The SIR Matlab Toolbox performs computation of time and frequency domain information measures of entropy rate (ER), mutual information rate (MIR) and O-Information rate (OIR), exploiting both parametric (i.e., vector autoregressive – VAR) and non-parametric (based on the weighted covariance estimator) estimation of the power spectral density of (blocks of) interacting stochastic processes. The toolbox provides algorithms for the identification of the VAR models from time series data and is completed with algorithms for model validation and for the estimation of time and frequency domain significance thresholds. It also contains a set of exemplary scripts illustrating the utilization of the various functions.

**DESCRIPTION OF THE TOOLBOX**

**Functions**

* sir\_VARspectra.m: performs frequency domain connectivity analysis from the parameters of a strictly causal vector autoregressive (VAR) model; returns spectral and transfer function matrices [1].
* sir\_WCspectra.m: performs frequency domain connectivity analysis using the weighted covariance (WC) estimator, which exploits the Fourier transform of the sample autocorrelation and cross-correlation functions of the data; returns spectral matrices [2].
* sir\_mir.m: performs computation of spectral and time domain entropy rate (ER) [3], [4] and mutual information rate (MIR) [5] for two (blocks of) processes given the spectral matrix, estimated using the parametric (sir\_VARspectra.m) or the non-parametric (sir\_WCspectra.m) approach.
* sir\_oir.m: performs computation of the gradient of the O-Information rate (OIR) and the OIR in the frequency and time domain for (blocks of) processes given the spectral matrix, estimated using the parametric (sir\_VARspectra.m) or the non-parametric (sir\_WCspectra.m) approach [6].
* sir\_deltaO.m: performs computation of the gradient of the OIR when the block is added to the lower order group . The function is called within sir\_oir.m [6].
* sir\_subindexes.m: performs extraction of indexes of the two (blocks of) processes to analyze. The function is called within sir\_oir.m [6].
* sir\_mos\_idMVAR.m: model order selection for the identification of the strictly causal VAR model, using Akaike Information Criterion [7] or Minimum Description Length criterion [8].
* sir\_idMVAR.m: identification of strictly causal VAR model; estimates model coefficients, innovations and innovation covariance from a given time series and a given model order. The default identification algorithm is the standard least squares method [9], but several other algorithms may be recalled [10].
* sir\_surrshuf.m: generates randomly shuffled surrogates, which are realizations of independent and identically distributed (IID) stochastic processes with the same mean, variance and probability distribution as the original series [11]. These surrogates are used to assess the statistical significance of ER measures.
* sir\_surriaafft.m: generates a given number of Iterative Amplitude Adjusted Fourier Transform (iAAFT) surrogates from a given original time series [12]. These surrogates are used to assess the statistical significance of MIR measures.
* sir\_block\_bootstrap.m: generates a given number of block bootstrap time series from a given original time series [13], [14]. Confidence intervals of the bootstrap distributions are used to assess the statistical significance of OIR measures.
* theoreticalVAR.m: generates the theoretical coefficients of some illustrative VAR processes.
* MVARfilter.m: yields a single realization of a strictly causal VAR process of assigned dimensionality and length, given strictly causal coefficients and residuals.
* AR\_filter.m: performs AR filtering (low-pass, high-pass) of an input data matrix, given the order of the filter.

**Examples**

* SIM1.m: runs the theoretical example of Sect. 4.1 of the main text, generating figures depicting the spectral and time domain ER, MIR and OIR values for an illustrative VAR process of 5 interacting stochastic scalar processes.
* SIM2.m: runs the theoretical example of Sect. 4.2 of the main text, generating figures depicting the spectral and time domain ER, MIR and OIR values for an illustrative VAR process of 6 interacting stochastic scalar processes grouped in 3 blocks.
* APPL1.m: applicates the ER, MIR and OIR measures to a physiological network of cardiovascular, cerebrovascular and respiratory time series extracted from one healthy subject in the supine resting state; the application is described in Sect. 5.1 of the main text.
* APPL2.m: applicates the ER, MIR and OIR measures to a network of electroencephalographic time series extracted from one healthy subject during a motor execution task; the application is described in Sect. 5.2 of the main text.

**External functions**

The toolbox makes use of external functions taken from existing MATLAB toolboxes:

* the BioSig Toolbox (<https://biosig.sourceforge.net/>), with one function used for providing a method for strictly causal VAR estimation (mvar.m).

**References**

[1] S. M. Kay, *Modern spectral estimation*. Pearson Education India, 1988.

[2] R. B. Blackman and J. W. Tukey, “The measurement of power spectra from the point of view of communications engineering—Part I,” *Bell Syst. Tech. J.*, vol. 37, no. 1, pp. 185–282, 1958.

[3] T. M. Cover, *Elements of information theory*. John Wiley & Sons, 1999.

[4] D. Chicharro, “On the spectral formulation of Granger causality,” *Biol. Cybern.*, vol. 105, no. 5–6, pp. 331–347, 2011.

[5] T. E. Duncan, “On the calculation of mutual information,” *SIAM J. Appl. Math.*, vol. 19, no. 1, pp. 215–220, 1970.

[6] L. Faes *et al.*, “A Framework for the Time- and Frequency-Domain Assessment of High-Order Interactions in Brain and Physiological Networks,” vol. XX, no. Xx, pp. 1–11, 2022.

[7] H. Akaike, “A new look at the statistical model identification,” *IEEE Trans. Automat. Contr.*, vol. 19, no. 6, pp. 716–723, 1974.

[8] J. Rissanen, “A universal prior for integers and estimation by minimum description length,” *Ann. Stat.*, vol. 11, no. 2, pp. 416–431, 1983.

[9] L. Faes and G. Nollo, “Multivariate frequency domain analysis of causal interactions in physiological time series,” *Biomed. Eng. Trends Electron. Commun. Softw.*, vol. 8, pp. 403–428, 2011.

[10] A. Schlögl, “A comparison of multivariate autoregressive estimators,” *Signal Processing*, vol. 86, no. 9, pp. 2426–2429, 2006.

[11] M. Paluš, “Detecting phase synchronization in noisy systems,” *Phys. Lett. Sect. A Gen. At. Solid State Phys.*, vol. 235, no. 4, pp. 341–351, 1997, doi: 10.1016/S0375-9601(97)00635-X.

[12] T. Schreiber and A. Schmitz, “Improved Surrogate Data for Nonlinearity Tests,” *Phys. Rev. Lett.*, vol. 77, no. 4, pp. 635–638, 1996, doi: 10.1152/ajpheart.1988.255.6.h1535.

[13] B. Efron, “Bootstrap methods: another look at the jackknife,” in *Breakthroughs in statistics: Methodology and distribution*, Springer, 1992, pp. 569–593.

[14] D. N. Politis, “The impact of bootstrap methods on time series analysis,” *Stat. Sci.*, pp. 219–230, 2003.