FINANCE WEB APP

COS METHOD-Lévy Process

A pricing method for European option based on Furier-Cosine series expansions

THIS section follows the celebrated works of F. Fang and C.W. Oosterlee in (2008) "A novel pricing method for european option based on fourier-cosine series expansions". The authors derives an option pricing method for European option based on Fourier-cosine series, called COS method as an alternative for the methods based on FFT. The key insight is the close relation of the characteristic function with the series of coefficients of the Fourier-cosine expasion of the function. This method can be adopted also in case of Lèvy processes and Heston stochastic volatility model and various types of option contracts.

Once the European option prices are determinated with COS method, there is the computation of implied volatilies term structure solving zero of the BS funciton with Brent's method.

In add is reported the return probability distribution related to of the four different Lévy-Processes selectable in the form: GBM (Gaussian Brownian Motion), VG (Variance-Gamma), NIG(Normal Inverse Gaussain) and CGMY.

Form Field

There are 3 different groups (fig 1 COS-method input form) of input parameters. Here we are going to explain the features of each one.

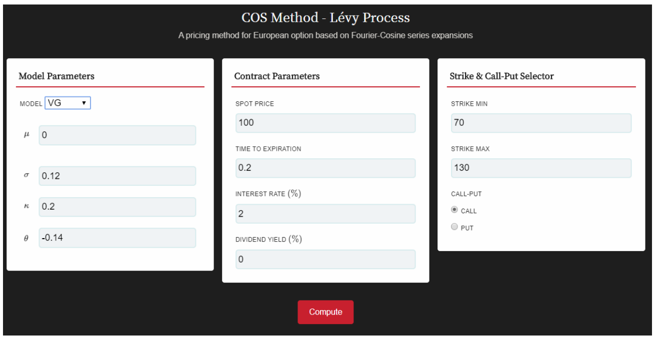

fig.1 COS-method input form

Model Parameters

In this subsection are listed the popular Lévy processes and their parameters selectable in the financewebapp. For more deatails look at the aforementioned paper or at "Lévy Processes in Finance: Pricing Financial Derivatives." Wim Schoutens (Copyright @ 2003 John Wiley Sons, Ltd).

- ❑ GBM (Gaussian Brownian Motion): μ the mean and σ the volatility on annual basis.
- ❑ VG (Variance Gamma): μ, σ, κ and θ
- ❑ NIG (Normal Inverse Gaussain): μ, σ, κ and θ on annual basis.
- ❑ CGMY: μ, C, G, M and Y on annual basis.

Contract Parameters

- ❑ Spot price: the price of underlying asset at to.
- ❑ Time to expiration: expiring date of the European option.
- ❑ Interest rate: it's an appropriate risk free rate on annual basis.
- ❑ Dividend yield: the dividend yield paid by the underlying asset (annual continuosly compounded).

Strike Call-Put Selector

- ❑ Strike Min - Strike Max: it's the range in which is computed the volatility term structure.
- ❑ Call-Put: A choice between call or put European option prices.

The output

The procedure implemented in this section lead the users to 3 main results. The first one (fig.2) is the graph of the comparison between the probability density function of the VG Lévy process (blue line) and the PDF in case of Normal assumption (red line). The second main plot represented the volatility term structure for different strike price (fig.6). Furthermore we have a pop-up table containing the prices of European Option estimated with the COS Method for different strike prices and related implied volatility(fig.8). The range of table's results depend on the value of strike min and strike max chosen by the users in the form field.

Recovering density function of Lévy Processes

In the fig.2 are reproduced two different PDF of returns. The blue line represent the PDF computed using the COS Method starting from the chosen Lévy process characteristic function and the related parameters. Conversely the red line, the benchmark normal, is the distribution followed by the underlying asset in Normal case, with mean and variance in fig.6 (Table of moments in VG model). This comparison serves to highlight how the distribution of the underlying deviates from the hypothesis of normality, resulting asymmetrical or having thick tails. This effect is reflected on the shape of implied volatility curve as already observed in the Shimko Theorethical and Shimko Market section.

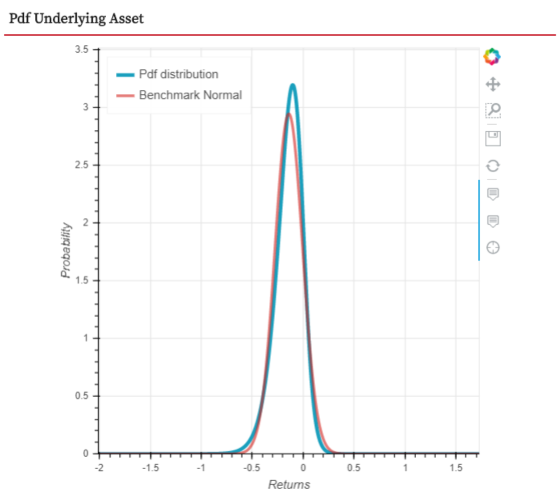

fig.2 Probability density function VG Model vs Benchmark Normal

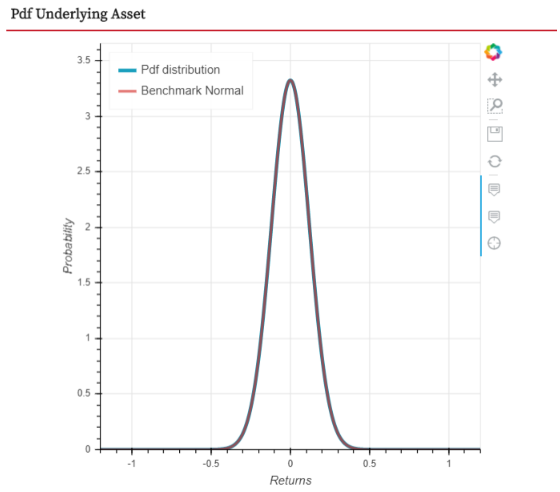

fig.3 Probability density function GBM Model ()μ = 0, σ = 0,12) vs Benchmark Normal

Table of Moments

The table contain the moments of the PDF of the model selected in the input form. Following our pilot case, we can observe the data relating to the VG model and the GBM model which are shown in the figures below.

Table of Moments

Model	Mean	Variance	Skewness	Kurtosis
VG	-0.14	0.0183	0.5303	8.4078

fig.4 Table of Moments in VG Model

Table of Moments

Model	Mean	Variance	Skewness	Kurtosis
Normal	0.0	0.04	0	3

fig.5 Table of Moments GBM model μ = 0, σ = 0.12

Implied Volatilty

The next step, starting from the prices obtained with the COS method, was to derive implied volatilities. The procedure implemented finds the zeros of the BS function using the Brent method varying the implied volatilitiy. It is observable that by selecting one of the Lévy processes (different from GBM) starting from the relative prices we can obtain a smiled volatility term structure more consistent with a case of non-normality. The user can also represent the implied volatility in Normal case selecting in the form the Gaussian Brownian Motion. As we well know in that case the resulting implied volatility is flat, as we can see in the fig.7 , since we have a symmetric distribution.

fig.6 Implied Volatility VG Model

fig.7 Implied Volatility GBM Model $\mu = 0$, $\sigma = 0.12$

Table of Volatility & Prices

Strike	Prices	Implied Volatility
70.0	31.441412	0.166152
71.0	30.473258	0.164894
72.0	29.507384	0.163645
73.0	28.544151	0.162405
74.0	27.583964	0.161172
75.0	26.627277	0.159947
76.0	25.674592	0.15873
77.0	24.726465	0.15752
78.0	23.783508	0.156318
79.0	22.846391	0.155124
80.0	21.915843	0.153937
81.0	20.992655	0.152758
82.0	20.077678	0.151586
83.0	19.171824	0.150422

Export Table

fig.8 Table of results in VG Model

These results are downloadable clicking on the export table bottom and stored in an Excel file.

	A	B	C
1	Strike	Prices	Implied Volatility
2	70	31,44141	0,166152
3	71	30,47326	0,164894
4	72	29,50738	0,163645
5	73	28,54415	0,162405
6	74	27,58396	0,161172
7	75	26,62728	0,159947
8	76	25,67459	0,15873

fig.9 Table of results in VG Model stored in an Excel File

Table of Volatility and Price

In the end the user can click on view details, this option reproduce a pop up table showing 3 columns

- Strike: the strikes price generated in a range between Strike Min - Strike Max.

- Prices : the prices of the European Option estimated with COS Method for each strikes.

- Implied Volatility : the implied volatility resulting from each pair of strike and price.

Chart Tools

In each chart there are interactive tools positioned at the top right. Here are listed all of them starting from the first one.

- Bokeh Logo: hyperlink to access the bokeh site. Bokeh is the library that used to create all interactive graph in the web-application.

- Pan Tool: the pan tool allows the user to pan the plot by left-dragging a mouse or dragging a finger across the plot region.

- Box Zoom: the box zoom tool allows the user to define a rectangular region to zoom the plot bounds too, by left-dragging a mouse, or dragging a finger across the plot area.

- ❑ Save: the save tool pops up a modal dialog that allows the user to save a PNG image of the plot.
- ❑ Reset: The reset tool turns off all the selected tools.
- ❑ Hoover Tool: the hover tool will generate a "tabular" tooltip where each row contains a label, and its associated value.

References

F. Fang And C.W. Oosterlee (2008), *"A novel pricing method for european oprions based on Fourier-Cosine series expansions B"*, SIAM J. SCI. COMPUT.

Wim Schoutens (2003), *"Lévy Processes in Finance: Pricing Financial Derivatives."*, (Copyright @ 2003 John Wiley Sons, Ltd).

`https://docs.bokeh.org/en/1.0.0/`

`https://webappfinance.pythonanywhere.com/`