Laboratory Session : April 27, 2023
Exercises due on : May 14, 2023

Exercise 1

- the number of claims received by an insurance company during a week follows a Poisson distribution with unknown mean (μ)
- the number of claims, per week, observed ovr a ten week period are:

  `5, 8, 4, 6, 11, 6, 6, 5, 6, 4`

(a) suppose to use a prior uniform distribution for μ

- find the posterior distribution for μ and compute the posterior mean, median and variance
- plot the posterior distribution and the 95% credibility interval

(b) suppose to use a Jeffreys' prior for μ ($g(\mu) \propto 1/sqrt\mu$)

- find the posterior distribution for μ and compute the posterior mean, median and variance
- plot the posterior distribution and the 95% credibility interval

(c) evaluate a 95% credibility interval for the results obtained with both priors. Compare the result with that obtained using a normal approximation for the posterior distribution, with the same mean and standard deviation

Exercise 2

- a well established and diffused method for detecting a disease in blood fails to detect the presence of disease in 15% of the patients that actually have the disease.
- A young UniPD startUp has developed an innovative method of screening. During the qualification phase, a random sample of $n = 75$ patients known to have the disease is screened using the new method.

(a) what is the probability distribution of y, the number of times the new method fails to detect the disease ?

(b) on the $n = 75$ patients sample, the new method fails to detect the disease in $y = 6$ cases. What is the frequentist estimator of the failure probability of the new method ?

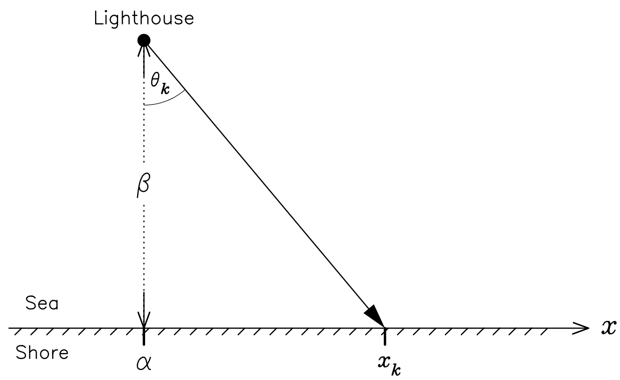
(c) setup a bayesian computation of the posterior probability, assuming a beta distribution with mean value 0.15 and standard deviation 0.14. Plot the posterior distribution for y, and mark on the plot the mean value and variance

(d) Perform a test of hypothesis assuming that if the probability of failing to the detect the desease in ill patients is greater or equal than 15%, the new test is no better that the traditional method. Test the sample at a 5% level of significance in the Bayesian way.

(e) Perform the same hypothesis test in the classical frequentist way.

Exercise 2

- given the problem of the lightouse discussed last week, study the case in which both the position along the shore (α) and the distance out at sea (β) are unknown



Exercise 3

- given the Signal over Background example discussed last week, analyze and discuss the following cases:

(a) vary the sampling resolution of used to generate the data, keeping the same sampling range

```
xdat <- seq(from=-7*w, to=7*w, by=0.5*w)
```

- change the resolution $w = \{0.1, 0.25, 1, 2, 3\}$
- Check the effect on the results

(b) change the ratio A/B used to simulate the data (keeping both positive in accordance with the prior)

- Check the effect on the results