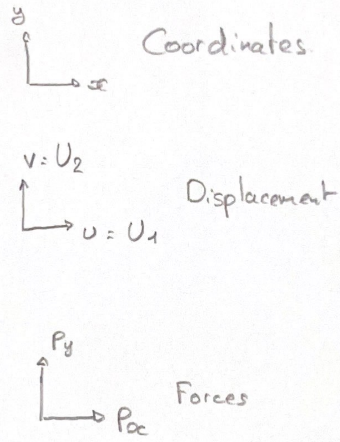
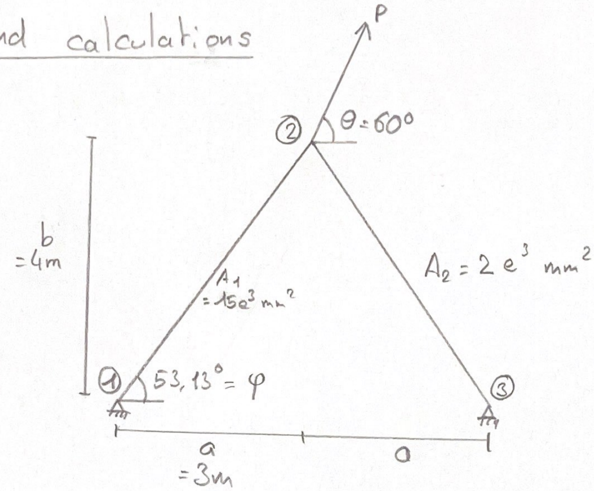
- Assignment 2 -

Exercise 1 - Hand calculations





A - System

E = 200 GPa

σ_y = 500 MPa

α = 0.02

Step 1: As a first step, we decided to apply a force $P = 10 \times 10^6$ N

Initial stiffnesses: $K_1 = \frac{E_1 A_1}{L_1} = 6 \times 10^5 \ \mathrm{N.mm^{-1}}$ $\qquad K_2 = \frac{E_2 A_2}{L_2} = 8 \times 10^4 \ \mathrm{N.mm^{-1}}$

$$\vec{P} = \begin{bmatrix} 5 \\ 8{,}66 \end{bmatrix} \times 10^6 \ \mathrm{N}$$

Connectivity: $\begin{bmatrix} [1,2] \\ [2,3] \end{bmatrix}$

$$F_y = \begin{pmatrix} 7{,}5 \\ 1 \end{pmatrix} \times 10^6 \ \mathrm{N}$$ Yielding forces in bars $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

B - Transformation matrices:

From assignment 1, $T = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 & 0 \\ -\sin\varphi & \cos\varphi & 0 & 0 \\ 0 & 0 & \cos\varphi & \sin\varphi \\ 0 & 0 & -\sin\varphi & \cos\varphi \end{pmatrix}$

With $\varphi = \pm 53{,}13°$ $\begin{pmatrix} + \rightarrow \text{Bar } 1 \\ - \rightarrow \text{Bar } 2 \end{pmatrix}$

C - Reduced stiffness matrices in global coordinates:

$$\left(K_{red} = \begin{bmatrix} 244\,801{,}75 & 249\,600{,}85 \\ 249\,600{,}85 & 435\,199{,}87 \end{bmatrix} \right) \cdot \underbrace{\begin{pmatrix} u \\ v \end{pmatrix}}_{U_1} = \begin{pmatrix} 5 \\ 8{,}66 \end{pmatrix} \times 10^6$$

$$\Rightarrow \begin{cases} u = 0.32535 \\ v = 19.71289 \end{cases} \quad [\mathrm{mm}]$$

Axial forces in elements at step 1; with step load P:

$P_1' = K_1' \cdot T_1 \cdot U_1$

$= \begin{pmatrix} -9,58 \\ 0 \\ 9,58 \\ 0 \end{pmatrix} \times 10^6 \, N$

$\boxed{\lambda = 0,7829} = \frac{7,5}{9,58}$

$P_2' = K_2' \cdot T_2 \cdot U_1$

$= \begin{pmatrix} -1,25 \\ 0 \\ 1,25 \\ 0 \end{pmatrix} \times 10^6 \, N$

$\lambda = 0,8 = \frac{1}{1,25}$

$P^1 = \lambda P = \begin{bmatrix} 3,915 \\ 6,780 \end{bmatrix} \times 10^6 \, N$ ← Applied load at element 1 plastification

With $K_{red} \cdot U = P^1$

$\Rightarrow U^1 = \begin{pmatrix} 0,25473 \\ 15,43394 \end{pmatrix} mm$, displacement of node 2 at plastification of bar 1.

Step 2: Second element plastification

New $k_1 = \alpha k_1 = 1,2 \times 10^4 \, N.mm^{-1}$

$\Rightarrow \left(K_{red} = \begin{pmatrix} 33120,24 & -32640,11 \\ -32640,11 & 58879,98 \end{pmatrix} \right) \cdot \underbrace{\begin{pmatrix} u \\ v \end{pmatrix}}_{\Delta U^2} = \underbrace{\begin{pmatrix} 5 \\ 8,66 \end{pmatrix}}_{\Delta P^2 = P} \times 10^6 \, N$

$\Rightarrow \Delta U^2 = \begin{pmatrix} 652,25 \\ 508,66 \end{pmatrix} mm$

Axial force in elements at step 2, with step load P:

$P_1' = K_1' \cdot T_1 \cdot U_2$

$= \begin{pmatrix} -9,58 \\ 0 \\ 9,58 \\ 0 \end{pmatrix} \times 10^6 \, N$

$P_2' = K_2' \cdot T_2 \cdot U_2$

$= \begin{pmatrix} -1,25 \\ 0 \\ 1,25 \\ 0 \end{pmatrix} \times 10^6 \, N \Rightarrow P_2^2 = 1,25 \times 10^6 \, N$

$\lambda = \frac{F_y - P_2^1}{P_2^2}$

$$P_2^{1'} = K_2' \cdot T_2 \cdot U^1 = \begin{pmatrix} -9,755 \\ 0 \\ 9,755 \\ 0 \end{pmatrix} \times 10^5 \, N$$

$$\Rightarrow P_2^1 = 9,755 \times 10^5 \, N$$

$$\Rightarrow \lambda = \frac{1 - 0,9755}{1,25} = 0,019626$$

$$\Rightarrow \Delta P^2 = \lambda P = \begin{bmatrix} 9,81244 \\ 16,99652 \end{bmatrix} \times 10^4 \, N.$$

$$\Rightarrow \Delta U^2 = \lambda \Delta U^2 = \begin{bmatrix} 12,80 \\ 9,98 \end{bmatrix} mm$$

$$\Rightarrow U_2 = U_1 + \Delta U^2 = \begin{bmatrix} 13,05 \\ 25,42 \end{bmatrix} mm \quad \& \quad P_2 = P_1 + \Delta P^2 = \begin{bmatrix} 4,013 \\ 6,950 \end{bmatrix} \times 10^6 \, N$$

Step 3 - Asymptotic behavior

New $k_2 = \alpha k_2 = 1,6 \times 10^3 \, N.mm^{-1}$

$$\Rightarrow \left(K_{red} = \begin{pmatrix} 4896,03 & 4992,02 \\ 4992,02 & 8704,00 \end{pmatrix} \right) \cdot \underbrace{\begin{pmatrix} u \\ v \end{pmatrix}}_{\Delta U^3} = \underbrace{\begin{bmatrix} 1,9626 \\ 3,3993 \end{bmatrix}}_{\Delta P^3 = 2 \cdot \Delta P^2} \times 10^5 \, N$$

Indicative load.

$$\Rightarrow \Delta U^3 = \begin{bmatrix} 0,639 \\ 38,688 \end{bmatrix} mm.$$

$$\Rightarrow U_3 = U_2 + \Delta U^3 = \begin{bmatrix} 13,694 \\ 64,105 \end{bmatrix} mm \quad \& \quad P_3 = P_2 + \Delta P^3 = \begin{bmatrix} 4,209 \\ 7,290 \end{bmatrix}$$

Graph: Points P: $\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} ; \begin{pmatrix} 3,915 \\ 6,760 \end{pmatrix} ; \begin{pmatrix} 4,013 \\ 6,950 \end{pmatrix} ; \begin{pmatrix} 4,209 \\ 7,290 \end{pmatrix} \right] \times 10^6 N \quad \begin{bmatrix} P_x \\ P_y \end{bmatrix}$

1^{st} element plastification — 2^{nd} elmt plastification — Indicative point for asymptotic behavior.

Points U: $\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} ; \begin{pmatrix} 0,25 \\ 15,43 \end{pmatrix} ; \begin{pmatrix} 13,06 \\ 25,42 \end{pmatrix} ; \begin{pmatrix} 13,69 \\ 64,11 \end{pmatrix} \right]$ mm $\begin{bmatrix} u \\ v \end{bmatrix}$