1 Modeling Euler-Langrangian(EL) equations

1.1 EL Equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0 \tag{1}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = (Bu)_i + d_i \tag{2}$$

for $i = 1, \ldots, n$ for which n is the number of **degrees of freedom**. The second equation accounts for non-conservative/external forces like disturbances d and the control u. We call $q \in \mathbb{R}^n$ the generalized coordinate.

1.2 Kinetic (co-)energy (mass)

$$T^*(\dot{q}) = \frac{1}{2}m\dot{q}^2 \tag{3}$$

1.3 Potential energy (spring)

$$E(q) = \frac{1}{2}kq^2 \tag{4}$$

are also called $V(q)$

1.4 Lagrangian function

$$L(q, \dot{q}) = T^*(\dot{q}) - E(q) \tag{5}$$

1.5 Rayleigh dissipation function

$$D(\dot{q}) = \frac{1}{2}c\dot{q}^2 \tag{6}$$

1.6 Euler-Lagrange Algorithm

1. Identify a generalised displacement vector $q \in \mathbb{R}^n$ (basic indep variables of the physical components)
2. Determine the kinetic co-energy 3 and potential energy 4 associated with elements respectively.
3. Determine the Lagrangian function 6
4. Differentiate the Lagrangian function with respect to q_i and $\dot{q}_i$
5. write the EL equations 1

1.7 Stability

If the Real part of the eigenvalue is smaller than 0 then is AS is = 0 is stable is > 0 then is unstable

If the imaginary part is not 0, there are oscillations is =0, there are no oscillations

1.8 Reachability matrix, feedback gain and reference gain

$$W_r = [A \quad AB] \tag{7}$$
$$K = [p_1 - a_1 \quad p_2 - a_2 \quad \cdots p_n - a_n] \tag{8}$$
$$Kr = \frac{-1}{C(A - BK)^{-1}B} \tag{9}$$
$$\tag{10}$$

1.9 State feedback controller

$$u = -Kx + K_r r \tag{11}$$

1.10 Observability matrix

We defined the observer state as

$$\dot{\hat{x}} = A\hat{x} + Bu + M(y - C\hat{x}) \tag{12}$$
$$\tilde{x} = x - \hat{x} \tag{13}$$
$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = (A - MC)\tilde{x} \tag{14}$$
$$W_o = \begin{bmatrix} C \\ CA \end{bmatrix} \tag{15}$$

1.11 Observer gain

$$M = W_o^{-1}\tilde{W}_o \begin{bmatrix} q_1 - a_1 \\ q_2 - a_2 \end{bmatrix} \tag{16}$$

1.12 Transfer function

$$\frac{Y(s)}{u(s)} = C(sI - A)^{-1}B + D \tag{17}$$

1.13 Get transfer function out of Bode diagram

If you have a pole (b) you will have a decreasing bode diagram, otherwise it corresponds to the zeros (a)

$$G(s) = \frac{s + a}{s + b} \tag{18}$$

we usually convert to decibel scale so $|G(i\omega)|_{dB} = 20$

$$G(s) = \frac{k(s+100)}{s(s+0.01)(s+1)} \tag{19}$$

$$G(i\omega) = \frac{k(i\omega+100)}{i\omega(i\omega+0.01)(i\omega+1)} \tag{20}$$

$$|G(i\omega)| = \sqrt{\frac{k^2(\omega^2+100^2)}{\omega^2(\omega^2+0.01)(\omega^2+1)}} \tag{21}$$

$$20\log(|G(i\omega)|) = 8.35 \tag{22}$$

$$k = 0.1 \tag{23}$$

1.14 Closed loop transfer function

$$H_{yr}(s) = \frac{C(s)P(s)}{1+C(s)P(s)H(s)} \tag{24}$$

1.14.1 Nyquist plot and Nyquist criteria

Nyquist criteria Z= N+P where n is the net clockwise encirclements of (-1,0), n is the positive poles for the open loop system and z is the positive poles of the closed loop system. If Z is something else than 0 it's unstable.

It's (-1,0) because in the open loop system we have a $\frac{1}{1+L(s)}$ somewhere.