Chapter 4: The Normal Distribution

TXCL7565/PHSC7565

What This Lecture Covers

- What is a normal distribution
- Identifying data that are not normally distributed
- Proportions of individuals within 1 SD or 2 SD of the mean
- Skewness and kurtosis
- Tests for normal distributions

WHAT IS A NORMAL DISTRIBUTION

Normal Distribution

- Many of the things we measure show a characteristic distribution, with the bulk of the data points clustered around the mean and data points become steadily rarer as we move further from the mean.
- When many (independent) random factors contribute to an observed value, the observed values tend to follow a normal distribution.
- Normal Distribution = Gaussian Distribution

Many Random Factors

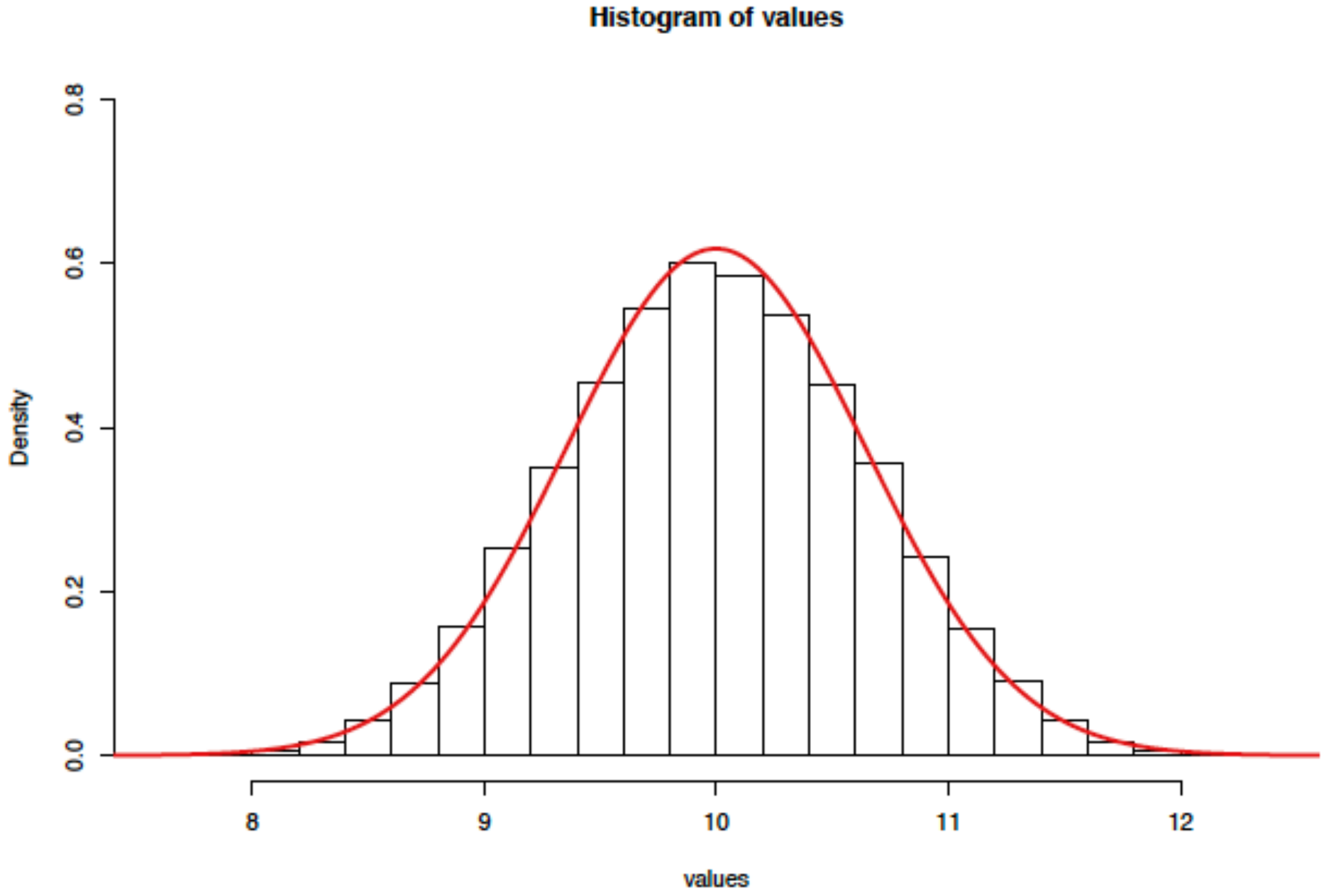
```
true_value = 10
e1 = runif(100000) - 0.5
e2 = runif(100000) - 0.5
e3 = runif(100000) - 0.5
e4 = runif(100000) - 0.5
e5 = runif(100000) - 0.5

values = true_value + e1 + e2 + e3 + e4 + e5
head(round(values,2))
```

```
## [1] 10.31  9.25  9.61  9.20  9.74 10.60
```

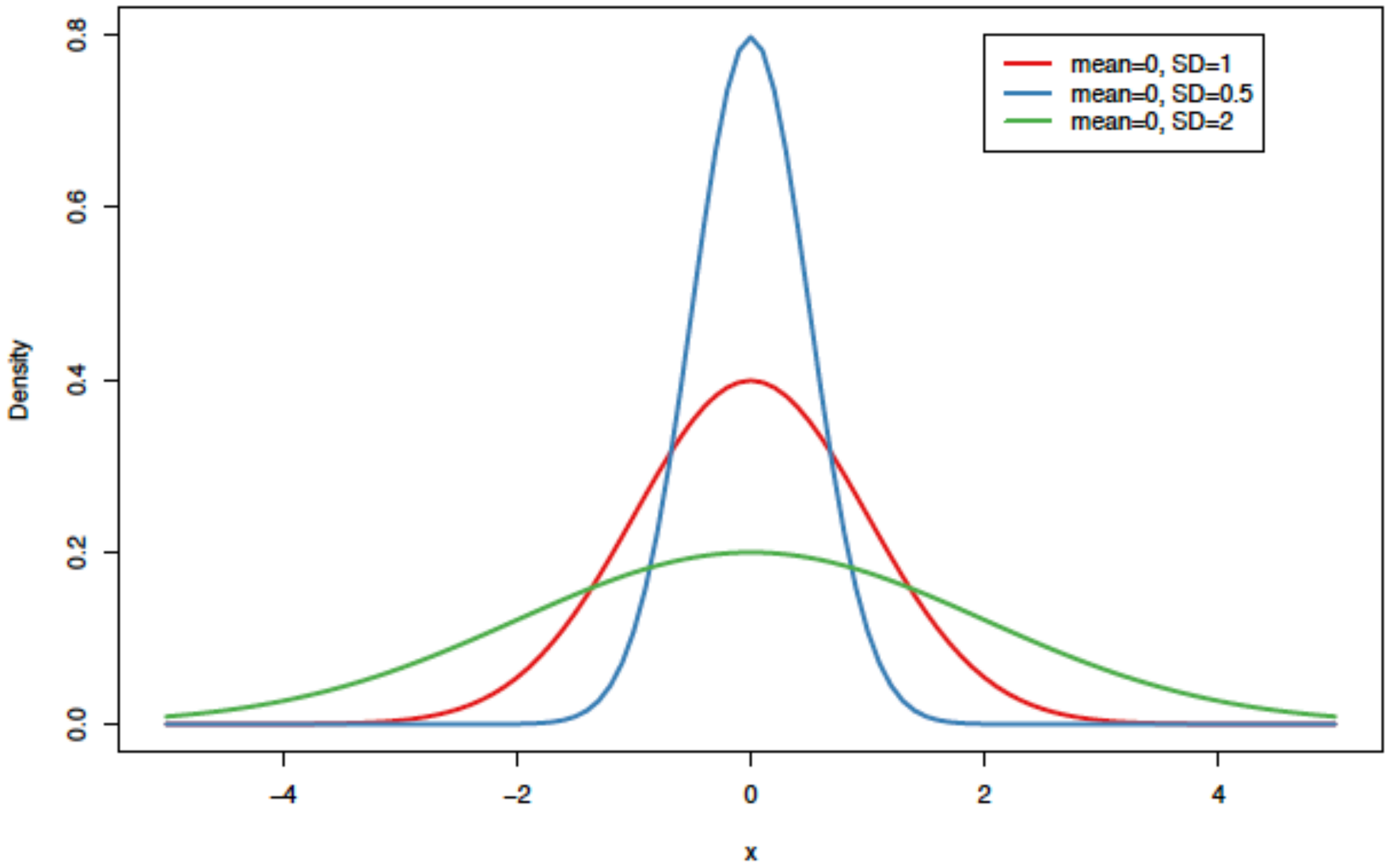
```
tail(round(values,2))
```

```
## [1]  8.57  9.06 10.63  9.87 10.81  9.57
```

Results of many random factors contributing



The Nature of the Normal Distribution



Why Does a Normal Distribution Matter?

Many commonly used statistical tests rely on the assumption that the data have been sampled from a population that follows a normal distribution.

This is often a reasonable assumption.

IDENTIFYING DATA THAT ARE NOT NORMALLY DISTRIBUTED

How to Spot Non-Normal Data

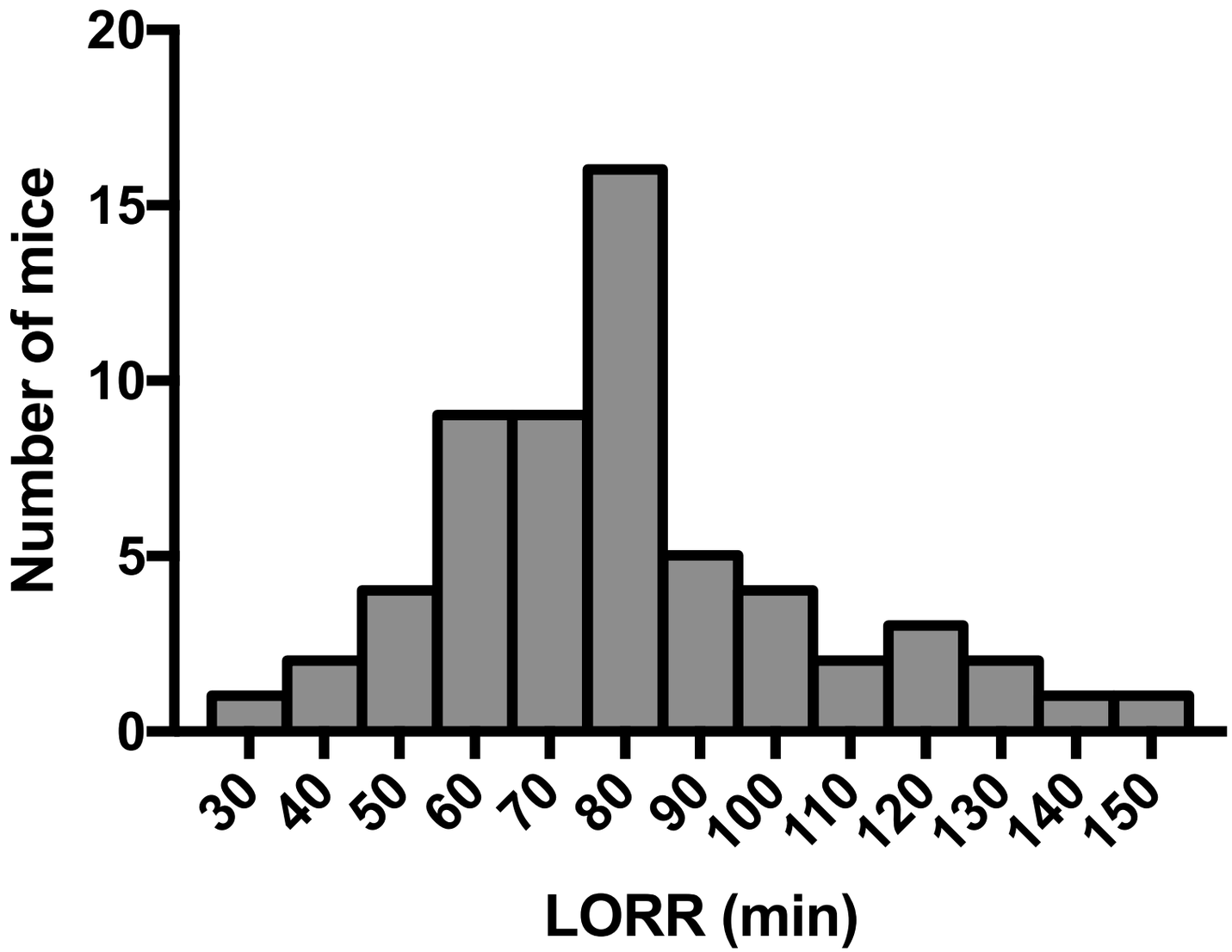
Most often, the easiest/best way to determine if data are normally distributed is to plot them in a histogram

Example Data Set - Sleep Time

One characteristic related to an individual's likelihood of developing an alcohol use disorder is how sensitive they are to alcohol. In the Radcliffe lab, how sensitive a particular mouse is to the sedative effects of alcohol is measured by giving them a large enough dose of alcohol to cause them to 'fall asleep' and then measuring the number of minutes that pass before they wake up.

Sleep time or **Loss of Righting Reflex (LORR)** is the number of minutes between the time point when the mouse first loses the ability to right themselves when placed on their back to the time when they can right themselves again.

Histogram of Sleep Time

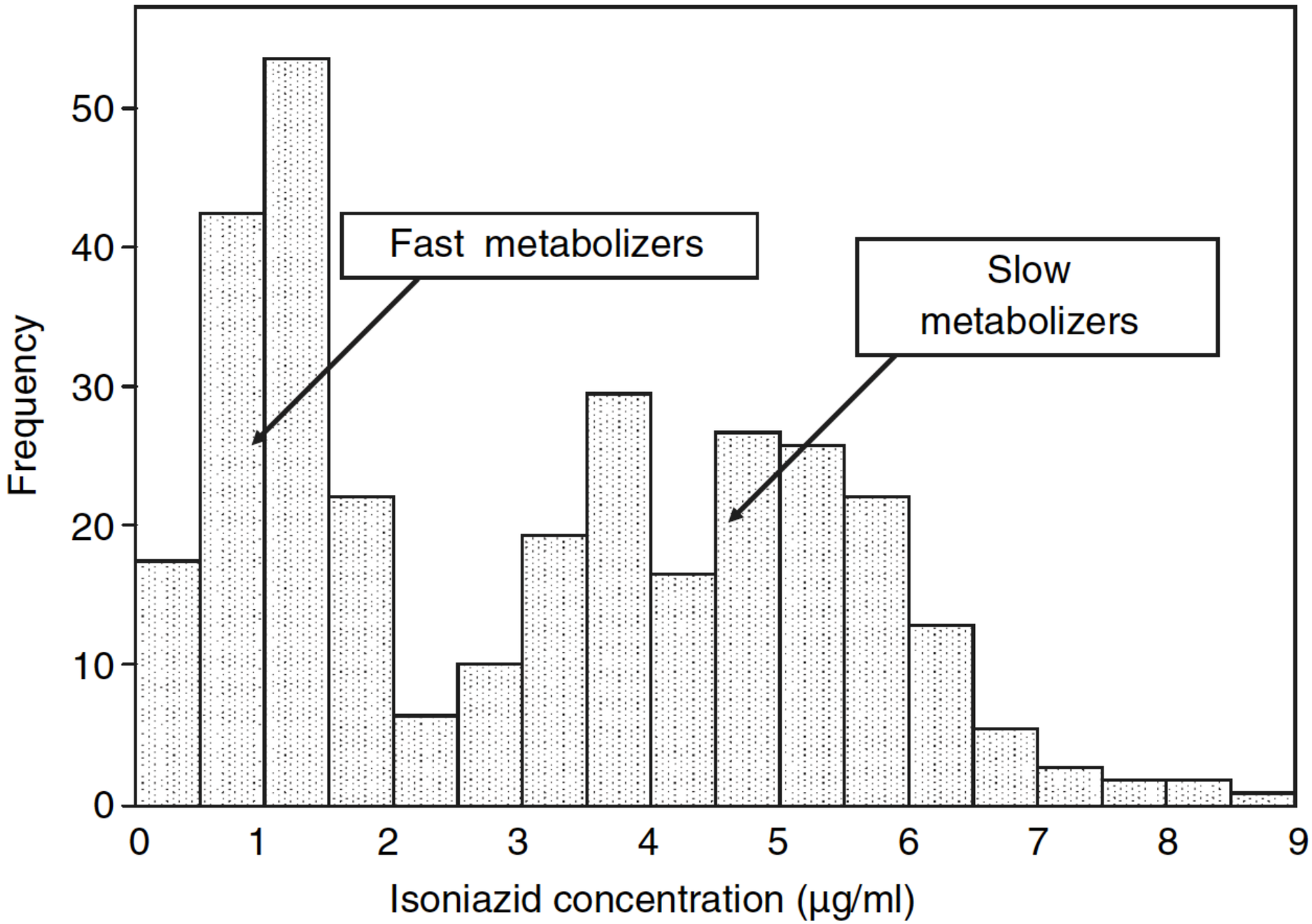


Characteristics of a Normal Distribution

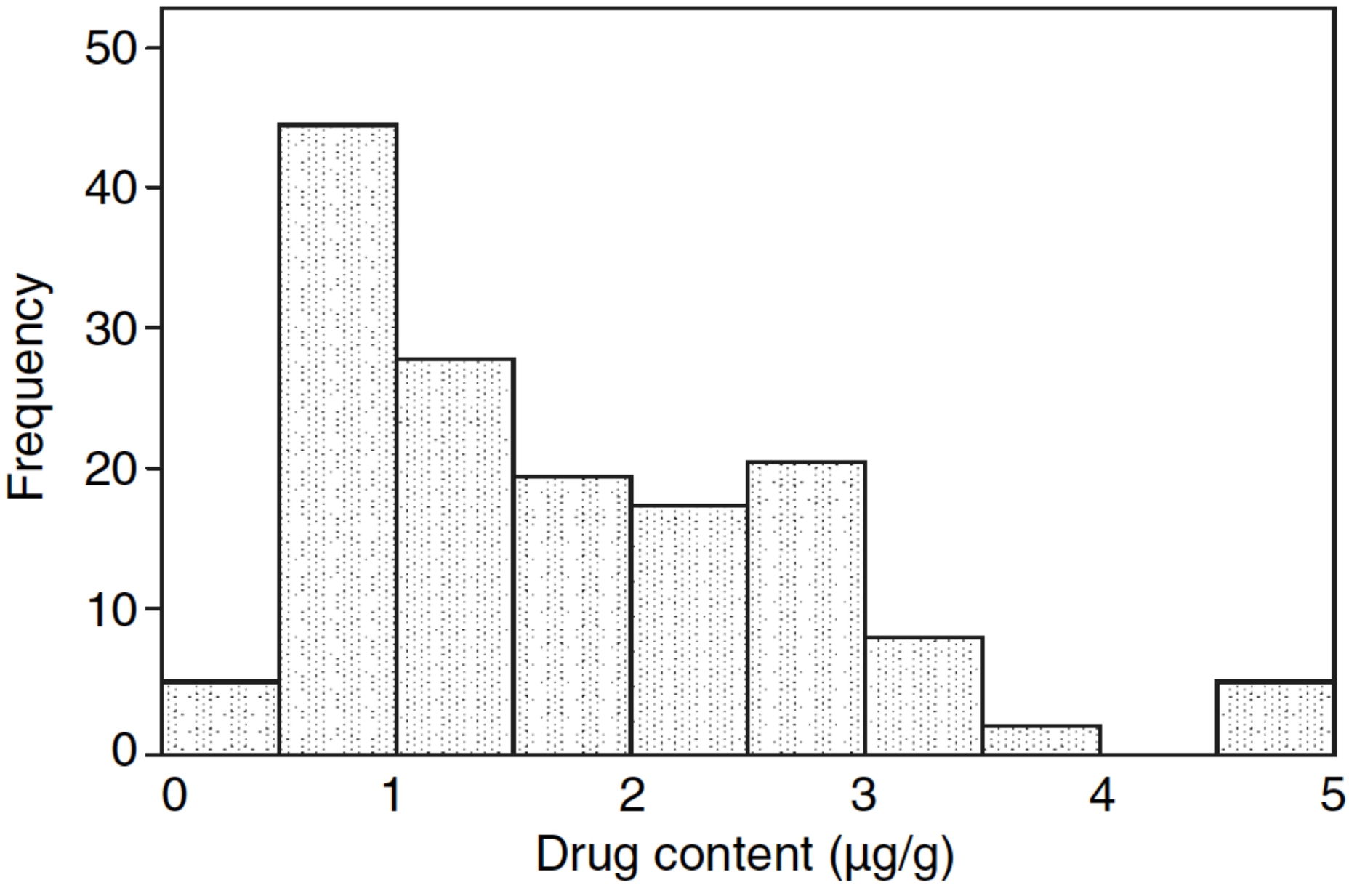
3 visual characteristics that any true normal distribution will possess:

1. The data are unimodal
2. The distribution is symmetrical
3. The frequencies decline steadily as we move towards higher or lower values, without any sudden, sharp cut-off

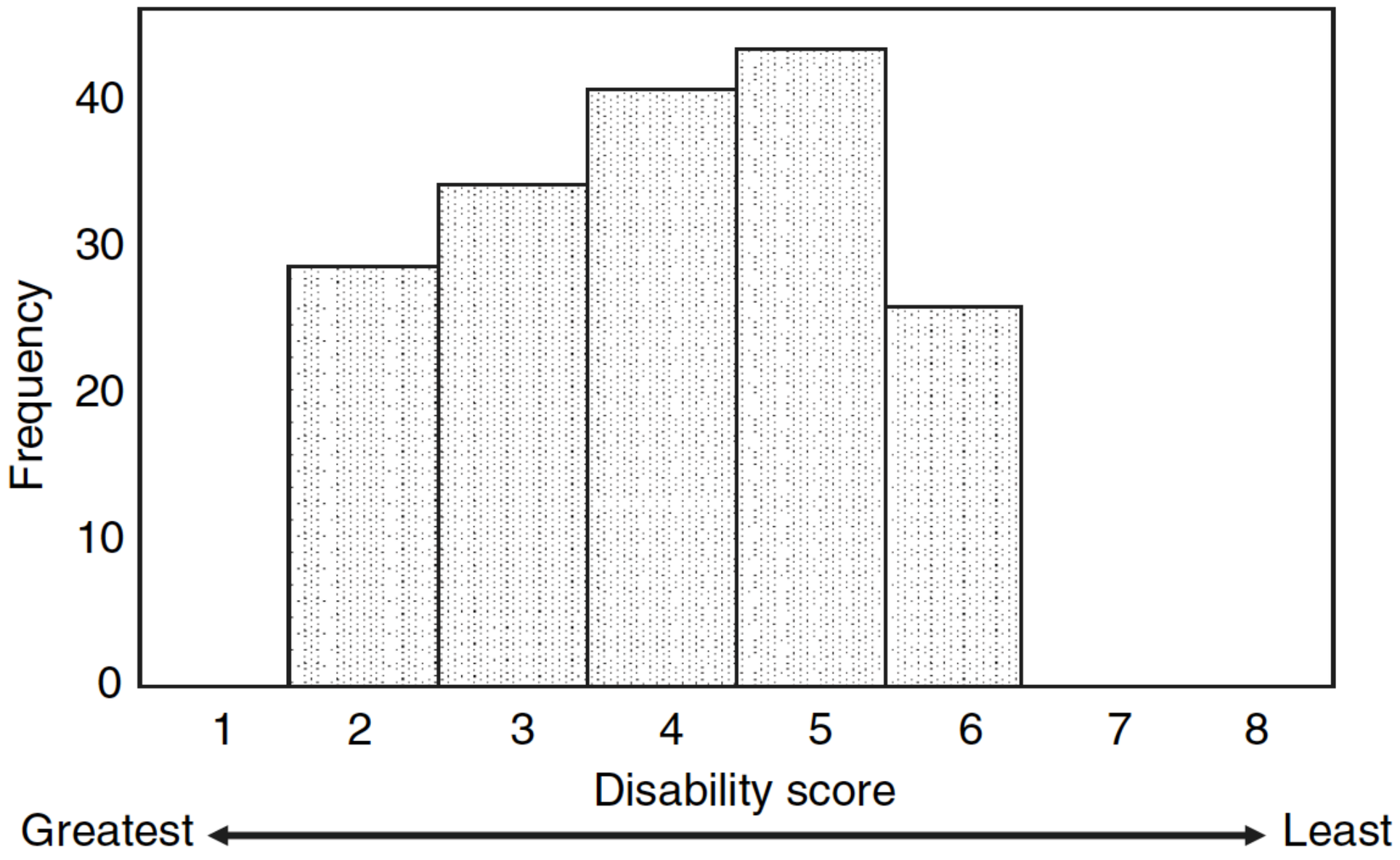
Unimodal vs. Polymodal



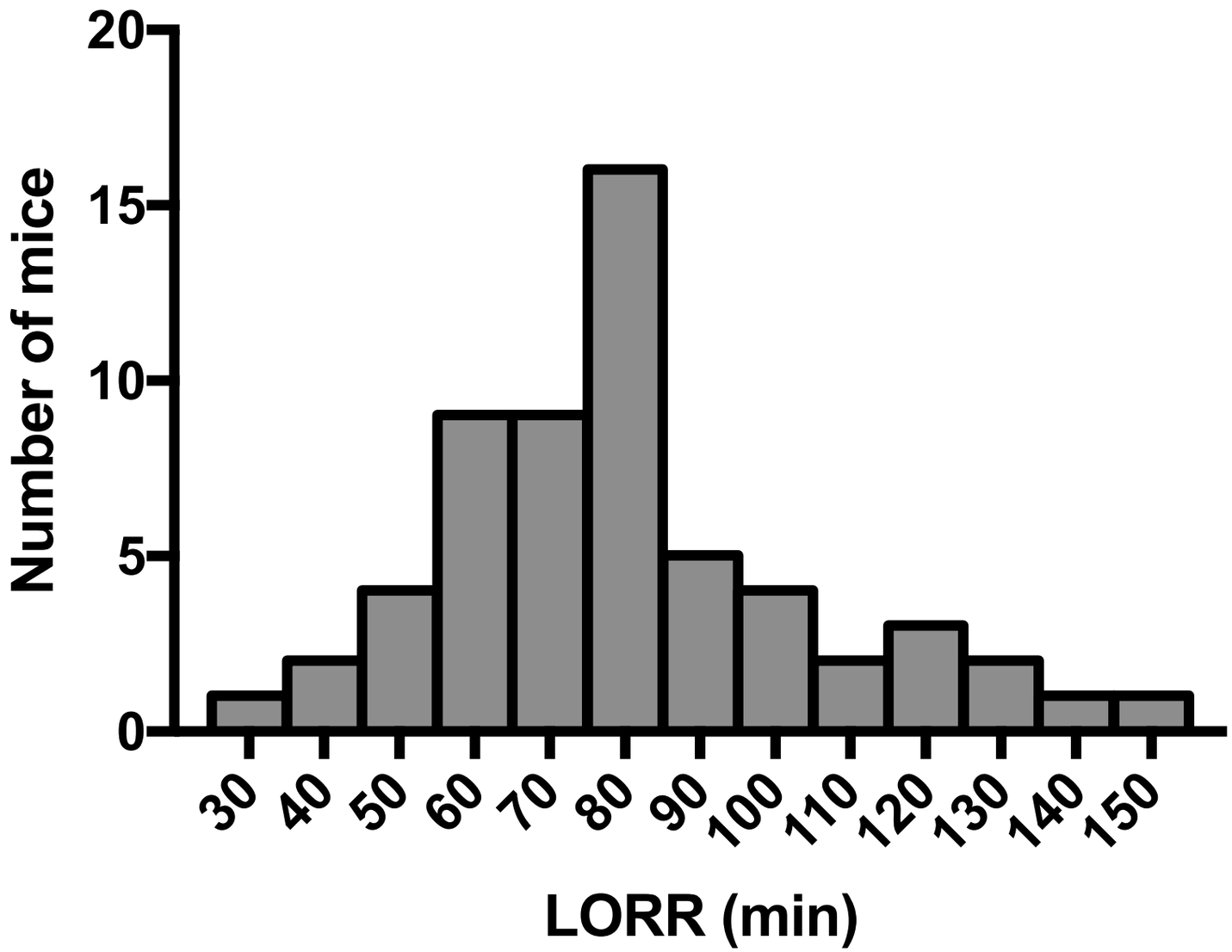
Asymmetrical Distribution



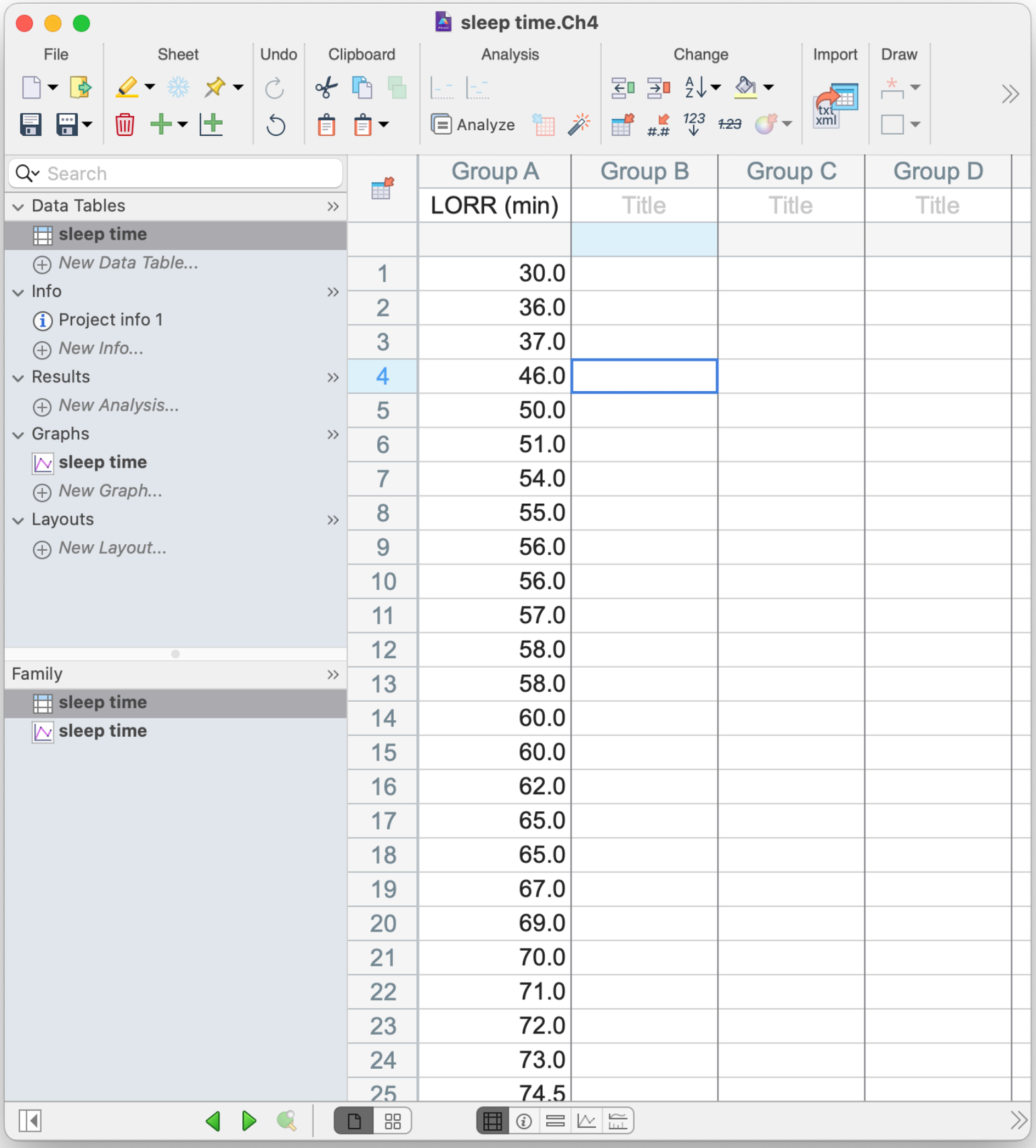
Sharp Cut-Offs



Histogram of Sleep Time



Enter in the LORR data using the ‘Column’ data table version

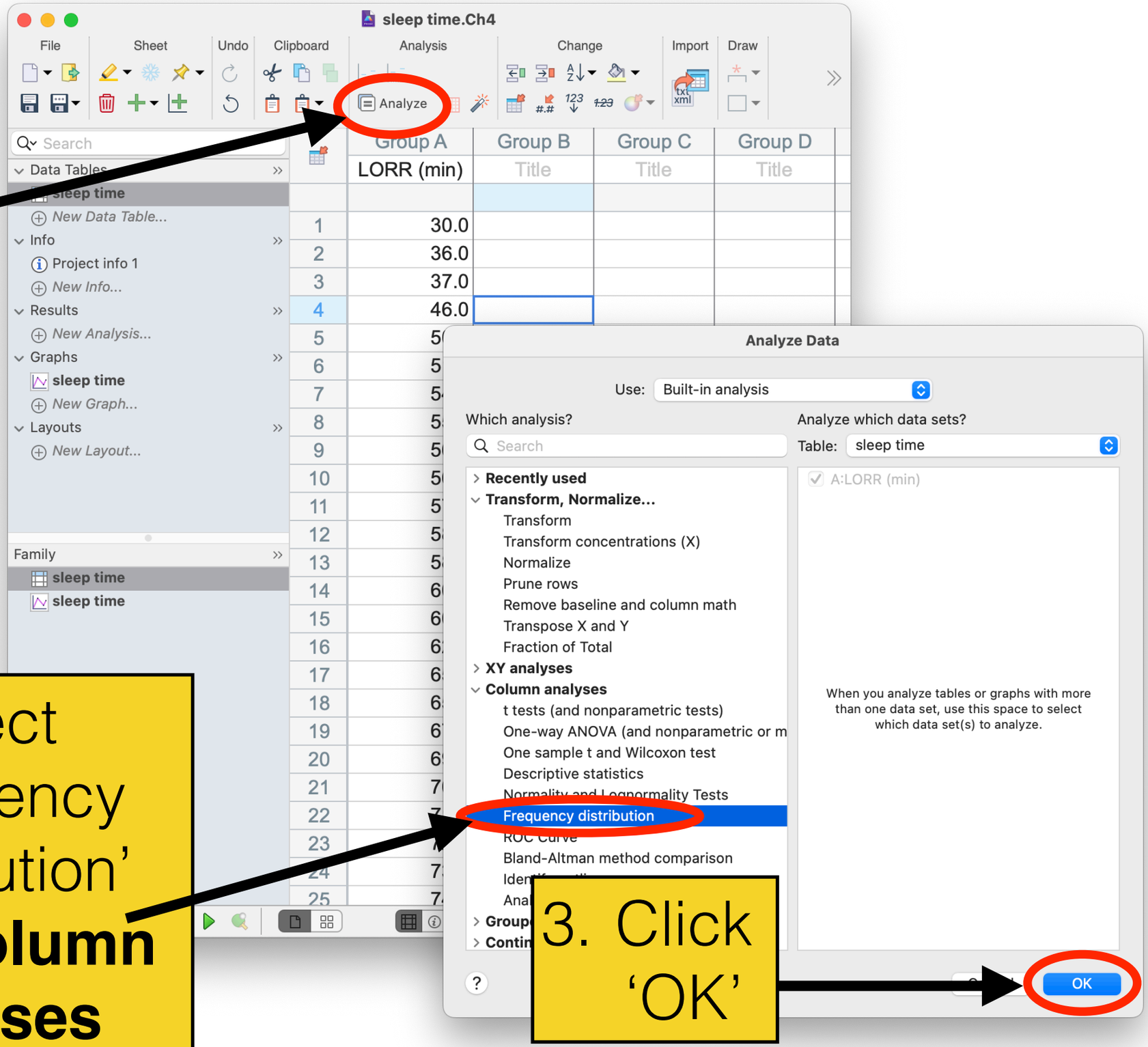


Create a histogram

1. Click the ‘Analyze’ button

2. Select ‘Frequency distribution’ from **Column analyses**

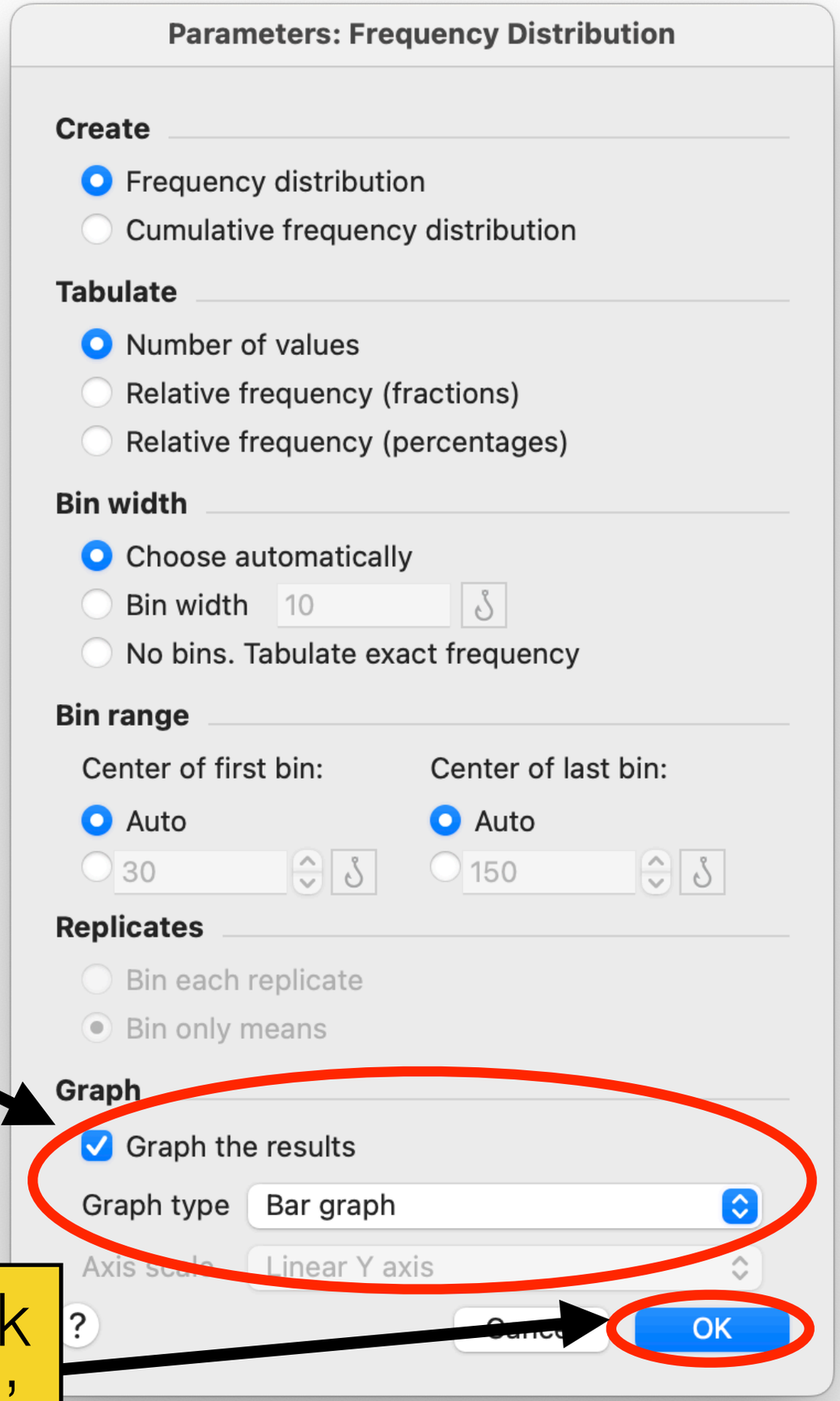
3. Click ‘OK’



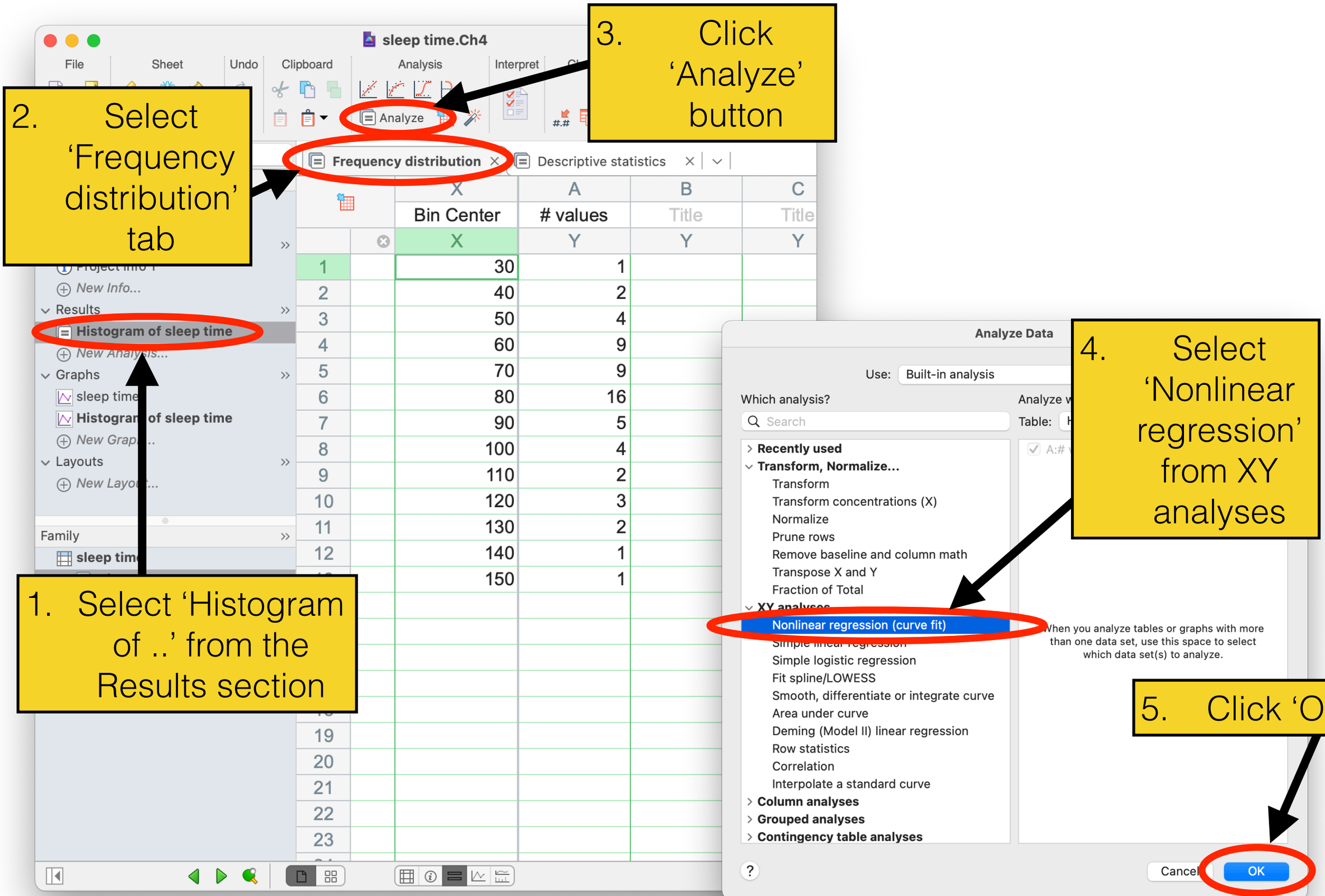
Create a histogram

1. Select ‘Graph the results’ and pick ‘Bar graph’ as the Graph type

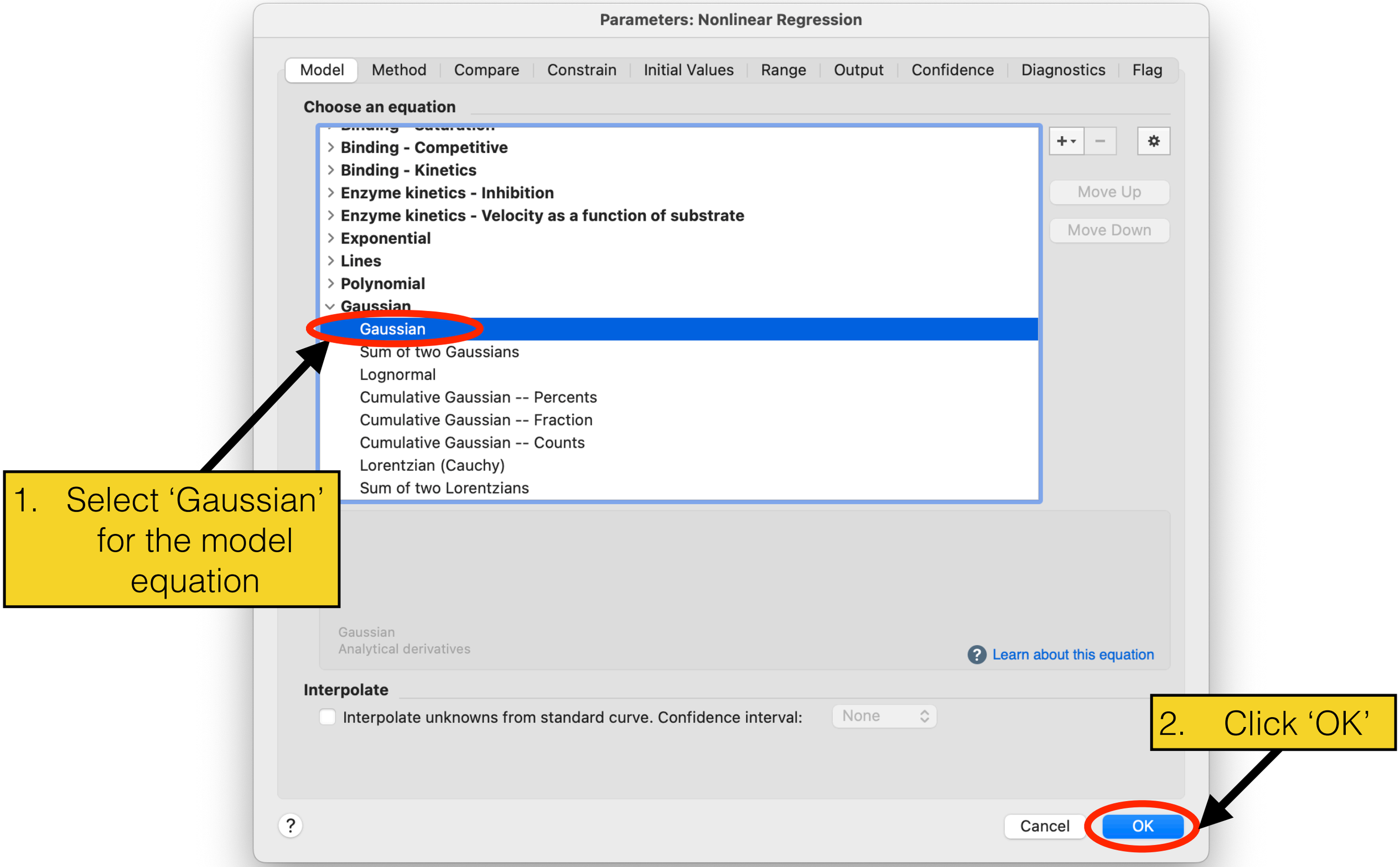
2. Click ‘OK’



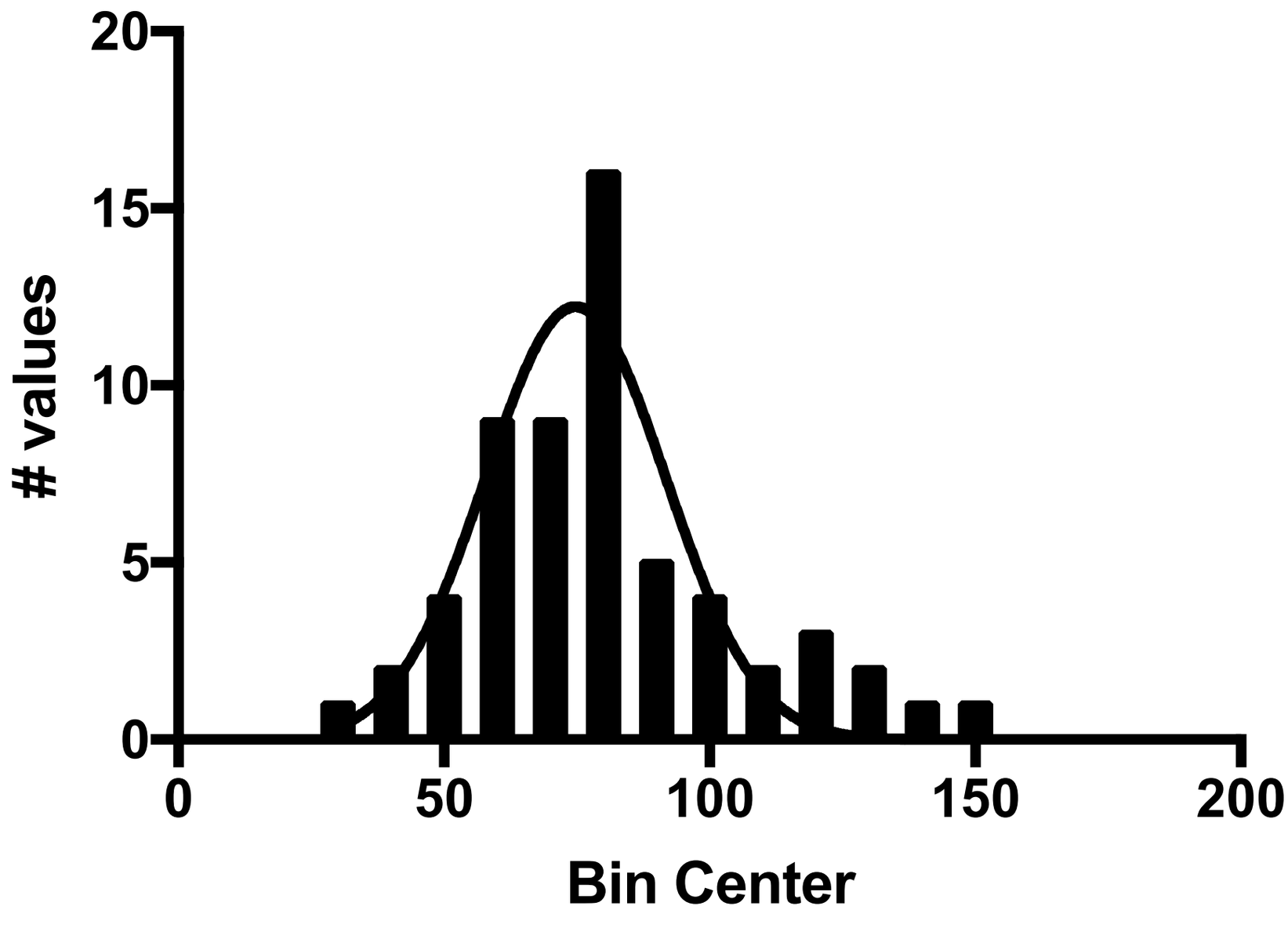
Adding a Normal Distribution Line - GraphPad



Adding a Normal Distribution Line - GraphPad



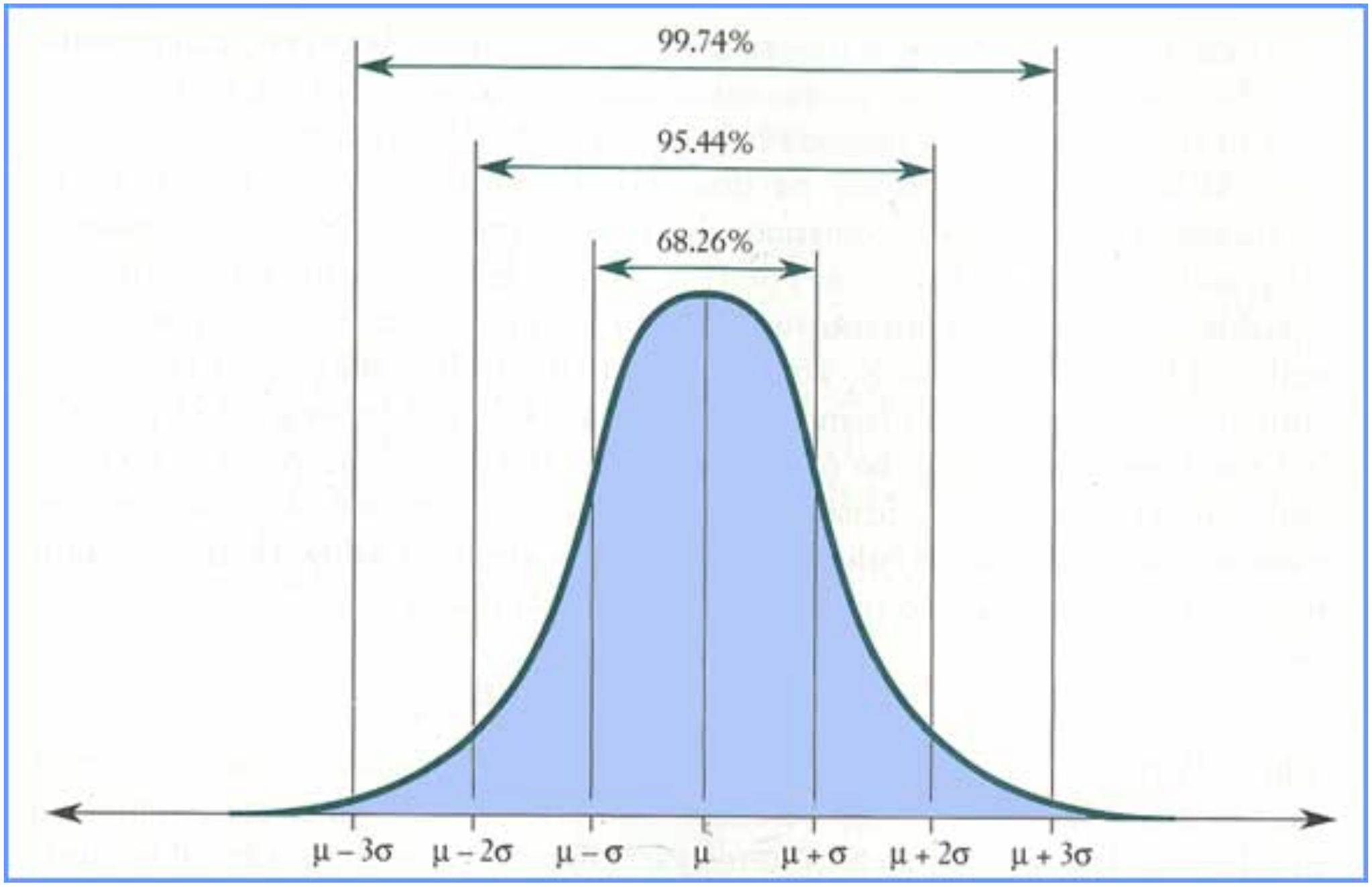
Histogram of Sleep Time With Normal Curve



PROPORTIONS OF INDIVIDUALS WITHIN 1 SD OR 2 SD OF THE MEAN

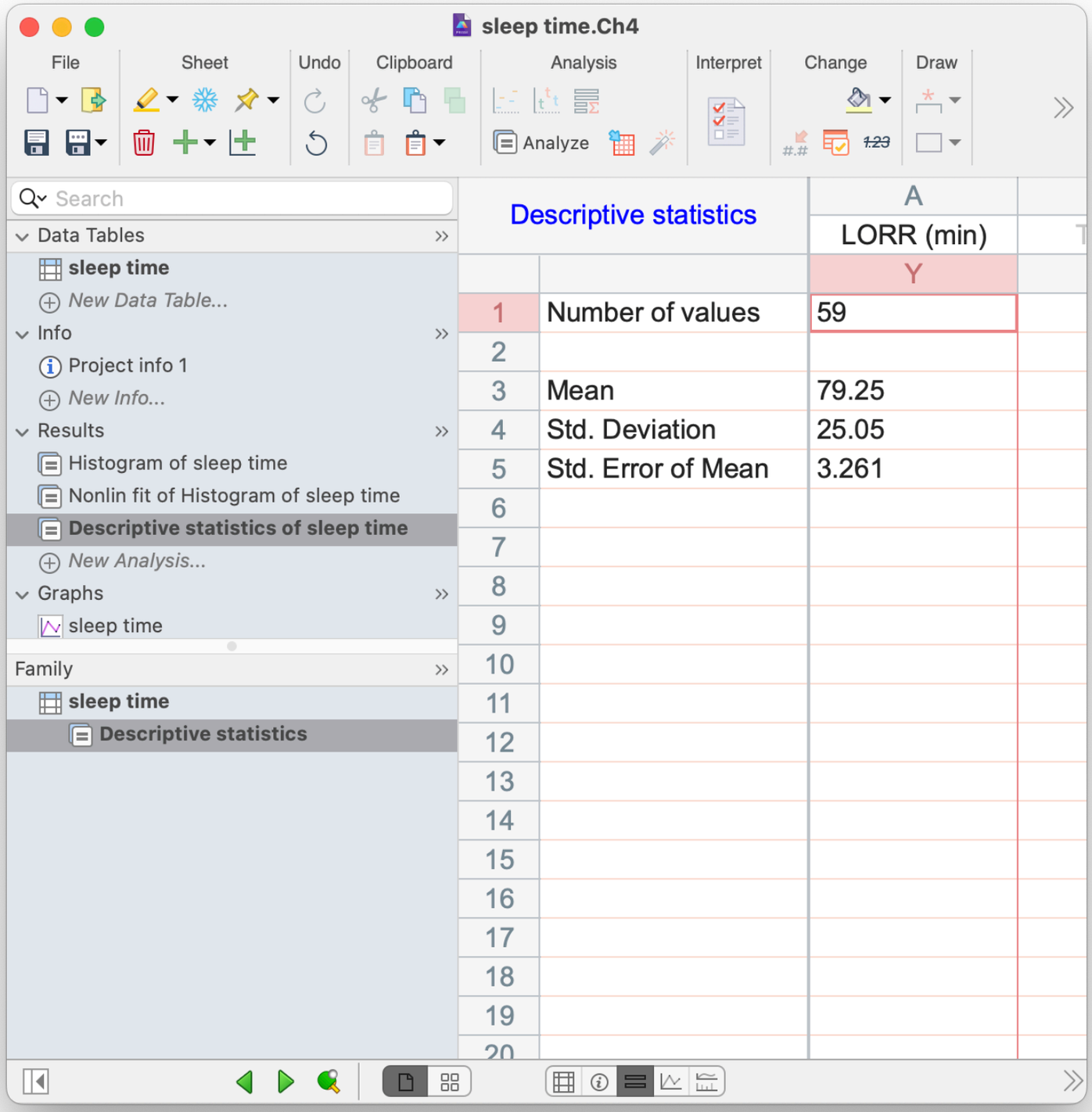
Proportions in Normal Distribution

Ideal Normal Distribution In Terms of SD



Sleep Time Example

- Mean = 79.25
- SD = 25.05
- Mean ‘±’ 1 SD = (54.2, 104.3)
- Proportion of values within 1 SD = 0.73
- Mean ‘±’ 2 SD = (29.2, 129.3)
- Proportion of values within 2 SD = 0.95



The Standard Normal

- Standard Normal - Normal distribution with mean = 0 and SD =1

- All normal distributions can be converted to a standard normal distribution using the following formula:

$$Z = \frac{Value - Mean}{SD}$$

- Z is the number of SD the value is away from the mean

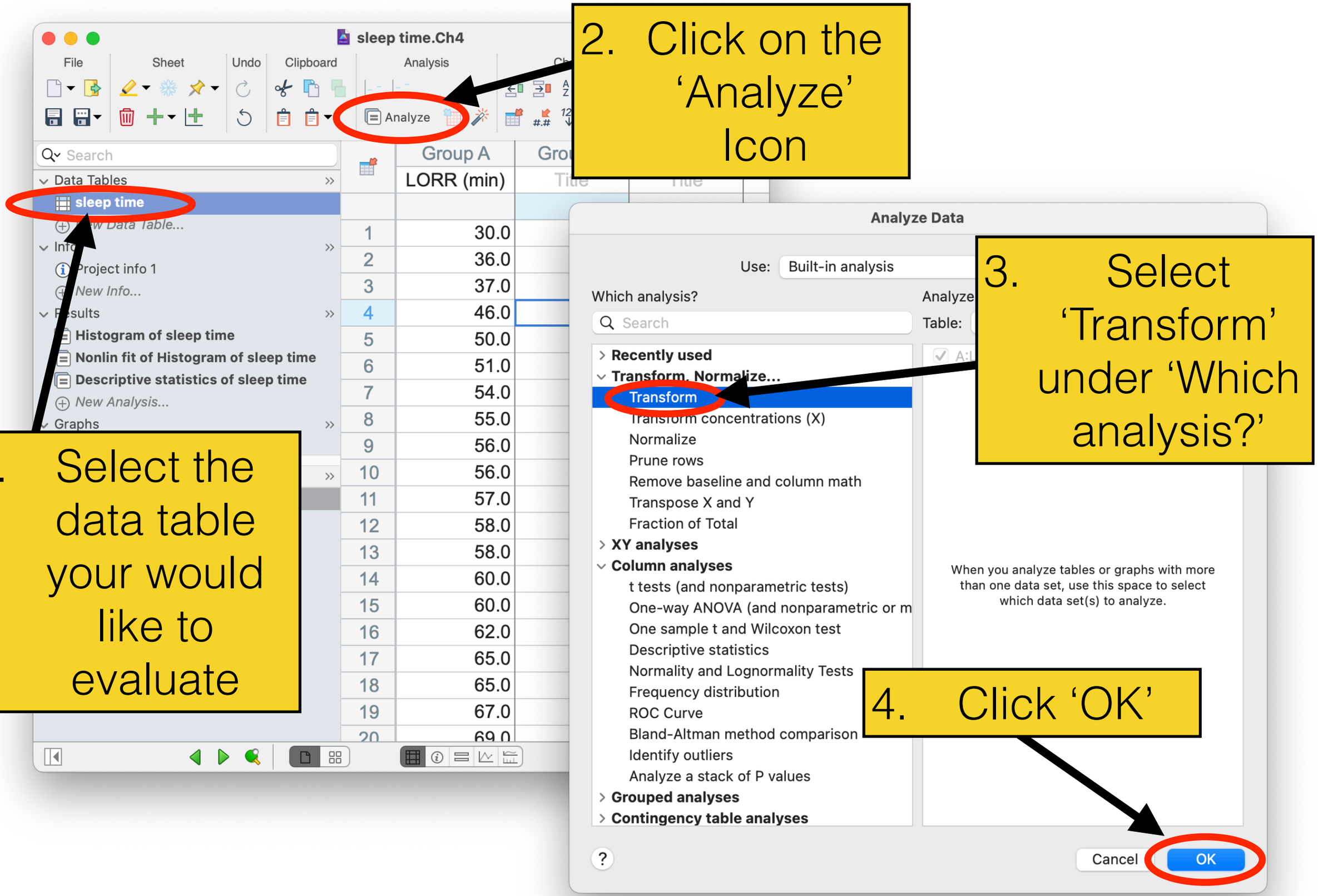
Calculate the proportion of samples within 1 SD and 2 SD - GraphPad

1. Calculate Z scores for each value using ‘Transform’

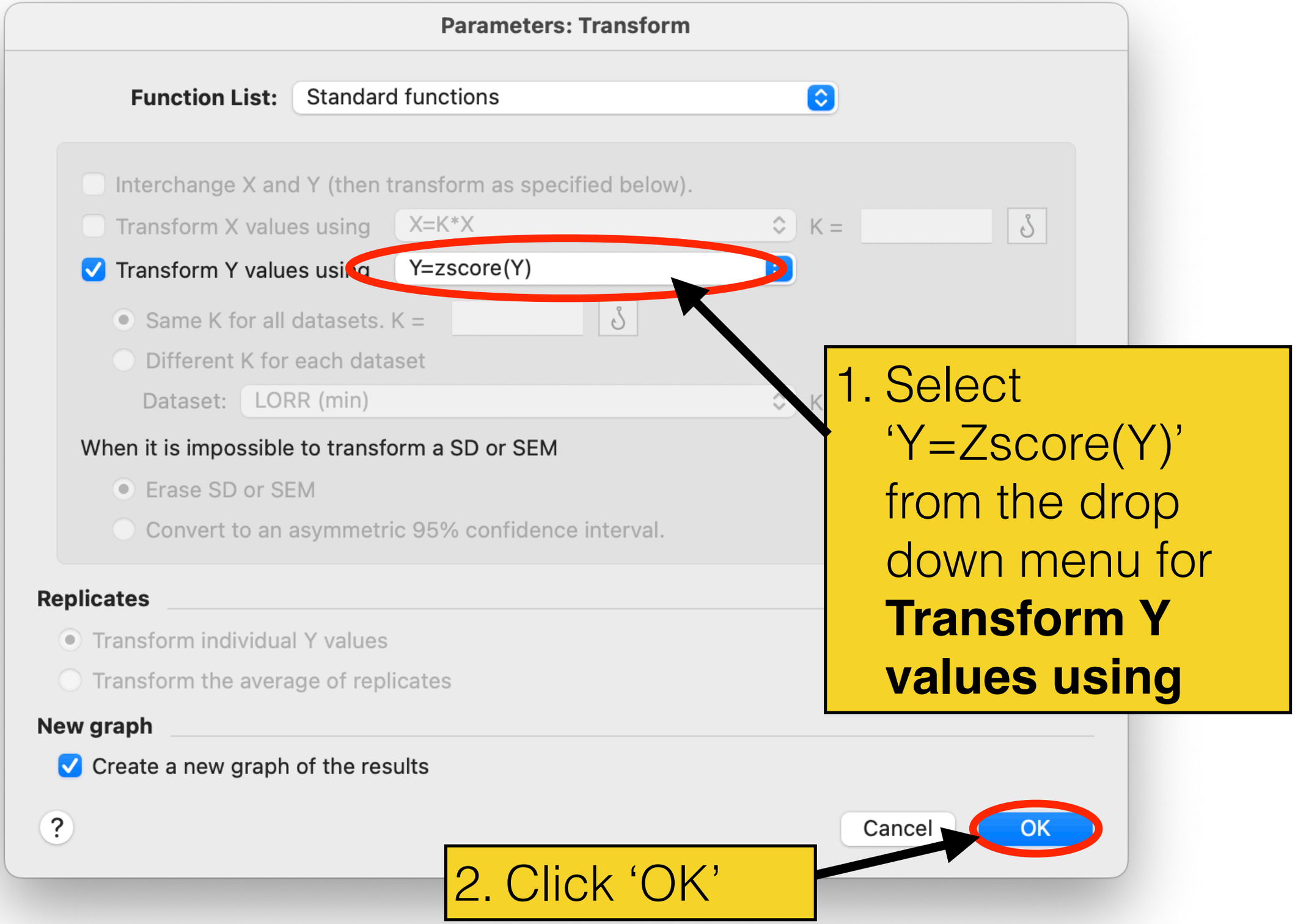
2. Calculate the absolute value of the z scores using ‘Transform’

3. Calculate the cumulative frequency based on the absolute values of the z scores

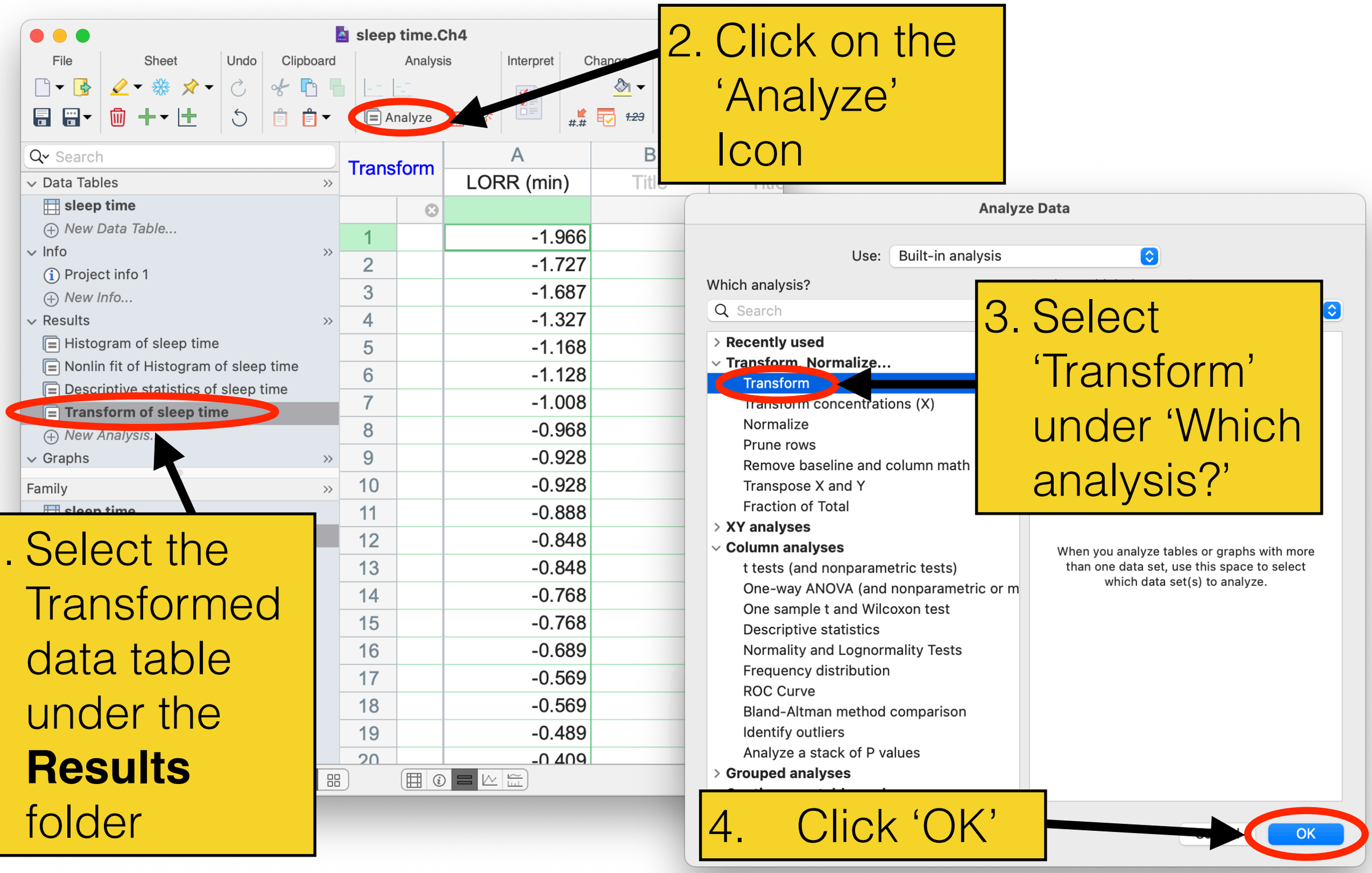
Calculate Z Scores



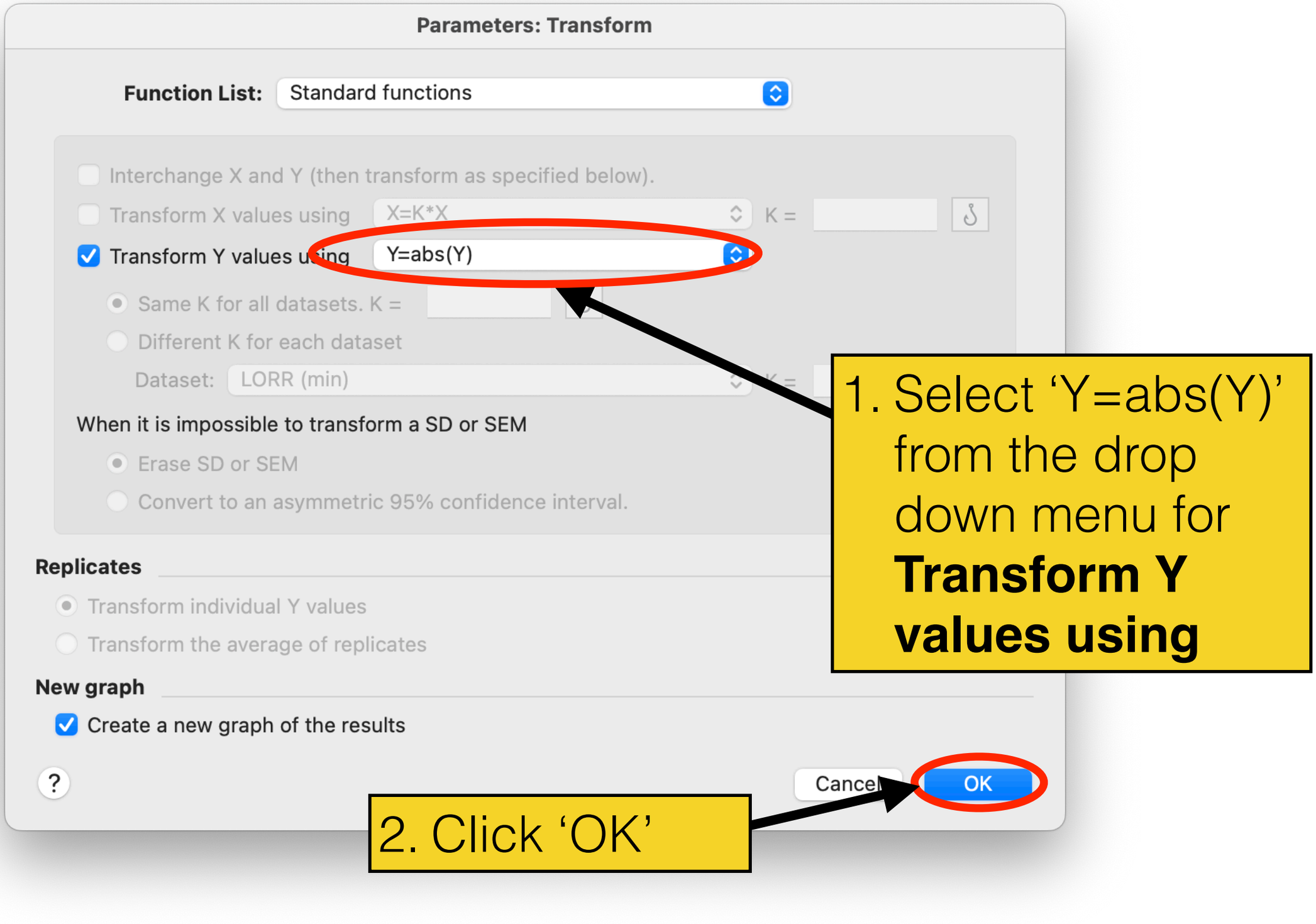
Calculate Z scores



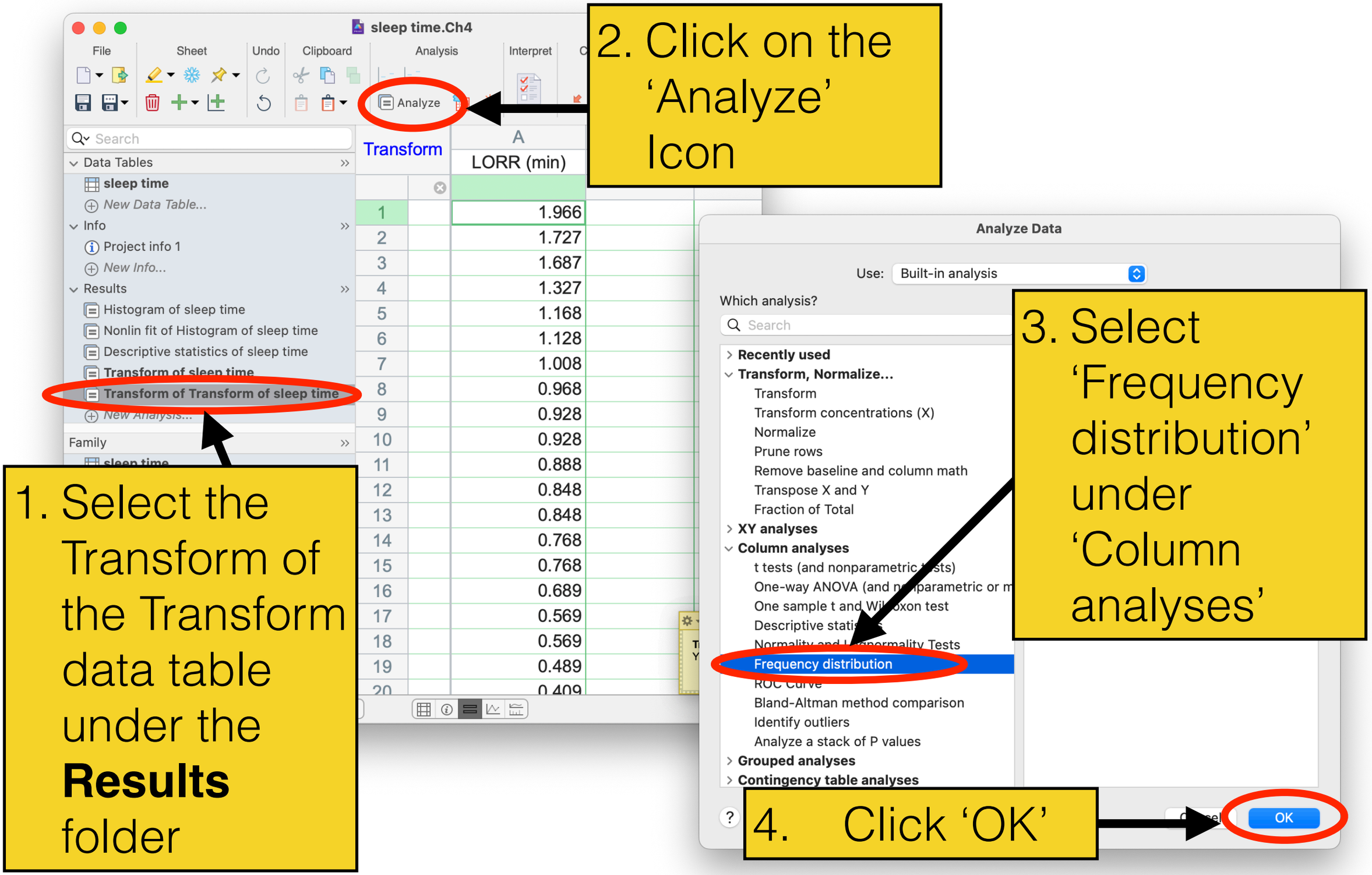
Calculate the absolute value of each Z score



Calculate the absolute value of each Z score



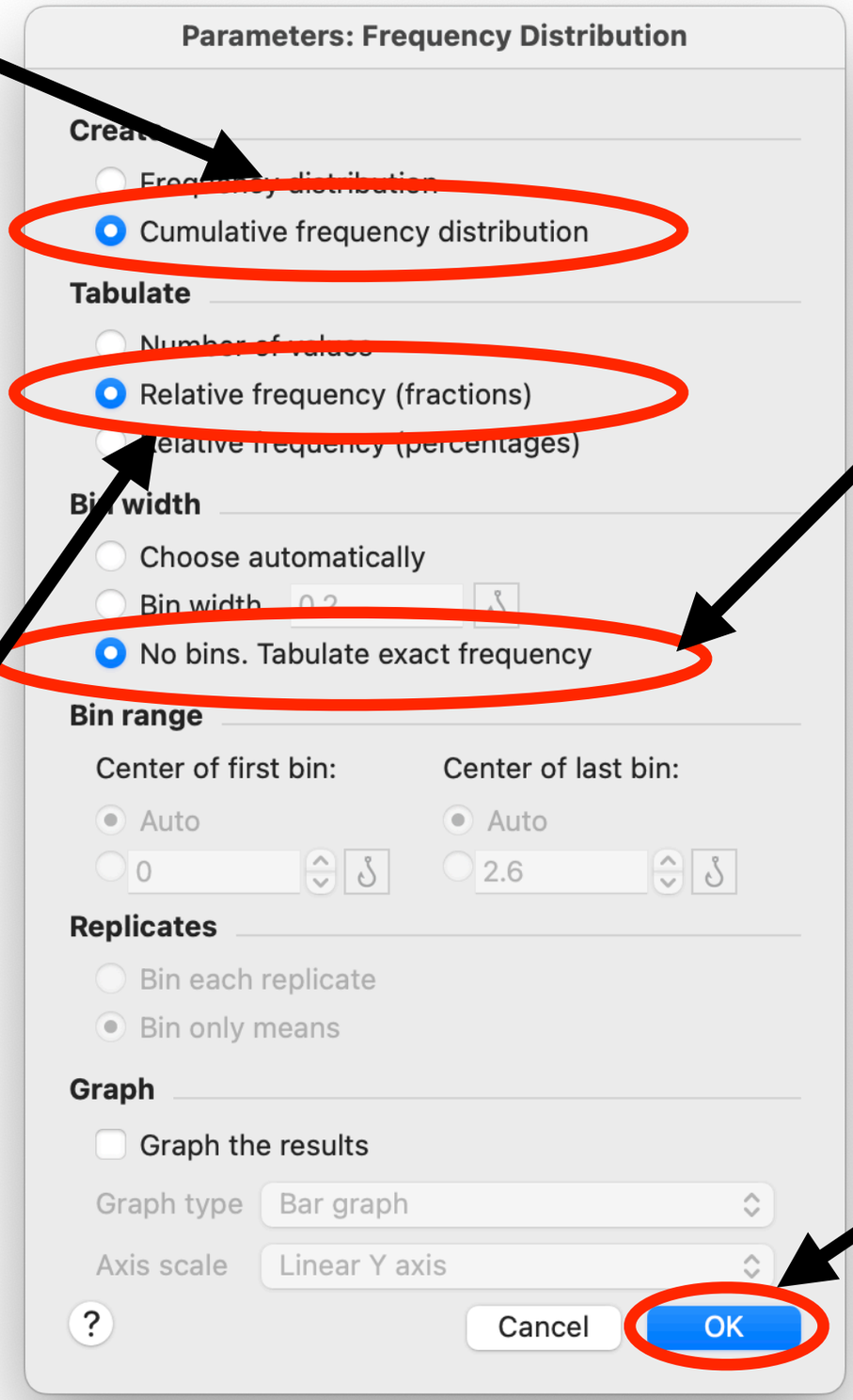
Calculate the cumulative frequency based on the absolute values of the z scores



Calculate the cumulative frequency based on the absolute values of the z scores

1. Select ‘Cumulative frequency distribution’ under **Create**

2. Select ‘Relative frequency (fractions)’ under **Tabulate**



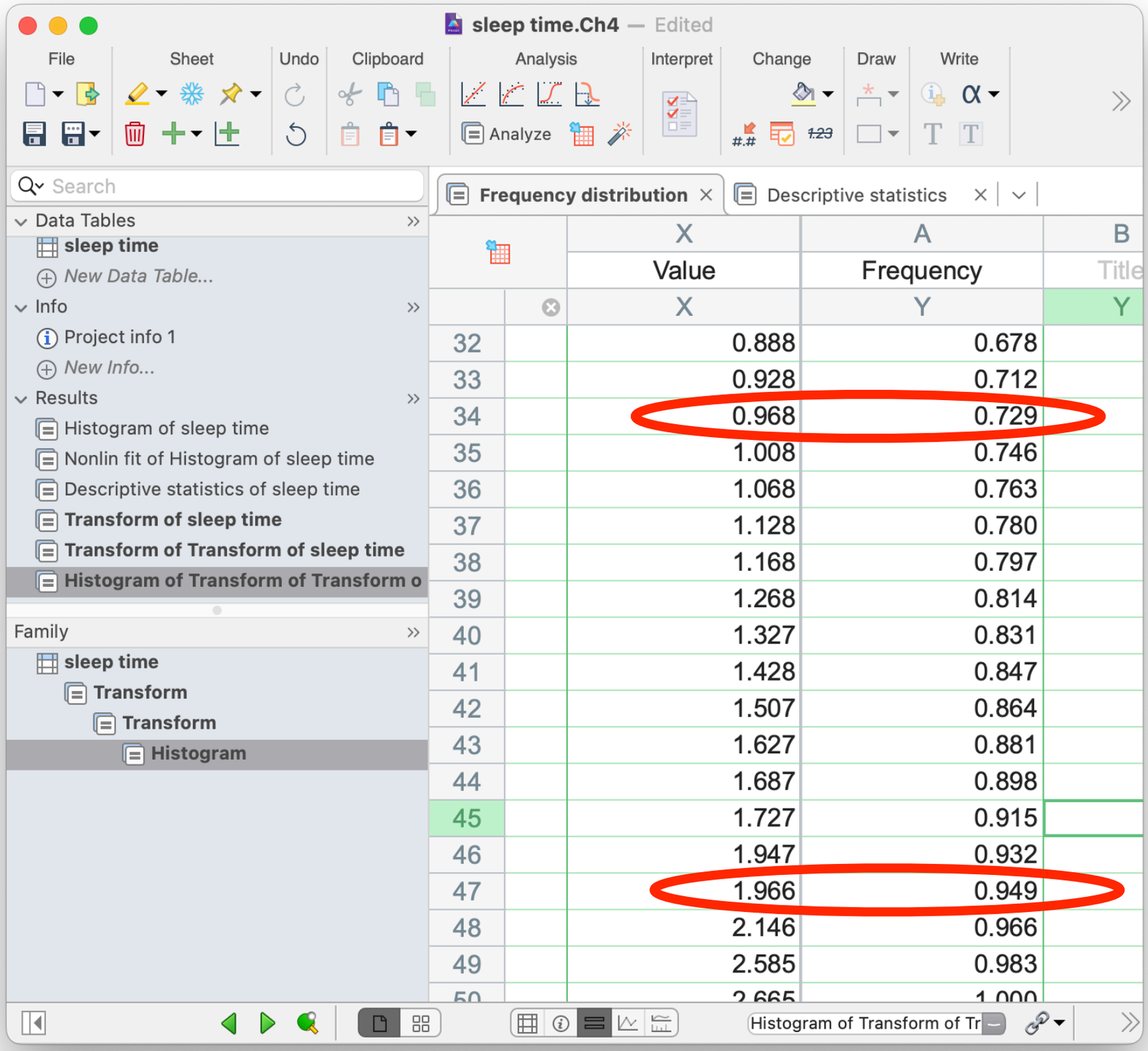
3. Select ‘No bins.’ under **Bin width**

4. Click ‘OK’

Calculate the proportion of samples within 1 SD and 2 SD - GraphPad

73% of the data points are less than 1 SD from the mean

95% of the data points are less than 2 SD from the mean



The Normal Distribution Does Not Define Normal Limits

- Normal distribution does not equate to ‘normal’ range.

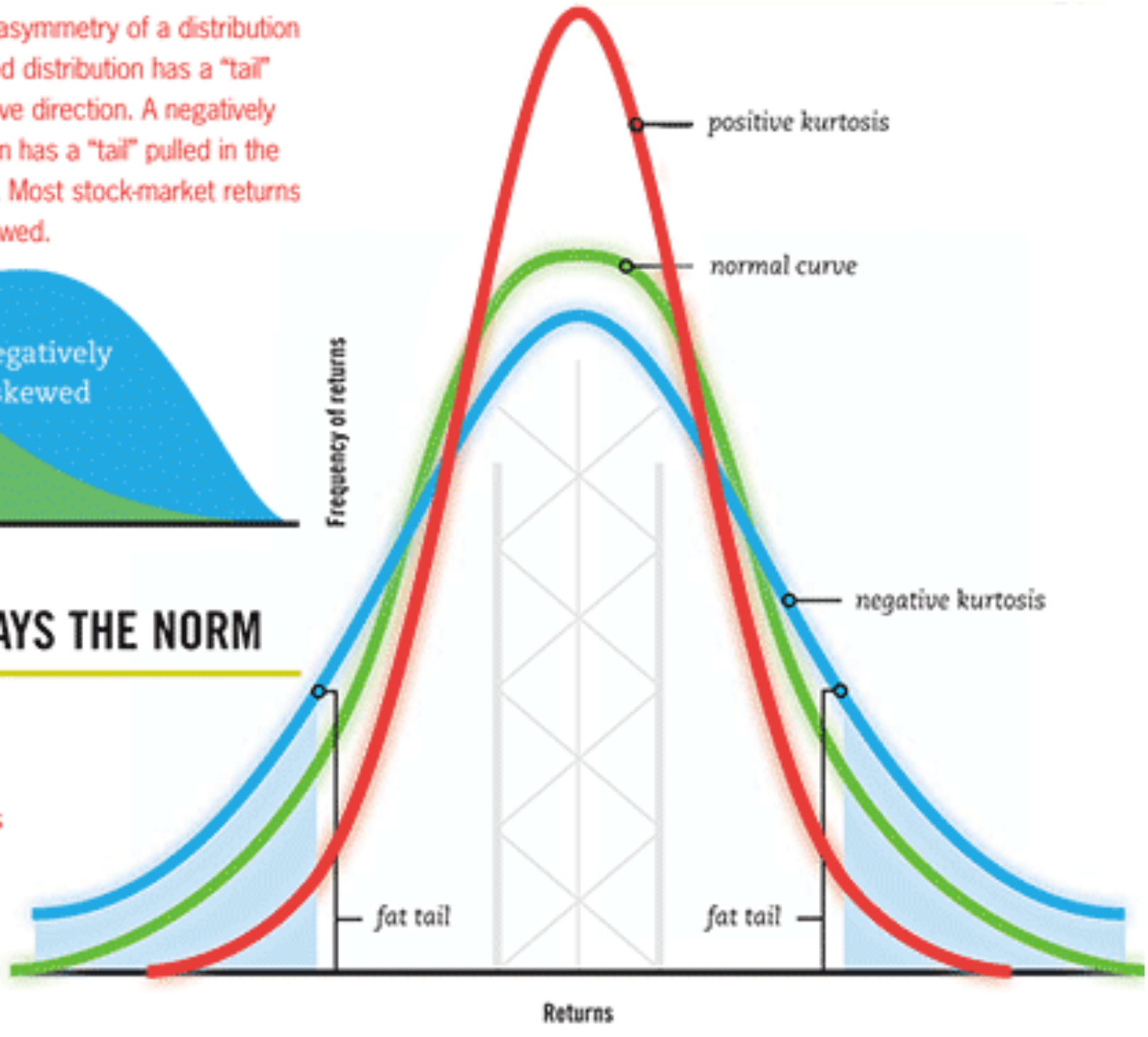
- Defining the normal limits of a clinical measurement is not straightforward and requires clinical thinking, not just statistics.

SKEWNESS AND KURTOSIS

Skewness is the asymmetry of a distribution
A positively skewed distribution has a "tail" pulled in the positive direction. A negatively skewed distribution has a "tail" pulled in the negative direction. Most stock-market returns are negatively skewed.

NORMAL NOT ALWAYS THE NORM

Kurtosis refers to how peaked the curve is: steeper means positive kurtosis and flatter means negative kurtosis. Fat tails occur when there are more outsize returns on the downside or upside, or both, than the normal curve suggests.



https://anotherbloodybullshitblog.wordpress.com/2016/01/18/statistics-how-to-test-if-your-data-follows-the-normal-distribution/

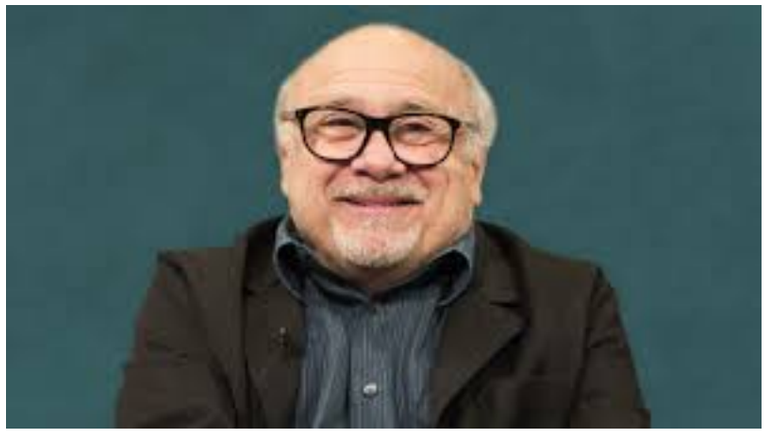
Skewness

Skewness can be quantified numerically by a number that ranges from negative infinity to positive infinity, with a value of 0 indicating that no skewness is present

Kurtosis

Like skewness, **kurtosis** can also be quantified numerically with a range of negative infinity to positive infinity, with a value of 0 indicating that no kurtosis is present.

- A *negative* kurtosis value indicates that the ‘shoulders’ of the distribution are too high and wide



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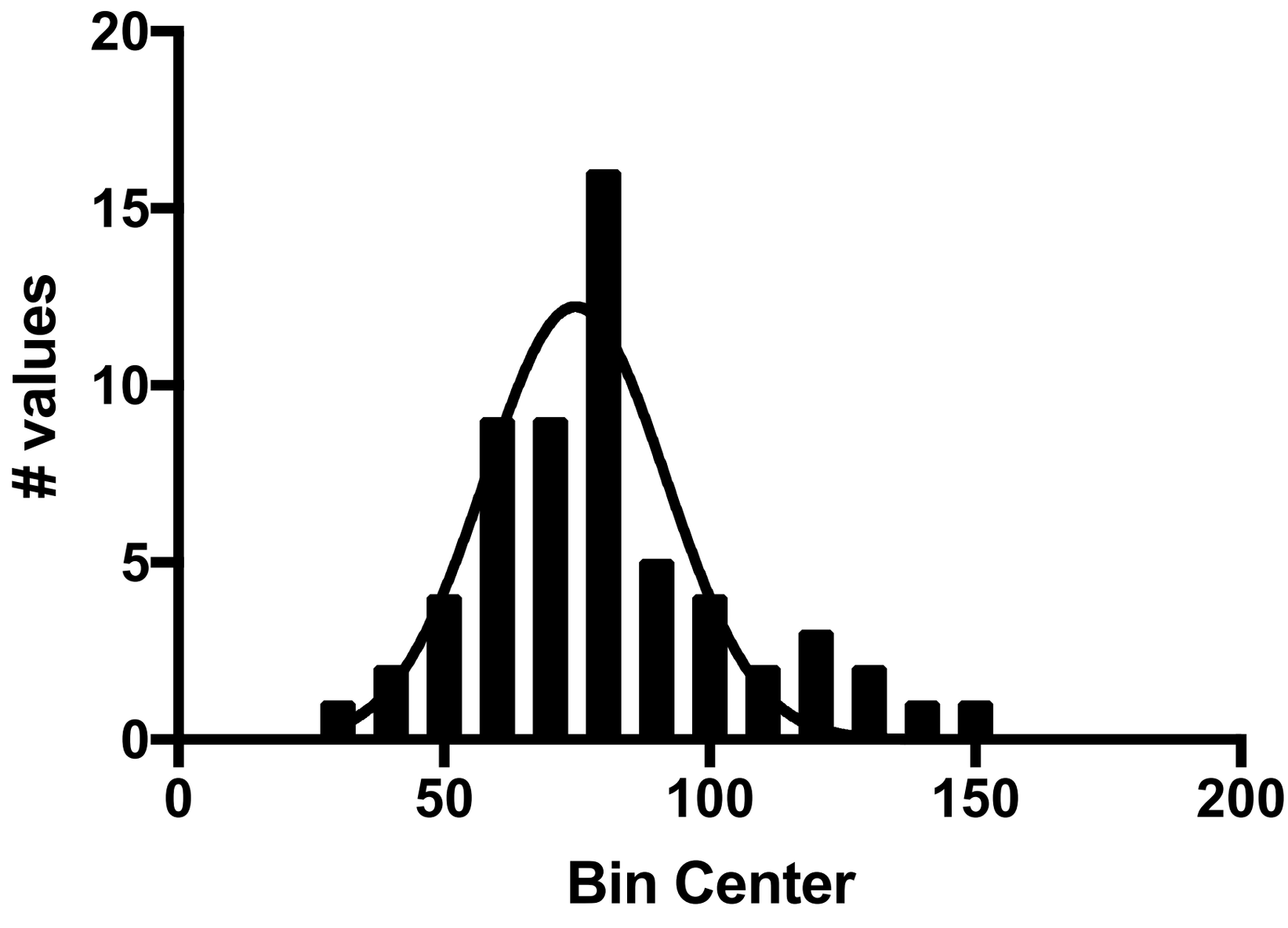
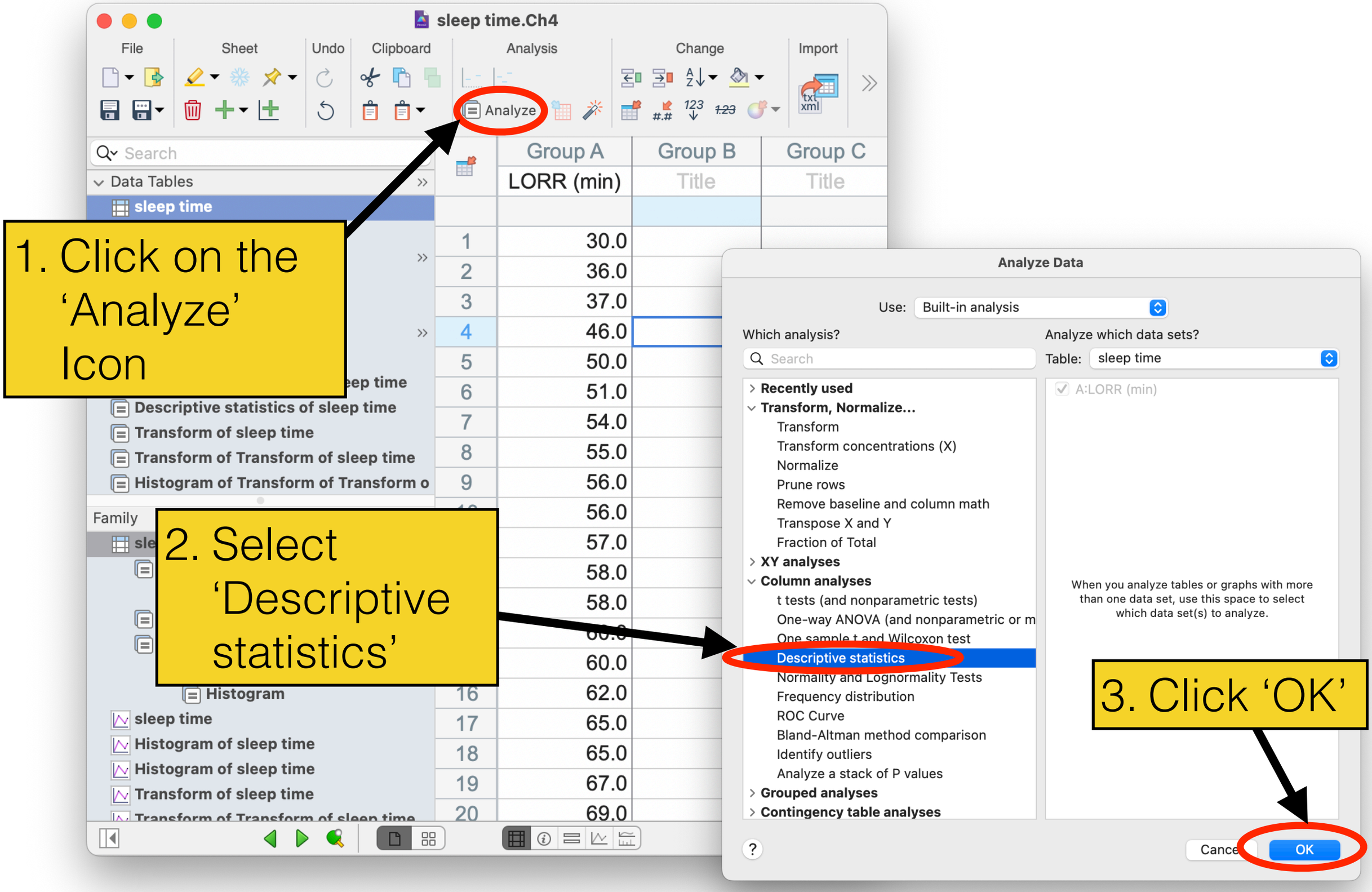
- A *positive* kurtosis value indicates that the ‘shoulders’ of the distribution are too low and narrow.

Skewness and Kurtosis in Sleep Time

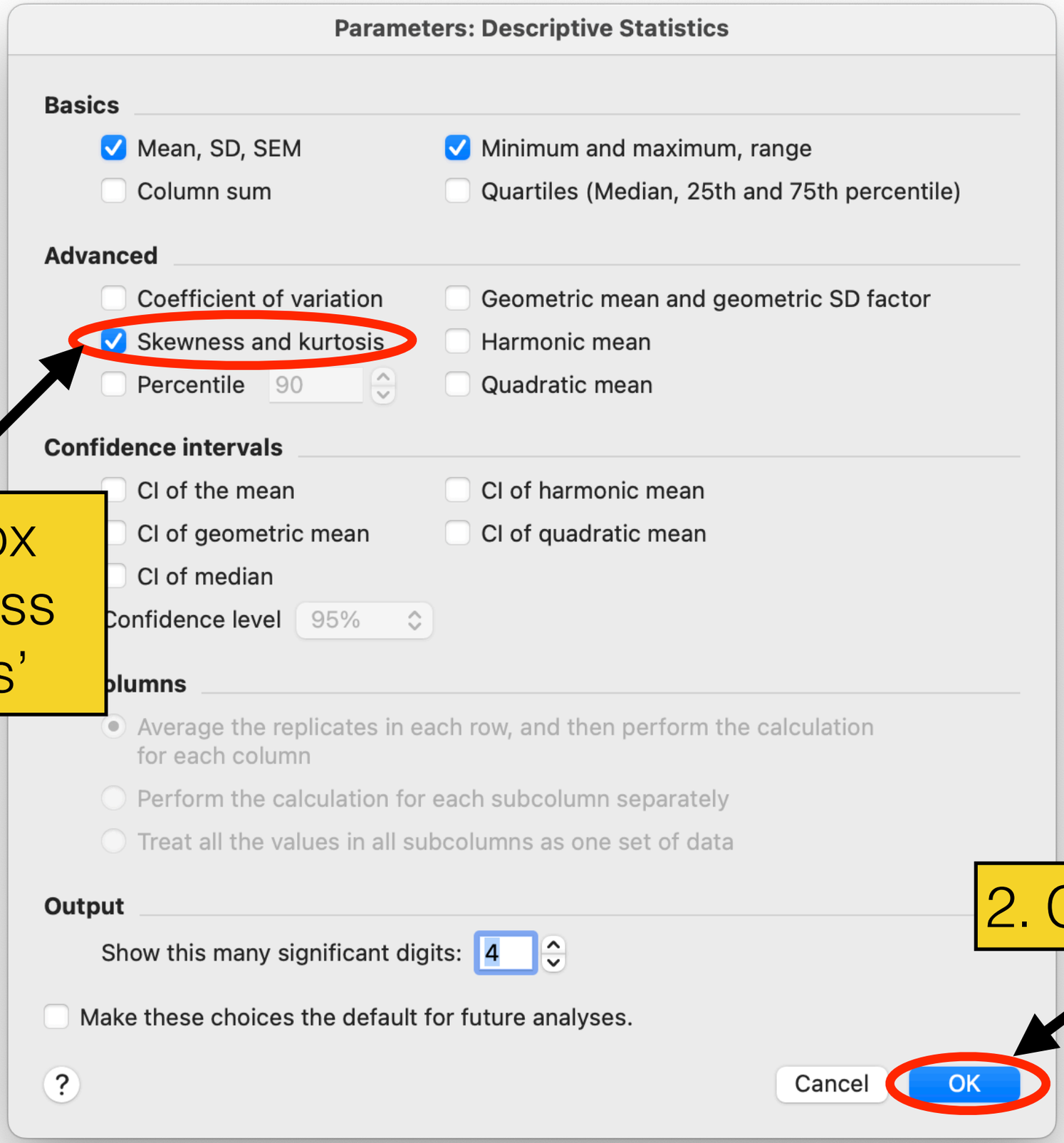
Skewness = 0.70 (positive skew); kurtosis = 0.61 (narrow peak)



Measuring Skewness and Kurtosis



Measuring Skewness and Kurtosis



1. Click the box for ‘Skewness and kurtosis’

2. Click ‘OK’

sleep time.Ch4

File | Sheet | Undo | Clipboard | Analysis | Interpret | Change | Draw

Analyze

Search

Data Tables
- New Data Table...

Info
- Project info 1
- New Info...

Results
- Histogram of sleep time
- Nonlin fit of Histogram of sleep time
- Descriptive statistics of sleep time
- Transform of sleep time
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- **Descriptive statistics of sleep time**

Family
- **sleep time**
  - **Descriptive statistics**

Descriptive statistics

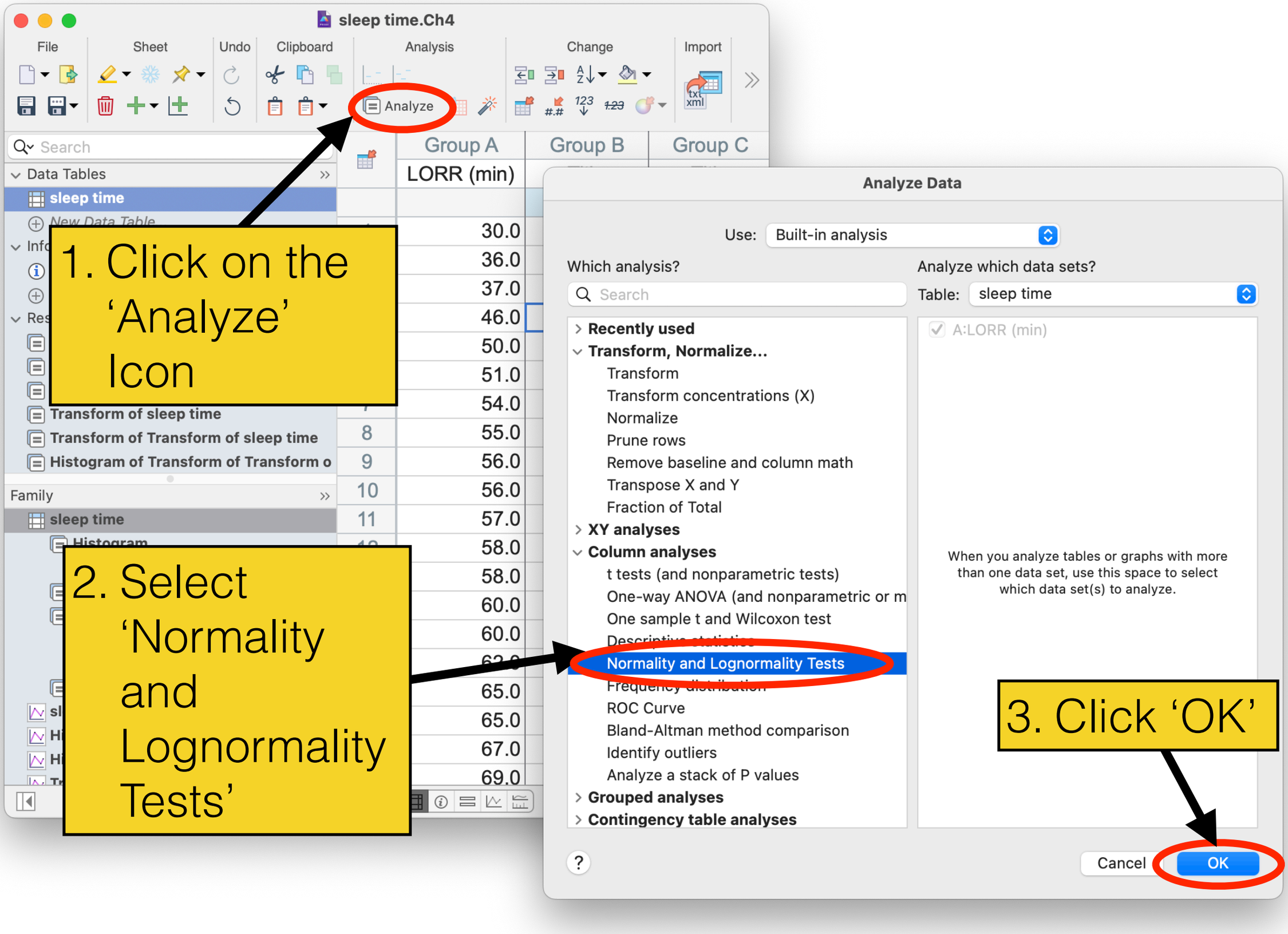
		A
		LORR (min)
		Y
1	Number of values	59
2		
3	Minimum	30.00
4	Maximum	146.0
5	Range	116.0
6		
7	Mean	79.25
8	Std. Deviation	25.05
9	Std. Error of Mean	3.261
10		
11	Skewness	0.6964
12	Kurtosis	0.6086
13		
14		
15		
16		
17		
18		
19		
20		

TESTS FOR NORMAL DISTRIBUTIONS

D'Agostino-Pearson Omnibus Normality Test

- Test for both skewness and kurtosis
- A significant p-value ($p<0.05$) indicates that the distribution is significantly different from normal, i.e., the data are non-normal.
- CAVEATS:
  - With a small sample size even large departures from normality may not be detected
  - With a large sample size even small departures from normality will be detected

Normality Test

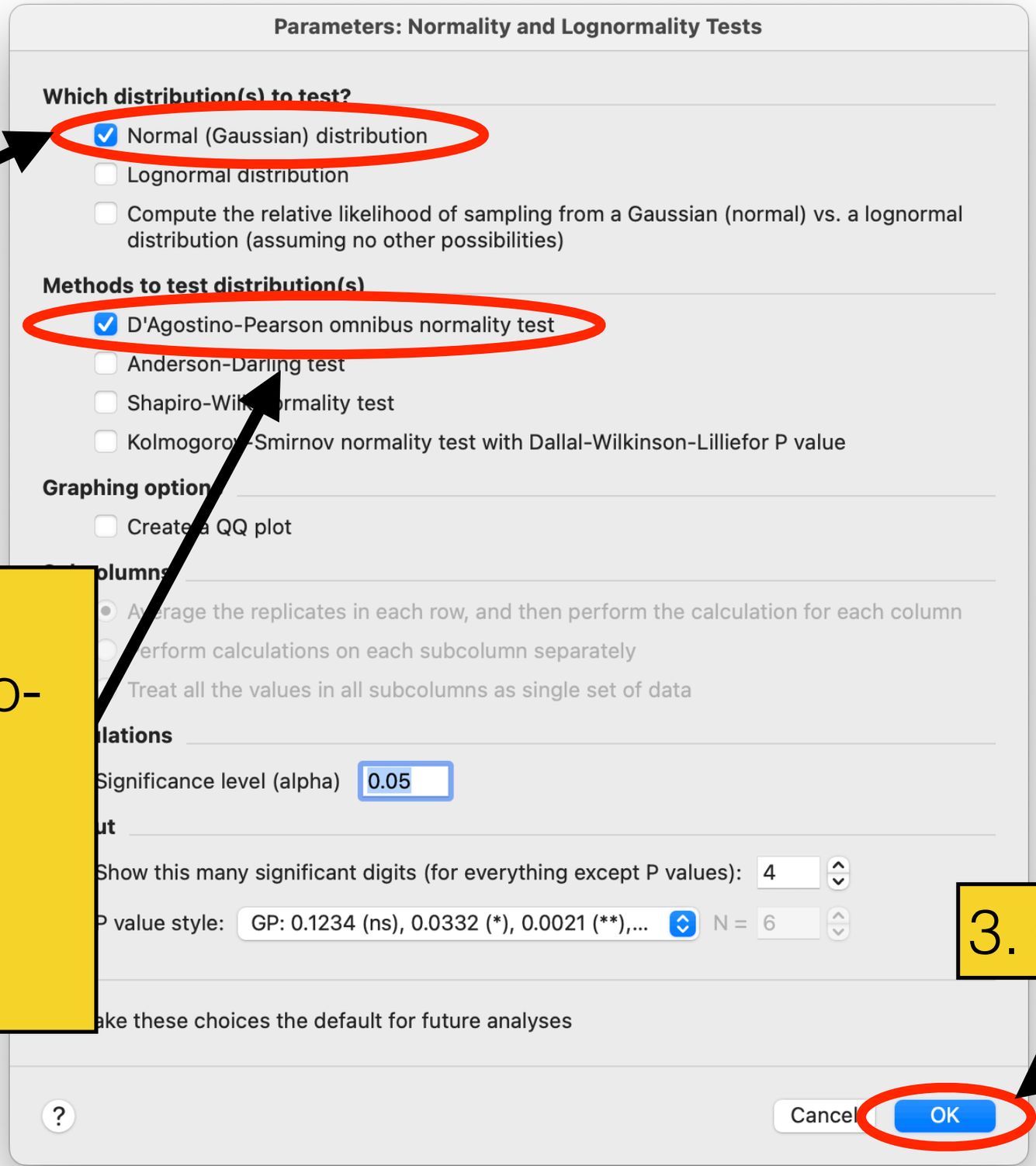


Normality Test

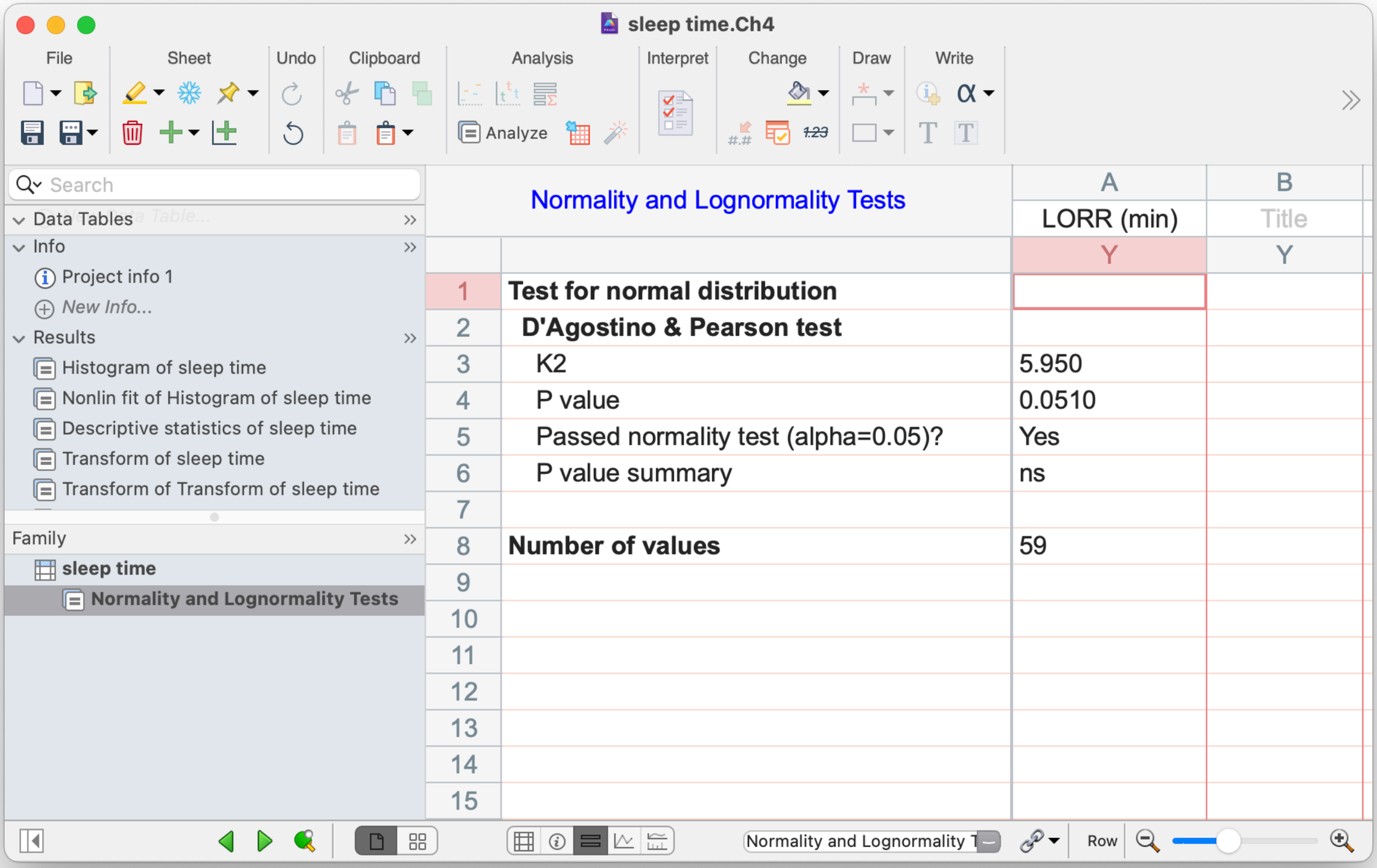
1. Select ‘Normal (Gaussian) distribution’

2. Select ‘D’Agostino-Pearson omnibus normality test’

3. Click ‘OK’



Normality Test



Is the sleep time data normally distributed?

What did we learn?

- The normal distribution often results from many random factors creating variability and therefore, is common in a research setting.
- 3 main characteristics of a normal distribution: 1) the data are unimodal, 2) the distribution is symmetric, and 3) the frequencies decline steadily as we move towards higher and lower values, without any sudden sharp cut-off.
- If the data are normally distributed, 68% of observations will fall within 1 SD of the mean and 95% of observations will fall within 2 SD.
- Skewness is a quantitative measure of the lack of symmetry in the data distribution and kurtosis is a quantitative measure of how closely the peak, shoulders, and tails of the distribution match a normal distribution.
- Use statistical tests of non-normality with caution.