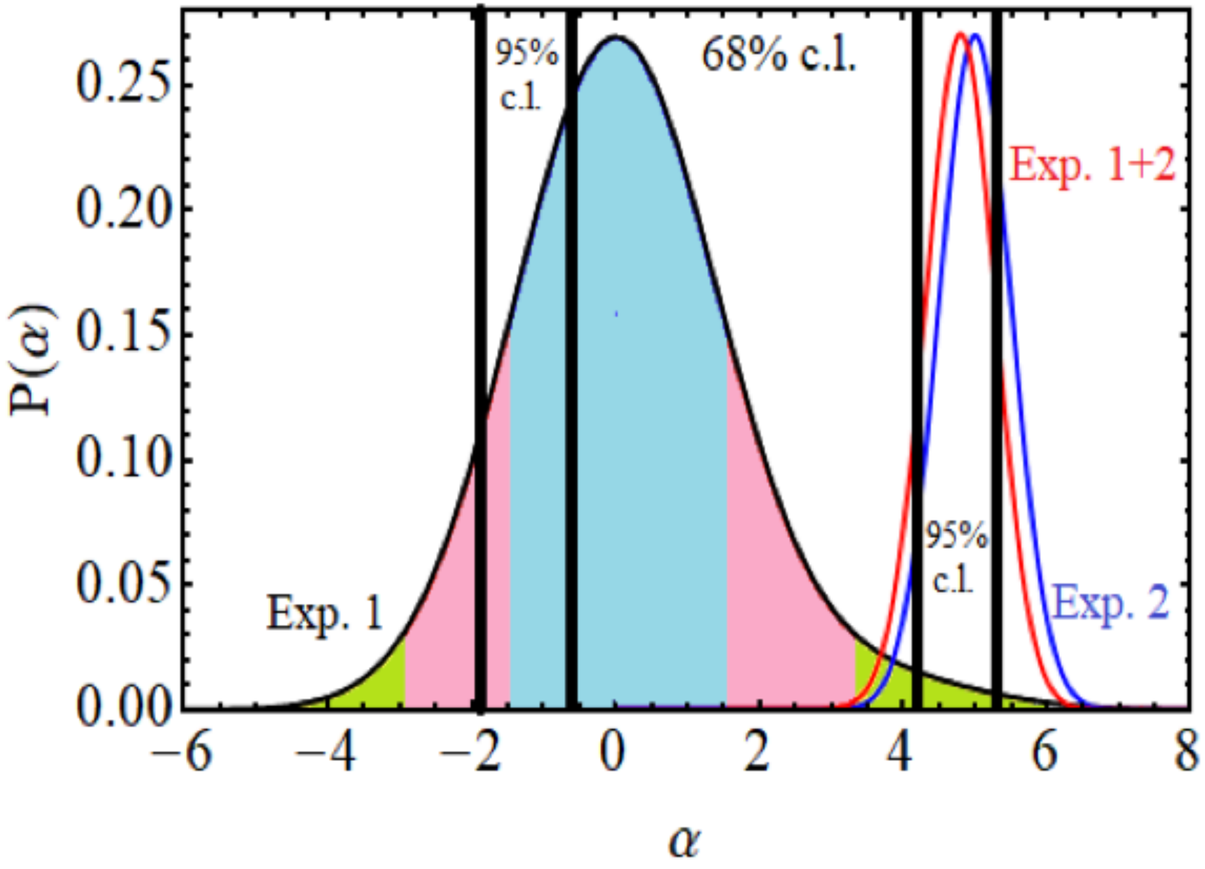


1D marginalized posterior of α



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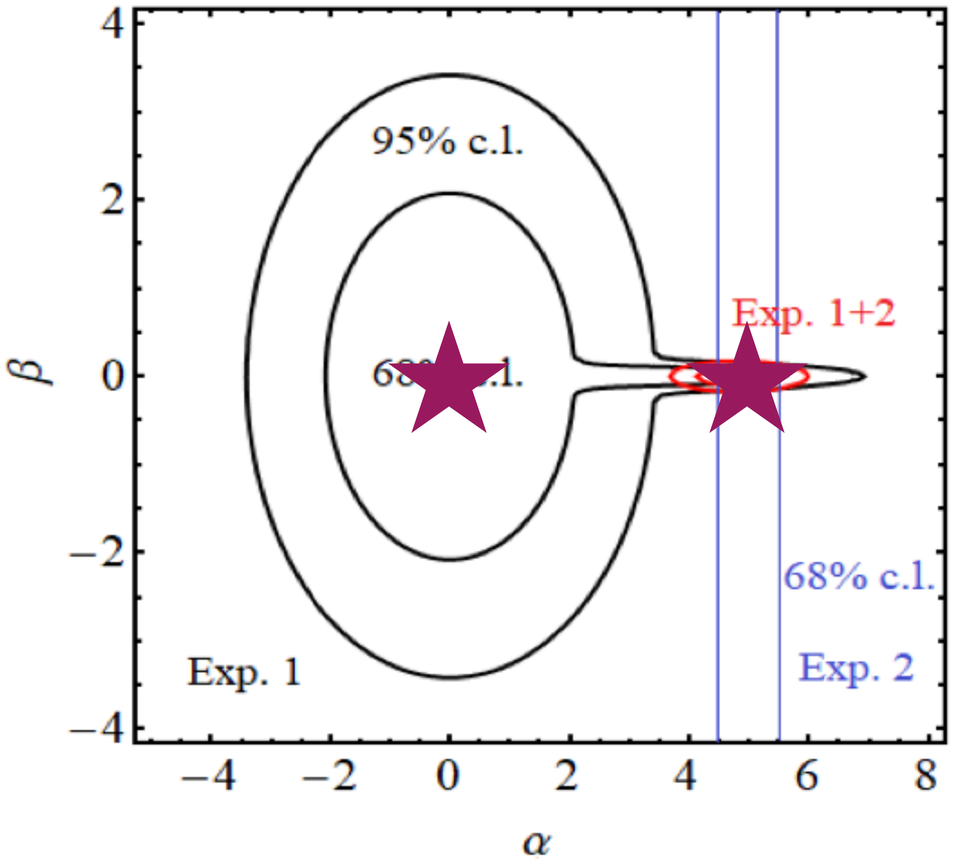

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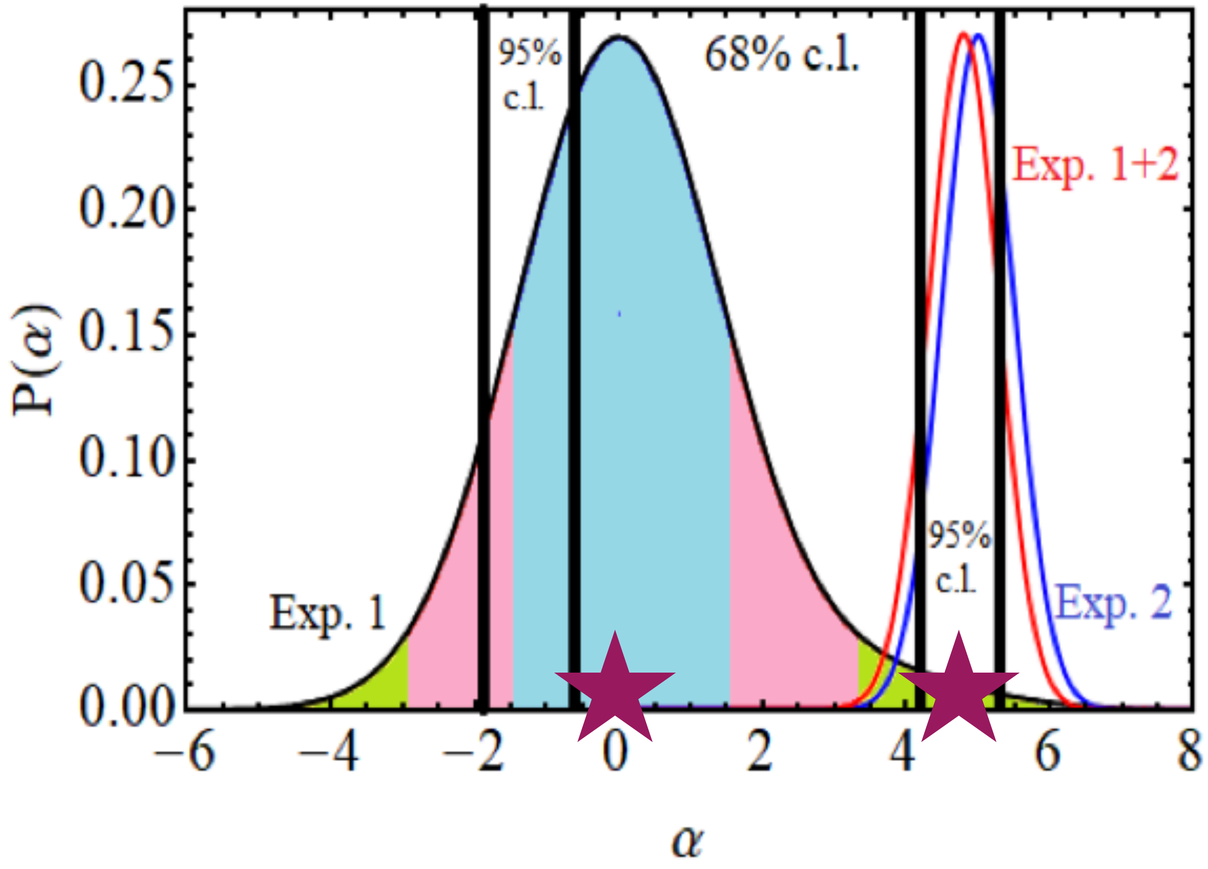
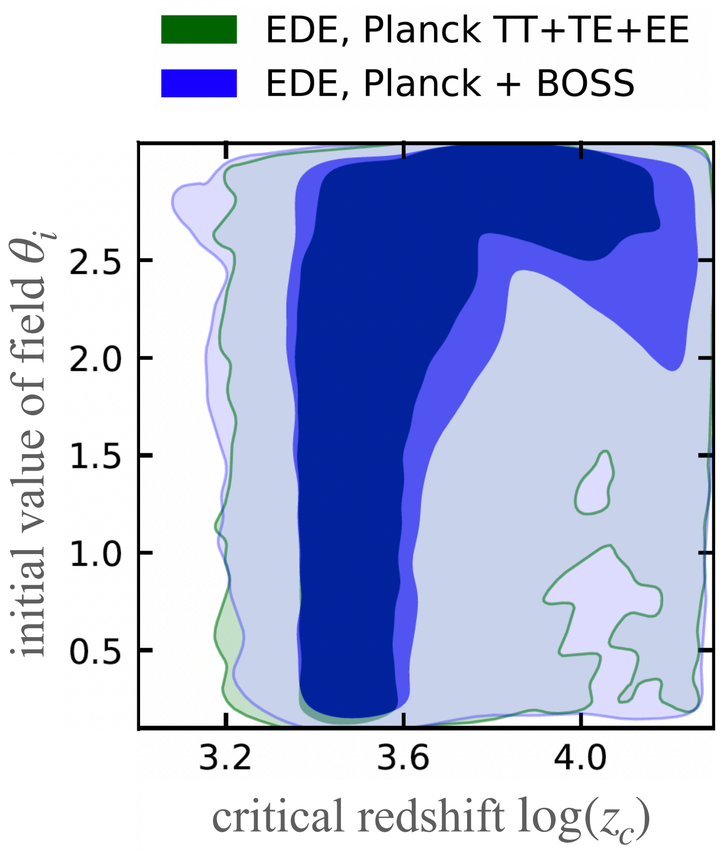


Prior volume / projection / marginalisation effects

…appear if the posterior is influenced by the prior volume.
Reasons:

- Model has too many parameters / data is not constraining.
- Posterior is very non-Gaussian.
- Parameter structure of the model generates large volume differences.

Ivanov et al. 2020



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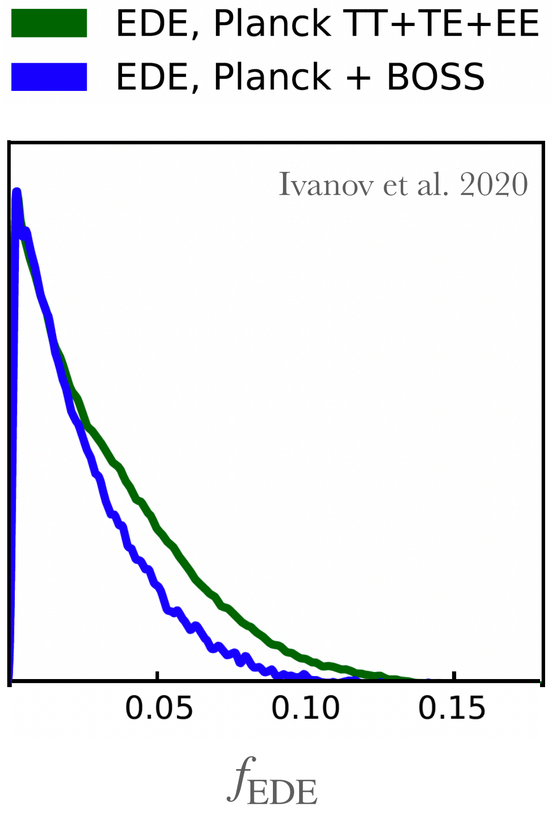

$f_{\rm EDE} \approx 0$: all values of z_c, θ_i unconstrained (ΛCDM limit)

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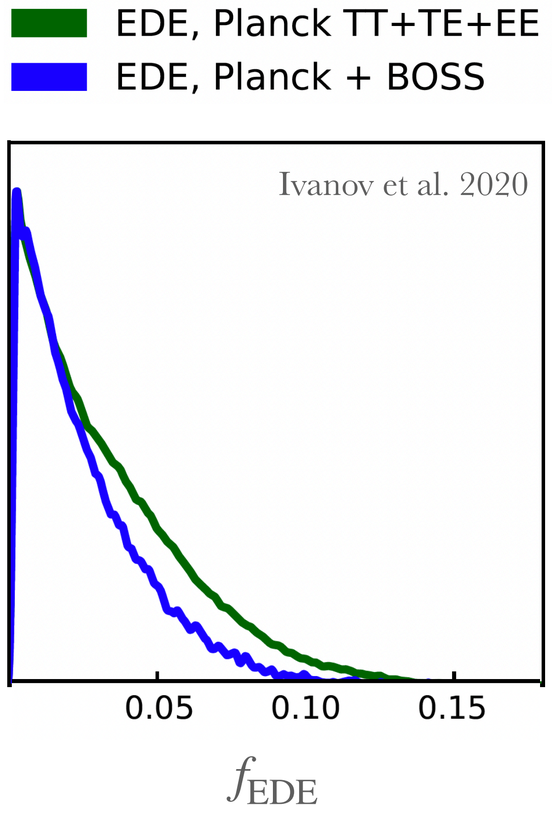
→ Bias in the marginalised posterior.



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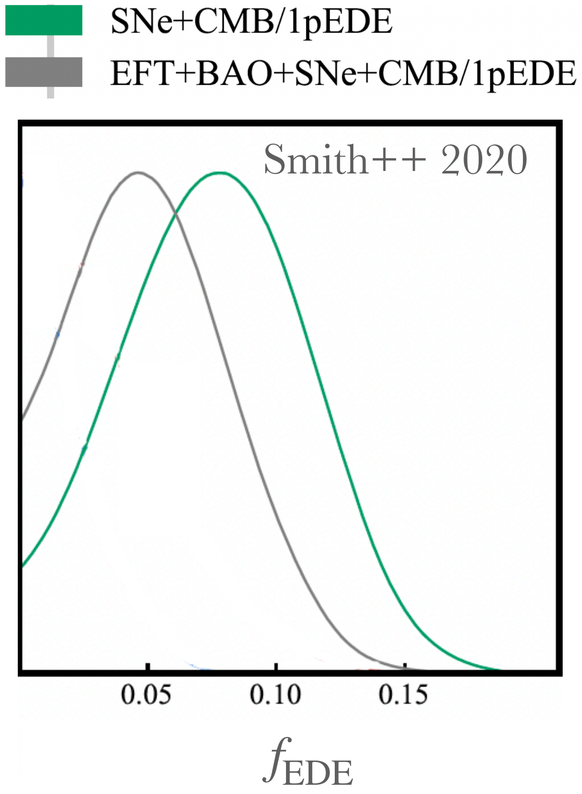
EDE is not ruled out by LSS?

Smith, Poulin, Bernal, Boddy, Kamionkowski, Murgia, 2020; Niedermann, Sloth, 2019 (for NEDE)

Data sets: Planck + BOSS DR12 BAO + full-shape analysis + Pantheon

- fixing z_c, θ_i to bestfit to *Planck* — "1-parameter model"
- $f_{\rm EDE} = 0.072 \pm 0.034$ (mean $\pm 1\sigma$)
- Same data set, but different conclusion than Ivanov et al., D'Amico et al., suspect volume effects



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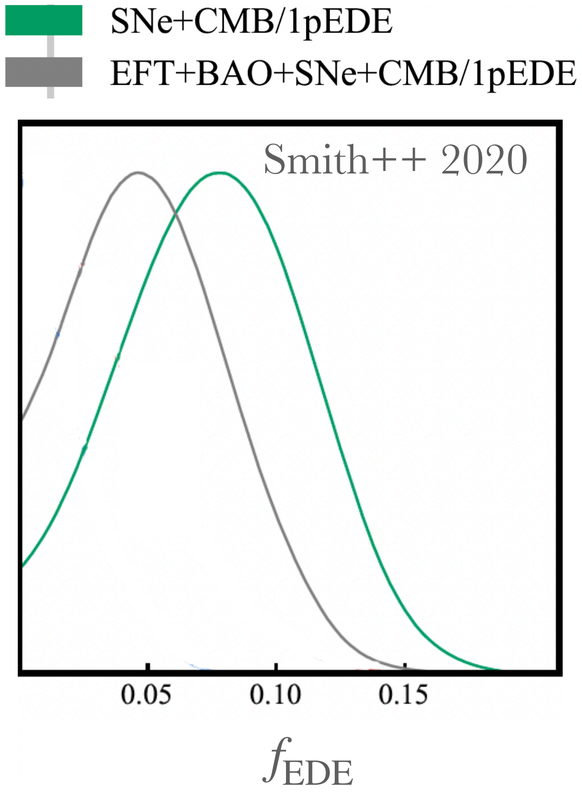
However: Not a full Bayesian analysis



Planck + BOSS (baseline data set)

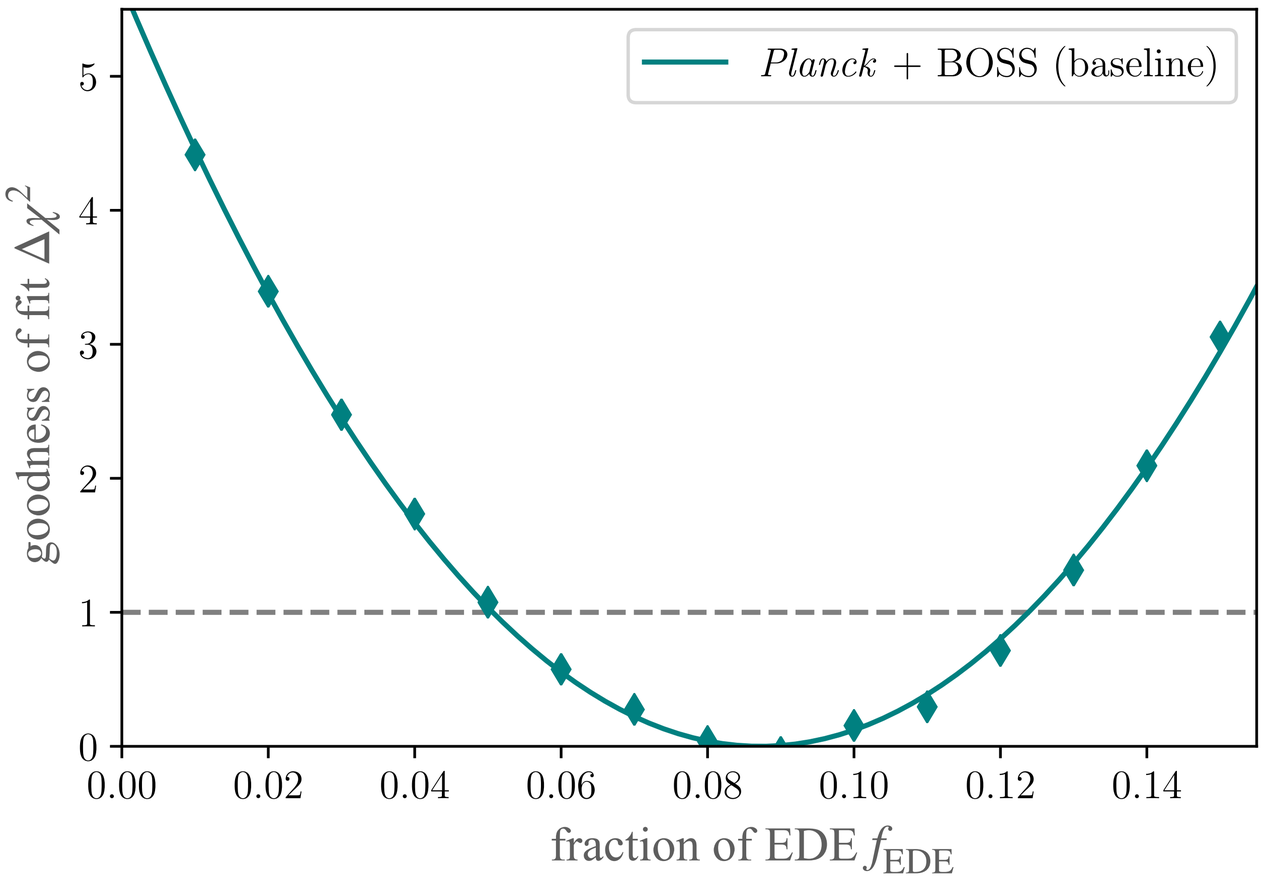
LH++ 2021; LH, Ferreira 2022

MCMC results:

- $f_{\rm EDE} < 0.072$ (95% C.L.), $H_0 = 68.55^{+0.62}_{-1.06}$ km/s/Mpc
- z_c and θ_i not well constrained

Profile likelihood results:

- $f_{\rm EDE} = 0.087 \pm 0.037$,



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LH, Ferreira 2022

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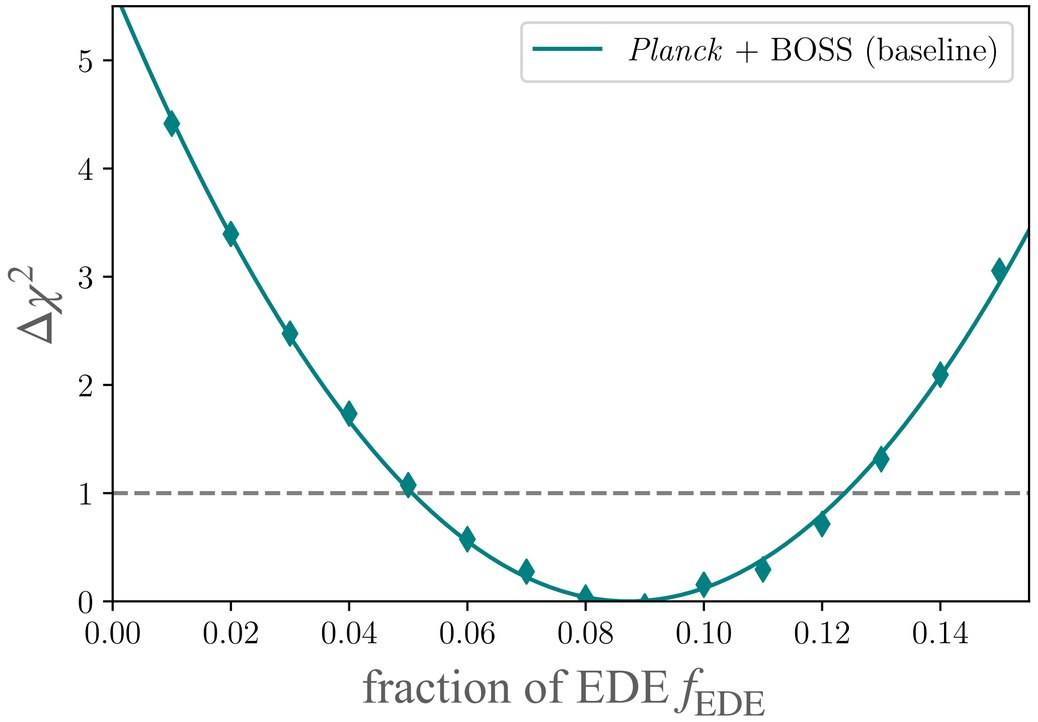
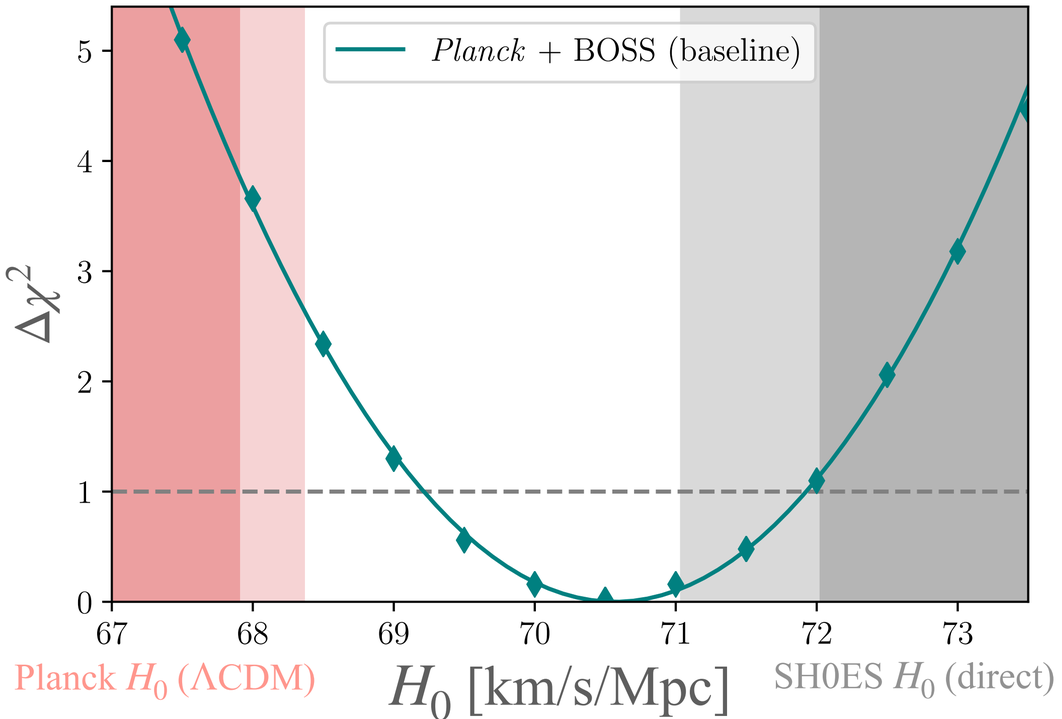
- z_c and θ_i not well constrained

Profile likelihood results:

- $f_{\rm EDE} = 0.087 \pm 0.037$,
  $H_0 = 70.57 \pm 1.36$ km/s/Mpc
- Consistency with SH0ES at $1.4\,\sigma$
- However: S_8 tension worsens slightly
  (ΛCDM: 0.828, EDE: 0.840, DES: 0.776)



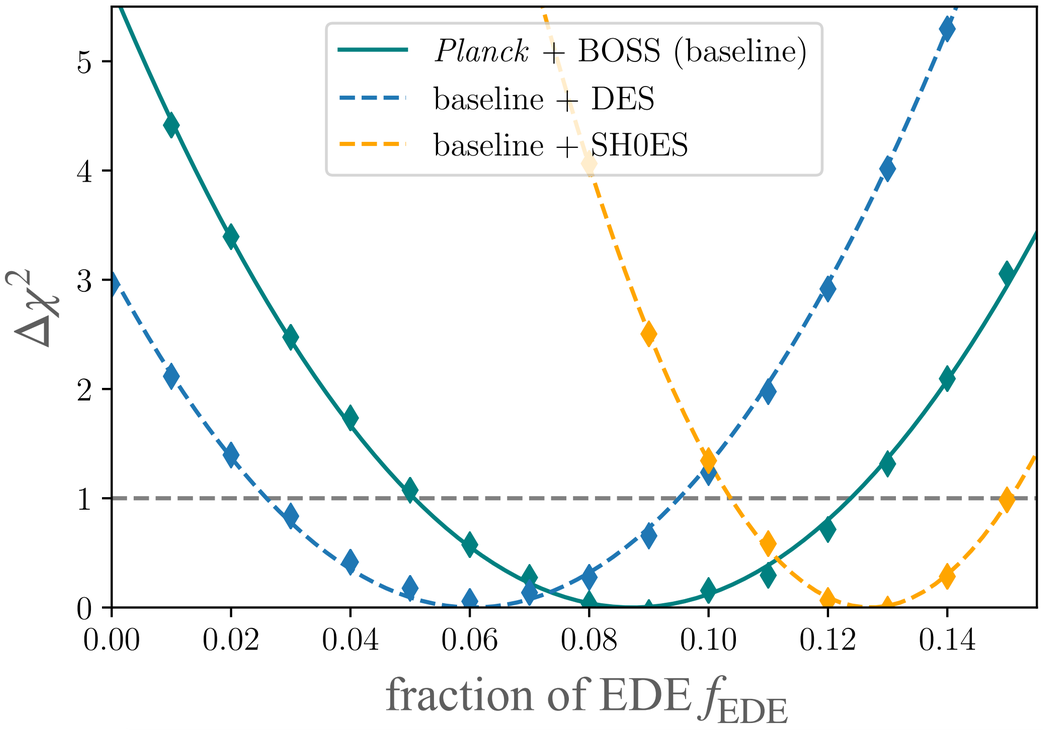


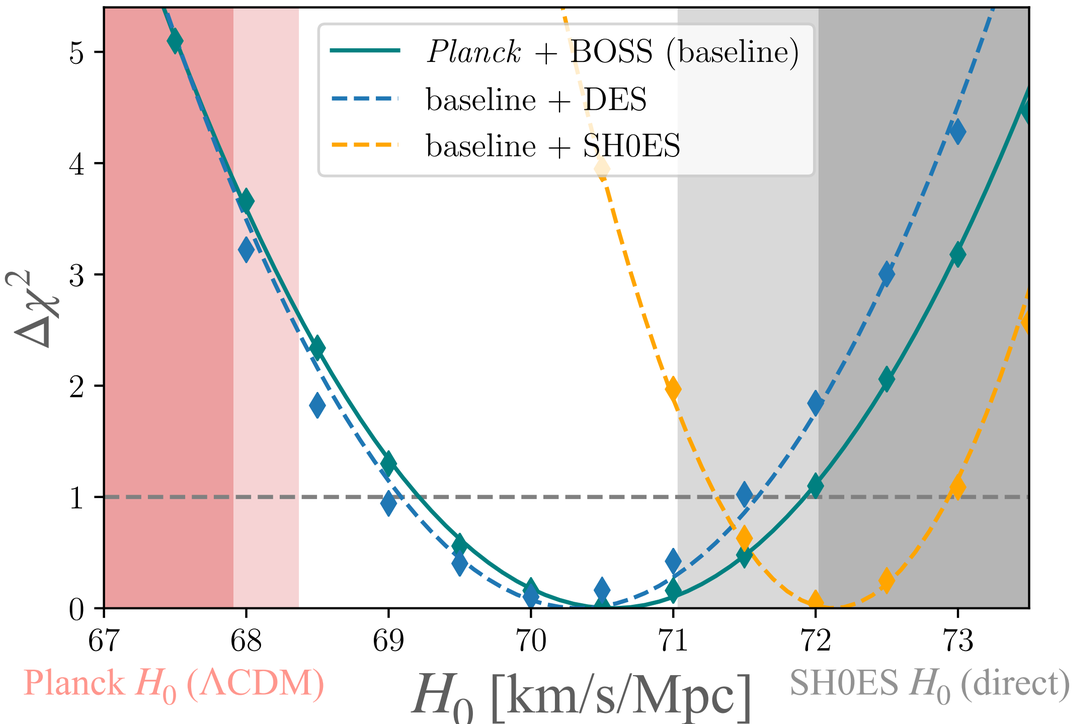
Baseline + DES

LH, Ferreira 2022

Baseline+DES ($S_8 = 0.776 \pm 0.017$):

- $f_{\rm EDE} = 0.061^{+0.035}_{-0.034}$,
  $H_0 = 70.28 \pm 1.33$ km/s/Mpc
- Consistency with SH0ES at $1.6\,\sigma$
- Including weak lensing reduces $f_{\rm EDE}$ and H_0 slightly but does not rule out EDE
- Prior volume effect seems to be the dominating effect (compared to clustering enhancement)



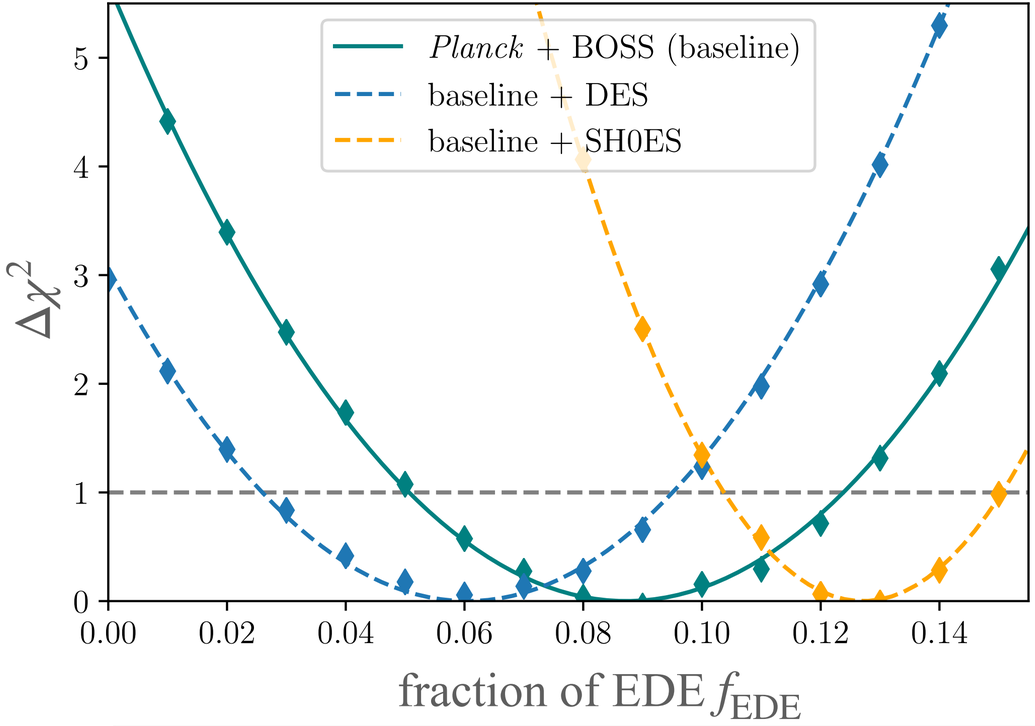


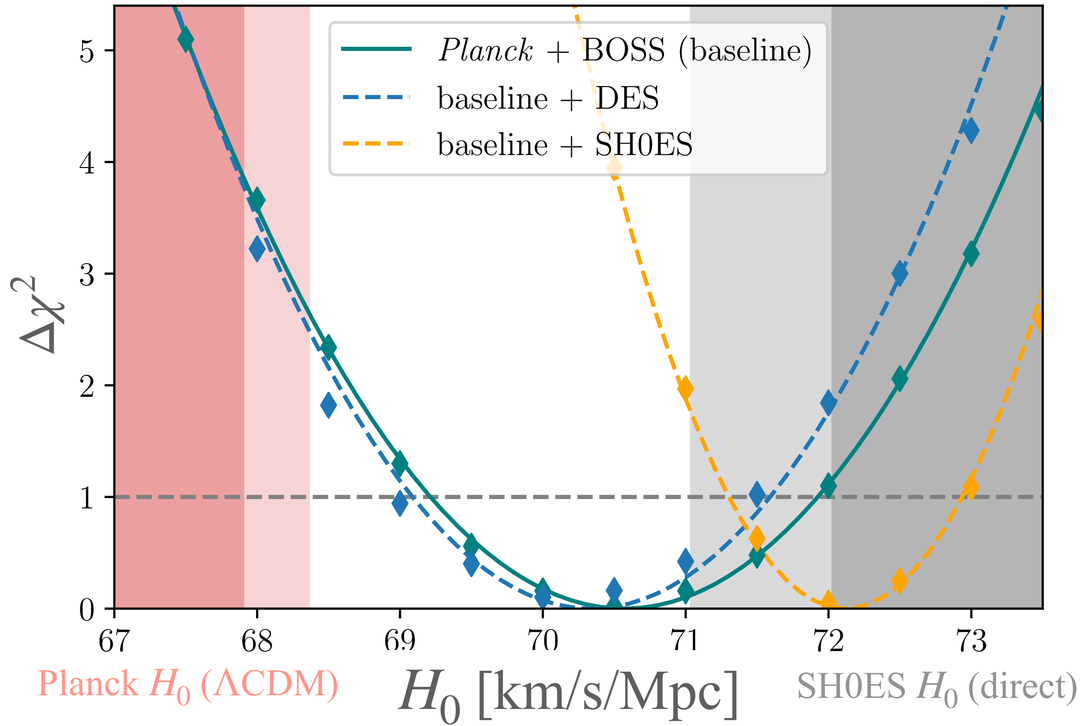
Baseline + SH0ES

LH, Ferreira 2022

Baseline+SH0ES
($H_0 = 73.04 \pm 1.04$ km/s/Mpc):

- Since we find that H_0 for the baseline data set is in agreement with SH0ES, we can combine both data sets
- $f_{\rm EDE} = 0.127 \pm 0.023$,
  $H_0 = 72.12 \pm 0.82$ km/s/Mpc
- Consistency with SH0ES at $0.69\,\sigma$



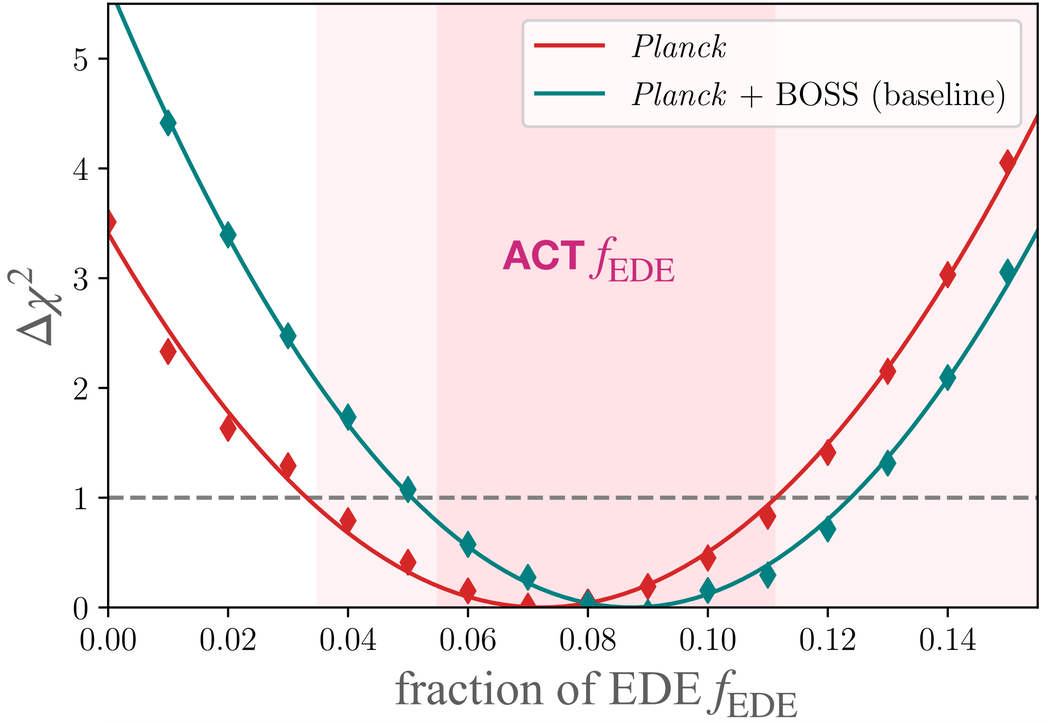


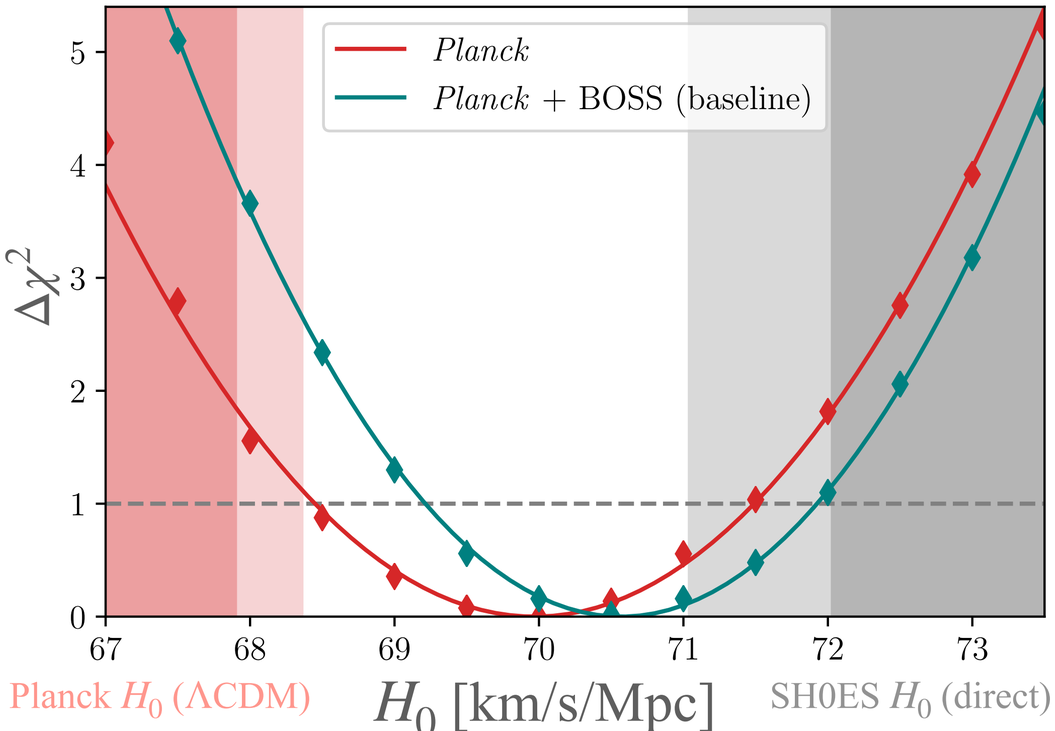
Planck only

LH, Ferreira 2022

Planck only:

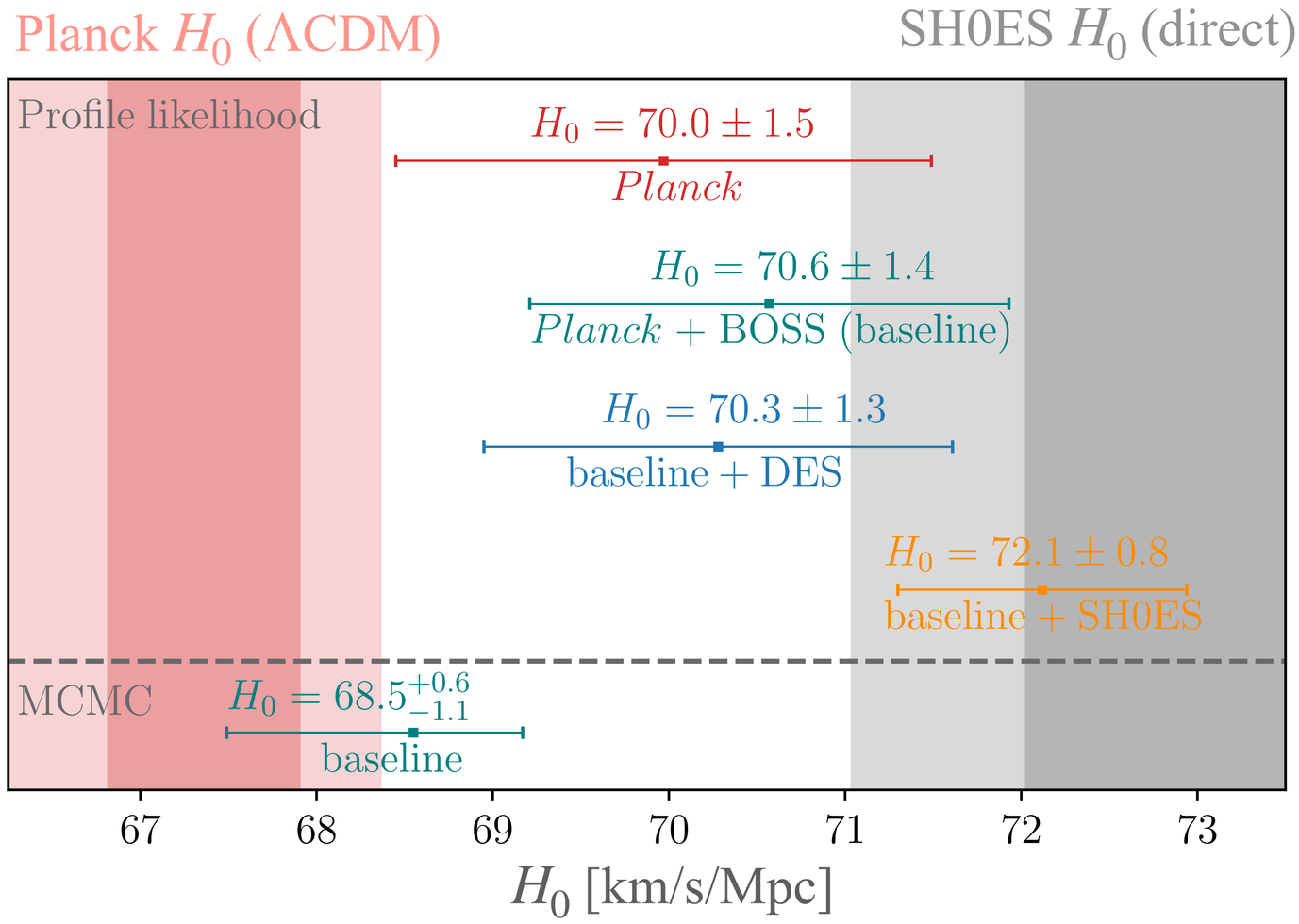
- $f_{\rm EDE} = 0.072 \pm 0.039$,
  $H_0 = 69.97 \pm 1.52$ km/s/Mpc
- Agreement with SH0ES at $1.7\,\sigma$
- Agrees with recent MCMC results from the Atacama cosmology telescope (ACT):
  $f_{\rm EDE} = 0.091^{+0.020}_{-0.036}$ (Hill++ 2022)





Profile likelihood — results

LH, Ferreira 2022



Results:

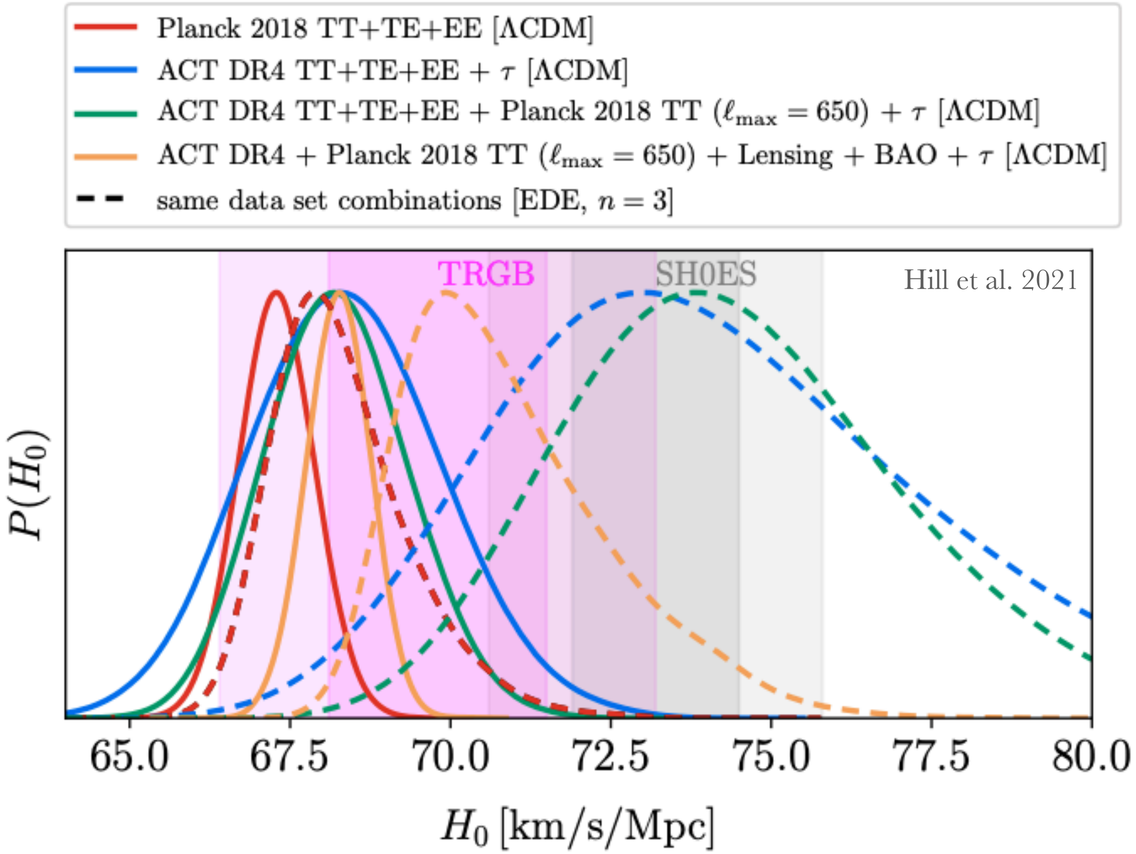
- Evidence for prior volume effects.
- H_0 in EDE model within $1.7\,\sigma$ of SH0ES measurement for all data sets (incl. galaxy clustering, weak lensing).
- EDE viable solution to Hubble tension.

Results from ACT

Hill et al. 2021, Poulin et al. 2021

Data sets: ACT DR4 + large-scale *Planck* TT+lensing + BOSS BAO (yellow line)

- prefers the EDE model over ΛCDM by 2-3 σ
- $H_0 = 70.9^{+1.0}_{-2.0}$ km/s/Mpc,
  $f_{\rm EDE} = 0.091^{+0.020}_{-0.036}$
- Driven by ACT *TE+EE* power spectra



- EDE constraints from
  - SPT
  - Lyman alpha forest
  - New Planck pipeline
  - DESI