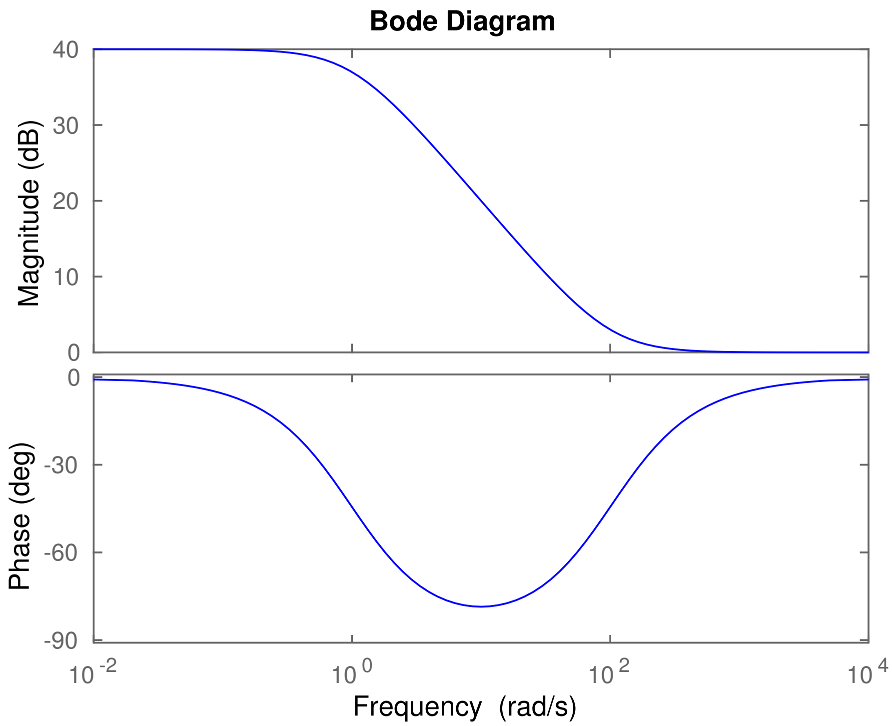
Control Engineering

Instruction Lecture 6

Frequency domain analysis | Chapter 9

Exercise 1. Bode diagrams

1. Consider a transfer function of the form $G_1(s) = \frac{s+a}{s+b}$, with $a, b > 0$. The Bode plot of $G_1(s)$ is shown below, determine a and b.



2. Consider a transfer function of the form $G_2(s) = \frac{c}{s(s+d)}$, with $c, d > 0$. The Bode plot of $G_2(s)$ is shown below, determine c and d.

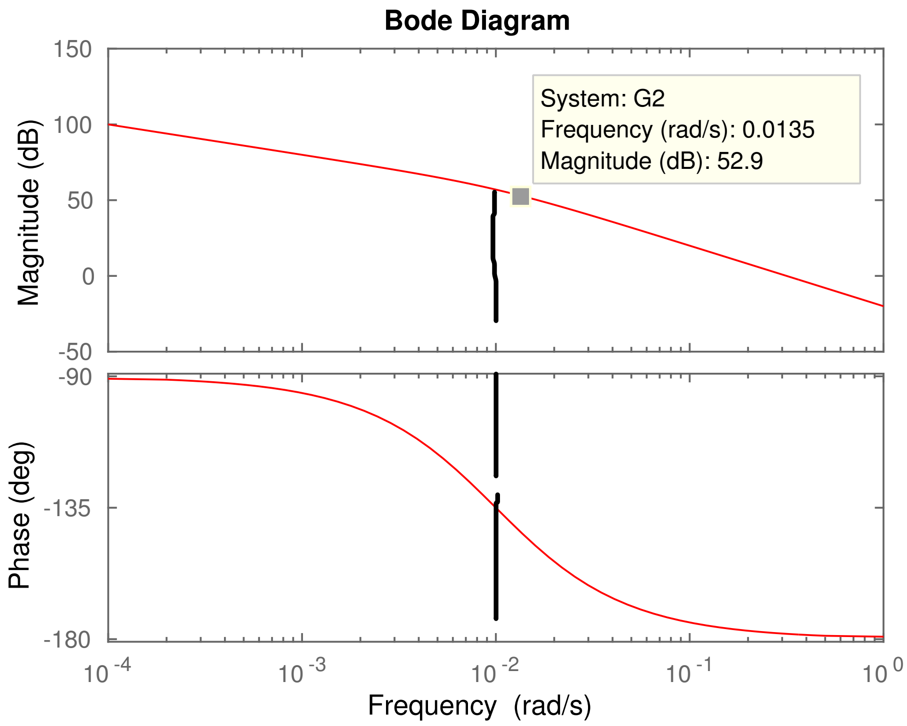
d = 0.01

$$G(s) = \frac{c}{s(s+0.01)}$$

$$G(i\omega) = \frac{c}{i\omega(i\omega+0.01}$$

$$|G(i\omega)| = \sqrt{\frac{c^2}{\omega^2(\omega^2+0.01)}}$$

$$20\log(|G(i\omega)|) =$$



3. Consider the Bode plot shown below and approximate the transfer function $G_3(s)$ from it.

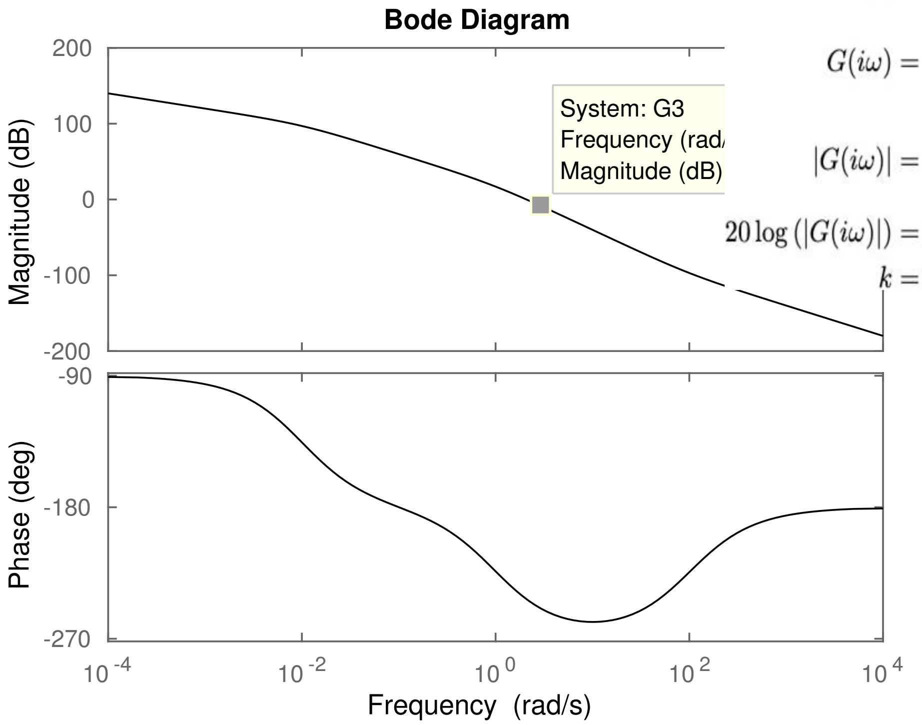
$$G(s) = \frac{k(s+100)}{s(s+0.01)(s+1)}$$

$$G(i\omega) = \frac{k(i\omega+100)}{i\omega(i\omega+0.01)(i\omega+1)}$$

$$|G(i\omega)| = \sqrt{\frac{k^2(\omega^2+100^2)}{\omega^2(\omega^2+0.01)(\omega^2+1)}}$$

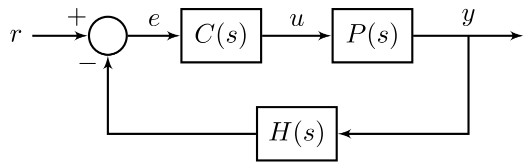
$$20\log\left(|G(i\omega)|\right) = 8.35$$

$$k = 0.1$$



4. Now calculate $G_4(s) = G_1(s)G_2(s)$ and compare your result to step 3. What do you notice?

Exercise 2. Loop transfer functions



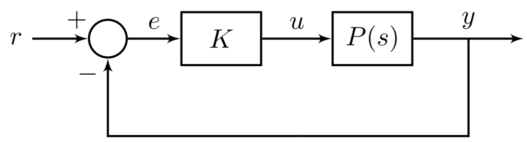
Consider the following feedback system above, with PI-controller $C(s)$, plant $P(s)$ and output filter $H(s)$ given by respectively

$$C(s) = k_p + \frac{k_i}{s}, \qquad P(s) = \frac{4}{s^2 + 2s + 3}, \qquad H(s) = \frac{1}{s+1}.$$

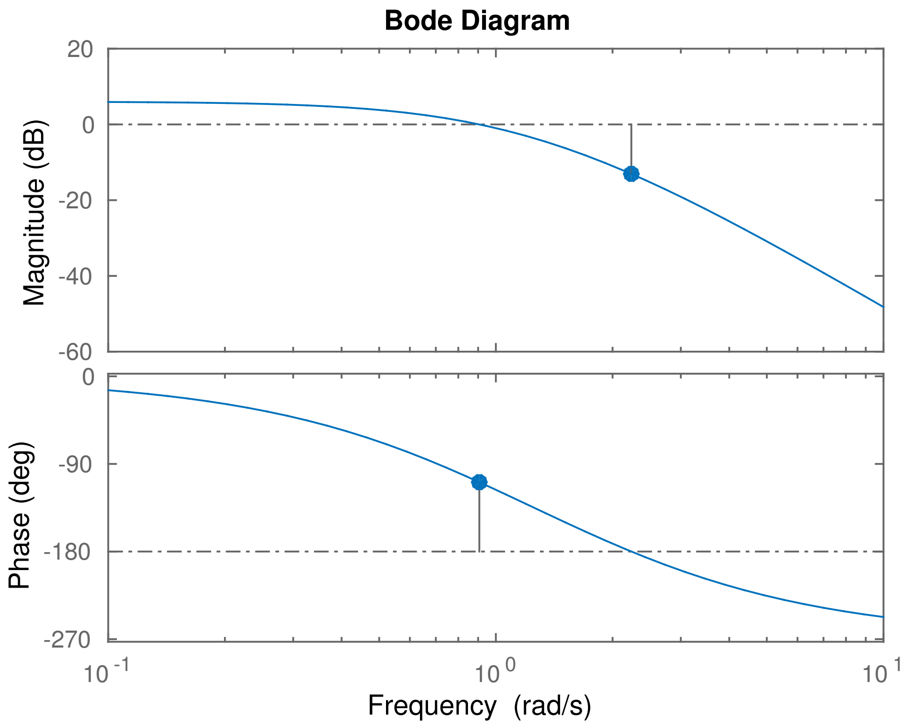
1. Determine the loop transfer function $L(s)$.
2. Determine the closed loop transfer function $H_{yr}(s)$.

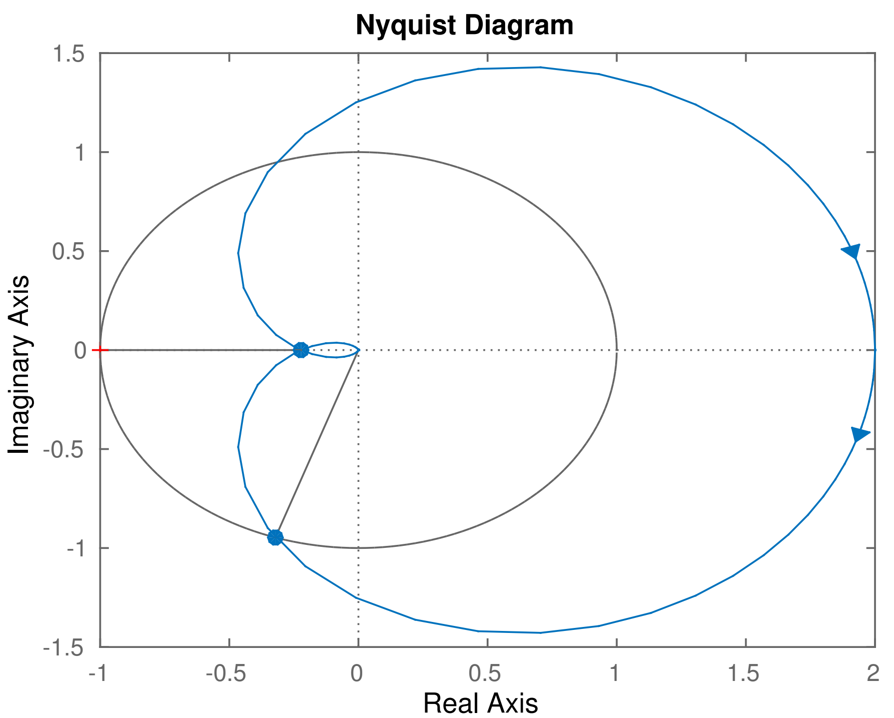
Exercise 3. Frequency domain stability

Consider a process $P(s)$ with a proportional controller $K > 0$ and unity feedback depicted below



1. Determine the loop transfer function $L(s)$.
2. Take
$$P(s) = \frac{4}{(s+1)^2(s+2)}.$$
Is the open loop system stable? Motivate!
3. The Bode plot and Nyquist plot for the loop transfer function $L(s)$ with $K = 1$ are depicted below. Indicate the gain margin and phase margin in both plots. Use the Nyquist criterion to determine if the closed loop system is stable.





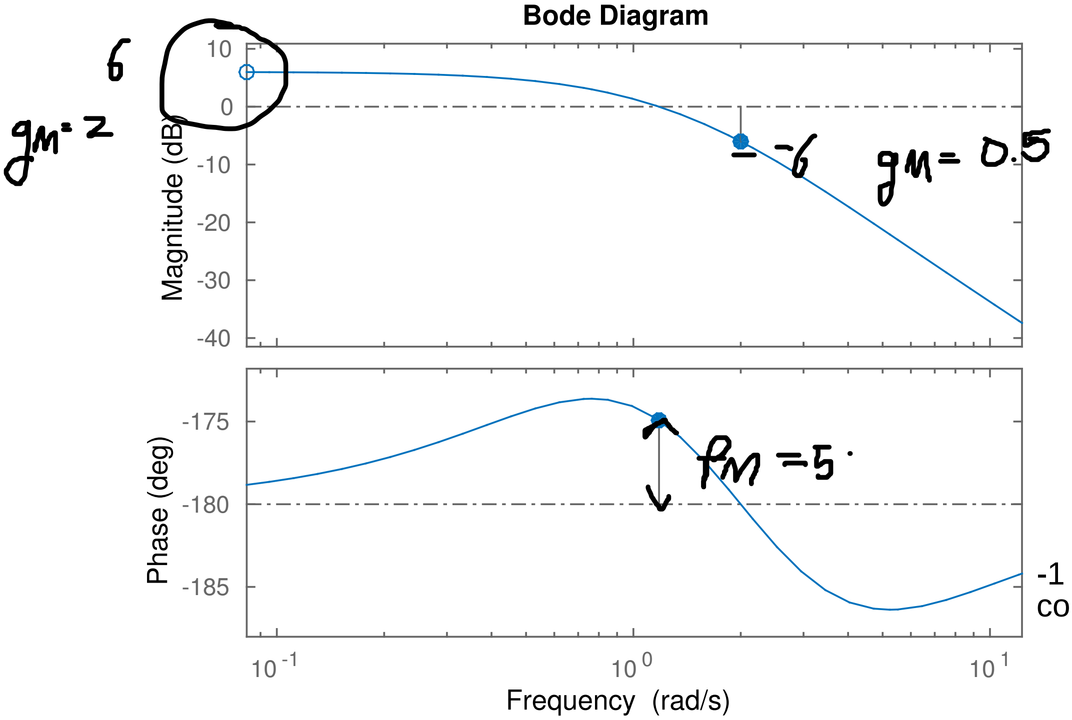
4. Now change the process transfer function to

$$P(s) = \frac{s+4}{(s+2)^2(s-1)}. \tag{1}$$

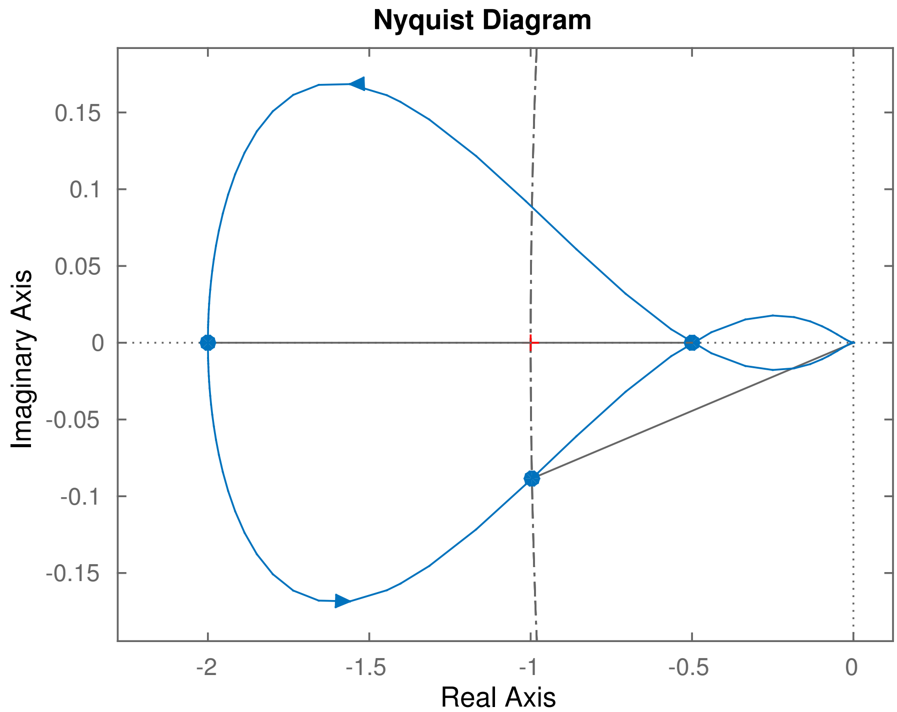
   Is the open loop system stable?

5. The corresponding Bode plot and Nyquist plot of the loop transfer function $L(s)$ for $K = 2$ is given as

notation with circle means convergence in 6



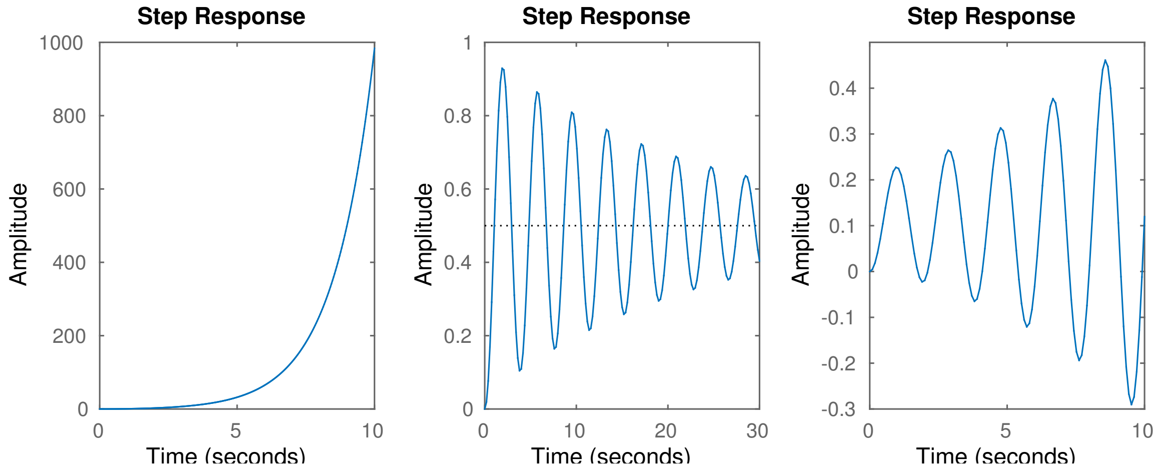
-1 because is counterclockwise



Indicate the gain margin and phase margin in both plots. Use the Nyquist criterion to determine if the closed loop system is stable.

6. In addition, determine for which values of K the closed-loop system is stable.

7. The closed loop step response for (1) is shown below for respectively $K = 1/2$, $K = 3$, and $K = 10$. Explain the behavior using your results from step 4-6.

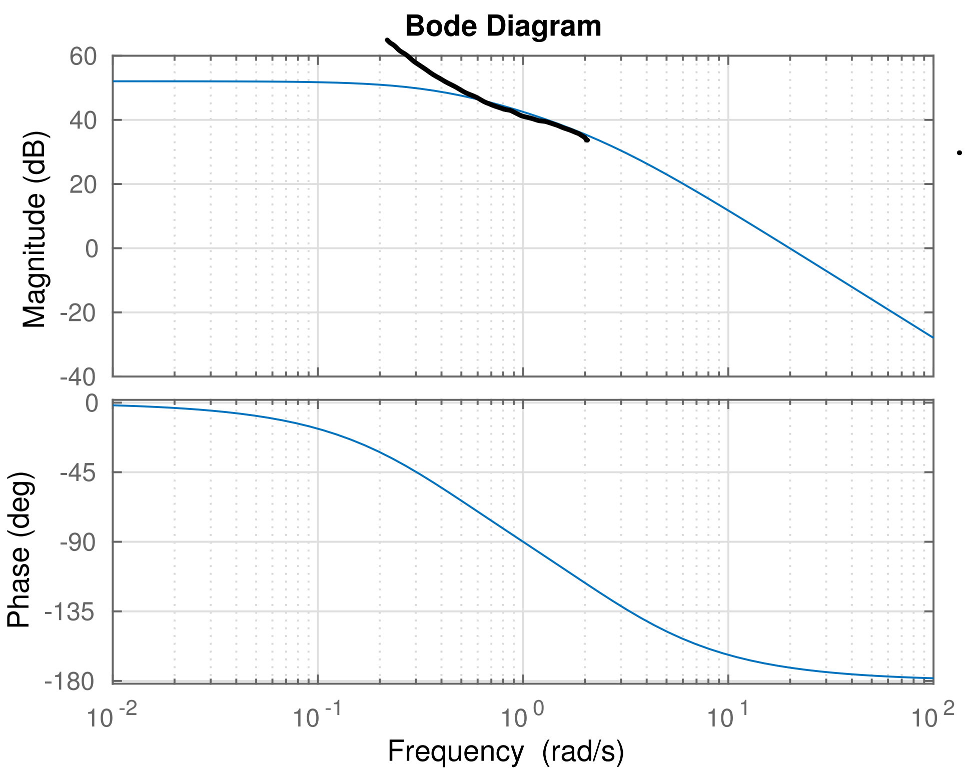


Exercise 4. Frequency domain tuning

Consider the feedback system of Exercise 3 with $K = 1$ and

$$P(s) = \frac{400}{s^2 + 3s + 1},$$

which represents a mass-spring-damper system. The corresponding Bode plot of the loop transfer function is depicted below.



1. Determine the phase margin.
2. Can you determine the gain margin from the Bode plot? If so, what is it? If not, why?
3. How should you change K to achieve a phase margin of 50°?