Análisis de un sistema de dos péndulos acoplados

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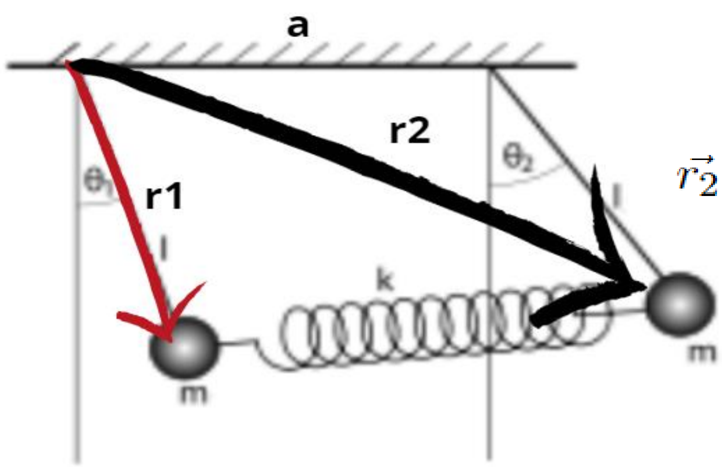


$$\vec{r_1} = bsen(\theta_1)\hat{i} - bcos(\theta_1)\hat{j}$$

$$\vec{r_2} = \vec{a} + \vec{b} = (a + bsen(\theta_2))\hat{i} - bcos(\theta_2)\hat{j}$$

$$T = \frac{1}{2}m\left(\dot{r}_1^2 + \dot{r}_2^2\right) = \frac{1}{2}mb^2\left(\dot{\theta}_1^2 + \dot{\theta}_2^2\right)$$

$$V = \frac{1}{2}k\Delta x^2 + mgb\,(1 - \cos\theta_1) + mgb\,(1 - \cos\theta_2)$$

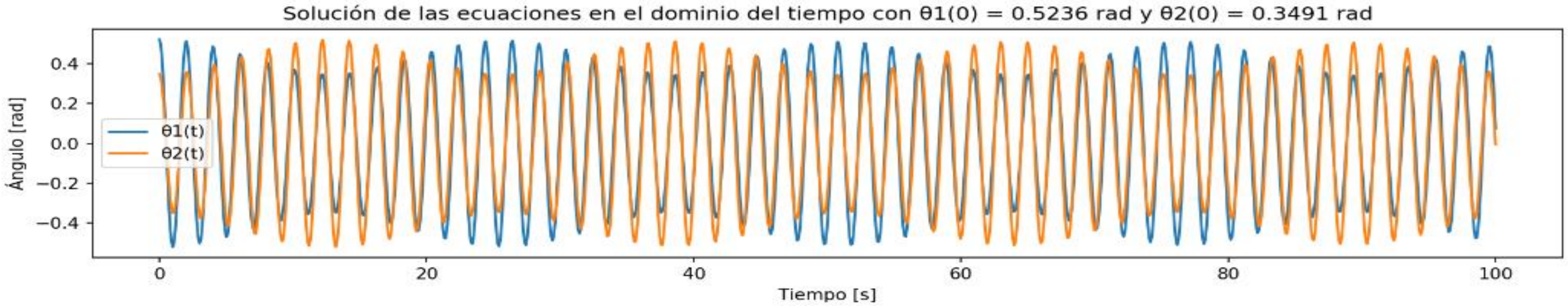
En donde nuestro Δx estará dado por:

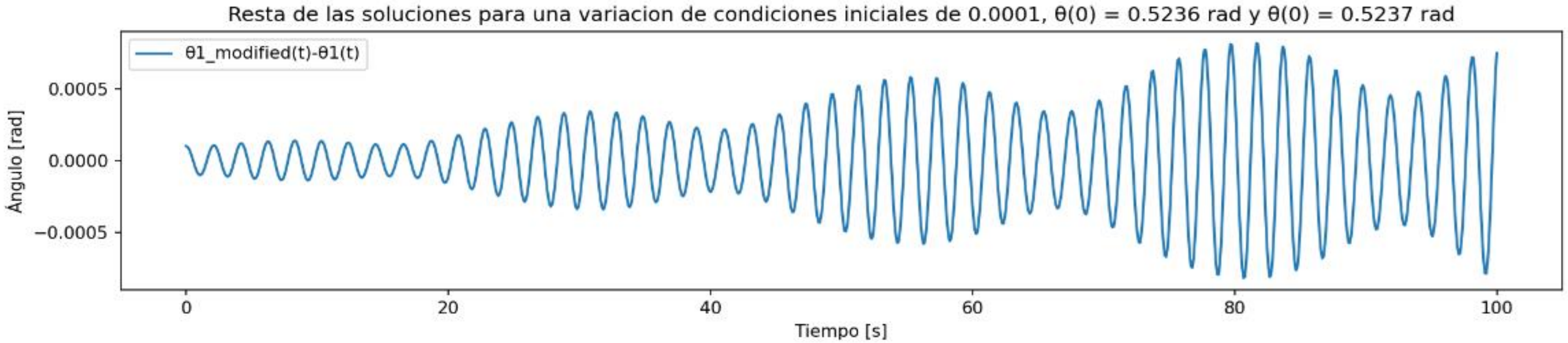
$$\Delta x = |\vec{r_1} - \vec{r_2}| - a \longrightarrow \left((b\,\mathrm{sen}\,\theta_1 - a - b\,\mathrm{sen}\,\theta_2)^2 + (b\cos\theta_2 - b\cos\theta_1)^2\right)^{1/2} - a$$

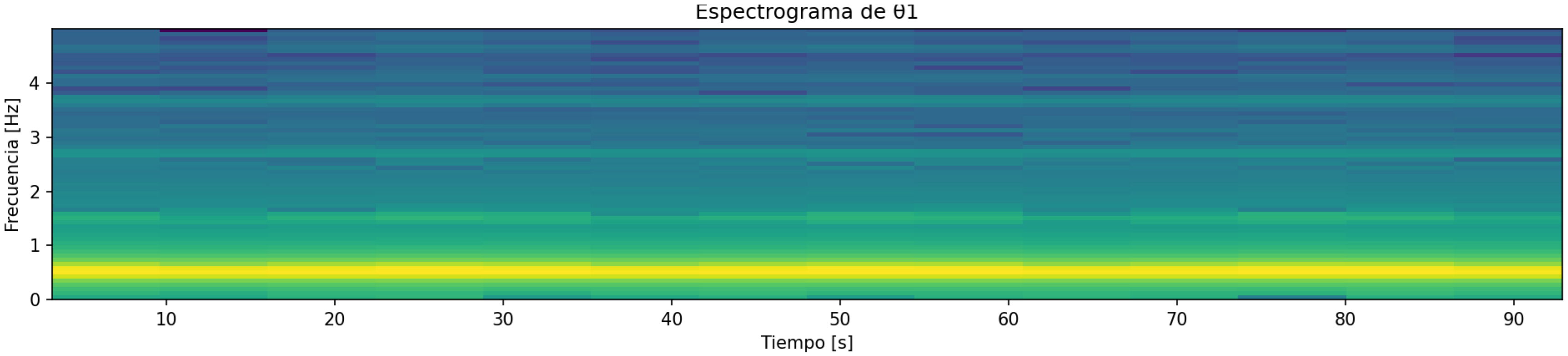
$$\mathscr{L} = \frac{1}{2}mb^2(\dot{\theta}_1^2 + \dot{\theta}_2^2) - mgb(1 - \cos\theta_1) - mgb(1 - \cos\theta_2) - -\frac{k}{2}((-2b^2(\cos(\theta_2 - \theta_1) + 2b^2 + a^2 + 2ba(\sin\theta_2 - \sin\theta_1)) - 2a(-2b^2\cos(\theta_2 - \theta_1) + 2b^2 + a^2 + 2ba(\sin\theta_2 - \sin\theta_1))^{1/2}) + a^2$$

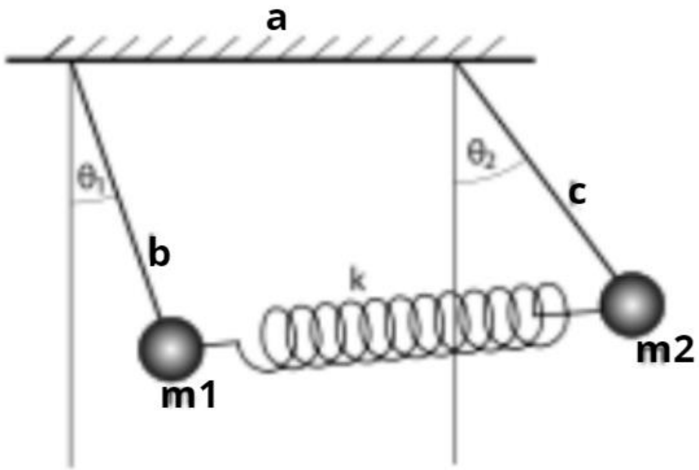
$$\theta_1 \longrightarrow mb\ddot{\theta}_1 + mgb\,\mathrm{sen}\,\theta_1 - k\left(b^2\,\mathrm{sen}\,(\theta_2 - \theta_1) + ba\cos\theta_1\right) + \frac{ak}{2}\left(2b^2 + a^2 + 2ab\,(\sin\theta_2 - \sin\theta_1)\right. - 2b^2\cos\left(\theta_2 - \theta_1\right)^{-1/2}\left(2b^2\,\mathrm{sen}\,(\theta_2 - \theta_1\,) + 2\text{ b a}\cos\theta_1)\right) = 0$$

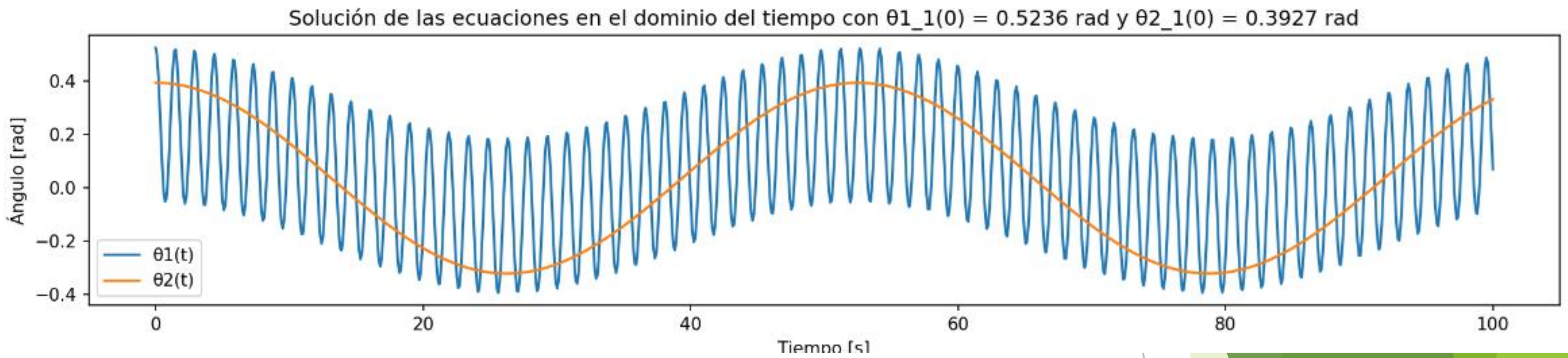
$$\theta_2 \longrightarrow mb\ddot{\theta}_2 + mgb\,\mathrm{sen}\,\theta_2 + k\left(b^2\,\mathrm{Sen}\,(\theta_2 - \theta_1) + ba\cos\theta_2\right) - \frac{ka}{2}\left(\left(2b^2 + a^2 + 2ba\,(\sin\theta_2 - \sin\theta_1\right)\right. - 2b^2\cos\left(\theta_2 - \theta_1\right)^{-1/2}\left(b^2\,\mathrm{sen}\,(\theta_2 - \theta_1) + 2ba\cos\theta_2\right) = 0$$

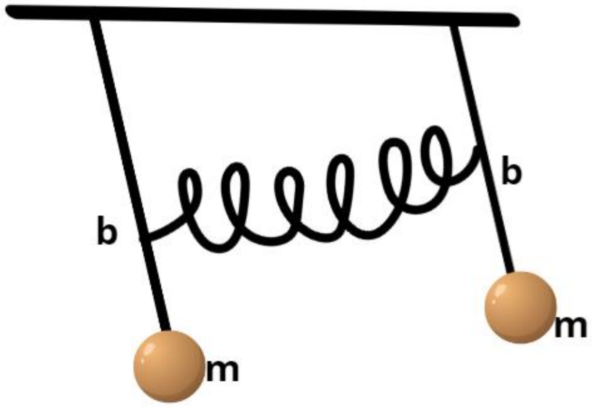

Solución de las ecuaciones en el dominio del tiempo con θ1(0) = 0.5236 rad y θ2(0) = 0.3491 rad
θ1(t)
θ2(t)
Ángulo [rad]
Tiempo [s]

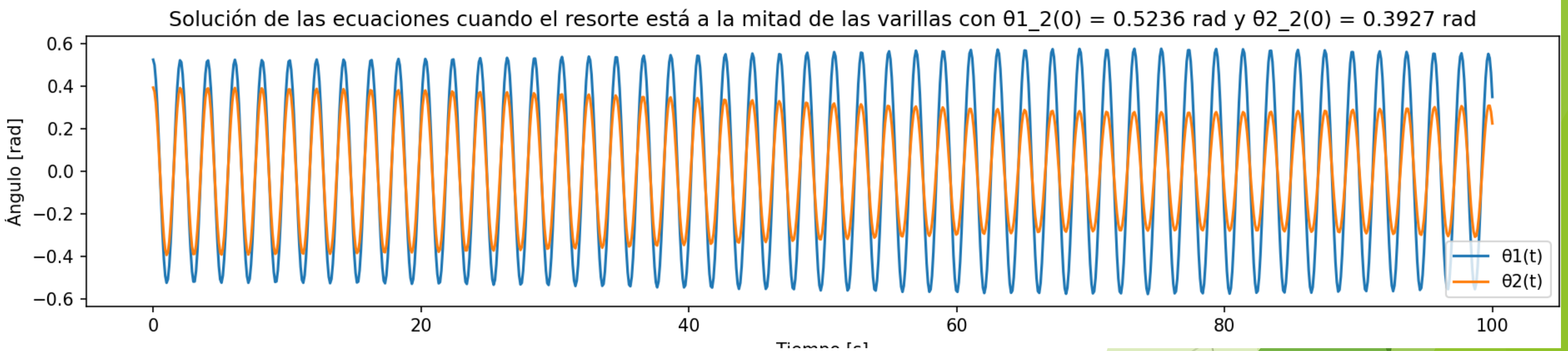

Resta de las soluciones para una variacion de condiciones iniciales de 0.0001, θ(0) = 0.5236 rad y θ(0) = 0.5237 rad
θ1_modified(t)-θ1(t)
Ángulo [rad]
Tiempo [s]


Espectrograma de θ1
Frecuencia [Hz]
Tiempo [s]


a
θ1
θ2
b
c
k
m1
m2


Solución de las ecuaciones en el dominio del tiempo con θ1_1(0) = 0.5236 rad y θ2_1(0) = 0.3927 rad
Ángulo [rad]
Tiempo [s]
θ1(t)
θ2(t)
0.4
0.2
0.0
−0.2
−0.4
0
20
40
60
80
100


b
b
m
m


Solución de las ecuaciones cuando el resorte está a la mitad de las varillas con θ1_2(0) = 0.5236 rad y θ2_2(0) = 0.3927 rad
Ángulo [rad]
θ1(t)
θ2(t)
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
20
40
60
80
100

$$\theta_1 \Rightarrow 2mb\ddot{\theta}_1 + mgb\theta_1 - kb^2\left(\theta_2 - \theta_1\right) + ba + \frac{ak}{2}\left(\left(2b^2 + a^2 + \right.\right.$$

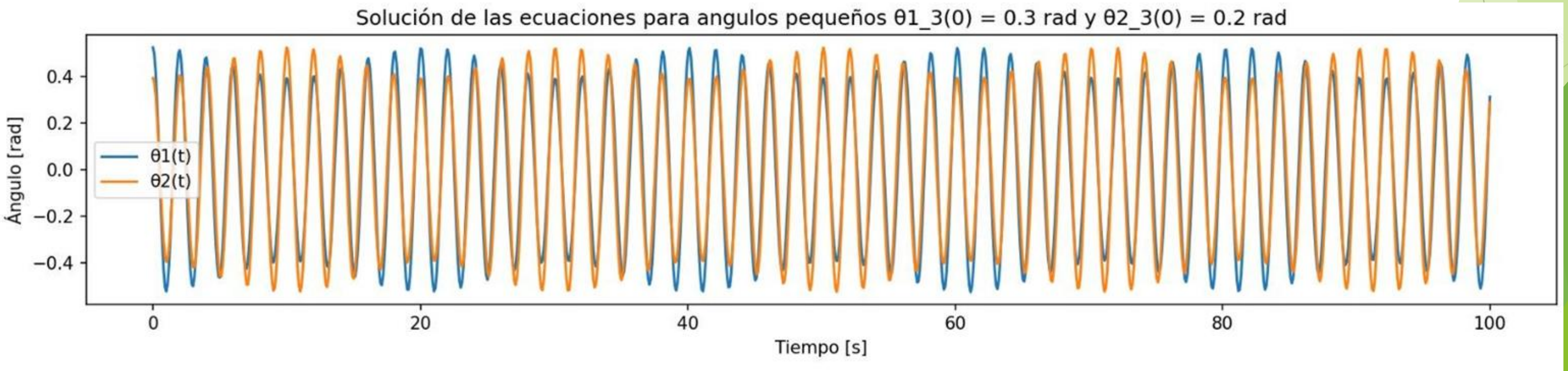
$$\left.2ab\left(\left(\theta_2 - \theta_1\right) - 2b^2\theta_2\theta_1\right)^{-1/2}\left(2b^2\left(\theta_2 - \theta_1\right) + 2ba\right) = 0$$

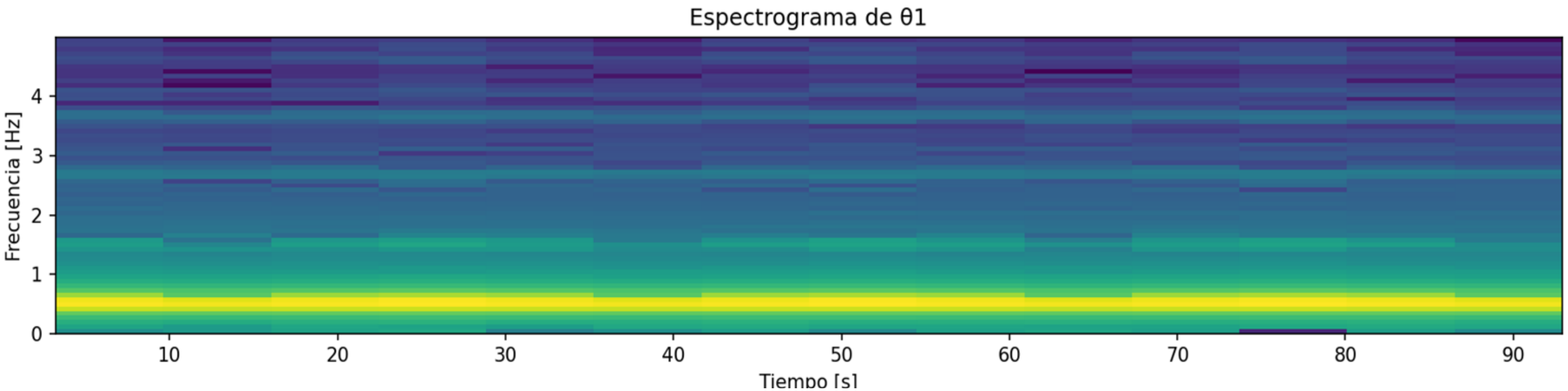
$$\theta_2 \Rightarrow mb\ddot{\theta}_2 + mgb\theta_2 + kb^2\left(\theta_2 - \theta_1\right) + ba - ka/2\left(2b^2 + a^2 + \right.$$

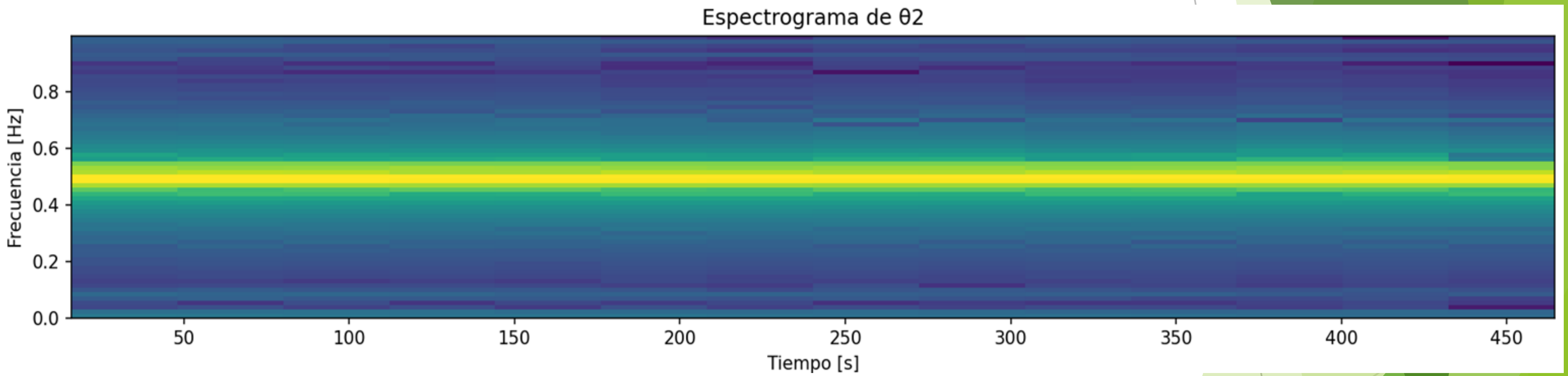
$$\left.2ba\left(\theta_2 - \theta_1\right) - 2b^2\theta_2\theta_1\right)^{-1/2}\left(b^2\left(\theta_2 - \theta_1\right) + 2ba\right) = 0$$

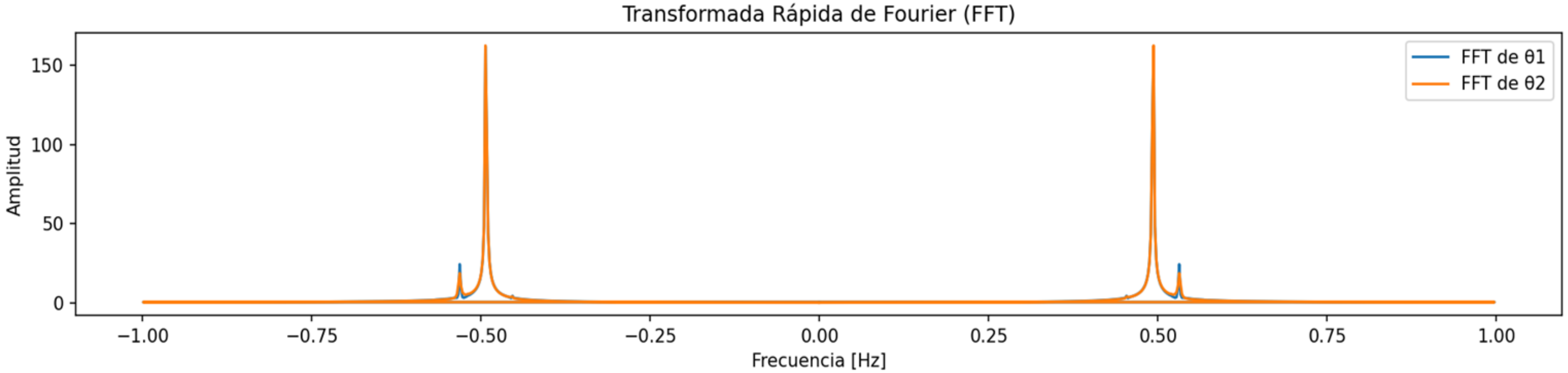
$$\begin{aligned}\theta_1 \Longrightarrow & m_1\ddot{\theta}_1 b^2 + m_1 g b\theta_1 - k\left(cb\left(\theta_2 - \theta_1\right) + ab\right)\\ & +ak/2\left(c^2 + a^2 + b^2 - 2cb\theta_2\theta_1\right) + 2a\left(c\theta_2 - b\theta_1\right)^{-1/2}\\ & \left(cb\left(\theta_2 - \theta_1\right) + ab\right) = 0\end{aligned}$$

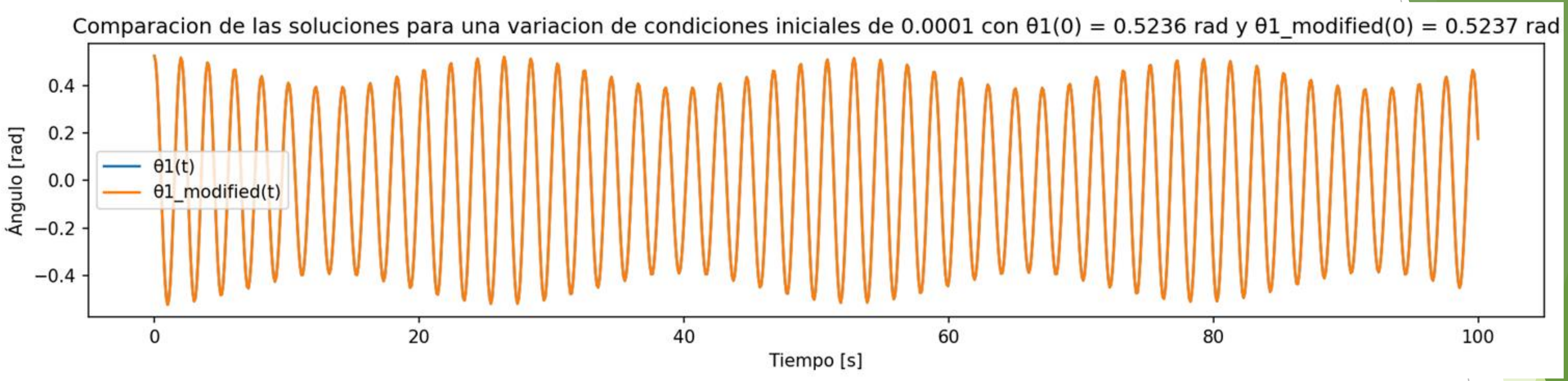
$$\begin{aligned}\theta_2 \Longrightarrow & m_2\ddot{\theta}_2 c^2 + m_2 g c\theta_2 + k\left(cb\left(\theta_2 - \theta_1\right) + ac\right)\\ & - k/2a(c^2 + a^2 + b^2 - 2bc\left(\theta_2\theta_1\right) + 2a\left(c\theta_2 - b\theta_1\right)^{-1/2}\\ & \left(cb\left(\theta_2 - \theta_1\right) + ac\right) = 0\end{aligned}$$

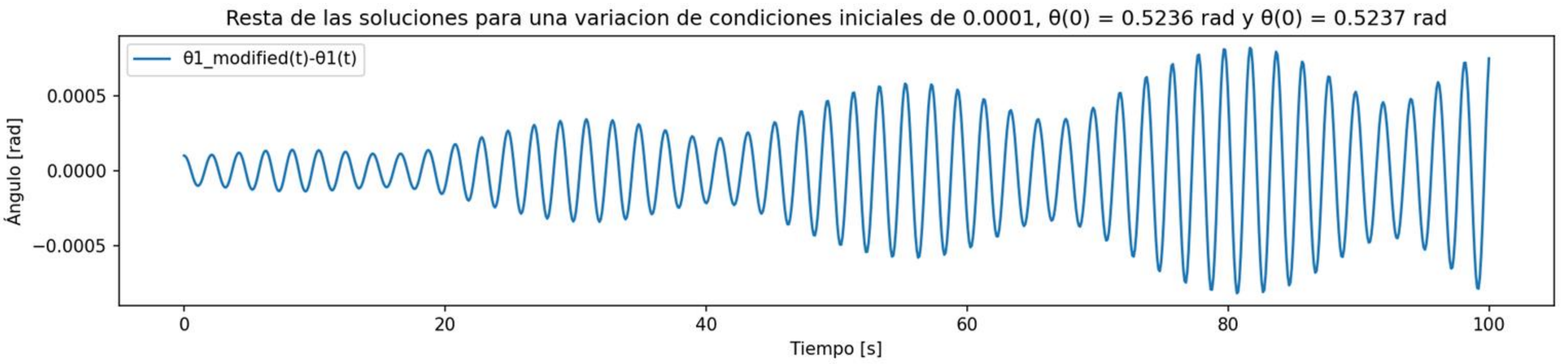



Espectrograma de θ1
Frecuencia [Hz]
Tiempo [s]
0
1
2
3
4
10
20
30
40
50
60
70
80
90


Espectrograma de θ2
Frecuencia [Hz]
Tiempo [s]
0.0
0.2
0.4
0.6
0.8
50
100
150
200
250
300
350
400
450


Transformada Rápida de Fourier (FFT)
Amplitud
Frecuencia [Hz]
FFT de θ1
FFT de θ2
0
50
100
150
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


Comparacion de las soluciones para una variacion de condiciones iniciales de 0.0001 con θ1(0) = 0.5236 rad y θ1_modified(0) = 0.5237 rad
θ1(t)
θ1_modified(t)
Ángulo [rad]
Tiempo [s]


Resta de las soluciones para una variacion de condiciones iniciales de 0.0001, θ(0) = 0.5236 rad y θ(0) = 0.5237 rad
θ1_modified(t)-θ1(t)
Ángulo [rad]
Tiempo [s]