university of groningen

faculty of science and engineering

Mechatronics

Week 2 Day 1

Introduction to modeling of dynamical systems

Motivation of Dynamical modeling

- Analysis - gain a deeper understanding of the underlying mechanisms and behavior of the system
- Prediction - forecast the future behavior of a system and anticipate outcomes
- Control - regulate system’s behavior to achieve desired outcomes

Representation of Dynamical Systems

There are 3 ways to represents dynamical systems:

- Ordinary differential equations,
- Transfer functions,
- State-space equations,

which give us the relationship between the input and output of the system.

Representation of systems by differential equations

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_{n-1} \frac{dy}{dt} + a_n y = b_0 \frac{d^n u}{dt^n} + b_1 \frac{d^{n-1} u}{dt^{n-1}} + \cdots + b_{n-1} \frac{du}{dt} + b_n u$$

- u is the input
- y is the output
- $a_1, \ldots, a_{n-1}, a_n$ and $b_0, b_1, \ldots, b_{n-1}, b_n$ are constants
- The left hand-side of the equation is related to the output of the system
- The right hand-side of the equation is related to the inputs of the system

Ref. Equation (3.15) of chapter 3 in the book.

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_{n-1} \frac{dy}{dt} + a_n y = b_0 \frac{d^n u}{dt^n} + b_1 \frac{d^{n-1} u}{dt^{n-1}} + \cdots + b_{n-1} \frac{du}{dt} + b_n u$$

The aim is to transform the differential equation into a transfer function, to do this we first have to take the Laplace transform:

$$s^n \hat{Y}(s) + a_1 s^{n-1} \hat{Y}(s) + \cdots + a_{n-1} s \hat{Y}(s) + a_n \hat{Y}(s) =$$

$$b_0 s^n \hat{U}(s) + b_1 s^{n-1} \hat{U}(s) + \cdots + b_{n-1} s \hat{U}(s) + b_n \hat{U}(s)$$

The transfer function of the system is:

Zeros of the system

$$G(s) = \frac{\hat{Y}(s)}{\hat{U}(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

Characteristic equation → poles of the system

REMARK

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_{n-1} \frac{dy}{dt} + a_n y = b_0 \frac{d^n u}{dt^n} + b_1 \frac{d^{n-1} u}{dt^{n-1}} + \cdots + b_{n-1} \frac{du}{dt} + b_n u$$

$$G(s) = \frac{\hat{Y}(s)}{\hat{U}(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{1 * s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

1. Differential equation $\xrightarrow{\text{Laplace Tranform}}$ Transfer Function

2. The coefficient of highest degree term (s^n) in the characteristic equation is 1

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_{n-1} \frac{dy}{dt} + a_n y = b_0 \frac{d^n u}{dt^n} + b_1 \frac{d^{n-1} u}{dt^{n-1}} + \cdots + b_{n-1} \frac{du}{dt} + b_n u$$

The transfer function of the system is:

Zeros of the system

$$G(s) = \frac{\hat{Y}(s)}{\hat{U}(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

Characteristic equation → poles of the system

- The left hand-side of the equation is related to the output of the system and gives the poles of the system
- The right hand-side of the equation is related to the inputs of the system and it gives the zeros of the system

State-Space Representation of Dynamical Systems

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

- $x \in R^n$, $u \in R$, $y \in R$, $A \in R^{nxn}$, $B \in R^{nx1}$, $C \in R^{1xn}$, and $D \in R$
- $u \in R$ and $y \in R$ means that we are considering a single-input-single-output (SISO) system.
- The case of multi-input-multi-output (MIMO) systems will be treated later on in the course

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

By taking the Laplace transform of the above state-space equation, we arrive at:

$$s\hat{X}(s) = A\hat{X}(s) + B\hat{U}(s) \quad \Rightarrow \hat{X}(s) = (sI - A)^{-1}B\hat{U}(s)$$
$$\hat{Y}(s) = C\hat{X}(s) + D\hat{U}(s)$$

Plugging $\hat{X}(s) = (sI - A)^{-1}B\hat{U}(s)$ in the equation above, we obtain:

$$\hat{Y}(s) = [C(sI - A)^{-1}B + D]\hat{U}(s)$$

$$\boxed{\frac{\hat{Y}(s)}{\hat{U}(s)} = C(sI - A)^{-1}B + D}$$

REMARK: State Space $\xrightarrow{\text{Laplace Tranform}}$ Transfer Function

Transformation of State Equations

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Original system

$$\downarrow \; z = Tx$$

$$\dot{z} = T\dot{x} = TAx + TBu$$
$$= TAT^{-1}z + TBu$$
$$y = Cx + Du = CT^{-1}z + Du$$

Transformed system

where:

- $T \in R^{nxn}$ is an invertible transformation matrix
- $TAT^{-1} = \bar{A},\ TB = \bar{B},\ CT^{-1} = \bar{C},\ D = \bar{D}$

The Transfer Function of the transformed state equation:

$$\bar{G}(s) = \bar{C}(sI - \bar{A})^{-1}\bar{B} + \bar{D} = CT^{-1}(sI - TAT^{-1})^{-1}TB + D =$$
$$C(sI - A)^{-1}B + D$$

Original system

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Transformed system

$$\dot{z} = \bar{A}z + \bar{B}u$$
$$y = \bar{C}z + \bar{D}u$$

$$C(sI - A)^{-1}B + D$$

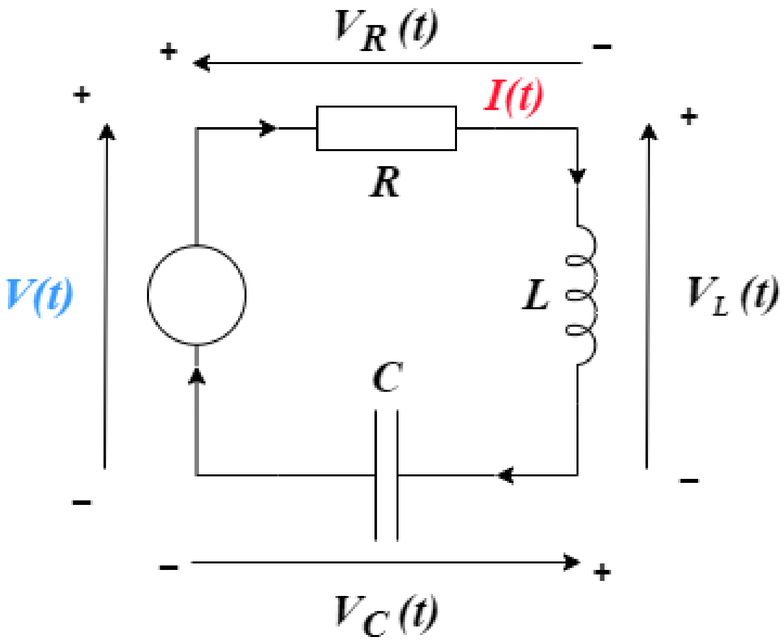
Same Transfer Function

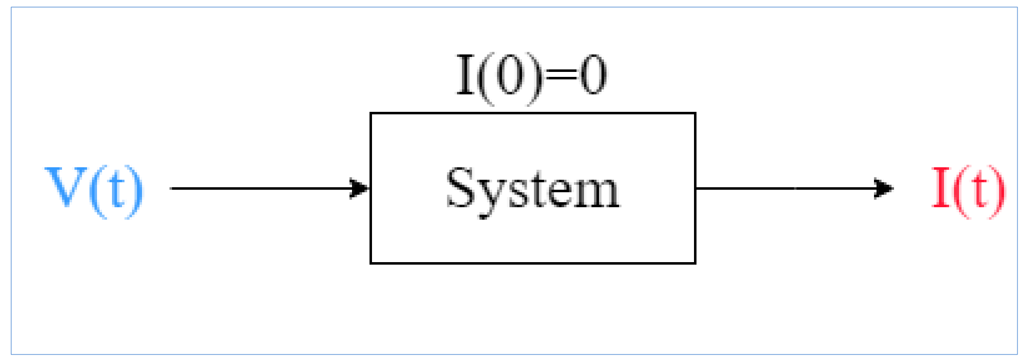
REMARK:

- The transfer function remain the same, i.e., the state-space is not unique

- There are infinite possibilities to express a given transfer function in state-space form

Example (Series RLC Circuit)

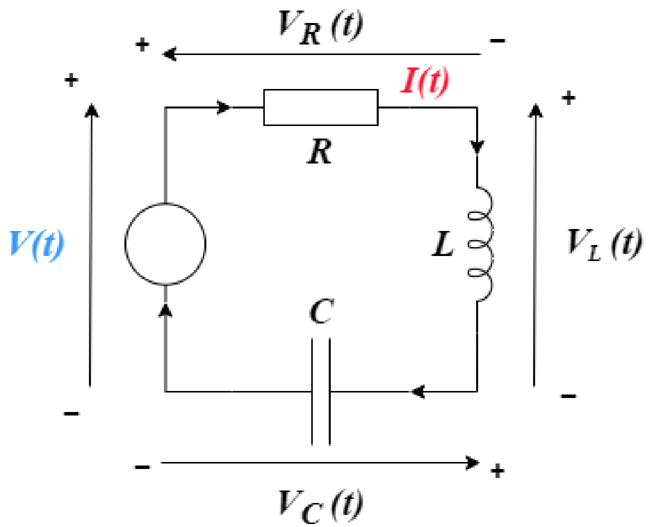


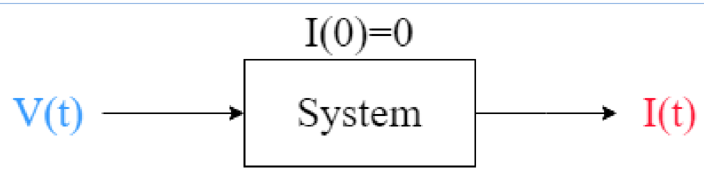


By Kirchoff's voltage law, the sum of voltages around the loop must be zero, i.e.,

$$V(t) - V_R(t) - V_L(t) - V_C(t) = 0 \qquad (1).$$

$$V(t) - V_R(t) - V_L(t) - V_C(t) = 0 \qquad (1)$$





Capacitor:

$$\frac{C dV_C(t)}{dt} = I(t) \;\Rightarrow$$

$$V_C(t) = \frac{1}{C}\int_0^t I(t)dt \qquad (2)$$

Inductor:

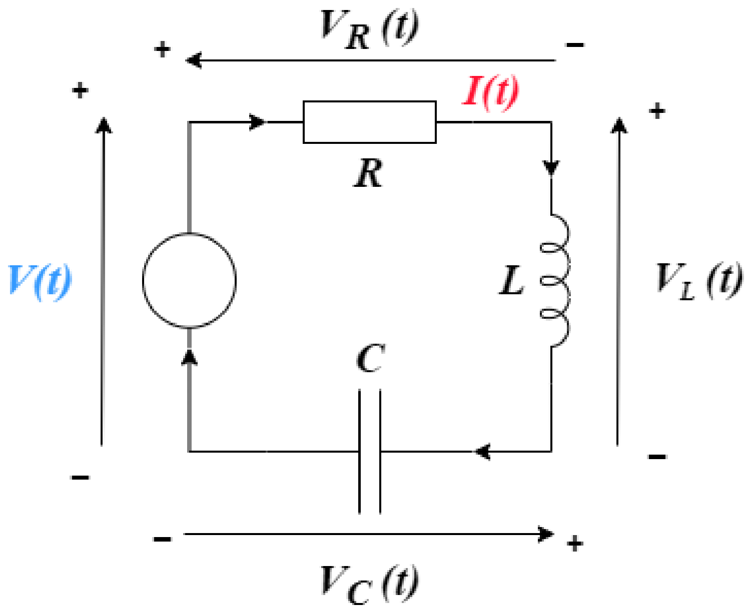
$$V_L(t) = L\frac{dI(t)}{dt} \qquad (3)$$

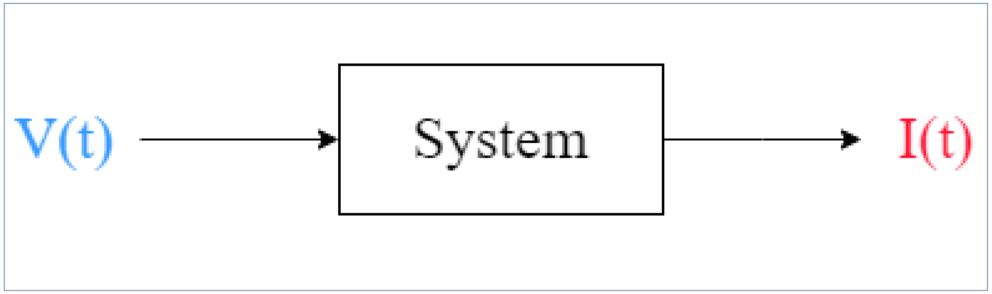
Resistor:

$$V_R(t) = RI(t) \qquad (4)$$

We obtain an integral-differential equation by plugging equations (2) through (4) in (1),

$$V(t) - RI(t) - L\frac{dI(t)}{dt} - \frac{1}{C}\int_0^t I(t)dt = 0 \qquad (5)$$





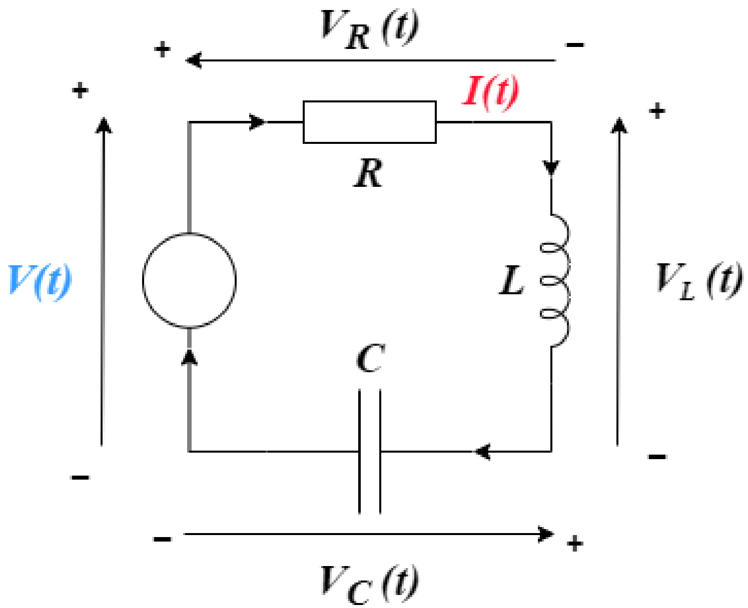
Consider the series RLC circuit model and its state space representation:

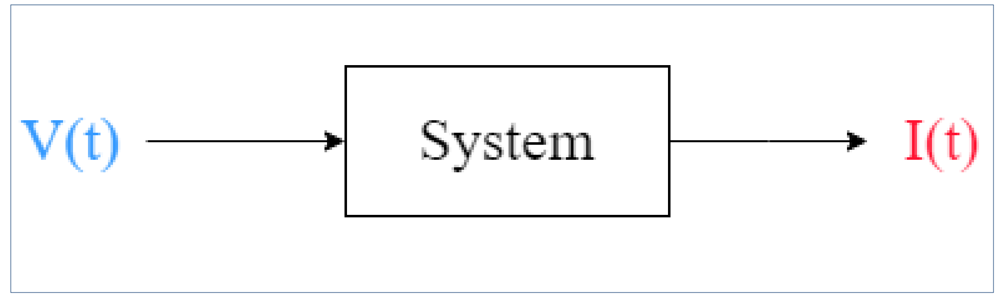
$$V(t) - RI(t) - L\frac{dI(t)}{dt} - \frac{1}{C}\int_0^t I(t)dt = 0$$

- $x_1 = \int_0^t I(t)dt$
- $x_2 = I(t)$

$\longrightarrow$

- $\dot{x}_1 = x_2$
- $\dot{x}_2 = -\frac{1}{LC}x_1 - \frac{R}{L}x_2 + \frac{1}{L}V(t),$





Now consider different state variables :

$$V(t) - RI(t) - L\frac{dI(t)}{dt} - \frac{1}{C}\int_0^t I(t)dt = 0$$

- $z_1 = -\frac{1}{LC}\int_0^t I(t)dt$
- $z_2 = I(t)$

$\longrightarrow$

- $\dot{z}_1 = -\frac{1}{LC}z_2$
- $\dot{z}_2 = z_1 - \frac{R}{L}z_2 + \frac{1}{L}V(t)$

State variables 1:

- $x_1 = \int_0^t I(t)dt$
- $x_2 = I(t)$

1. $\dot{x}_1 = x_2$
2. $\dot{x}_2 = -\frac{1}{LC}x_1 - \frac{R}{L}x_2 + \frac{1}{L}V(t)$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = 0$$

State variables 2:

- $z_1 = -\frac{1}{LC}\int_0^t I(t)dt$
- $z_2 = I(t)$

1. $\dot{z}_1 = -\frac{1}{LC}z_2$
2. $\dot{z}_2 = z_1 - \frac{R}{L}z_2 + \frac{1}{L}V(t)$

$$\bar{A} = \begin{bmatrix} 0 & -{}^1/_{LC} \\ 1 & -{}^R/_L \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ {}^1/_L \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \bar{D} = 0$$

$$\mathrm{T} = \begin{bmatrix} -\frac{1}{LC} & 0 \\ 0 & 1 \end{bmatrix}$$

$$G(s) = \frac{Cs}{1+RCs+LCs^2}$$

Different state space representations lead to the same transfer function, state-space is not unique!

Controller Canonical Form (CCF)

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_{n-1} \frac{dy}{dt} + a_n y = b_0 \frac{d^n u}{dt^n} + b_1 \frac{d^{n-1} u}{dt^{n-1}} + \cdots + b_{n-1} \frac{du}{dt} + b_n u$$

$$G(s) = \frac{\hat{Y}(s)}{\hat{U}(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [b_n - a_n b_0 \quad b_{n-1} - a_{n-1} b_0 \quad \ldots \quad b_2 - a_2 b_0 \quad b_1 - a_1 b_0]\mathrm{x} + b_0 u$$

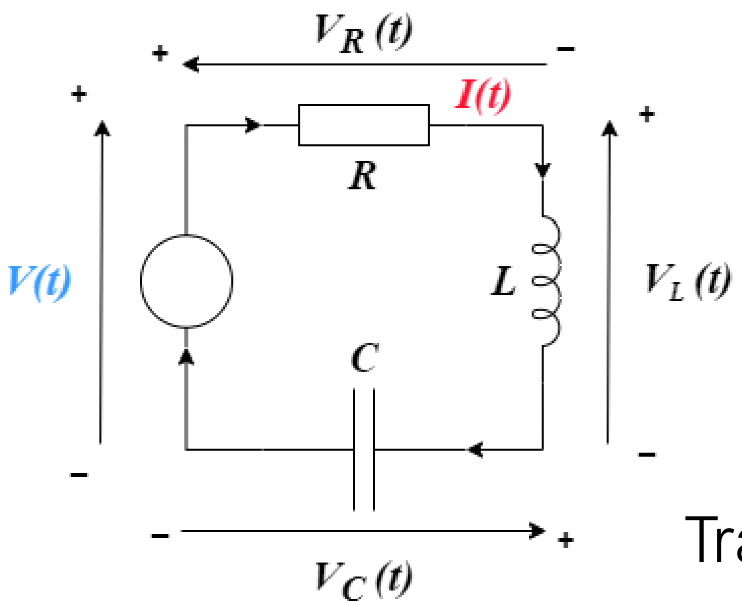
$$G(s)=\frac{\hat{Y}(s)}{\hat{U}(s)}=\frac{b_0 s^n+b_1 s^{n-1}+\cdots+b_{n-1}s+b_n}{s^n+a_1 s^{n-1}+\cdots+a_{n-1}s+a_n}$$

$$\dot{x}=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}\boldsymbol{x}+\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}\boldsymbol{u}$$

- The control directly influences the characteristic equation as shown by the arrow

- The control can change the poles of your system, or in other words, the behavior of the system

Example



$$V(t) - RI(t) - L\frac{dI(t)}{dt} - \frac{1}{C}\int_0^t I(t)dt = 0$$

State variables

- $x_1 = \int_0^t I(t)dt$
- $x_2 = I(t)$
- $\dot{x}_1 = \bar{x}_2$
- $\dot{x}_2 = -\frac{1}{LC}\bar{x}_1 - \frac{R}{L}\bar{x}_2 + \frac{1}{L}V(t),$

Transfer function: $G(s) = \dfrac{Cs}{1 + RCs + LCs^2}$

State-space representation in controllable form:

GENERAL FORM

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -{}^{1}/_{LC} & -{}^{R}/_{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -{}^{1}/_{LC} & -{}^{R}/_{L}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

controller: $u = -[k_1 \quad k_2]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + R$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -{}^{1}/_{LC} - k_1 & -{}^{R}/_{L} - k_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}R$$

Transfer function: $$G(s) = \frac{\frac{1}{L}s}{s^2 + \underbrace{\left(\frac{R}{L} + k_2\right)}_{\text{damping}} s + \underbrace{\left(\frac{1}{LC} + k_1\right)}_{\text{stiffness}}}$$

By controller canonical form you can change the poles of the system directly to achieve desired **damping** and **stiffness**.

Observer Canonical Form (OCF)

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_{n-1} \frac{dy}{dt} + a_n y = b_0 \frac{d^n u}{dt^n} + b_1 \frac{d^{n-1} u}{dt^{n-1}} + \cdots + b_{n-1} \frac{du}{dt} + b_n u$$

$$G(s) = \frac{\hat{Y}(s)}{\hat{U}(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots 0 & -a_n \\ 1 & 0 & \cdots 0 & -a_{n-1} \\ 0 & 1 & \cdots 0 & -a_{n-2} \\ \vdots & \vdots & \ddots \vdots & \vdots \\ 0 & 0 & \cdots 1 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_n - a_n b_0 \\ b_{n-1} - a_{n-1} b_0 \\ b_{n-2} - a_{n-2} b_0 \\ \vdots \\ b_1 - a_1 b_0 \end{bmatrix} u$$

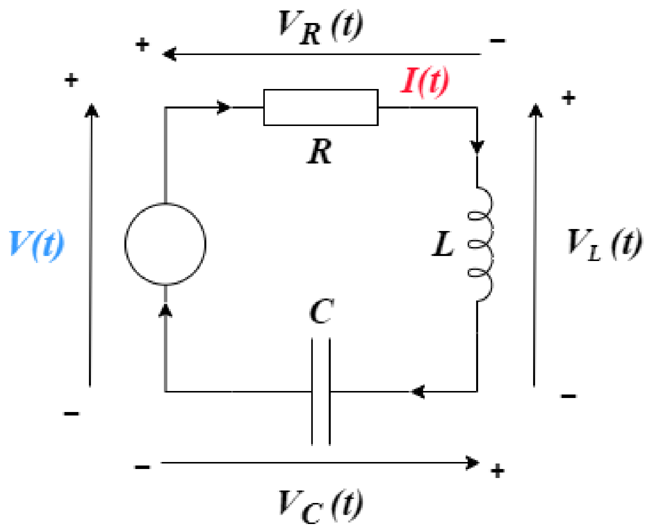
$$y = \begin{bmatrix} 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + b_0 u$$

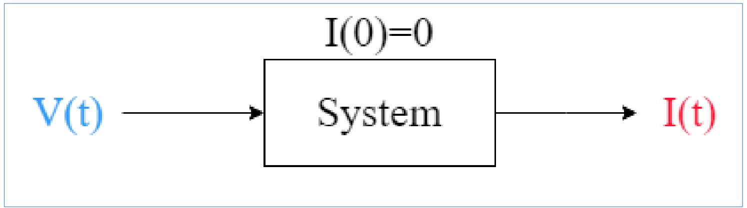
Advantage: Looking at just one state, namely, x_n, you have knowledge on all the other states

Remarks on Choosing State Variables

Example (Series RLC Circuit)

$$V(t) - V_R(t) - V_L(t) - V_C(t) = 0 \qquad (1)$$





Capacitor:

$$\frac{C dV_C(t)}{dt} = I(t) \Rightarrow$$

$$V_C(t) = \frac{1}{C}\int_0^t I(t)dt \qquad (2)$$

Inductor:

$$V_L(t) = L\frac{dI(t)}{dt} \qquad (3)$$

Resistor:

$$V_R(t) = RI(t) \qquad (4)$$

We obtain an integral-differential equation by plugging equations (2) through (4) in (1),

$$V(t) - RI(t) - L\frac{dI(t)}{dt} - \frac{1}{C}\int_0^t I(t)dt = 0 \qquad (5)$$

$$V(t) - RI(t) - L\frac{dI(t)}{dt} - \frac{1}{C}\int_0^t I(t)dt = 0 \tag{5}$$

We wish to convert **(5)** into a pure differential equation, therefore we take the derivative of (5):

$$\frac{dV(t)}{dt} - R\frac{dI(t)}{dt} - L\frac{d^2I(t)}{dt^2} - \frac{1}{C}\ I(t) = 0 \tag{6}$$

Using the notation:

- $\dot{V}\ (t) = \frac{dV(t)}{dt}$
- $\dot{I}\ (t) = \frac{dI(t)}{dt}$
- $\ddot{I}\ (t) = \frac{d^2I(t)}{dt^2}$, we have from (6) that

$$\dot{V}(t) - R\dot{I}(t) - L\,\ddot{I}(t) - \frac{1}{C}I(t) = 0 \tag{7}$$

Remark

$$V(t) - RI(t) - L\frac{dI(t)}{dt} - \frac{1}{C}\int_0^t I(t)dt = 0 \qquad (*)$$

$$\dot{V}(t) - R\,\dot{I}(t) - L\,\ddot{I}(t) - \frac{1}{C}I(t) = 0 \qquad (**)$$

State-space representation of (**):

- $x_1(t) = I(t)$
- $x_2(t) = \dot{I}(t)$

$\longrightarrow$

- $\dot{x_1}(t) = \dot{I}(t) = x_2(t)$
- $\dot{x_2}(t) = \ddot{I}(t) = \frac{1}{L}\,\dot{V}(t) - \frac{R}{L}\,\dot{I}(t) - \frac{1}{LC}\,I(t)$

$$\dot{x_1}(t) = x_2(t) \qquad (8)$$

$$\dot{x_2}(t) = -\frac{1}{LC}x_1(t) - \frac{R}{L}x_2(t) + \frac{1}{L}\,\dot{V}(t), \qquad (9)$$

where the last equation (9) is derived using (**)

$$\dot{V}(t) - R\,\dot{I}(t) - L\,\ddot{I}(t) - \frac{1}{C} I(t) = 0 \qquad (**)$$

So, the state-space representation of (**) is given by

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -\frac{1}{LC} x_1(t) - \frac{R}{L} x_2(t) + \frac{1}{L} \dot{V}(t)$$

The state-space representation contains the derivative of the input V(t), which is undesirable. This happened because of an unsuitable choice of state variables.

Remark

$$V(t) - RI(t) - L\frac{dI(t)}{dt} - \frac{1}{C}\int_0^t I(t)dt = 0 \qquad (*)$$

$$\dot{V}(t) - R\,\dot{I}(t) - L\,\ddot{I}(t) - \frac{1}{C}I(t) = 0 \qquad (**)$$

To avoid this issue, let us define a different pair of state-space variables, and use the integral-differential equation:

State-space representation of (*):

- $\overline{x_1}(t) = \int_0^t I(t)dt$
- $\overline{x_2}(t) = I(t)$

$\longrightarrow$

- $\dot{\overline{x_1}}(t) = I(t) = \overline{x_2}(t)$
- $\dot{\overline{x_2}}(t) = \frac{1}{L}V(t) - \frac{R}{L}I(t) - \frac{1}{LC}\int_0^t I(t)dt$

$$\dot{\overline{x_1}}(t) = \overline{x_2}(t) \qquad (10)$$

$$\dot{\overline{x_2}}(t) = -\frac{1}{LC}\overline{x_1}(t) - \frac{R}{L}\,\overline{x_2}(t) + \frac{1}{L}V(t), \qquad (11)$$

where the last equation (11) is derived using (*)

$$V(t) - RI(t) - L\frac{dI(t)}{dt} - \frac{1}{C}\int_0^t I(t)dt = 0 \qquad (*)$$

$$\dot{V}(t) - R\,\dot{I}(t) - L\,\ddot{I}(t) - \frac{1}{C}I(t) = 0 \qquad (**)$$

State-Space Representation of (**)

$$\dot{x_1}(t) = x_2(t)$$

$$\dot{x_2}(t) = -\frac{1}{LC}x_1(t) - \frac{R}{L}x_2(t) + \frac{1}{L}\dot{V}(t)$$

with variables 1:

- $x_1(t) = I(t)$
- $x_2(t) = \dot{I}(t)$

State-Space Representation of (*)

$$\dot{\overline{x_1}}(t) = \overline{x_2}(t)$$

$$\dot{\overline{x_2}}(t) = -\frac{1}{LC}\overline{x_1}(t) - \frac{R}{L}\,\overline{x_2}(t) + \frac{1}{L}V(t)$$

with variables 2:

- $\overline{x_1}(t) = \int_0^t I(t)dt$
- $\overline{x_2}(t) = I(t)$

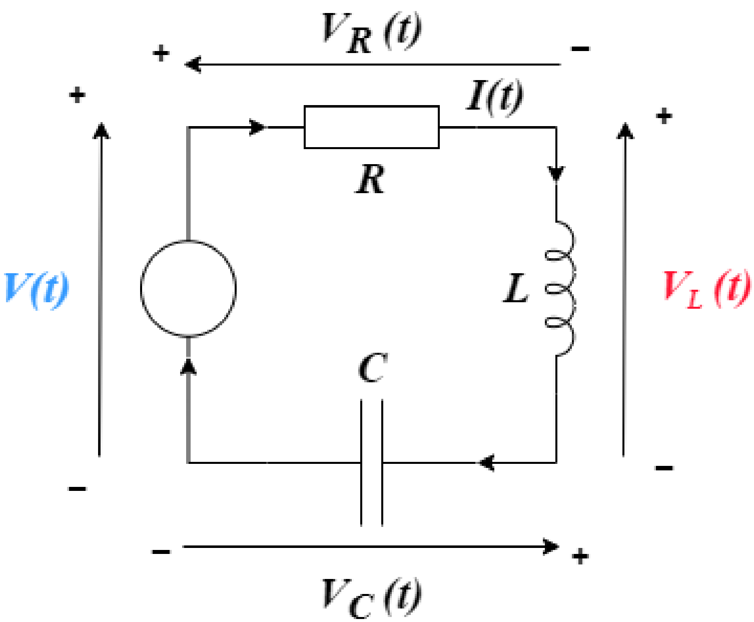
$$V(t) - RI(t) - L\frac{dI(t)}{dt} - \frac{1}{C}\int_0^t I(t)dt = 0 \qquad (*)$$

$$\dot{V}(t) - R\,\dot{I}(t) - L\,\ddot{I}(t) - \frac{1}{C}I(t) = 0 \qquad (**)$$

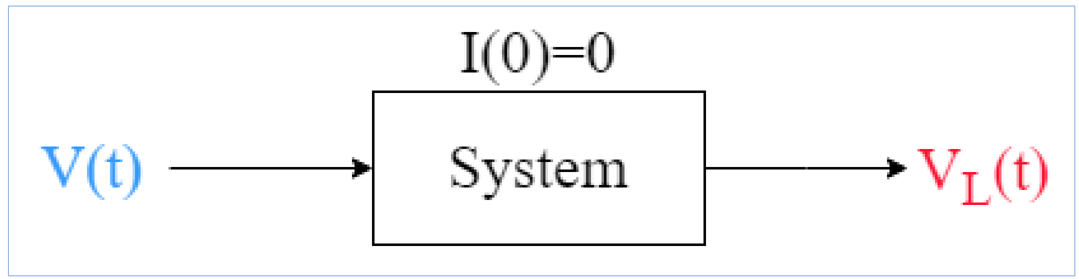
The lesson to take home is:

- Don't convert the integral-differential equation (*) into a pure differential one (**)
- Consider the integral-differential equation and choose the state-variables in a clever way to avoid appearance of input derivatives in the state-space representation

Another Example (Series RLC Circuit with Output as Voltage across Inductor)



Let us analyze the RLC Circuit using a different output, namely $V_L(t)$, instead of I(t).

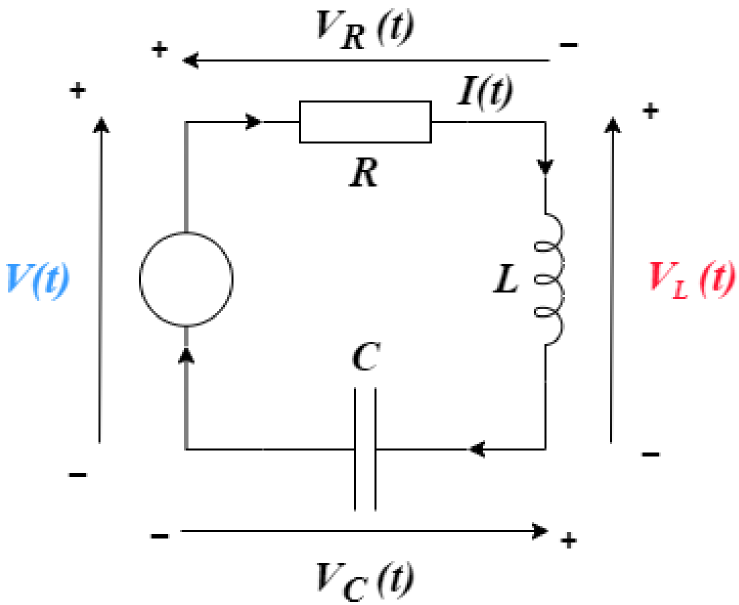


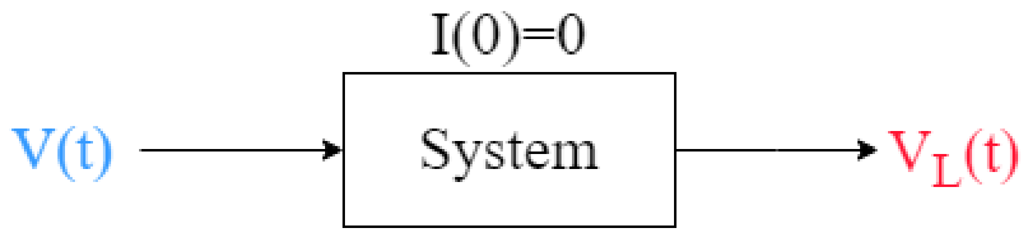
Once again, by KVL, i.e., Kirchoff's voltage law, we have:

$$V(t) - RI(t) - V_L(t) - \frac{1}{C}\int_0^t I(t)dt = 0 \qquad (12)$$

Replacing $V_L(t) = L\frac{dI(t)}{dt}$ or $I(t) = \frac{1}{L}\int_0^t V_L(t)dt$ in equation (12) to eliminate current I(t):

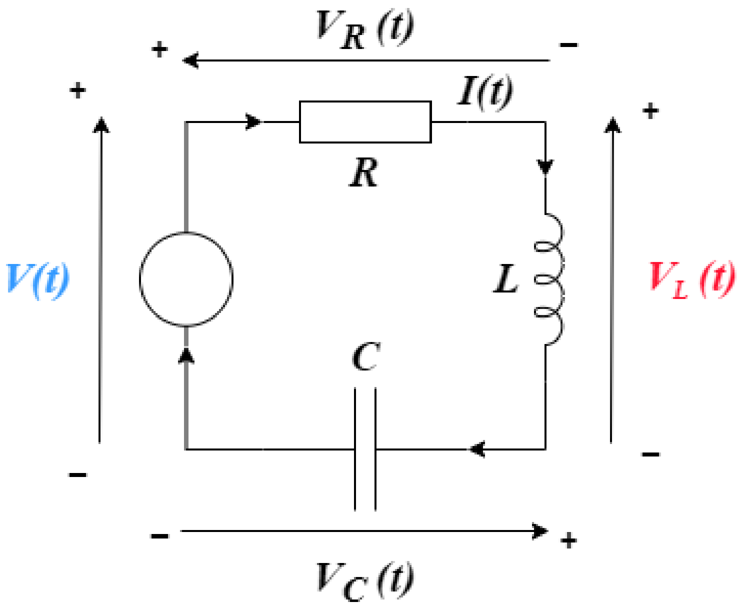
$$\mathrm{V(t)} - \frac{R}{L}\int_0^t V_L(t)dt - V_L(t) - \frac{1}{C}\int_0^t\int_0^t V_L(t)dtdt = 0. \qquad (13)$$

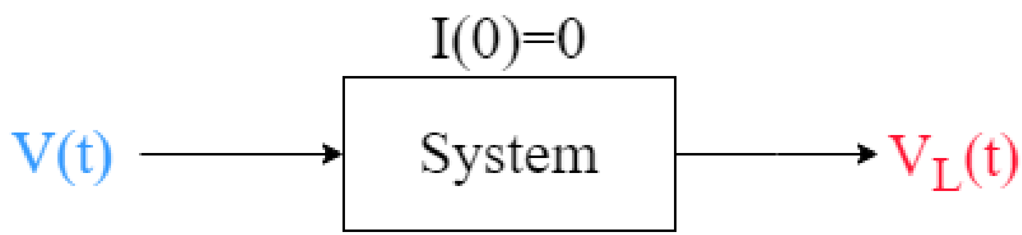




$$V(t) - \frac{R}{L}\int_0^t V_L(t)dt - V_L(t) - \frac{1}{C}\int_0^t\int_0^t V_L(t)dtdt = 0. \tag{13}$$

How to avoid the appearance of the input derivatives in the state-space representation?





$$V(t) - \frac{R}{L}\int_0^t V_L(t)dt - V_L(t) - \frac{1}{C}\int_0^t\int_0^t V_L(t)dtdt = 0. \qquad (13)$$

- Use the integral-differential equation without converting it to a pure differential equation

- Proper choice of the state-space variables

$$V(t) - \frac{R}{L}\int_0^t V_L(t)dt - V_L(t) - \frac{1}{C}\int_0^t\int_0^t V_L(t)dtdt = 0. \tag{13}$$

State-space representation of (13):

- $x_1(t) = \int_0^t\int_0^t V_L(t)dtdt$
- $x_2(t) = \int_0^t V_L(t)dt$

$\longrightarrow$

- $\dot{x}_1(t) = \int_0^t V_L(t)dt = x_2(t)$
- $\dot{x}_2(t) = V_L(t) = -\frac{R}{L}x_2(t) - \frac{1}{LC}x_1(t) + V(t)$

$$\dot{x}_1(t) = x_2\,(t) \tag{14}$$

$$\dot{x}_2(t) = -\frac{R}{L}x_2(t) - \frac{1}{LC}x_1(t) + V(t) \tag{15}$$

FINAL REMARK

- Traditionally, to get state-space representation for second-order differential equations, we usually assign the first state to be $V_L(t)$ (or I(t)) and second state to be $\dot{V}_L(t)\left(\text{or } \dot{I}(t)\right)$
- However, when there are zeros in the system, using the described method leads to the appearance of the derivatives of input in state-space representation, which is undesirable
- Therefore, choose state-space variables cleverly!

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Next lecture:

State-space representation and A, T, and D-type variables