PROJECT FOR EVALUATION # 1

Pricing Financial Derivatives

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The total grade of the 3 projects for evaluation will count as 30% of the final grade (10% each project). The projects have to be done in groups of 2-4 students. They have to be submitted in box as a unique pdf file. The code needs to be included. The answers need to be justified.

Deadline for submission : Monday 27th February 2017 at 11h a.m.

Exercise 1 : Black-Scholes model

Consider the Black-Scholes model :

$$dX_t = 2X_t dt + 1.5X_t dB_t, \qquad X_0 = 1$$

Write its solution. Generate on an interval a Brownian path. Using that path plot a trajectory of the solution. Using the same Brownian path use the Euler scheme approximation to plot another trajectory of the equation. Compare these plots with the simulation of the exact solution.

Exercise 2 : Vasicek model for interest rates

Consider the SDE

$$dX_t = a\,(m - X_t)\,dt + \sigma dB_t \quad X_0 = x,$$

where $a > 0, \sigma > 0$ and $m \geq 0$. This process tends to drift towards its long-term mean m : such a process is called mean-reverting.

1. Differentiating $e^{at}X_t$, find the solution to this SDE.
2. Compute $E(X_t)$ and $\mathrm{V}ar(X_t)$. Take the limit of the expectation as $t \to \infty$ to observe the mean-reverting phenomena.
3. Using the Euler's scheme on an interval simulate several trajectories of the process for different values of a, σ, m. Do you observe the mean-reverting phenomena ?

Exercise 2 : The Cox-Ingersoll-Ross (CIR) model

The CIR model for the interest rate process $(r_t, t \in [0, T])$ is

$$dr_t = (\alpha - \beta r_t)dt + \sigma\sqrt{r_t}dB_t, \quad r_0 = 0,$$

where α, β and σ are positive constants.

1. Applying Itô's formula to the function $f(t, x) = e^{\beta t}x$, compute the expectation of r_t. Take the limit of the expectation as $t \to \infty$.
2. Using the Euler's scheme on an interval simulate several trajectories of the process for different values of α, β, σ. Take into account that this scheme does not necessarily preserve positivity.