Control Engineering
Lecture 7
ver. 1.5.2

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Today

Last lecture

- Step response
- Harmonic response
- Reachability
- Reachable canonical form

Today

- An output regulation problem
- Solution to the output regulation problem
- Eigenvalue assignment
- Second order systems

A first control problem

Problem Given the system

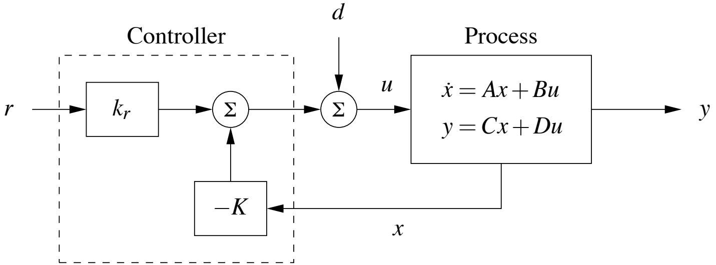
$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$$

find a state feedback control of the form

$$u = -Kx + k_r r$$

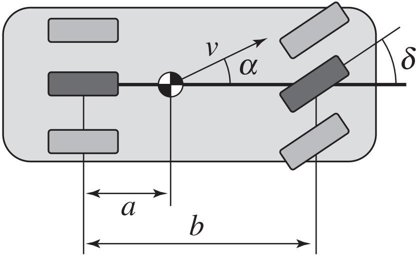
such that the output response of the closed-loop system converges to r, i.e.

$$y(t) \to r \quad \text{as} \quad t \to +\infty.$$



Control of the lateral deviation of a vehicle

Problem Linearized & normalized vehicle steering dynamics [Textbook, Example 2.8 and 5.12]



$x = \begin{bmatrix} y & \theta \end{bmatrix}^\top$

y vertical position of the c.m.v.

θ vehicle heading angle

c.m.v. $=$ center of mass of the vehicle

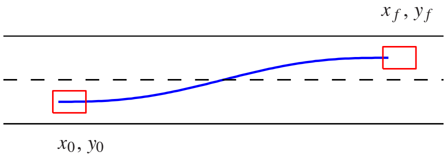
$u = \delta$ angle of the front wheel

$\gamma = a/b$

Change of lane manoeuvre find a state feedback control $u = -Kx + k_r r$ such that the vertical position of the c.m.v. $y(t) \to r$ as $t \to +\infty$, where r is the vertical position to which the vehicle should converge

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} \gamma \\ 1 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} x + \underbrace{0}_{D} u$$



A first control problem

Closed-loop system ($D = 0$)

$$
\begin{aligned}
\dot{x} &= Ax + B(-Kx + k_r r) = (A - BK)x + Bk_r r \\
y &= Cx
\end{aligned}
$$

What is the equilibrium $\bar{x}$ that returns the output

$$\bar{y} = C\bar{x} = r \ ?$$

It is the state $\bar{x}$ that satisfies

$$
\begin{aligned}
0 = \dot{\bar{x}} &= (A - BK)\bar{x} + Bk_r r \\
r = \bar{y} &= C\bar{x}
\end{aligned}
$$

If $A - BK$ is asymptotically stable (hence, nonsingular), then $\bar{x} = -(A - BK)^{-1}Bk_r r$ and

$$r = C\bar{x} = -C(A - BK)^{-1}Bk_r r \quad \Leftrightarrow \quad k_r = -\frac{1}{C(A - BK)^{-1}B}$$

A first control problem

Consider the closed-loop system

$$\begin{aligned} \dot{x} &= Ax + B(-Kx + k_r r) = (A - BK)x + Bk_r r \\ y &= Cx \end{aligned}$$

In the new coordinates

$$\tilde{x} = x - \bar{x}$$

the system becomes

$$\begin{aligned} \dot{\tilde{x}} &= (A - BK)\tilde{x} \\ y &= C\tilde{x} + r \end{aligned}$$

Conclusion

If K is designed so that $(A - BK)$ has all the eigenvalues with strictly negative real part then the system $\dot{\tilde{x}} = (A - BK)\tilde{x}$ is asymptotically stable and therefore

$$\tilde{x}(t) \to 0 \quad \Leftrightarrow \quad x(t) \to \bar{x} \quad \Rightarrow \quad y(t) \to r, \quad \text{as } t \to +\infty$$

Today

- How to design K such that $(A - BK)$ has all the eigenvalues with strictly negative real part? (eigenvalue assignment via state feedback)
- The design is possible provided that the system $\dot{x} = Ax + Bu$ is reachable
- The procedure is constructive: we design K
- We already know how to design $k_r = -\frac{1}{C(A-BK)^{-1}B}$

Today

- Solution to the output regulation problem
- **Eigenvalue assignment**
- Second order systems

Reachability form

Consider the reachable system

$$\dot{x} = Ax + Bu$$

Based on characteristic polynomial

$$\det(sI - A) = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

one constructs the canonical form:

$$\dot{z} = \underbrace{\begin{pmatrix} -a_1 & -a_2 & \cdots & -a_n \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{pmatrix}}_{\tilde{A}} z + \underbrace{\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{\tilde{B}} u$$

To obtain this representation, in Lecture 6 we have design the change of coordinates $z = Tx$, with

$$T = \tilde{W}_r W_r^{-1}, \quad W_r = \begin{bmatrix} B & AB & \ldots & A^{n-1}B \end{bmatrix}, \quad \tilde{W}_r = \begin{bmatrix} \tilde{B} & \tilde{A}\tilde{B} & \ldots & \tilde{A}^{n-1}\tilde{B} \end{bmatrix}$$

In the new coordinates the system is

$$\dot{z} = T\dot{x} = T(Ax + Bu) = T(AT^{-1}z + Bu) = \underbrace{TAT^{-1}}_{\tilde{A}} z + \underbrace{TB}_{\tilde{B}} u$$

State feedback for systems in reachable form

Consider the feedback control law in the coordinates $z = Tx$ ($x = T^{-1}z$)

$$\begin{aligned} u &= -Kx + k_r r = -KT^{-1}z + k_r r = -\tilde{K}z + k_r r \\ &= -\underbrace{\begin{pmatrix} \tilde{k}_1 & \tilde{k}_2 & \dots & \tilde{k}_n \end{pmatrix}}_{\tilde{K}} \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} + k_r r \\ &= -\tilde{k}_1 z_1 - \tilde{k}_2 z_2 \dots - \tilde{k}_n z_n + k_r r \end{aligned}$$

Then the **closed-loop** system becomes

$$\begin{aligned} \dot{z} &= \tilde{A}z + \tilde{B}(-\tilde{K}z + k_r r) = (\tilde{A} - \tilde{B}\tilde{K})z + \tilde{B}k_r r \\ &= \begin{pmatrix} -a_1 - \tilde{k}_1 & -a_2 - \tilde{k}_2 & \cdots & -a_n - \tilde{k}_n \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{pmatrix} z + \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} k_r r \end{aligned}$$

because $\tilde{B}\tilde{K} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} \tilde{k}_1 & \tilde{k}_2 & \dots & \tilde{k}_n \end{pmatrix} = \begin{pmatrix} \tilde{k}_1 & \tilde{k}_2 & \cdots & \tilde{k}_n \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$

State feedback for systems in reachable form

Closed-loop system

$$
\begin{aligned}
\dot{z} &= \tilde{A}z + \tilde{B}(-\tilde{K}z + k_r r) \\
&= \begin{pmatrix} -a_1 - \tilde{k}_1 & -a_2 - \tilde{k}_2 & \cdots & -a_n - \tilde{k}_n \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{pmatrix} z + \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} k_r r
\end{aligned}
$$

Since $(\tilde{A} - \tilde{B}\tilde{K}, \tilde{B})$ is in reachable canonical form, the characteristic polynomial is

$$\det(sI - (\tilde{A} - \tilde{B}\tilde{K})) = s^n + (a_1 + \tilde{k}_1)s^{n-1} + \cdots + (a_{n-1} + \tilde{k}_{n-1})s + a_n + \tilde{k}_n$$

If we want this to be equal to a <u>desired polynomial</u>

$$p_{des}(s) = s^n + p_1 s^{n-1} + \cdots + p_{n-1}s + p_n$$

we must choose $\tilde{K}$ such that

$$
\begin{aligned}
p_1 &= a_1 + \tilde{k}_1 \\
&\cdots \\
p_{n-1} &= a_{n-1} + \tilde{k}_{n-1} \\
p_n &= a_n + \tilde{k}_n
\end{aligned}
\quad \Leftrightarrow \quad
\begin{aligned}
\tilde{k}_1 &= p_1 - a_1 \\
&\cdots \\
\tilde{k}_{n-1} &= p_{n-1} - a_{n-1} \\
\tilde{k}_n &= p_n - a_n
\end{aligned}
$$

Eigenvalue assignment

For a reachable system not in reachable canonical form

$$\dot{x} = Ax + Bu, \quad y = Cx + Du$$

one converts the system into reachable canonical form via $z = Tx$. The feedback becomes

$$u = -\tilde{K}z + k_r r = \underbrace{-\tilde{K}T}_{-K} x + k_r r$$

where

$$K = \tilde{K}T = \begin{pmatrix} p_1 - a_1 & p_2 - a_2 & \dots & p_n - a_n \end{pmatrix} \tilde{W}_r W_r^{-1}$$

and (prove that $\tilde{W}_r$ has the expression below – see [Textbook, Exercise 6.7])

$$W_r = \begin{pmatrix} B & AB & \cdots & A^{n-1}B \end{pmatrix}, \qquad \tilde{W}_r = \begin{pmatrix} 1 & a_1 & a_2 & \cdots & a_{n-1} \\ 0 & 1 & a_1 & \cdots & a_{n-2} \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & a_1 \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}^{-1}$$

The feedback gain K assign the characteristic polynomial $p_{des}(s)$ to the closed-loop matrix $A - BK$. Hence, the feedback gain K makes the roots of $p_{des}(s) = 0$ the eigenvalues of the closed-loop matrix $A - BK$

Eigenvalue assignment

Example [Textbook, Example 6.4] Lateral vehicle steering

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} \gamma \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad D = 0$$

Rechability matrix

$$W_r = (B\ AB) = \begin{pmatrix} \gamma & 1 \\ 1 & 0 \end{pmatrix}$$

The system is reachable because $\det W_r = \gamma \neq 0$.
Characteristic polynomial open-loop system

$$p(s) = s^2 + a_1 s + a_2 = s^2$$

Find K, k_r in $u = -Kx + k_r r$ such that

- polynomial characteristic of $A - BK$ (closed-loop) is

$$p(s) = s^2 + p_1 s + p_2 = s^2 + 2\zeta_c \omega_c s + \omega_c^2$$

- $\bar{y} = r$ ($\Leftrightarrow k_r = -\frac{1}{C(A-BK)^{-1}B}$)

Eigenvalue assignment

To assign the desired characteristic polynomial we use

$$K = \tilde{K}T = \begin{pmatrix} p_1 - a_1 & p_2 - a_2 & \dots & p_n - a_n \end{pmatrix} \tilde{W}_r W_r^{-1}$$

For this example

- $n = 2$ and

$$\tilde{K} = \begin{pmatrix} p_1 - a_1 & p_2 - a_2 \end{pmatrix} = \begin{pmatrix} 2\zeta_c\omega_c - 0 & \omega_c^2 - 0 \end{pmatrix}$$

- The system in reachable canonical form is (recall that $p(s) = \det(sI - A) = s^2$)

$$\begin{pmatrix} -a_1 & -a_2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

  Hence, the reachability matrix $\tilde{W}_r$ is

$$\tilde{W}_r = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

Eigenvalue assignment

Replacing the previous expressions in K gives

$$\begin{aligned} K &= \tilde{K}T = \begin{pmatrix} p_1 - a_1 & p_2 - a_2 & \dots & p_n - a_n \end{pmatrix} \tilde{W}_r W_r^{-1} \\ &= \begin{pmatrix} 2\zeta_c\omega_c & \omega_c^2 \end{pmatrix} I_2 \begin{pmatrix} \gamma & 1 \\ 1 & 0 \end{pmatrix}^{-1} \\ &= \begin{pmatrix} 2\zeta_c\omega_c & \omega_c^2 \end{pmatrix} I_2 \begin{pmatrix} 0 & 1 \\ 1 & -\gamma \end{pmatrix} \\ &= \begin{pmatrix} \omega_c^2 & 2\zeta_c\omega_c - \gamma\omega_c^2 \end{pmatrix} \end{aligned}$$

Moreover

$$k_r = -\frac{1}{C(A-BK)^{-1}B} = \omega_c^2$$

State feedback

The overall state feedback control is given by

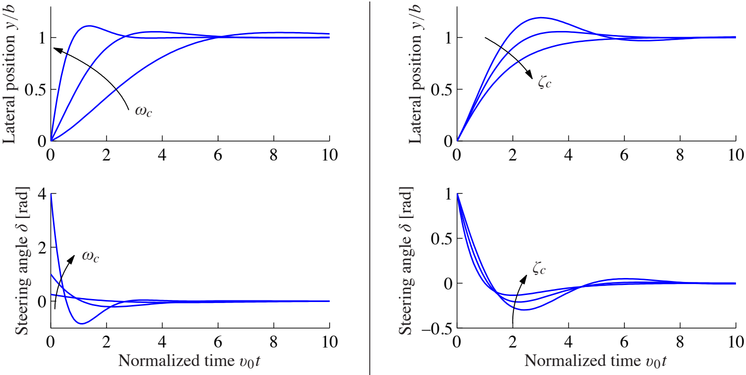
$$\begin{aligned} u &= -\begin{pmatrix} \omega_c^2 & 2\zeta_c\omega_c - \gamma\omega_c^2 \end{pmatrix} x + \omega_c^2 r \\ &= -\omega_c^2 x_1 - (2\zeta_c\omega_c - \gamma\omega_c^2)x_2 + \omega_c^2 r \end{aligned}$$

Eigenvalue assignment

State feedback control

$$u = -\omega_c^2 x_1 - (2\zeta_c\omega_c - \gamma\omega_c^2)x_2 + \omega_c^2 r$$

It describes the evolution of the steering angle $u = \delta$ to enforce the transition of the vertical position of the c.m.v. towards the new value r during a lateral manoeuvre



(a) Step response for varying ω_c (b) Step response for varying ζ_c

Today

- Solution to the output regulation problem
- Eigenvalue assignment
- **Second order systems**

Second-order systems

Second-order systems model several important physical systems. They are important to study their response as their eigenvalues change.

$$\frac{d^2q(t)}{dt^2} + 2\zeta\omega_0 \frac{dq(t)}{dt} + \omega_0^2 q(t) = k\omega_0^2 u(t)$$

$x_1 = q$, $x_2 = \dot{q}$

$$\begin{aligned} \begin{bmatrix} \dfrac{dx_1(t)}{dt} \\ \dfrac{dx_2(t)}{dt} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\zeta\omega_0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ k\omega_0^2 \end{bmatrix} u(t) \\ y(t) &= q(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

Second-order system

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_0\frac{dy(t)}{dt} + \omega_0^2 y(t) = k\omega_0^2 u(t)$$

$y = q$

$\omega_0 > 0$ **undamped natural frequency**
$\zeta > 0$ **damping ratio**

Second-order system

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_0\frac{dy(t)}{dt} + \omega_0^2 y(t) = k\omega_0^2 u(t)$$

Characteristic equation:

$$\lambda^2 + 2\zeta\omega_0\lambda + \omega_0^2 = 0$$

Characteristic roots/poles:

$$\lambda_1 = -\zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1}, \quad \lambda_2 = -\zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1}$$

Free response

$u = 0$

- **Overdamped response $\zeta > 1$**
  Two real and distinct roots $\lambda_{1/2} = -\zeta\omega_0 \pm \omega_0\sqrt{\zeta^2 - 1}$
  $\lambda_1 \neq \lambda_2$
  The homogeneous output response is

$$y_h(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

  In fact

$$\begin{aligned} y_h(t) &= C\mathrm{e}^{At}x_0 \\ &= CT^{-1}\begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix} Tx_0 \\ &= \tilde{C}\begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix} z_0 \\ &= \tilde{C}_1 e^{\lambda_1 t} z_{01} + \tilde{C}_2 e^{\lambda_2 t} z_{02} \end{aligned}$$

Free response

$u = 0$

- **Critically damped response $\zeta = 1$**
  One root with multiplicity two $\lambda_{1/2} = -\zeta\omega_0 \pm \omega_0\sqrt{\zeta^2 - 1} \Rightarrow$
  $\lambda_1 = \lambda_2 = -\zeta\omega_0$

$$y_h(t) = e^{-\zeta\omega_0 t}(C_1 + C_2 t)$$

In fact (Jordan form)

$$\tilde{A} = TAT^{-1} = \begin{bmatrix} -\zeta\omega_0 & 1 \\ 0 & -\zeta\omega_0 \end{bmatrix}$$

and therefore

$$\begin{aligned}
y_h(t) &= C\mathrm{e}^{At}x_0 = CT^{-1}\mathrm{e}^{\tilde{A}t}Tx_0 \\
&= CT^{-1}\begin{bmatrix} e^{-\zeta\omega_0 t} & te^{-\zeta\omega_0 t} \\ 0 & e^{-\zeta\omega_0 t} \end{bmatrix} Tx_0 \\
&= \tilde{C}\begin{bmatrix} e^{-\zeta\omega_0 t} & te^{-\zeta\omega_0 t} \\ 0 & e^{-\zeta\omega_0 t} \end{bmatrix} z_0 \\
&= \tilde{C}_1 e^{-\zeta\omega_0 t}(z_{01} + t\, z_{02}) + \tilde{C}_2 e^{-\zeta\omega_0 t} z_{02} \\
&= e^{-\zeta\omega_0 t}[(\tilde{C}_1 z_{01} + \tilde{C}_2 z_{02}) + t\, \tilde{C}_1 z_{02}]
\end{aligned}$$

Free response

$u = 0$

- **Underdamped response** $0 < \zeta < 1$
  Two imaginary and distinct roots $\lambda_{1/2} = -\zeta\omega_0 \pm \omega_0\sqrt{\zeta^2 - 1} \Rightarrow$
  $\lambda_{1/2} = -\zeta\omega_0 \pm i\omega_0\sqrt{1-\zeta^2}$

  $$y_h(t) = e^{-\zeta\omega_0 t}(C_1 \cos\omega_d t + C_2 \sin\omega_d t)$$

  where $\omega_d = \omega_0\sqrt{1-\zeta^2}$ is the **damped frequency**.

  In fact

  $$\tilde{A} = TAT^{-1} = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix} = \begin{bmatrix} -\zeta\omega_0 & \omega_0\sqrt{1-\zeta^2} \\ -\omega_0\sqrt{1-\zeta^2} & -\zeta\omega_0 \end{bmatrix}$$

  and therefore

  $$\begin{aligned} y_h(t) &= CT^{-1}e^{-\zeta\omega_0 t}\begin{bmatrix} \cos(\omega_d t) & \sin(\omega_d t) \\ -\sin(\omega_d t) & \cos(\omega_d t) \end{bmatrix} Tx_0 \\ &= \tilde{C}e^{-\zeta\omega_0 t}\begin{bmatrix} \cos(\omega_d t) & \sin(\omega_d t) \\ -\sin(\omega_d t) & \cos(\omega_d t) \end{bmatrix} z_0 \\ &= e^{-\zeta\omega_0 t}(\underbrace{(\tilde{C}_1 z_{01} + \tilde{C}_2 z_{02})}_{C_1}\cos\omega_d t + \underbrace{(\tilde{C}_1 z_{02} - \tilde{C}_2 z_{01})}_{C_2}\sin\omega_d t) \end{aligned}$$

Step response

$u(t) = 1(t)$ (i.e. $u(t) = 0$ for all $t < 0$ and $u(t) = 1$ for all $t \geq 0$)

Recall the expression of the step response ($x_0 = 0$, $D = 0$)

$$r(t) = \underbrace{\overbrace{C\mathrm{e}^{At}x_0}^{=0} + CA^{-1}\mathrm{e}^{At}B}_{\text{transient response}} \underbrace{-CA^{-1}B}_{\text{steady state}}$$

Consider the overdamped homogeneous response ($\zeta > 1$)

$$C\mathrm{e}^{At}x_0 = \tilde{C}_1\mathrm{e}^{\lambda_1 t}z_{01} + \tilde{C}_2\mathrm{e}^{\lambda_2 t}z_{02}$$

Similarly

$$CA^{-1}\mathrm{e}^{At}B = \hat{C}_1\mathrm{e}^{\lambda_1 t}\hat{b}_1 + \hat{C}_2\mathrm{e}^{\lambda_2 t}\hat{b}_2$$

Hence

$$\begin{aligned} r(t) = &\quad (\tilde{C}_1 z_{01} + \hat{C}_1\hat{b}_1)\mathrm{e}^{\lambda_1 t} + (\tilde{C}_2 z_{02} + \hat{C}_2\hat{b}_2)\mathrm{e}^{\lambda_2 t} - CA^{-1}B \\ = &\quad \underbrace{-CA^{-1}B}_{k}\left(1 - \underbrace{\frac{\tilde{C}_1 z_{01} + \hat{C}_1\hat{b}_1}{CA^{-1}B}}_{c_1}\mathrm{e}^{\lambda_1 t} - \underbrace{\frac{\tilde{C}_2 z_{02} + \hat{C}_2\hat{b}_2}{CA^{-1}B}}_{c_2}\mathrm{e}^{\lambda_2 t}\right) \end{aligned}$$

Step response

- Overdamped response $\zeta > 1$

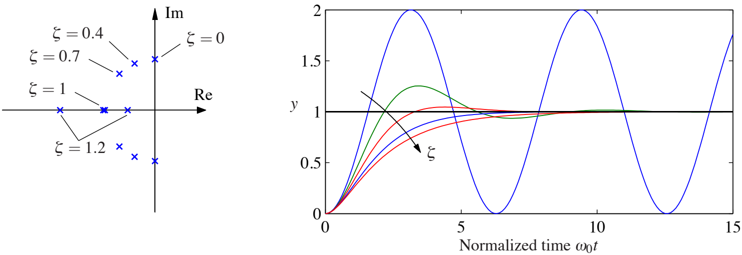
$$r(t) = k(1 - C_1 e^{-(\zeta - \sqrt{\zeta^2 - 1})\omega_0 t} - C_2 e^{-(\zeta + \sqrt{\zeta^2 - 1})\omega_0 t})$$

- Critically damped response $\zeta = 1$

$$r(t) = k(1 - e^{-\zeta\omega_0 t}(1 + \omega_0 t))$$

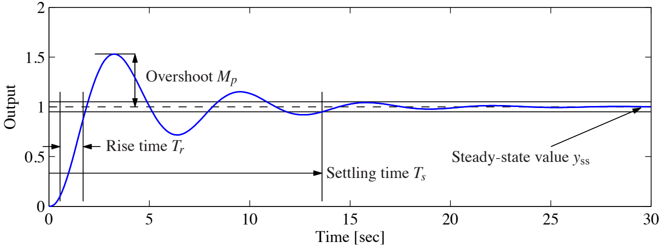
- Underdamped response $0 < \zeta < 1$

$$r(t) = k(1 - e^{-\zeta\omega_0 t}\cos\omega_d t - \frac{\zeta}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t}\sin\omega_d t)$$



(a) Eigenvalues (b) Step responses

Step response characterization



- **steady state value** y_{st} final value where the output converges
- **rise time** T_r amount of time required for the output to go from 10% to 90% of its final value
- **overshoot** M_p percentage of the final value by which the signal rises above the final value
- **settling time** T_s amount of time required for the output to stay within 2% of its final value

Properties of the step response

Table 6.1: Properties of the step response for a second-order system with $0 < \zeta < 1$.

Property	Value	$\zeta = 0.5$	$\zeta = 1/\sqrt{2}$	$\zeta = 1$
Steady-state value	k	k	k	k
Rise time	$T_r \approx 1/\omega_0 \cdot e^{\varphi/\tan\varphi}$	$1.8/\omega_0$	$2.2/\omega_0$	$2.7/\omega_0$
Overshoot	$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$	16%	4%	0%
Settling time (2%)	$T_s \approx 4/\zeta\omega_0$	$8.0/\omega_0$	$5.9/\omega_0$	$5.8/\omega_0$

$$\varphi = \arccos\zeta$$

$$T_r = \frac{\pi - \arctan\frac{\sqrt{1+\zeta^2}}{\zeta}}{\omega_0\sqrt{1+\zeta^2}}$$

Recapitulation

The contents of this lecture are covered by

- Sections 6.1–6.3

Reading assignments

- pp. 187–190 (**Higher-order Systems)**
- Section 6.4 (**Integral Action**)

Next lecture

- Recapitulation lecture