

$$n_i = y_i - y_{0i}$$

$$T = \frac{1}{2} m (\dot{y}_1^2 + \dot{y}_2^2)$$

$$V = \frac{1}{2} k (\sqrt{a^2 + y_1^2} - a)^2 + \frac{1}{2} k (\sqrt{a^2 + y_2^2} - a)^2 + \frac{1}{2} k (\sqrt{a^2 + (y_1 - y_2)^2} - a)^2 - mg(y_1 + y_2)$$

$$V = \frac{1}{2} k (a^2 + y_1^2) + \frac{1}{2} k (a^2 + y_2^2) + \frac{1}{2} k ((y_1 - y_2)^2 + a^2) - mg(y_1 + y_2)$$

$$L = T - V = \frac{1}{2} m (\dot{y}_1^2 + \dot{y}_2^2) - \frac{1}{2} k (a^2 + y_1^2) - \frac{1}{2} k (a^2 + y_2^2) - \frac{1}{2} k ((y_1 - y_2)^2 + a^2) + mg(y_1 + y_2)$$

$$m\ddot{y}_1 = -k \left[ \left( \frac{y_1}{\sqrt{a^2 + y_1^2}} \right) + \left( \frac{y_1 - y_2}{\sqrt{a^2 + (y_1 - y_2)^2}} \right) \right]$$

$$m\ddot{y}_1 = -k \left[ \left( 1 - \frac{a^2}{2y_1^2} \right) y_1 + \left( 1 - \frac{a^2}{2(y_1 - y_2)^2} \right) (y_1 - y_2) \right]$$

$$m\ddot{y}_2 = -k \left[ \left( 1 - \frac{a^2}{2y_2^2} \right) y_2 + \left( 1 - \frac{a^2}{2(y_1 - y_2)^2} \right) (y_1 - y_2) \right]$$

Para Pequeñas Oscilaciones  $y_1 \approx y_2 \approx y$   $n_i = y_i - y_{0i}$

$$m\ddot{n}_1 = -k \left[ \left( 1 - \frac{a^2}{2y^2} \right) n_1 + \left( 1 - \frac{a^2}{2y^2} \right) (n_1 - n_2) \right] \quad k' = k \left( 1 - \frac{a^2}{2y^2} \right)$$

$$m\ddot{n}_2 = -k \left[ \left( 1 - \frac{a^2}{2y^2} \right) n_2 + \left( 1 - \frac{a^2}{2y^2} \right) (n_1 - n_2) \right]$$

$$m\ddot{n}_1 = -k' n_1 + k' (n_2 - n_1)$$

$$m\ddot{n}_2 = -k' n_2 - k' (n_2 - n_1)$$

$$n_j = a_j e^{i\omega t}$$

$$\ddot{n}_j = -\omega^2 a_j e^{i\omega t}$$

$$\begin{pmatrix} -m\omega^2 \\ -m\omega^2 \end{pmatrix} = \begin{pmatrix} -2k' & k' \\ k' & -2k' \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$



$$\begin{bmatrix} -2k' & k' \\ k' & -2k' \end{bmatrix} + \begin{bmatrix} m\omega^2 & 0 \\ 0 & -m\omega^2 \end{bmatrix} = \begin{bmatrix} -2k' + m\omega^2 & k' \\ k' & -2k' - m\omega^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 0$$

$$(-2k' + m\omega^2)^2 - k'^2 = 0$$

$$-2k' + m\omega^2 = \pm k'$$

$$\omega^2 = \frac{2k' \pm k'}{m}$$

$$\omega_1 = \sqrt{\frac{3k(1 - a/L)}{m}}$$

$$\omega_2 = \sqrt{\frac{(1 - a/L)k}{m}}$$

$$(-2k' + m\omega^2)a_1 + k'a_2 = 0$$

$$\text{Para } \omega_1 \quad a_1 = a_2$$

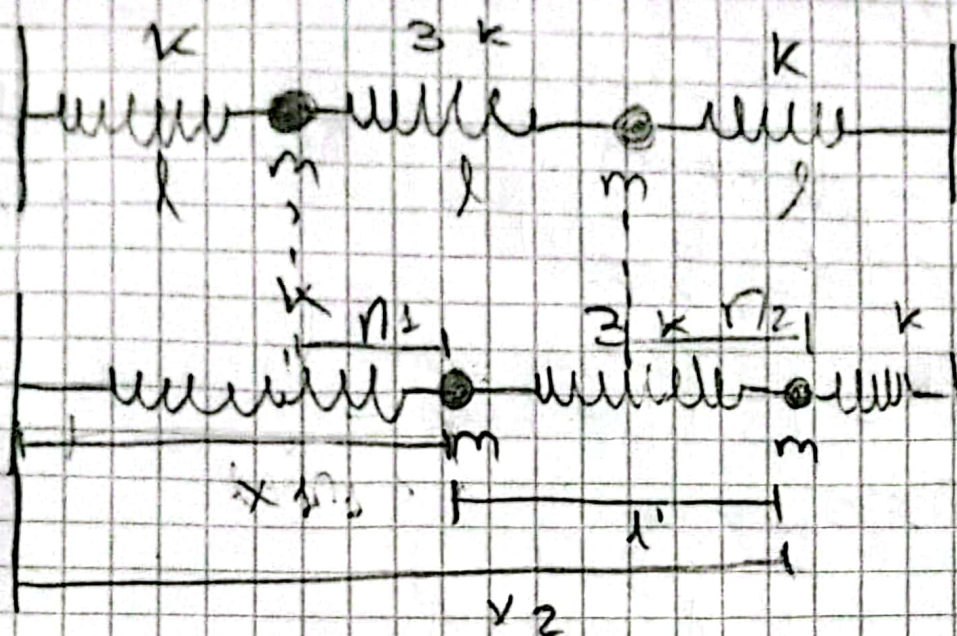
$$(-2k' + m\omega^2)a_2 + k'a_1 = 0$$

$$ka_1 = ka_2$$

$$\text{Para } \omega_2 \quad a_1 = -a_2$$

$$-ka_1 = ka_2$$





$$T = \frac{1}{2} m \dot{n}_1^2 + \frac{1}{2} m \dot{n}_2^2$$

$$V = \frac{1}{2} k n_1^2 + (l' - l) \frac{3k}{2} + n_2^2 \frac{k}{2}$$

$$V = \frac{k}{2} n_1^2 + \frac{3}{2} k (n_2 - n_1)^2 + n_2^2 \frac{k}{2}$$

$$V = \frac{k}{2} n_1^2 + \frac{3}{2} k (n_2^2 - 2n_2 n_1 + n_1^2) + n_2^2 \frac{k}{2}$$

$$V = k (2n_1^2 - 3n_2 n_1 + 2n_2^2)$$

$$V = \frac{1}{2} \sum_{i,j} V_{ij} n_i n_j$$

$$V_{11} = 4k \quad V_{22} = 4k$$

$$V_{21} = -3k \quad V_{12} = -3k$$



$$V = \begin{pmatrix} 4k & -3k \\ -3k & 4k \end{pmatrix}$$

$$T = \frac{1}{2} \sum T_{ij} \hat{n}_i \hat{n}_j$$

$$T_{11} = m$$

$$T_{12} = T_{21} = 0$$

$$T = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$T_{22} = m$$

$$\det |V_{ij} - \omega^2 T_{ij}| = 0$$

$$\begin{vmatrix} 4k - \omega^2 m & -3k \\ -3k & 4k - \omega^2 m \end{vmatrix} = 0$$

$$(4k - \omega^2 m)^2 - (3k)^2 = 0$$

$$4k - \omega^2 m = \pm 3k$$

$$\omega^2 = \frac{4k \pm 3k}{m}$$

$$\omega_1 = \sqrt{\frac{k}{m}}$$

$$\omega_2 = \sqrt{\frac{7k}{m}}$$

$$i=1: (4k - \omega_1^2 m) a_1 - 3k a_2 = 0$$

$$i=2: (-3k) a_1 + (4k - \omega_2^2 m) a_2 = 0$$



Con  $w_1$

$$(4k - k)a_1 - 3ka_2 = 0 \quad \Rightarrow (-3k)a_1 + (4k - k)a_2 = 0$$

$$3ka_1 - 3ka_2 = 0$$

$$a_1 - a_2 = 0$$

$$a_1 = a_2 = a_2$$

$$1 - 1 = 0$$

$$a_1 = 0$$

$$-3ka_2 + (3k)a_2 = 0$$

$$a_2(-3k + 3k) = 0$$

$$a_2 = 0$$

Con  $w_2$

$$(4k - 7k)a_1 - 3ka_2 = 0$$

$$\Rightarrow -3ka_1 - 3ka_2 = 0$$

$$a_2 = -a_1$$



$$2k + \frac{2\epsilon\alpha}{l^2} - \omega^2 m = \pm \left( -k - \frac{2\epsilon\alpha}{l^2} \right)$$

$$\omega^2 = \left( \frac{2\epsilon\alpha}{l^2} + 2k \pm \left( -k - \frac{2\epsilon\alpha}{l^2} \right) \right) \frac{1}{m}$$

$$\omega_1^2 = \frac{3k}{m} + \frac{4\epsilon\alpha}{l^2 m}$$

$$\omega_2^2 = \frac{k}{m}$$

$$\left( 2k + \frac{2\epsilon\alpha}{l^2} - \omega^2 m \right) a_1 + \left( -k - \frac{2\epsilon\alpha}{l^2} \right) a_2 = 0$$

$$\left( -k - \frac{2\epsilon\alpha}{l^2} \right) a_1 + \left( 2k + \frac{2\epsilon\alpha}{l^2} - \omega^2 m \right) a_2 = 0$$

For  $\lambda = 1$

$$\left( 2k + \frac{2\epsilon\alpha}{l^2} - 3k - \frac{4\epsilon\alpha}{l^2} \right) a_1 + \left( -k - \frac{2\epsilon\alpha}{l^2} \right) a_2 = 0$$

$$\left( -k - \frac{2\epsilon\alpha}{l^2} \right) a_1 + \left( -k - \frac{2\epsilon\alpha}{l^2} \right) a_2 = 0$$

$$a_1 = -a_2$$

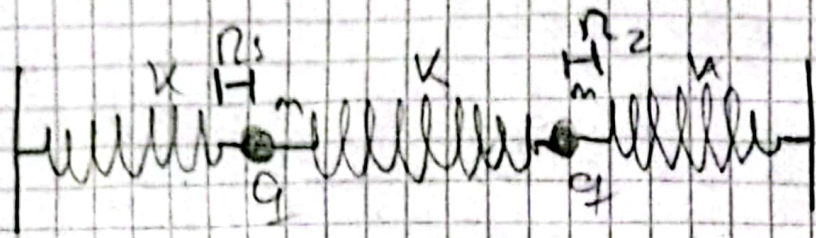
For  $\lambda = 2$

$$\left( 2k + \frac{2\epsilon\alpha}{l^2} - k \right) a_2 + \left( -k - \frac{2\epsilon\alpha}{l^2} \right) a_1 = 0$$

$$\left( k + \frac{2\epsilon\alpha}{l^2} \right) a_2 - \left( k + \frac{2\epsilon\alpha}{l^2} \right) a_1 = 0$$

$$a_2 = a_1$$





$$T = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2$$

$$V = \underbrace{\frac{1}{2} k x_1^2 + \frac{k}{2} (x_2 - x_1)^2 + x_2^2 \frac{k}{2}}_{V_{res}} + \underbrace{\frac{6q}{(l + x_2 - x_1)}}_{V_{el}}$$

$$V_{(res)} = \begin{pmatrix} 2k & -k \\ -k & 2k \end{pmatrix} \quad V_{(el)} = \frac{\partial^2 V_{ij}}{\partial x_i \partial x_j}$$

$$V = \begin{pmatrix} 2k + \frac{2 \cdot 6q}{l^2} & -k - \frac{2 \cdot 6q}{l^2} \\ -k - \frac{2 \cdot 6q}{l^2} & 2k + \frac{2 \cdot 6q}{l^2} \end{pmatrix} \quad V_{(el)} = \begin{pmatrix} \frac{2 \cdot 6q}{l^2} & -\frac{2 \cdot 6q}{l^2} \\ -\frac{2 \cdot 6q}{l^2} & \frac{2 \cdot 6q}{l^2} \end{pmatrix}$$

$$\det |V_{ij} - \omega^2 T_{ij}| = 0$$

$$\begin{vmatrix} (2k + \frac{2 \cdot 6q}{l^2}) - \omega^2 m & -k - \frac{2 \cdot 6q}{l^2} \\ -k - \frac{2 \cdot 6q}{l^2} & \omega^2 m + 2k + \frac{2 \cdot 6q}{l^2} \end{vmatrix} = 0$$

$$(2k + \frac{2 \cdot 6q}{l^2} - \omega^2 m)^2 - (-k - \frac{2 \cdot 6q}{l^2})^2 = 0$$