MC-homework

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1 Statistics: MonteCarlo Methods

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1.1 1. Metropolis for hard disks

*Two disks of radius **R** in a square box of side **L** which each walker is made of 4 coordinates. Start with several walkers and iterate checking the convergence of quantities such as the distance between them. Repeat for disks in a sphere of radius **L***

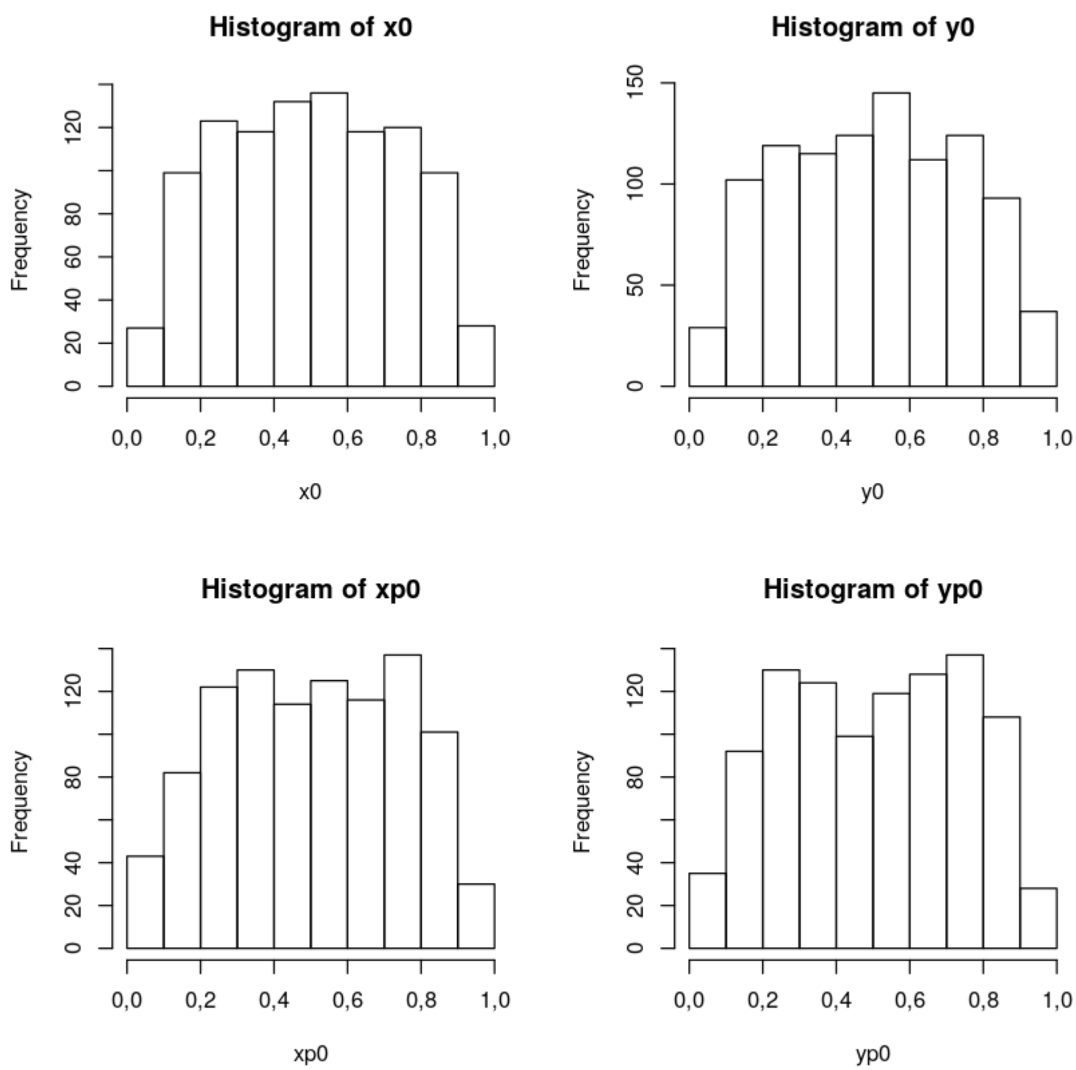
[230]:
```
#Parameters
N <- 1000
L <- 1
R <- 0.2

#Function distance
d <- function(x1, x2, y1, y2){
    X <- (x2 - x1)^2
    Y <- (y2 - y1)^2
    return(sqrt(X + Y))
}
```

[274]:
```
#Walkers
delta_x <- runif(N, 0, 0.2 * L)
delta_y <- runif(N, 0, 0.2 * L)
r0 <- runif(N, 0,  0.8 * L)
rp0 <- runif(N, 0, 0.8 * L)

#Inital positions
x0 <- ifelse(r0 < L, r0 + delta_x, r0) #This always be true since r0, rp0 max
 ↪value is 0.8.
y0 <-  ifelse(r0 < L, r0 + delta_y, r0)

xp0 <-  ifelse(r0 < L, rp0  + delta_x, rp0)
```

```
yp0 <-  ifelse(r0 < L , rp0 + delta_y, rp0)
```

[276]:

```
par(mfrow=c(2,2))
hist(x0)
hist(y0)
hist(xp0)
hist(yp0)
```



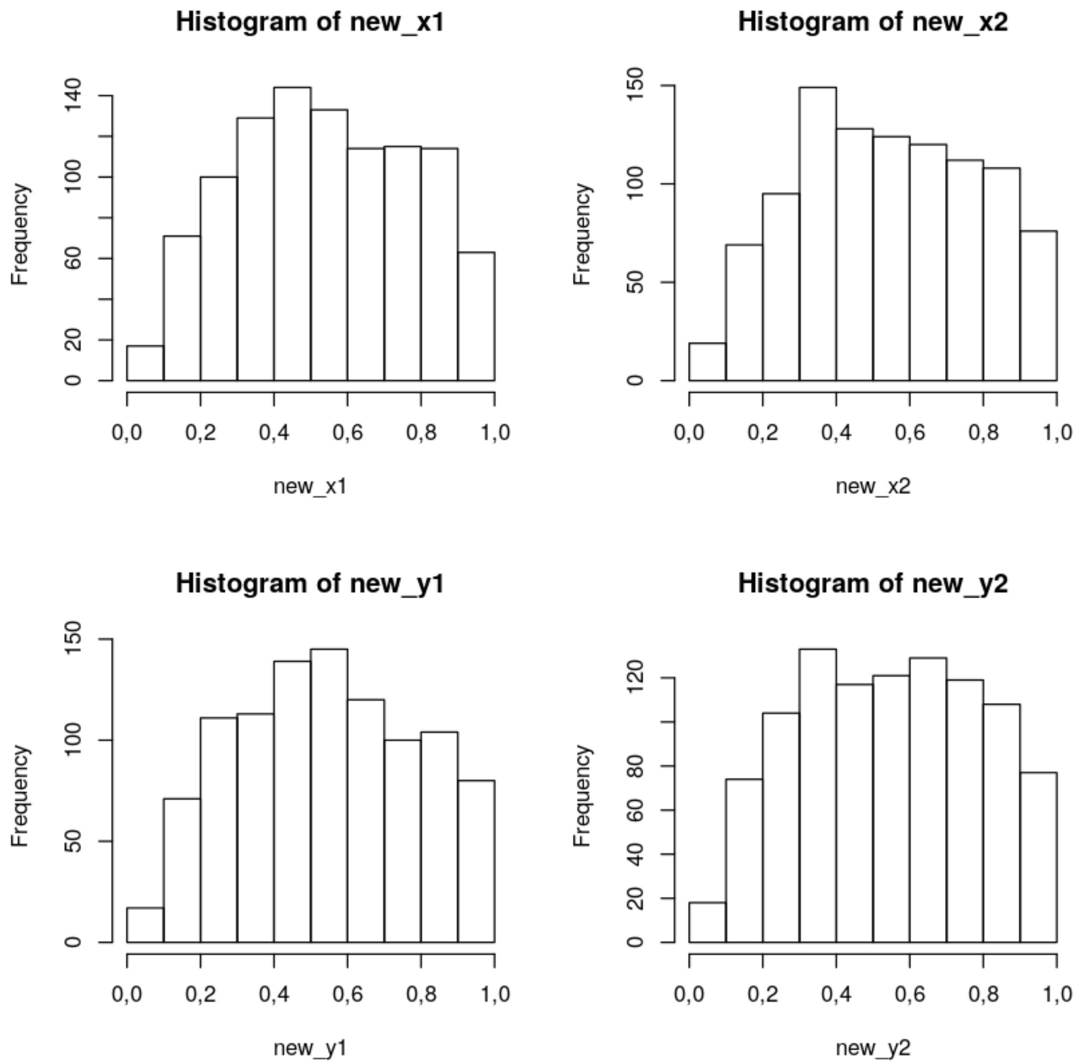
[277]:

```
# Disks' new positions
d0 <- d(x0, xp0, y0, yp0)

x1 <-  ifelse(d0 > (2 * R) & x0 < L, x0+ delta_x, x0)
y1 <-  ifelse(d0 > (2 * R) & y0 < L , y0+ delta_y, y0)
```

```
x2 <-  ifelse(d0 > (2 * R)  & xp0 < L  , xp0+ delta_x, xp0)
y2 <-  ifelse(d0 > (2 * R)  & yp0 < L  , yp0+ delta_y, yp0)
```

[279]:
```
# accepted positions
d1 <- d(x1, x2, y1, y2)

new_x1 <- ifelse(x1 < L & d1 > (2 * R), x1, x0)
new_y1 <- ifelse(y1 < L & d1 > (2 * R), y1, y0)

new_x2 <- ifelse(x2 < L & d1 > (2 * R), x2, xp0)
new_y2 <- ifelse(y2 < L & d1 > (2 * R), y2, yp0)
```

[280]:
```
par(mfrow=c(2,2))
hist(new_x1)
hist(new_x2)
hist(new_y1)
hist(new_y2)
```

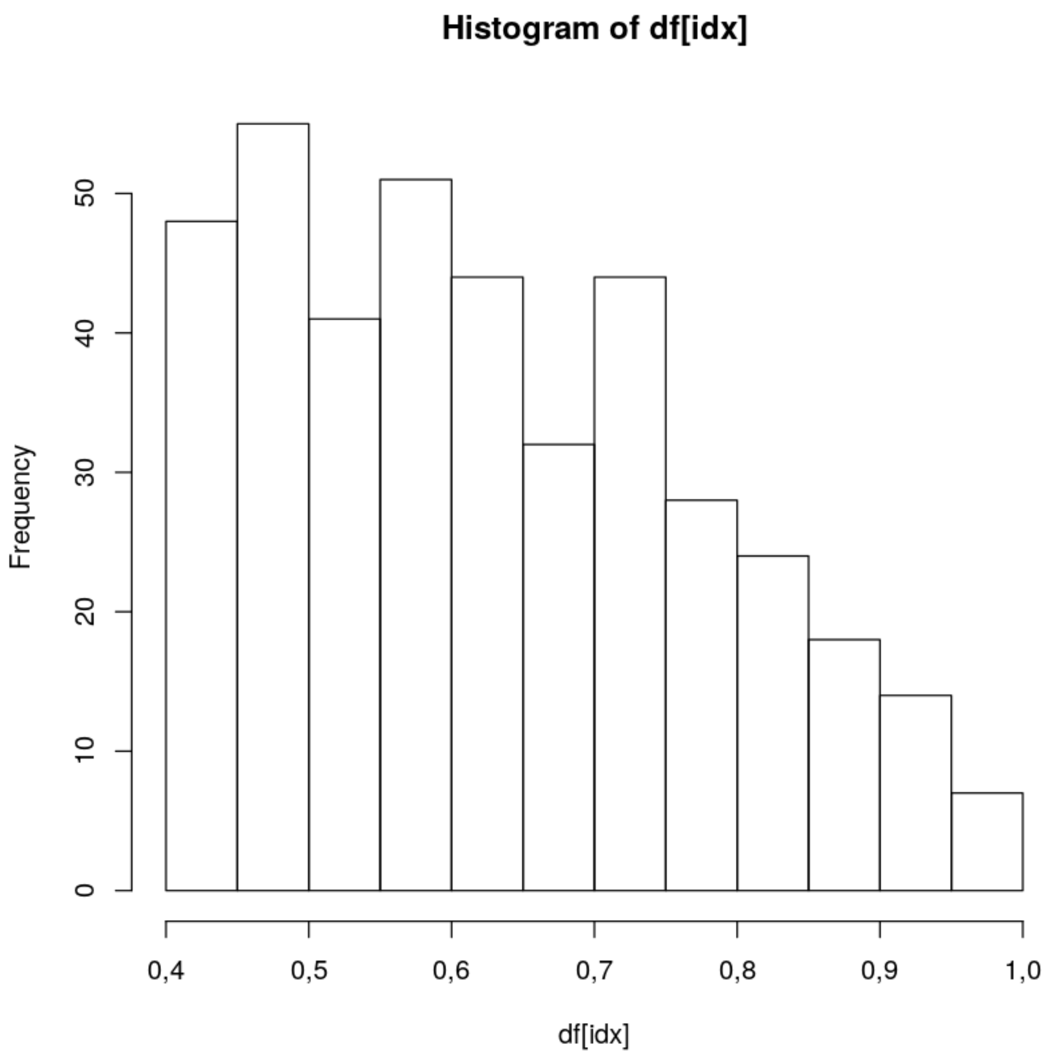


[281]:

```
df <- d(new_x1, new_x2, new_y1, new_y2)
```

[282]:

```
idx <- which(df > (2 * R) & df <= L)
hist(df[idx])
```

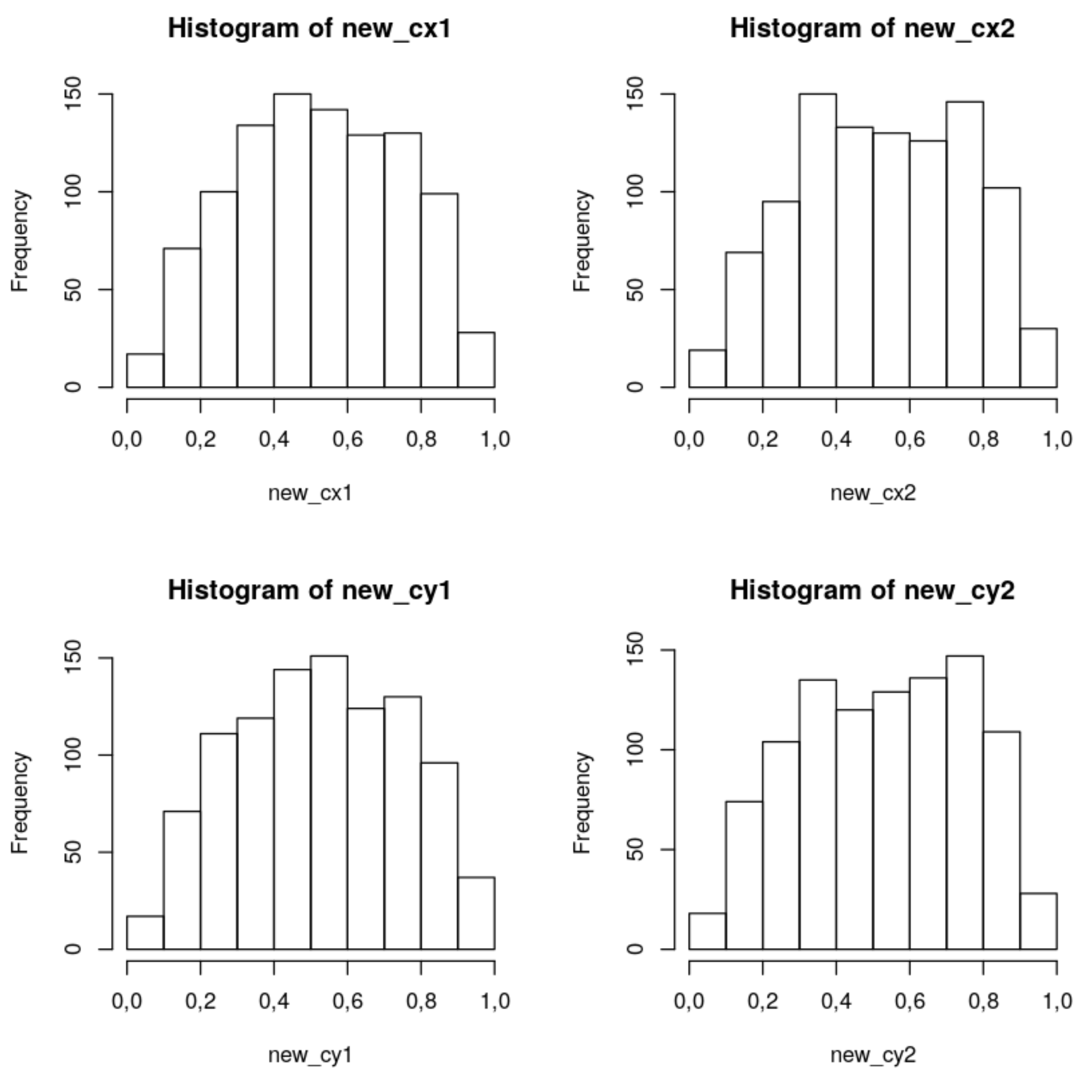


If the disks were in a circle of radius L

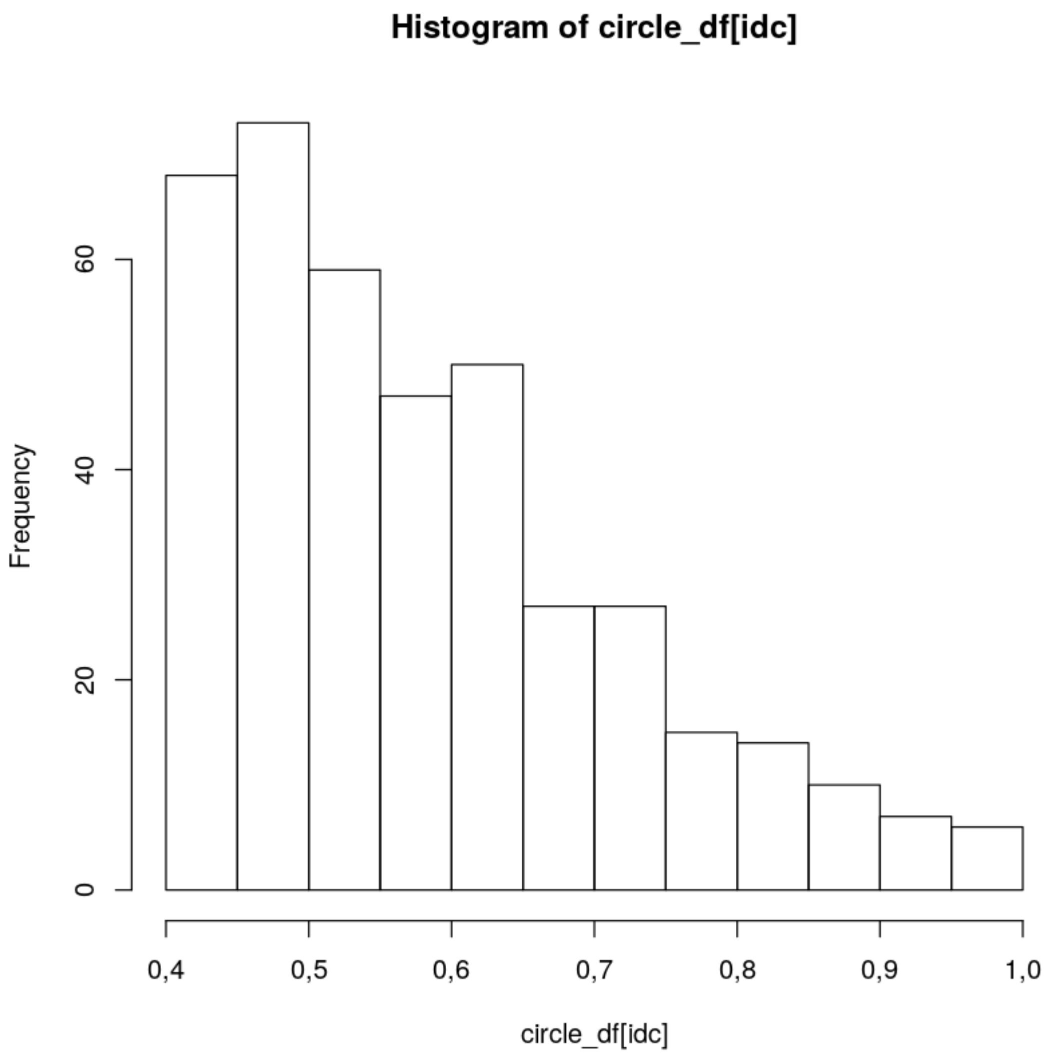
[291]:
```
condition <- function(x, y, R){
    return(x^2 + y^2 < R^2)
}

cx1 <-  ifelse(d0 > (2 * R) & condition(x0,y0,L), x0+ delta_x, x0)
cy1 <-  ifelse(d0 > (2 * R) & condition(x0,y0,L), y0+ delta_y, y0)

cx2 <-  ifelse(d0 > (2 * R)  & condition(xp0,yp0, L), xp0+ delta_x, xp0)
cy2 <-  ifelse(d0 > (2 * R)  & condition(xp0,yp0, L), yp0+ delta_y, yp0)
```

[292]:
```
# accepted positions
dc1 <- d(cx1, cx2, cy1, cy2)

new_cx1 <- ifelse(condition(cx1,cy1,L) & dc1 > (2 * R), cx1, x0)
new_cy1 <- ifelse(condition(cx1,cy1, L) & dc1 > (2 * R), cy1, y0)

new_cx2 <- ifelse(condition(cx2,cy2,L) & dc1 > (2 * R), cx2, xp0)
new_cy2 <- ifelse(condition(cx2,cy2,L) & dc1 > (2 * R), cy2, yp0)
```

[293]:
```
par(mfrow=c(2,2))
hist(new_cx1)
hist(new_cx2)
hist(new_cy1)
hist(new_cy2)
```



[295]:
```
circle_df <- d(new_cx1, new_cx2, new_cy1, new_cy2)
idc <- which(circle_df > (2 * R) & circle_df <= L)
hist(circle_df[idc])
```



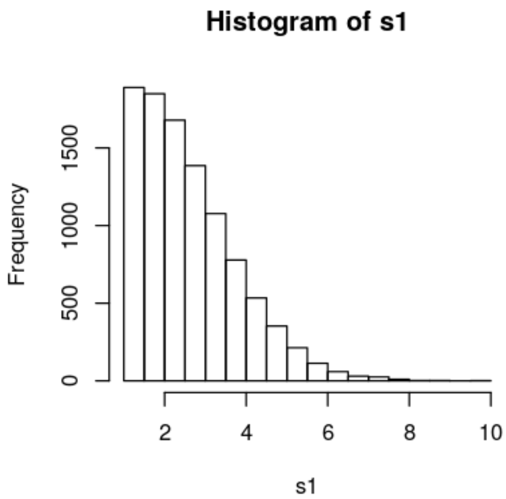
1.2 2. An example of MC-based interference

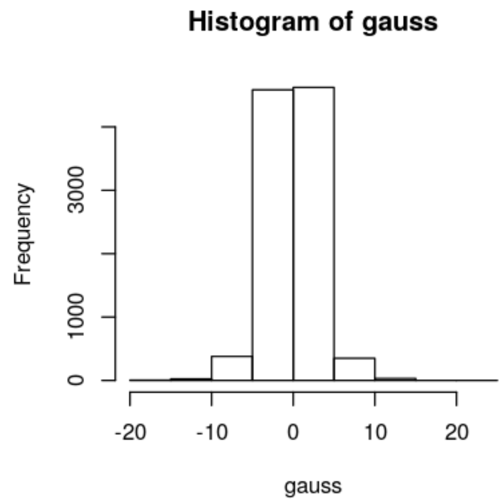
We have N measurements that follow a gaussian law whose σ *is also gaussian* $1 + |\mu|$ *N(0,1). Use a KS to find the optimal* μ

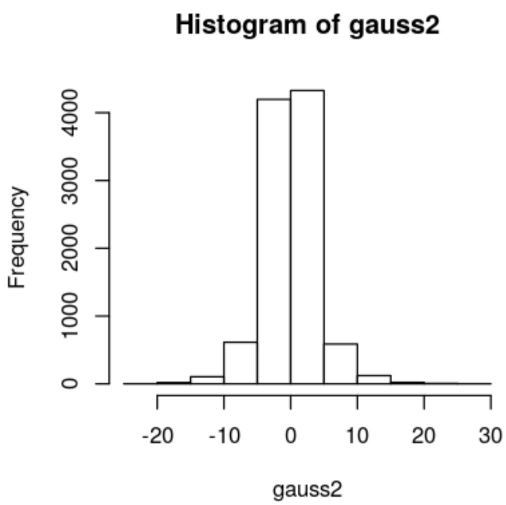
[21]:
```
sigma2 <- function(N, nd){
    return(1 + (abs(nd * rnorm(N, 0, 1))))
}
```

[41]:
```
N <- 1e4
nd <- 2
s1 <- sigma2(N, nd)

gauss <- rnorm(N, 0, s1)

gauss2 <- rnorm(N, 0, sigma2(N, 3))
```

[42]:
```
par(mfrow=c(2,2))
hist(s1)
hist(gauss)
hist(gauss2)
```







[43]:
```
mean(gauss)
```

-0,0267861232716946

[44]:
```
mean(gauss2)
```

0,0434642541848007

[45]:
```
ks.test(gauss, gauss2)
```

```
Two-sample Kolmogorov-Smirnov test

data:  gauss and gauss2
D = 0,0504, p-value = 1,859e-11
```

```
alternative hypothesis: two-sided
```

```
[ ]:
```