Detecting Corner Features in Images

Recognize & retrieve image content

- image descriptions so far:
  - frequencies → base functions & coefficients
  - image regions → shapes of binary images
  - edges & boundaries
- all have their benefits & applications
- shared drawback: none of them stable & easier to retrieve

Properties of good image descriptors

- *localizable* $\longrightarrow$ spatial coordinates

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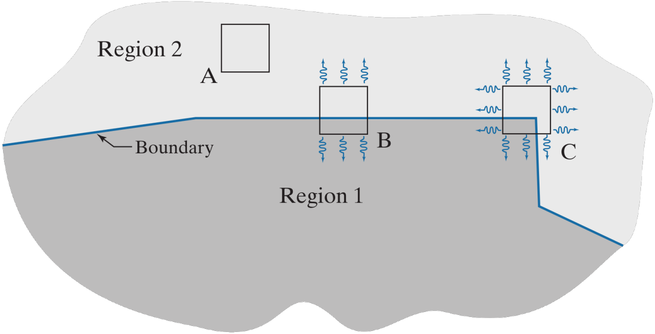
Conclusion: none of the (topologically) high-dimensional *features* fulfills those requirements

Corners - low-dimensional significant points

Compact, significant & localizable structure: corner point



Corners - low-dimensional significant points

Compact, significant & localizable structure: corner point

- intuition: meeting point of orthogonal edges
- edge definition → derivative filters ($1^{st}/2^{nd}$ *order*)
- *neighbourhood* of drastic intensity changes

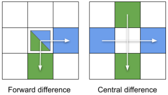
Arithmetic expression of above conditions → *Harris-Stephens corner detector*

Harris-Stephens detector - Formulation

Let f be an image, and we consider an image patch $(x, y) \in W$
We compare this to its shifted version by $(\Delta x, \Delta y)$
Then the weighted sum of squared differences can be computed as

$$C(\Delta x, \Delta y) = \sum_{(\Delta x, \Delta y) \in W} w(\Delta x, \Delta y) \left[f(x + \Delta x, y + \Delta y) - f(x, y) \right]^2 ,$$

which is the sum of 1^{st}-order squared forward-differences (i.e. 1^{st}-order derivatives) of $\mathcal{N}(f(x, y))$.



Harris-Stephens detector - $1^{st}/2^{nd}$ order derivatives

Observation: via a Taylor-expansion, we can approximate $f(x + \Delta x, y + \Delta y)$ with the 1^{st}-order partial derivative:

$$f(x + \Delta x, y + \Delta y) \approx f(x, y) + \Delta x \frac{\partial f}{\partial x} + \Delta y \frac{\partial f}{\partial y},$$

resulting in the following:

$$C(\Delta x, \Delta y) = \sum_{(\Delta x, \Delta y) \in W} w(\Delta x, \Delta y) \left[\Delta x \frac{\partial f}{\partial x} + \Delta y \frac{\partial f}{\partial y} \right]^2 \rightarrow \partial^2 f$$

Harris-Stephens detector - $1^{st}/2^{nd}$ order derivatives

Matrix reformulation:

$$C(\Delta x, \Delta y) = [\Delta x \Delta y]\, M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}; \qquad M = \sum_{(\Delta x, \Delta y) \in W} w(\Delta x, \Delta y) A,$$

with $A = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} \rightarrow$ looks familiar ?

Harris-Stephens detector - $1^{st}/2^{nd}$ order derivatives

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Reminder - 2^{nd} order derivatives:

$$D_{xx}(x, y) = -D_x(x-1, y) + D_x(x, y)$$

$$D_{xy}(x, y) = -D_x(x, y-1) + D_x(x, y)$$

$$D_{yy}(x, y) = -D_y(x, y-1) + D_y(x, y)$$

here: $\frac{\partial^2 f}{\partial x^2} = D_{xx}$, $\frac{\partial^2 f}{\partial y^2} = D_{yy}$, $\frac{\partial^2 f}{\partial x \partial y} = D_{xy}$

Harris-Stephens detector - Weighting Matrix

What about weighting matrix $w(\Delta x, \Delta y)$?
2 options:

- box filter: $w(\Delta x, \Delta y) = 1 \;\forall (\Delta x, \Delta y) \in W$
- Gaussian filter: $w(\Delta x, \Delta y) = e^{-\frac{\Delta x^2 + \Delta y^2}{2\sigma^2}}$

Lastly:

$$C(\Delta x, \Delta y) = [\Delta x \Delta y]\, M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}; \qquad M \rightarrow R = \det(M) - k \cdot \mathrm{trace}^2(M)$$

Harris-Stephens detector - Algorithm

1. Calculate 1D gradient images D_x and D_y (as you would for Prewitt operator)
2. Filter gradient images again to obtain D_{xx}, D_{yy} and D_{xy}
3. From the 2^{nd}-order derivatives, construct a Hessian matrix $H\ f(x, y)$ for every pixel
4. iterate over the image, correlate $M = w(\Delta x, \Delta y)H\ f(x + \Delta x, y + \Delta x)$
5. compute determinant $det(M)$ and square-trace $trace^2(M)$
6. compute R for a given constant $k \in \{0...0.25\}$
7. Threshold result $R(x, y)$ for highest responses

Properties of good image descriptors

- **localizable** → corner; single point
- **significant** → max. of 2^{nd}-order derivative
- *invariant* to distortion or geometric pertubation
- accurate or *unique*, for easy separation
- **compact** → 3 values: (x, y, R)

Our new image feature descripion: (a) corner point(s)

That's it for this week!