Ejercicios
Raíces

1) a) $2i$

$z = \sqrt{2i}$

$(a+ib)^2 = 2i$

$a^2 + 2iab - b^2 = 2i + 0$

$\hookrightarrow 2ab = 2 \Rightarrow ab = 1 \Rightarrow a = 1/b$

$\hookrightarrow a^2 - b^2 = 0$

$\frac{1}{b^2} - b^2 = 0$

$\frac{1}{b^2} = b^2$

$1 = b^2 \cdot b^2$

$1 = b \qquad a = \frac{1}{1} = 1$

Como $a = 1$ y $b = 1$ entonces $\sqrt{2i} = \pm(1 + i)$

b) $1 - \sqrt{3}i$

$(a+ib)^2 = 1 - \sqrt{3}i$

$a^2 + 2iab - b^2 = 1 - \sqrt{3}i$

$\hookrightarrow a^2 - b^2 = 1 \Rightarrow a^2 = 1 + b^2$

$\hookrightarrow 2ab = -\sqrt{3}$

$4a^2b^2 = 3$

$4(1+b^2)b^2 = 3$

$b^2 + b^4 = 3/4$

$b^2 + b^4 - 3/4 = 0$

llamando a $t = b^2$ se tiene $t^2 + t - 3/4 = 0$

$b = \frac{\sqrt{2}}{2}$

$$a^2 = 1 + \left(\frac{\sqrt{2}}{2}\right)^2$$

$$a^2 = 1 + \frac{2}{4}$$

$$4a^2 = 4 + 2$$

$$a^2 = 6/4$$

$$a = \sqrt{\frac{3}{2}}$$

- $\frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{2}}{2} i = \frac{2\sqrt{3} + 2i}{2\sqrt{2}} = \frac{2(\sqrt{3} + i)}{2\sqrt{2}} = \frac{\sqrt{3} + i}{\sqrt{2}}$

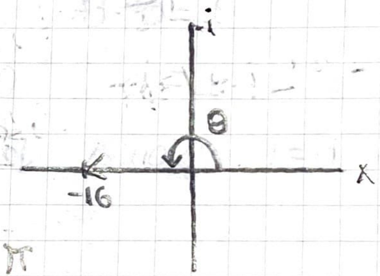
Como $a = \sqrt{\frac{3}{2}}$ y $b = \frac{\sqrt{2}}{2}$ entonces $\sqrt{1 - \sqrt{3}\, i} = \pm \frac{\sqrt{3} + i}{\sqrt{2}}$

2) $(-16)^{1/4}$

$n = 4$

$r = \sqrt{0 + (-16)^2} = 16$

$\theta = \arctan\left(\frac{0}{-16}\right) = 0 + 180^\circ = \pi$



$z_0 = 2\left[\cos\left(\frac{\pi + 0}{4}\right) + i \operatorname{Sen}\left(\frac{\pi + 0}{4}\right)\right] = \sqrt{2} + i\sqrt{2} = \sqrt{2}(1 + i)$

$z_1 = 2\left[\cos\left(\frac{3\pi}{4}\right) + i \operatorname{Sen}\left(\frac{3\pi}{4}\right)\right] = -\sqrt{2} + i\sqrt{2} = -\sqrt{2}(1 - i)$

$z_4 = 2\left[\cos\left(\frac{9\pi}{4}\right) + i \operatorname{Sen}\left(\frac{9\pi}{4}\right)\right] = \sqrt{2} + \sqrt{2} = \sqrt{2}(1 + i)$

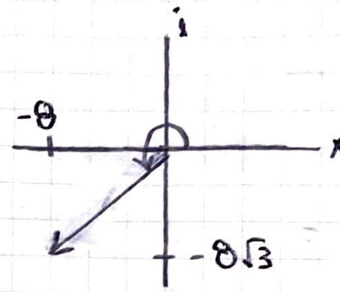
- $\pm \sqrt{2}(1 + i)$ • Principal $= \sqrt{2}(1 + i)$
- $\pm \sqrt{2}(1 - i)$

b) $(-8-8\sqrt{3}i)^{1/4}$

$n=4$

$r=\sqrt{(-8)^2+(-8\sqrt{3})}=16$

$\theta=\arctan\left(\frac{-8\sqrt{3}}{-8}\right)=\frac{1}{3}\pi+\pi=\frac{4}{3}\pi$



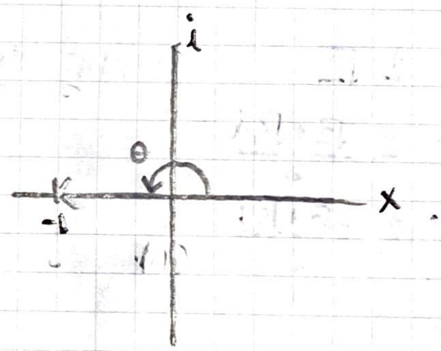
$z_0=2\left[\text{Cos}\left(\frac{\pi}{3}\right)+i\,\text{Sen}\left(\frac{\pi}{3}\right)\right]=1+\sqrt{3}i$

$z_1=2\left[\text{Cos}\left(\frac{5\pi}{6}\right)+i\,\text{Sen}\left(\frac{5\pi}{6}\right)\right]=-\sqrt{3}+i=-(\sqrt{3}-i)$

$z_4=2\left[\text{Cos}\left(\frac{7\pi}{3}\right)+i\,\text{Sen}\left(\frac{7\pi}{3}\right)\right]=1+\sqrt{3}i$

- $\pm(1+\sqrt{3}i)$ • Principal $=1+\sqrt{3}i$
- $\pm(\sqrt{3}-i)$

3) $(-1)^{1/3}$



$n=3$

$r=\sqrt{1^2+0}=1$

$\theta=\arctan\left(\frac{0}{1}\right)=0^\circ=\pi$

$z_0=\left[\text{Cos}\left(\frac{\pi}{3}\right)+i\,\text{Sen}\left(\frac{\pi}{3}\right)\right]=\frac{1+i\sqrt{3}}{2}$

$z_1=\left[\text{Cos}(\pi)+i\,\text{Sen}(\pi)\right]=-1$

$z_2=\left[\text{Cos}\left(\frac{5\pi}{3}\right)+i\,\text{Sen}\left(\frac{5\pi}{3}\right)\right]=\frac{1}{2}-\frac{\sqrt{3}}{2}i=\frac{1-\sqrt{3}i}{2}$

$z_3=\left[\text{Cos}\left(\frac{7\pi}{3}\right)+i\,\text{Sen}\left(\frac{7\pi}{3}\right)\right]=\frac{1}{2}+\frac{\sqrt{3}}{2}i=\frac{1+i\sqrt{3}}{2}$

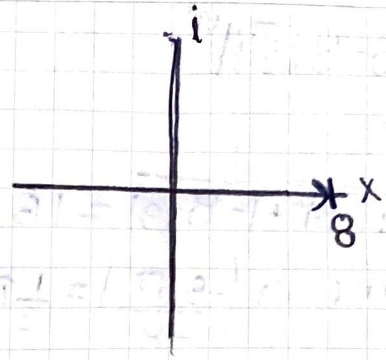
- $\frac{1+i\sqrt{3}}{2}$ • Principal: $\frac{1+i\sqrt{3}}{2}$
- $\frac{1-i\sqrt{3}}{2}$
- -1

b) $8^{1/6}$

$n=6$

$r = \sqrt{8^2+0} = 8$

$\theta = \arctan\left(\frac{0}{8}\right) = 0^\circ = 0$



$z_0 = 8^{1/6}[\cos(0)+i\,\text{Sen}(0)] = 8^{1/6} = \sqrt{2}$

$z_1 = 8^{1/6}\left[\cos\left(\frac{2\pi}{6}\right) + i\,\text{Sen}\left(\frac{2\pi}{6}\right)\right] = \sqrt{2}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}+\sqrt{6}\,i}{2}$

$z_2 = 8^{1/6}\left[\cos\left(\frac{4\pi}{6}\right) + i\,\text{Sen}\left(\frac{4\pi}{6}\right)\right] = \sqrt{2}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{-\sqrt{2}+\sqrt{6}\,i}{2}$

$z_6 = 8^{1/6}\left[\cos\left(\frac{12\pi}{6}\right) + i\,\text{Sen}\left(\frac{12\pi}{6}\right)\right] = \sqrt{2}$

- $\pm\sqrt{2}$
- $\pm \frac{\sqrt{2}+\sqrt{6}\,i}{2}$
- $\pm \frac{-\sqrt{2}+\sqrt{6}\,i}{2}$

- Principal $= \sqrt{2}$