TIP8419 - Tensor Algebra
Homework 4

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2019.2

Least Squares Kronecker Product Factorization (LSKronF)

Problem 1 Generate $\mathbf{X} = \mathbf{A} \otimes \mathbf{B} \in \mathbb{C}^{24\times 6}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{4\times 2}$ e $\mathbf{B} \in \mathbb{C}^{6\times 3}$. Then, implement the Least Square Kronecker Product Factorization (LSKronF) algorithm that estimate $\mathbf{A}$ and $\mathbf{B}$ by solving the following problem

$$(\hat{\mathbf{A}}, \hat{\mathbf{B}}) = \min_{\mathbf{A},\mathbf{B}} \|\mathbf{X} - \mathbf{A} \otimes \mathbf{B}\|_F^2.$$

Compare the estimated matrices $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ with the original ones. What can you conclude? Explain the results.

Hint: Use the file "kronf_matrix.mat" to validate your result.

Problem 2 Assuming 1000 Monte Carlo experiments, generate $\mathbf{X}_0 = \mathbf{A} \otimes \mathbf{B} \in \mathbb{C}^{IJ\times PQ}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{I\times P}$ and $\mathbf{B} \in \mathbb{C}^{J\times Q}$, whose elements are drawn from a normal distribution. Let $\mathbf{X} = \mathbf{X}_0 + \alpha\mathbf{V}$ be a noisy version of $\mathbf{X}_0$, where $\mathbf{V}$ is the additive noise term, whose elements are drawn from a normal distribution. The parameter α controls the power (variance) of the noise term, and is defined as a function of the signal to noise ratio (SNR), in dB, as follows

$$\text{SNR}_{\text{dB}} = 10\log_{10}\left(\frac{||\mathbf{X}_0||_F^2}{||\alpha\mathbf{V}||_F^2}\right). \tag{1}$$

Assuming the SNR range [0, 5, 10, 15, 20, 25, 30] dB, find the estimates $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ obtained with the LSKronF algorithm for the configurations: I.$(I, J) = (2, 4)$, $(P, Q) = (3, 5)$ and II.$(I, J) = (4, 8)$,$(P, Q) = (3, 5)$.

Let us define the normalized mean square error (NMSE) measure as follows

$$\text{NMSE}(\mathbf{X}_0) = \frac{1}{1000}\sum_{i=1}^{1000} \frac{\|\hat{\mathbf{X}}_0(i) - \mathbf{X}_0(i)\|_F^2}{\|\mathbf{X}_0(i)\|_F^2},$$

where $\mathbf{X}_0(i)$ e $\hat{\mathbf{X}}_0(i)$ represent the original data matrix and the reconstructed one at the ith experiment, respectively. For each SNR value and configuration, plot the NMSE vs. SNR curve. Discuss the obtained results.