P01 - Learning the distribution of gravitational waves sources

Consider a population of sources emitting gravitational waves with n, the density of the population. Let's assume that n is a low density so that the number of sources, even in large volumes, remains relatively small. For the exercise, we ignore cosmological effects (let's consider redshift $z \ll 1$), and we assume that the position of the sources are statistically independent. Let's build a statistical model of the pupulation:

1. given a spherical shell with radius R and thickness ΔR centered on the Sun, what is the probability distribution of the number of sources in our shell ?

2. and, according to that probability distribution, what is the average number and variance of the sources in the shell ?

3. with increasing distance from the Sun, the detection efficiency of the number of sources is decreasing; let's suppose to characterize the amplitude of the gravitational radiation through the maximum strain, h, and we also assume we can neglet the difference on polarization and orientation of the sources. With this assumptions, $h \propto 1/r$.

4. let's assume that the detection efficiency is a sigmoid function that can be approximated with the following Gaussian integral

$$\epsilon(h) = \int_{-\infty}^{h} \frac{\exp\left[-(h' - h_\circ)^2/(2w^2)\right]}{\sqrt{2\pi w^2}} dh'$$

   with $h_\circ$ the strain produced by a source located at distance $r_\circ$ corresponding to a detection efficiency $\epsilon(h_\circ) = 0.5$.

Perform an analysis of the detection efficiency and on its uncertainty as a function of the source density n.