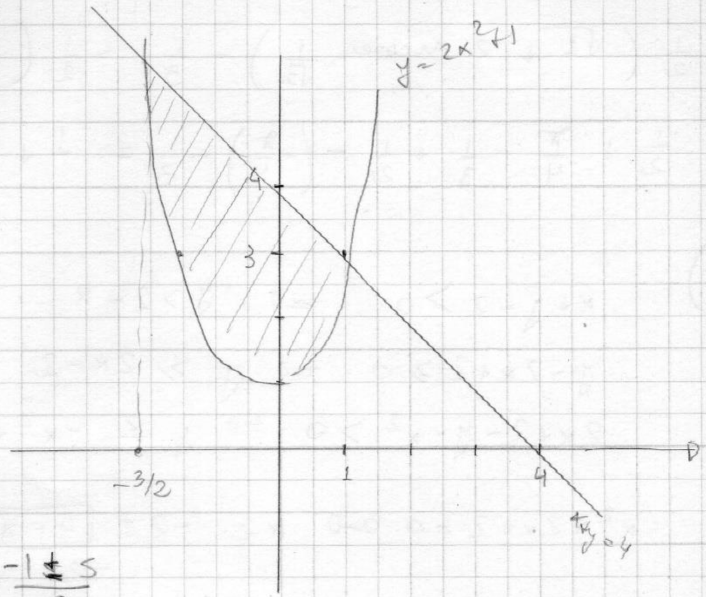
RESOLUCIÓN EJERCICIOS TP9

01)

a) $y \geq 2x^2+1$

$x + y \leq 4$



$$\left.\begin{array}{l} y = 2x^2+1 \\ y = 4-x \end{array}\right\} \Rightarrow$$

$\Rightarrow 2x^2+1 = 4-x \Rightarrow$

$\Rightarrow 2x^2 + x - 3 = 0 \Rightarrow$

$\Rightarrow x = \dfrac{-1 \pm \sqrt{1+24}}{4} = \dfrac{-1 \pm 5}{4}$

$\Rightarrow x_1 = -\dfrac{6}{4} \wedge x_2 = 1 \Rightarrow x_1 = -\dfrac{3}{2} \wedge x_2 = 1$

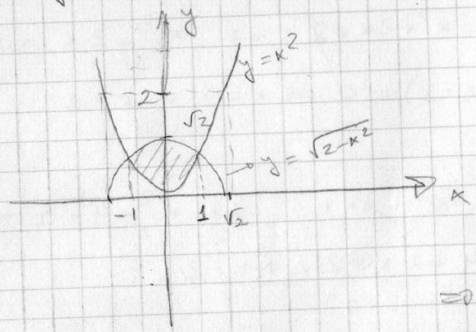
Area $= \iint_D dx\,dy = \displaystyle\int_{-3/2}^{1} dx \int_{2x^2+1}^{4-x} dy = \int_{-3/2}^{1} dx \left(4 - x - 2x^2 - 1\right) =$

$= \left[-\dfrac{x^2}{2} - \dfrac{2x^3}{3} + 3x \Big|_{-3/2}^{1} \right] = -\dfrac{1}{2} - \dfrac{2}{3} + 3 + \dfrac{9/4}{2} + \dfrac{2\left(-27/8\right)}{3}$

$- 3\left(-\dfrac{3}{2}\right) = -\dfrac{1}{2} - \dfrac{2}{3} + 3 + \dfrac{9}{8} - \dfrac{54}{24} + \dfrac{9}{2} = \dfrac{125}{24}$

1) b) $x^2 \le y \le \sqrt{2-x^2}$

$y = \sqrt{2-x^2} \Rightarrow y^2 = 2 - x^2 \Rightarrow x^2 + y^2 = 2$



$$\left.\begin{array}{l} x^2 = y \\ \sqrt{2-x^2} = y \end{array}\right\} \Rightarrow x^2 = \sqrt{2-x^2} \Rightarrow$$

$\Rightarrow x^4 = 2 - x^2 \Rightarrow$

$\Rightarrow x^4 + x^2 - 2 = 0 \Rightarrow$

$$\Rightarrow x^2 = \frac{-1 \pm \sqrt{1+8}}{2} \begin{array}{l} \nearrow -2 \text{ absurdo} \\ \searrow 1 \end{array}$$

$\Rightarrow x^2 = 1 \Rightarrow x = 1 \vee x = -1$

$$\text{Área} = \int_{-1}^{1} dx \int_{x^2}^{\sqrt{2-x^2}} dy = \int_{-1}^{1} \left(\sqrt{2-x^2} - x^2 \right) dx =$$

$$= \left[\frac{1}{2} \left(x\sqrt{2-x^2} + 2 \operatorname{arcsen} \frac{x}{\sqrt{2}} \right) - \frac{x^3}{3} \Big|_{-1}^{1} \right] =$$

$$= \frac{1}{2} \left(\sqrt{1} + 2 \operatorname{arcsen} \frac{1}{\sqrt{2}} \right) - \frac{1}{3} - \frac{1}{2} \left(-1\sqrt{1} + 2 \operatorname{arcsen} \left(\frac{-1}{\sqrt{2}} \right) \right) + \frac{(-1)}{3} =$$

$$= \frac{1}{2} + \frac{\pi}{4} - \frac{1}{3} + \frac{1}{2} - \left(-\frac{\pi}{4} \right) - \frac{1}{3} = 1 + \frac{\pi}{2} - \frac{2}{3} = \frac{\pi}{2} + \frac{1}{3}$$

1c) $x+y-2>0 \Rightarrow y>2-x$

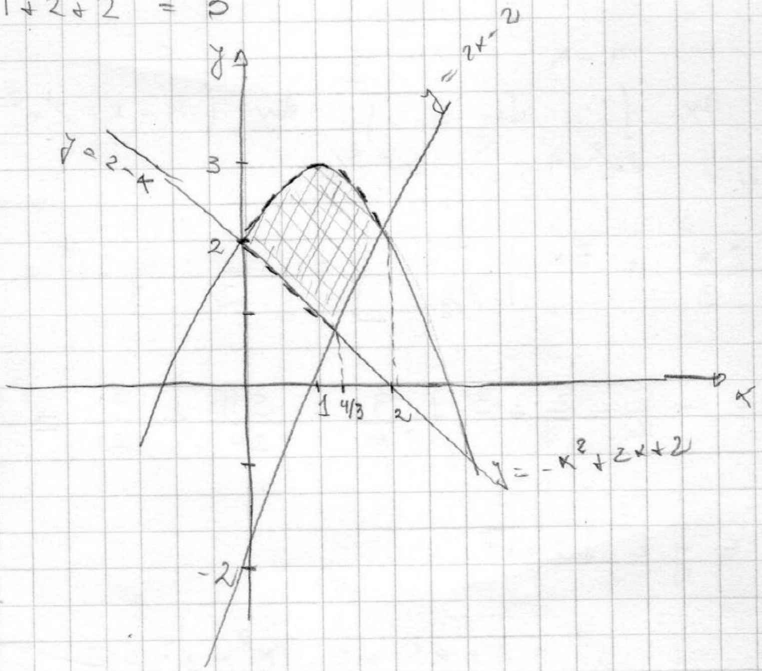
$y-2x+2 \geq 0 \Rightarrow y \geq 2x-2$

$2x+2-y-x^2>0 \Rightarrow y < -x^2+2x+2$

$-x^2+2x+2=0 \Rightarrow x = \dfrac{-2 \pm \sqrt{4+8}}{-2} = \dfrac{-2 \pm \sqrt{12}}{-2} = 1 \pm \sqrt{3}$

$x_v = \dfrac{1+\sqrt{3}+1-\sqrt{3}}{2} = 1$

$y_v = -1+2+2 = 3$



$\left.\begin{array}{l} y=2-x \\ y=-x^2+2x+2 \end{array}\right\} \Rightarrow \cancel{2}-x=-x^2+2x+\cancel{2} \Rightarrow x^2-3x=0 \Rightarrow$

$\Rightarrow x(x-3)=0 \Rightarrow x=0 \vee x=3$

$\left.\begin{array}{l} y=2x-2 \\ y=-x^2+2x+2 \end{array}\right\} \Rightarrow \cancel{2x}-2=-x^2+\cancel{2x}+2 \Rightarrow x^2-4=0 \Rightarrow$

$\Rightarrow x^2=4 \Rightarrow x=2 \vee x=-2$

$$\left.\begin{array}{l} y = 2x - 2 \\ y = 2 - x \end{array}\right\} \Rightarrow 2x - 2 = 2 - x \Rightarrow 3x - 4 = 0 \Rightarrow x = \frac{4}{3}$$

$$\text{Area} = \int_0^{4/3} dx \int_{2-x}^{-x^2+2x+2} dy + \int_{4/3}^{2} dx \int_{2x-2}^{-x^2+2x+2} dy =$$

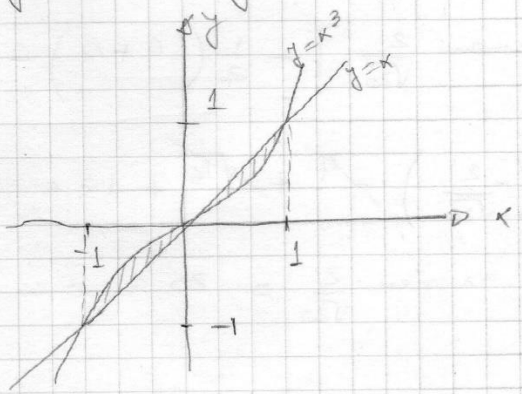
$$= \int_0^{4/3} dx \left(-x^2 + 2x + \cancel{2} - \cancel{2} + x\right) + \int_{4/3}^{2} dx \left(-x^2 + \cancel{2x} + 2 - \cancel{2x} + 2\right) =$$

$$= \left(-\frac{x^3}{3} + \frac{3x^2}{2}\Bigg|_0^{4/3}\right) + \left(-\frac{x^3}{3} + 4x\Bigg|_{4/3}^{2}\right) =$$

$$= -\frac{1}{3}\,\frac{4^3}{3^3} + \frac{3}{2}\,\frac{16}{9} + \left(-\frac{8}{3}\right) + 8 + \left(\frac{4}{3}\right)^3 \frac{1}{3} - 4 \cdot \frac{4}{3} =$$

$$= -\cancel{\frac{64}{81}} + \frac{48}{18} - \frac{8}{3} + 8 + \cancel{\frac{64}{81}} - \frac{16}{3} = \frac{8}{3}$$

1 d) $y = x^3 \qquad y = x$



$$\left.\begin{aligned} y &= x \\ y &= x^3 \end{aligned}\right\} \Rightarrow \quad x = x^3 \Rightarrow x^3 - x = 0 \Rightarrow x(x^2-1) = 0 \Rightarrow$$

$$\Rightarrow x = 0 \ \vee \ x = 1 \ \vee \ x = -1$$

$$A = \int_{-1}^{0} dx \int_{x}^{x^3} dy + \int_{0}^{1} dx \int_{x^3}^{x} dy = \int_{-1}^{0} dx \left(x^3 - x\right) +$$

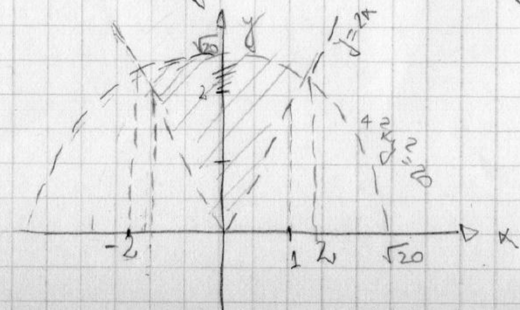
$$+ \int_{0}^{1} dx \left(x - x^3\right) = \left[\frac{x^4}{4} - \frac{x^2}{2} \Big|_{-1}^{0}\right] + \left[\frac{x^2}{2} - \frac{x^4}{4} \Big|_{0}^{1}\right] =$$

$$= 0 - \frac{(-1)^4}{4} + \frac{(-1)^2}{2} + \frac{1}{2} - \frac{1}{4} = -\frac{1}{4} + \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{1}{2}$$

e) positivo $\to f(x,y) = (y-2|x|)\underbrace{\sqrt{20-x^2-y^2}}_{\geq 0}$

i) $y-2|x|>0 \Rightarrow y>2|x|$

ii) $20-x^2-y^2>0 \Rightarrow x^2+y^2<20$



$\left.\begin{array}{l} y=2x \\ x^2+y^2=20 \end{array}\right\} \Rightarrow x^2+4x^2=20 \Rightarrow$

$\Rightarrow x^2=\frac{20}{5} \Rightarrow x=2 \vee x=-2$

Se $y=-2x \to y=4 \vee y=-4$

$$\text{Área} = \int_{-2}^{0} dx \int_{-2x}^{\sqrt{20-x^2}} dy + \int_{0}^{2} dx \int_{2x}^{\sqrt{20-x^2}} dy =$$

$$= \int_{-2}^{0} dx \left(\sqrt{20-x^2}+2x\right) + \int_{0}^{2} dx \left(\sqrt{20-x^2}-2x\right) =$$

$$= \left(\frac{1}{2}\left[x\sqrt{20-x^2}+20\,\text{arcosen}\,\frac{x}{\sqrt{20}}\right]\Big|_{-2}^{0}\right) + \left(x^2\Big|_{-2}^{0}\right) +$$

$$+ \left(\frac{1}{2}\left[x\sqrt{20-x^2}+20\,\text{arcosen}\,\frac{x}{\sqrt{20}}\right]\Big|_{0}^{2}\right) - \left(x^2\Big|_{0}^{2}\right) =$$

$$= \frac{1}{2}(0+0) - \frac{1}{2}\left(-2\sqrt{20-4}+20\,\text{arcosen}\left(-\frac{2}{\sqrt{20}}\right)\right) + 0 - (-2)^2 +$$

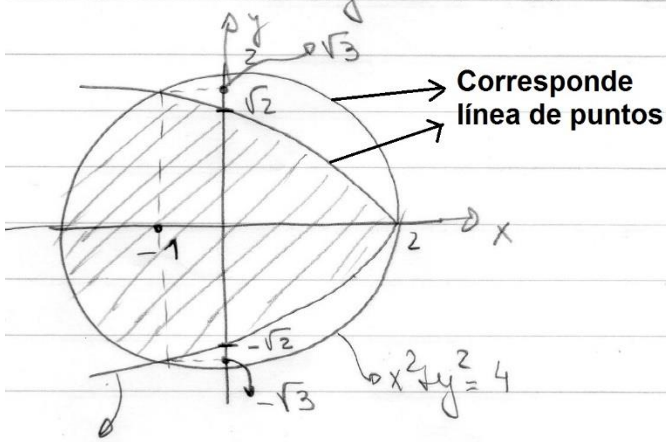
$$+ \frac{1}{2}\left[2\sqrt{20-4}+20\,\text{arcosen}\,\frac{2}{\sqrt{20}}\right] - \frac{1}{2}(0+0) - 4 =$$

$$= \cancel{\sqrt{16}} - 10\,\text{arcosen}\left(-\frac{2}{\sqrt{20}}\right) \cancel{-4} + \cancel{\sqrt{16}} + 10\,\text{arcosen}\,\frac{2}{\sqrt{20}} \cancel{-4} =$$

$$= 10\,\text{arcosen}\,\frac{2}{\sqrt{20}} + 10\,\text{arcosen}\,\frac{2}{\sqrt{20}} = 20\,\text{arcosen}\,\frac{2}{\sqrt{20}}$$

1) f) $4-x^2-y^2>0 \wedge 2-x-y^2>0$

$x^2+y^2<4 \wedge x<2-y^2$



$x=2-y^2$

$$\left.\begin{array}{l}2-y^2=x\\ x^2+y^2=4\end{array}\right\}\Rightarrow$$

$\Rightarrow x^2+2-x=4 \Rightarrow$

$\Rightarrow x^2-x-2=0$ $\to x_1=2$, $\to x_2=-1$

$2-y^2=0 \Rightarrow |y|=\sqrt{2}$ $\to y_1=\sqrt{2}$, $\to y_2=-\sqrt{2}$

$x=-1 \to -1=2-y^2 \Rightarrow y^2=3$ $\to y_1=\sqrt{3}$, $\to y_2=-\sqrt{3}$

$$\text{Área}=\int_{-\sqrt{3}}^{\sqrt{3}} dy \int_{-\sqrt{4-y^2}}^{2-y^2} dx=\int_{-\sqrt{3}}^{\sqrt{3}}\left(2-y^2+\sqrt{4-y^2}\right)dy=$$

$$=\left[2y-\frac{y^3}{3}+\frac{1}{2}\left(y\sqrt{4-y^2}+4\,\text{arcosen}\left(\frac{y}{2}\right)\right)\right]_{-\sqrt{3}}^{\sqrt{3}}=$$

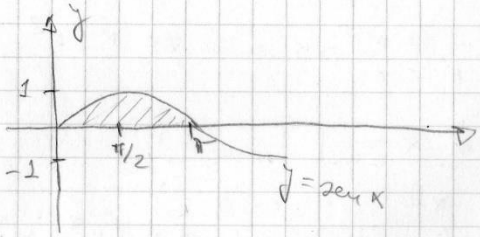
$$=2\sqrt{3}-\frac{3\sqrt{3}}{3}+\frac{1}{2}\left(\sqrt{3}\sqrt{1}+4\,\text{arcosen}\left(\frac{\sqrt{3}}{2}\right)\right)-2(-\sqrt{3})+$$

$$+\frac{3(-\sqrt{3})}{3}-\frac{1}{2}\left(-\sqrt{3}\sqrt{1}+4\,\text{arcosen}\left(\frac{-\sqrt{3}}{2}\right)\right)=$$

$$=\cancel{2\sqrt{3}}-\cancel{\sqrt{3}}+\frac{\sqrt{3}}{2}-\cancel{\frac{1}{2}}+2\cdot\frac{\pi}{3}+2\sqrt{3}-\cancel{\sqrt{3}}+\frac{\sqrt{3}}{2}+\cancel{\frac{1}{2}}-$$

$$-2\left(-\frac{\pi}{3}\right)=3\sqrt{3}+\frac{4\pi}{3}$$

2 a)



$$\iint_D dx\,dy = \int_0^{\pi} dx \int_0^{\operatorname{sen} x} dy = \int_0^{\pi} \operatorname{sen} x\, dx = \left[-\cos x \Big|_0^{\pi} \right] =$$

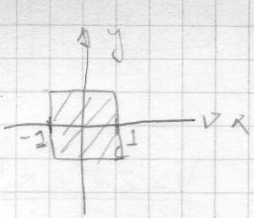
$$= -\cos \pi + \cos 0 = -(-1) + 1 = 2$$

$$= \int_0^1 dy \int_{\operatorname{arcosen} y}^{\pi - \operatorname{arcosen} y} dx = \int_0^1 dy \left(\pi - \operatorname{arcosen} y - \operatorname{arcosen} y \right) =$$

$$= \left[\pi y \Big|_0^1 \right] - 2 \left[y \operatorname{arcosen} y + \sqrt{1-y^2} \Big|_0^1 \right] = \pi - 2 \cdot$$

$$\left(\frac{\pi}{2} + \sqrt{0} - 0 - 1 \right) = \pi - \pi + 2 = 2$$

2) b)



$$\iint_D x\,dx\,dy = \int_{-1}^{1} dx \int_{-1}^{1} x\, dy =$$

$$= \int_{-1}^{1} x\, dx\, (1+1) = 2 \left[\frac{x^2}{2} \Big|_{-1}^{1} \right] =$$

$$= 2 \left[\frac{1}{2} - \frac{1}{2} \right] = 0$$

$$\iint_D x\,dx\,dy = \int_{-1}^{1} dy \int_{-1}^{1} x\, dx = \int_{-1}^{1} dy \left[\frac{x^2}{2} \Big|_{-1}^{1} \right] =$$

$$= \int_{-1}^{1} dy \left(\frac{1}{2} - \frac{1}{2} \right) = 0$$

2)c) $\iint_D |x|\,dx\,dy = \int_{-1}^{1} dx \int_{-1}^{1} |x|\,dy = \int_{-1}^{1} |x|\,dx\,(1+1) =$

$= 2\left[\int_{-1}^{0}(-x)\,dx + \int_{0}^{1} x\,dx\right] =$

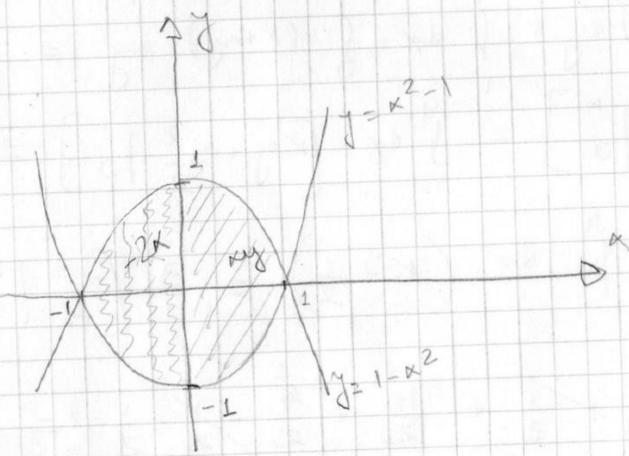
$= 2\left[-\frac{x^2}{2}\Big|_{-1}^{0}\right] + 2\left[\frac{x^2}{2}\Big|_{0}^{1}\right] =$

$= 2\left(-0+\frac{1}{2}\right) + 2\left(\frac{1}{2}-0\right) = 1+1 = 2$

$\iint_D |x|\,dx\,dy = \int_{-1}^{1} dy \int_{-1}^{1} |x|\,dx = \int_{-1} dy\left(\int_{-1}^{0} -x\,dx + \int_{0}^{1} x\,dx\right) = \int_{-1}^{1} dy\left(-\frac{x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_{0}^{1}\right) =$

$= \int_{-1}^{1} dy\left(-0+\frac{1}{2}+\frac{1}{2}-0\right) = \int_{-1}^{1} dy = 1+1 = 2$

2)d)



$y = x^2-1 = 0 \quad x^2 = y+1$

$|x| = \sqrt{y+1}$

$y = 1-x^2 = 0 \quad x^2 = 1-y$

$|x| = \sqrt{1-y}$

$\iint_D f(x,y)\,dx\,dy = \int_{-1}^{0} dx \int_{x^2-1}^{1-x^2} -2x\,dy + \int_{0}^{1} dx \int_{x^2-1}^{1-x^2} xy\,dy =$

$= \int_{-1}^{0} -2x\,dx\,(1-x^2-x^2+1) + \int_{0}^{1} x\,dx\left[\frac{y^2}{2}\Big|_{x^2-1}^{1-x^2}\right] =$

$$= \int_{-1}^{0} \left(-4x + 4x^3\right) dx + \int_0^1 x \, dx \left[\frac{(1-x^2)^2}{2} - \frac{(x^2-1)^2}{2} \right] =$$

$$= \left[-2x^2 + x^4 \Big|_{-1}^{0} \right] + \int_0^1 dx \; x \; \frac{1 - \cancel{2x^2} + \cancel{x^4} - \cancel{x^4} - 1 + \cancel{2x^2}}{2} =$$

$$= 0 + 2(-1)^2 - (-1)^4 + 0 = 2 - 1 = 1$$

$$\iint_D f(x,y)\, dx\, dy = \int_{-1}^{0} dy \int_{-\sqrt{y+1}}^{0} dx\,(-2x) + \int_0^1 dy \int_{-\sqrt{1-y}}^{0} dx\,(-2x) +$$

$$+ \int_{-1}^{0} dy \int_0^{\sqrt{y+1}} dx \; xy + \int_0^1 dy \int_0^{\sqrt{1-y}} xy \, dx =$$

$$= \int_{-1}^{0} dy \left[-x^2 \Big|_{-\sqrt{y+1}}^{0} \right] + \int_0^1 dy \left[-x^2 \Big|_{-\sqrt{1-y}}^{0} \right] +$$

$$+ \int_{-1}^{0} y \, dy \left[\frac{x^2}{2} \Big|_0^{\sqrt{y+1}} \right] + \int_0^1 y \, dy \left[\frac{x^2}{2} \Big|_0^{\sqrt{1-y}} \right] =$$

$$= \int_{-1}^{0} (-0 + y + 1)\, dy + \int_0^1 [-0 + 1 - y]\, dy + \int_{-1}^{0} y \, \frac{(y+1)}{2}\, dy +$$

$$+ \int_0^1 y \, \frac{(1-y)}{2}\, dy = \left[\frac{y^2}{2} + y \Big|_{-1}^{0} \right] + \left[y - \frac{y^2}{2} \Big|_0^1 \right] + \frac{1}{2} \left[\frac{y^3}{3} + \frac{y^2}{2} \Big|_{-1}^{0} \right] +$$

$$+ \frac{1}{2} \left[\frac{y^2}{2} - \frac{y^3}{3} \Big|_0^1 \right] = 0 \cancel{- \frac{1}{2}} \cancel{+1} + 1 \cancel{- \frac{1}{2}} + \cancel{\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right)} + \cancel{\frac{1}{2}\left(\frac{1}{2} - \frac{1}{3}\right)} = 1$$

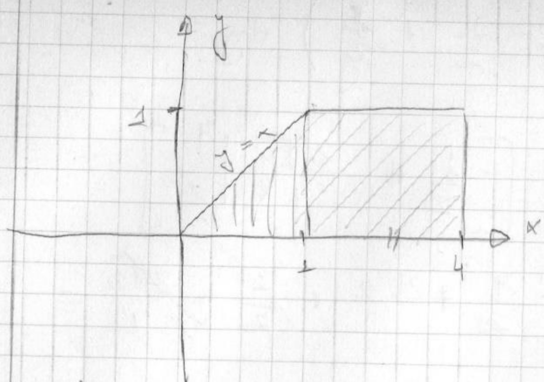
2e) $\int_0^1 dx \int_0^x (x+y)\,dy + \int_1^4 dx \int_0^1 (x+y)\,dy =$

$= \int_0^1 dx \left[xy + \frac{y^2}{2}\Big|_0^x\right] = \int_1^4 dx \left[xy + \frac{y^2}{2}\Big|_0^1\right] =$

$= \int_0^1 \left(x^2 + \frac{x^2}{2}\right)dx + \int_1^4 dx \left(x + \frac{1}{2}\right) = \frac{3}{2}\left[\frac{x^3}{3}\Big|_0^1\right] +$

$+ \left[\frac{x^2}{2} + \frac{x}{2}\Big|_1^4\right] = \cancel{\frac{3}{2}\cdot\frac{1}{3}} + \overset{8}{\frac{16}{2}} + \overset{2}{\frac{4}{2}} - \cancel{\frac{1}{2}} - \frac{1}{2} =$

$= 10 - \frac{1}{2} = \frac{19}{2}$

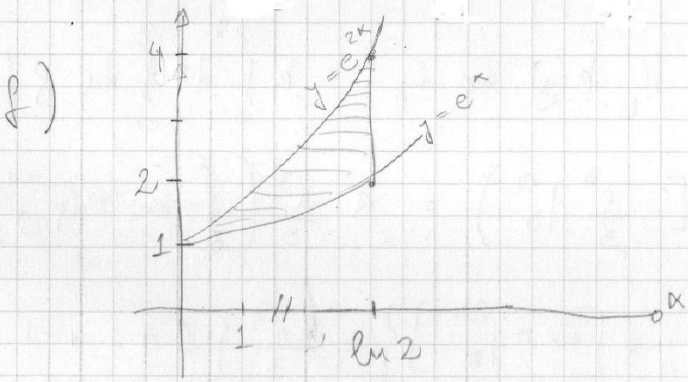


$\int_0^1 dy \int_y^4 dx\,(x+y) = \int_0^1 dy \left[\frac{x^2}{2} + yx\Big|_y^4\right] =$

$= \int_0^1 \left(\frac{16}{2} + 4y - \frac{y^2}{2} - y^2\right) dy = \left[8y + 2y^2 - \frac{3}{2}\frac{y^3}{\cancel{3}}\Big|_0^1\right]$

$= 8 + 2 - \frac{1}{2} = 10 - \frac{1}{2} = \frac{19}{2}$

f)



$e^{\ln 2} = 2$

$e^{2\ln 2} = (e^{\ln 2})^2 = 4$

$y = e^{2x} \Rightarrow \ln y = 2x$

$y = e^{x} \Rightarrow \ln y = x$

$$\iint_D e^{-x}\,dx\,dy = \int_0^{\ln 2} dx \int_{e^x}^{e^{2x}} e^{-x}\,dy =$$

$$= \int_0^{\ln 2} e^{-x}(e^{2x}-e^{x})\,dx = \int_0^{\ln 2} (e^{x}-1)\,dx =$$

$$= \left[e^{x} - x \Big|_0^{\ln 2}\right] = 2 - \ln 2 - 1 + 0 =$$

$$= 1 - \ln 2$$

$$\int_1^2 dy \int_{\frac{\ln y}{2}}^{\ln y} e^{-x}\,dx + \int_2^4 dy \int_{\frac{\ln y}{2}}^{\ln 2} e^{-x}\,dx =$$

$$= \int_1^2 dy \left[-e^{-x} \Big|_{\frac{\ln y}{2}}^{\ln y}\right] + \int_2^4 dy \left[-e^{-x} \Big|_{\frac{\ln y}{2}}^{\ln 2}\right] =$$

$$= \int_1^2 dy \left[-e^{\overbrace{-\ln y}^{\ln y^{-1}}} + e^{\underbrace{-\frac{\ln y}{2}}_{\ln y^{-1/2}}}\right] + \int_2^4 dy \left[-e^{-\ln 2} + e^{-\frac{\ln y}{2}}\right] =$$

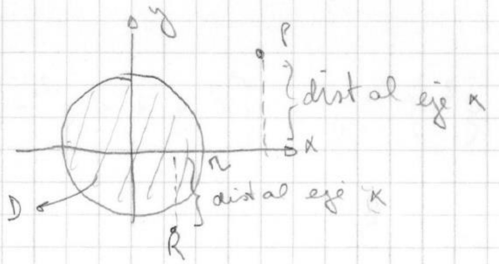
$$= \int_1^2 \left(-\frac{1}{y} + \frac{1}{\sqrt{y}}\right) dy + \int_2^4 \left(2 - \frac{1}{2} + \frac{1}{\sqrt{y}}\right) dy =$$

$$= \left[-\ln|y| + 2\sqrt{y} \Big|_1^2 \right] + \left[-\frac{y}{2} + 2\sqrt{y} \Big|_2^4 \right] =$$

$$= -\ln 2 + \cancel{2\sqrt{2}} + 0 - 2 - \frac{4}{2} + 2 \cdot 2 + \frac{2}{2} - \cancel{2\sqrt{2}} =$$

$$= -\ln 2 - \cancel{2} - \cancel{2} + \cancel{4} + 1 = 1 - \ln 2$$

3)



$$\delta = k\,|y| \quad (k \in R)$$

$$m = \iint_D \delta \, dx\,dy = \iint_D k|y| \, dx\,dy = k \int_0^{2\pi} d\theta \int_0^r d\rho \, \rho \, |\rho \operatorname{sen}\theta| =$$

$$= k \int_0^{2\pi} |\operatorname{sen}\theta| \, d\theta \left[\frac{\rho^3}{3} \Big|_0^r \right) = k \, \frac{r^3}{3} \left[\int_0^{\pi} \operatorname{sen}\theta \, d\theta + \int_{\pi}^{2\pi} (-\operatorname{sen}\theta) \, d\theta \right]$$

$$= k \frac{r^3}{3} \left\{ \left[-\cos\theta \Big|_0^{\pi} \right] + \left[\cos\theta \Big|_{\pi}^{2\pi} \right] \right\} = \frac{k r^3}{3} \left[1 + 1 + 1 + 1 \right] =$$

$$= \frac{4}{3} k r^3$$

$$x_G = \frac{\iint_D x \, \delta \, dx\,dy}{\iint_D \delta \, dx\,dy} = \frac{\iint_D k x |y| \, dx\,dy}{\frac{4}{3} k r^3} =$$

$$= \frac{\cancel{k}\int_0^{2\pi} d\theta \int_0^{r} \rho^2 \cos\theta\, |\operatorname{sen}\theta|\; \rho\; d\rho}{4/3\; \cancel{k}\; r^3} = \frac{3}{4}\,\frac{\int_0^{2\pi} \cos\theta\, |\operatorname{sen}\theta| \left[\frac{\rho^4}{4}\Big|_0^r\right] d\theta}{r^3} =$$

$$= \frac{3}{4\cancel{r^3}} \cdot \frac{r^{\cancel{4}}}{4} \left[\int_0^{\pi} \cos\theta \operatorname{sen}\theta\, d\theta + \int_{\pi}^{2\pi} -\operatorname{sen}\theta\cos\theta\, d\theta \right] =$$

$$= \frac{3}{16} r \left[\frac{\operatorname{sen}^2\theta}{2}\Big|_0^{\pi} - \frac{\operatorname{sen}^2\theta}{2}\Big|_{\pi}^{2\pi} \right] =$$

$$= \frac{3}{16} r \left[0 - 0 - 0 + 0 \right] = 0$$

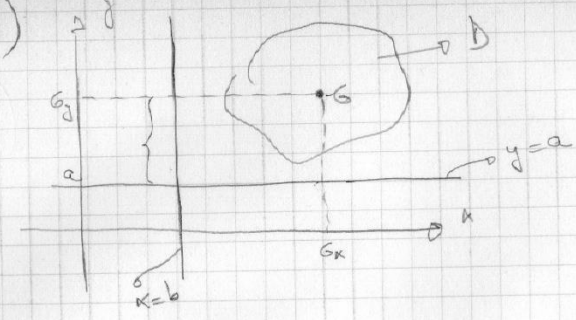
$$y_G = \frac{\iint_D y\, \delta\, dx\, dy}{4/3\; k r^3} = \frac{\iint_D y\, k|y|\; dx\, dy}{4/3\; k r^3} = \frac{\cancel{k}\int_0^{2\pi} d\theta \int_0^{r} \operatorname{sen}\theta\, |\operatorname{sen}\theta|\, \rho^2\, d\rho\, \rho}{4/3\; \cancel{k}\, r^3}$$

$$= \frac{3}{4r^3} \int_0^{2\pi} \operatorname{sen}\theta\, |\operatorname{sen}\theta| \left[\frac{\rho^4}{4}\Big|_0^r\right] = \frac{3 r^{\cancel{4}}}{4 \cancel{r^3}\, 4}$$

$$\cdot \underbrace{\left[\int_0^{\pi} \operatorname{sen}^2\theta\, d\theta + \int_0^{2\pi} -\operatorname{sen}^2\theta\, d\theta \right]}_{=0} = 0$$

$$G = (x_G,\; y_G) = (0, 0)$$

04)



i) $I_a = \iint_D |G_y - a|^2 \delta \, dx \, dy = \iint_D (G_y - a)^2 \delta \, dx \, dy =$

$\iint_D (G_y^2 + a^2 - 2G_y a) \delta \, dx \, dy = \iint_D G_y^2 \delta \, dx \, dy +$

$+ \iint_D a^2 \delta \, dx \, dy - 2\iint_D G_y a \, dx \, dy \geq 0$ (por ser

un momento de inercia)

Si la recta pasa por G → su ecuac es : $y = G_y$ $(a = G_y)$

Luego la expresión se reduce a :

$\iint_D G_y^2 \cancel{\delta \, dx \, dy} + \iint_D \cancel{G_y^2 \delta \, dx \, dy} - 2\iint_D \cancel{G_y G_y \, dx} \, dy =$

$= 0 \rightarrow$ donde 0 es el mínimo valor posible que toma el momento de inercia.

Por lo tanto, si la recta pasa por $G \rightarrow y = G_y$ el I_{G_y} es mínimo y vale 0

ii) $I_b = \iint_D |G_x - b|^2 \delta \, dx \, dy = \iint_D (G_x - b)^2 \delta \, dx \, dy \geq 0$ (por ser un momento de inercia)

Si la recta pasa por $G \rightarrow$ su ecuación es: $x = G_x$ ($b = G_x$)

Luego la expresión se reduce a:

$\iint_D (G_x - G_x)^2 \delta \, dx \, dy = 0 \rightarrow$ donde 0 es el mínimo valor posible que toma el momento de inercia.

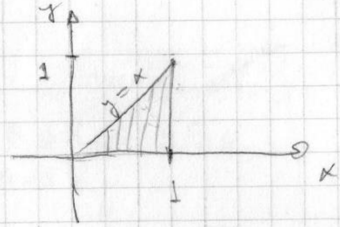
5)

a) $$\int_0^1 \int_0^{\sqrt{1-x^2}} x \, dy \, dx = \int_0^1 x \, dx \int_0^{\sqrt{1-x^2}} dy = \int_0^1 x \sqrt{1-x^2} \, dx =$$

$$= \left[-\frac{1}{3} \sqrt{(1-x^2)^3} \Big|_0^1 \right] = -0 + \frac{1}{3} \sqrt{1} = \frac{1}{3}$$

5 b) $\int_0^1 \int_y^1 e^{x^2}\, dy\, dx = \int_0^1 dy \int_y^1 \underbrace{e^{x^2}\, dx}_{\text{no tiene primitiva}} = ?$

Veo aplicando un cambio en el orden de integración:



$$\int_0^1 dx \int_0^x e^{x^2}\, dy = \int_0^1 dx\ x e^{x^2} = (A)$$

$u = e^{x^2}$ $\qquad$ $x = 0 \Rightarrow u = e^{0^2} = 1$

$du = \underbrace{e^{x^2}}_{u} \cdot 2x\ dx$ $\qquad$ $x = 1 \Rightarrow u\ e^{1^2} = e$

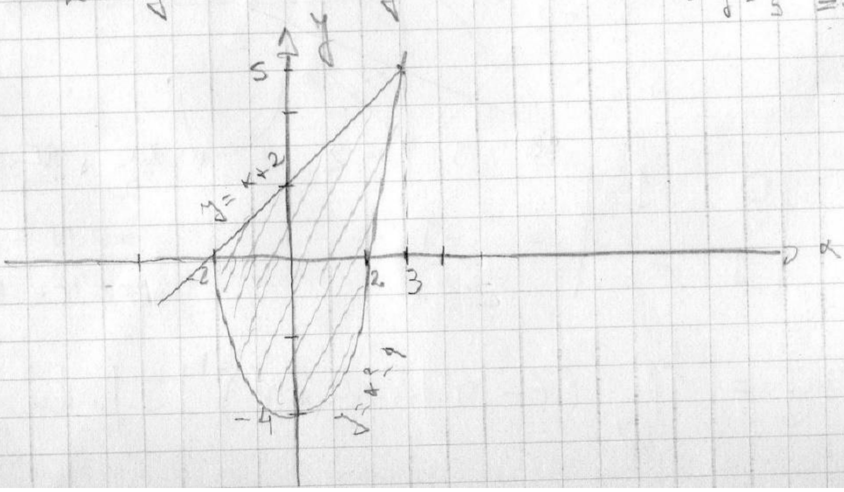
$$(A) = \int_1^e \frac{du}{\cancel{u}\, 2\cancel{x}}\ \cancel{x}\ \cancel{u} = \frac{1}{2}\left[u \Big|_1^e\right] = \frac{1}{2}(e-1)$$

5 c) $\int_{-4}^{0} dy \int_{-\sqrt{y+4}}^{\sqrt{y+4}} dx + \int_{0}^{5} dy \int_{y-2}^{\sqrt{y+4}} dx$

Veo cambiando el orden de integración

$x = \sqrt{y+4} \Rightarrow x^2 - 4 = y$ — $y = 5 \Rightarrow x = 3$; $y = -4 \Rightarrow x = 0$; $y = 0 \Rightarrow x = \pm 2$

$x = y - 2 \Rightarrow y = x + 2$ — $y = 5 \Rightarrow x = 3$



$\int_{-2}^{3} dx \int_{x^2-4}^{x+2} dy = \int_{-2}^{3} dx \left(x+2-x^2+4\right) = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x \Big|_{-2}^{3}\right] =$

$= -\frac{\cancel{27}^{9}}{\cancel{3}} + \frac{9}{2} + 18 + \frac{-8}{3} - \frac{\cancel{4}^{2}}{\cancel{2}} - (-12) = -9 + \frac{9}{2} + 18 -$

$- \frac{8}{3} - 2 + 12 = \frac{9}{2} - \frac{8}{3} + 19 = \frac{27-16}{6} + 19 = \frac{11}{6} + 19$

6) a) $\iint_D (6-x-y)^{-1}\, dx\, dy$

D: $|x+y| \leq 2 \ \wedge \ y \leq x+2 \leq 4$

usando $(x,y) = (v, u-v) \longrightarrow \begin{cases} x = v \\ y = u-v \end{cases}$

Hallamos el recinto transformado de D

$|x+y| \leq 2 \Rightarrow -2 \leq x+y \leq 2$

i) $-2 \leq x+y \rightarrow -2 \leq \cancel{v} + u - \cancel{v} \Rightarrow \boxed{u \geq -2}$

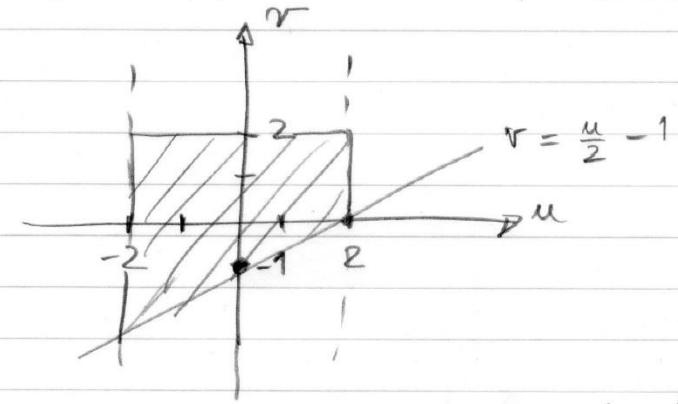
ii) $x+y \leq 2 \rightarrow \cancel{v} + u - \cancel{v} \leq 2 \Rightarrow \boxed{u \leq 2}$

$y \leq x+2 \leq 4$

i) $y \leq x+2 \rightarrow u - v \leq v+2 \Rightarrow 2v \geq u-2$

$\Rightarrow \boxed{v \geq \frac{u}{2} - 1}$

ii) $x+2 \leq 4 \Rightarrow x \leq 2 \rightarrow \boxed{v \leq 2}$



$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x'_u & x'_v \\ y'_u & y'_v \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} = -1$$

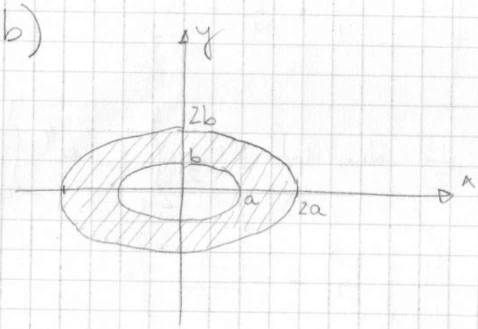
$|J| = |-1| = 1$

$$\iint_D (6-x-y)^{-1}\, dx\, dy = \int_{-2}^{2} du \int_{\frac{u}{2}-1}^{2} dv\, \overbrace{\frac{1}{6-\cancel{v}-u+\cancel{v}}}^{1/(6-u)} \cdot 1$$

$$= \int_{-2}^{2} du \, \frac{1}{6-u}\left(2-\frac{u}{2}+1\right) = \int_{-2}^{2} \frac{3-u/2}{6-u}\, du =$$

$$= \int_{-2}^{2} \frac{1}{2}\, \frac{\cancel{6-u}}{\cancel{6-u}}\, du = \frac{1}{2}\left(2-(-2)\right) = 2$$

b)



$$\begin{cases} x = a\rho\cos\theta \\ y = b\rho \,\text{sen}\,\theta \end{cases}$$

$$1 \le \rho \le 2$$

$$0 \le \theta \le 2\pi$$

$$J = \frac{\partial(x,y)}{\partial(\rho,\theta)} = \begin{vmatrix} a\cos\theta & -a\rho\,\text{sen}\,\theta \\ b\,\text{sen}\,\theta & b\rho\cos\theta \end{vmatrix} = ab\rho\cos^2\theta + ab\cdot$$

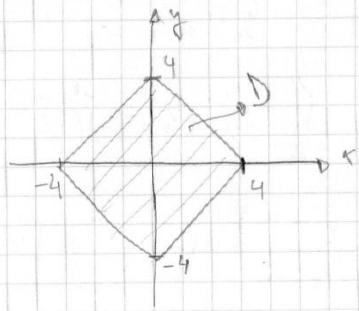
$$\cdot\rho\,\text{sen}^2\theta =$$

$$= ab\rho$$

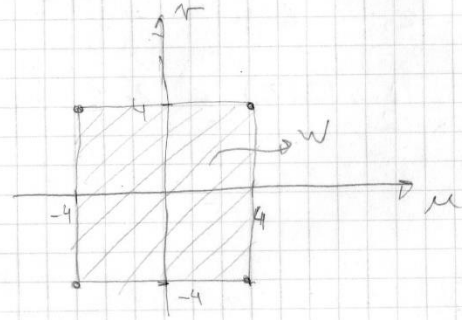
$$\text{Area} = \iint_R dx\,dy = \iint_W ab\rho\, d\rho\, d\theta = ab\int_1^2 d\rho\,\rho \int_0^{2\pi} d\theta =$$

$$= ab\,2\pi\left[\frac{\rho^2}{2}\Big|_1^2\right] = \cancel{2}ab\pi\left(\frac{4}{\cancel{2}}-\frac{1}{\cancel{2}}\right) = 3ab\pi$$

c) $|x| + |y| \leq 4$



$$\begin{cases} x - y = u \\ x + y = v \end{cases}$$



$T(4,0) = (4,4)$

$T(0,4) = (-4,4)$

$T(-4,0) = (-4,-4)$

$T(0,-4) = (4,-4)$

$$\begin{cases} x - y = u \\ x + y = v \end{cases} \Rightarrow \oplus \text{ MAM} \rightarrow 2x = u + v \Rightarrow x = \frac{u+v}{2}$$

$$\ominus \text{ MAM} \rightarrow -2y = u - v \Rightarrow y = \frac{u-v}{-2} = \frac{v-u}{2}$$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\iint_D (x-y)^4 \, dx\, dy = \iint_W u^4 \, \frac{1}{2} \, du\, dv = \frac{1}{2} \int_{-4}^{4} du\, u^4 \int_{-4}^{4} dv =$$

$$= \frac{1}{2} (4+4) \left[\frac{u^5}{5} \Big|_{-4}^{4} \right] = 4 \left[\frac{4^5}{5} - \frac{(-4)^5}{5} \right] =$$

$$= 4 \cdot 2 \cdot \frac{4^5}{5} = 2 \cdot \frac{4^6}{5}$$

6) d) $\iint_D (x+y-2)^2 \, dx\, dy \qquad \begin{cases} x = u+v \\ y = u-v \end{cases}$

$D = \{ (x,y) \in \mathbb{R}^2 / \ y \geq |x| \ , \ x+2y \leq 3 \}$

Hallamos el recinto transformado de D

i) $|x| \leq y \longrightarrow |u+v| \leq u-v \Longrightarrow$

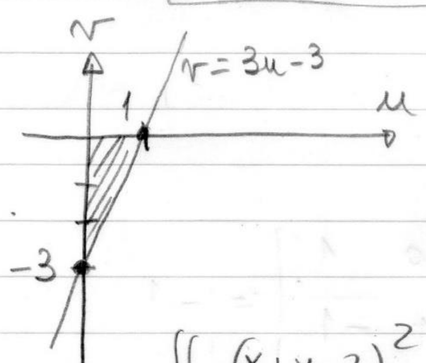
$\Longrightarrow \underbrace{-(u-v) \leq u+v}_{①} \leq u-v$, where ② is $u+v \leq u-v$

①: $-u+\cancel{v} \leq u+\cancel{v} \Longrightarrow 2u \geq 0 \Longrightarrow \boxed{u \geq 0}$

②: $\cancel{u}+v \leq \cancel{u}-v \Longrightarrow 2v \leq 0 \Longrightarrow \boxed{v \leq 0}$

ii) $x+2y \leq 3 \longrightarrow u+v+2(u-v) \leq 3 \Longrightarrow$

$\Longrightarrow \boxed{3u-3 \leq v}$



$J = \dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x'_u & x'_v \\ y'_u & y'_v \end{vmatrix} =$

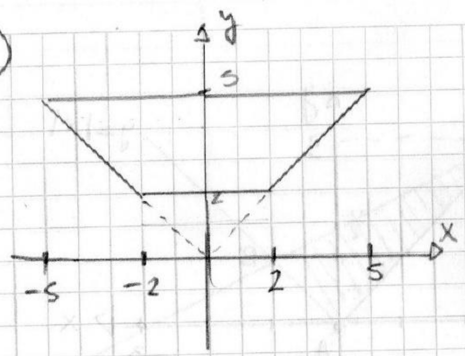
$= \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2 \Longrightarrow |J| = 2$

$\iint_D (x+y-2)^2 \, dx\, dy = \int_0^1 du \int_{3u-3}^{0} dv \, (u+\cancel{v}+u-\cancel{v}-2)^2 \cdot \overbrace{2}^{|J|} =$

$= 2 \int_0^1 du \, (2u-2)^2 \left[v \Big|_{3u-3}^{0} \right] = 2 \int_0^1 du \, (2u-2)^2 (-3u+3) =$

$= 24 \int_0^1 (-u^3+3u^2-3u+1) \, du = -\dfrac{u^4}{4} + u^3 - \dfrac{3u^2}{2} + u \Big|_0^1 = 6$

e)



$$\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$$

$$y = 2 \to \rho\, \text{sen}\,\theta = 2 \Rightarrow \rho = \frac{2}{\text{sen}\,\theta}$$

$$y = 5 \to \rho\, \text{sen}\,\theta = 5 \Rightarrow \rho = \frac{5}{\text{sen}\,\theta}$$

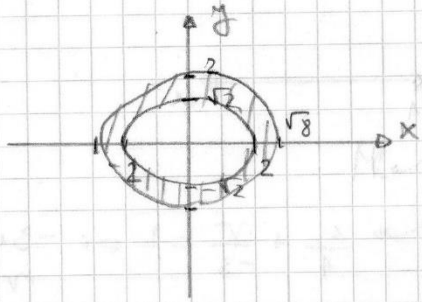
$$\iint_D y\,(x^2+y^2)^{-1}\,dx\,dy = \int_{\pi/4}^{3\pi/4} d\theta \int_{\frac{2}{\text{sen}\,\theta}}^{5/\text{sen}\,\theta} \frac{\cancel{\rho}\,\text{sen}\,\theta}{\cancel{\rho^2}}\,\cancel{\rho}\,d\rho =$$

$$= \int_{\pi/4}^{3\pi/4} \cancel{\text{sen}\,\theta}\left(\frac{5}{\cancel{\text{sen}\,\theta}} - \frac{2}{\cancel{\text{sen}\,\theta}}\right) d\theta = 3\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) =$$

$$= 3\,\frac{2\pi}{4} = \frac{6\pi}{4} = \frac{3\pi}{2}$$

7) a) $f(x,y) = e^{x^2+2y^2} = e^4 \Rightarrow x^2+2y^2 = 4 \Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{2} = 1$

$e^{x^2+2y^2} = e^8 \Rightarrow x^2+2y^2 = 8 \Rightarrow \dfrac{x^2}{8} + \dfrac{y^2}{4} = 1$



$$1 \le \frac{x^2}{4} + \frac{y^2}{2} \le 2$$

$0 \le \theta \le 2\pi$

$1 \le \rho \le \sqrt{2}$

Aplicando ejº 6) b)

$x = 2\rho\cos\theta$

$y = \sqrt{2}\,\rho\,\text{sen}\,\theta$

$J = 2\sqrt{2}\,\rho$

$$\text{Área } D = \iint_D dx\,dy = \iint_W 2\sqrt{2}\,\rho\, d\rho\, d\theta = \int_0^{2\pi} d\theta \int_1^{\sqrt{2}} 2\sqrt{2}\,\rho\, d\rho =$$

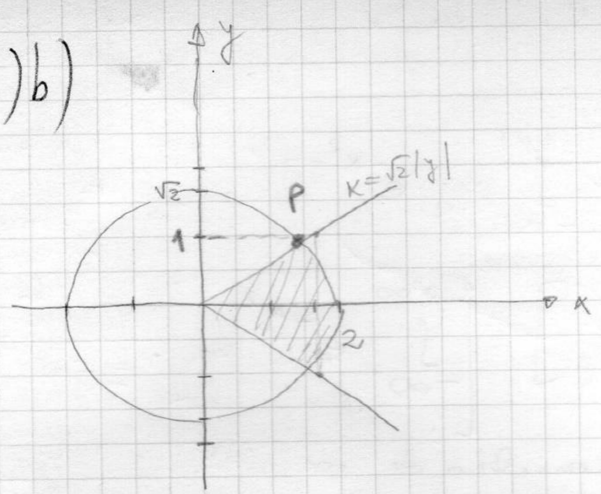
$$= 2\sqrt{2}\cdot 2\pi \left[\frac{\rho^2}{2}\Big|_1^{\sqrt{2}}\right] = 4\sqrt{2}\,\pi\left(\frac{2}{2} - \frac{1}{2}\right) = \frac{4}{2}\sqrt{2}\,\pi = 2\sqrt{2}\,\pi$$

07) b) $\iint_D e^{x^2+2y^2}\,dx\,dy$ con $x^2+2y^2 \leq 4$

$x \geq \sqrt{2}\,|y|$

$x^2+2y^2=4 \Rightarrow \frac{x^2}{4}+\frac{y^2}{2}=1$

7) b)



$x=\sqrt{2}|y| \Rightarrow x^2=2y^2$

en la elipse $\rightarrow$ $2y^2+2y^2=4 \Rightarrow |y|=1$

$\Rightarrow y=1 \vee y=-1$

$P=(\sqrt{2},1)$

$\sqrt{2}=2.1.\cos\theta \Rightarrow \cos\theta=\frac{\sqrt{2}}{2}$

$1=\sqrt{2}.1.\operatorname{sen}\theta \Rightarrow \operatorname{sen}\theta=\frac{1}{\sqrt{2}}$

$\Downarrow$

$\theta=\frac{\pi}{4}$

$$\begin{cases} x = 2\rho\cos\theta \\ y=\sqrt{2}\,\rho\operatorname{sen}\theta \end{cases} \qquad \begin{matrix} 0\leq\rho\leq 1 \\ -\pi/4 \leq \theta \leq \pi/4 \end{matrix}$$

$J=2\sqrt{2}\,\rho$ (según ejercicio 6)b)

$$\iint_D e^{x^2+2y^2}\,dx\,dy = \int_0^1 d\rho \int_{-\pi/4}^{\pi/4} d\theta\; e^{4\rho^2\cos^2\theta+2.2\rho^2\operatorname{sen}^2\theta}\, 2\sqrt{2}\rho =$$

$$2\sqrt{2}\int_0^1 d\rho\int_{-\pi/4}^{\pi/4} d\theta\; e^{4\rho^2}\rho = \cancel{2}\sqrt{2}\,\frac{\pi}{\cancel{2}}\int_1^{e^4}\cancel{u}\,\cancel{\rho}\,\frac{du}{\cancel{u}\,8\cancel{\rho}} =$$

$e^{4\rho^2}=u$

$e^{4\rho^2}.8\rho\,d\rho=du$

$\rho=0\rightarrow u=e^0=1$

$\rho=1\rightarrow u=e^4$

$$=\frac{\sqrt{2}\,\pi}{8}\left(e^4-1\right)$$

8)

a) $\iint_D e^{-x^2-y^2}\, dx\, dy = \int_0^{2\pi} d\theta \int_0^{\infty} d\rho\, e^{-\rho^2}\, d\rho\, \rho =$

$= 2\pi \int_1^0 \cancel{u} \frac{du}{-2\cancel{u}\cancel{\rho}} \cancel{\rho} = -\pi \int_1^0 du =$

$e^{-\rho^2} = u$

$e^{-\rho^2}(-2\rho)\, d\rho = du$

$\rho = 0 \Rightarrow u = e^0 = 1$

$\rho \to \infty \Rightarrow u = \lim_{\rho\to\infty} e^{-\rho^2} = 0$

$= -\pi\,(0-1) = \pi$

8) b) $\left(\int_{-\infty}^{\infty} e^{-u^2} du\right)^2 = \left(\int_{-\infty}^{\infty} e^{-u^2} du\right)\cdot\left(\int_{-\infty}^{\infty} e^{-u^2} du\right) = A$

$B = \iint_D e^{-x^2-y^2}\, dx\, dy = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy\, e^{-x^2-y^2} = \int_{-\infty}^{+\infty} dx \left[\int_{-\infty}^{+\infty} dy\, e^{-x^2-y^2}\right]$

$= \int_{-\infty}^{+\infty} dx \left[\int_{-\infty}^{+\infty} e^{-x^2}\cdot e^{-y^2} dy\right] = \left[\int_{-\infty}^{+\infty} e^{-x^2} dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^{-y^2} dy\right] =$

Haciendo en c/caso el cambio de variables $y = u$, $x = u$ obtenemos

$B = \left(\int_{-\infty}^{+\infty} e^{-u^2} du\right)\cdot\left(\int_{-\infty}^{+\infty} e^{-u^2} du\right) = \left[\int_{-\infty}^{+\infty} e^{-u^2} du\right]^2$

8c) $\text{Área} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}z^2} dz =$

Sustitución

$u = \frac{1}{\sqrt{2}} z$

$du = \frac{1}{\sqrt{2}} dz$

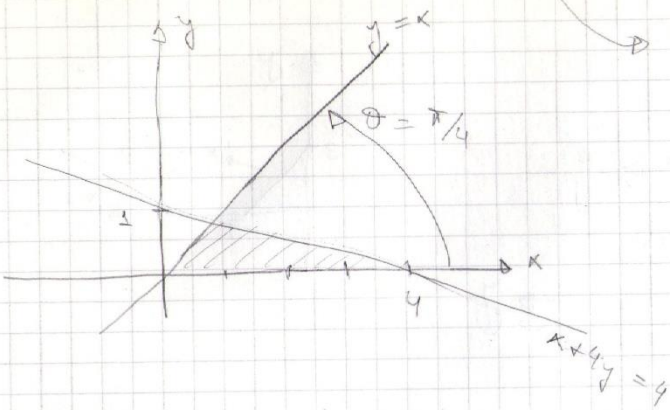
$z \to \pm\infty \Rightarrow u \to \pm\infty$

$$= \frac{1}{\sqrt{2}\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-u^2} \sqrt{2}\, du =$$

$$= \frac{\sqrt{2}}{\sqrt{2}\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-u^2} du =$$

$$= \frac{\sqrt{2}}{\sqrt{2}\sqrt{\pi}} \cdot \sqrt{\pi} = 1$$

09) $x \geq y \quad x + 4y \leq 4 \quad y \geq 0 \Rightarrow \frac{x}{4} + y \leq 1$



$$\begin{cases} x = \rho\cos\theta \\ y = \rho\,\text{sen}\,\theta \end{cases}$$

$x + 4y = 4 \Rightarrow \rho\cos\theta + 4\rho\,\text{sen}\,\theta = 4 \Rightarrow$

$\Rightarrow \rho = \frac{4}{\cos\theta + 4\,\text{sen}\,\theta}$

$$\iint_D \frac{x+4y}{x^2}\, dx\, dy = \int_0^{\pi/4} d\theta \int_0^{\frac{4}{\cos\theta + 4\,\text{sen}\,\theta}} \frac{\cancel{\rho}\cos\theta + 4\cancel{\rho}\,\text{sen}\,\theta}{\cancel{\rho^2}\cos^2\theta} \cancel{\rho}\, d\rho$$

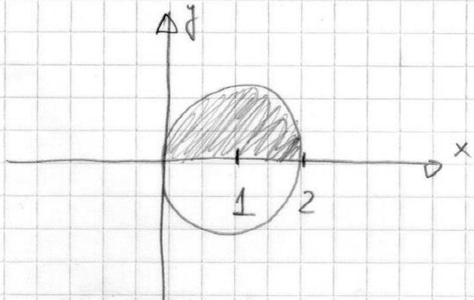
$$= \int_0^{\pi/4} d\theta \left(\frac{\cancel{\cos\theta + 4\,\text{sen}\,\theta}}{\cos^2\theta}\right) \frac{4}{\cancel{\cos\theta + 4\,\text{sen}\,\theta}} =$$

$$= 4\left[\text{tg}\,\theta \Big|_0^{\pi/4}\right] = 4(1-0) = 4$$

10) a) $\int_0^{\pi/2} d\varphi \int_0^{2\cos\varphi} \rho^3 \, d\rho \qquad \underbrace{\rho}_{J} \cdot \underbrace{\rho^2}_{x^2+y^2}$

$0 \leq \varphi \leq \pi/2 \qquad 0 \leq \rho \leq 2\cos\varphi$

$\rho = 2\cos\varphi \Rightarrow \rho^2 = 2\rho\cos\varphi \Rightarrow x^2+y^2 = 2x \Rightarrow$
$\Rightarrow x^2 - 2x + 1 + y^2 = 1 \Rightarrow (x-1)^2 + y^2 = 1$



$\int_0^2 dx \int_0^{\sqrt{2x-x^2}} (x^2+y^2) \, dy$

Resuelvo en polares:

$\int_0^{\pi/2} d\varphi \int_0^{2\cos\varphi} \rho^3 \, d\rho = \int_0^{\pi/2} d\varphi \left[\frac{\rho^4}{4} \Big|_0^{2\cos\varphi} \right] =$

$= \int_0^{\pi/2} d\varphi \left[\frac{16}{4} \cos^4\varphi \right] = 4 \int_0^{\pi/2} \cos^4\varphi \, d\varphi =$

$= 4 \left[\frac{3}{8}\varphi + \frac{\operatorname{sen}(2\varphi)}{4} + \frac{\operatorname{sen}(4\varphi)}{32} \Big|_0^{\pi/2} \right] =$

$= \cancel{4}^{1} \left[\frac{3}{\cancel{8}_2} \frac{\pi}{2} + \cancel{\frac{\operatorname{sen}\pi}{4}} + \cancel{\frac{\operatorname{sen}(2\pi)}{32}} - 0 \right] = \frac{3}{4}\pi$

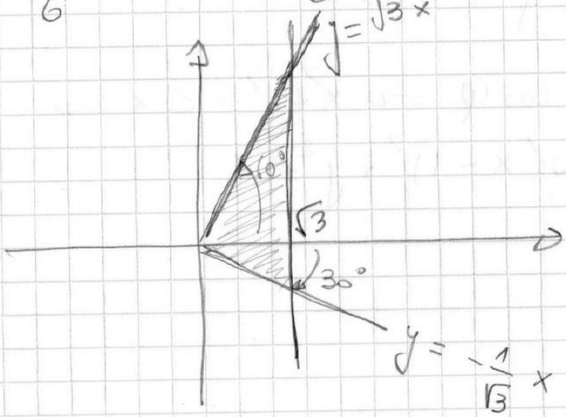
b) $\int_{-\pi/6}^{\pi/3} d\varphi \int_0^{\frac{\sqrt{3}}{\cos\varphi}} \underbrace{\rho}_{}{}^2 \cos\varphi \; d\rho$

$\underbrace{\rho}_{J} \cdot \underbrace{\rho\cos\varphi}_{x}$

$-\frac{\pi}{6} \le \varphi \le \pi/3$

$0 \le \underbrace{\rho \le \frac{\sqrt{3}}{\cos\varphi}}$

$\rho\cos\varphi \le \sqrt{3} \Rightarrow x \le \sqrt{3}$



$\int_0^{\sqrt{3}} dx \int_{-\frac{1}{\sqrt{3}}x}^{\sqrt{3}x} x \; dy$ Resuelvo en cartesianas

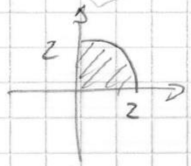
$\int_0^{\sqrt{3}} dx \; x \left[\sqrt{3}x + \frac{1}{\sqrt{3}}x\right] = \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) \int_0^{\sqrt{3}} x^2 \; dx =$

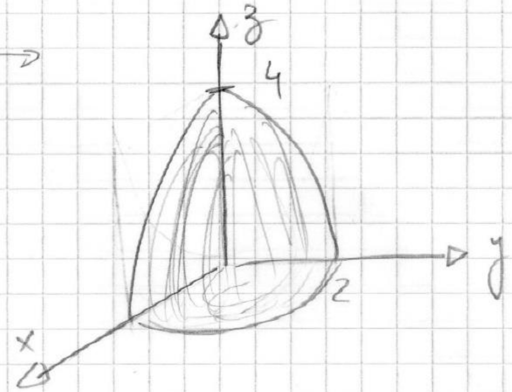
$= \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) \left[\frac{x^3}{3}\Big|_0^{\sqrt{3}}\right] = \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) \frac{\cancel{3}\sqrt{3}}{\cancel{3}} =$

$= \frac{3+1}{\cancel{\sqrt{3}}} \cdot \cancel{\sqrt{3}} = 4$

11) $\int_0^{\pi/2} d\varphi \int_0^2 d\rho \int_0^{4-\rho^2} \underbrace{\rho^2}_{\underbrace{\rho}_{\text{jacob}} \cdot \underbrace{\rho}_{\sqrt{x^2+y^2}}} dz$

$0 \le \varphi \le \pi/2 \qquad 0 \le \rho \le 2 \qquad 0 \le z \le \underbrace{4-\rho^2}_{4-x^2-y^2}$





$$\int_0^2 dx \int_0^{\sqrt{4-x^2}} dy \int_0^{4-x^2-y^2} \sqrt{x^2+y^2}\, dz$$

Resuelvo en polares:

$$\int_0^{\pi/2} d\varphi \int_0^2 d\rho \int_0^{4-\rho^2} \rho^2\, dz = \int_0^{\pi/2} d\varphi \int_0^2 \underbrace{\rho^2(4-\rho^2)}_{4\rho^2-\rho^4} d\rho$$

$$= \pi/2 \left[\frac{4\rho^3}{3} - \frac{\rho^5}{5} \Big|_0^2 \right] = \frac{\pi}{2}\left[\frac{4 \cdot 8}{3} - \frac{32}{5}\right] =$$

$$= \frac{\pi}{2} \cdot \frac{64}{15} = \frac{32\pi}{15}$$

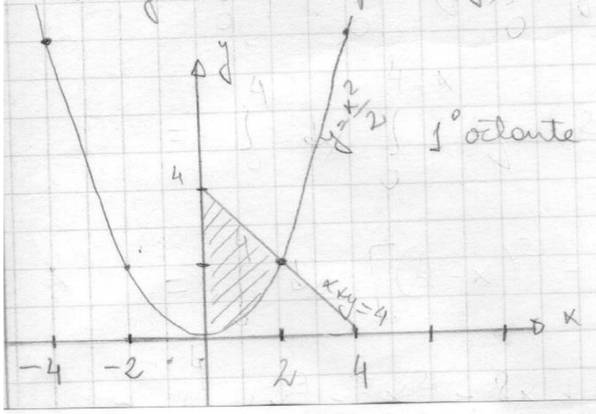
12) a) $2y \geq x^2 + z$; $x + y \leq 4$; 1° octante

$\Downarrow$

$z \leq 2y - x^2$

si $z = 0 \Rightarrow 2y - x^2 \geq 0 \Rightarrow y \geq \frac{x^2}{2}$

Proyecc. en pl (xy)



$$\begin{cases} y = \frac{x^2}{2} \\ y = 4 - x \end{cases} \Rightarrow \frac{x^2}{2} - 4 - x = 0$$

$\Rightarrow \frac{x^2}{2} + x - 4 = 0 \Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow x = \frac{-2 \pm \sqrt{4+32}}{2} \begin{array}{l} \nearrow 2 \\ \searrow -4 \end{array}$

$$\text{Vol } V = \int_0^2 dx \int_{\frac{x^2}{2}}^{4-x} dy \int_0^{2y-x^2} dz = \int_0^2 dx \int_{x^2/2}^{4-x} (2y - x^2)\, dy =$$

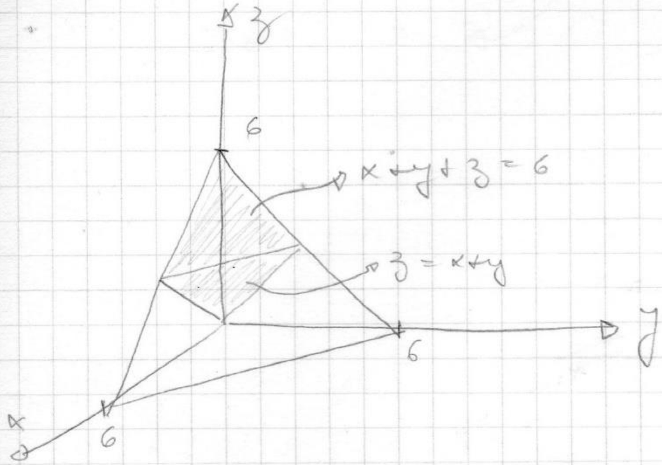
$$= \int_0^2 dx \left[y^2 - x^2 y \Big|_{x^2/2}^{4-x} \right] = \int_0^2 dx \left[(4-x)^2 - x^2(4-x) - \frac{x^4}{4} + \right.$$

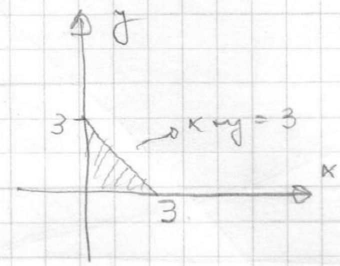
$$\left. + x^2 \frac{x^2}{2} \right] = \int_0^2 \left(16 + x^2 - 8x - 4x^2 + x^3 + \frac{x^4}{4} \right) dx =$$

$$= \left[16x - x^3 - 4x^2 + \frac{x^4}{4} + \frac{x^5}{20} \Big|_0^2 \right] = 32 - 8 - 16 + 4 + 1{,}6 =$$

$$= 13{,}6$$

12) b) $x + y + z \leq 6 \qquad z \geq x+y \qquad x \geq 0 \quad y \geq 0$





$$\left.\begin{array}{l} x+y+z=6 \\ z=x+y \end{array}\right\} \Rightarrow x+y+x+y=6 \Rightarrow 2(x+y)=6 \Rightarrow$$

$$\Rightarrow x+y=3$$

$$\text{Vol } V = \int_0^3 dx \int_0^{3-x} dy \int_{x+y}^{6-x-y} dz = \int_0^3 dx \int_0^{3-x} dy \left[6-x-y-x-y\right] =$$

$$= \int_0^3 dx \int_0^{3-x} \left[6-2x-2y\right] dy = \int_0^3 dx \left[6y-2xy-y^2 \Big|_0^{3-x}\right] =$$

$$= \int_0^3 dx \left[6(3-x)-2x(3-x)-(3-x)^2\right] =$$

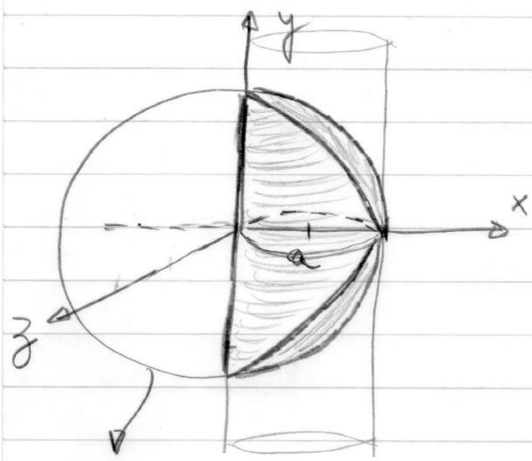
$$= \int_0^3 \left[18-6x-\cancel{6x}+2x^2-9-x^2+\cancel{6x}\right] dx =$$

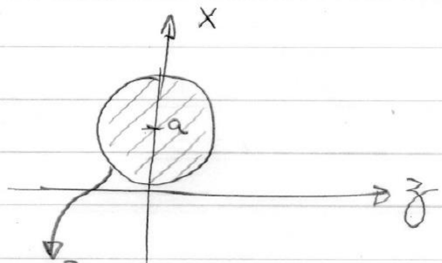
$$= \int_0^3 \left[x^2-6x+9\right] dx = \left[\frac{x^3}{3}-3x^2+9x \Big|_0^3\right] =$$

$$= \frac{\cancel{27}^{\,9}}{\cancel{3}} - \cancel{27} + \cancel{27} = 9$$

12) c) $x^2+z^2 \leq 2ax$ interior a la esfera de radio $2a$ con centro en $\bar{O}$

$x^2+z^2=2ax \Rightarrow x^2-2ax+z^2=0 \Rightarrow x^2-2ax+a^2+z^2=a^2$
$\Rightarrow (x-a)^2+z^2=a^2$





$x^2+z^2=2ax \Rightarrow$
$\Rightarrow \rho^2 = 2a\rho \operatorname{sen}\theta \Rightarrow$
$\Rightarrow \rho = 2a \operatorname{sen}\theta$

$x^2+y^2+z^2=4a^2 \Rightarrow |y| = \sqrt{4a^2-x^2-z^2} \Rightarrow |y| = \sqrt{4a^2-\rho^2}$

$$\operatorname{Vol} V = \iiint_V dx\,dy\,dz = \int_0^{\pi} d\theta \int_0^{2a\operatorname{sen}\theta} d\rho \int_{-\sqrt{4a^2-\rho^2}}^{\sqrt{4a^2-\rho^2}} \rho\, dy =$$

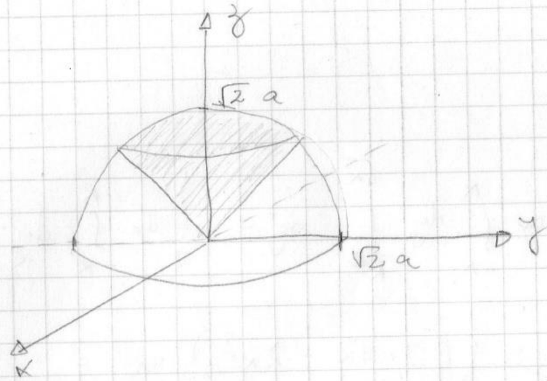
$$= \int_0^{\pi} d\theta \int_0^{2a\operatorname{sen}\theta} d\rho \left[\rho y \Big|_{-\sqrt{4a^2-\rho^2}}^{\sqrt{4a^2-\rho^2}} \right] =$$

$$= \int_0^{\pi} d\theta \int_0^{2a\operatorname{sen}\theta} 2\rho\sqrt{4a^2-\rho^2}\, d\rho = \int_0^{\pi} d\theta \left[-\frac{2}{3}\sqrt{(4a^2-\rho^2)^3} \Big|_0^{2a\operatorname{sen}\theta} \right. =$$

$$= \int_0^{\pi} d\theta \left[-\frac{2}{3}\sqrt{(4a^2-4a^2\operatorname{sen}^2\theta)^3} + \frac{2}{3}\sqrt{(4a^2)^3} \right] =$$

$$= \frac{16}{9} a^3 (3\pi - 4)$$

d) $z \geq \sqrt{x^2+y^2}$; $x^2+y^2+z^2 \leq 2a^2$



$$Vol = \iiint_V dx\,dy\,dz = \int_0^{2\pi} d\theta \int_0^{\pi/4} d\lambda \int_0^{\sqrt{2}a} d\rho\,.\,\rho^2 \operatorname{sen}\lambda =$$

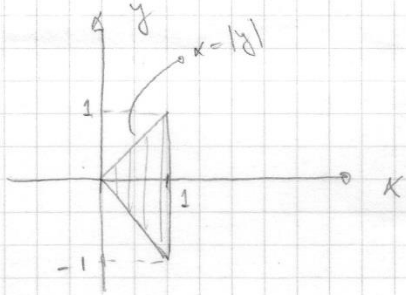
$$= 2\pi \int_0^{\pi/4} \operatorname{sen}\lambda\, d\lambda \left[\frac{\rho^3}{3}\Big|_0^{\sqrt{2}a}\right] = 2\pi\,\frac{2\sqrt{2}\,a^3}{3}\left[-\cos\lambda\Big|_0^{\pi/4}\right] =$$

$$= \frac{4\pi\sqrt{2}\,a^3}{3}\left(-\frac{\sqrt{2}}{2}+1\right) = \frac{4\pi\sqrt{2}\,a^3}{3}\left(1-\frac{1}{\sqrt{2}}\right) = (\sqrt{2}-1)\,\frac{4\pi\cdot a^3}{3}$$

e) $z \geq x^2$; $x \geq z^2 \to |z| \leq \sqrt{x}$

$$\left.\begin{array}{l} z = x^2 \\ x = z^2 \end{array}\right\} \Rightarrow z = z^2 \Rightarrow z(z-1) = 0 \Rightarrow \underbrace{z = 0 \ \vee \ z = 1}$$

luego $z \geq 0$, entonces es $z \leq \sqrt{x}$

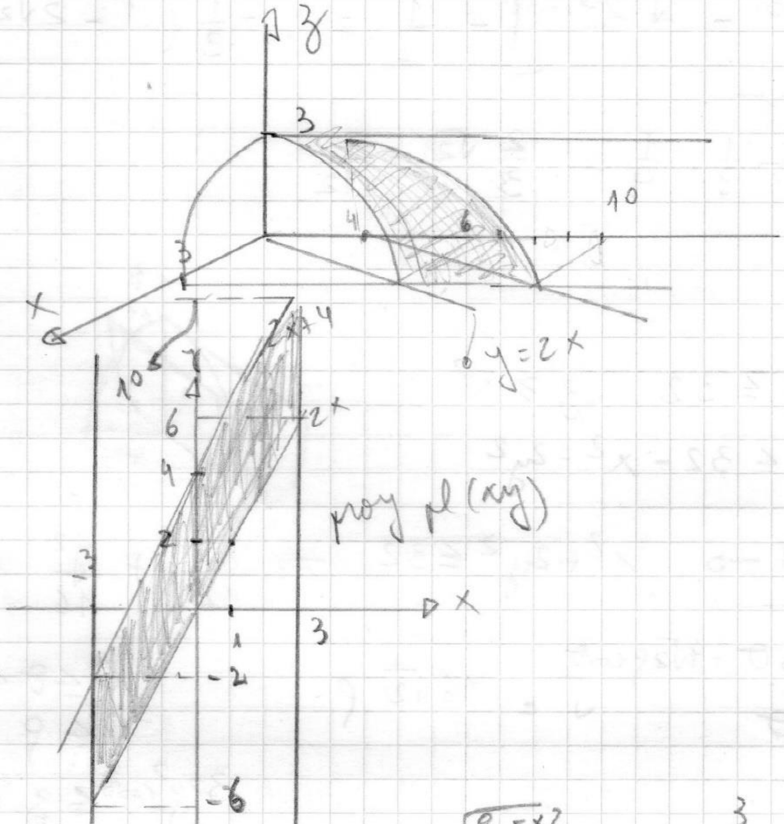


$$\text{Vol } V = \iiint_V dx\, dy\, dz = \int_0^1 dx \int_{-x}^{x} dy \int_{x^2}^{\sqrt{x}} dz = \int_0^1 dx \int_{-x}^{x} dy \left(\sqrt{x} - x^2\right)$$

$$= \int_0^1 dx\, (x + x)\left(\sqrt{x} - x^2\right) = \int_0^1 \Big(2 \underbrace{x\sqrt{x}}_{x^{3/2}} - 2x^3\Big)\, dx =$$

$$= \left[2\, \frac{2}{5}\, x^{5/2} - 2\, \frac{x^4}{4} \Big|_0^1 \right] = \frac{4}{5} - \frac{1}{2} = \frac{8-5}{10} = \frac{3}{10}$$

12) f) $x^2+z^2 \le 9$, $y \ge 2x$, $y \le 2x+4$



$$\mathrm{Vol}\ H = \int_{-3}^{3} dx \int_{2x}^{2x+4} dy \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} dz = \int_{-3}^{3} dx\,(2x+4-2x)\cdot$$

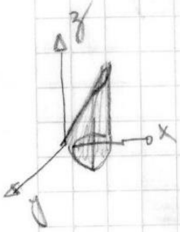
$$\cdot\left(\sqrt{9-x^2}+\sqrt{9-x^2}\right) = \int_{-3}^{3} 4\cdot 2\sqrt{9-x^2}\,dx = 8\left[\frac{1}{2}\left(x\sqrt{9-x^2}+\right.\right.$$

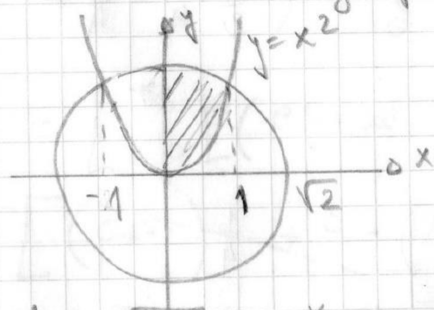
$$\left.\left.+9\,\text{arcosen}\,\frac{x}{3}\right)\right] = \frac{8}{2}\left[\ \cdot 3\cdot 0 + 9\,\text{arcosen}\,1 - \ (-3)\cdot 0\right.$$

$$\left. - 9\,\text{arcosen}\,(-1)\right] = 4\cdot 9\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right) = 36\,\frac{\pi}{2}\cdot 2 = 36\pi$$

9) $y \geq x^2 \quad x^2+y^2 \leq 2 \quad z \geq 0 \quad z \leq x \to x \geq 0$

gráfico sólo proyección





$$\begin{cases} y = x^2 \\ x^2+y^2 = 2 \end{cases} \Rightarrow y + y^2 = 2 \Rightarrow$$

$$\Rightarrow y^2+y-2=0 \quad \begin{cases} y = 1 \to x^2 = 1 \begin{cases} x = 1 \\ x = -1 \end{cases} \\ y = -2 \to x^2 = -2 \text{ abs} \end{cases}$$

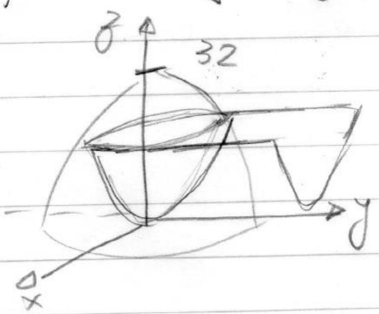
$$\text{Vol } H = \int_0^1 dx \int_{x^2}^{\sqrt{2-x^2}} dy \int_0^x dz = \int_0^1 \left(\sqrt{2-x^2} - x^2\right) x \, dx = \int_0^1 x\sqrt{2-x^2}\, dx -$$

$$- \int_0^1 x^3 dx = -\frac{1}{3}\left[\sqrt{(2-x^2)^3}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1 =$$

$$= -\frac{1}{3}\left[\sqrt{(2-1)^3} - \sqrt{2^3}\right] - \frac{1}{4} = -\frac{1}{3}\left(1 - 2\sqrt{2}\right) - \frac{1}{4} =$$

$$= \frac{2}{3}\sqrt{2} - \frac{1}{3} - \frac{1}{4} = \frac{2}{3}\sqrt{2} - \frac{7}{12}$$

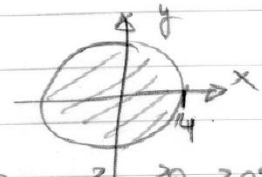
h) $x^2+2y^2+z \leq 32 \wedge z \geq x^2$



Analizo la proyección de la curva de corte en el pl (xy)

$$\begin{cases} z = 32-x^2-2y^2 \\ z = x^2 \end{cases} \Rightarrow$$

$\Rightarrow 32-x^2-2y^2 = x^2 \Rightarrow x^2+y^2=16$



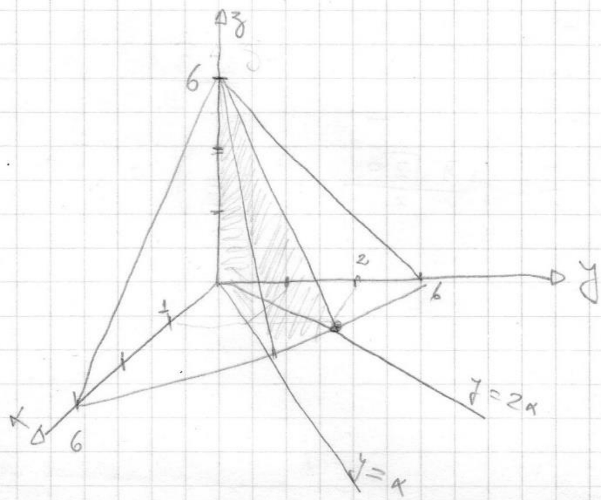
$$\text{Vol } H = \iiint_V dx\,dy\,dz = \int_0^{2\pi} d\theta \int_0^4 d\rho \int_{\rho^2\cos^2\theta}^{32-\rho^2\cos^2\theta-2\rho^2\operatorname{sen}^2\theta} \rho\, dz =$$

$$= \int_0^{2\pi} d\theta \int_0^4 d\rho\ \rho\left[32-\rho^2\cos^2\theta-2\rho^2\operatorname{sen}^2\theta-\rho^2\cos^2\theta\right] =$$

$$= \int_0^{2\pi} d\theta \int_0^4 d\rho\ \rho\left[32-2\rho^2\right] = \int_0^{2\pi} d\theta \left[\frac{32\rho^2}{2}-\frac{2\rho^4}{4}\Big|_0^4\right] =$$

$$= \int_0^{2\pi} d\theta\left(16\cdot 16-\frac{1}{2}256\right) = 2\pi\cdot 128 = 256\pi$$

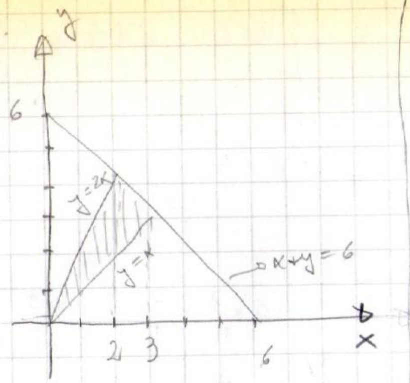
13)



$\delta = k \text{ dist pl } (yz)$

$\delta = k|x| \quad (k \in \mathbb{R})$

$\delta = kx$ por ser 1° octante



$$\text{Masa} = \iiint_V \delta \, dx\, dy\, dz = \iiint_V kx \, dx\,dy\,dz =$$

$$= \int_0^2 dx \int_x^{2x} dy \int_0^{6-x-y} kx \, dz + \int_2^3 dx \int_x^{6-x} dy \int_0^{6-x-y} kx =$$

$$= \frac{15}{2}$$

$$\left.\begin{array}{l} y = x \\ x+y = 6 \end{array}\right\} \Rightarrow 2x = 6 \Rightarrow x = 3$$

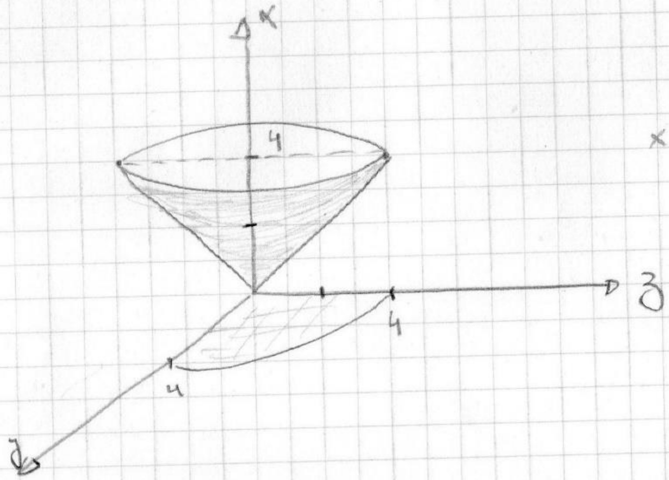
$$\left.\begin{array}{l} y = 2x \\ x+y = 6 \end{array}\right\} \Rightarrow x + 2x = 6 \Rightarrow x = \frac{6}{3} = 2$$

14)

$\sqrt{y^2+z^2} \leq x$ $\quad$ frontiera $\quad$ $x = \sqrt{y^2+z^2} \Rightarrow x^2 = y^2+z^2$

$x = 4 \Rightarrow y^2+z^2 = 16$

$x = \sqrt{\underbrace{\rho^2 \sin^2\theta}_{z^2} + \underbrace{\rho^2\cos^2\theta}_{y^2}} = \sqrt{\rho^2} = \rho$



$$I_x = \iiint_V \delta (y^2+z^2)\, dx\, dy\, dz = \iiint_V k|y|\,(y^2+z^2)\, dx\, dy\, dz$$

$$= k\int_0^{2\pi} d\theta \int_0^4 d\rho \int_\rho^4 dx \; |\rho\cos\theta|\, \rho^2 \, \rho =$$

$$= k\int_0^{2\pi} d\theta \int_0^4 d\rho \int_\rho^4 \rho^4\, |\cos\theta|\, dx =$$

$$= k \int_0^{2\pi} d\theta \int_0^4 d\rho \ (4-\rho)\rho^4 |\cos\theta| = k\int_0^{2\pi} |\cos\theta| \, d\theta \int_0^4 (4\rho^4 - \rho^5) \, d\rho$$

$$= k \int_0^{2\pi} |\cos\theta| \, d\theta \left[\frac{4\rho^5}{5} - \frac{\rho^6}{6} \Big|_0^4 \right] = k \left(\frac{4096}{5} - \frac{4096}{6} \right) \cdot$$

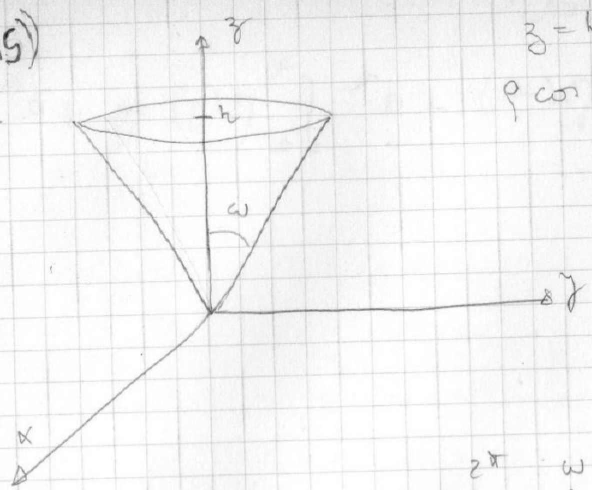
$$\cdot \left[\int_0^{\pi/2} \cos\theta \, d\theta + \int_{\pi/2}^{3\pi/2} - \cos\theta \, d\theta + \int_{3\pi/2}^{2\pi} \cos\theta \, d\theta \right] =$$

$$= k \cdot \frac{2048}{15} \left[\operatorname{sen}\theta \Big|_0^{\pi/2} - \operatorname{sen}\theta \Big|_{\pi/2}^{3\pi/2} + \operatorname{sen}\theta \Big|_{3\pi/2}^{2\pi} \right] =$$

$$= k \ \frac{2048}{15} \Big[\underbrace{(1-0)}_{1} - \underbrace{(-1-1)}_{+\ 2} + \underbrace{(0+1)}_{+\ 1} \Big] = k \ \frac{2048}{15} \cdot 4 =$$

$$= \frac{8192 k}{15}$$

15)



$$z = h$$

$$\rho \cos\lambda = h \Rightarrow \rho = \frac{h}{\cos\lambda}$$

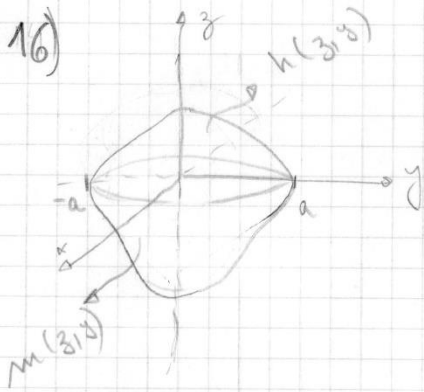
$$\text{Vol } V = \iiint_V dx\,dy\,dz = \int_0^{2\pi} d\theta \int_0^{\omega} d\lambda \int_0^{\frac{h}{\cos\lambda}} d\rho\ \rho^2 \operatorname{sen}\lambda =$$

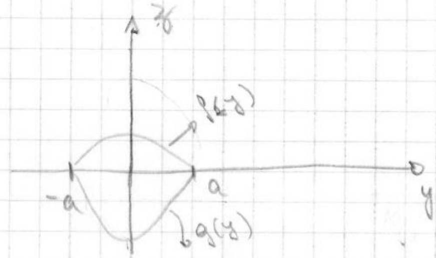
$$= 2\pi \int_0^{\omega} d\lambda\ \operatorname{sen}\lambda \left[\frac{\rho^3}{3} \Big|_0^{h/\cos\lambda} \right] = \frac{2\pi}{3} \int_0^{\omega} \operatorname{sen}\lambda \frac{h^3}{\cos^3\lambda}\, d\lambda =$$

$$= \frac{2\pi h^3}{3} \left[\frac{1}{2} \frac{1}{\cos^2\lambda} \Big|_0^{\omega} \right] = \frac{\cancel{2}\pi h^3}{3} \left[\frac{1}{\cancel{2}} \frac{1}{\cos^2\omega} - \frac{1}{\cancel{2}} \right] =$$

$$= \frac{\pi h^3}{3} \left[\frac{1}{\cos^2\omega} - 1 \right] = \frac{\pi h^3}{3} \frac{1 - \cos^2\omega}{\cos^2\omega} = \frac{\pi h^3}{3} \frac{\operatorname{sen}^2\omega}{\cos^2\omega}$$

16)





proyección de V sobre el plano (z y)

$$\iiint_V y^n \, dx\, dy\, dz = \int_{-a}^{a} dy \int_{g(y)}^{f(y)} dz \int_{h(z,y)}^{m(z,y)} dx \; y^n =$$

$$= \int_{-a}^{a} dy \; y^n \underbrace{\int_{g(y)}^{f(y)} dz \left[m(z,y) - h(z,y) \right]}_{\text{esta integral es una función de } y \text{ sea } \beta(y)}$$

$$= \int_{-a}^{a} y^n \beta(y) \, dy = \frac{1}{2}\left(\frac{y^{n+1}}{n+1} \beta(y) + \int \beta(y)\, dy \cdot y^n \right) \Bigg]_{-a}^{a}$$

$$= \frac{1}{2}\left(\frac{a^{n+1}}{n+1} \beta(a) + \int_{-a}^{a} \beta(y)\, dy \cdot a^n - \frac{(-a)^{n+1}}{n+1} \beta(-a) - (-a)^n \int_{-a}^{a} \beta(y)\, dy \right)$$

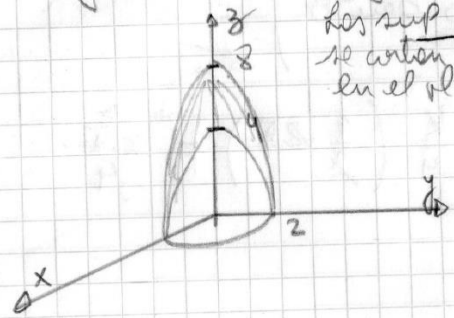
n impar $\Rightarrow$ n+1 es par

$$\frac{1}{2}\left(\frac{a^{n+1}}{n+1} \beta(a) - \frac{a^{n+1}}{n+1} \beta(-a) + a^n \int_{-a}^{a} \beta(y)\, dy + a^n \int_{-a}^{a} \beta(y)\, dy \right)$$

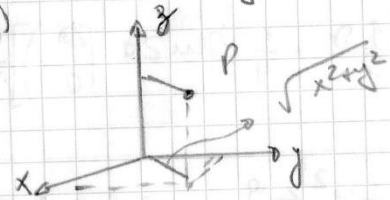
17) a) $z = 4 - x^2 - y^2$ $\qquad z = 8 - 2x^2 - 2y^2$

Busco la proyección en el pl(xy) de la curva intersecc de las superficies

$$4 - x^2 - y^2 = 8 - 2x^2 - 2y^2 \Rightarrow \boxed{x^2 + y^2 = 4}$$



Los sup se cortan en el pl(xy) $\rightarrow z = 4 - \underbrace{4}_{x^2+y^2} = 0$



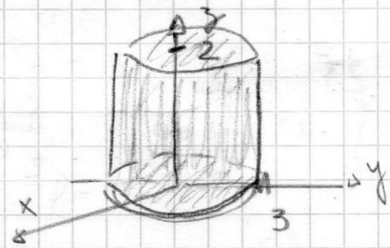
$\delta = k\sqrt{x^2+y^2} \qquad (k \in \mathbb{R}^+)$

$$M = \iiint_V \delta \, dx \, dy \, dz = k \int_0^{2\pi} d\theta \int_0^2 d\rho \int_{4-\rho^2}^{8-2\rho^2} dz \; \rho \sqrt{\rho^2} =$$

$$= k \, 2\pi \int_0^2 d\rho \, \rho^2 \left(8 - 2\rho^2 - 4 + \rho^2\right) = 2k\pi \int_0^2 d\rho \left(4\rho^2 - \rho^4\right) =$$

$$= 2k\pi \left(\frac{4\rho^3}{3} - \frac{\rho^5}{5} \Big|_0^2 \right) = 2k\pi \left(\frac{32}{3} - \frac{32}{5} \right) = 2k\pi \frac{64}{15} = \frac{128k\pi}{15}$$

17) c) $x^2 + y^2 \leq 9 \qquad 0 \leq z \leq 2$



$\delta = k \cdot |y| \qquad k \in \mathbb{R}^+$

$M = \iiint_V \delta \, dx\, dy\, dz = k \int_0^{2\pi} d\theta \int_0^3 d\rho\, \rho \int_0^2 dz\, |\rho \operatorname{sen} \theta| =$

$= k \cdot 2 \int_0^{2\pi} |\operatorname{sen}\theta| \, d\theta \int_0^3 d\rho\, \rho^2 = 2k \cdot \left[\frac{\rho^3}{3} \Big|_0^3 \right] \cdot$

$\cdot \left(\int_0^{\pi} \operatorname{sen}\theta \, d\theta + \int_{\pi}^{2\pi} -\operatorname{sen}\theta \, d\theta \right) = 2k \cdot 9 \left[-\cos\theta \Big|_0^{\pi} \right] +$

$+ \left[\cos\theta \Big|_{\pi}^{2\pi} \right] = 18k \left[-(-1) + 1 + 1 - (-1) \right] =$

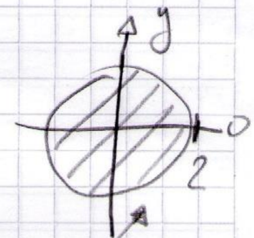
$= 18k \; 4 = 72k$

18) $\text{Vol } V = \iiint_V dx\,dy\,dz = \int_0^{2\pi} d\theta \int_0^{2} d\rho \int_{8\rho^2-\rho^4}^{16} \rho \; dz =$

$$= \int_0^{2\pi} d\theta \int_0^{2} d\rho \; \rho\,(16 - 8\rho^2 + \rho^4) = 2\pi \cdot \left[\frac{16\rho^2}{2} - \frac{8\rho^4}{4} + \frac{\rho^6}{6} \Big|_0^2 \right] = 2\pi \left[8\cdot 4 - 2\cdot 16 + \frac{64}{6} \right]$$

$$= 2\pi \cdot \frac{32}{3} = \frac{64}{3}\pi$$

Busco el máximo de la función



$f(x,y) = 8(x^2+y^2) - (x^2+y^2)^2$

$$\begin{cases} f'_x = 8\cdot 2x - 2(x^2+y^2)\cdot 2x = 0 & (1) \\ f'_y = 8\cdot 2y - 2(x^2+y^2)\cdot 2y = 0 & (2) \end{cases}$$

(1) $\to 4x\,(4 - x^2 - y^2) = 0$ $\to x = 0 \to$ descarto (es mín

$\to 4 - x^2 - y^2 = 0 \Rightarrow x^2+y^2 = 4$

(2) $\to 4y\,(4 - x^2 - y^2) = 0$ $\to y = 0 \to$ descarto (es mín)

$$z_{máx} = 8 \cdot \underbrace{4}_{x^2+y^2} - \underbrace{4}_{x^2+y^2}^{2} = 32 - 16 = 16$$