FINANCE WEB APP

COS METHOD - Heston Model

A pricing method for European option based on Fourier-Cosine series expansions

This section follows the celebrated works of F. Fang and C.W. Oosterlee in (2008) "A novel pricing method for european option based on fourier-cosine series expansions". The authors derives an option pricing method for European option based on Fourier-cosine series, called COS method as an alternative for the methods based on FFT. The key insight is the close relation of the characteristic function with the series of coefficients of the Fourier-cosine expasion of the function. This method can be adopted also in case of Lévy processes and Heston stochastic volatility model and various types of option contracts. Once the European option prices are determinated with COS method, there is the computation of implied volatilies term structure solving zero of the BS function with Brent's method. This section is dedicated to a stochastic volatility model the popular Heston Model. This model captures the volatility smiles considering a random distributed volatility instead of Black and Sholes Model where the curve is flat and volatility constant.

Form Field

There are 3 different groups (fig 1 COS-method input form) of input parameters. Here we are going to explain the features of each one.

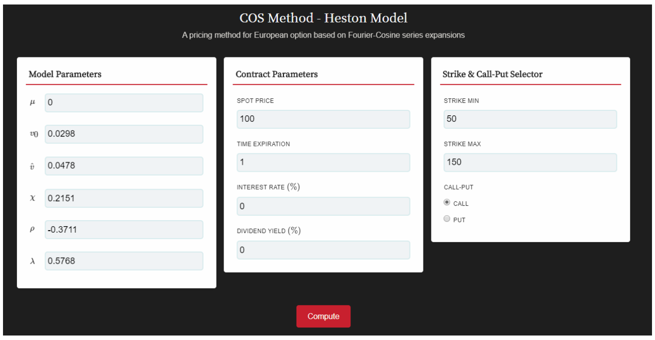

fig.1 COS-method input form

Model Parameters

The Heston model assumes that the underlying stock price, follows a Black- Scholes–type stochastic process, but with a stochastic variance that follows a Cox, Ingersoll, and Ross (1985) process. Hence, the Heston model is represented by the bivariate system of stochastic differential equations (SDEs):

- ❏ v0 : istantaneous variance.
- ❏ $\hat{v}$: mean level of variance/ long rumn variance.
- ❏ ρ : correlation between two Brownian motions(between volatility and the spot asset's price).
- ❏ χ : volatility of volatility.
- ❏ λ : variance mean reversion speed parameter.

Contract Parameters

- ❏ Spot price: the price of underlying asset at t0.
- ❏ Time to expiration: expiring date of the European option.
- ❏ Interest rate: it's an appropriate risk free rate on annual basis.
- ❏ Dividend yield: the dividend yield paid by the underlying asset (annual continuosly compounded).

Strike Call-Put Selector

- ❏ Strike Min - Strike Max: it's the range in which is computed the volatility term structure.
- ❏ Call-Put: A choice between call or put European option prices.

The output

The procedure implemented in this section lead the users to 3 main results. The first one is the graph of the comparison between the probability density function of the stochastic volatility model and the PDF in case of Normal assumption. The second plot represented the volatility term structure for different strike price. Furthermore we have a pop-up table containing the prices of European Option estimated with the COS Method for different strike prices and implied volatilities. The range of table's results depend on the value of strike min and strike max chosen by the users in the form field. The data used for the computation are rapresented in fig.1 (Cos-method input form).

Recovering density function of Heston Model

In the figure number are reproduced two different PDF of returns. The blue line represent the PDF computed using the COS Method starting from the Heston characteristic function and the related parameters. Conversely the red line, the benchmark normal, is the distribution followed by the underlying asset in Normal case, with mean and constant standard deviation resulting from the first and the second moment of the Heston model PDF. This comparison allows to highlight how much the distribution of the underlying deviates from the hypothesis of normality, resulting asymmetrical or having thick tails. This effect is reflected also on the form of implied volatility, as already observed in the Shimko Theorethical and Shimko Market section, isn't flat but smiled.

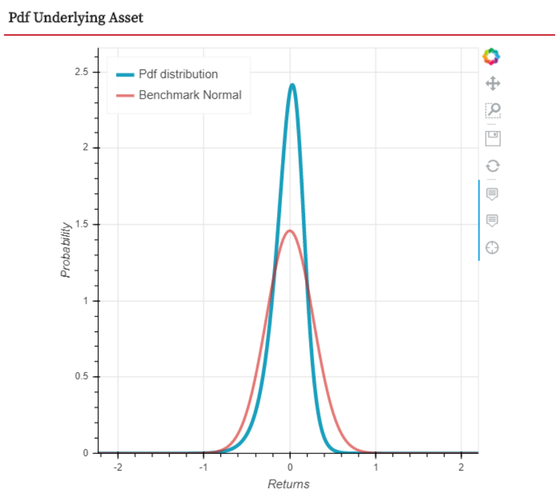

fig.2 Probability density function Heston model

Table of Moments

The table contain the moments of the Heston PDF. Following our pilot case thhe assimetry is rapresented by a negative skewness of -0,069. For this reason the blue line in the plot (fig.2 Probability densitity function Heston model) is negative skewed.

Table of Moments

Mean	Variance	Skewness	Kurtosis
0.0	0.0748	-0.0697	3.0202

fig.3 Table of moments in Heston model

Implied Volatilty

The next step, starting from the prices obtained with the COS method, is to derive the implied volatilities. The procedure implemented finds the zeros of the BS function using the Brent method by varying the implied volatilitiy. It is observable how with the Heston we can obtain a smiled volatility term structure, more rapresentative of market conditions, since the model can capture the typical negative asymmetry. However by reducing the impact of negative correlation (ρ) and the volatility of volatility (χ) we can also reproduce the normal case with an appropriate set of parameters(ρ=-0.00711, χ = 0.00151 and and λ = 0.05768). As we well know in that case the resulting implied volatility is flat, as we can see in the fig., since we have a skewness close to zero.

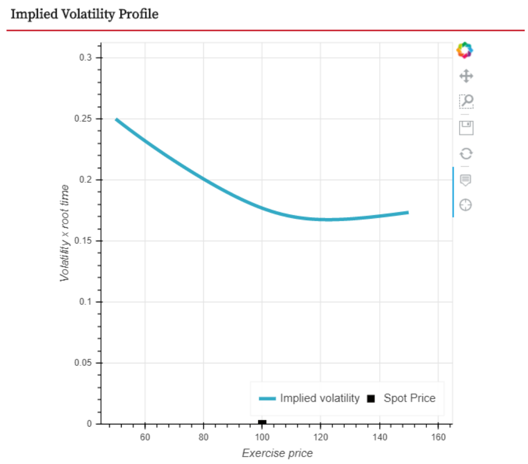

fig.4 Implied volatility vs exercise price in Heston model smiled case

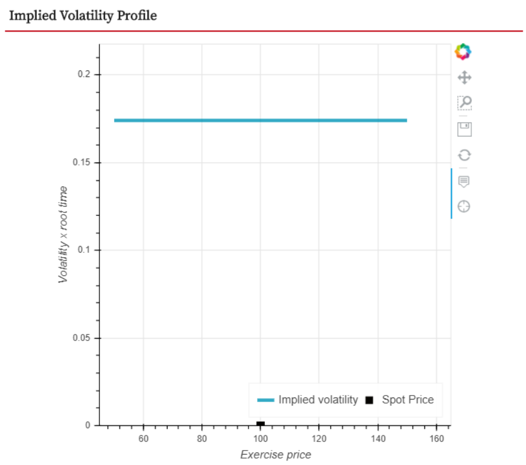

fig.5 Implied volatility vs exercise price in Heston model flat case

Table of Volatility and Price

In the end the user can click on view details this option reproduce a pop up table showing 3 columns

- ❑ Strike: the strikes price generated in a range between Strike Min - Strike Max.
- ❑ Prices : the prices of the European Option estimated with COS Method for each strikes.
- ❑ Implied Volatility : the implied volatility resulting from each pair of strike and price.

Table of Volatility & Prices ×

Strike	Prices	Implied Volatility
50.0	50.014669	0.250034
51.0	49.01786	0.248151
52.0	48.021647	0.24629
53.0	47.026122	0.244449
54.0	46.031388	0.242627
55.0	45.037562	0.240825
56.0	44.044773	0.239041
57.0	43.053165	0.237274
58.0	42.062897	0.235525
59.0	41.074145	0.233793
60.0	40.087102	0.232077
61.0	39.101981	0.230377
62.0	38.119014	0.228692
63.0	37.138452	0.227023

Export Table

fig.6 Table of results in Heston model

These results are downloadable, clicking on the export table bottom, and stored in an Excel file.

	A	B	C
1	Strike	Prices	Implied Volatility
2	50	50,01467	0,250034
3	51	49,01786	0,248151
4	52	48,02165	0,24629
5	53	47,02612	0,244449
6	54	46,03139	0,242627
7	55	45,03756	0,240825
8	56	44,04477	0,239041

data

fig.7 Table of results in Heston model stored in an Excel file

Chart Tools

In each chart there are interactive tools positioned at the top right. Here are listed all of them starting from the first one.

- ❑ Bokeh Logo: hyperlink to access the bokeh site. Bokeh is the library that used to create all interactive graph in the web-application.
- ❑ Pan Tool: the pan tool allows the user to pan the plot by left-dragging a mouse or dragging a finger across the plot region.
- ❑ Box Zoom: the box zoom tool allows the user to define a rectangular region to zoom the plot bounds too, by left-dragging a mouse, or dragging a finger across the plot area.
- ❑ Save: the save tool pops up a modal dialog that allows the user to save a PNG image of the plot.
- ❑ Reset: The reset tool turns off all the selected tools.
- ❑ Hoover Tool: the hover tool will generate a “tabular” tooltip where each row contains a label, and its associated value.

References

F. Fang And C.W. Oosterlee (2008), *"A novel pricing method for european oprions based on Fourier-Cosine series expansions B"*, SIAM J. SCI. COMPUT.

Heston, Steven L. (1993), . *"A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options"*,The Review of Financial Studies.327–343.

`https://docs.bokeh.org/en/1.0.0/`

`https://webappfinance.pythonanywhere.com/`