MACH: Digital Field Plotting

2nd Year Electronics Lab

IMPERIAL COLLEGE LONDON

v2.01

Table of Contents

Equipment

- Lab computer with MATLAB
- Magnetic Circuit
- 20A Pulsed PSU
- Oscilloscope
- 20V, 20A Meters
- Hall Effect Flux Probe

Aim

To study the distribution of the magnetic field in the air-gap of a simple magnetic circuit. The air-gap shape is complex enough to demonstrate how useful a numerical field-solver technique can be. The accuracy of the program solution and its limitations are investigated by comparison, with experimental measurements of the magnetic flux distribution and approximate analytical solutions.

Objective

The magnetic circuit under study is a C-core with the following characteristics:

- 350 turn coil wound on to the core.
- The cross-sectional dimensions of the pole face are 70.0 mm by approx. 42 mm.
- The dimensions of the air-gap are shown in Figure 1 below.

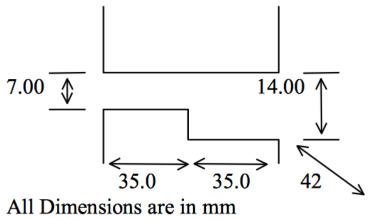

Figure 1 – Front view of the core

The test is divided into three major parts:

1. A hand calculation. Sometimes, it is very useful to know what to expect before embarking to any experiment. This part of the experiment will give that insight to the rest of the problem
2. An experimental activity where you will measure the flux density distribution in the air-gap of the C-core and the inductance associated to this circuit.
3. A computer simulation where you will explore the distribution of the scalar magnetic potential and the flux density in the core, the air-gap and the fringing fields around them.

Background

This experiment relies on basic magneto-statics. This is covered in the 1st year Energy Conversion (EE1-5) course and reviewed in the 2nd year Fields (EE2-10B) course during the spring term. Any textbook on basic fields will provide equivalent background in greater detail, for those who need more than is provided in this sheet.

Recommended Timetable

This experiment consists of 2 parts. There are 4 timetabled lab sessions for this experiment. You should aim to complete both parts within set time.

1. Introduction

For many years, the design of electrical machines has relied heavily on a simplistic view of the complex electric/magnetic interactions which determine their behaviour. This, combined with the designer's own experience and several empirically derived parameters, normally resulted in the production of a satisfactory machine.

Over the past couple of decades, the enormous increase in processing speed of commonly available computers has allowed the designer to examine the field variations within the machine in greater detail using numerical methods. This enables the designer to refine the design in terms of material costs and overall machine performance. This in turn has promoted increased interest in the development of the largescale numerical techniques required to solve the equations governing electric/magnetic field distributions within a wide variety of devices.

In this laboratory, you will gain some experience in the use of a numerical field-solver program. You will also be able to verify the accuracy of your program solutions and examine their limitations, by comparing the predicted and experimental measurements of the magnetic flux distribution in and around the air-gap of a simple magnetic circuit (Figure 2).

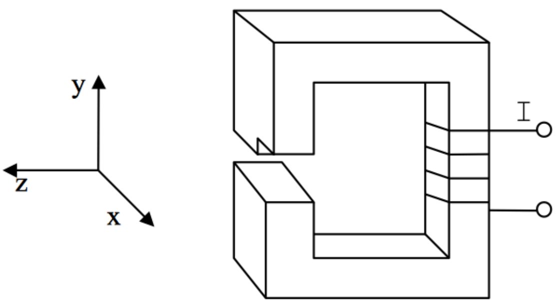

Figure 2 – Schematic of the core

2. Hand Calculation

To draw an accurate flux plot by hand is not possible for this mixed material model. However, you can make a good estimate of the flux density in the two parts of the air-gap using basic formulae introduced in the fields lecture course.

2.1 Flux Density

First, calculate the magnitude of the magnetic field (H) as:

$$H = \frac{NI}{g} \qquad \textbf{(Eqn. 2.1)}$$

Where:

- NI = Ampere-turns (AT)
- g = Air gap length (m)

Note that the air-gap length is 7.0mm or 14.0mm. (Remember to use metres in your calculation).

Then calculate the magnitude of the magnetic flux density (B):

$$B = \mu_0 H \qquad \textbf{(Eqn. 2.2)}$$

Where $\mu_0 = 4\pi \times 10^{-7} H/m$

These results come from a rather simplistic model of the C-core. However, they do provide a good insight on what to expect for the following steps.

Q Write down all the assumptions you think have been made to conclude that the flux density in the air-gap can be estimated with Eqn. 2.1 and Eqn. 2.2.

2.2 Inductance

For a linear magnetic circuit, the inductance is defined by:

$$L = \frac{N\phi}{I} \qquad \textbf{(Eqn. 2.3)}$$

Where ϕ is the flux linked with each turn of the coil.

Q Calculate the total flux (from the estimates of flux density) and then calculate the inductance.

3 Experimental Procedure

In the first part of the experiment, measurements are taken from the magnetic circuit.

3.1 Flux Density at Centre of Slot

1. Connect the coil to the D.C. supply via an appropriate ammeter to measure the current.
2. By adjusting the supply voltage, excite the coil with 5 Amps and keep this level throughout the test, checking the current before flux measurement.
3. Insert the flux probe into the centre slot. Take an initial set of approximately 12-15 readings of the flux density and the position of the probe.
4. Plot these results on graph paper, draw a curve through them. Based on these measurements take several more readings in the areas where significant changes in the flux density are apparent. You should be able to account for the shape of the graph.

Q Is the highest flux density in the region expected?

Q Why is the flux density non-zero outside the air-gap?

5. Compare the magnitude of the flux densities in each air-gap with the value calculated in section 3 (assuming that all AT are dropped across the gap).

3.2 Flux Density at Edge of Slot

Repeat the flux density measurements outlined in Section 3.1, using the slot along the edge of the pole piece rather than at the centre of the air-gap. Plot these values on your graph.

Q Is the distribution different from that obtained in Section 3.1? Suggest an explanation for this.

3.3 Assessment of Core Linearity

Return to the slot at the centre of the air-gap. Increase the winding current to 14 Amps. Take flux density measurements at one or two positions in the slot.

Take your readings as quickly as possible to avoid overheating the coil. By comparing these readings with those obtained at the same position in Section 3.1, determine whether the flux density scales linearly with current. If not, suggest a reason for this.

3.4 Inductance Measurement

The inductance of the C-core winding can be measured with the circuit shown in Figure 3:

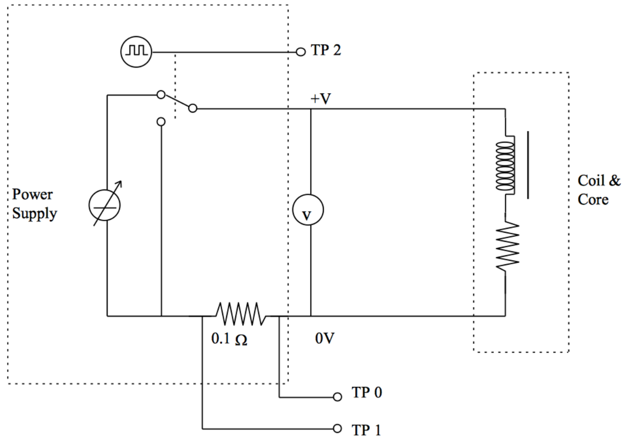

Figure 3 – Circuit diagram for measuring core inductance

The inductance L is obtained by recording the initial rate of decay of current in the inductor with an oscilloscope when the supply voltage is removed and the current decays in an alternative path.

When the supply voltage is removed, the instantaneous current $i(t)$ is therefore:

$$i(t) = I_0 e^{-t/T}$$

Where $I_0 = \frac{V_{on}}{R}$ and V_{on} is the voltage applied while the pulse is on at $t = 0$.

$$\frac{di}{dt} = -\frac{V_{on}}{L}\frac{1}{T}$$

And $T = {}^{L}/_{R}$ so:

$$\frac{di}{dt} = -\frac{V_{on}}{L}$$

Thus:

$$L = -{}^{V_{on}}\Big/\frac{di}{dt}$$

The power supply has a built-in timer to give pulses of voltage of about half second. The power supply contains a current sense resistor of 0.1Ω.

The oscilloscope may be arranged to give a record of a single event by:

- Connecting TP1 to CH1 and TP2 to CH2
- Set trigger source to CH2

- Set time base to 200 ms
- Set trigger mode to normal
- Set trigger slope to falling edge
- Set trigger level to about 2 V
- Set the power supply to pulse mode and a trace should be displayed (if not adjust the trigger level)
- If the trace is noisy use display averaging
- You can obtain $\frac{dv}{dt}$ (and hence $\frac{di}{dt}$) by inspection, by using curors or by using the $\frac{dv}{dt}$ function available on the menu called up by the MATH button.

You will also need to measure the voltage produced by the power supply. Compare your answer with the value given by the numerical solution obtained in Section 4.

4. Finite Difference Numerical Field Solution

4.1 Background

The magnetic field in and round the air-gap is described by Laplace's equation. This is a particular form of Poisson's equation dealing with regions that do not contain distributed currents. The solution of this equation is used in the laboratory to obtain both the magnetic equipotentials and the magnetic flux distributions and show that the potential and flux functions are orthogonal, i.e. flux lines and lines of equipotential intersect each other at right angles.

In the x-y plane, Laplace's equation for the magnetic potential distribution (V_m) is given by:

$$\frac{\partial^2 V_m}{\partial x^2} + \frac{\partial^2 V_m}{\partial y^2} = 0 \qquad \textbf{(Eqn. 4.1)}$$

or alternatively for the magnetic flux distribution (Ψ):

$$\frac{\partial^2 \Psi_m}{\partial x^2} + \frac{\partial^2 \Psi_m}{\partial y^2} = 0 \qquad \textbf{(Eqn. 4.2)}$$

Laplace's equations can be solved analytically or numerically. It is very unusual to find an electromagnetics related problem that can be solved purely by analytical methods. In particular, analytical approaches may fail if:

- The problem is non-linear and cannot be linearised without seriously affecting the result.
- The problem involves an irregular geometry
- The boundary conditions are very complex
- The problem media are inhomogeneous or anisotropic.

Numerical techniques should be used whenever a problem of such complexity arises (Sadiku, 1992). Most of these methods solve two-dimensional field problems. Three-dimensional solvers are available, but as the number of nodes is high they are relatively computationally expensive and have not yet come into common use. In particular, the Finite Difference Method is one of easiest numerical techniques to understand and employ (within its own limitations).

The Finite Difference techniques are based on approximations which allow replacing differential equations by finite difference equations. In general, a finite difference method involves three basic steps:

1. Divide the solution region into a grid of nodes.
2. Approximate the related differential equation by finite differences and relate the value of a variable at a point in the solution region to the values at some neighbouring points.
3. Solve the finite difference equations subject to a set of prescribed boundary and initial conditions.

Differentials of $V_m(x, y)$ (although potential V_m is used from here onwards, the method applies equally to the flux function, Ψ) are approximated by the finite differences to yield Laplace's equation relating the potential at any node to those around it. For a single surface (i.e., without involving the interface of two or more different materials) a finite-difference molecule may be defined as the 5-node square grid as shown in Figure 4.

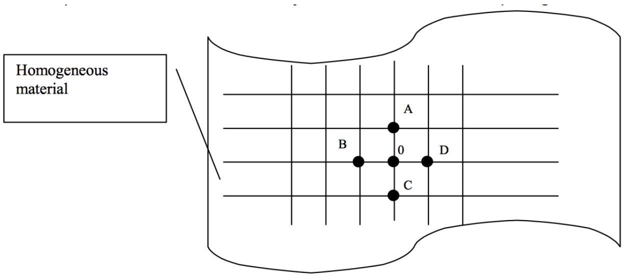

Figure 4 – Finite Difference molecule model

The potential of the middle node (V_0) then can be estimated in terms of the potential at the nodes around it by the following expression:

$$V_0 = \frac{1}{4}(V_A + V_B + V_C + V_D) \qquad \textbf{(Eqn. 4.3)}$$

Q Consider that the nodes in Figure 4 are spaced at intervals of h, write simple expressions for the gradient of V_m in the x and y directions.

Q Write expressions for the second derivatives and substitute into Eqn 4.3. Rearrange the expression to obtain Eqn 4.3.

In the experiment, you were required to maintain a constant current in the coil. Assuming that the iron core remains unsaturated and is of high permeability, this means that the total MMF (magnetomotive force) acting across the air-gap remains constant and that all the ampere-turns (At) of the coil are available to drive flux across the air-gap. The magnetic potential (V_m) gradually drops from the value at one iron surface to the value at the other. Equipotential lines (V_m contours) show how the potential changes. The magnetic flux will be at right angles to equipotentials. The magnetic field strength that drives the flux is found from the gradient of the potential:

$$H = -\nabla(V_m) \qquad \textbf{(Eqn. 4.4)}$$

Two components at right angles can be found from the scalar potential as:

$$H_x = -\frac{\partial V_m}{\partial x} \quad \text{and} \quad H_y = -\frac{\partial V_m}{\partial y}$$

And so the flux density is given by:

$$B_x = \mu_0 H_x \qquad B_y = \mu_0 H_y$$

$$B_x = -\mu_0 \frac{\partial V_m}{\partial x} \qquad B_y = -\mu_0 \frac{\partial V_m}{\partial y}$$

Thus, if V_m is changing rapidly, that is the contour lines are close together, then the gradient of V_m is large and B is large. If you find this all very difficult, think of one end of the core as a plateau high above the other one with a slope in the air-gap. Equipotentials are then contour lines and flux lines represent the directions of the greatest slope.

4.2 Software

The field distribution in the C-core will be studied using a model and finite difference algorithm written for MATLAB, which is a widespread CAD in research and industry. Both, the programming style and the electromagnetic model have been kept as simple as possible for a good reason: you will be able to interact with the main program and with the Electromagnetics-related equations easily and freely.

1. Find the 'MATLAB numerical simulation scripts for N' (expN_files.exe) on the MACH handout web page.
2. Run the executable to copy ALL the MATLAB files into a sub-directory in your home drive, e.g.: h:\my_directory\2ndyear\expN\
3. Start a MATLAB session from the windows task bar (Start/Programs/MATLAB)
4. Within the MATLAB command-line move to the directory you chose e.g.:
   `cd h:\my_directory\2ndyear\expN\`
5. Open the file 'define_problem.m'. Introduce the dimensions of the core according to Figure 5.

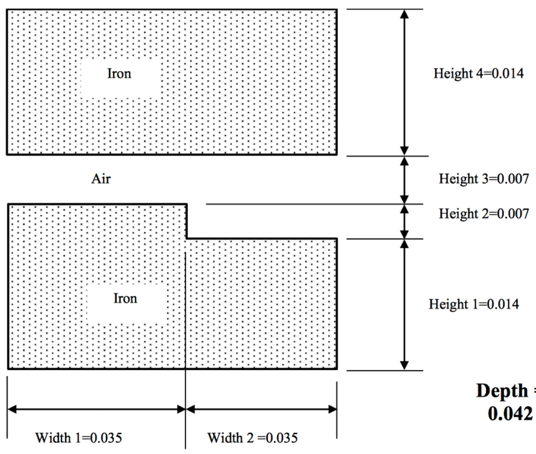

Figure 5 – Front view of the core

Please note that only a small section of the C-core is simulated. 'Height 1' and 'Height 4' could take the real dimensions of the core, at the expense of increasing the computing time, as you will find out very soon. Suggestion: leave these values to the default 14mm.

6. Introduce also the excitation parameters: current in amperes (`I=5;`) and coil turns (`nTurns=350;`).
7. Save all the files you edit before starting any simulation. Keep this in mind for the whole experiment.

4.3 Simulating an Ideal-core

In an ideal-core simulation, the solver assumes iron to be a perfect conductor of magnetic flux and only calculates changes in potential in the air-gap and in the air regions surrounding the core. The potential in the iron parts are set to some initial conditions and are not processed by the solver, they preserve their initial condition throughout the simulation.

1. Open the file 'define_problem.m' and change the following:
   - Set the flag-type variable '**`IdealCore`**' to 1.
   - Set '**`NumIterations`**' to 500
   - Choose a '**`NodeDensity`**' of 1000
2. Identify the two places where Eqn. 4.3 should be processed within the file 'solve_air_only.m' and incorporate it into the code. The places are marked with the following comment line:

   '**`%Type the finite difference equation below this line`**'.

   The stereotype of the variables and matrix indexes may be taken from previous lines within the same file.
3. Start a fresh simulation by executing from the MATLAB command line:

```
close all; % This will close all the figures
clear all; % Clear all the variables.
```

4. Run the program from the MATLAB command line by executing:

```
c_core; % Calling the main program
```

NOTE: Although some plots appear soon after the program call, wait for the iteration process to finish before attempting any plot manipulation.

After the program has performed as many iterations as requested, various plots will appear. Use the table in the appendix as a guideline to know what to look for.

- Feel free to rotate the 3D plots and observe the behavior of $\boldsymbol{V_m}$ and $\boldsymbol{H}$ from several perspectives. Although using the mouse to do this is very easy, the command view provides a more accurate camera position (type help view if you want to know how to use this command). Use the zoom in and out for the 2D plots.
- Pay particular attention to all the regions in and around the air-gap.
- Plot the simulation time against the node density, and comment on the trade-off between the computation time and the quality of the plots obtained.
- Build the following plot: Node density versus simulation time. The procedure is as follows:

1. Edit the file "define_problem.m" to change the node density to 250 nodes/meter. Leave 500 iterations.
2. Execute all these three instructions:

```
close all;
clear all;
c_core;
```

3. Wait for the simulation to finish
4. Take note of the simulation time (measured between the instructions '**`tic;`**' and '**`toc;`**' in the file 'c_core.m') and observe the quality of the plots.
5. Repeat all 4 previous steps for 500, 1000, 1500 and 2000 nodes/meter.
6. Plot the simulation time against the node density. Can you work-out an expression for the computing power demanded as a function of the size of the problem? Comment also on the quality of the plots as a function of the node density.

IMPORTANT: It is especially important for this section of the experiment not to distribute the computing power of your PC among several tasks (i.e. do not write your report or read your e-mail while MATLAB is iterating)

4.4 Simulating a non-ideal core

In the case of a non-ideal-core, the air and iron regions are solved together. Only the first and the last rows of nodes in the iron are set to an initial condition and kept in that condition throughout the simulation (i.e., only the first and last rows are kept at a fixed potential). The rest of the mesh is fully processed taking into account the material interfaces. As you will realize, the solution will take a bit longer since we have increased both the size of the problem and its complexity.

1. Open the file 'define_problem.m' and change the following:
   - Set the flag-type variable '**`IdealCore`**' to 0.
   - Set '**`NumIterations`**' to 500
   - Choose a '**`NodeDensity`**' of 1000
2. Open the file 'c_core.m' and comment out the function **`plot_results(...)`** by typing a '`%`' in front of it.
3. Start a fresh simulation by executing from the MATLAB command line:
   ```
   close all;
   clear all;
   ```
4. Re-start the simulation:
   ```
   c_core;
   ```
5. Wait for the simulation to finish and write down the **Inductance value** and the **Error** reported by the program. What does this value of error refer to? (Open file 'solve_whole_core.m' to find out).
6. Repeat steps 4 and 5 to complete 2000 iterations. DO NOT close any window (as you will need them all) or clear any variable (so that the previous solution is left in place as a starting point for the next set of iterations).
7. Uncomment the function:
   `plot_results(...)` in 'c_core.m'
8. Re-start the simulation:
   ```
   c_core;
   ```

At the end of this last simulation, you should have a set of values for Inductance, Error and V_m plots for 2500 iterations in steps of 500. Additionally, you should have a full set of plots for the last iteration.

- Plot the value of inductance and Error against the number of iterations. Describe how they change.
- Observe how the magnetic potential distribution has changed from one simulation to the other. At this stage you should be convinced that the number of iterations does change the accuracy of the simulation.
- Print some plots of the magnetic potential distribution to complement your report.
- Consider the table in the appendix again to explore the results obtained in this section

5. Conclusions

You should be able to discuss the following.

1. Compare your experimental results of flux density (B) with those from the numerical solution.
2. Appreciate the relationship between accuracy, iteration number, solution time and node density
3. You have found some values for the core inductance and flux density by three different approaches: hand calculation, experimental measurement and numerical solution. Discuss any discrepancy that you may have found between the results obtained by these different methods.

6. Extra activities for brave students

1. Iteration Error
   - Is the formula to calculate the iteration error suitable for a case where V_m may take a value of zero (such as in the case of an AC power supply)?
   - Is this definition for 'Error' unique? Can you think of any other?
   - The program here uses a fixed number of iteration. How could you use the error definition to stop the program after a certain accuracy had been reached?
2. Media with different permeabilities
   - With a grid involving surfaces with different permeabilities, the finite-

difference molecule may be set as shown in Figure 6.

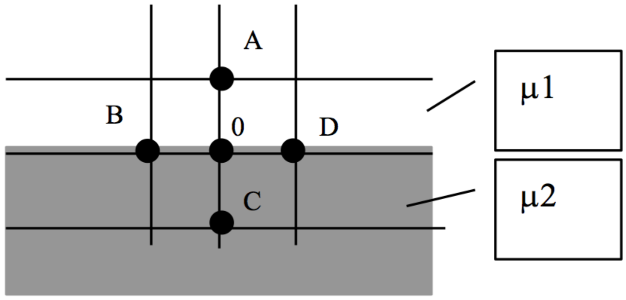

Figure 6 – Finite Difference molecule model

The potential at node 0 equation for this problem may be written as:

$$V_0 = \left(\frac{V_A}{2}\frac{\mu_1}{\mu_1+\mu_2} + \frac{V_B}{4} + \frac{V_C}{2}\frac{\mu_2}{\mu_1+\mu_2} + \frac{V_D}{4}\right)$$

Derive the expression for the case where there are 4 materials (all with different permeabilities) involved. The appropriate expression is contained in the solution package but it would be SUCH A SATISFACTION for you if you find it by yourself without looking at the answer first.

References and Bibliography to Probe Further

Binns, K., & Lawrenson, P. (1973). *Analysis of Computation of Electric and Magnetic Problems.* Pergamon.

Sadiku, M. (1992). *Numerical Techniques in Electromagnetics.* CRC Press Inc.

Appendix

Plot	Some features to observe
Physical problem mesh	The mesh of nodes in the core in Figure 5 (in particular note the number of nodes assigned to the air-gap according to the node density declared)
Scalar magnetic potential V_m	The magnetic potential distribution in the core and air-gap at the start and end of every simulation
Equipotential lines	Equipotential lines in the air-gap. Recall at this point that the magnetic flux lines will be perpendicular to the equipotential contours
Directional vectors for H	How the magnetic field behaves in the air-gap. Fringing effects. Zoom in and out to observe a particular area.
Magnitude of the Magnetic field strength	The magnitude of the magnetic field strength (H) in the core and air-gap. Take a moment to think of a graphical explanation for the expression: $H = -gradient(V_m)$. Acquaint yourself with the relationship between the plot of 'Scalar magnetic potential' and this plot.
Magnetic flux density in the middle of the air-gap	The magnetic flux density distribution in the middle of the air-gap (where the magnetic flux density distribution probe slides in the experimental rig)