TIP8419 - Tensor Algebra
Homework 9

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Multidimensional Least-Squares Kronecker Factorization (MLS-KronF)

On practice 4 we implement the LS-KronF(Least Square Kronecker Factorization) algorithm, now we will go to implement the MLS-KronF(Multidimensional Least Square Kronecker Factorization) algorithm. Then, Let $\mathbf{X} \approx \mathbf{A}^{(1)} \otimes \mathbf{A}^{(2)} \cdots \otimes \mathbf{A}^{(N)} \in \mathbb{C}^{I_1 I_2 \dots I_N \times J_1 J_2 \dots J_N}$ be a matrix composed by Kronecker product of N matrices $\mathbf{A}^{(n)} \in \mathbb{C}^{I_n \times J_n}$, with $n = 1, 2, \dots, N$. For $N = 3$ and I_n and J_n arbitrary implement the MLS-KronF for that estimate $\mathbf{A}^{(1)}, \mathbf{A}^{(2)}$ and $\mathbf{A}^{(3)}$ by solving the following problem.

$$(\hat{\mathbf{A}}^{(1)}, \hat{\mathbf{A}}^{(2)}, \hat{\mathbf{A}}^{(3)}) = \min_{\mathbf{A},\mathbf{B}} \|\mathbf{X} - \mathbf{A}^{(1)} \otimes \mathbf{A}^{(2)} \otimes \mathbf{A}^{(3)}\|_F^2.$$

Compere the estimate matrices $\hat{\mathbf{A}}^{(1)}, \hat{\mathbf{A}}^{(2)}$ and $\hat{\mathbf{A}^{(3)}}$ with the original ones. What can you conclude? Explain the results.

Hint: Use the file "kronf_matrix_3D.mat" to validate your result.

Problem 1 Assuming 1000 Monte Carlo experiments, generate $\mathbf{X}_0 = \mathbf{A} \otimes \mathbf{B} \otimes \mathbf{C} \in \mathbb{C}^{I_1 I_2 I_3 \times J_1 J_2 J_3}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{I_1 \times J_1}$, $\mathbf{B} \in \mathbb{C}^{I_2 \times J_2}$ and $\mathbf{C} \in \mathbb{C}^{I_3 \times J_3}$, whose elements are drawn from a normal distribution. Let $\mathbf{X} = \mathbf{X}_0 + \alpha \mathbf{V}$ be a noisy version of $\mathbf{X}_0$, where $\mathbf{V}$ is the additive noise term, whose elements are drawn from a normal distribution. The parameter α controls the power (variance) of the noise term, and is defined as a function of the signal to noise ratio (SNR), in dB, as follows

$$\mathrm{SNR}_{\mathrm{dB}} = 10\log_{10}\left(\frac{||\mathbf{X}_0||_F^2}{||\alpha\mathbf{V}||_F^2}\right). \tag{1}$$

Assuming the SNR range [0, 5, 10, 15, 20, 25, 30] dB, find the estimates $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ obtained with the MLS-KRF algorithm for the configurations $I_1 = J_1 = 2$,$I_2 = J_2 = 3$ and $I_3 = J_3 4$. Let us define the normalized mean square error (NMSE) measure as follows

$$\mathrm{NMSE}(\mathbf{X}_0) = \frac{1}{1000}\sum_{i=1}^{1000}\frac{\|\hat{\mathbf{X}}_0(i) - \mathbf{X}_0(i)\|_F^2}{\|\mathbf{X}_0(i)\|_F^2}, \tag{2}$$

where $\mathbf{X}_0(i)$ e $\hat{\mathbf{X}}_0(i)$ represent the original data matrix and the reconstructed one at the ith experiment, respectively. For each SNR value and configuration, plot the NMSE vs. SNR curve. Discuss the obtained results.

Note: For a given SNR (dB), the parameter α to be used in your experiment is determined from equation (1).