2)

a) Para demostrar que una cantidad es conservada se debe cumplir:

$$\frac{df}{dt} = 0$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \{f, H\} = 0$$

$$= -2H + \{pq - 2Ht, H\} = 0$$

$$= -2H + \{pq, H\} - 2t\cancel{\{H, H\}} = 0$$

$$\{pq, H\} = \frac{\partial pq}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial H}{\partial q}$$

$$= p(p) - q\left(\frac{1}{q^3}\right)$$

$$= p^2 - \frac{1}{q^2} = 2H$$

$$\frac{df}{dt} = -2H + 2H = 0$$

b) $Q = \lambda q \qquad P = \lambda^{-1} p$

$$\{Q, P\} = \frac{\partial Q}{\partial q}\frac{\partial P}{\partial p} - \frac{\partial P}{\partial q}\frac{\partial Q}{\partial p} = \lambda\lambda^{-1} - 0 = 1$$

$$\{Q, Q\} = \frac{\partial Q}{\partial q}\cancel{\frac{\partial Q}{\partial p}} - \frac{\partial Q}{\partial q}\cancel{\frac{\partial Q}{\partial p}} = 0$$

$$\{P, P\} = \cancel{\frac{\partial P}{\partial q}}\frac{\partial P}{\partial p} - \cancel{\frac{\partial P}{\partial q}}\frac{\partial P}{\partial p} = 0$$

$f_1(q_1, p_1)$ $\quad H = H(f(q_1, p_1), q_2, q_3 \dots p_1, p_2, \dots p_n)$

a. demostrar que $f(q_1, p_1)$ es una constante del movimiento

$$\{f, H\} = 0 \quad \left(\frac{\partial f}{\partial q_1}\frac{\partial H}{\partial p_1} - \frac{\partial f}{\partial p_1}\frac{\partial H}{\partial q}\right) = 0$$

$$\frac{df}{dt} = \frac{\partial f}{\partial q_1}\frac{\partial H}{\partial q_1} - \frac{\partial f}{\partial p_1}\frac{\partial H}{\partial q_1} + \frac{\partial f}{\partial t} = 0$$

$$\frac{\partial f}{\partial q_1}\frac{\partial H}{\partial p_1} = \frac{\partial f}{\partial p_1}\frac{\partial H}{\partial q_1} \longrightarrow \frac{\partial f}{\partial q_1}\left(\frac{\partial H}{\partial f}\frac{\partial f}{\partial q_1}\right) = \frac{\partial f}{\partial p_1}\left(\frac{\partial H}{\partial f}\frac{\partial f}{\partial p_1}\right)$$

then: $\frac{\partial H}{\partial f}\left(\frac{\partial f}{\partial q_1}\frac{\partial f}{\partial p_1} - \frac{\partial f}{\partial p_1}\frac{\partial f}{\partial p_1}\right) = 0$

Porque $\left.\frac{\partial H}{\partial q_1}\right|_f = \frac{\partial H}{\partial p_1} = 0.$

b. Partícula sometida a $\quad V = \frac{\vec{a} \cdot \vec{r}}{r^3} \qquad a = a_z \hat{z}$

$$\vec{r} = r\sin\theta\cos\phi\,\hat{x} + r\sin\theta\sin\phi\,\hat{y} + r\cos\theta\,\hat{z}$$

$\hat{r}_z = \cos\theta\,\hat{z}$ $\qquad a_z \cdot \hat{r} \rightarrow \frac{a_z\cos\theta}{r^2}$

$$L = 1/2\,m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin\theta\,\dot{\phi}^2) - \frac{a_z\cos\theta}{r^2}$$

$\frac{\partial L}{\partial \dot{r}} = m\dot{r} = P_r$

$\frac{\partial L}{\partial \dot{\theta}} = r^2 m\dot{\theta} = P_\theta$ $\qquad H = \frac{P_r^2}{2m} + \frac{P_\theta^2}{2m^2r^2} + \frac{a_z\cos\theta}{r^2}$

$\frac{\partial L}{\partial \dot{\phi}} = r^2\sin\theta\, m\, 1/2\,\dot{\phi} = P_\phi$ $\qquad$ buscar la función $f(q_1, p_1)$

$$f(\theta, P_\theta) = \left[\frac{P_\theta^2}{2m} + a_z\cos\theta\right] \qquad \frac{P_r^2}{2m} + \frac{1}{r^2}\left[\frac{P_\theta^2}{2m} + a_z\cos\theta\right]$$

Por el planteamiento anterior, $f(\theta, P_\theta)$ es una constante de movimiento, a su vez H y E son constantes

$$H = \frac{P_r^2}{2m} + \frac{P_\theta^2}{2mr^2} + \frac{P_\phi^2}{2mr^2\sin^2\theta} + V(r,\theta)$$

Con el Potencia $V(\vec{r}) = \vec{a} \cdot \vec{r}$

Calcular $\{\vec{r} \cdot \vec{P}, H\} \rightarrow r^{13}\{P, H\} + P\{r, H\}$ $\hat{r}r$

$$L = 1/2\, m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin\theta\,\dot{\phi}^2) - \frac{a_z\cos\theta}{r^2}$$

$$\hat{r}\left(-r\frac{\partial H}{\partial r} + P_r\frac{\partial H}{\partial P_r}\right) = \{\vec{r} \cdot \vec{p}, H\}$$ → momento radial

$$(\dot{P}_r r + P_r^2/m)\hat{r}$$

Con lo anterio, $\{P_\phi, H\} = 0$ Es una cantidad Conservada

Como el momento radial no le Conserva

$H = cste = H$

$P_\phi = F_2 + \epsilon\, G$ → función generadora infinitesimal

$P_\phi = m r^2 \sin\theta\, \dot{\phi} = L_\phi$

$\phi \rightarrow \phi + \epsilon f$ $\quad P_\phi \rightarrow P_\phi + \epsilon g$ $\quad$ 1 Consideremos la siguiente transformación infintesimal

$$F_2(\phi, P_\phi) = \Sigma\, \phi P_\phi + \epsilon\, G$$

$$P_\phi = \frac{\partial F_2}{\partial \phi} = P_\phi - \epsilon\frac{\partial G}{\partial \phi} \qquad \phi = \frac{\partial F_2}{\partial P_\phi} = \phi + \epsilon\frac{\partial G}{\partial P_\phi}$$

Comparando:

$$f = \frac{\partial G}{\partial P_\phi} \qquad g = \frac{\partial G}{\partial \phi}$$

$$\{K, P_\phi\} = 0$$

$$\frac{\partial K}{\partial \phi}\frac{\partial P_\phi}{\partial P_\phi} - \frac{\partial K}{\partial P_\phi}\frac{\partial P_\phi}{\partial \phi} = 0$$