1) Determine la proyección estereográfica

a. $i \rightarrow 0+1i$

- $x_1 = \dfrac{2x}{(x^2+y^2)+1} = \dfrac{2\cdot 0}{0^2+1^2+1} = 0$
- $x_2 = \dfrac{2y}{(x^2+y^2)+1} = \dfrac{2\cdot 1}{(0^2+1^2)+1} = \dfrac{2}{2} = 1$
- $x_3 = \dfrac{(x^2+y^2)-1}{(x^2+y^2)+1} = \dfrac{1-1}{1+1} = 0$

La proyección estereográfica es: $(0,1,0)$

b. $6-8i$

- $x_1 = \dfrac{2\cdot 6}{(6^2+(-8)^2)+1} = \dfrac{12}{101}$
- $x_2 = \dfrac{2(-8)}{(6^2+(-8)^2)+1} = -\dfrac{16}{101}$
- $x_3 = \dfrac{(6^2+(-8)^2)-1}{(6^2+(-8)^2)+1} = \dfrac{99}{101}$

La proyección es: $\left(\dfrac{12}{101}, \dfrac{-16}{101}, \dfrac{99}{101}\right)$

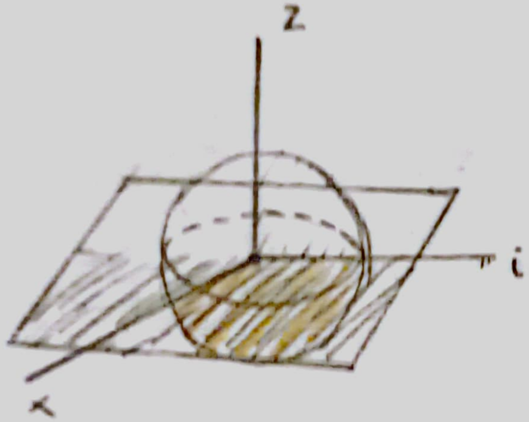
c. $-\dfrac{3}{10} + \dfrac{2}{5}i$

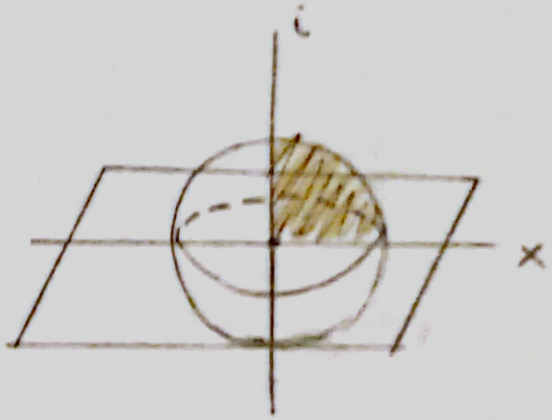
- $x_1 = \dfrac{2\cdot\left(\frac{-3}{10}\right)}{\left(\left(\frac{-3}{10}\right)^2+\left(\frac{2}{5}\right)^2\right)+1} = \dfrac{-\frac{3}{5}}{\frac{5}{4}} = -\dfrac{12}{25}$
- $x_2 = \dfrac{2\cdot\left(\frac{2}{5}\right)}{\left(\left(\frac{-3}{10}\right)^2+\left(\frac{2}{5}\right)^2\right)+1} = \dfrac{\frac{4}{5}}{\frac{5}{4}} = \dfrac{16}{25}$
- $x_3 = \dfrac{\left(\left(\frac{-3}{10}\right)^2+\left(\frac{2}{5}\right)^2\right)-1}{\left(\left(\frac{-3}{10}\right)^2+\left(\frac{2}{5}\right)^2\right)+1} = \dfrac{-\frac{3}{4}}{\frac{5}{4}} = -\dfrac{3}{5}$

La proyección es: $\left(-\dfrac{12}{25}, \dfrac{16}{25}, -\dfrac{3}{5}\right)$

2) a. $\{z : \operatorname{Re} z > 0\}$

x_1 positivo





b. $\{z : |z| < 1/2\}$

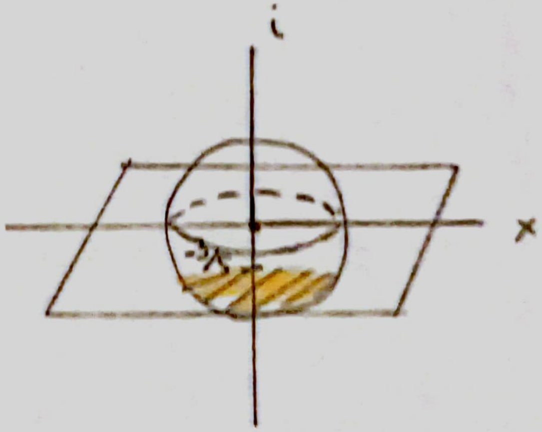
$\sqrt{x^2+y^2} < \frac{1}{2}$

$x^2+y^2 < \frac{1}{4}$

tenemos $x^2+y^2 = \frac{1}{4}$ en x_2

$$x_3 = \frac{(x^2+y^2)-1}{(x^2+y^2)+1} = \frac{\frac{1}{4}-1}{\frac{1}{4}+1} = -\frac{3}{5}$$

Luego $x_3 < -\frac{3}{5}$



c. $\{z : |z| > 3\}$

$\sqrt{x^2+y^2} > 3$

$x^2+y^2 > 9$

$x_3 = \frac{9-1}{9+1} = \frac{4}{5}$

$x_3 > \frac{4}{5}$