

TIP8419 - Tensor Algebra

Homework 0

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Kronecker Product

Problem 1 For randomly generated $\mathbf{A} \in \mathbb{C}^{N \times N}$ and $\mathbf{B} \in \mathbb{C}^{N \times N}$, evaluate the computational performance (run time) of the following matrix inversion formulas:

(a) Method 1: $(\mathbf{A}_{N \times N} \otimes \mathbf{B}_{N \times N})^{-1}$

Method 2: $\mathbf{A}_{N \times N}^{-1} \otimes \mathbf{B}_{N \times N}^{-1}$

for $n \in \{2, 4, 8, 16, 32, 64\}$.

(b) Method 1: $\left(\mathbf{A}_{4 \times 4}^{(1)} \otimes \mathbf{A}_{4 \times 4}^{(2)} \otimes \dots \otimes \mathbf{A}_{4 \times 4}^{(K)}\right)^{-1} = \left(\bigotimes_{i=1}^K \mathbf{A}_{4 \times 4}^{(i)}\right)^{-1}$

Method 2: $\left(\mathbf{A}^{(1)}\right)_{4 \times 4}^{-1} \otimes \left(\mathbf{A}^{(2)}\right)_{4 \times 4}^{-1} \otimes \dots \otimes \left(\mathbf{A}^{(K)}\right)_{4 \times 4}^{-1} = \bigotimes_{i=1}^K \left(\mathbf{A}^{(i)}\right)_{4 \times 4}^{-1}$

for $K \in \{2, 4, 6, 8, 10\}$.

Problem 2 Let $\text{eig}(\mathbf{X})$ be the function that returns the matrix $\Sigma_{K \times K}$ of eigenvalues of \mathbf{X} . Show algebraically that $\text{eig}(\mathbf{A} \otimes \mathbf{B}) = \text{eig}(\mathbf{A}) \otimes \text{eig}(\mathbf{B})$.

Hint: Use the property $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}$.

\otimes Denotes the Kronecker Product.