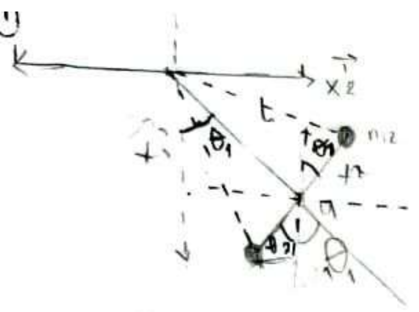
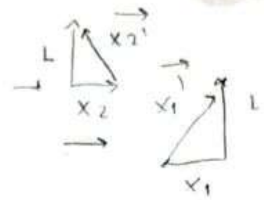


1 Ligadura $\rightarrow$

$\vec{x_2'} = \vec{x_2} + \vec{L}$

$\vec{x_1'} = \vec{x_1} + \vec{l}$



$2(2)-(2) \rightarrow$ 2 grados de libertd

dado qe mi valor de x_2 y $x_1 = a/2$

Usando Coordenadas Polares $\rightarrow$ $\vec{l} = \text{Sen}\theta_1 l \hat{i} - \cos\theta_1 l \hat{j}$

$\vec{x_2} = \text{Sen}\theta_2 x_2 \hat{i} + \cos\theta_2 x_2 \hat{j}$

$\vec{x_1} = -\text{Sen}\theta_2 x_1 \hat{i} + \cos\theta_2 x_1 \hat{j} \rightarrow y_2$

$\vec{x_2'} = \vec{x_2} + \vec{l} \rightarrow (\text{Sen}\theta_2 x_2 + \text{Sen}\theta_1 l)\hat{i} + (-\cos\theta_2 x_2 - \cos\theta_1 l)\hat{j} \rightarrow y_1$

$\vec{x_1'} = \vec{x_1} + \vec{l} \rightarrow (-\text{sen}\theta_1 x_1 + \text{Sen}\theta_1 l)\hat{i} + (-\cos\theta_1 l - \cos\theta_2 x_1)\hat{j}$

definiendo el lagrangiano: $T - V$ $\qquad T = \frac{1}{2} m_1 (\dot{x}_1')^2 + \frac{1}{2} m_2 \dot{x}_2'^2$

$V = -m_1 g y_1 - m_2 g y_2$

$\dot{x}_1' = (\cos\theta_1 \dot{\theta}_1 l - \cos\theta_2 \dot{\theta}_2 a/2)\hat{i} + (\text{sen}\theta_1 \dot{\theta}_1 l + \text{Sen}\theta_2 \dot{\theta}_2 a/2)\hat{j}$

$\dot{x}_2' = (\cos\theta_1 \dot{\theta}_1 l + \cos\theta_2 \dot{\theta}_2 a/2)\hat{i} + (l\,\text{sen}\theta_1 \dot{\theta}_1 l - \text{sen}\theta_2 \dot{\theta}_2 a/2)\hat{j}$

• $L = \frac{1}{2} m_1 \underset{①}{(\dot{x}_1')^2} + \frac{1}{2} m_2 \underset{②}{(\dot{x}_2')^2} - (m_2 g(-\cos\theta_2 a/2 - \cos\theta_1 l) - m_1 g(-\cos\theta_1 l - \cos\theta_2 a/2)$

① $1/2\, m_1 ((\cos\theta_1 \dot{\theta}_1 l) - (\cos\theta_2 \dot{\theta}_2 a/2))^2 + ((\text{Sen}\theta_1 \dot{\theta}_1 l + \text{Sen}\theta_2 \dot{\theta}_2 a/2)^2)$

② $\frac{1}{2} m_2 ((\cos\theta_1 \dot{\theta}_1 l + \cos\theta_2 \dot{\theta}_2 a/2)^2 + (l\,\text{sen}\theta_1 \dot{\theta}_1 l - \text{Sen}\theta_2 \dot{\theta}_2 a/2)^2$

$L = \frac{1}{2} m_1 (l^2 \dot{\theta}^2 - a l \cos(\theta_2 + \theta_1)\dot{\theta}_1 \dot{\theta}_2 + \frac{a^2}{4}\dot{\theta}_2^2) + \frac{1}{2} m_2 (l^2 \dot{\theta}_1^2 + a l \cos(\theta_1 + \theta_2)\dot{\theta}_1 \dot{\theta}_2$

$+ \frac{a^2}{4}\dot{\theta}_2^2) - m_1 g(-l\cos(\theta_1) - a/2 \cos\theta_2)) - m_2 g(-l\cos(\theta_1) + a/2 \cos\theta_2)$

Sabiendo qe las ligaduras son holómicas, usamos

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0 \quad ①$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0. \quad ②$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2} m_2 \left(2 L^2 \dot{\theta}_1 + aL\cos(\theta_1 + \theta_2)\right)\dot{\theta}_2 + \tfrac{1}{2} m_1 \left(2 L^2 \dot{\theta}_1 - aL\cos(\theta_1 + \theta_2)\right)\dot{\theta}_1$$

$$\frac{\partial L}{\partial \theta} = \frac{1}{2} m_2 \left(-aL\,\mathrm{Sen}(\theta_1 + \theta_2)\dot{\theta}_1 \dot{\theta}_2\right) - \tfrac{1}{2} m_1 \left(aL\,\mathrm{Sen}(\theta_1 + \theta_2)\dot{\theta}_1 \dot{\theta}_2\right.$$

$$- m_1 g L\,\mathrm{sen}\theta_1 - m_2 g L\,\mathrm{sen}\theta_1 = 0$$

$$\frac{d}{dt}\left(\tfrac{1}{2} m_2 \left(2 L^2 \dot{\theta}_1 + aL\cos(\theta_1 + \theta_2)\right)\dot{\theta}_2 + \tfrac{1}{2} m_1 \left(2 L^2 \dot{\theta}_1 - aL\cos(\theta_1 - \theta_2)\dot{\theta}_2\right)\right.$$

$$- \tfrac{1}{2} m_2 \left(aL\,\mathrm{Sen}(\theta_1 + \theta_2)\right)\dot{\theta}_1 \dot{\theta}_2 - \tfrac{1}{2} m_1 \left(aL\,\mathrm{Sen}(\theta_1 + \theta_2)\right)\dot{\theta}_1 \dot{\theta}_2$$

$$- m_1 g L\,\mathrm{Sen}\theta_1 - m_2 L g\,\mathrm{Sen}\theta_2 = 0$$

Realizando las derivadas y factorizando.

$$(m_2 - m_1)\tfrac{1}{2} aL\left(\ddot{\theta}_2 \cos(\theta_1 + \theta_2) + \dot{\theta}_2\left(-\mathrm{sen}(\theta_1 + \theta_2)\right)\dot{\theta}_1\right.$$

$$\left.- \mathrm{Sen}(\theta_1 + \theta_2)\dot{\theta}_2\right) + \tfrac{1}{2}(m_2 - m_1)\, aL\,\mathrm{Sen}(\theta_1 + \theta_2)\dot{\theta}_1 \dot{\theta}_2 +$$

$$(m_1 + m_2)\, g L\,\mathrm{Sen}\theta_1 = 0$$

---

(2) $$\frac{\partial}{\partial \dot{\theta}_2} = \tfrac{1}{2} m_1 \left(-aL\cos(\theta_1 + \theta_2)\dot{\theta}_1 + \frac{a^2}{2}\dot{\theta}_2\right) + \frac{1}{2} m_2 \left(aL\cos(\theta_1 + \theta_2)\dot{\theta}_1 + \right.$$

$$\left.\frac{a^2}{2}\dot{\theta}_2\right)$$

$$\frac{\partial}{\partial \theta_2} = \tfrac{1}{2} m_2 \left(-aL\,\mathrm{Sen}(\theta_2 + \theta_1)\right)\dot{\theta}_1 \dot{\theta}_2 - \tfrac{1}{2} m_1 \left(aL\,\mathrm{Sen}(\theta_1 + \theta_2)\right)\dot{\theta}_1 \dot{\theta}_2$$

$$- m_1 g \,a/2\,\mathrm{sen}\theta_2 + m_2 g\, a/2\,\mathrm{sen}\theta_2$$

$$\frac{d}{dt}\left(\tfrac{1}{2} m_1 \left(-aL\cos(\theta_1 + \theta_2)\dot{\theta}_1 + \frac{a^2}{2}\dot{\theta}_2\right) + \tfrac{1}{2} m_2\left(aL\cos(\theta_1 + \theta_2) + \frac{a^2}{2}\dot{\theta}_2\right)\right.$$

$$- m_1 g\, a/2\,\mathrm{Sen}\theta_2 + m_2 g\, a/2\,\mathrm{Sen}\theta_2 + \tfrac{1}{2} m_2 \left(aL\,\mathrm{Sen}(\theta_1 + \theta_2)\right)\dot{\theta}_1 \dot{\theta}_2$$

$$- \mathrm{Sen}(\theta_1 + \theta_2)\,\tfrac{1}{2} m_1 \dot{\theta}_1 \dot{\theta}_2$$

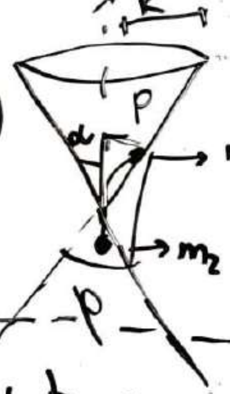
Realizando las derivadas y factorizando

$$\frac{1}{2}(m_2 - m_1) aL\,\mathrm{Sen}(\theta_1 + \theta_2)\dot{\theta}_1 \dot{\theta}_2 + (m_1 - m_2)\, g\, a/2\,\mathrm{Sen}\theta_2 + \tfrac{1}{2}(m_2 - m_1)$$

$$aL\left(\ddot{\theta}_1 \cos(\theta_1 + \theta_2) + \dot{\theta}_1\left(-\mathrm{sen}(\theta_1 + \theta_2)\right)\dot{\theta}_1 - \mathrm{Sen}(\theta_1 + \theta_2)\dot{\theta}_2\right)$$

$$+ \tfrac{1}{4}(m_1 + m_2)\frac{a^2}{2}\ddot{\theta}_2 = 0$$

2



$m_1 \rightarrow$ $x = \cos\varphi\, \hat{p} - \operatorname{Sen}\varphi\, \hat{e}$

$y = \operatorname{Sen}\varphi\, \hat{p} + \cos\varphi\, \hat{\varphi}$

$3(2) - 4 = 2$ grados de libertad

$z\ \varphi$

$m_2 \rightarrow$ $x = 0$ $\quad z = z\hat{z}$

$y = 0$ $\quad \rightarrow \hat{\varphi} = 0$ (1) $\cdot\ r_1 + r_2 = l$ (3)

$z_2 = z_2 \hat{z}$ $\quad \hat{p} = 0$ (2) $\quad z_1 = P\ \ h/R$

Variación lineal de m_1

$|\vec{r_1}| + |\vec{r_2}| = L$ $\qquad |\vec{r_1}| =$

$z = P \cot\alpha/2$ $\qquad \tan = \frac{R}{h}$ $\quad \frac{co}{ca}$

$\vec{r_2} = z_2\, \hat{k}$

$\vec{r_1} = P(\cos\varphi\, \hat{i} + P\operatorname{Sen}\varphi\, \hat{j} + z\, \hat{k}$

$\vec{r_1} = P(\cos\varphi\, \hat{i} + \operatorname{Sen}\varphi\, \hat{j}) + z\hat{z}$ $\quad \rightarrow \vec{r_1} = P\hat{p} + P\ h/R\ \hat{k}$

$\vec{r_1} = \underbrace{P(\cos\varphi\, \hat{i} + \operatorname{Sen}\varphi\, \hat{j}}_{\hat{p}}$ $\qquad \vec{v} = \dot{P}\hat{p} + P\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$

$L = \frac{1}{2} m_1 (\dot{P}^2 \hat{P} + \dot{P}\dot{\theta}^2 \hat{\theta} + \dot{z}^2\hat{z}) + \frac{1}{2} m_2 |\dot{P}^2\hat{P} + \dot{p}\dot{\theta}\hat{\theta} + \dot{z}\hat{z}| - mgz - mgz_2$

como mis restricciones son holonómicas

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \qquad q_i = (P, z)$$

$z_2 = P\ h/R$

$\vec{r_1} + \vec{r_2} = l$

$l = z_2^2 + P^2 (1 + \cot\alpha$

$z_2 = P\sqrt{1 + \cot\alpha}$

$+\ l$

$$L = \frac{1}{2} m_1 \left(\dot{p}^2 + p^2\dot{\theta}^2 + (\dot{p}h/R)^2 \right) + \frac{1}{2} m_2 \dot{p}^2 (1+\cot^2\alpha) - m_1 g\, p h/R - m_2 g \left[p\sqrt{1+\cot^2\alpha} + z_1 \right]$$

dado que $\dot{z}_1 = \dot{p}\, h/R$

dado que Fenki $m_2 = 0$

dado que $\dot{z}_2 = \dot{p}\sqrt{1+\cot^2\alpha}$

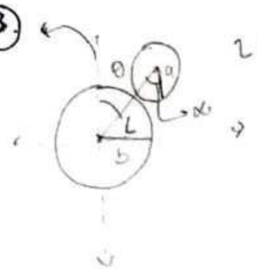
Para P: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{p}}\right) - \frac{\partial L}{\partial p} = 0 \rightarrow m_1 \ddot{p} + m_1 \ddot{p}\, h^2/R^2 - m_1 p \dot{\theta}^2 - m_1 g\, h/R - m_2 g\sqrt{1+\cot^2\alpha} = 0$ (1)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \rightarrow m_1 p^2 \ddot{\theta} = 0$$

(1)

1 ligadura $\rightarrow$ $\vec{v}$ +1   L

(3)



$l = a+b$

Por la condición de rodar sin deslizar
$\dot{\alpha} a = \dot{\theta} l \rightarrow$ integrando $\rightarrow$ pq no puede depender de α y θ

$\alpha a - \theta l = 0$ $\rightarrow$ la ligadura de $l = a+b$ estará metida implícitamente en α y θ

$x = l \operatorname{sen}\theta$ $\dot{x} = l\cos\theta\dot{\theta}$

$y = l\cos\theta$ $\dot{y} = -l\operatorname{sen}\theta\dot{\theta}$

$T = 1/2\, m a^2 \dot{\alpha}^2 + 1/2\, m\, l^2 \dot{\theta}^2$ $V = +mg\, y$

rotacional   traslacional $V = +mg\, l\cos\theta$

$L = \frac{1}{2} m a^2 \dot{\alpha}^2 + \frac{1}{2} m l^2 \dot{\theta}^2 - mg\, l\cos\theta$

Por euler lagrange

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = -\sum^{m} \frac{\partial f_\alpha}{\partial q_i} \lambda_\alpha$ con $\dot{\alpha} a = \dot{\theta} l$

Para θ: $m l^2 \ddot{\theta} - mg\, l \operatorname{sen}\theta = -\frac{\lambda_1}{l}$

$\lambda = \frac{m l^2 \ddot{\theta}}{a}$

Para α $m a \ddot{\alpha} = -\lambda_1$

$m a \ddot{\alpha} = mg \operatorname{sen}\theta - m l \ddot{\theta}$

$a\ddot{\alpha} = g \operatorname{sen}\theta - l\ddot{\theta}$ $\rightarrow$ ecuación que describe el sistema; el cilindro a se separa de b cuando $\dot{\theta} = 0$

$\frac{a\, d\dot{\alpha}}{dt} = g\operatorname{sen}\theta - l\frac{d\dot{\theta}}{dt}$ $\ddot{\alpha} = \frac{\ddot{\theta} l}{a}$

$(\ddot{\theta} l) = g\operatorname{sen}\theta - l\ddot{\theta}$

$2\ddot{\theta} l - g\operatorname{sen}\theta = 0$

$\ddot{\theta} = \frac{g\operatorname{sen}\theta}{2l}$

$= \frac{d}{dt}(\dot{\theta}^2) \Rightarrow 2\dot{\theta}\ddot{\theta}$

$(\dot{\theta})\frac{d}{dt}\dot{\theta} = (\dot{\theta})\frac{g\operatorname{sen}\theta}{2l}$

$\frac{1}{2}\frac{d}{dt}(\dot{\theta})^2 = \frac{g\dot{\theta}\operatorname{sen}\theta}{2l}$

$\int \frac{d}{dt}\dot{\theta}^2 = \int g \frac{\sin\theta}{l} d\theta$

$\dot{\theta}^2 = -\frac{g}{l}\cos\theta + C_1$

$0 = -\frac{g}{l} + C_1$, $C_1 = \frac{g}{l}$

$\rightarrow \dot{\theta}^2 = \frac{g}{l}(1-\cos\theta)$

$\ddot{\theta} = \frac{d}{dt}\sqrt{\frac{g}{l}(1-\cos\theta)}$

ara calclar el angulo donde se separa

se ultilizará q e

$F(r, \theta, \alpha)$ hay otra λ_1

$$\frac{\partial L}{\partial r} + \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) = -r m\dot{\theta}^2 + mg\cos\theta = \lambda_2$$

λ_1 es la fuerza normal, λ_1

$$mg\cos\theta - r\, m\dot{\theta}^2 = 0$$

$$\cancel{m}g\cos\theta = 2r\left(\cancel{m}\frac{g}{\cancel{r}}\right)(1-\cos\theta$$

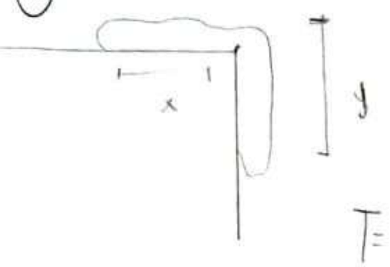
$$\cos\theta = (1-\cos\theta)$$

$$2\cos\theta = 1 \quad \approx 1/3\pi$$

(4)

$\lambda = \frac{M}{y} = \frac{dm}{dy}$ $\quad dv = dm\, g\, y \rightarrow V = y^2\, 1/2\, \lambda g$

$dv = \lambda\, dy\, g\, y$



$l = x + y$

$\lambda(1) - 1 = 1$

$T = \frac{1}{2} M \dot{y}^2 - \frac{1}{2} g \lambda y^2$

como $l = x+y$ es holonómica y no afecta a l, entonces:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

$$\frac{\partial L}{\partial y} = -g \frac{M}{l} y \qquad \frac{\partial L}{\partial \dot{y}} = M\dot{y}$$

$$M\ddot{y} - g\frac{M}{l} y = 0$$

$$\ddot{y} = g/l\, y$$

$$\dot{y} \frac{d}{dt}\dot{y} = \frac{g}{l} y \dot{y} \quad \rightarrow \frac{d}{dt}(\dot{y}^2) = 2\dot{y}\ddot{y}$$

$$\frac{d}{dt}(\dot{y}^2) = g y \dot{y} \qquad \dot{y} = \sqrt{\frac{g}{l}}\, e^{\sqrt{g/l}\, t} \qquad \ddot{y} = \left(\frac{g}{l}\right) e^{\sqrt{g/l}\, t}$$

$$\dot{y}^2 = g\, 1/l\, y^2 \qquad \left(\frac{g}{l}\right) e^{\sqrt{g/l}\, t} = g/l\, e^{\sqrt{g1/l}\, t}$$

$$\dot{y} = y\sqrt{g\, 1/l} \qquad e^{\sqrt{g/l}\, t}(g/l - g/l) = 0$$

$$\frac{dy}{y} = \sqrt{g1/l}\, dt$$

$$\ln y = \sqrt{g1/l}\, t$$

$$y = e^{\sqrt{g1/l}\, t}$$