ANALISIS MATEMATICO II 2DO PARCIAL U.T.N.

ALUM[redacted]

FECHA: 11/7/22

3 (tres)

B

T1) Enuncie el teorema de GREEN. Apliquelo para calcular $\oint_{C^+} \bar{f} \cdot \overline{ds}$ siendo C: $x^2 - 2x + y^2 = 0$, $\bar{f}(x, y) = (xy, x^2)$

T2) Enuncie el teorema de cambio de variable para integrales dobles. Dada $\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} d\varphi \int_0^2 \rho^2 \, d\rho$ planteada en coordenadas polares, dibuje la región de integración y plantee la integral en coordenadas cartesianas

R E1) Dado $f(x, y, z) = (e^x + y - z, e^y + x - z, e^z + y - x)$, calcule la circulación de $\bar{f}$ a lo largo de la curva intersección de la superficie de ecuación $z = 4 - x^2 - y^2$ con los planos coordenados en el 1er octante. Indique gráficamente con que orientación ha decidido realizar la circulación pedida.

R E2) Dado $f(x, y, z) = (x + sen(yz), cos(xz) - y, z^3)$, calcule el flujo de $\bar{f}$ a través de la superficie S abierta de ecuación $z = x^2 + y^2$ con $z \le 9$ .Indique gráficamente como ha decidido orientar a S

M E3) Calcule el volumen del cuerpo definido por $z \le 4 - \sqrt{x^2 + y^2}$, $2z \ge x^2 + y^2$, 1er octante

E4) Dado $\bar{f}(x, y) = (y, 2x)$ calcule $\int_C \vec{f} \cdot \overrightarrow{ds}$ con C curva integral de y´´+y´=4 recorrida desde (0,2) hasta (1,6)

Rec 2º parcial cast 2020

P1) $f(x) \leq y \leq 5$

$y'' + y' = 2x$ $\qquad$ $r^2 + r = 0 \Rightarrow r(r+1) = 0 < \begin{matrix} r=0 \\ r=-1 \end{matrix}$

$y_H = C_1 + C_2 e^{-x}$ $\qquad$ $y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

$2A + 2Ax + B = 2x \Rightarrow 2A = 2 \rightarrow A = 1 \rightarrow 2A + B = 0 \Rightarrow B$

$y_p = x^2 - 2x \Rightarrow y_G = C_1 + C_2 e^{-x} + x^2 - 2x$

$y = -2x + 2 \Rightarrow y' = -2 \Rightarrow y' = -C_2 e^{-x} + 2x - 2$

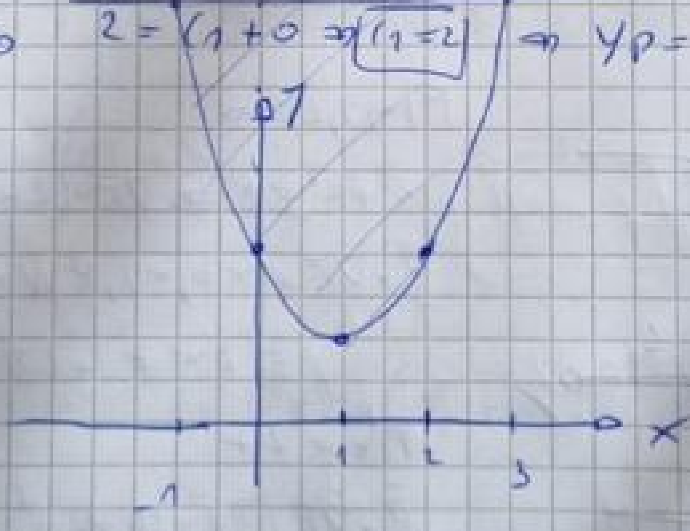
$-2 = -C_2 - 2 \Rightarrow \boxed{C_2 = 0}$

$y_0 = -2 \cdot 0 + 2 \Rightarrow y_0 = 2$

$(0,2) \Rightarrow 2 = C_1 + 0 \Rightarrow \boxed{C_1 = 2} \Rightarrow y_p = 2 + x^2 - 2x = x^2 - 2x + 2$

$x_v = -\frac{b}{2a}$

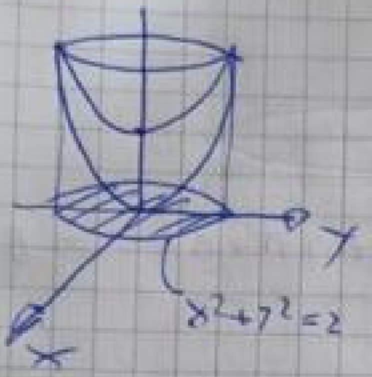
$x = 3 \rightarrow 9 - 6$



$$A = \int_{-1}^{3} \left(\int_{x^2-2x+2}^{5} dy \right) dx = \int_{-1}^{3} (5 - x^2 + 2x - 2)\, dx = \frac{32}{3}$$

P2) $M = k\int_0^{2\pi}\left(\int_0^{2}\int_{2\rho^2}^{4+\rho^2}\left(\underbrace{\sqrt{x^2+y^2}}_{\rho}\,\rho\,dz\right)d\rho\right)d\theta$

$4+x^2+y^2 = 2x^2+2y^2 \Rightarrow x^2+y^2=2$

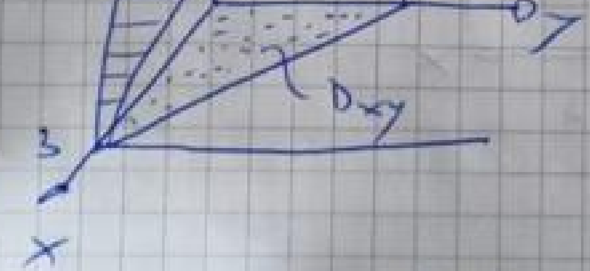


$M = 2\pi k\int_0^2 \rho^2(4+\rho^2-2\rho^2)\,d\rho$

$M = 2\pi k\int_0^2 (4\rho^2-\rho^4)\,d\rho$

$M = 2\pi k\left(\frac{4\rho^3}{3}-\frac{\rho^5}{5}\right|_0^2 = 2\pi k\left(\frac{32}{3}-\frac{32}{5}\right) = \frac{128}{15}k$

P3)



$F(x,y,z)=0$

$x^2+z^2=9 \Rightarrow \bar{\nabla}F=(2x,0,2z)$

$\bar{f}\cdot\bar{\nabla}F = (xz,xy,z^2)(2x,0,2z)$

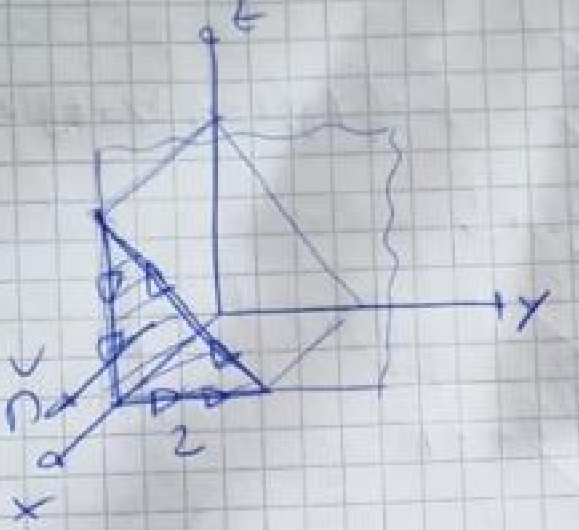
$= 2x^2z+2z^3$

$F'_z = 2z$

$\frac{\bar{f}\cdot\bar{\nabla}F}{|F'_z|} = \frac{2x^2z+2z^3}{2z} = x^2+z^2=9$

$\phi = \iint \frac{\bar{f}\cdot\bar{\nabla}F}{|F'_z|}\,dx\,dy = \int_0^3\int_0^{3-x} 9\,dx\,dy = 9\cdot\frac{3\cdot 3}{2} = \frac{81}{2}$

P4)



$$\bar{f}(x,y,z) = (\overset{P}{\widehat{yz}}, \overset{Q}{\widehat{xz}}, \overset{R}{\widehat{\varphi(y,z)}})$$

$$\oint_{C^+} \bar{f}\, d\bar{l} = \iint \operatorname{rot} \bar{f} \cdot \bar{\nabla} F \cdot \frac{dz\, dy}{|F'_x|}$$

$$\operatorname{rot} \bar{f} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = (R'_y - Q'_z ; P'_z - R'_x , Q'_x - P'_y)$$

$$\operatorname{rot} \bar{f} = (0 - x, y - \varphi'_x, z - z) = (-x, y - \varphi'_x ; 0)$$

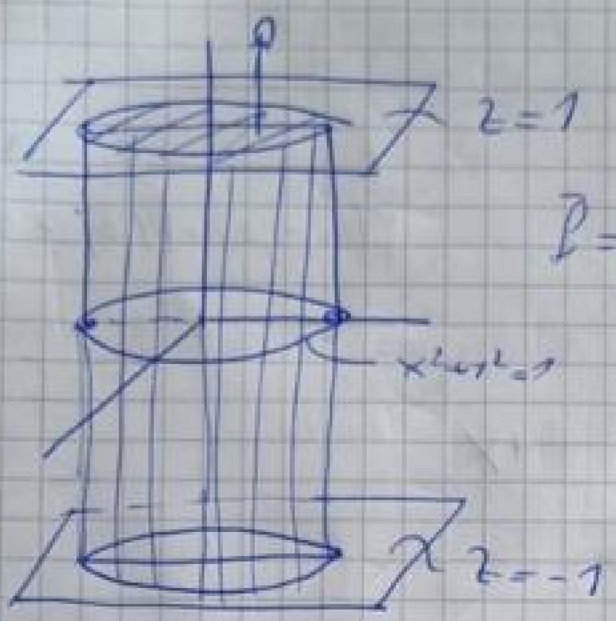
$F(x,y,z) = 0$

$x - 4 = 0 \Rightarrow \bar{\nabla} F = (1,0,0)$

$$\oint_{C^+} \bar{f}\, d\bar{l} = \iint \frac{(-x, y-\varphi'_x, 0)\cdot(1,0,0)}{\underset{F'_x}{(1)}} dz\, dy =$$

$$-4 \underbrace{\iint dz\, dy}_{\text{Area}} = -4 \left(\frac{2 \cdot 2}{2}\right) = -8$$

2)



$\bar{f} = \left(y\,g(z-x),\ 3y + z\,g(z-x),\ y\,g(z-x) \right)$

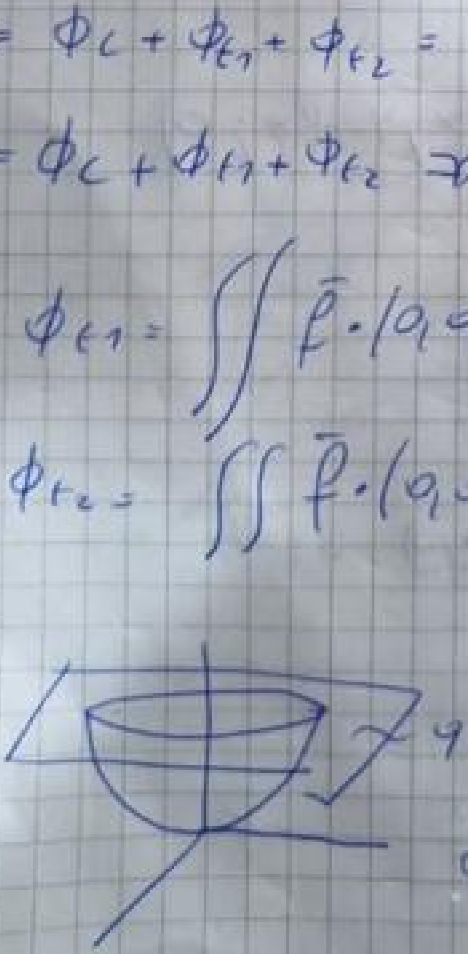
$$\Phi_T = \iiint \operatorname{div} \bar{f}\, dV = 3\,\pi \cdot 1^2 \cdot 2 = 6\pi$$

$$\operatorname{div} \bar{f} = \cancel{y\,g'(z-x)\cdot(-1)} + 3 + \cancel{y\,g'(z-x)} = 3$$

$$\Phi_T = \Phi_C + \Phi_{t_1} + \Phi_{t_2} =$$

$$6\pi = \Phi_C + \Phi_{t_1} + \Phi_{t_2} \Rightarrow \boxed{\Phi_C = 6\pi}$$

$$\left.\begin{aligned} \Phi_{t_1} &= \iint \bar{f}\cdot(0,0,1)\, dx\,dy = \iint y\,g(z-x)\, dx\,dy \\ \Phi_{t_2} &= \iint \bar{f}\cdot(0,0,-1)\, dx\,dy = -\iint y\,g(z-x)\, dx\,dy \end{aligned}\right\} = 0$$



$$\Phi_T = 3 \iiint dV$$

$$\Phi_T = 3\int_0^{2\pi}\int_0^{3}\int_{\rho^2}^{9} \rho\, dz\, d\rho\, d\theta = 6\pi \int_0^3 \rho(9-\rho^2)\, d\rho = \frac{243}{2}\pi$$

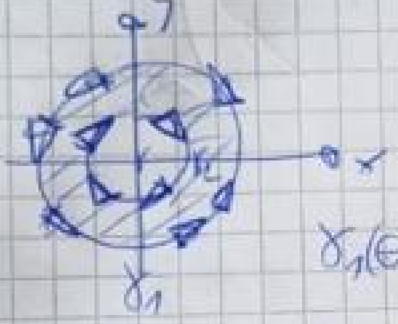
T1) Area $= \oint_{C^+} \bar{f}\cdot d\bar{\ell} = \iint \underbrace{(Q'_x - P'_y)}_{1}\, dx\,dy$

$\bar{f} = (0, x)$ pues $Q'_x - P'_y = 1$

$\bar{f}(x,y) = (4-2y;\ 4+4x)$

$Q'_x - P'_y = 4 - (-2) = 6$

$\oint \bar{f}\, d\bar{\ell} = \pi 1^2 = 6\pi 1^2$



Area

$\gamma_1(\theta) = (\sqrt{2}\cos\theta, \sqrt{2}\operatorname{sen}\theta) \Rightarrow \gamma'_1(\theta) = (-\sqrt{2}\operatorname{sen}\theta, \sqrt{2}\cos\theta)$

$\gamma_2(\theta) = (\cos\theta, \operatorname{sen}\theta) \Rightarrow \gamma'_2(\theta) = (-\operatorname{sen}\theta, \cos\theta)$

$$\text{Area} = \oint_{C_1} \bar{f}(\gamma_1(\theta))\cdot\bar{\gamma}'_1(\theta)\, d\theta - \oint_{C_2} \bar{f}(\gamma_2(\theta))\cdot\bar{\gamma}'_2(\theta)\, d\theta$$

$$\text{Area} = \oint (0, \sqrt{2}\cos\theta)\cdot(-\sqrt{2}\operatorname{sen}\theta, \sqrt{2}\cos\theta)\, d\theta$$

$$- \oint (0, \cos\theta)\cdot(-\operatorname{sen}\theta, \cos\theta)\, d\theta$$

$$\text{Area} = \int_0^{2\pi} 2\cos^2\theta\, d\theta - \int_0^{2\pi} \cos^2\theta\, d\theta$$

$$2\cdot\frac{6{,}258}{2} - \frac{6{,}258}{2} = \pi \qquad = \pi(\sqrt{2})^2 - \pi$$

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