Introduction to Logic 2023/24

Formative homework assignment 1

- ☞ This homework is to be made in pairs: together you hand in one set of solutions.
- ☞ Hand in your homework on paper on **Monday, 4 December 2023, by 13:00**, the start of the lecture, in the lecture room. This deadline is strict. In case of late submission, you might not receive feedback.
- ☞ Write your student numbers, your names, and your tutorial group (or groups if you belong to different ones) on the first page. The reviewed homework will be handed back in the tutorial group of the student that is mentioned first.
- ☞ Make sure that multiple pages are stapled together. Do not use paperclips or tape.
- ☞ Write with a blue or black pen (so no pencil or red pen).
- ☞ Leave the first ten lines of the first page blank, for feedback.
- ☞ Only collaborate with your partner.
- ☞ The assignment consists of 3 exercises and a bonus exercise.
- ☞ You will receive formative feedback for the homework, not a grade.

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1. Translate the following sentences to *propositional logic*, where atomic sentences are represented by uppercase letters, **if possible**. Do not forget to provide the translation key (one key for the whole exercise). One or more of these sentences cannot be translated to propositional logic. In these cases, *briefly* explain why not.

   (a) City traffic can be reduced only if parking gets more expensive.

   (b) If the city council had not been incompetent, then they would have chosen a different policy.

   (c) Provided that parking gets more expensive, it will be harder to travel to the city by car, but air quality will improve.

   (d) Air quality will improve because people will cycle.

   (e) People will not cycle unless air quality will improve.

2. Translate the following sentences to *first-order logic*. Show as much logical structure as possible. Do not forget to provide the translation key (one key for the whole exercise).

   (a) If Pythagoras is a philosopher, then Aristotle or Plato is a philosopher.

   (b) Pythagoras influenced both Aristotle and Plato, however, Aristotle and Plato did not agree with each other.

   (c) Although both Aristotle and Plato lived long, Plato lived longer than Aristotle.

3. Give formal proofs for the following inferences. Do not forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule:

   (a)
   $$\begin{array}{|l} R(a,b) \\ a = b \\ \hline R(b,a) \end{array}$$

   (b)
   $$\begin{array}{|l} (\neg P \vee Q) \leftrightarrow P \\ \hline P \end{array}$$

   (c)
   $$\begin{array}{|l} A \vee \bot \\ \hline A \end{array}$$

   (d)
   $$\begin{array}{|l} (A \wedge B) \vee (\neg B \wedge C) \\ \hline A \vee C \end{array}$$

   (e)
   $$\begin{array}{|l} A \to \neg B \\ \neg A \to \neg B \\ \hline \neg B \end{array}$$

4. (Bonus exercise) Give a formal proof for $\neg(P \leftrightarrow Q) \to (\neg P \leftrightarrow Q)$. Do not forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.