Exercise 3

- File `Exercise3_Data.xls` in Aula Virtual

- In that file there is a table that contains the number of nights lost to bad weather in a given astronomical observatory, in months and years.
- In this exercise we aim to analyse both the possible seasonal variation and variations in longer timescales of the number of nights lost to bad weather

Exercise 3.1

1. Considering the different months of each year as statistically independent samples, calculate the fraction of nights lost to bad weather each year (the varying number of nights per month have to be taken into account), and estimate the root mean square in that fraction, assuming gaussianity. With this, a sequence of tuples $\{x_i, y_i, \sigma_i\}$ (where x_i is the year, y_i is the fraction of lost nights and σ_i its statistical error) is obtained, to which we will apply the χ^2 statistic.
   a. Assuming that the fraction of lost nights does not vary year-to-year (constant) estimate that constant value, and provide its 90% confidence interval using the method of varying χ^2.
   b. Calculate the goodness of that fit, i.e., the probability that the χ^2 is larger than the measured value under the assumption that the model is correct.
   c. A simple model of climate change predicts that the fraction of nights lost should grow linearly with time (years). Fit a straight line (constant plus slope) and estimate the best-fit values of the two parameters minimizing χ^2. Give 90% and 99% confidence intervals for the slope in that fit.
   d. Using the F-test, what is the probability that the improvement in χ^2 produced by the model of the previous part improves on the constant model?

Exercise 3.2

2. Consider now the seasonal variation, i.e., month to month, calculating for each month the fraction of lost nights, using all available years, and estimating the root mean square of this fraction assuming gaussianity.
   a. Are the data compatible with no seasonal variation (month to month)? To answer this question fit a constant and calculate the probability that the χ^2 value is higher than the measured value if the model were correct.
   b. Assume now a model in which the fraction of lost nights is a constant plus a sinusoidal with a period of one year (12 months). Calculate the best-fit values of the parameters of this model (3 free parameters in total: the constant fc, the amplitude of the sinusoidal function f_e and the phase t_0). Estimate the 90% confidence intervals of the seasonal and constant fractions independently

$$f=f_c+f_e \sin(2\pi(t-t_0)/\tau)$$

   c. Considering the two amplitude parameters f_c and f_e simultaneously, find (using the method of χ^2 variation) the region of that parameter space that contains 95.4% of the probability.