Laboratory Session : April 6, 2023

Exercises due on: April 23, 2023

Exercise 1 - Discrete random variable

• the probability distribution function of a discrete variable k is given by the following:

$$p(k) = \begin{cases} \frac{k}{15} & \text{with } k = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

- 1) write the R probability functions for the probability density and cumulative distribution functions, following the R naming convention
- 2 produce two plots showing the pdf and cdf, separately
- 3) compute, using R, the mean value and variance of the probability distribution
- 4) compute, using R, the expected value E[k(6-k)]
- 5) write the R function that allows to sample random numbers from the probability distribution
- 6) using the implemented function (point (5)), sample 10⁵ random numbers from this distribution and plot them in a graph showing the distribution of the numbers superimposed to the pdf (normalize properly the plots with random numbers)

Exercise 2 - Continuous random variable

• The triangular distribution, in the interval (a, b), is given by the following:

$$f(X) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & a \le x < c \\ \frac{2(b-x)}{(b-a)(b-c)} & c \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

where $c \in [a, b]$.

- a) plot the function, given the interval (a, b)
- b) and write an algorithm to generate random numbers from the triangular distribution
- c) generate 10^4 random number from the distribution, show them in an histogram and superimpose the analytical curve

Exercise 3

- \bullet the waiting time, in minutes, at the doctor's is about 30 minutes, and the distribution follows an exponential pdf with rate 1/30
- A) simulate the waiting time for 60 people at the doctor's office and plot the relative histogram
- B) what is the probability that a person will wait for less than 12 minutes?
- C) evaluate the average waiting time from the simulated data and compare it with the expected value (calculated from theory and by manipulating the probability distributions using R)
- B) what is the probability for waiting more than one hour before being received?

Exercise 4 - Multiple choices exams

- the final exam of a course is given to the students in the format of a multiple choice written test: for each questions there are five possible alternatives
- a student either knows the answer, or selects randomly the answer among the five possible choices
- assuming p = 0.7 the probability that the student knows the answer, once a correct answer is given, what it the probability that the student really knew the correct answer?
- write a small R program to provide the answer

Exercise 5 - Waiting time

- starting from 5:00 in the morning, every half an hour there is a train from Milano Centrale to Roma Termini. We assume there is always an available seat on a train leaving from Milano
- assuming a person arrives at a random time between 10:45 and 11:45 and compute the probability that she has to wait
- a) at most 10 minutes
- b) at least 15 minutes
- c) what is the average time spent waiting?

Exercise 6 - stock investment

- \bullet the annual return rate for a specific stock on the market is a normal variable with a 10% mean and a 12% standard deviation
- Mr X decides to buy 200 share of that specific stock at a price of 85€ per share
- what is the probability that after a year his net profit from the investment is at least 800€?
- (suppose there are no transaction costs and that there are no annual dividend for the stock)

References

[1] https://cran.r-project.org/web/packages/nycflights13/nycflights13.pdf