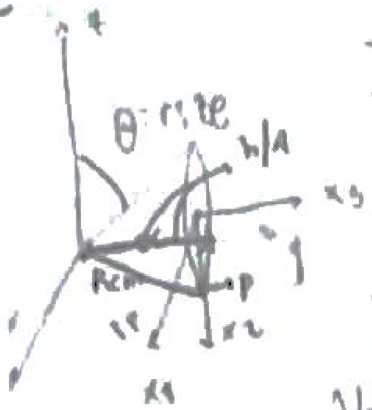


$$I_{1k} = I^{cm}_{k1} + m(d^2 \delta_{1k} - d_1 d_k)$$

$$I_{11} \cdot I_{22} = \frac{3}{20} M (R^2 + 4h^2)$$

$$I_{33} = \frac{3}{10} M R^2$$

Ligaduras

$V_p = 0 \cdot W = 0 \qquad |R_{cm}| = \frac{3}{4} h \ \hat{x}_3 \to \frac{3}{4} h [\operatorname{sen}\phi \operatorname{sen}\theta \ \hat{i} - \cos\phi \operatorname{sen}\theta \ \hat{j} + \cos\theta \ \hat{z}]$

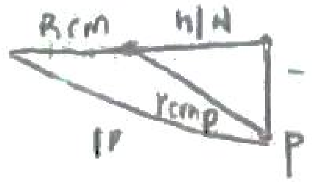
$x = \operatorname{sen}\phi \operatorname{sen}\theta \ 3/4 h \qquad \dot{x} = \dot{\phi} \operatorname{sen}\phi \operatorname{sen}\theta \ 3/4 h$

$y = \cos\phi \operatorname{sen}\theta \ 3/4 h \qquad \dot{y} = +\dot{\phi} \operatorname{sen}\phi \operatorname{sen}\theta \ 3/4 h$

$z = \cos\theta \ 3/4 h \qquad \dot{z} = 0$

$$r_p = R_{cm} + r_{cmp} \to 0 = \dot{R}_{cm} + \dot{r}_{cmp} \to \dot{R}_{cm} + (\tilde{\Omega} \times r_{cmp})$$

$$\begin{vmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 \\ \Omega_1 & \Omega_2 & \Omega_3 \\ 0 & -d & h/4 \end{vmatrix}$$



$r_{cmp} = -d \hat{x}_2 + h/4 \hat{x}_3$

$$0 = \dot{R}_{cm} + [(\Omega_2 h/4 + \Omega_3 d)\hat{x}_1 + (\Omega_1 h/4)\hat{x}_2 + (-\Omega_1 d)\hat{x}_3]$$

proyectamos al sistema laboratorio

$$0 = \hat{x} \dot{x} + [\Omega_2 h/4 + \Omega_3 d (\hat{x} \cdot \hat{x}_1) + (\Omega_1 h/4)(\hat{x} \cdot \hat{x}_2) - (\Omega_1 d)(\hat{x} \cdot \hat{x}_3)]$$

$\dot{x} + [\frac{h}{4}(\dot{\phi} \sin\theta \cos\psi - \dot{\theta} \sin\psi)(\cos\psi \cos\phi - \operatorname{sen}\psi \cos\theta \operatorname{sen}\phi) + (d \ \dot{\psi} + \dot{\phi} \cos\theta)$

$(\cos\psi \cos\phi - \cos\psi \cos\theta \operatorname{sen}\phi) + ((\dot{\phi} \operatorname{sen}\theta \operatorname{sen}\psi + \dot{\theta} \cos\psi \ h/4))$

$(-\operatorname{sen}\psi \cos\phi - \cos\psi \cos\theta \operatorname{sen}\phi) - (d \ \dot{\phi} \operatorname{sen}\theta \operatorname{sen}\psi + \dot{\theta} \cos\psi)(\operatorname{sen}\theta \operatorname{sen}\phi)]$

$\dot{x} + [h/4(\dot{\phi} \sin\theta \cos\psi - \dot{\theta} \sin\psi)(\cos\psi \cos\phi - \operatorname{sen}\psi \cos\theta \operatorname{sen}\phi) + (\dot{\phi} \operatorname{sen}\theta \operatorname{sen}\psi$

$+ \dot{\theta} \cos\psi)((\cos\psi \cos\theta - \cos\psi \cos\theta \operatorname{sen}\phi)]$

$+[d(\dot{\psi} + \dot{\phi} \cos\theta(\cos\psi \cos\theta - \cos\psi \cos\theta \operatorname{sen}\phi) - (\dot{\phi} \operatorname{sen}\theta \operatorname{sen}\psi$

$+ \dot{\theta} \cos\psi)(\operatorname{sen}\theta \operatorname{sen}\phi] = 0$

$$0 = \dot{\hat{y}} \cdot \dot{j} + \left[(\Omega_2 h/A + \Omega_3 d)(\hat{y} \cdot \hat{x}_1) + (\Omega_1 h/A)(\hat{y} \cdot \hat{x}_2) - (\Omega_1 d)(\hat{y} \cdot \hat{x}_3) \right]$$

$$\dot{j} + [h/A(\dot{\phi} \operatorname{sen}\theta \cos\psi - \dot{\theta} \operatorname{sen}\psi)(\cos\psi \operatorname{sen}\phi + \operatorname{sen}\psi \cos\theta \cos\phi)$$

$$+(\dot{\phi} \operatorname{sen}\theta \operatorname{sen}\psi + \dot{\theta}\cos\psi)(-\operatorname{sen}\psi \operatorname{sen}\phi + \cos\psi\cos\theta\cos\phi]$$

$$+[d(\dot{\psi} + \dot{\phi}\cos\theta)(\cos\psi \operatorname{sen}\phi + \operatorname{sen}\psi \cos\theta\cos\phi) -$$

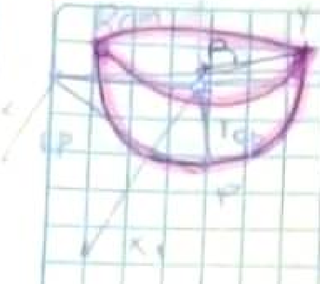
$$(\dot{\phi} \operatorname{sen}\theta \operatorname{sen}\psi + \dot{\theta}\cos\psi)(-\operatorname{sen}\theta\cos\phi] = 0$$

$$0 = \dot{\hat{z}} \cdot \dot{z} + \left[(\Omega_2 h/A + \Omega_3 d)(\hat{z} \cdot \hat{x}_1) + (\Omega_1 h/A)(\hat{z} \cdot \hat{x}_2) - (\Omega_1 d)(\hat{z} \cdot \hat{x}_3) \right.$$

$$\dot{z} + [h/A((\dot{\phi} \operatorname{sen}\theta\cos\psi - \dot{\theta}\operatorname{sen}\psi)(\operatorname{sen}\psi \operatorname{sen}\theta) + (\dot{\phi}\operatorname{sen}\theta \operatorname{sen}\psi + \dot{\theta}\cos\psi)$$

$$(\cos\psi \operatorname{sen}\theta)] + [d(\dot{\psi} + \dot{\phi}\cos\theta)(\operatorname{sen}\psi \operatorname{sen}\theta) - (\dot{\phi} \operatorname{sen}\theta \operatorname{sen}\psi + \dot{\theta}\cos\psi$$

$$(-\operatorname{sen}\theta \cos\phi)] = 0$$



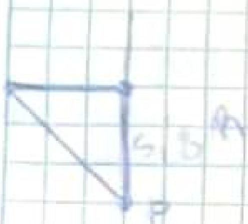
$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2} (I_{11} \Omega_1^2 + I_{22} \Omega_2^2 + I_{33} \Omega_3^2)$$

$V + W$

$v_{cm} + r_{cp}$

$0 = v_{cm} + (\Omega \times r_{cp})$

$$\begin{vmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 \\ \Omega_1 & \Omega_2 & \Omega_3 \\ 0 & -5/8R & 0 \end{vmatrix} \quad \Omega_3 \frac{5}{8} R \hat{x}_1 - \Omega_1 \frac{5}{8} R \hat{x}_3$$



$0 = \hat{x} (\dot{x} + \Omega_3 5/8R \hat{x}_1 - \Omega_1 5/8R \hat{x}_3)$

$0 = \hat{y} (\dot{y} + \Omega_3 5/8R \hat{x}_1 - \Omega_1 5/8R \hat{x}_3)$

$0 = \hat{z} (\dot{z} + \Omega_3 5/8R \hat{x}_1 - \Omega_1 5/8R \hat{x}_3)$

$\dot{x} + 5/8R (\Omega_3 (\hat{x} \cdot \hat{x}_1) - \Omega_1 (\hat{x} \cdot \hat{x}_3)) = 0$

$\dot{y} + 5/8R (\Omega_3 (\hat{y} \cdot \hat{x}_1) - \Omega_1 (\hat{y} \cdot \hat{x}_3)) = 0$

$\dot{z} + 5/8R (\Omega_3 (\hat{z} \cdot \hat{x}_1) - \Omega_1 (\hat{z} \cdot \hat{x}_3)) = 0$

$\dot{x} + 5/8R (\dot{\phi} \cos\phi \cos\theta - \dot{\theta} \operatorname{sen}\phi \operatorname{sen}\theta + \dot{\psi} \cos\phi) = 0$

$\dot{y} + 5/8R (\dot{\phi} \operatorname{sen}\phi \cos\theta + \dot{\theta} \cos\phi \operatorname{sen}\theta + \dot{\psi} \operatorname{sen}\phi) = 0$

$\dot{z} - 5/8R \, \dot{\theta} \cos\theta = 0$

$$T = \frac{1}{2} m \Big[\big(\tfrac{5}{8} R (\dot{\phi} \cos\phi \cos\theta - \dot{\theta} \operatorname{sen}\phi \operatorname{sen}\theta + \dot{\psi} \cos\phi)\big)^2 +$$
$$5/8R (\dot{\phi} \operatorname{sen}\phi \cos\theta + \dot{\theta} \cos\phi \operatorname{sen}\theta + \dot{\psi} \operatorname{sen}\phi)^2$$
$$+ (5/8R \, \dot{\theta} \cos\theta)^2 + \frac{1}{2} \Big(\frac{2}{5} m R^2 (\dot{\phi} \sin\theta \sin\psi + \dot{\theta} \cos\psi)^2$$
$$+ \frac{2}{5} m R^2 (\dot{\phi} \sin\theta \cos\psi - \dot{\theta} \sin\psi)^2$$
$$+ \frac{2}{3} m R^2 (\dot{\psi} + \dot{\phi} \cos\theta)^2$$

$\dot{z} = 5/8 R \dot{\theta} \cos\theta$

$z = 5/8R \operatorname{sen}\theta$

$V = mg(5/8R \operatorname{sen}\theta)$

Para ángulos pequeños

$\operatorname{sen}\theta \approx \theta \qquad \cos\theta \approx 1$