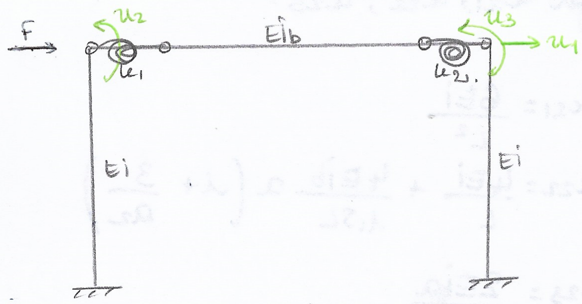
Problem 1:

The two first questions are answered by hand.

Question 1: Computation of the condensed stiffness matrix Ktotal (3x3).

Generalities:
- the flexural stiffness of the beam is very high, therefore there is no rotation stiffness
- as the springs have a different stiffness, the rotations are different at each extremity of the beam.



$$K = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \end{array}\begin{array}{c} \begin{array}{ccc} u_1 & u_2 & u_3 \end{array} \\ \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} \end{array}$$

- For an elastic beam element with rotational springs, we assume:

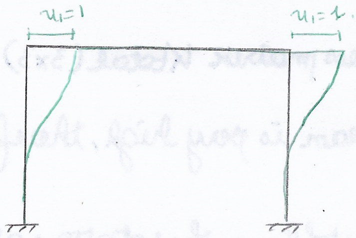
$k_1 = a_1 \frac{EI_b}{L_b}$ and $k_2 = a_2 \frac{EI_b}{L_b} = 2k_1$.

Therefore:
$$\begin{cases} a_1 = \dfrac{k_1 \cdot L_b}{EI_b} = \dfrac{8{,}1 \cdot 10^{10} \cdot 1{,}5 \cdot 3000}{200000 \cdot 3{,}67 \cdot 10^9} = 0{,}497\ rad^{-1} \\ a_2 = \dfrac{k_2 L_b}{EI_b} = 2\left(\dfrac{k_1 L_b}{EI_b}\right) = 2a_1 = 0{,}993\ rad^{-1} \end{cases}$$

$$a = \frac{a_1 a_2}{a_1 a_2 + 4a_1 + 4a_2 + 12} = 0{,}027.$$

- We determine the different stiffness k_{ij} using abacus and the "spring and beam" element stiffness matrix seen in class.

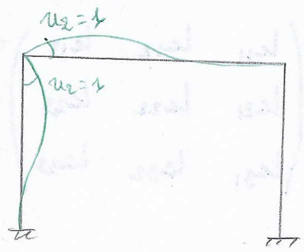
- if $u_1 = 1$, $u_2 = 0$ and $u_3 = 0$, we get k_{11}, k_{12}, k_{13}:



$$k_{11} = 2 \cdot \frac{12 E i_c}{L^3}$$

$$k_{12} = k_{13} = \frac{6Ei}{L^2}$$

- if $u_1 = 0$, $u_2 = 1$, $u_3 = 0$, we get k_{21}, k_{22}, k_{23}:

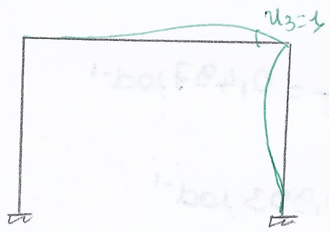


$$k_{21} = \frac{6Ei}{L^2}$$

$$k_{22} = \frac{4Ei}{L} + \frac{4Ei_b}{1{,}5L} a \left(1 + \frac{3}{a_2}\right)$$

$$k_{23} = \frac{2Ei_b a}{1{,}5L}$$

- if $u_1 = 0$, $u_2 = 0$, $u_3 = 1$, we get k_{31}, k_{32}, k_{33}:



$$k_{31} = \frac{6Ei}{L^2}$$

$$k_{32} = \frac{2Ei_b a}{1{,}5L}$$

$$k_{33} = \frac{4Ei}{L} + \frac{4Ei_b a}{1{,}5L} \left(1 + \frac{3}{a_1}\right)$$

$$K_{tot} = \begin{pmatrix} \frac{24Ei}{L^3} & \frac{6Ei}{L^2} & \frac{6Ei}{L^2} \\ \frac{6Ei}{L^2} & \frac{4Ei}{L} + \frac{4Ei_b}{1{,}5L} a \left(1 + \frac{3}{a_2}\right) & \frac{2Ei_b a}{1{,}5L} \\ \frac{6Ei}{L^2} & \frac{2Ei_b a}{1{,}5L} & \frac{4Ei}{L} + \frac{4Ei_b a}{1{,}5L} \left(1 + \frac{3}{a_1}\right) \end{pmatrix}$$

With numerical values:

$$K_{tot} = \begin{pmatrix} 65244 & 4{,}893 \cdot 10^7 & 4{,}893 \cdot 10^7 \\ 4{,}893 \cdot 10^7 & 1{,}687 \cdot 10^{11} & 8{,}808 \cdot 10^9 \\ 4{,}893 \cdot 10^7 & 8{,}808 \cdot 10^9 & 2{,}218 \cdot 10^{11} \end{pmatrix} [N/mm].$$

Question 2 : Computation of lateral displacement

As the global stiffness matrix is known, as well as the lateral force applied on the MRF, the displacements can be calculated as follow:

$$F = K \cdot \Delta \Leftrightarrow \Delta = K^{-1} F$$

The inverse matrix of K is calculated with Python and $F = \begin{pmatrix} 500 \\ 0 \\ 0 \end{pmatrix} kN$

$$\Delta = K^{-1} \cdot \begin{pmatrix} 500 \cdot 10^3 \\ 0 \\ 0 \end{pmatrix} \rightarrow \Delta x = \underline{12{,}1 \ mm}$$

The horizontal displacement is 12.1mm.

3. <u>Verification of the results using OpenSeesNavigator:</u>

The steel MRF had been modelled in OpenSeesNavigator, as shown below:

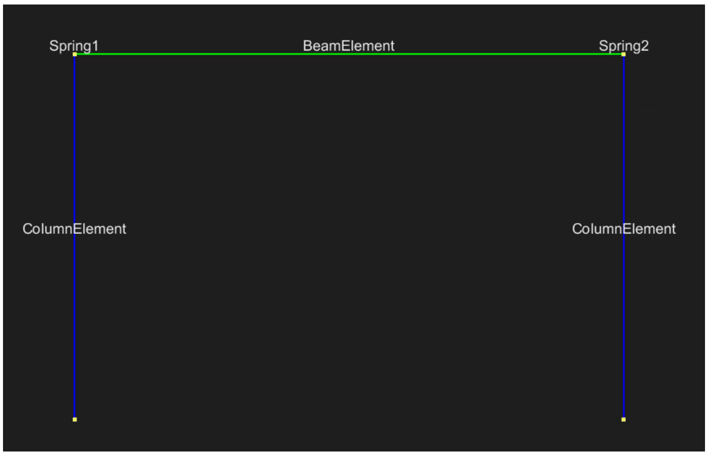


Figure 1: Elements defined in OpenSeesNavigator for the steel MRF

In this problem, as linear geometric transformation are considered, the columns and beam elements are considered elastic (the stresses in the elements are always below the yield limit of the material). The spring are modelled as Zero-Length elastic elements. Their respective stiffnesses have been defined in the software.

Applying a lateral load of 500kN on the frame, a displacement of 12.35mm is obtained.

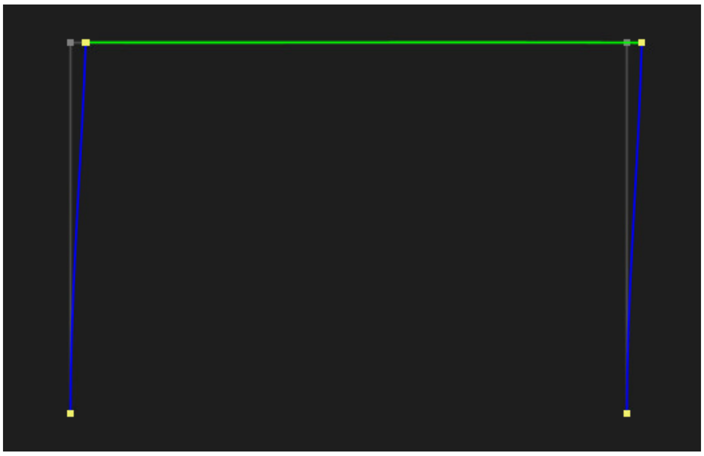


Figure 2: Displacement of the MRF under a lateral load of 500kN (the displacement has been increased by a factor 10)

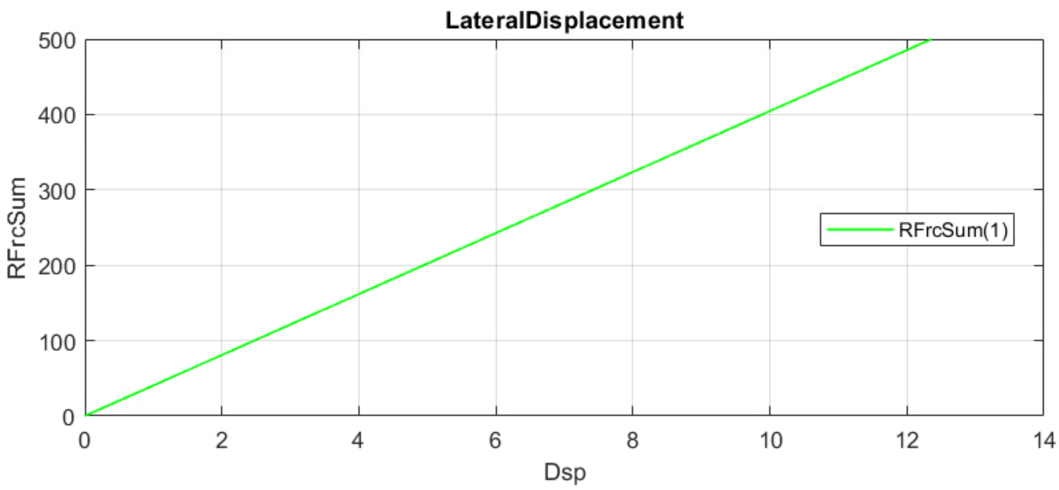


Figure 3: Lateral displacement as a function of the lateral load applied on the MRF

The displacement obtained is: Δ=12.36mm, which is very similar to the displacement calculated by hand calculation (difference of 2.1%).

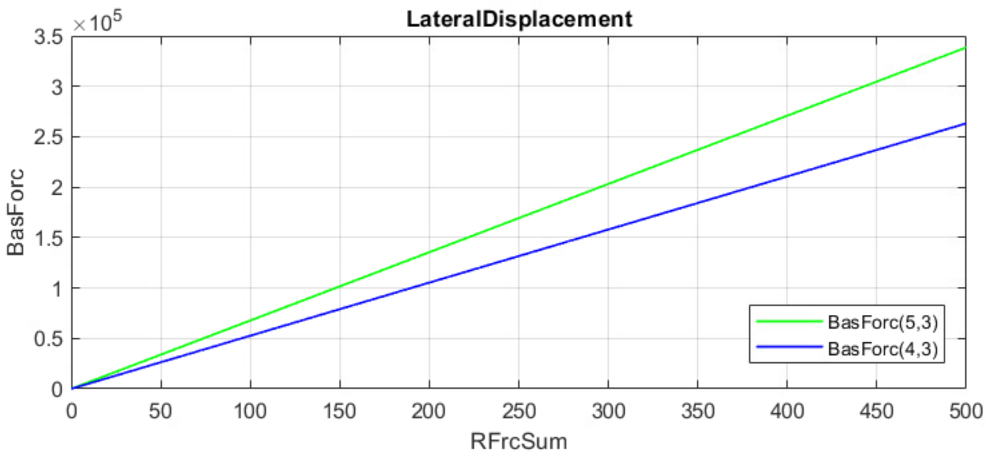


Figure 4: Force inducing bending moments in the springs (blue graph for the left spring and green graph for the right spring)

For a load of 500 kN, the maximum moment obtained in the springs is 263323 kNm for left spring and 338659 kNm for right spring.

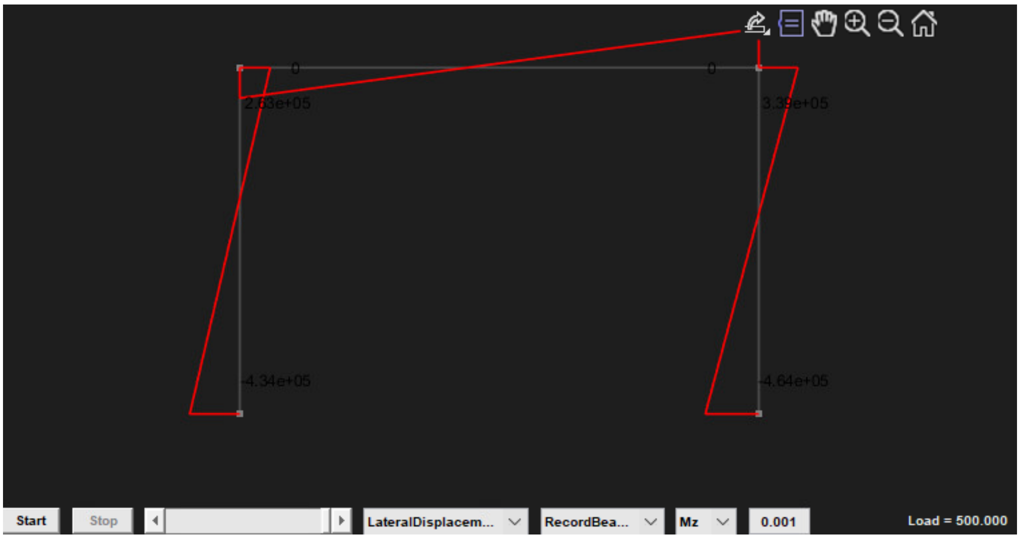


Figure 5: Bending moment diagram in the MR frame

4. For a uniform elastic element, the area and second moment of inertia are constant. Assuming a very large area for the elements enables to ignore possible small changes in the cross-section area of the elements. Moreover, the axial stresses can be ignores as for a very large area, axial stresses would not be predominant.

Problem 2:

The geometric model (elements, supports conditions) from problem 1 is conserved for the second problem.

As a degrading model is implemented, displacement-based calculations are introduced, by displacement of 1mm increments on the node at the top left of the frame (node 2) in the horizontal direction. The analysis is done with 600 displacement increments, in order to give a broad range of analysis of the response of the structure, for a total roof displacement of 600 mm.

Degradation material modelization:

The empirical relationships for non-composite steel beams gives:

$$M_y^* = 1.15 \cdot \gamma_{rm} \cdot W_{pl,y} \cdot f_y = 1.00 \cdot 10^9 \, Nmm$$

$\gamma_{rm} = 1.25$ for S355 steel.

$$M_u = 1.11 \cdot M_y^* = 1.11 \cdot 10^9 \, Nmm$$

The residual moment M_r is set equal to 0.

The associated plastic deformation parameters are computed as following:

$$\theta_p = 0.0885 \left(\frac{h_1}{t_w}\right)^{-0.365} \left(\frac{b}{2 \cdot t_f}\right)^{-0.14} \left(\frac{L_o}{h}\right)^{0.34} \left(\frac{h}{533}\right)^{-0.721} \left(\frac{f_y}{355}\right)^{-0.23}$$

$$\theta_{pc} = 5.63 \left(\frac{h_1}{t_w}\right)^{-0.565} \left(\frac{b}{2 \cdot t_f}\right)^{-0.80} \left(\frac{h}{533}\right)^{-0.28} \left(\frac{f_y}{355}\right)^{-0.43}$$

$$\theta_u = 0.20 rad$$

Giving, with the parameters given, and L_0 the length of the beam:

$$\theta_p = 0.0415$$

$$\theta_{pc} = 0.1175$$

The considered loading is monotonic, therefore these equations are correct.

The yield stain θ_y is computed using

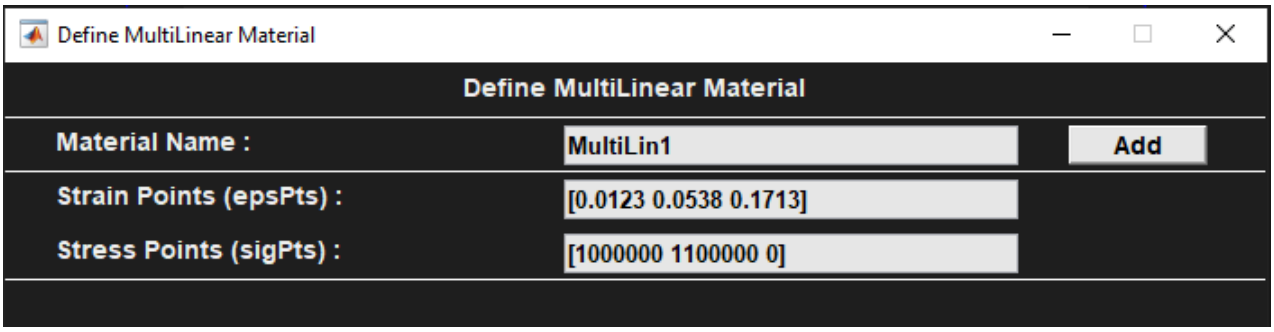
$$\theta_{y_1} = \frac{M_y^*}{k_1} = 0.0123$$

$$\theta_{y_2} = \frac{M_y^*}{k_2} = 0.0247$$

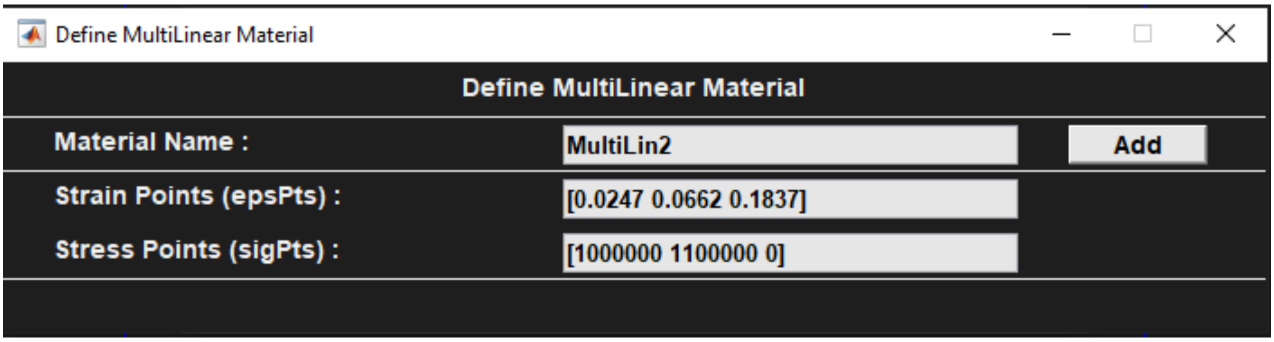
As the stiffness for each spring is conserved in this second part. It does not influence the yield moment, ultimate moment and residual moment values.

Using the parameters defined previously, two multi-linear materials are defined, one for each connection, in order to model the plastic behaviour :

Right connection zero-element material:



Left connection zero-element material:



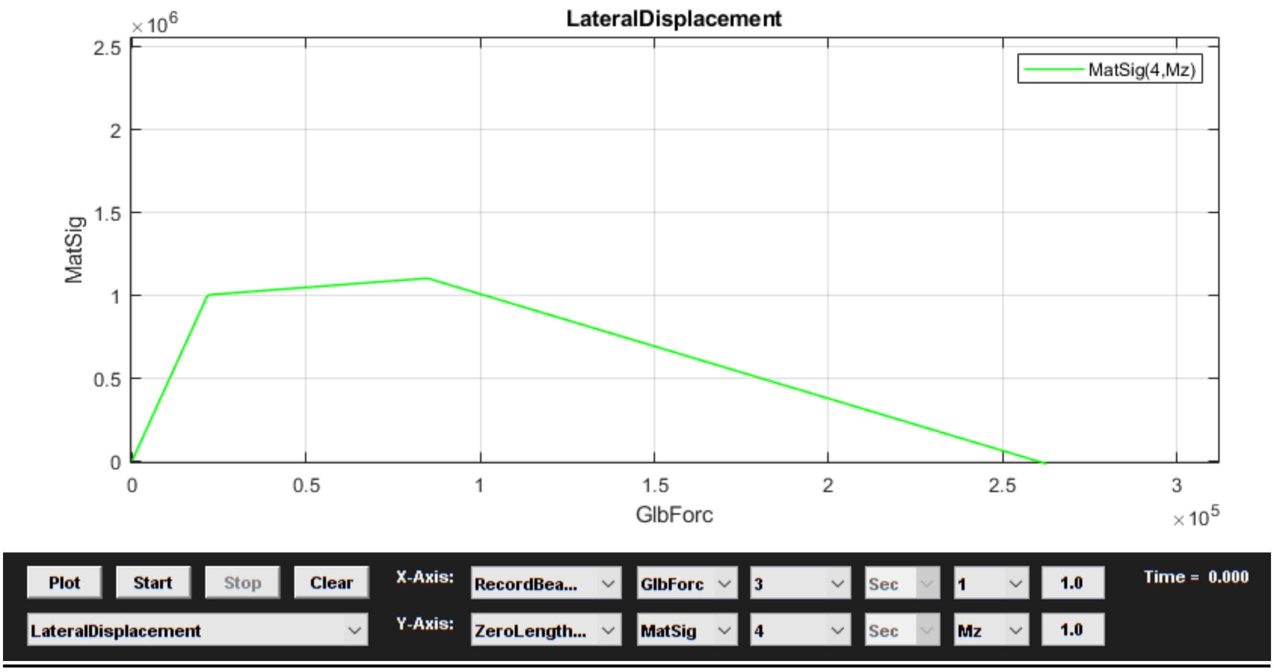
The displacement-increment analysis is then computed.

Results:

The left joint is expected to be the first to reach strength degradation, as the strain θ_{sd} needed is smaller for this element than for the right connection. It is indeed observed in the model.

In order to compute the applied load at which degradation start, moment in the left connection as function of load in the first direction of the beam element is computed.

The load in the first direction of the beam element is equal to the applied load by the displacements in the first direction at node 2.



We can observe that the point where strength degradation begins is reached for an horizontal applied load of 84450 kN, or 84.5 MN. Which corresponds to a roof displacement of 341 mm.

Story shear strength:

The story shear strength is defined as the maximal lateral load stress the frame can withstand before rupture.

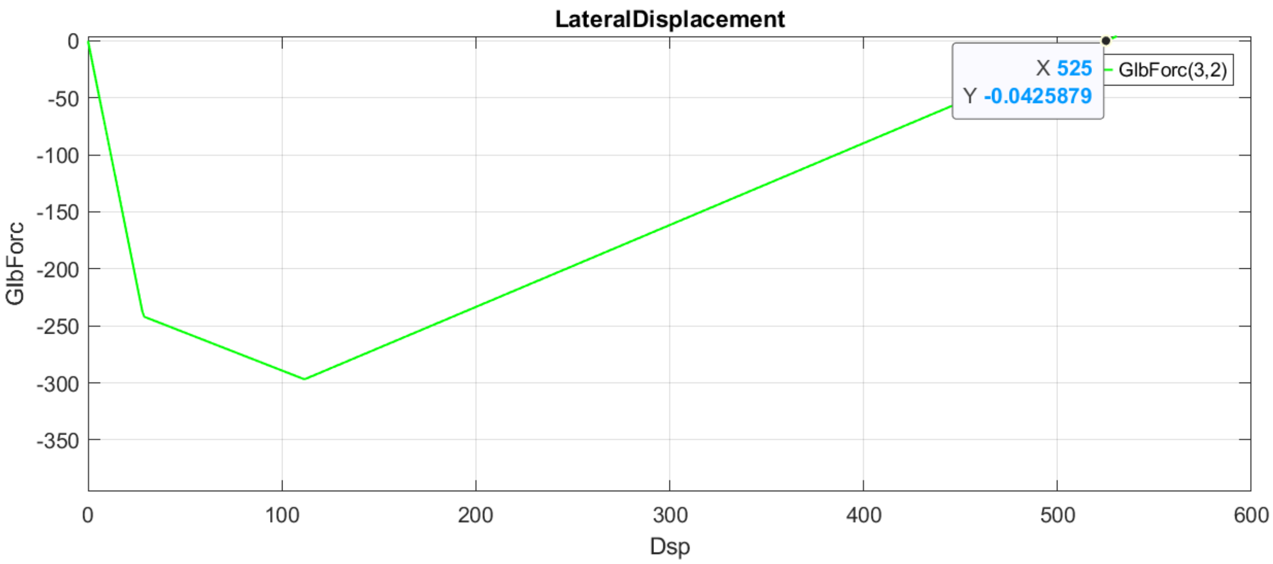
In order to determine the story shear strength of the steel beam element, the strength of the non-zero connection elements must be determined, as well as the shear resistance of the beam in itself.

Using the same analysis as for the previous step, the global force of the beam element for the second and third (degree of freedom are plotted. Second degree of freedom corresponds to the shear of the beam, and the third to the rotation of the left connection.

Theses plots allow to observe the roof displacement at which the degrading effect reduce the strength of the considered element in the considered direction to zero.

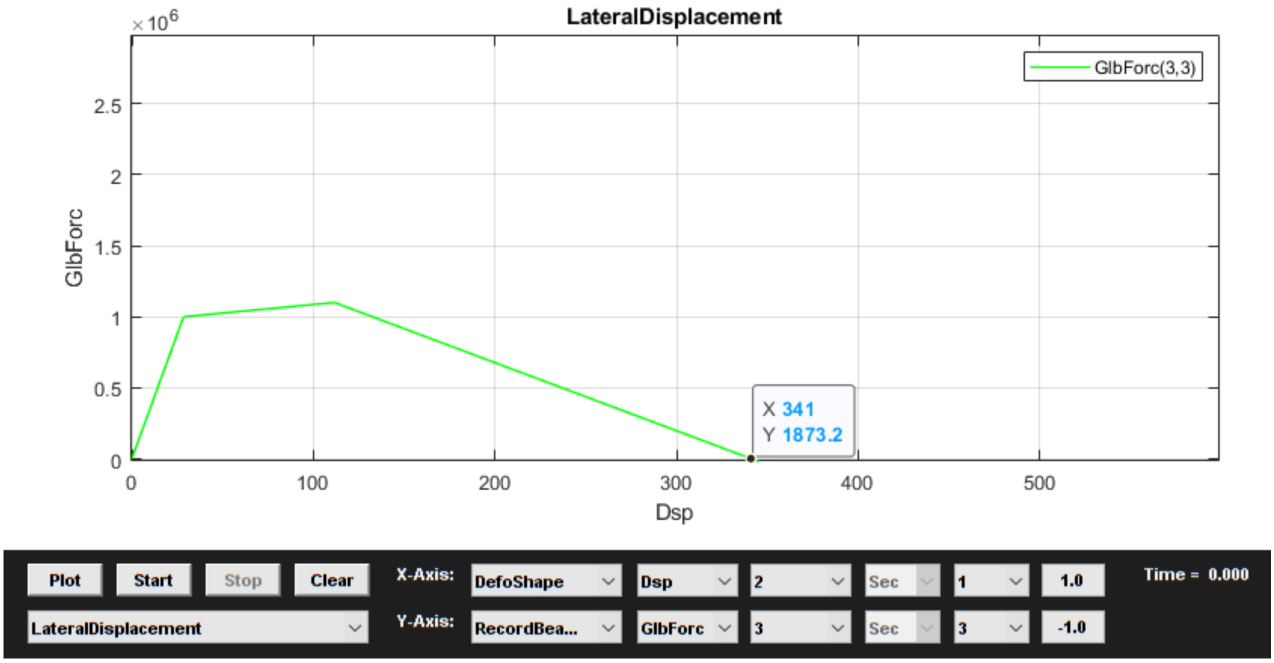
The smaller of theses two values is determinant for the story shear resistance of the steel beam element.

<u>Second degree of freedom (shear of the beam):</u>



The shear strength of the beam is reduced to zero for an horizontal displacement of the roof equal to 525 mm.

<u>Third degree of freedom (rotational resistance of the beam):</u>



The rotational strength of the beam is reduced to zero for a displacement of 341 mm.

As the rotational strength is determinant here, the story shear strength of the beam is reduced to zero for an horizontal displacement of the roof equal to 341 mm.