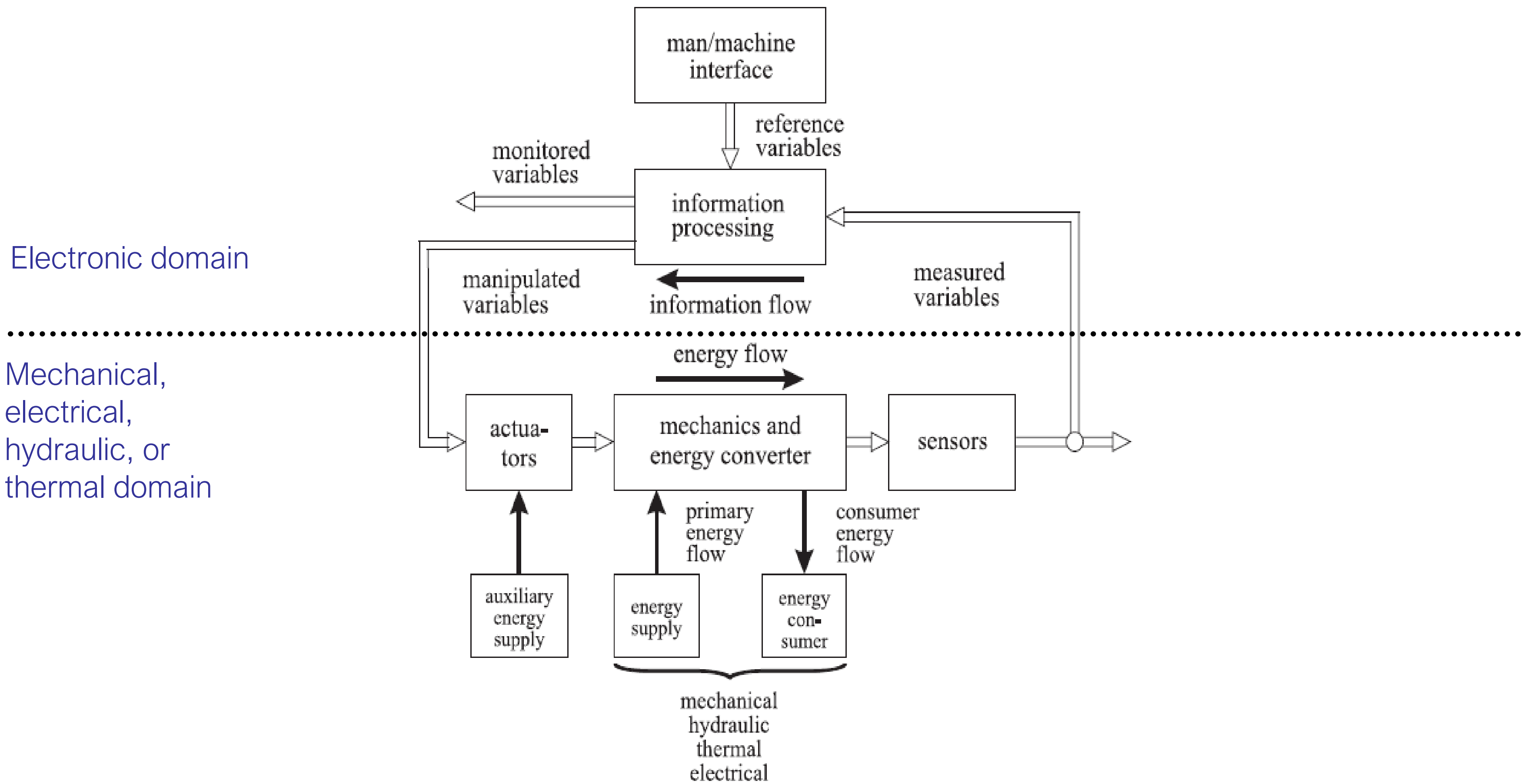
Mechatronics

Week 3 Day 1

Components of a Mechatronics System



The figure is taken from (Isermann, 2008).

Previous lecture

- You were introduced to A, T, D-type elements and A,T-variables
- You learned that A-type and T-type variables are suitable state variables
- You learned to describe
  - Energy-storing elements in terms of A-type and T-type elements by differential equations
  - Dissipative elements in terms of static relationship between A-type and T-type variables

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Today's lecture:

Modeling of interconnected multidomain systems

Learning objectives

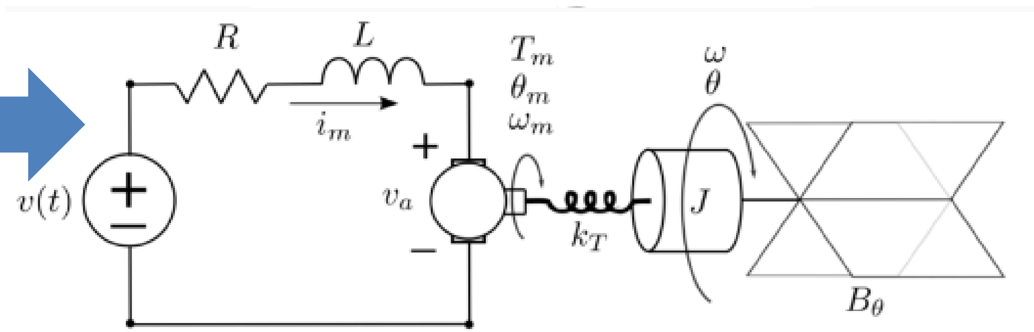
After today's lecture, you will be able to

- Formulate the state-space representation of multidomain systems

Let's learn the steps to follow using an example…

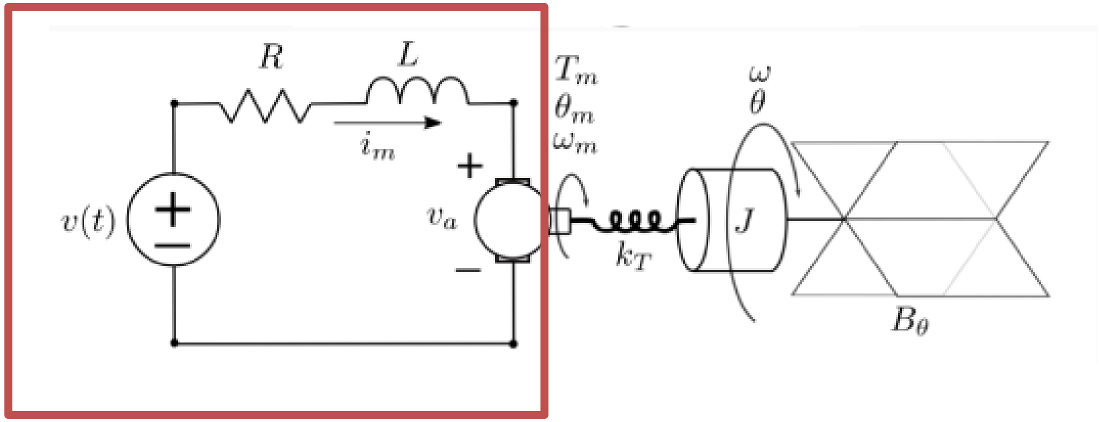
Example 1: Modelling of a fan

An electric fan is an interconnected multidomain system comprising of an electrical and a mechanical domain connected via magnetic coupling



Example 1: Modelling of a fan

Electrical system: RL circuit that controls motor



$\boldsymbol{v(t)}$: input voltage

$\boldsymbol{R}$: resistance

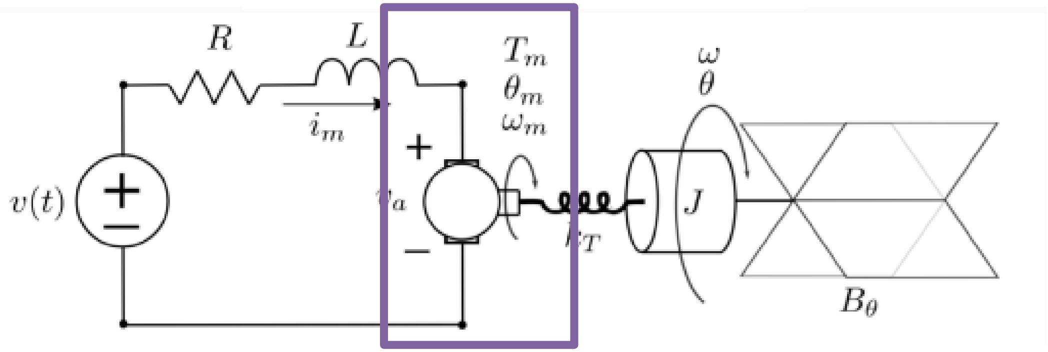
$\boldsymbol{L}$: inductance

$\boldsymbol{v_a}$: voltage across motor terminals

$\boldsymbol{i_m}$: current through the circuit

Example 1: Modelling of a fan

Magnetic coupling: converts electrical energy to mechanical energy



Coupling equations (given in exam)

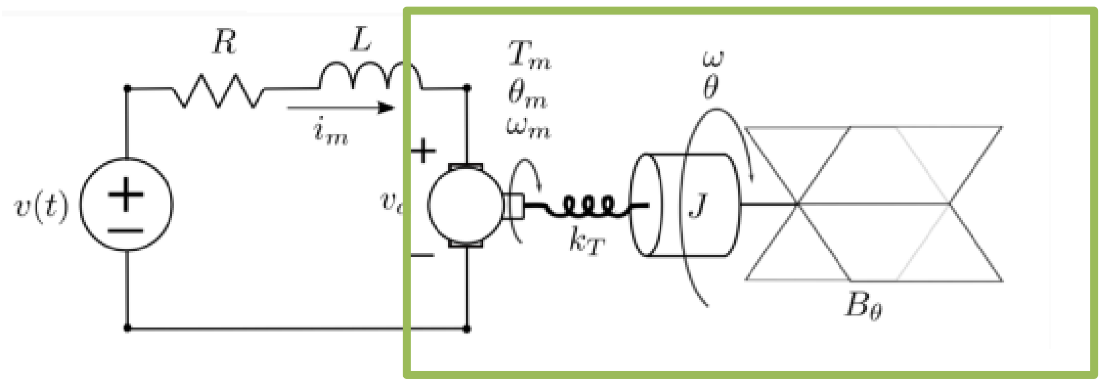
$$v_a = k_a \omega_m$$

$$T_m = k_f i_m$$

with k_a and k_f coupling constants

Example 1: Modelling of a fan

Mechanical system: motor+propeller



Motor shaft

$\boldsymbol{\theta_m}$: angular position

$\boldsymbol{\omega_m}$: angular velocity

$\boldsymbol{T_m}$: torque

Propeller

$\boldsymbol{J}$: moment of inertia

$\boldsymbol{\theta}$: angular position

$\boldsymbol{\omega}$: angular velocity

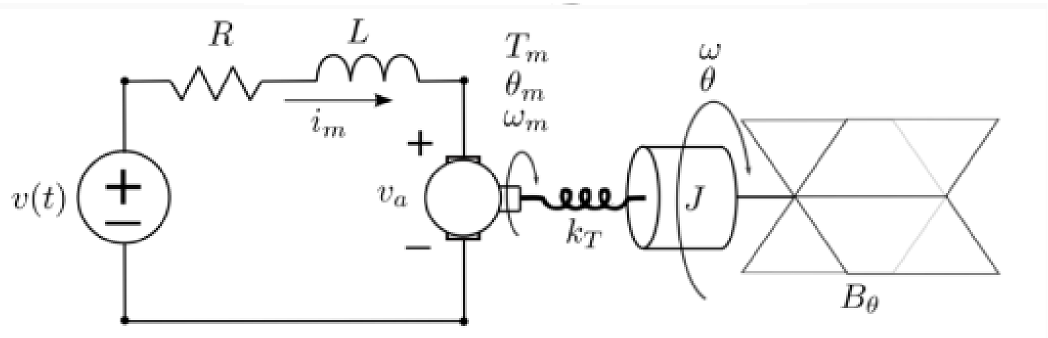
$\boldsymbol{B_\theta}$: damping coefficient due to air

Rotational spring (motor-propeller connection)

$\boldsymbol{k_T}$: constant of the rotational spring

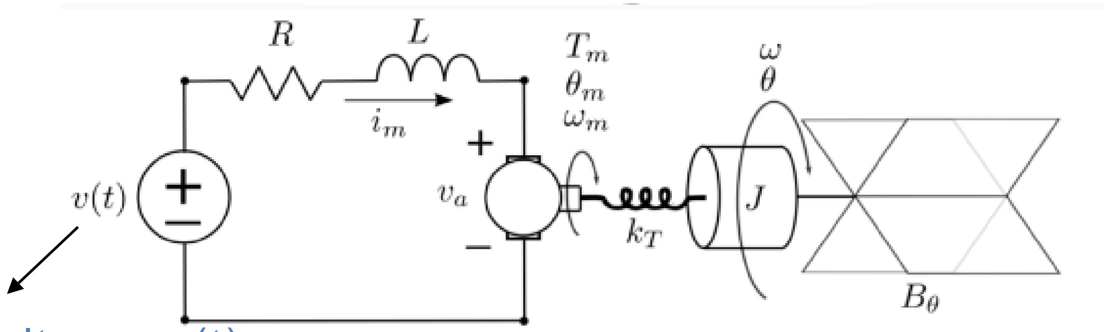
Example 1: Modelling of a fan

Step 1. Identify energy storing elements, dissipative elements and inputs



Example 1: Modelling of a fan

Step 1. Identify energy storing elements, dissipative elements and inputs



Input: voltage v(t)

Energy storing elements (A and T-type)

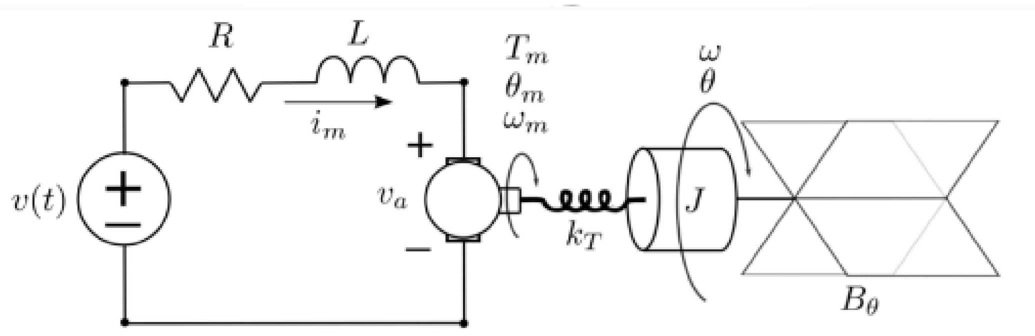
Element	Variable	Type
Inductor L	Current $i_L = i_m$	T-type
Rotational spring k_T	Torque of the spring T_{k_T}	T-type
Moment of inertia of propeller J	Angular velocity of propeller ω	A-type

Dissipative elements (D-type)

Element	Type
Resistor R	D-type
Rotational damper B_θ	D-type

Example 1: Modelling of a fan

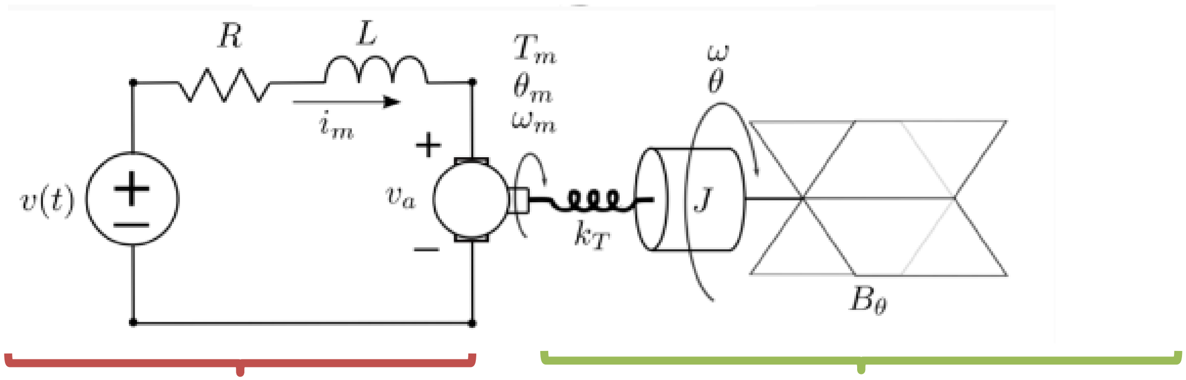
Step 2. i) Elemental equations for A,T,D-type elements



Element	Equation	Type
Inductor L	$V_L = L\frac{di_L}{dt}$	T-type
Rotational spring k_T	$\frac{1}{k_T}\frac{dT_k}{dt} = \omega_m - \omega$	T-type
Moment of inertia J of propeller	$T_J = J\frac{d\omega}{dt}$	A-type
Resistor R	$v_R = Ri_R$	D-type
Rotational damper B_θ	$T_{B_\theta} = B_\theta\omega$	D-type

Example 1: Modelling of a fan

Step 2. ii) Describe interaction between elements in each domain



Electrical domain

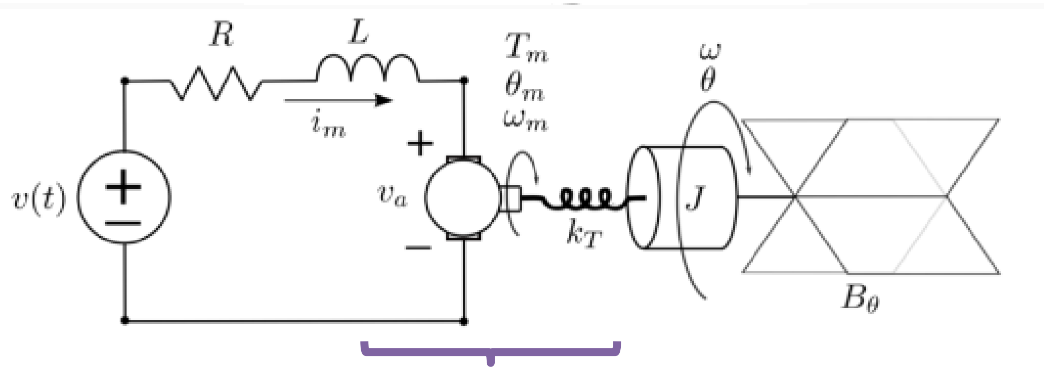
$$i_R = i_L = i_m$$
$$v(t) = v_R + v_L + v_a$$

Mechanical domain

$$T_m = T_{k_T}$$
$$T_{k_T} = T_J + T_{B_\theta}$$

Example 1: Modelling of a fan

Step 2. iii) Coupling Equations



Coupling equations (given in exam)

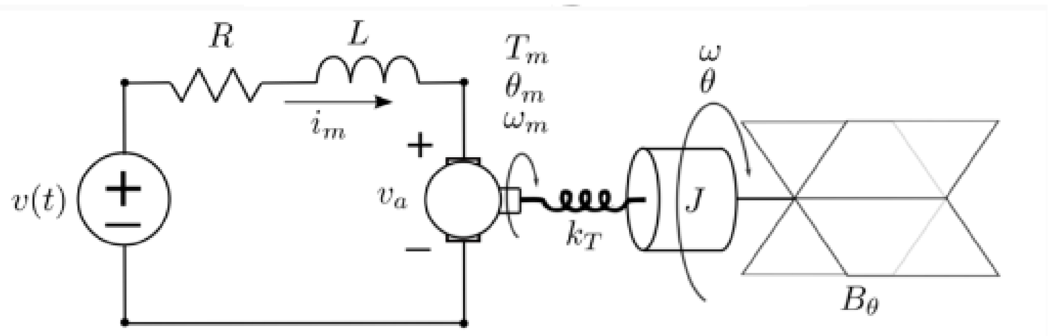
$$v_a = k_a \omega_m$$

$$T_m = k_f i_m$$

with k_a and k_f coupling constants

Example 1: Modelling of a fan

Step 3. Propose A-type and T-type variables as state variables



Variable	Type
Current $i_L = i_m$	T-type
Torque of the spring T_{k_T}	T-type
Angular velocity ω of propeller	A-type

Propose Independent energy storing elements as state variables

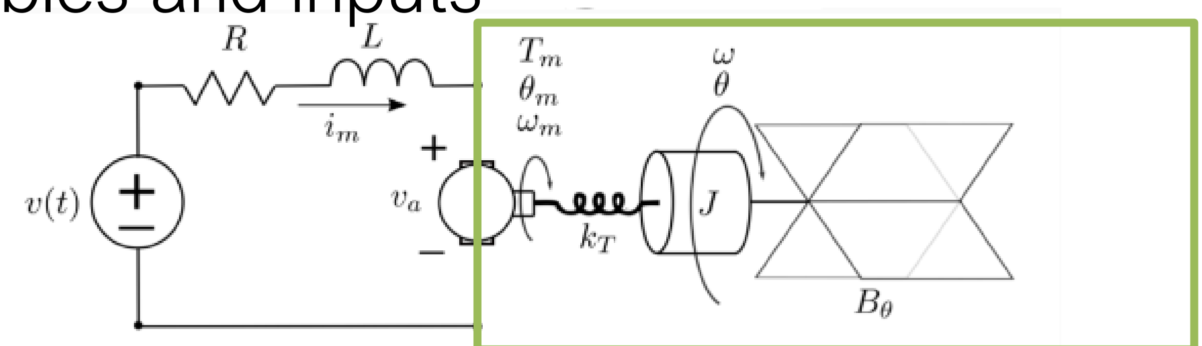
$$x_1 = i_L$$

$$x_2 = \omega$$

$\frac{1}{k_T}\frac{dT_k}{dt} = \omega_m - \omega$, so it is not independent!!

Example 1: Modelling of a fan

Step 4. Divide into subsystems and use equations i), ii) and iii) to express the dynamics only in terms of state variables and inputs



Mechanical subsystem

i) Elemental

$$T_J = J\frac{d\omega}{dt}$$

$$T_{B_\theta} = B_\theta \omega$$

ii) Interconnection

$$T_m = T_{k_T}$$

$$T_{k_T} = T_J + T_{B_\theta}$$

iii) Coupling

$$T_m = k_f i_m = k_f i_L$$

All state variables!!

$$\boxed{\frac{d\omega}{dt} = \frac{k_f}{J} i_L - \frac{B_\theta}{J}\omega}$$

$\dot{x}_2$ x_1 x_2

Example 1: Modelling of a fan

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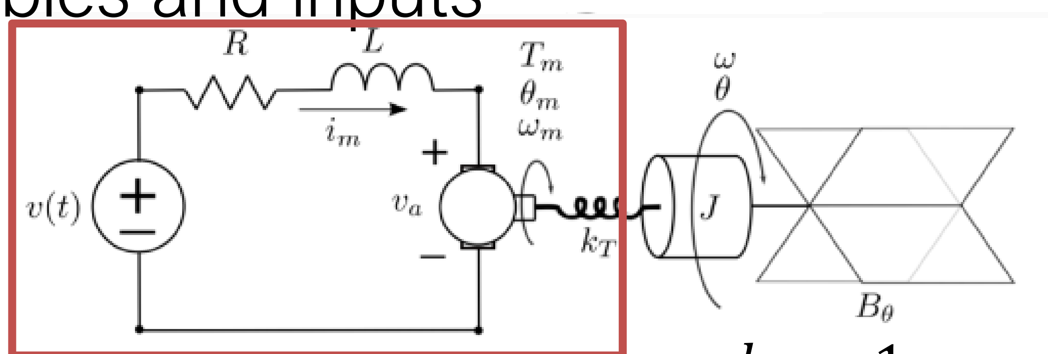

Electrical subsystem

i) Elemental

$$v_R = R i_L$$

$$v_L = L \frac{d_{iL}}{dt}$$

ii)Physics

$$v(t) = v_R + v_L + v_a$$

$$i_R = i_L = i_m$$

iii)Coupling

$$v_a = k_a \omega_m$$

Issue

$$\frac{d_{iL}}{dt} = \frac{1}{L} v(t) - \frac{R}{L} i_L - \frac{k_a}{L} \boxed{\omega_m}$$

Example 1: Modelling of a fan

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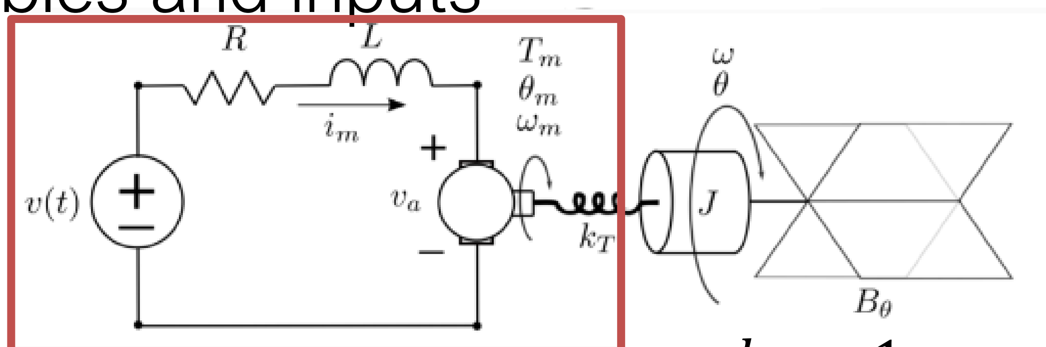

Electrical subsystem

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iii)Coupling

$$v_a = k_a\omega_m$$

$$\frac{d_{iL}}{dt} = \frac{1}{L}v(t) - \frac{R}{L}i_L - \frac{k_a}{L}\omega_m$$

Using i) $\frac{1}{k_T}\frac{dT_k}{dt} = \omega_m - \omega$

iii) $T_{k_T} = T_m = k_f i_m = k_f i_L$

$$\boxed{\frac{d_{iL}}{dt} = \frac{K_T}{LK_T + k_a k_f}v(t) - \frac{K_T R}{LK_T + k_a k_f}i_L - \frac{K_T k_a}{LK_T + k_a k_f}\omega}$$

Example 1: Modelling of a fan

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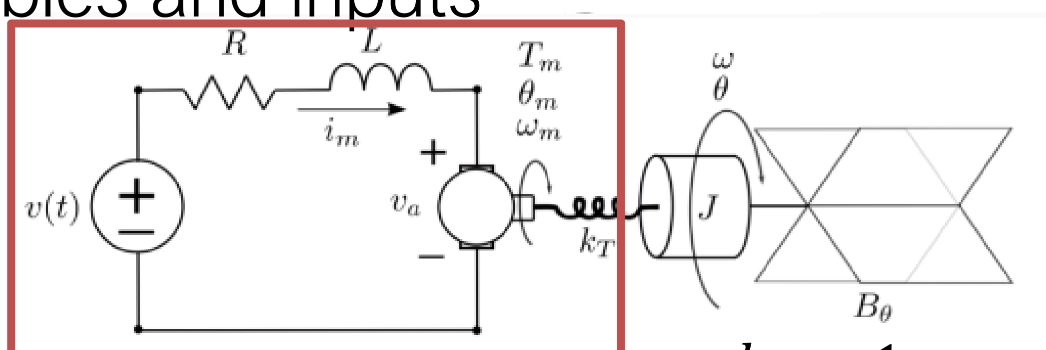

Electrical subsystem

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iii)Coupling

$$v_a = k_a \omega_m$$

$$\frac{d_{iL}}{dt} = \frac{1}{L}v(t) - \frac{R}{L}i_L - \frac{k_a}{L}\omega_m$$

All state variables and input!!

$\dot{x}_1$ Input x_1 x_2

$$\boxed{\frac{d_{iL}}{dt} = \frac{K_T}{LK_T + k_a k_f}v(t) - \frac{K_T R}{LK_T + k_a k_f}i_L - \frac{K_T k_a}{LK_T + k_a k_f}\omega}$$

Example 1: Modelling of a fan

Step 5. Express system as $\dot{x} = Ax + B$ if its linear or $\dot{x} = f(x) + g(x)u$ if not

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$$\frac{d\omega}{dt} = \frac{k_f}{J} i_L - \frac{B_\theta}{J}\omega$$

$$\frac{d_{iL}}{dt} = \frac{K_T}{LK_T + k_a k_f} v(t) - \frac{K_T R}{LK_T + k_a k_f} i_L - \frac{K_T k_a}{LK_T + k_a k_f}\omega$$

$$\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{d_{iL}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{B_\theta}{J} & \frac{k_f}{J} \\ -\frac{K_T k_a}{LK_T + k_a k_f} & -\frac{K_T R}{LK_T + k_a k_f} \end{bmatrix} \begin{bmatrix} \omega \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_T}{LK_T + k_a k_f} \end{bmatrix} v(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \omega \\ i_L \end{bmatrix}$$

Example 1: Modelling of a fan

Step 5. Express system as $\dot{x} = Ax + B$ if its linear or $\dot{x} = f(x) + g(x)u$ if not

$$\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{d_{iL}}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{B_\theta}{J} & \frac{k_f}{J} \\ -\frac{K_T k_a}{LK_T + k_a k_f} & -\frac{K_T R}{LK_T + k_a k_f} \end{bmatrix}}_{A} \begin{bmatrix} \omega \\ i_L \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{K_T}{LK_T + k_a k_f} \end{bmatrix}}_{B} v(t)$$

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Summary of steps

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    i. Equations describing interaction among elements in each domain

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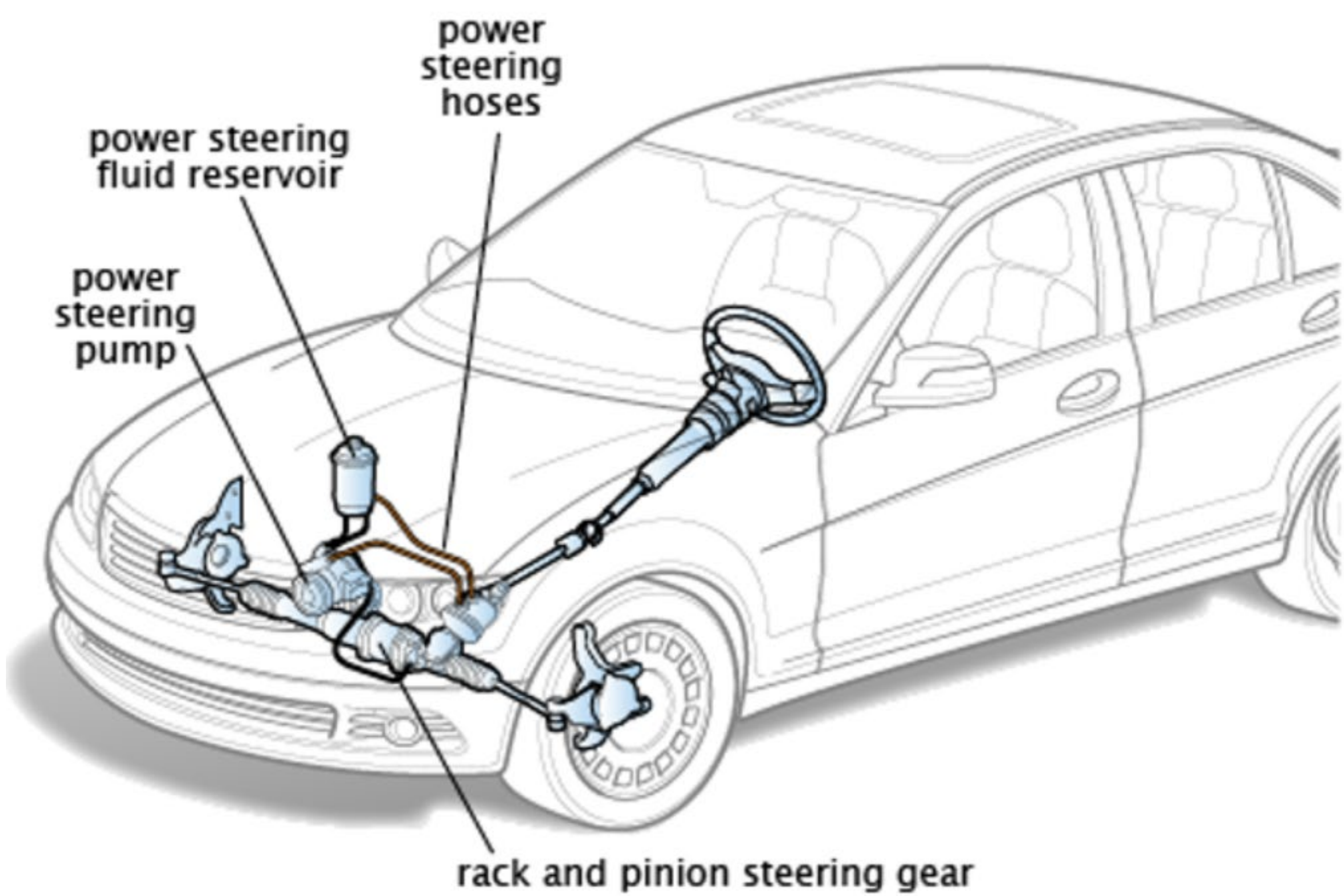
6. State-space to Transfer Fn: $G(s) = \frac{\hat{Y}(s)}{\hat{U}(s)} = C(sI - A)^{-1}B + D$

Example 2: Hydraulic assisted steering wheel mechanism in a car



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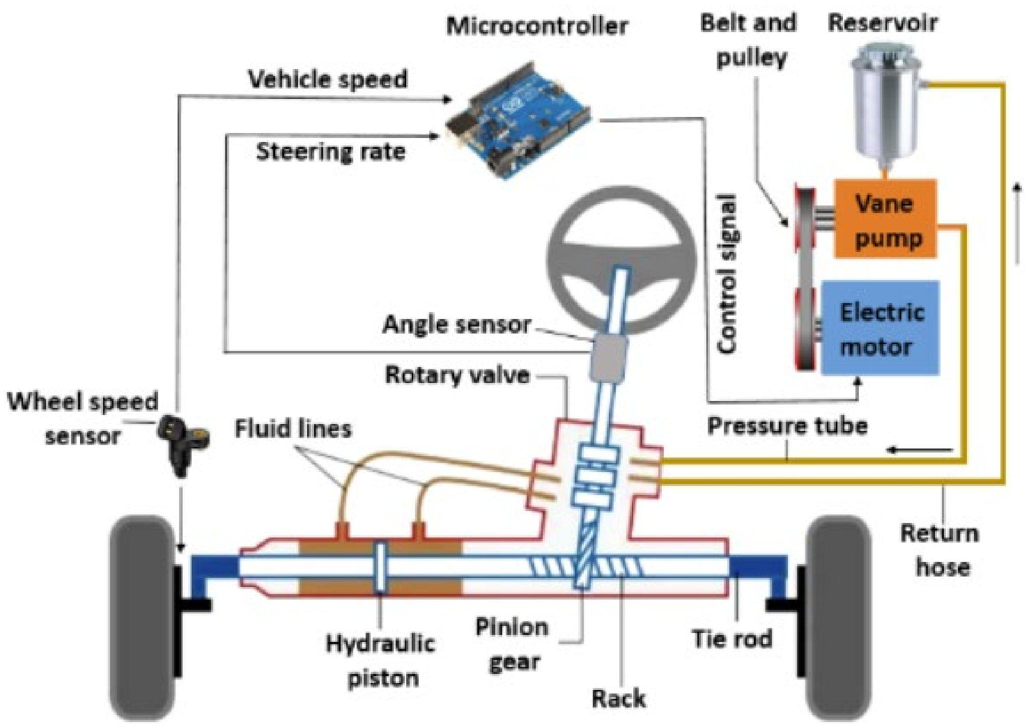
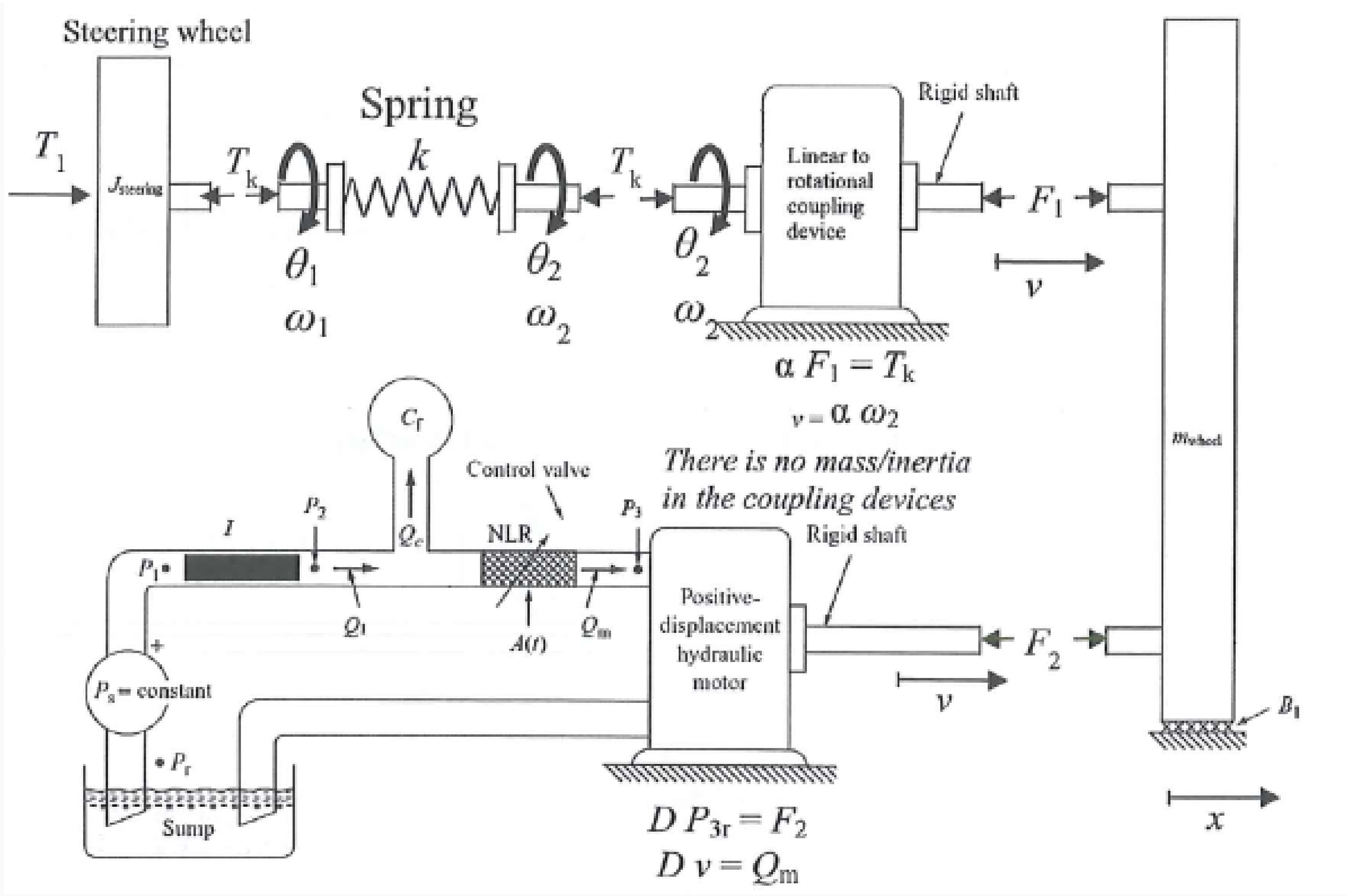


Example 2: Steering wheel

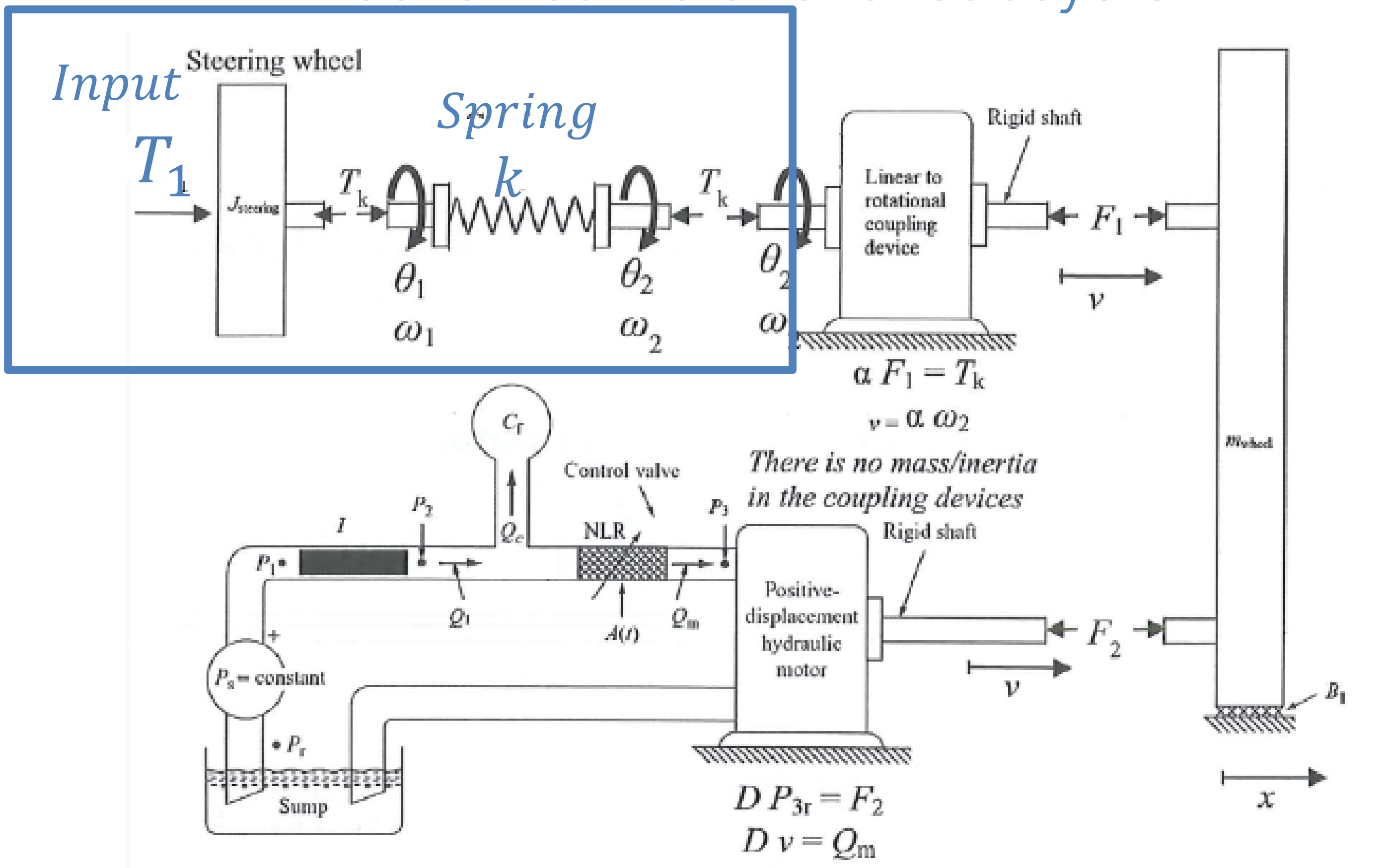
Steering wheel as interconnected multidomain system



Let's understand it!

Example 2: Steering wheel

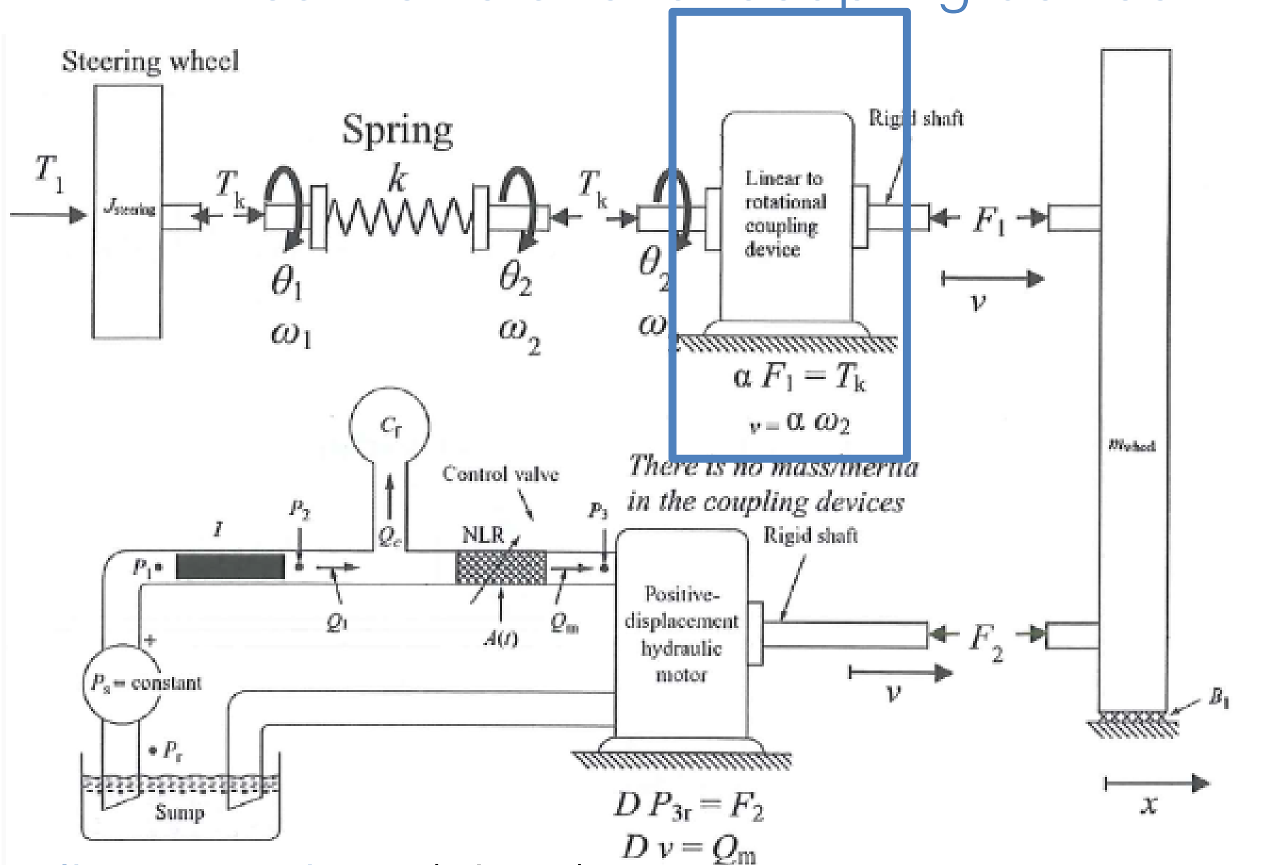
Mechanical-rotational subsystem



- Torque applied by driver: T_1 with inertia $J_{steering}$
- Connected to spring with constant k and torque T_k

Example 2: Steering wheel

Linear-to-rotational coupling device

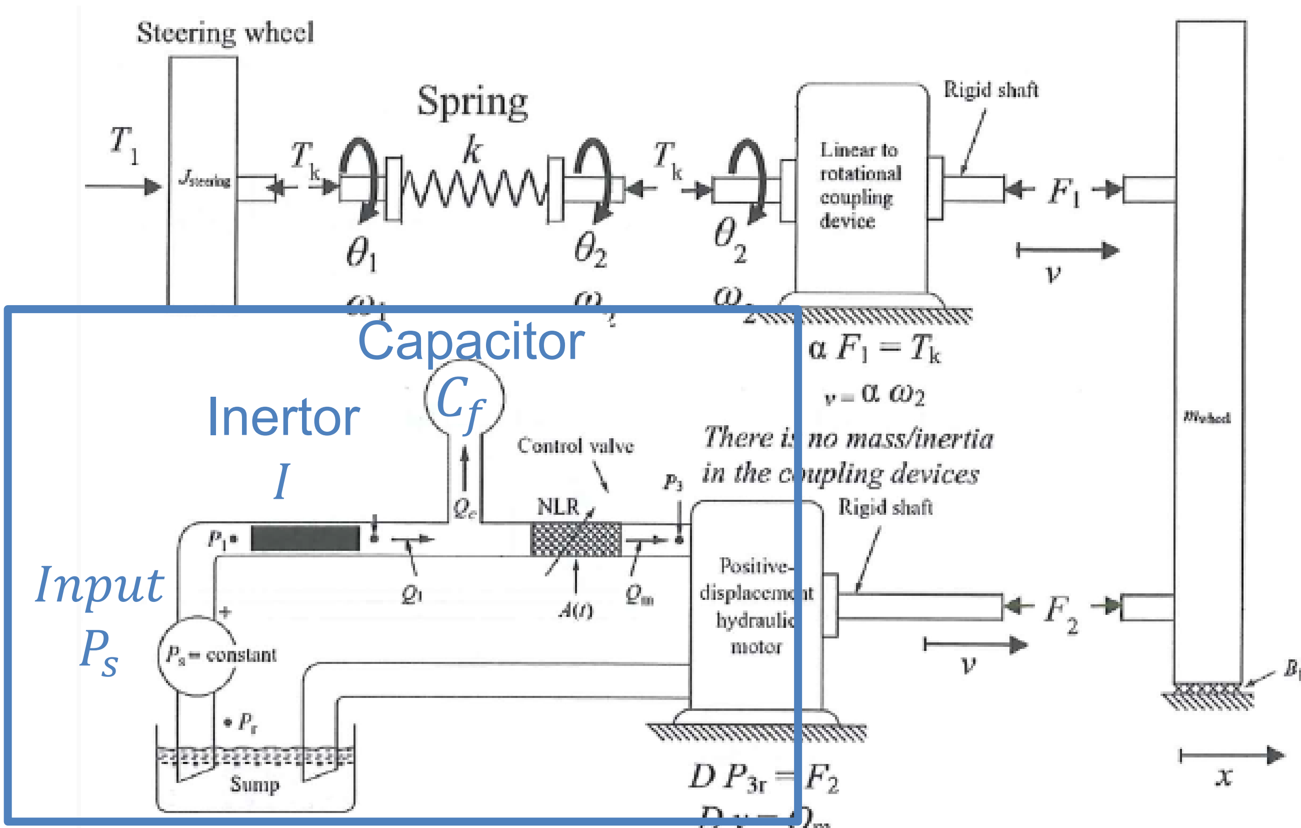


Coupling equations (given)

- Transform torque to linear force: $T_k = \alpha F_1$
- Transform angular velocity to linear velocity: $\alpha\omega_2 = v$

Example 2: Steering wheel

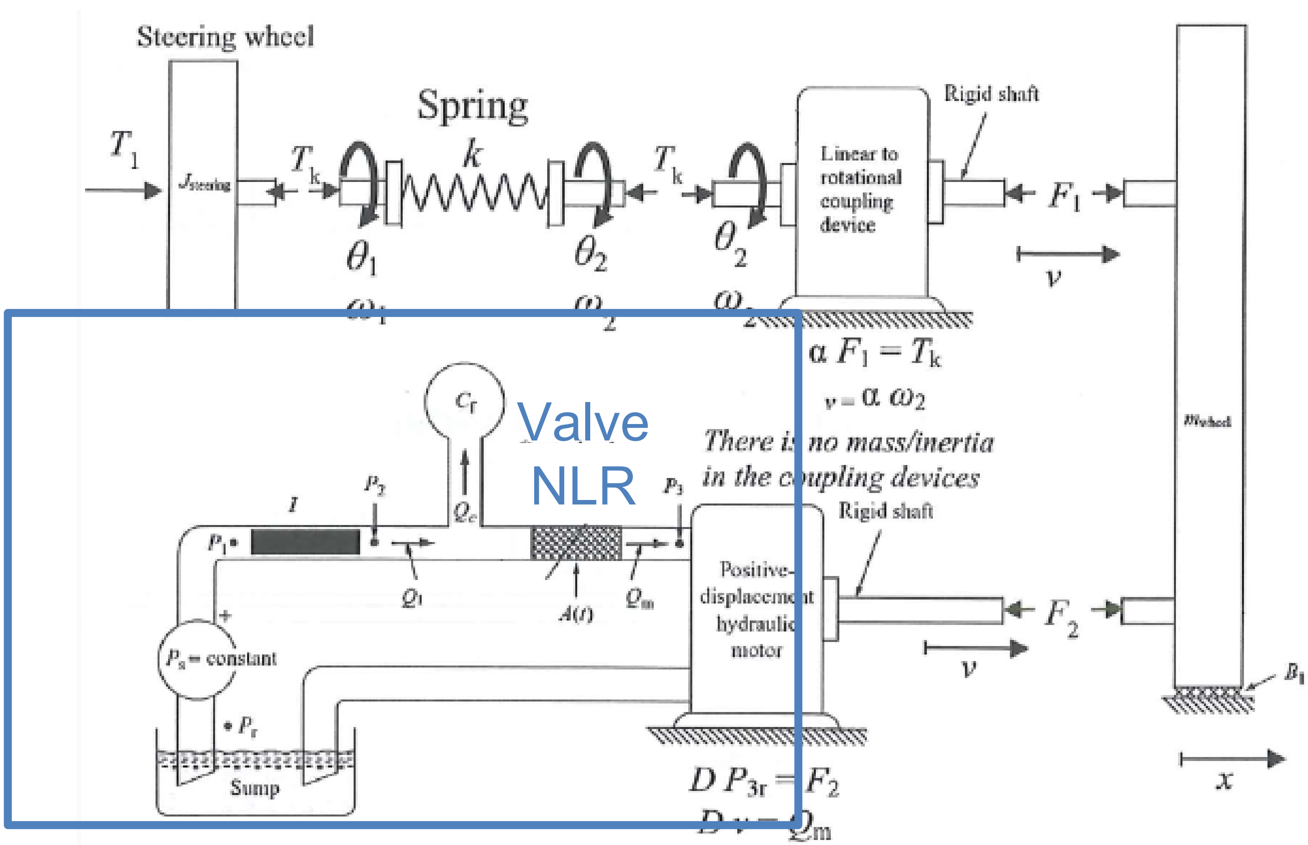
Fluid subsystem



- Pressure P_s is regulated by valve NLR to provide ‘hydraulic assisted forces’
- Pressure P_{12} across inertor with inertance I
- Flow Q_c through fuid capacitor with capacitance C_f

Example 2: Steering wheel

Fluid subsystem

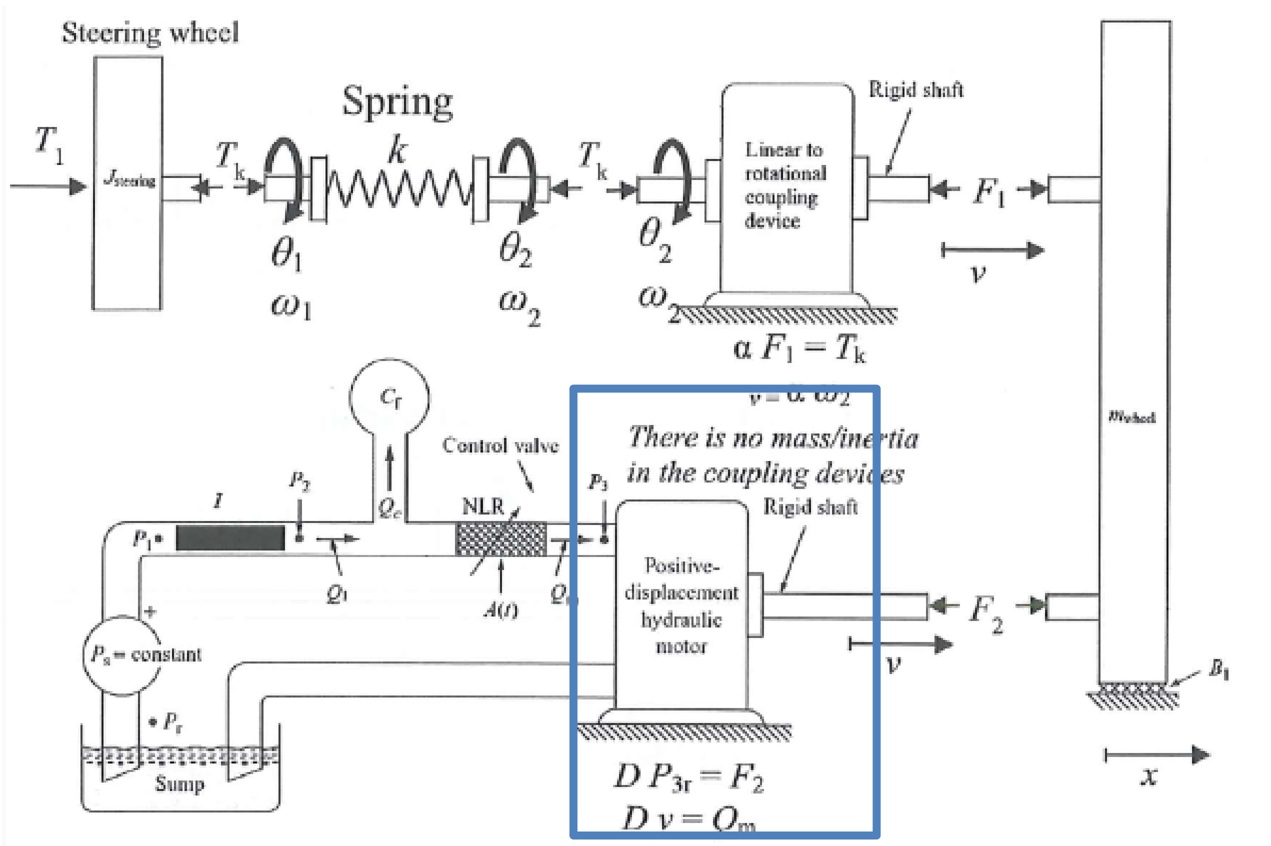


Pressure P_{23} across valve NLR and fluid flow Q_m through valve NLR related by $\frac{1}{cA^2} {Q_m}^2 = P_{23}$ with c being the valve constant

A is regulated based on amount of spring torsion in steering wheel bar T_2 as $A = f(T_2)$

Example 2: Steering wheel

Fluid-to-linear coupling

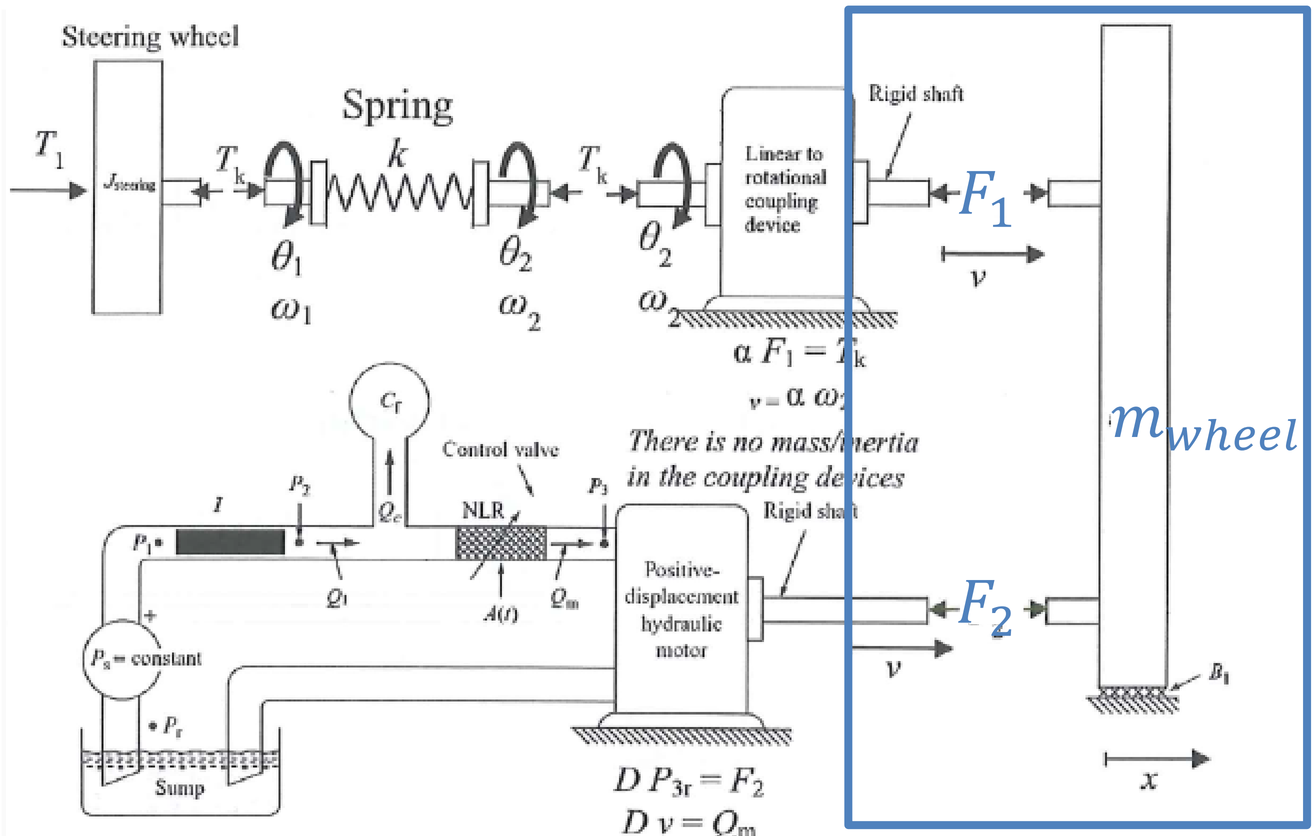


Coupling equations (given)

- Pressure accros coupling device P_{3r} converted to linear force $DP_{3r} = F_2$
- Fluid Flow through coupling device Q_m converted to linear velocity $Dv = Q_m$

Example 2: Steering wheel

Linear-mechanical system (wheels)



- Linear force due to driver F_1 and due to hydraulic pressure F_2 drive the wheel mass
- Wheel mass interacts with road friction, modelled as damper with damping coefficient B_1

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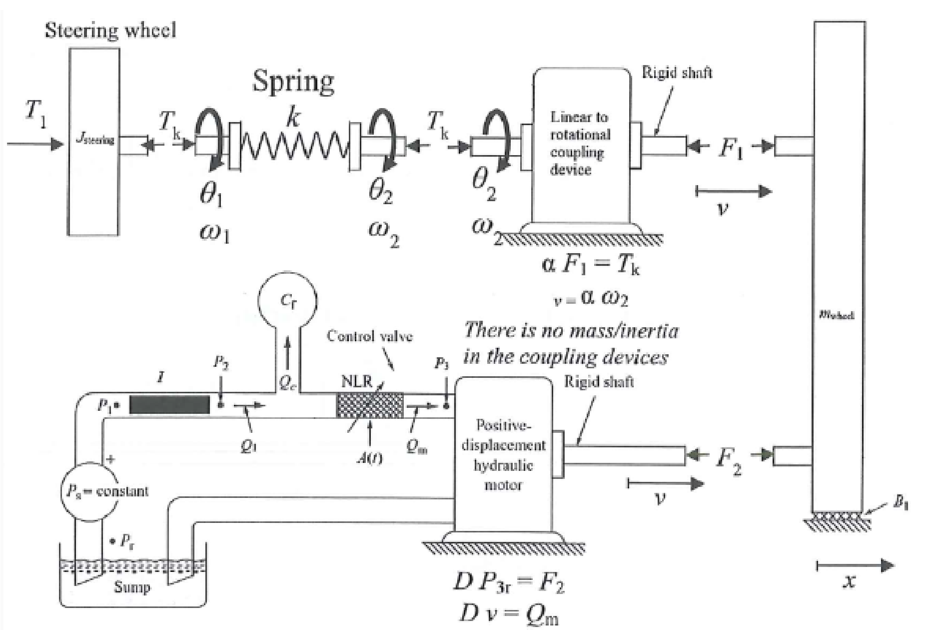
Example 2: Steering wheel

Step 1. Identify energy storing elements, dissipative elements and inputs



Inputs:

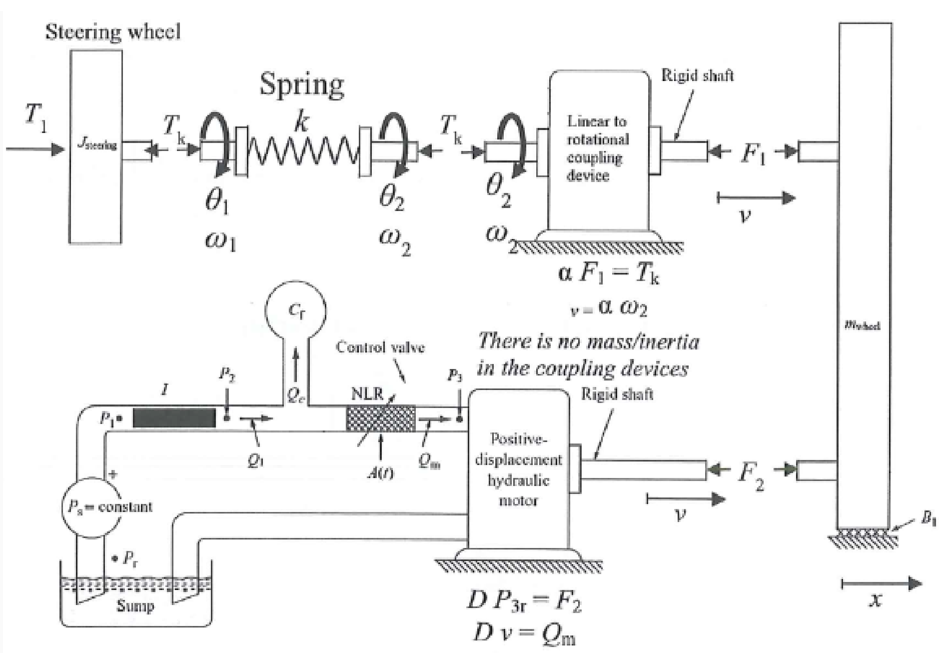
Torque applied by driver: T_1

Pressure source: P_s

Element	Type
Inertia $J_{steering}$	A-type
Spring compliance k	T-type
Fluid capacitor C_f	A-type
Fluid inertor I	T-type
Wheel mass m_{wheel}	A-type
Linear damper B_1	D-type

Example 2: Steering wheel

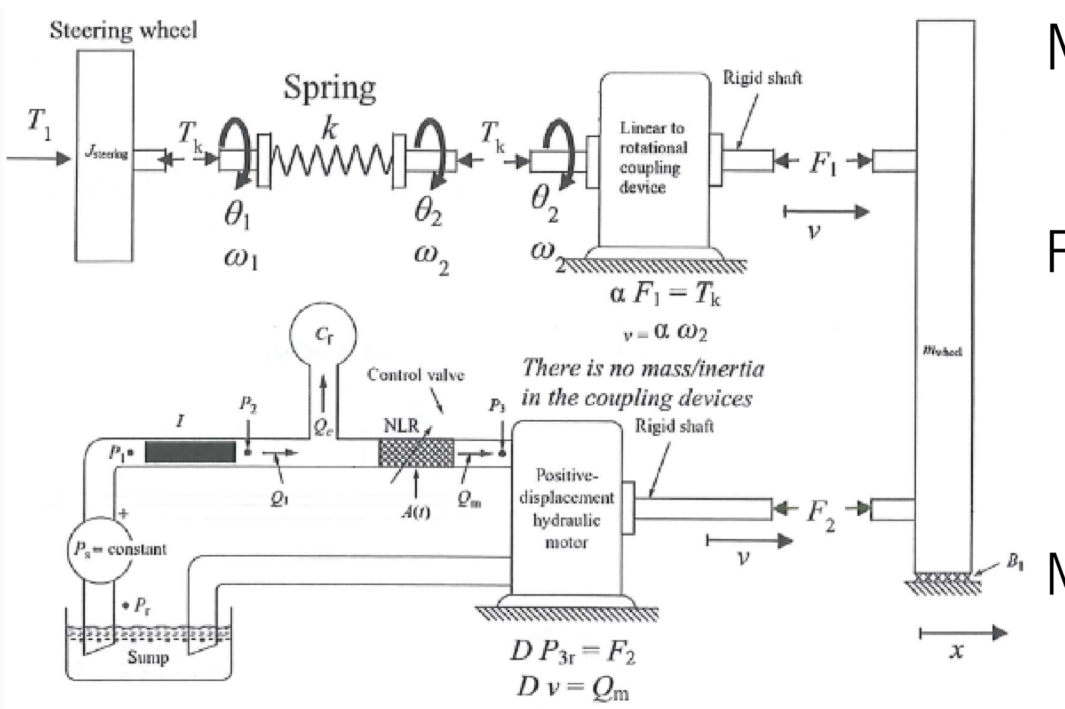
Step 2. i) Elemental equations for A,T,D-type elements



Element	Equation	Type
Inertia $J_{steering}$	$T_{steering} = J_{steering}\frac{d\omega_1}{dt}$	A-type
Spring compliance k	$\frac{1}{k}\frac{dT_k}{dt} = \omega_1 - \omega_2$	T-type
Fluid capacitor C_f	$Q_c = C_f\frac{dP_{2r}}{dt}$	A-type
Fluid inertor I	$I\frac{dQ_1}{dt} = P_{21}$	T-type
Wheel mass m_{wheel}	$F_{wheel} = m_{wheel}\frac{dv}{dt}$	A-type
Linear damper B_1	$F_{B1} = B_1 v$	D-type

Example 2: Steering wheel

Step 2. ii) Describe interaction between elements in each domain



Mechanical rotational system

$$T_1 = T_{steering} + T_k$$

Fluid system

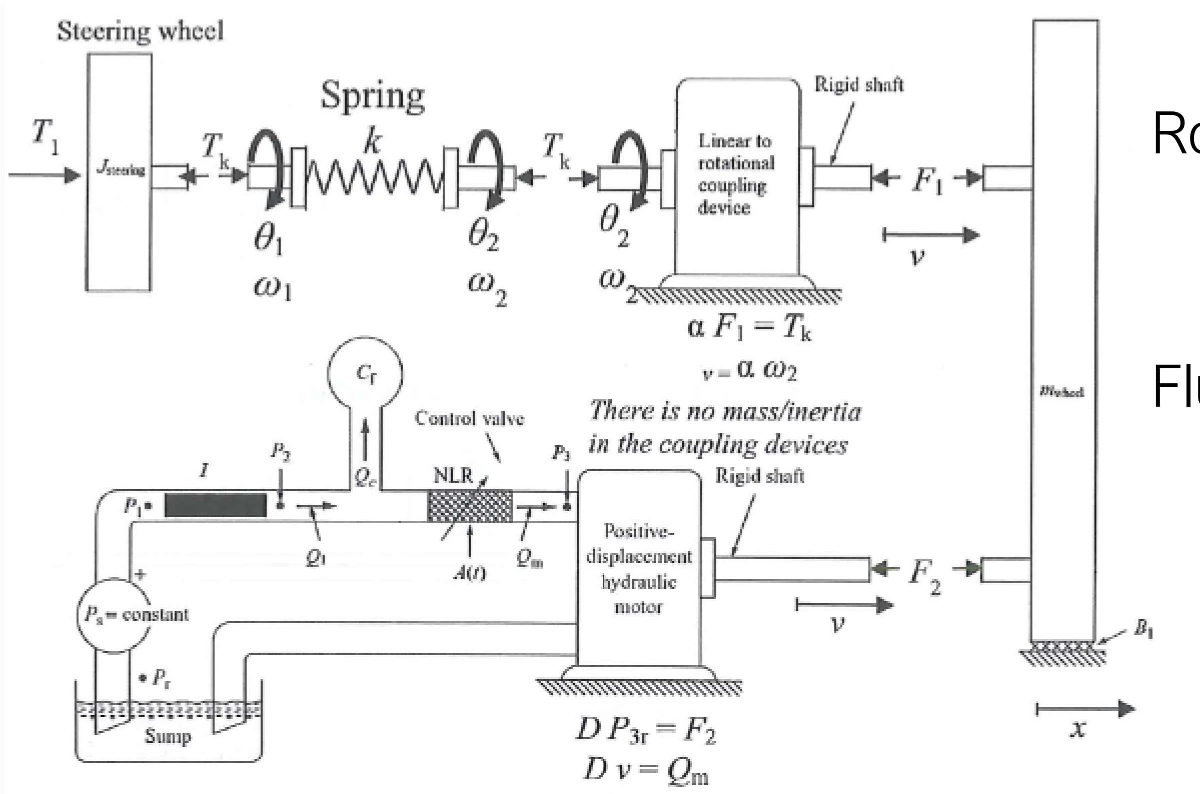
$$P_{21} = P_s - P_{2r}$$
$$Q_c = Q_1 - Q_m$$
$$P_{3r} = P_{2r} - P_{23}$$

Mechanical linear system

$$F_{wheel} = F_1 + F_2 - F_{B1}$$

Example 2: Steering wheel

Step 2. iii) Coupling equations (given)



Rotational-linear coupling:

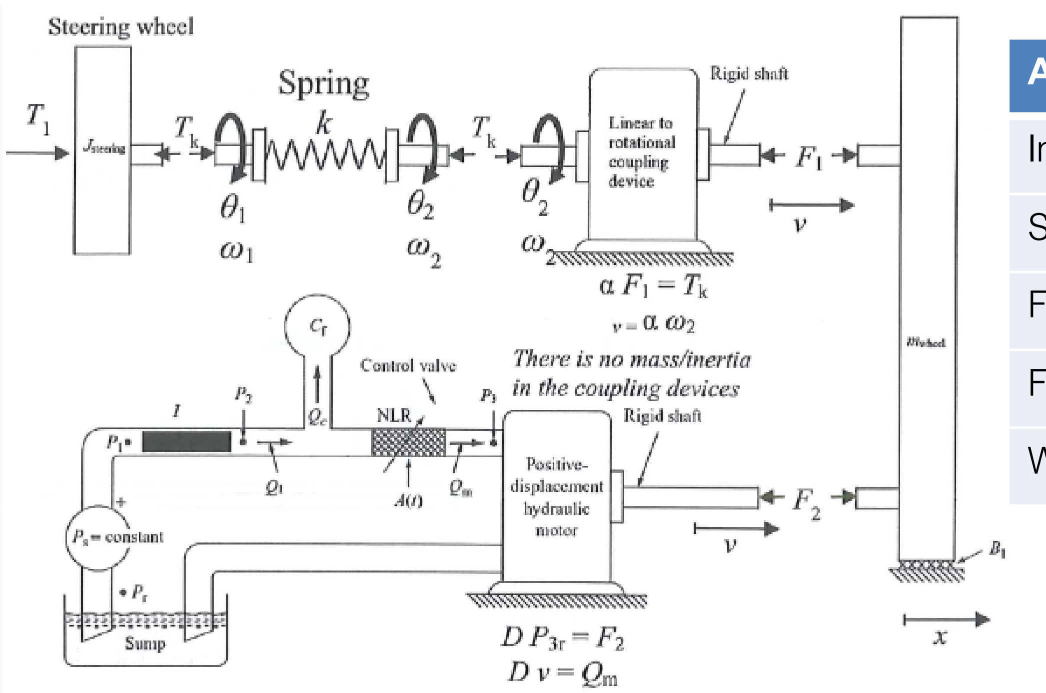
$$T_k = \alpha F_1$$
$$\alpha\omega_2 = v$$

Fluid-linear coupling:

$$DP_{3r} = F_2$$
$$Dv = Q_m$$

Example 2: Steering wheel

Step 3. Propose A-type and T-type variables as state variables

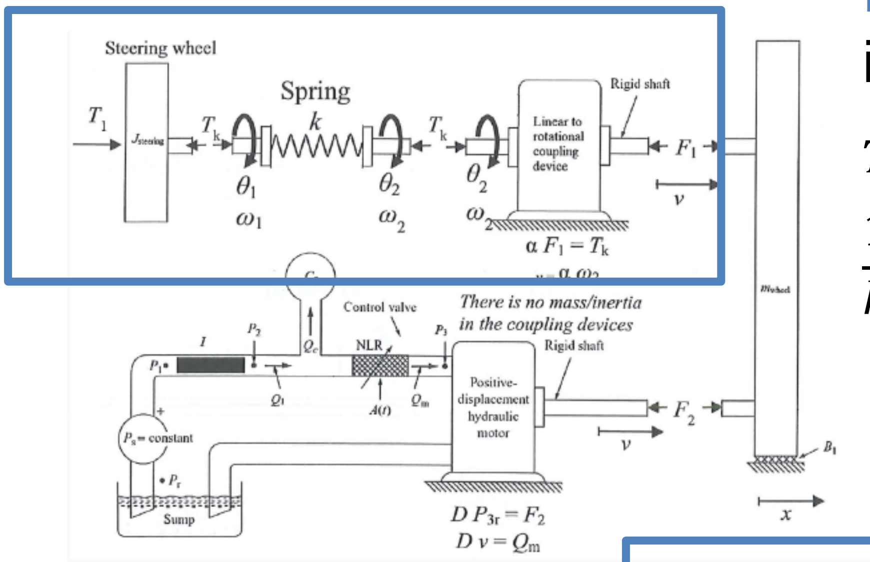


A,T-type elements	Variables
Inertia $J_{steering}$	ω_1
Spring compliance k	T_k
Fluid capacitor C_f	P_{2r}
Fluid inertor I	Q_1
Wheel mass m_1	v

Inputs $[T_1\ P_s]^T$
States $[Q_1\ P_{2r}\ T_k\ \omega_1\ v]^T$

Example 2: Steering wheel

Step 4. Divide into subsystems and use equations i), ii) and iii) to express the dynamics only in terms of state variables and inputs



Mechanical rotational system

i)Elemental

$$T_{steering} = J_{steering}\frac{d\omega_1}{dt}$$

$$\frac{1}{k}\frac{dT_k}{dt} = \omega_1 - \omega_2$$

ii)Interconect

$$T_1 = T_{steering} + T_K$$

iii)Coupling

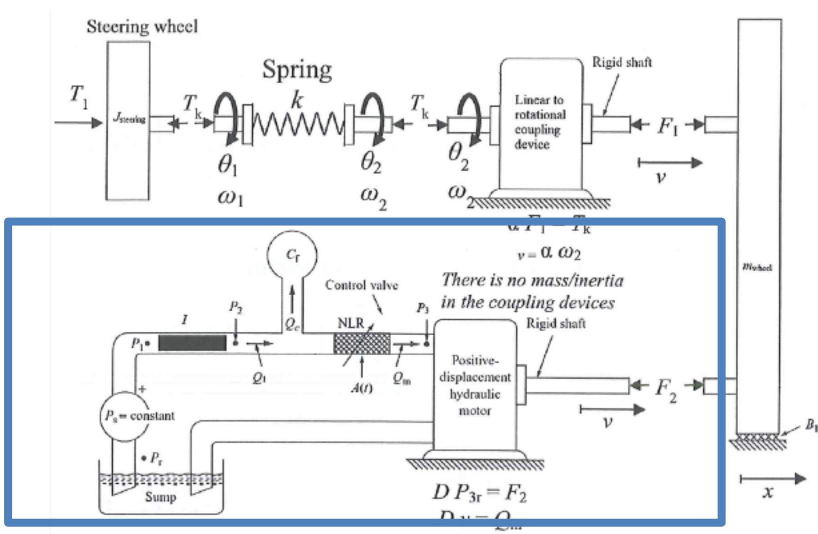
$$\alpha\omega_2 = v$$

$$\boxed{\begin{aligned}\frac{d\omega_1}{dt} &= \frac{T_1}{J_{steering}} - \frac{T_K}{J_{steering}}\\ \frac{dT_k}{dt} &= k\omega_1 - \frac{kv}{\alpha}\end{aligned}}$$

All state variables and inputs!!

Example 2: Steering wheel

Step 4. Divide into subsystems and use equations i), ii) and iii) to express the dynamics only in terms of state variables and inputs



Fluid system

i)Elemental

$$Q_c = C_f \frac{dP_{2r}}{dt}$$

$$I \frac{dQ_1}{dt} = P_{21}$$

ii)Intercon.

$$P_{21} = P_s - P_{2r}$$

$$Q_c = Q_1 - Q_m$$

iii)Coupling:

$$Dv = Q_m$$

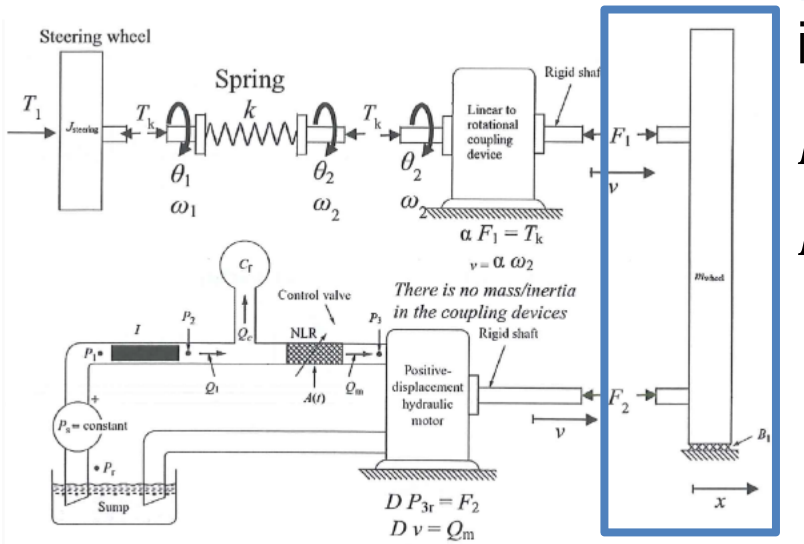
$$\boxed{\begin{aligned} \frac{dP_{2r}}{dt} &= \frac{Q_1}{C_f} - \frac{Dv}{C_f} \\ \frac{dQ_1}{dt} &= \frac{P_s}{I} - \frac{P_{2r}}{I} \end{aligned}}$$

All state variables and inputs!!

Example 2: Steering wheel

Step 4. Divide into subsystems and use equations i), ii) and iii) to express the dynamics only in terms of state variables and inputs

+



Mechanical linear system

i)Elemental

$$F_{wheel} = m_{wheel}\frac{dv}{dt}$$

$$F_{B1} = B_1 v$$

ii)Intercon.

$$F_{wheel} = F_1 + F_2 - F_{B1}$$

iii)Coupling:

$$T_k = \alpha F_1$$

$$DP_{3r} = F_2$$

where $P_{3r} = P_{2r} - P_{23}$ and $P_{23} = \frac{1}{cA^2}{Q_m}^2$

$$\boxed{\frac{dv}{dt} = \frac{\frac{T_k}{\alpha} + D\left(P_{2r} - \frac{1}{cA^2}D^2v^2\right) - B_1 v}{m_{wheel}}}$$

All state variables and inputs!!

Example 2: Steering wheel

Step 5. Express system as $\dot{x} = Ax + Bu$ if linear or $\dot{x} = f(x) + g(x)u$ if not

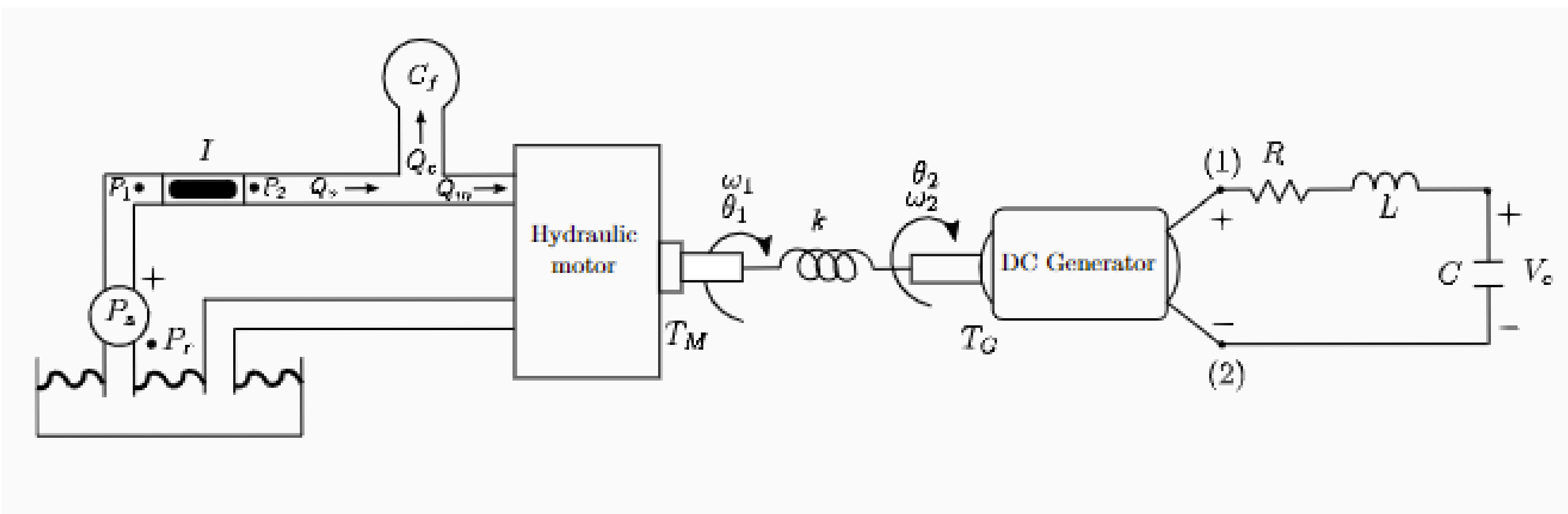
$$\begin{bmatrix} \frac{dQ_1}{dt} \\ \frac{dP_{2r}}{dt} \\ \frac{dT_k}{dt} \\ \frac{d\omega_1}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} \frac{P_s}{I} - \frac{P_{2r}}{I} \\ \frac{Q_1}{C_f} - \frac{Dv}{C_f} \\ k\omega_1 - \frac{kv}{\alpha} \\ \frac{T_1}{J_{steering}} - \frac{T_K}{J_{steering}} \\ \frac{\frac{T_k}{\alpha} + D\left(P_{2r} - \frac{1}{cA^2} D^2 v^2\right) - B_1 v}{m_{wheel}} \end{bmatrix}$$

Next lecture:

Guest lecture: Modelling via Euler Lagrange

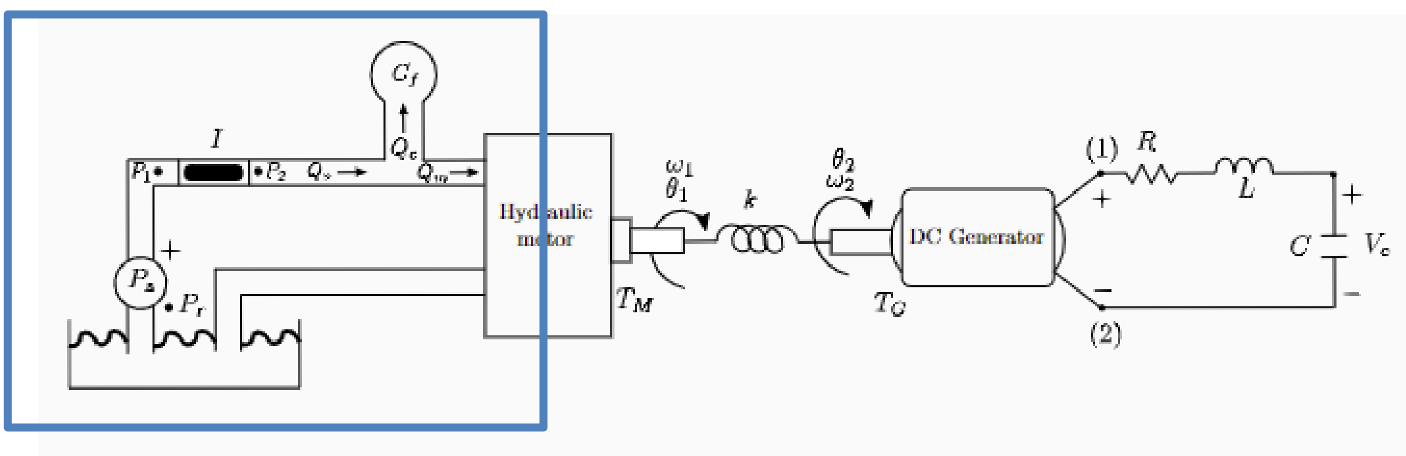
Homework Example: Hydraulic motor and DC generator supplying RLC circuit

In this multidomain system pressure $P_{2r} = P_2 - P_r$ generates torque T_M coupled through rotational spring to a generator to produce a voltage that modifies dynamics of RLC circuit.



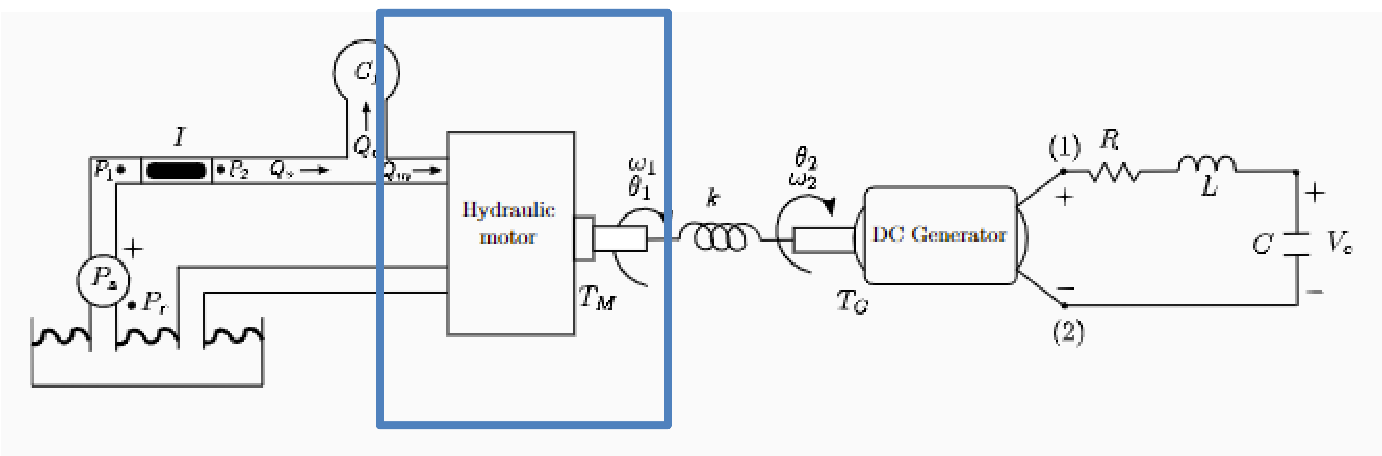
Homework Example: Hydraulic motor and DC generator supplying RLC circuit

- Fluid system: fluid inertor with inertance I, fluid capacitor with capacitance C_f



Homework Example: Hydraulic motor and DC generator supplying RLC circuit

- Fluid system: fluid inertor with inertance I, fluid capacitor with capacitance C_f
- Rotational mechanical system 1 (Hydraulic motor): J_M moment of intertia, ω_1 angular velocity, θ_1 angular position, T_M torque



Homework Example: Hydraulic motor and DC generator supplying RLC circuit

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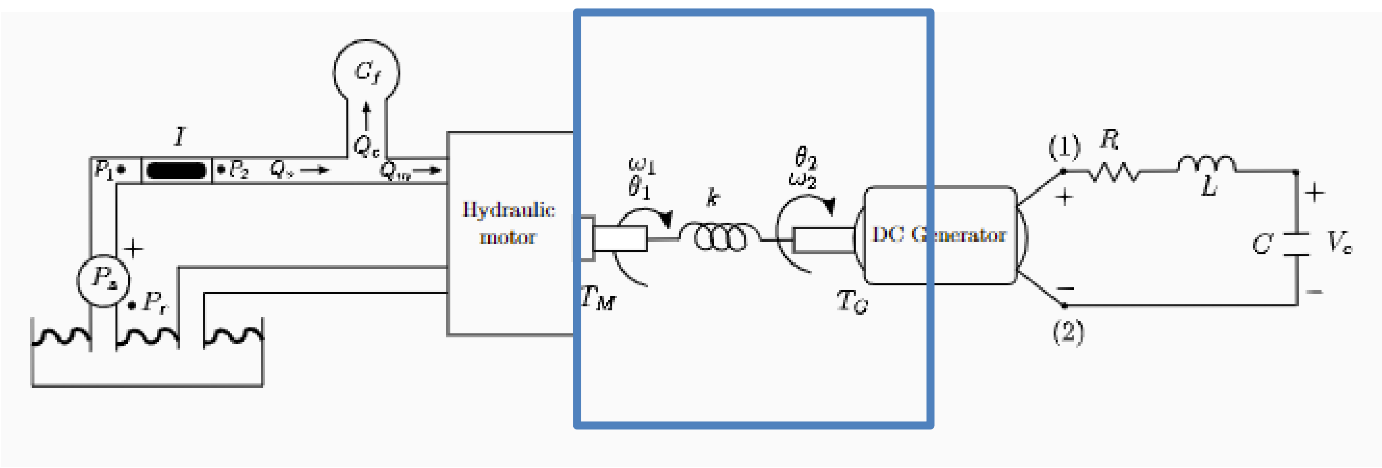
- Rotational mechanical system 2 (Spring): Torque T_k with spring constant k



Homework Example: Hydraulic motor and DC generator supplying RLC circuit

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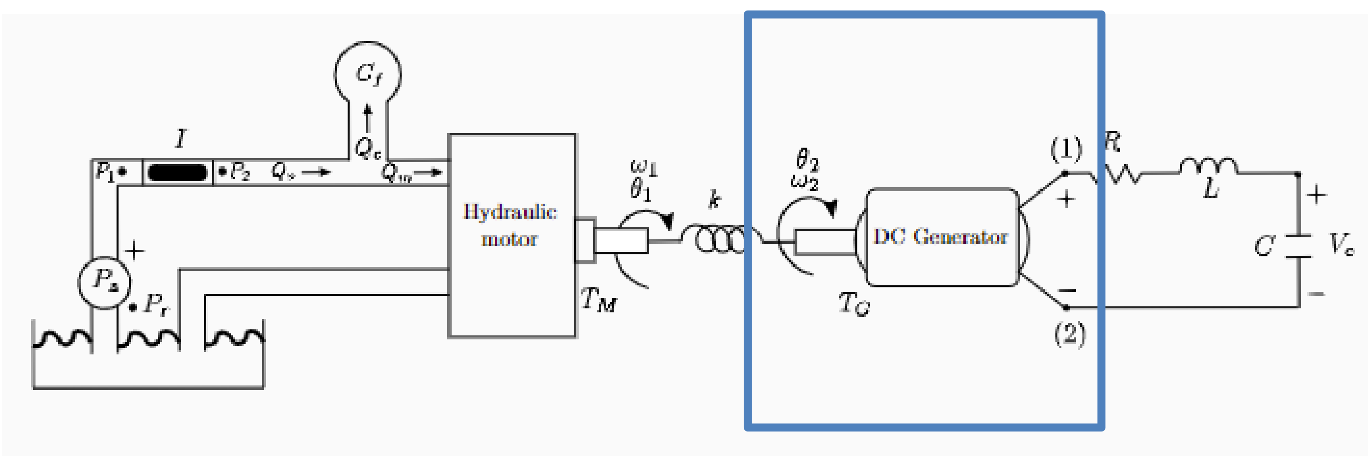
- Rotational mechanical system 2 (Spring): Torque T_k with spring constant k
- Rotational mechanical system 3 (generator): J_G moment of intertia, ω_2 angular velocity, θ_2 angular position, T_G torque



Homework Example: Hydraulic motor and DC generator supplying RLC circuit

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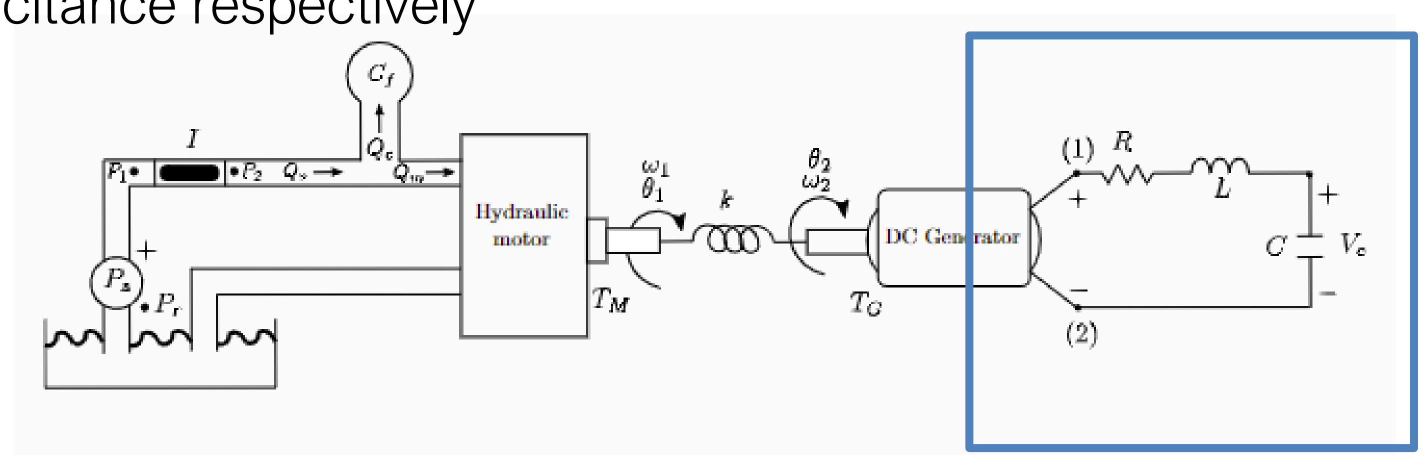
- Rotational mechanical system 2 (Spring): Torque T_k with spring constant k
- Rotational mechanical system 3 (generator): J_G moment of intertia, ω_2 angular velocity, θ_2 angular position, T_G torque
- Electrical system: R, L and C are resistance, inductance and capacitance respectively



Homework Example: Hydraulic motor and DC generator supplying RLC circuit

Coupling with coupling constants D_r and α_G

Fluid to mechanical

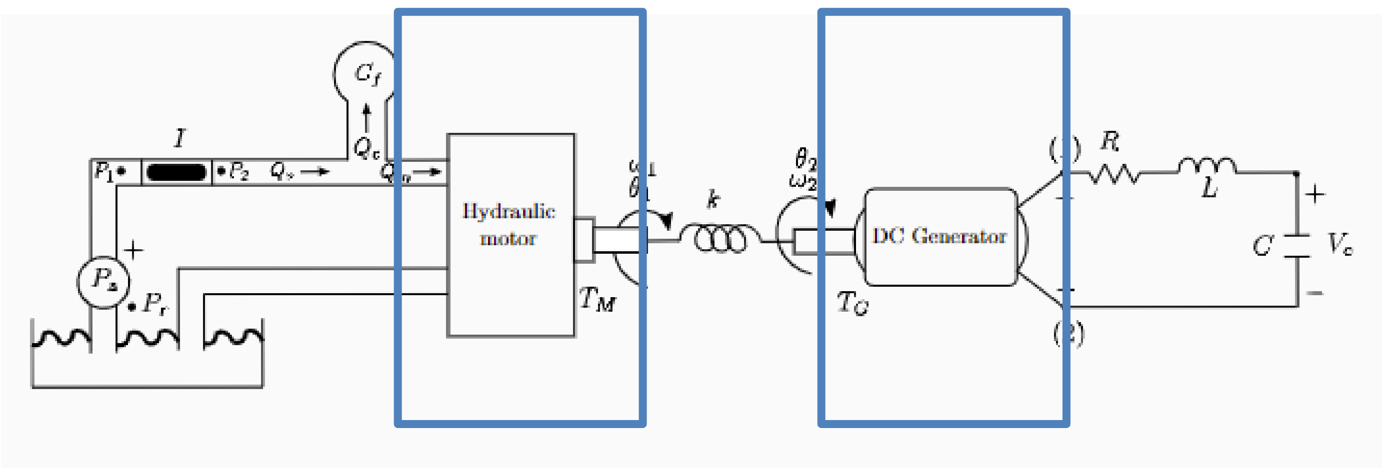
$$T_M = D_r P_{2r}$$

$$\omega_1 = \frac{1}{D_r} Q_m$$

Mechanical to electrical

$$T_G = \frac{1}{\alpha_G} i_L$$

$$\omega_2 = \alpha_G V_{12}$$



Homework Example: Hydraulic motor and DC generator supplying RLC circuit

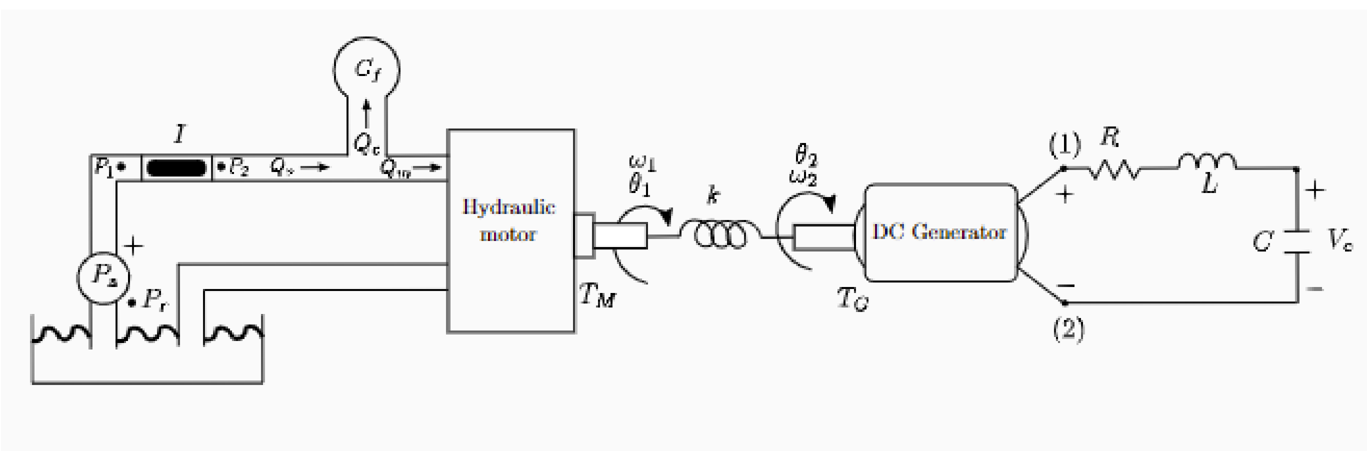
Step 1. Identify A-type and T-type elements and variables

A-type

- Fluid capacitor: P_{2r}
- Mass moment of inertia 1: ω_1
- Mass moment of inertia 2: ω_2
- Capacitor: V_C

T-type

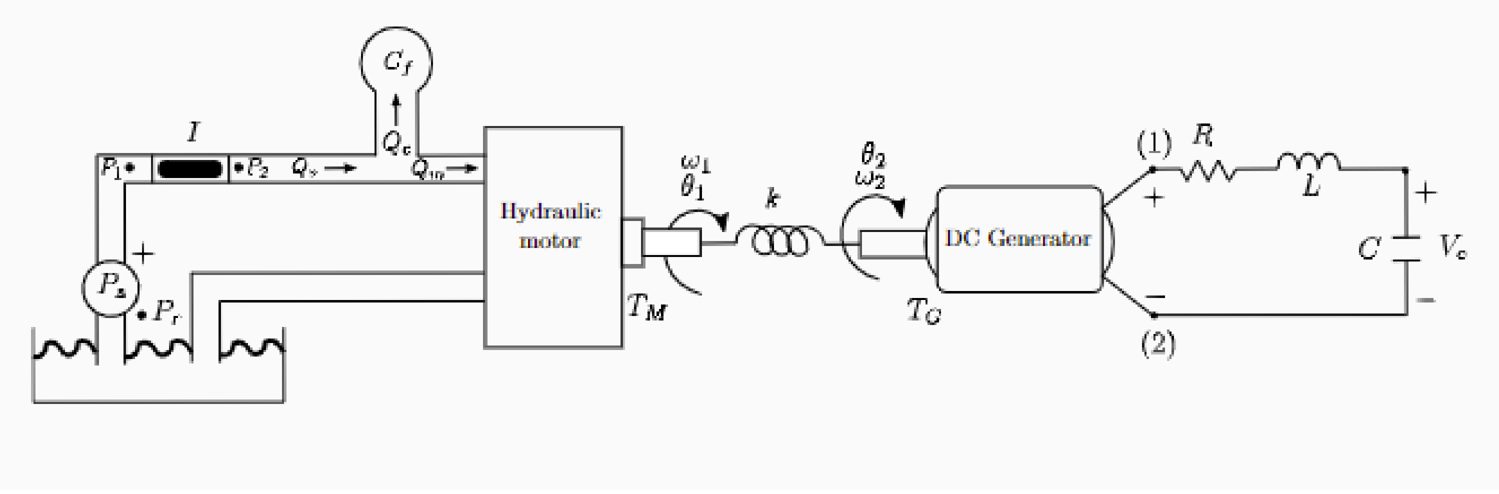
- Inertor: Q_V
- Spring (compliance): T_k
- Inductor: i_L



Homework Example: Hydraulic motor and DC generator supplying RLC circuit

Step 2. i) Elemental equations (A-type, T-type, D-type)

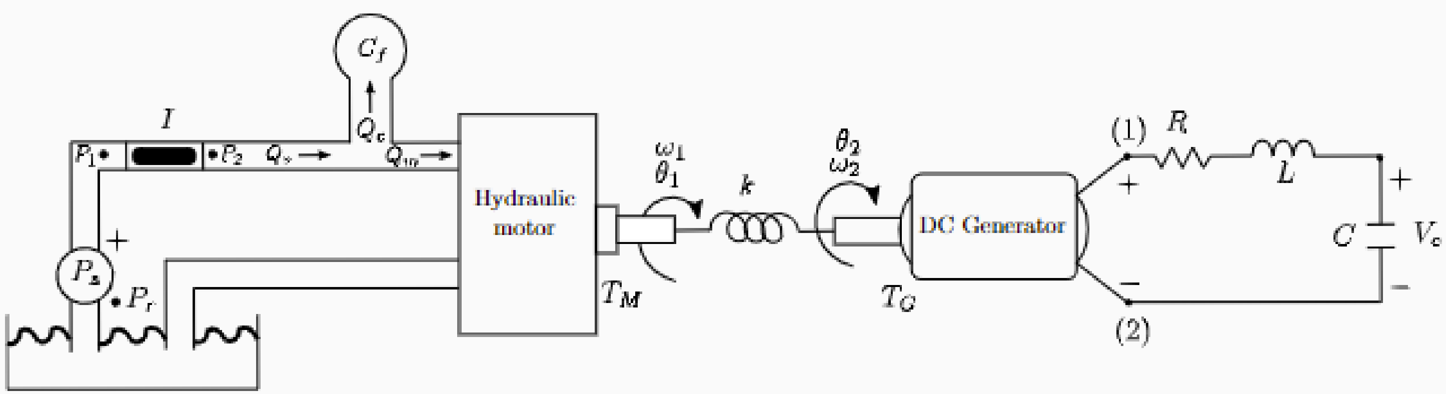
$$
\begin{aligned}
Q_c &= C_f \dot{P}_{2r}, & P_{12} &= I\dot{Q}_v, \\
T_{m_1} &= J_M \dot{\omega}_1, & T_{m_2} &= J_G \dot{\omega}_2, \\
T_k &= k(\theta_1 - \theta_2), & V_R &= R i_R, \\
V_L &= L i'_L, & i_C &= C\dot{V}_c.
\end{aligned}
$$



Homework Example: Hydraulic motor and DC generator supplying RLC circuit

Step 2. ii) Interaction equations (Kirchoff's law, Newton's law…)

$$
\begin{aligned}
P_s &= P_{12} + P_{2r}, & Q_v &= Q_c + Q_m, \\
T_G &= T_{m_2} - T_k, & T_M &= T_{m_1} + T_k, \\
i_L &= i_C = i_R, & V_{12} &= V_c + V_R + V_L.
\end{aligned}
$$



Homework Example: Hydraulic motor and DC generator supplying RLC circuit

Step 3. Propose state variables

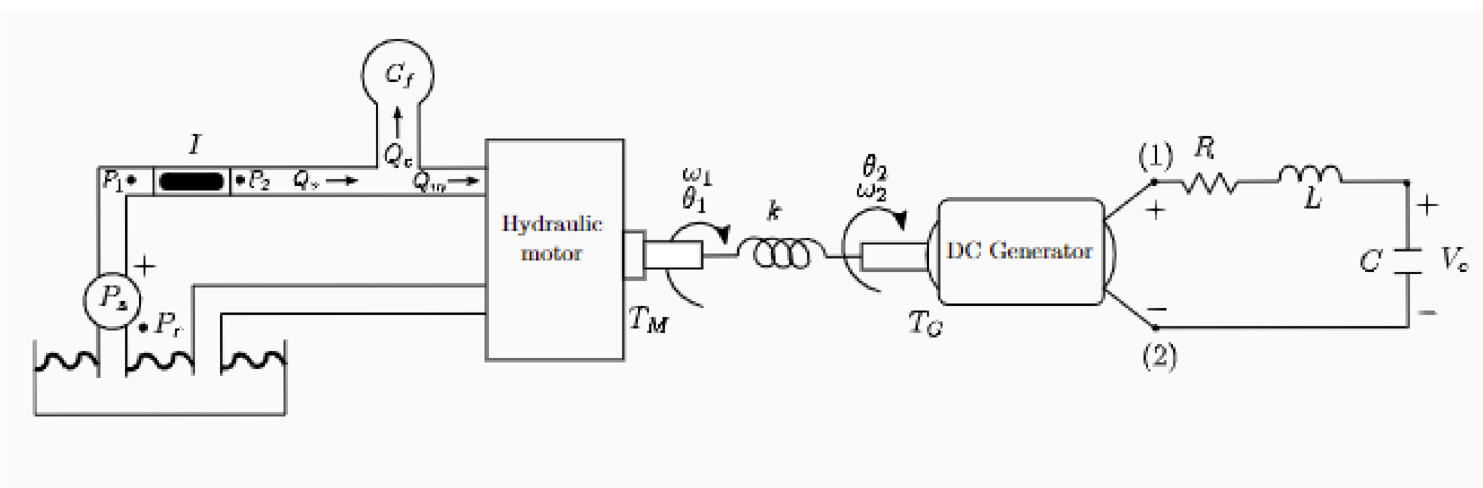
A-type and

- P_{2r}
- w_1
- w_2
- V_C

T-type

- Q_V
- i_L

$T_k = k(\theta_1 - \theta_2)$ not independent, so not necessarily a state
We can take θ_1 and θ_2 as states, defining
$\dot{\theta}_1 = \omega_1$ and $\dot{\theta}_2 = \omega_2$



Homework Example: Hydraulic motor and DC generator supplying RLC circuit

Step 4. Combine equations for each domain and coupling equations to represent system with state variables, input P_s, and output V_c

$$
\begin{aligned}
\dot{Q}_v &= -\tfrac{1}{I}P_{2r} + \tfrac{1}{I}P_s, \\
\dot{P}_{2r} &= \tfrac{1}{C_f}Q_v - \tfrac{D_r}{C_f}\omega_1, \\
\dot{\theta}_1 &= \omega_1, \\
\dot{\omega}_1 &= \tfrac{D_r}{J_M}P_{2r} - \tfrac{k}{J_M}(\theta_1 - \theta_2)
\end{aligned}
$$

$$
\begin{aligned}
\dot{\theta}_2 &= \omega_2, \\
\dot{\omega}_2 &= \tfrac{1}{J_G\alpha_G}i_L + \tfrac{k}{J_G}(\theta_1 - \theta_2) \\
\dot{V}_c &= \tfrac{1}{C}i_L \\
i'_L &= \tfrac{1}{L\alpha_G}\omega_2 - \tfrac{R}{L}i_L - \tfrac{1}{L}V_c
\end{aligned}
$$