Exam Image Processing
April 7 2022, 12:15-14:15

- This exam consists of 5 problems. Problem 1 is worth 20 points, problem 2 is worth 25 points, and the other problems are worth 15 points each. You get 10 points for not misspelling your name and student number.
- Write neatly and carefully. If the handwriting is unreadable, or needs guessing to make something out of it, then the answer is rejected.
- There will be no points awarded for correct answers without explanation (if asked for).
- The answers must be given in the boxes. There is a blank page added at the end, in case you need more space.

Name:	
Student number:	

Problem 1: Point wise operations [20 points]
Consider the following (small) grey scale image:

8	6	3	1
7	7	2	2
8	5	1	6
1	1	5	3

(a) (5 points) Which global threshold value would be returned by *Otsu's* algorithm if we apply it to the given image? Explain your answer. [Hint: there is no need to simulate the algorithm to answer this question.]

Answer:

(b) (5 points) Determine the output image if we use *linear contrast stretching* to change the dynamic range of this image to $[2..16]$. Explain how you arrived at your answer.

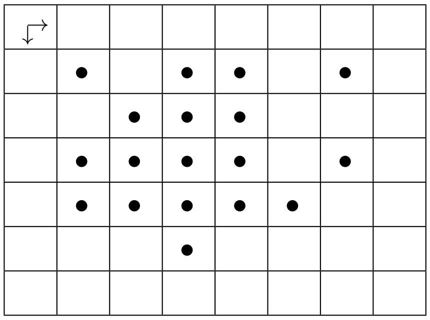
Answer:

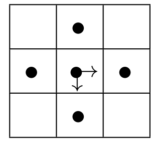
(c) (10 points) Determine the output image if we use *histogram equalization* to change the dynamic range of this image to $[0..8]$. Explain how you arrived at your answer.

Answer:

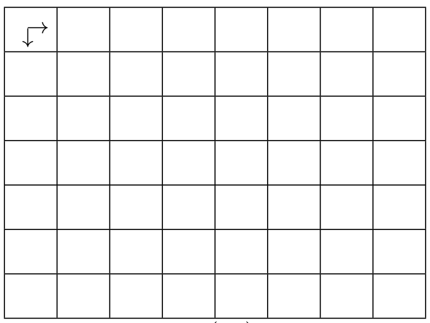
Problem 2: Morphological image processing [25 points]

(a) ($4 \times 2 = 8$ points) Consider the following binary image X (left) and the structuring element B (right).

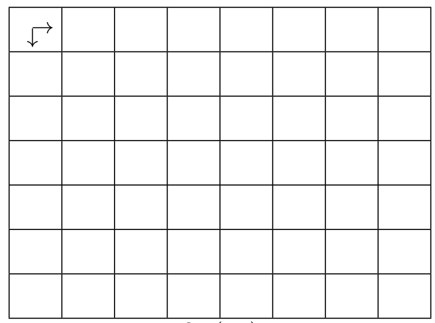




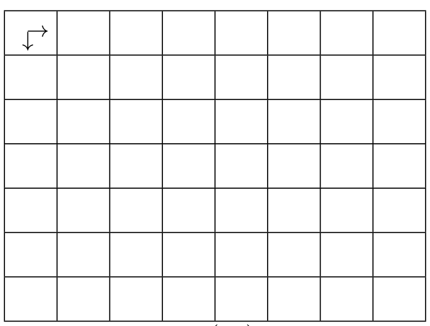
Draw in the empty images the erosion $\varepsilon_B(X)$, the dilation $\delta_B(X)$, the opening $\gamma_B(X)$, and the closing $\phi_B(X)$.



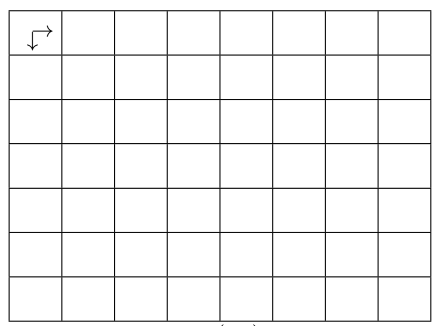
$\varepsilon_B(X)$



$\delta_B(X)$



$\gamma_B(X)$

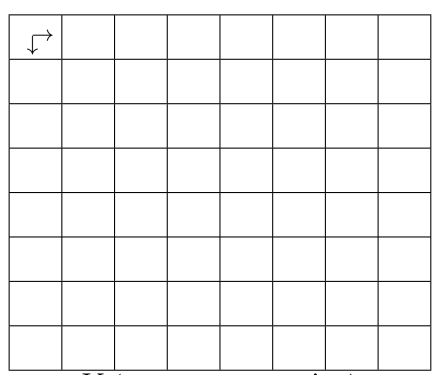


$\phi_B(X)$

(b) (6 points) Below, on the left, you find the *morphological skeleton function* $skf(Y)$ that was obtained with the structuring element that was given in part (a). Draw in the empty image (right) the reconstruction of Y.

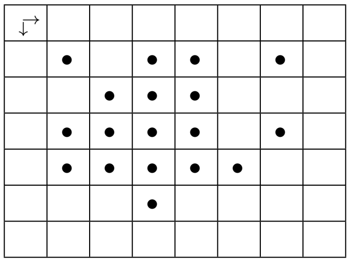
			4				
			3				
	1	1		1	1		

$skf(Y)$

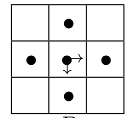


Y (your reconstruction)

(c) (6 points) For your convenience, the image and structuring element of part (a) is copied to this page:

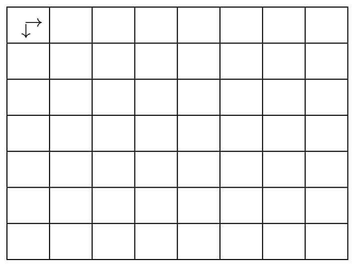


X



B

Compute the *morphological skeleton function* $skf(X)$ and place it in the empty grid below. Also, give the relevant sets $S_k(X)$ that you encounter in the construction of $skf(X)$ in the answer box.



Answer:

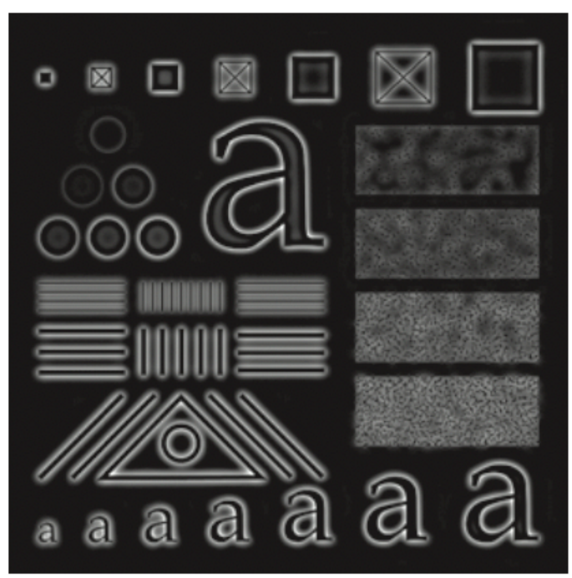
(d) (3 points) Explain what the expression $X \ominus \{(-1,-1)\}$ returns. Given an equivalent expression using the operation $\oplus$.

Answer:

(e) (2 points) Assuming the availability of a procedure that can efficiently compute distance transforms of binary images using a Manhattan metric or a chessboard metric, how can this be used to efficiently compute the erosion of a binary set X with the structuring element that was used in parts (a-c)?

Answer:

Problem 3: Frequency domain filtering [15 points]



Top: Original image; Bottom left:IHPF-filtered, $D_0 = 60$; Bottom right:BHPF-filtered, $D_0 = 60$

An example of frequency domain filtering is the ideal highpass filter (IHPF), defined by the transfer function

$$H(\mu, v) = \begin{cases} 0 & \text{if } D(\mu, v) \leq D_0 \\ 1 & \text{if } D(\mu, v) > D_0, \end{cases}$$

$D(\mu, v)$ is the distance of the point (μ, v) to the origin of the frequency domain, and D_0 is is the cut-off radius.

(a) (5 points) What is the purpose of highpass filtering?

Answer:

(b) (5 points) What is the artefact caused by IHPF in comparison to Butterworth high pass filter (BHPF)?

Answer:

(c) (5 points) What are potential applications for highpass filtering?

Answer:

Problem 4: Image compression [15 points]

Consider the following image:

125	80	80	80
125	255	255	80
125	255	255	80
200	255	80	80

(a) (5 points) What is the entropy of this image? What does this value indicate?

Answer:

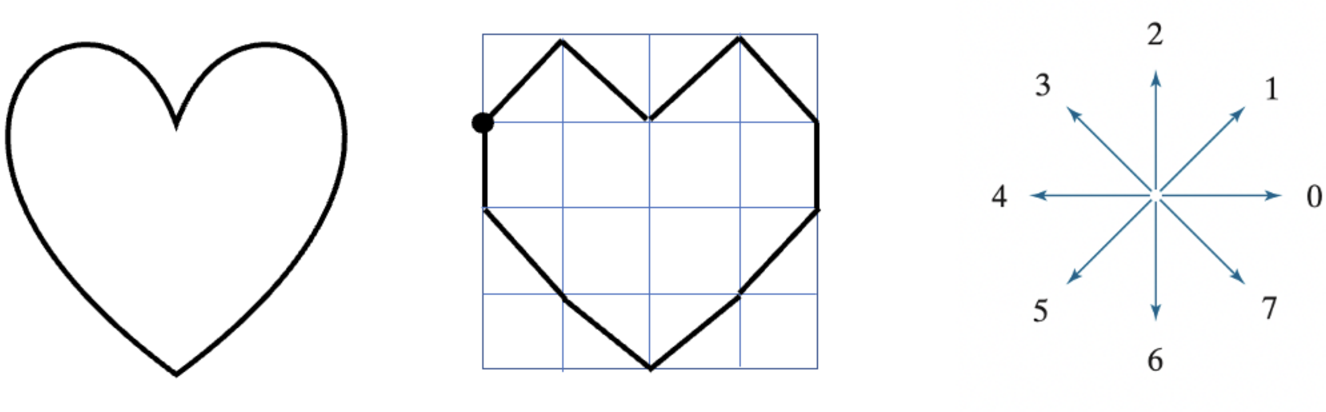
(b) (8 points) If we use Huffman coding for this image, what would be the code book? Give the derivation, not only the result.

Answer:

(c) (2 points) This image is originally represented by 8-bit fixed-length encoding. What is the average number of bits per pixel of the image after Huffman coding? Does Huffman coding reduce the redundancy of the image?

Answer:

Problem 5: Boundary descriptor [15 points]



Consider the above figures. On the left, you see the boundary of an object. In the middle, you see a simplified digital version of the boundary. On the right, you see direction numbers that are used in 8-directional chain codes.

(a) (5 points) What is the 8-directional Freeman chain code of the simplified boundary? The dot indicates the starting point.

Answer:

(b) (5 points) What is the first difference of the chain code? What is the shape number of this boundary?

Answer:

(a) (5 points) Describe another boundary descriptor and explain how the descriptor works.

Answer:

Answer:

Formula sheet

Co-occurrence matrix $g(i,j) = \{\text{no. of pixel pairs with grey levels } (z_i, z_j) \text{ satisfying predicate } Q\},\ 1 \leq i,j \leq L$

Convolution, 2-D discrete $(f \star h)(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n)\, h(x-m, y-n)$,
for $x = 0,1,2,\ldots,M-1$, $y = 0,1,2,\ldots,N-1$

Convolution Theorem, 2-D discrete $\mathcal{F}\{f \star h\}(u,v) = F(u,v)\, H(u,v)$

Distance measures Euclidean: $D_e(p,q) = \sqrt{(p_1-q_1)^2 + (p_2-q_2)^2}$, City-block: $D_4(p,q) = |p_1 - q_1| + |p_2 - q_2|$,
Chessboard: $D_8(p,q) = \max(|p_1 - q_1|, |p_2 - q_2|)$

Entropy, source $H = -\sum_{j=1}^{J} P(a_j) \log P(a_j)$

Entropy, estimated for L-level image: $\tilde{H} = -\sum_{k=0}^{L-1} p_r(r_k) \log_2 p_r(r_k)$

Error, root-mean square $e_{\text{rms}} = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(\widehat{f}(x,y) - f(x,y) \right)^2 \right]^{\frac{1}{2}}$

Exponentials $e^{ix} = \cos x + i \sin x$; $\cos x = (e^{ix} + e^{-ix})/2$; $\sin x = (e^{ix} - e^{-ix})/2i$

Filter, inverse $\widehat{\mathbf{f}} = \mathbf{f} + \mathbf{H}^{-1}\mathbf{n}$, $\widehat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$

Filter, parametric Wiener $\widehat{\mathbf{f}} = \left(\mathbf{H}^t\mathbf{H} + K\,\mathbf{I}\right)^{-1} \mathbf{H}^t\mathbf{g}$, $\widehat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + K} \right] G(u,v)$

Fourier series of signal with period T: $f(t) = \sum_{n=-\infty}^{\infty} c_n\, e^{i\frac{2\pi n}{T}t}$, with Fourier coefficients:
$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t)\, e^{-i\frac{2\pi n}{T}t}\, dt, \quad n = 0, \pm 1, \pm 2, \ldots$

Fourier transform 1-D (continuous) $F(\mu) = \int_{-\infty}^{\infty} f(t)\, e^{-i2\pi\mu t}\, dt$

Fourier transform 1-D, inverse (continuous) $f(t) = \int_{-\infty}^{\infty} F(\mu)\, e^{i2\pi\mu t}\, d\mu$

Fourier Transform, 2-D Discrete $F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-i2\pi(u\,x/M + v\,y/N)}$
for $u = 0,1,2,\ldots,M-1$, $v = 0,1,2,\ldots,N-1$

Fourier Transform, 2-D Inverse Discrete $f(x,y) = \frac{1}{M\,N} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v)\, e^{i2\pi(u\,x/M + v\,y/N)}$
for $x = 0,1,2,\ldots,M-1$, $y = 0,1,\ldots,N-1$

Fourier spectrum Fourier transform of $f(x,y)$: $F(u,v) = R(u,v) + i\,I(u,v)$, Fourier spectrum: $|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}$, phase angle: $\phi(u,v) = \arctan\left(\frac{I(u,v)}{R(u,v)}\right)$

Gaussian function mean μ, variance σ^2: $G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-(x-\mu)^2/2\sigma^2}$

Gradient $\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$

Histogram $h(m) = \#\{(x,y) \in D : f(x,y) = m\}$. Cumulative histogram: $\mathsf{P}(\ell) = \sum_{m=0}^{\ell} h(m)$

Impulse, discrete $\delta(0) = 1, \delta(x) = 0$ for $x \in \mathbb{N} \setminus \{0\}$

Impulse, continuous $\delta(0) = \infty, \delta(x) = 0$ for $x \neq 0$, with $\int_{-\infty}^{\infty} f(t)\, \delta(t - t_0)\, dt = f(t_0)$

Impulse train $s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$, with Fourier transform $S(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \delta(\mu - \frac{n}{\Delta T})$

Laplacian $\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

Laplacian-of-Gaussian $\nabla^2 G_\sigma(x,y) = -\frac{2}{\pi\sigma^4} \left(1 - \frac{r^2}{2\sigma^2} \right) e^{-r^2/2\sigma^2} \quad (r^2 = x^2 + y^2)$

Median The median of N numerical values is found by ordering all values from lowest to highest and picking the middle one (when N is odd), or the average of the two middle values (when N is even).

Morphology Let E denote the universal set.

Dilation $\delta_A(X) = X \oplus A = \bigcup_{a \in A} X_a = \bigcup_{x \in X} A_x = \{h \in E : \check{A}_h \cap X \neq \emptyset\}$,
where $X_h = \{x + h : x \in X\}$, $h \in E$ and $\check{A} = \{-a : a \in A\}$

Erosion $\varepsilon_A(X) = X \ominus A = \bigcap_{a \in A} X_{-a} = \{h \in E : A_h \subseteq X\}$

Opening $\gamma_A(X) = X \circ A := (X \ominus A) \oplus A = \delta_A \varepsilon_A(X)$

Closing $\phi_A(X) = X \bullet A := (X \oplus A) \ominus A = \varepsilon_A \delta_A(X)$

Hit-or-miss transform $X \circledast (A_1, A_2) = (X \ominus A_1) \cap (X^c \ominus A_2)$

Thinning $X \otimes A = X \backslash (X \circledast A)$, **Thickening** $X \odot A = X \cup (X \circledast A)$

Morphological boundary $\beta_A(X) = X \backslash (X \ominus A)$

Morphological reconstruction Marker F, mask G, structuring element A:
$X_0 = F$, $X_k = (X_{k-1} \oplus A) \cap G, \quad k = 1, 2, 3, \ldots$

Morphological skeleton Image X, structuring element A: $SK(X) = \bigcup_{n=0}^{N} S_n(X)$,
$S_n(X) = X \ominus_n A \setminus (X \ominus_n A) \circ A$, where $X \ominus_0 A = X$ and N is the largest integer such that $S_N(X) \neq \emptyset$

Morphological skeleton function Image X, structuring element A: $[skf(X)](x, y) = k + 1$ if $(x, y) \in S_k(X)$, $[skf(X)](x, y) = 0$ if $(x, y) \notin S_k(X)$ for any k,
where $S_n(X) = X \ominus_n A \setminus (X \ominus_n A) \circ A$, and $X \ominus_0 A = X$.

Grey value dilation $(f \oplus b)(x, y) = \max\limits_{(s,t) \in B} [f(x - s, y - t) + b(s, t)]$

Grey value erosion $(f \ominus b)(x, y) = \min\limits_{(s,t) \in B} [f(x + s, y + t) - b(s, t)]$

Grey value opening $f \circ b = (f \ominus b) \oplus b$

Grey value closing $f \bullet b = (f \oplus b) \ominus b$

Morphological gradient $g = (f \oplus b) - (f \ominus b)$

Top-hat filter $T_{\text{hat}} = f - (f \circ b)$, **Bottom-hat filter** $B_{\text{hat}} = (f \bullet b) - f$

Sampling of continuous function $f(t)$: $\tilde{f}(t) = f(t)\, s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t)\, \delta(t - n\Delta T)$.
Fourier transform of sampled function: $\tilde{F}(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} F(\mu - \frac{n}{\Delta T})$

Sampling theorem Signal $f(t)$, bandwidth $\mu_{\max}$: If $\frac{1}{\Delta T} \geq 2\mu_{\max}$, $f(t) = \sum_{n=-\infty}^{\infty} f(n\Delta T)\, \mathrm{sinc}\left[\frac{t - n\Delta T}{\Delta T}\right]$.

Sampling: downsampling by a factor of 2: $\downarrow_2 (a_0, a_1, a_2, \ldots, a_{2N-1}) = (a_0, a_2, a_4, \ldots, a_{2N-2})$

Sampling: upsampling by a factor of 2: $\uparrow_2 (a_0, a_1, a_2, \ldots, a_{N-1}) = (a_0, 0, a_1, 0, a_2, 0, \ldots, a_{N-1}, 0)$

Set, circularity ratio $R_c = \frac{4\pi A}{P^2}$ of set with area A, perimeter P

Set, diameter $\mathrm{Diam}(B) = \max\limits_{i,j} [D(p_i, p_j)]$ with p_i, p_j on the boundary B and D a distance measure

Sinc function $\mathrm{sinc}\,(x) = \frac{\sin(\pi x)}{\pi x}$ when $x \neq 0$, and $\mathrm{sinc}\,(0) = 1$. If $f(t) = A$ for $-W/2 \leq t \leq W/2$ and zero elsewhere (block signal), then its Fourier transform is $F(\mu) = A\, W \mathrm{sinc}\,(\mu\, W)$

Spatial moments of an $M \times N$ image $f(x, y)$: $m_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} x^p\, y^q\, f(x, y), \quad p, q = 0, 1, 2, \ldots$

Statistical moments of distribution $p(i)$: $\mu_n = \sum_{i=0}^{L-1} (i - m)^n\, p(i)$, $m = \sum_{i=0}^{L-1} i\, p(i)$

Signal-to-noise ratio, mean-square $\mathrm{SNR}_{\text{rms}} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \widehat{f}(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(\widehat{f}(x,y) - f(x,y)\right)^2}$

Wavelet decomposition with scaling function h_ϕ, wavelet function h_ψ. For $j = 1, \ldots, J$:

Approximation: $c_j = \mathbf{H}c_{j-1} = \downarrow_2 (h_\phi * c_{j-1})$; Detail: $d_j = \mathbf{G}c_{j-1} = \downarrow_2 (h_\psi * c_{j-1})$

Wavelet reconstruction with dual scaling function $\tilde{h}_\phi$, dual wavelet function $\tilde{h}_\psi$. For $j = J, J-1, \ldots, 1$:

$c_{j-1} = \tilde{h}_\phi * (\uparrow_2 c_j) + \tilde{h}_\psi * (\uparrow_2 d_j)$

Wavelet, Haar basis $h_\phi = \frac{1}{\sqrt{2}}(1, 1)$, $h_\psi = \frac{1}{\sqrt{2}}(1, -1)$, $\tilde{h}_\phi = \frac{1}{\sqrt{2}}(1, 1)$, $\tilde{h}_\psi = \frac{1}{\sqrt{2}}(1, -1)$