Homework 1
(due on Friday 3 May)

Chapters covered: 1, 2, 3 and 4.

Upload your solutions on Brightspace until **Friday 3 May, 23:59**. If you hand in late then that will count as not handed in.

For each problem you need to justify your result, in a clear way. If you provide a numerical result without explanations then your answer will be ignored.

Problem 1; 2 points Ruben broke into a blank. He must deactivate the alarm system by entering a pin number. Before the robbery he has been able to gather the following information about the pin number: It consists of 4 digits, the digit 3 is used exactly 2 times and the other digits are distinct (for example 3319 or 2303 but not 3363 or 5353). He has made a list of all the possible combinations and constructed a little device that tries each of them in a random order, but never twice the same combination. When it inputs the wrong pin, nothing happens, and the device tries the next number exactly 0.1 second later. If it inputs the correct pin, the alarm is deactivated immediately and the device stops. As soon as it enters the first pin, a countdown starts. If the pin number is not found after 30.01 seconds, the alarm will go off.

What is the probability that the alarm will go off?

Problem 2; 3 points An investor has 19,000 euro to invest among 4 possible investments. Each investment must be in units of 1000 euro (0 euro, 1000 euro, 2000 euro, ...).

(a) If the total 19,000 euro is to be invested, how many different investment strategies are possible?

(b) What if not all the money needs to be invested?

Problem 3; 3 points. A bin contains 3 types of disposable flashlights. The probability that a type 1 flashlight will give more than 100 hours of use is .7, with the corresponding probabilities for type 2 and type 3 flashlights being .4 and .3, respectively. Suppose that 20 percent of the flashlights in the bin are type 1, 30 percent are type 2, and 50 percent are type 3.

(a) What is the probability that a randomly chosen flashlight will give more than 100 hours of use?

(b) Given that a flashlight lasted more than 100 hours, what is the conditional probability that it was a type j flashlight, $j = 1, 2, 3$?

Hint: Consider the events

$$A := \{\text{the chosen flashlight will give more than 100 hours of use}\},$$
$$F_j := \{\text{the chosen flashlight is of type } j\}, \quad j = 1, 2, 3.$$

Problem 4; 2 points Let λ be a positive constant. The probability mass function of a random variable X taking values in $\mathbb{N}_0$ is given by $p(i) = c\lambda^{2i}/i!$, $i = 0, 1, 2, ...$, where c is some constant.

(a) Find the value of c.

(b) Find $\mathbb{P}(X = 0)$.

(c) Assume $\lambda = 1$. Find $\mathbb{P}(X > 2)$.