Part A: The Newton Raphson method, applied to a linear material with a non-linear geometric behaviour

The same method used in assignment 2 is used.

However, the geometric behaviour of the bars is non-linear, therefore the length of the bars varies over time. At each iteration, each length is calculated according to the displacements.

Furthermore, the resisting force in each bar is calculated differently as the strain of a bar subjected to a change in length $L-L_0$ is: $W_{int}=0.5*k*(L-L_0)^2$, with $k=E*A_0/E_0$. The resisting force is the derivative of Wint with respect to the displacements.

As the geometric behaviour is non-linear, the tangent stiffness matrix is the sum of the material tangent stiffness (same as the one calculated for a linear geometric behaviour) and the geometric tangent stiffness.

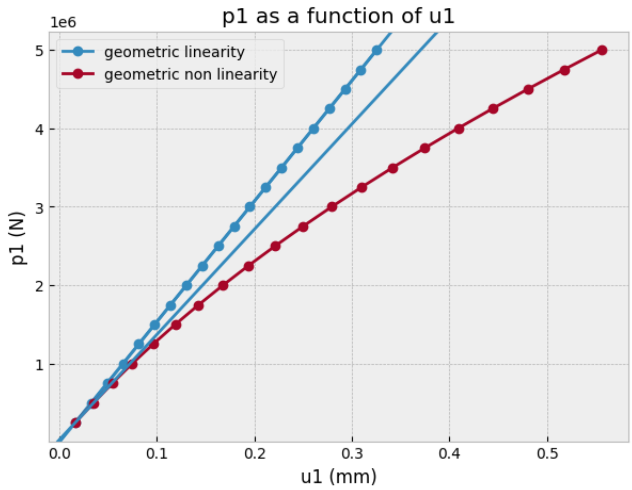
Part B: Displacements linear geometric behaviour and non-linear geometric behaviour

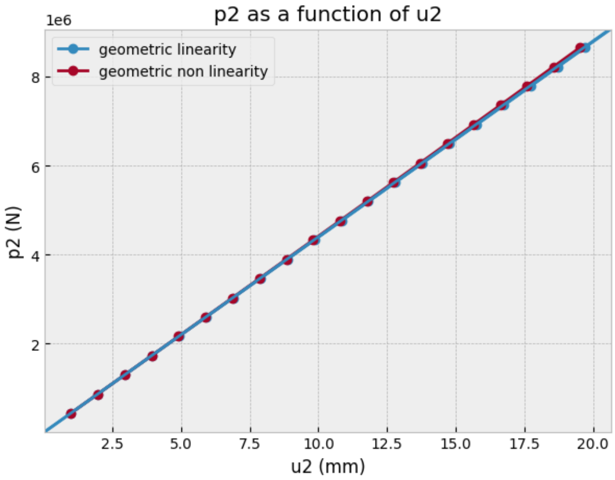
Load	Displacements [mm]			
	Linear Geometric Behaviour		Non linear geometric behaviour	
	U1	U2	U1	U2
10^7 N	0.326	19.71	0.556	19.53
10^8 N	3.255	192.13	21.946	182.58
10^9 N	32.55	1971.3	666.63*	1489.5*

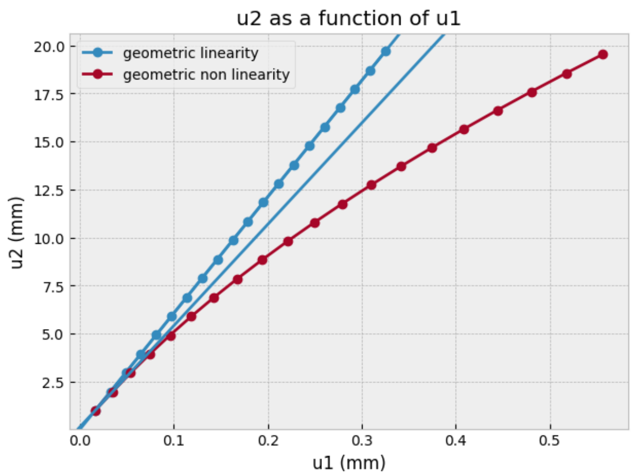
* Approximative results, with a large tolerance considered in order to have convergence of the Newton-Raphson method. If not, the results do not converge in reasonable computational times.

Graphical comparisons considering linear and non-linear geometric behaviour, with linear element properties:

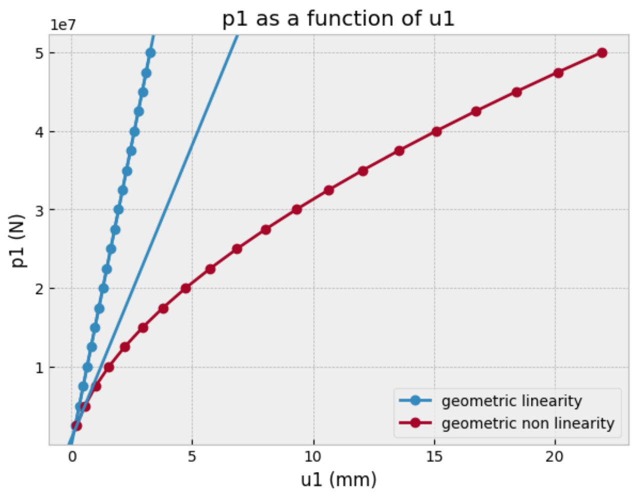
For load 10^7 N:

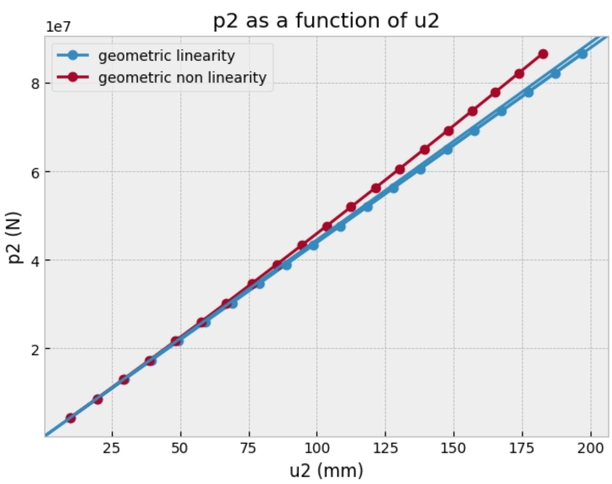


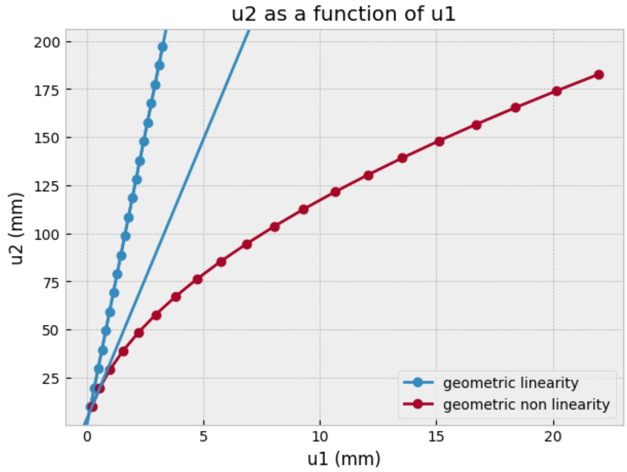




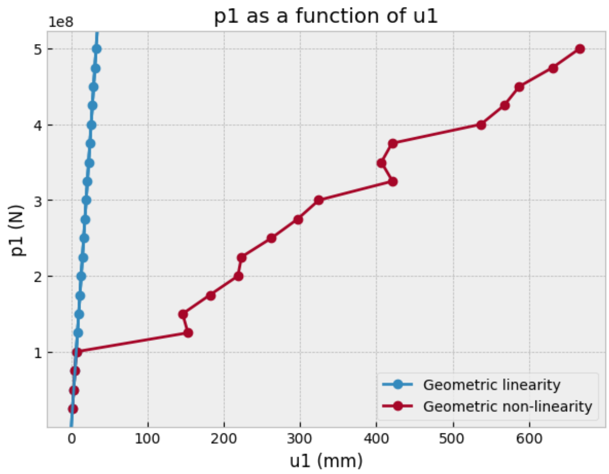
For load 10^8 N:

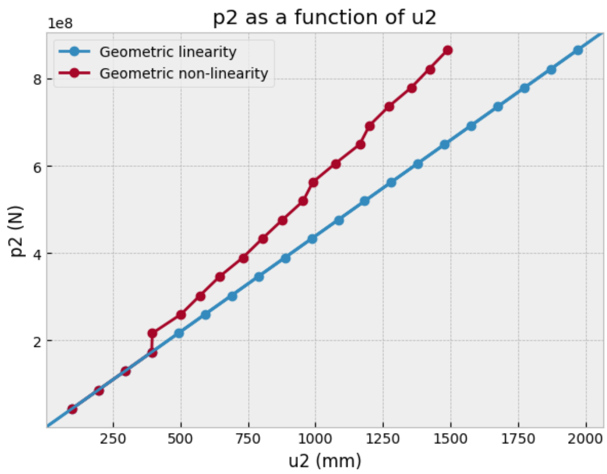


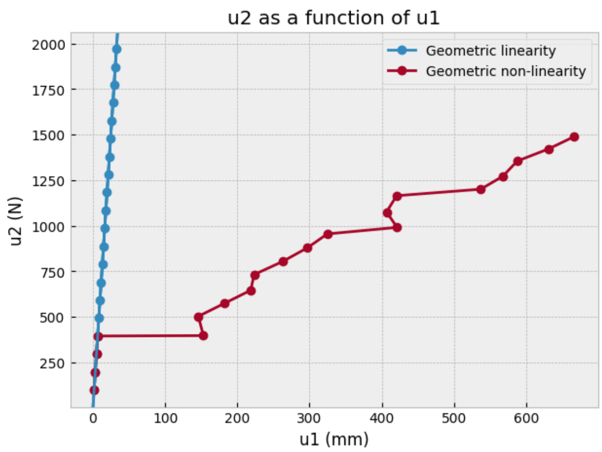




For load 10^9 N:







In order to obtain convergence with the load-increment Newton-Raphson method, the tolerance for the residual force had to be increased to 1/100 of the applied load.

Therefore, there is an important imprecision in the results for a 10^9 applied load.

The global behavior of the node is however trustworthy, with large deformations to be expected. Also, the results for 1/10 of the load are similar to the ones obtained with a 10^8 applied load.

Part C: Comparison between linear geometric behaviour and non-linear geometric behaviours.

The **relative change** is computed as following:

$$diff = \frac{u - u_{ref}}{u_{ref}}$$

With u_{ref} the maximal linear deformation and u the maximal non-linear deformation.

The relative change indicate the change applied to the linear displacements when considering the non-linear geometric behaviour.

Load	Relative change between largest displacements [%]	
	U1	U2
10^7 N	70.92	- 0.92
10^8 N	574.14	- 7.38
10^9 N	1947.82*	- 24.44*

* Approximative results, with a large tolerance considered in order to have convergence of the Newton-Raphson method. If not, the results do not converge in reasonable computational times.

Another good indicator of changes is the **percentage difference**, computed as following:

$$diff = \frac{|u_1 - u_2|}{(u_1 + u_2)/2}$$

With u_1 the maximal linear deformation and u_2 the maximal non-linear deformation.

The percentage difference compare the displacement for each case relative to the mean of the displacements.

Load	Percentage difference between largest displacements [%]	
	U1	U2
10^7 N	26.18	0.46
10^8 N	74.16	3.83
10^9 N	90.69*	13.92*

* Approximative results, with a large tolerance considered in order to have convergence of the Newton-Raphson method. If not, the results do not converge in reasonable computational times.

As showed by the previous tables, the geometric nonlinearity is correlated to the displacement magnitude, growing in an exponential way. (If the displacement gets larger, the influence of geometric non linearity gets larger, and therefore the displacement become even larger). This behaviour is seen with the increase of the differences between the maximal displacements for increasing loads.

However, for the very large displacements such as load case 3, the results must be mitigated by the fact that the elements cannot elongate enough for theses displacements. Furthermore, the Newton-Raphson method with load increments appears to be limited for very large displacement, failing to converge adequately.

Therefore, the results for this load case should not be trusted, and the model must be improved to have trustworthy results.