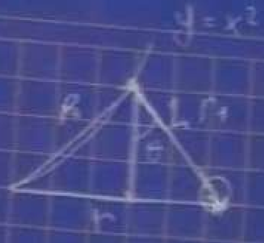

$$\vec{r}_1 = \vec{R} - \vec{r}$$

$$x = x' + L\,\text{sen}\,\theta$$

$$y = y' - L\cos\theta$$

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) \rightarrow \frac{1}{2}m\left((\dot{x} + L\dot{\theta}\cos\theta)^2 + (2x\dot{x} + \dot{\theta}L\,\text{sen}\,\theta)^2\right)$$

$$\rightarrow \frac{1}{2}m\left[\dot{x}^2 + 2L\dot{\theta}\cos\theta\dot{x} + L^2\dot{\theta}^2\cos\theta^2 + (2x\dot{x})^2 + 2\dot{\theta}\dot{x}xL\,\text{sen}\,\theta + \dot{\theta}^2L^2\,\text{sen}\,\theta^2\right]$$

$$T = \left[\dot{x}^2 + L^2\dot{\theta}^2 + 4x^2\dot{x}^2 + 2L\dot{x}\dot{\theta}(\cos\theta + 2x\,\text{sen}\,\theta)\right]m/2$$

$$V = mgy \rightarrow x^2 - L\cos\theta$$

$$L = 1/2\,m\left[\dot{x}^2 + L^2\dot{\theta}^2 + 4x^2\dot{x}^2 + 2L\dot{x}\dot{\theta}(\cos\theta + 2x\,\text{sen}\,\theta)\right]$$
$$- x^2 + L\cos\theta$$

$$H = \dot{q}p - L$$

$$\frac{\partial L}{\partial \dot{q}} = p_i \qquad \frac{\partial L}{\partial \dot{x}} = p_x \rightarrow m\dot{x} + 4x^2\dot{x} + 2L\dot{\theta}(\cos\theta + 2x\,\text{sen}\,\theta) = p_x$$

$$\dot{x} = \frac{p_x - 2L\dot{\theta}(\cos\theta + 2x\,\text{sen}\,\theta)}{(m + 4x^2)}$$

$$\frac{\partial L}{\partial \dot{\theta}} = p_\theta = mL^2\dot{\theta} + 2L\dot{x}(\cos\theta + 2x\,\text{sen}\,\theta)$$

$$\dot{\theta} = \frac{p_\theta - 2L\dot{x}(\cos\theta + 2x\,\text{sen}\,\theta)}{mL^2}$$

$$\dot{x} = (4p_\theta x\sin\theta)/(Lm^2 + 4Lmx^2 - 16Lx^2\sin\theta^2) -$$
$$16Lx\sin\theta\cos\theta - 4L\cos\theta^2 - 2p_\theta\cos\theta/(Lm^2 + 4Lmx^2$$
$$- 16Lx^2\sin\theta^2 - 16Lx\sin\theta - 4L\cos\theta^2 + p_x Lm/(Lm^2 +$$
$$4Lmx^2 - 16Lx^2\sin\theta^2 - 16Lx\sin\theta\cos\theta - 4L\cos\theta^2)$$

$\dot{\theta} = (P_\theta\, m) / (l^2 m^2 + 4 l^2 m x^2 - 16 l^2 x^2 \sin\theta^2 -$

$16 l^2 x \sin\theta \cos\theta - 4 l^2 \cos\theta^2) + 4 P_\theta x^2 / (l^2 m^2 + 4 l^2 m x^2$

$- 16 l^2 x^2 \sin\theta^2 - 16 l^2 x \sin\theta \cos\theta - 4 l^2 \cos\theta^2)$

$- 4 P_x l x \sin\theta / (l^2 m^2 + 4 l^2 m x^2 - 16 l^2 x^2 \sin\theta^2$

$- 16 l^2 x \sin\theta \cos\theta - 4 l^2 \cos\theta^2) - 2 P_x l \cos\theta / (l^2 m^2 +$

$4 l^2 m x^2 - 16 L^2 x^2 \sin\theta^2 - 16 l^2 x \sin\theta \cos\theta$

$- 4 l^2 \cos\theta^2$

$H(x, \theta, P_x, P_\theta) = mg(x^2 - l\cos\theta) + [P_x^2 + \frac{2}{l}(\cos\theta + 2x\sin\theta) P_x P_\theta$

$+ P_\theta^2 (1 + 4x^2) / l^2] [2m^{-1} [\sin\theta - 2x\cos\theta]^2]$

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i} \qquad p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$H = \frac{1}{2}\left(q p^3 + \frac{q}{p}\right) \rightarrow \frac{\partial H}{\partial p} = \frac{1}{2}\left(\frac{\partial (q p^3)}{\partial p} + \frac{\partial\left(\frac{q}{p}\right)}{\partial p}\right)$$

$$\frac{\partial H}{\partial p} = \frac{1}{2}\left(q\, 3p^2 - q\, p^{-2}\right)$$

$$\dot{q} = \frac{1}{2} q\, 3p^2 - q\, p^{-2} \qquad \dot{p} = -1/2\, p^3 + 1/p$$

$$\frac{\partial H}{\partial q_i} = \left(\frac{1}{2} p^3 + 1/p\right) \qquad \text{Ecua } \frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) = -1/2\, p^3 + 1/p$$

$$\dot{p} = -1/2\, p^3 + 1/p$$

→ Ecuación de movimiento dada q

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right)$$

$$H = 1/2\left(q p^3 + q/p\right) \rightarrow H' = \frac{1}{2}\left(P^2/2m + k Q^2\right)$$

$$P^2 = \frac{p^3 q}{2} \qquad Q^2 = \frac{q}{2p}$$

Para saber si es canónica, aplicamos los corchetes de Poisson.

$$\{P, Q\} = 1 \qquad \frac{\partial P}{\partial p}\frac{\partial Q}{\partial q} - \frac{\partial P}{\partial q}\frac{\partial Q}{\partial p}$$

$$\frac{\partial P}{\partial p} = 1/2\left(\frac{p^3 q}{2}\right)^{-1/2} \cdot \frac{3p^2 q}{2} \qquad \frac{\partial P}{\partial q} = 1/2\left(\frac{p^3 q}{2}\right)^{-1/2} \cdot \frac{p^3}{2}$$

$$\frac{\partial Q}{\partial p} = 1/2\left(\frac{q}{2p}\right)^{-1/2} \cdot \left(-\frac{q}{2p^2}\right) \qquad \frac{\partial Q}{\partial q} = 1/2\left(\frac{q}{2p}\right)^{-1/2} \frac{1}{2p}$$

② Sea el Hamiltoniano

$$\mathcal{H} = \frac{p^2}{2m} - A\left(\frac{p}{m}\cos\gamma t + \gamma q \sin\gamma t\right) + \frac{1}{2}kq^2 \quad | \; A, \gamma \text{ y } k \text{ son ctes}$$

a) Hallar el lagrangiano $\mathcal{L}$

① partiendo de esta ecuación:

$$H(p_i, q_i, t) = \sum_i p_i \dot{q}_i - \mathcal{L}(p_i, q_i, t)$$

$$\mathcal{L} = \sum_i p_i \dot{q}_i - H(p_i, q_i, t)$$

$$① \Rightarrow (p)(\dot{q}) - \left(\frac{p^2}{2m} - A\left(\frac{p}{m}\cos\gamma t + \gamma q \sin\gamma t\right) + \frac{1}{2}kq^2\right)$$

(solamente hay un momento y una coordenada).

Dado que $\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{m} - \frac{A}{m}\cos\gamma t$

$$\Rightarrow p = \left(\dot{q} + \frac{A}{m}\cos\gamma t\right)m$$

$$p = \dot{q}m + A\cos\gamma t$$

Reemplazando en ① se obtiene el lagrangiano asociado

$$\mathcal{L} = \frac{1}{2}\dot{q}^2 m + \dot{q} A \cos \gamma t + \frac{1}{2} A^2 \frac{\cos \gamma^2 t}{m}$$
$$+ A \gamma q \sin \gamma t + \frac{1}{2} k q^2$$

Utilizando el hecho que dos Lagrangianos son indistintos entre sí por una derivada total temporal, es decir

$$\mathcal{L} = \mathcal{L}' + \frac{d\Delta}{dt}$$

$$\mathcal{L}' = \frac{1}{2}\dot{q}^2 m + \frac{1}{2} k q^2 \quad \Bigg| \quad \frac{\partial L}{\partial \dot{q}} = p = \dot{q} m \Rightarrow \dot{q} = \frac{p}{m}$$

Este será el nuevo lagrangiano, que ya no depende del tiempo y es equivalente a $\mathcal{L}$

d) El nuevo Hamiltoniano será:

$$H = \sum p\dot{q} - \mathcal{L}' = \frac{p^2}{m} - \frac{p^2}{2m} + \frac{1}{2} k q^2$$

$$\Delta = \frac{1}{2}\frac{p^2}{m} - \frac{1}{2} k q^2$$