Introduction to Logic Summary
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C. LIGHT AILSMAN

7.	ngre dients
Fi	st-Order Logic Ingredients
<u> </u>	al 1
Ψ	constants a.b.c
Q	variables xxx
]	·
3	functions figh
(4)	tems a, f(a), f(g(a))
(\$)	predicates 1,0
1	
φ.	connectives 7,1, V
(7)	grantifiers 4.3
(3)	Sentences P(a)
(3)	vell-formed formulas
Prop	positional Logic Ingredients
	•
Ψ	propositions PO
2	connectives 7,1, V
Ĭ,	55711001WO3 F 66
(3)	sertences P^Q

Negation 7 not	PIZP
3	TF
	FIT
Conjunction 1 and	PIQIPAQ
moreover	
althorb	TFF
b.t	T F F F F F F F
honever	FFF
Disjunction v or	PIQIPYQ
unless	
1100	T T T
	FTT
	FFF
Conditions >	P Q 17Q
	TTT
	TFF
	FTT
	FFT
Bicanditional () if and only if precisely if	PQ P Q
precisely if	TTT
	TFF
	FTF
	FFT

In	ference Schemes	
∧ I,	ntro	
		Û
i. P		P is the
j. Q		Q is the
K. P^	Q 1Intro: i,j	Therefore, (Pand Q) is true
^ Elin	1	
	_	
i. f^0	2	(Pand Q) is tre
j. P	∧Elim: i	Therefore, P is true
VIn	10	
i. P		P is true
j. P V	Q VIntro:i	Therefor, (Por Q) is the
7		7 -
VElim		
i P ~ (Q	(Por Q) is the
j, P		,
		Pimplies R
I, R		The state of the s
]. Q		Q implies R
m.R.		or relibios
n. R	VFlimiti-Ll-m	Therefore, R is true
	· · · · · · · · · · · · · · · · · · ·	10000

7 Into	0	
i. P		
;		Pleads to a contradiction
li T		
K.TP	7 Intro: i-j	Therefore, of is true
~ E!-		
7 Elim		
i 77		(not not P) is tree
	7 Elin: i	Therefore, Pistive
		hearts of the
I Intro		
i. P		P is the
j. ¬P		(not P) is the
k. L	L Intro: inj	Therefore there is a contradiction
1 7.1		
1 Elin		
i. L		1. contradiction
j. P	LElin: i	2. 222
10	T (14/1-)	3. PROFIT
		,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,

→ Intro	
1 1:0	
	Q follows from P
li Q	
RP > Q -> Intro: i-j	Therefore, Pinglies Q
	·
> Elim	
\. D. 3 @	1. 0
i. P→Q	P implies Q
j. P	Pis fire Therefore, Q is the
KQ >Elimilij	herefore Q & The
en Intro	
]i. P	
:	a follows from P
j- Q	
K.Q	
	P follows from Q
l b	
nesa es Interi-j. k-1	therefore, Pit and only if a
•	
to Elim	
166 (or (66)	Pistre it and only it Q is true
KQ + Elim: ici	Po tre
K.Q & Elim: i,j	Therefore, Os tre

+ Elim		
i. ∀×P(×)		Francis DCJ 5 dese
j. P(c)	¥ Elim:j	Therefore, P(c) is also tre
- U		
Y Intro		
 C		for an orbitory c,
: i. P(c)		P(c) is true
. ∀ ×ρ(×)	*Into in:	P(c) is true Therefore P(x) is true for every x
, , , , , , ,	<u> </u>	Module 10.75 hos 10 800 1
Intro		
i. P(c)		P(c) is true
1 J×P(x)	3 Intro: i	Therefore, there exists on x such that
		P(x) is true
7 Elin		
7 6/100		
i. 7xPcx)		There exists an x such that pcx) is true
E P(c)		for an orbitrory c,
:		
k Q		a is true
JI. Q	JElin: ij-k	Therefore, Q is true

Proof Strategy Start Focus on subproof Does conclusion have Introduce it 3,63,4X as main operator? JNo Does a premise have Eliminate it as main operator? No No Done? Does conclusion have Introduce it a main operator? Yes J, No Infer conclusion informally. Translate to formal proof. I Not working Prove conclusion by contradiction You nock.

	7ruth Tables
Def.	tactology (sentence) glungs tre
Def.	contradiction (sentence) always false
Oct.	logical toth/necessity (sentence) always true on non-sourious rows
Def.	
Def.	logical possibility - (sentence) the on at least one non-sprias row
	tautological equivalence (sentences) always have some truth values
	logical equivalence (sentences) always have some fruth values on non-spurious nows
Def.	tartological consequence - (sentence) always true when all previses true
Jef.	logical consequence - (sentence) always five when all previews free
	on non-spurious rows
_	

	Set Theory	
Def.	set sig	collection of elements
Def.	emoty set Ø	set with no elements
1	L 10 2125	ordered pair
net.	typle <1,2>	a series hall
Def.	relation {<1,2>}	set of types
Def.	is element of e (X)	is the element part of the set?
No.		in a new place of the left of
Det.	is susset of a	is every element in the left set on element of the right set?
		accept) of the Aids act c
Def.	is proper subset of G (XXX)	is every element in the left set an
		element of the right set and does the
		right set have elements which are not
		in the left set?
Nan	The A X X	
uer,	intersection 1 (xxxx)x	set of all commonly shared elements
		of 2 sets
Def.	union U X X X X	set of all elements of 2 sets
	×	pt together
> 0		
Def.	difference \ (xxx) x	set of all elements which are part
		of the left set but are not part
	_	of the right set
Def.	cartesion product × XX	set of all possible pairs composed
	$\times \rightarrow \times$	of an element from the left set
		and an element of the right set
		, ,

	Rel	lation Properties	S	
	(1)	Reflexivity	× 6.4 => < x, x> eR	()
	1.8	•		
		•	x,yeA, <x,y>ER => <y,x>ER</y,x></x,y>	
	3	Transitivity	$x_{1,2}$ $x_{1,2}$ $x_{1,2}$ $x_{1,2}$ $x_{1,2}$ $x_{1,2}$ $x_{2,2}$ $x_{1,2}$ $x_{2,2}$ x_{2	
(9	Density	xy,264, <x,2>6R => <x,y>,<y,2>6R</y,2></x,y></x,2>	<i>3</i>
	5	Functionality	×y,264, <×,y>,<×,2>6R =>y=2	·<"
		·		

Jef.	negation normal form > only 1, 1,7 connectives
	negations directly in front of atom
N C	
Jef.	conjunction normal form > negation normal form conditions
	conjunction of disjunctions of literal
	Conjunction or disjunctions so there
Def.	disjunction normal form > negation normal form conditions
	disjunction of conjunctions of litera
	Equivalences
	D Double negation elimination 77900
	1) Nouble negation elimination 77P(=> P
	1) Double negation elimination 77P P 1) Lempotency of 1 and V PAP P
	1) Double negation elimination 77P@P 1) Lempotency of 1 and V PAP @ P PVP@P
	1 Double negation elimination 77P(=) P 1 Dempotercy of 1 and V PAP (=) P 1 Propose 1
	1 Double negation elimination 77P(=) P 1 Dempotercy of 1 and V PAP (=) P 1 Propose 1
	1) Double negation elimination 77P(3) P 1) Distributivity of 1 and V PAP(3) P 1) Proportion of 1 and V PAR(3) (PAR) 1) Commutativity of 1 and V PAR(3) RP 1) Proportion of
	1) Double negation elimination 77P(3) P 1) Distributivity of 1 and V PAP(3) P 1) Proportion of 1 and V PAR(3) (PAR) 1) Commutativity of 1 and V PAR(3) RP 1) Proportion of
	1 Double negation elimination 77P(=) P 1 Distributivity of 1 and V PAP(=) P 1 Pr(arr)=(Pra)r(PAR) 1 Pr(arr)=(Pra)r(Prr) 2 Commutativity of 1 and V PAR= RP 1 Pr(arr)=(Pra)r(Prr) 2 Preserve
	Double negation elimination 77P0 P Dempotency of A and V PAP (2) P Propose Distributivity of A and V PAR(2) (PAR) Propose (Pra) (PAR) Propose (Pra) (Pra) Preserve S Associativity of A and V PAR(2) PA(2) PA(2) (Pra) MR(2) PA(2) PA(2) (Pra) MR(2) (Pra) MR(2)
	1 Double negation elimination 77P(3) P 1 Lempotency of A and V PAP(3) P 1 Pre>P 1 Pre>P 1 Distributivity of A and V PAR(3) P
	1) Double negation elimination 77P(3) P 1) L'dempotency of A and V PAP(3) P 1) Pre>P 1) Distributivity of A and V PA(2) PAQ) V (PAR) 1) Commutativity of A and V PAR(3) RP 1) Pre>P 1) Commutativity of A and V PAR(3) RP 1) Pre>P 1) Commutativity of A and V PAR(3) RP 1) Pre>P 1) Commutativity of A and V (PAQ) AR(3) PA(2) PA(2) 1) Commutativity of A and V (PAQ) AR(3) PA(2) 1) PAR(3) PAR(3) PAR(3) PAR(3) 1) PAR(3) PAR(3) PAR(3) PAR(3) PAR(3) 1) PAR(3) PAR(3) PAR(3) PAR(3) PAR(3) PAR(3) 1) PAR(3) PAR

	Semantics of FOL
Jef.	model - like a dictionary for FOL "words"
_	·
	or mapping of symbols to the objects they signify
	a mapping of a language to a world
	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	language world
	c > M(c) a constant
	c > M(c) a constant
	symbols of M(f) a function objects
	signs
	P > M(P) a predicate
Def.	M(c) = "interpretation" of c in M
Det.	variable assignment — like a dictionary for FOL "names"
	ما ما امری ما امری کو هندووس و
	a mapping of variable symbols to their values
	3 rich value so
	designed to hold temporary interpretations
	9
Def.	h (x) -> "interpretation" of x under h
Def.	h[x/d] > new variable assignment in which variable x
	is assigned value d
Def.	hø -> empty vaviable ossignment
	The section of the se

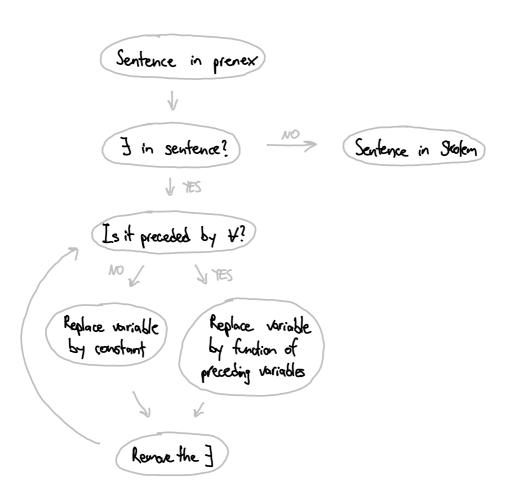
Def.	interpretation of term t in model M under variable assignment h MEALH formula A is satisfied in model M by variable assignment h
Oef.	formula A is not satisfied in model M by variable assignment h
	Equivalences
) M= P(+,+n)[h] (=> <[+]], [+n], > e M(P)
(2) M= 7ALIJ (=> M\faci)
	3) M=(Ang)[h] (=) M=A[h] and M=B[h]
	1) M= (AVB)[h] (=> M=A[h] or M=B[h]
	5) M=(A->B)(h) (=> MKA[h] or MFB[h]
	ME(1408)[h] (=) MEA[h] and MEB[h] or MEA[h] and MEB[h]
	ME YXA[4] (=> for all LEM(4), MEA[LLX/4]]
(§	M=J×A[h] (=) there is a d=M(V) such that M=A[h[×/d]]

	Quantifier Normal Forms
	variables
Oef.	Prenex Normal Form > Q.V Q.V.
	grantifiers grantifier-free broads
	2 April 1927 April 194 - (Less De Man Cl
Def.	Skolen Normal Form prener normal form conditions
	only universal quantifiers
Def.	Horn torm Skolem normal form conditions
	grantifier-free formulas in CNF
	Equivalences literal per conjunct
	India (or fine)
	1) Vaccous grantification QxP = P if no free x in P
	<u> </u>
	(2) Renaming QxP(x) (>> QyP(y)
	3 De Morgan 74xP(x) (=> 1/x7P(x)
	$(9) \text{ d}$ and $(1) Q_{\times} P \wedge Q_{\times} R \text$
	QxPVQxR >> Ox(PVR)
	QxPAR (=) Qx(PAR), no free x in R
	QxPVR (=) Qx(PVR), no free x in R 5) # and > $\forall xP \rightarrow R \Leftrightarrow \exists x(P \rightarrow R)$, no free x in R
	TXP->R(=) YX(P->R) no free x in R
	P > OxR (>) Ox(P>R), no free x in P

Compar, son Feature To sms Normal

Horn Seren Seren NNF Touly accurs in fraut of about soutences only universal grantifiers トラン max. 1 positive literal Scentifies in front conjunction of disjunctions lis, inclian of dsjundian conjunctions only uses ي

Skolenization



Horn's algorithm

Write futh table header for sentence

Find values of atomic conjuncts so that the sentence may be true

Are there still atoms without values?

V YES

Analyze consequences of previously added values

Find values of atomic conjuncts so that the sentence may be true

forced to assign F to a renjunct?

10/ JYES

sentence satisfiable sentence NOT sotisfiable