Algorithms: Basic Concepts Analysis and Design of Algorithms

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Basic concepts

Correctness and Loop Invariants







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2 Correctness and Loop Invariants







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Definition (Program)

A **program** is a sequence of **instructions** written to perform a specified task using a computer.

- A computer requires programs to function.
- The source code of a computer is written by computer programmers.
- The source code is written in a programming language
- The source code may be converted into an executable file by a compiler or an interpreter (intermediate language).





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Example

The sequence (31, 41, 59, 26, 41, 58) is an instance of the sorting problem





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- Formally establishes the relationship between the input and the output of an algorithm.

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Input: A sequence of *n* numbers $\langle a_1, a_2, \ldots, a_n \rangle$

Output: A permutation (reordering) $\langle a'_1, a'_2, \dots, a'_n \rangle$ of the input, such that $a'_1 \leq a'_2 \leq \dots \leq a'_n$.







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- Algorithms are essential in Computer Science.
- They provide a step-by-step procedure that can be translated into a computer program. It can be executed by a computer.
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 - Pseudocode
 - Data Flow Diagrams
 - Computer program
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- Which algorithm is better for a given application? It depends of:



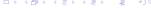




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 - Possible restrictions on the item values.







Definition (Data structures)

A **data structure** is a way to store and organize data in order to facilitate access and modifications.

- No single data structure works well for all purposes
- It is important to know the strengths and limitations of several of them.

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Definition (Efficiency)

The **algorithmic efficiency** is the property of an algorithm related to the number of computational resources used to solve a computational problem.

- An algorithm must be analyzed to determine the resources it uses, since different algorithms designed to solve the same problem often differ dramatically in their efficiency.
- These differences can be much more significant than differences due to hardware and software.

Examples

Insert-sort takes $c_1 n^2$ instructions; Merge-sort takes $c_2 n \log n$ instructions The number of instructions is proportional to the time required. Generally

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- Suppose the time required for Insertion-sort is $3.6n^2$ instructions and it is run on computer A.
- Suppose the time required for Merge-sort is 200 n log n instructions and it is run on computer B.
- Computer A processes 1.000.000.000 instructions per second.
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- How much time in seconds is required by each one of them, when n = 1,000,000?





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3 Analysis and Design of Algorithms







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An algorithm is said to be **correct** if, for every input instance, it halts with the correct output.

We say that a correct algorithm solves the given computational problem.







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A statement (property about a loop) is said to be a **loop invariant** if we can show following three properties.

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The loop invariants to help us understand why an algorithm is correct.





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Resources such as memory, bandwidth, power or computer hardware are important, but most of the time what we mainly want to measure is **computational time**.

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Implementation Technology

- Before we can analyze an algorithm, we must have a model of the implementation technology that will be used.
- We will assume a generic one processor, random-access machine (RAM) model of computation (instructions are executed one after another).
- Strictly speaking, one should precisely define the instructions of the RAM model and their costs. Tedious!
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- We need a means for summarizing that behavior in simple, easily understood formulas.
- There are many choices in deciding how to express our analysis.
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Input size

- For many problems, such as sorting the most natural measure is the number of items in the input.
- For multiplying two integers, the best measure of input size is the total number of bits needed to represent the input in ordinary binary notation.
- Sometimes, it is more appropriate to describe the size of the input with two numbers rather than one. For example, in graphs order (|V|) and size (|E|).







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- It is convenient to define the notion of step so that it is as machine-independent as possible.
- Each execution of the i-th line takes time c_i where c_i is a constant Some operations like calling a subroutine may take more than constant time.
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- We want to know about its behavior on the best, worst and average-case.
- What mathematical functions describe its behavior?
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Exercise

For each function f(n) and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes f(n) microseconds.

f(n)	1 second	1 minute	1 hour	1 day	1 month	1 year	1 century
lg n							
\sqrt{n}							
n							
n lg n							
n ²							
n ³							
2 ⁿ							
n!							

Solution

	1 second	1 minute	1 hour	1 day	1 month	1 year	1 century
$\lg n$	2^{10^6}	$2^{6\times 10^7}$	$2^{3.6\times10^9}$	$2^{8.64 \times 10^{10}}$	$2^{2.59 imes 10^{12}}$	$2^{3.15 imes 10^{13}}$	$2^{3.15 \times 10^{15}}$
\sqrt{n}	10^{12}	$3.6 imes 10^{15}$	$1.3 imes 10^{19}$	$7.46 imes 10^{21}$	$6.72 imes 10^{24}$	$9.95 imes 10^{26}$	$9.95 imes 10^{30}$
n	10^{6}	$6 imes10^7$	$3.6 imes 10^9$	$8.64 imes 10^{10}$	$2.59 imes 10^{12}$	$3.15 imes 10^{13}$	$3.15 imes 10^{15}$
	$6.24 imes 10^4$	$2.8 imes10^6$	$1.33 imes 10^8$	$2.76 imes 10^9$	$7.19 imes 10^{10}$	$7.98 imes 10^{11}$	$6.86 imes10^{13}$
n^2	1000	7745	60000	293938	1609968	5615692	56156922
n^3	100	391	1532	4420	13736	31593	146645
2^n	19	25	31	36	41	44	51
n!	9	11	12	13	15	16	17





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Design of Algorithms

Definition (Design of Algorithms)

Is the strategy thought, used and designed to implementing a solution of a problem by means of an algorithm. For example: Divide and conquer, Greedy programming, Dynamic programming, Linear programming, Randomized algorithms, etc.



