11- Teoremas integrales

① Teorema de Green

$$\oint_{C^+} \bar{f}\, d\bar{s} = \iint_D (Q'_x - P'_y)\, dx\, dy$$

$$\Rightarrow \bar{f}\, d\bar{s} = (P, Q, R)(dx, dy, \cancel{dz}) = P\,dx + Q\,dy$$

$$Q'_x - P'_y = K$$

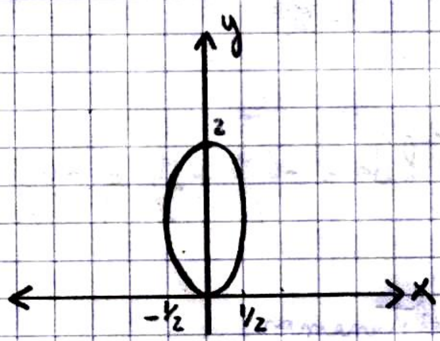
$$dx\,dy = d\bar{s}$$

$$\Rightarrow \oint_{C^+} \bar{f}\, d\bar{s} = \iint_D K\, dx\,dy = K \overbrace{\iint_D dx\,dy}^{A(D)}$$

$$\boxed{A(D) = \frac{1}{K} \oint_{C^+} \bar{f}\, d\bar{s}}$$

③ $\bar{f}_{(x,y)} = (x^2+y^2;\ 3xy + \ln(y^2+1))$

$4x^2 + (y-1)^2 \leq 1$ recorrido en sentido positivo



$$\begin{cases} x = u \\ y = 2v + 1 \end{cases} \Rightarrow u^2 + v^2 \leq \frac{1}{4}$$

$$\begin{cases} u = r\cos(\theta) \\ v = r\,\text{sen}(\theta) \end{cases} \rightarrow \begin{matrix} x = r\cos\theta \\ y = 2r\,\text{sen}\,\theta + 1 \end{matrix}$$

$|Df| = 2r \qquad 0 \leq r \leq 1/2 \qquad 0 \leq \theta \leq 2\pi$

$Q'_x - P'_y = y = 2r\,\text{sen}(\theta) + 1$

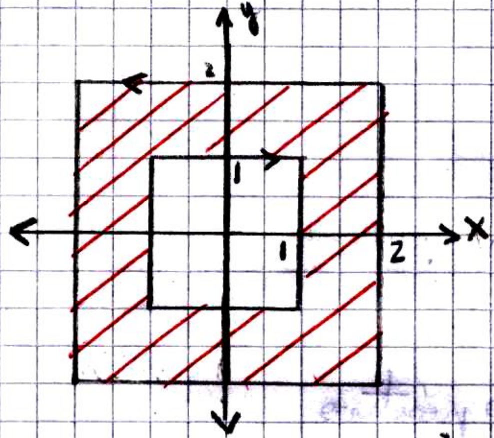
$$\iint_D (Q'_x - P'_y)\, dx\, dy$$

$$\int_0^{2\pi}\int_0^{1/2} 4r^2 \operatorname{sen}(\theta) + 2r \, dr\, d\theta = \int_0^{2\pi} 4 \operatorname{sen}(\theta)\frac{r^3}{3} + r^2 \Big|_0^{1/2} d\theta$$

$$\frac{1}{2}\left(-\frac{\cos\theta}{3} + \frac{\theta}{2}\right)\Big|_0^{2\pi} = \boxed{\frac{\pi}{2}}$$ Hecha mayormente por calculadora.

④ $f(x,y) = (x^2y, y^2)$ en la region plan $D = D_1 - D_2$

$D_1 = [-2,2]\times[-2,2]$ $D_2 = [-1,1]\times[-1,1]$



$$\oint_C f\,ds = \iint_{D_1} (Q'_x - P'_y)\,dy\,dx - \iint_{D_2} (Q'_x - P'_y)\,dy\,dx$$

$$Q'_x - P'_y = -x^2$$

$$\iint_{D_1} -x^2\,dy\,dx = \int_{-2}^{2}\int_{-2}^{2} -x^2\,dy\,dx$$

$$-\int_{-2}^{2} x^2 y\Big|_{-2}^{2} dx = -\int_{-2}^{2} 4x^2\,dx = -\frac{4}{3}x^3\Big|_{-2}^{2} = \frac{-64}{3}$$

$$\int_{-1}^{1}\int_{-1}^{1} -x^2\,dy\,dx = -\int_{-1}^{1} 2x^2\,dx = -\frac{4}{3}$$ $$\left[\oint_C f\,ds = \frac{-64}{3} + \frac{4}{3} = \frac{-60}{3} = -20\right]$$

⑤ Para empezar por la matriz jacobiana obtenemos

$Q'_x = 3x - 1$ $P'_y = 3x + 2$ $\longrightarrow$

$$\oint_{ACB} f\,ds = \oint_C f\,ds + \underbrace{\int_{AB} f\,ds}_{17}$$

$$\Downarrow$$

$$\iint_D (Q'_x - P'_y)\,dx\,dy$$

NOTA

HOJA

FECHA

$y = x^2 - x^4 \Rightarrow x^2 - x^4 = 0 \quad x^2(1-x^2) = 0 < \begin{matrix} x_1 = 1 \\ x_2 = -1 \end{matrix}$

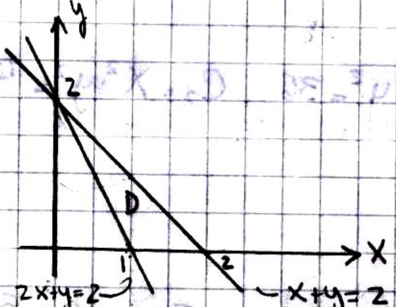
$0 \le y \le x^2 - x^4$

$$\iint_D 3x - 1 - (3x+2)\,dy\,dx = \int_{-1}^{1}\int_{0}^{x^2-x^4} -3\,dy\,dx$$

$$\int_{-1}^{1} -3(x^2-x^4)\,dx = -x^3 + \frac{3x^5}{5}\Big|_{-1}^{1} = -\frac{4}{5}$$

$$\int_{ACB} f\,d\bar{s} = -\frac{4}{5} + 17 = \boxed{\frac{81}{5}}$$

6 a) $f(x,y) = (2y - g(x), 5x - h(y)) \quad x + y \le 2 \quad 2x + y \ge 2$



$$\oint_C f\,d\bar{s} = \iint_D (Q'_x - P'_y)\,dx\,dy$$

$$\begin{cases} 0 \le x \le 2 \\ 2-2x \le y \le 2-x \end{cases}$$

$y = 2 - x$

$y = 2 - 2x$

$Q'_x - P'_y = 5 - 2 = 3$

$$\int_0^2\int_{2-2x}^{2-x} 3\,dy\,dx = \int_0^2 3y\Big|_{2-2x}^{2-x} dx = \int_0^2 3(2-x) - 3(2-2x)\,dx$$

$$\int_0^2 \cancel{6} - 3x - \cancel{6} + 6x\,dx = \int_0^2 3x\,dx = \frac{3}{2}x^2\Big|_0^2 = 6$$

ME DA 6
NO ENCUENTRO
EL ERROR

b) $f(x,y) = (2y + g(x-y), 2x - g(x-y))$

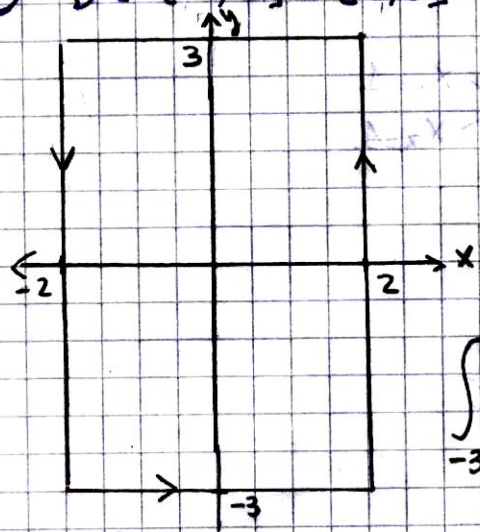
$Q'_x = 2 - g'(x-y) \qquad P'_y = (2 + g'(x-y)(-1))$

$Q'_x - P'_y = 2 - g'(x-y) - 2 + g'(x-y) = 0$

$$\left[\oint_C f\,d\bar{s} = \boxed{0}\right]$$

NOTA

⑦ $D = [-2;2] \times [-3,3]$ $\quad \bar{f}(x,y) = (1-y\,;\, h(x))$



$-2 \leq x \leq 2$

$-3 \leq x \leq 3$

$Q'_x = h'(x) \quad P'_y = -1$

$Q'_x - P'_y = h'(x) + 1$

$$\int_{-3}^{3}\int_{-2}^{2} h'(x)+1 \, dx\, dy = \int_{-3}^{3} h(x)+x \Big|_{-2}^{2} dy$$

$$\int_{-3}^{3} \cancel{h(2)}+2-\cancel{h(-2)}+2 \, dy$$

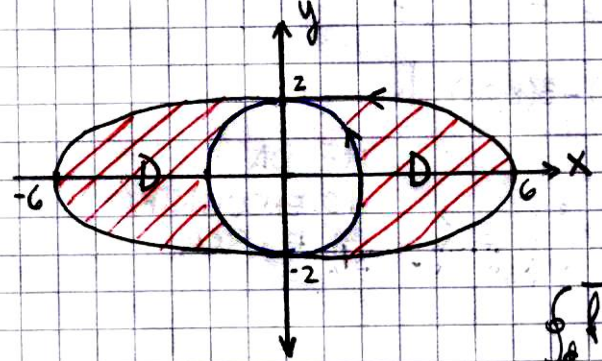
~~h(x) par ⇒~~

$h(x) = h(-x)$

$$\int_{-3}^{3} 4 \, dy = 4y \Big|_{-3}^{3} = \boxed{24}$$

⑧ $\bar{f} = (P,Q) \in C^1 \quad Q'_x - P'_y \equiv 5 \quad C_1: x^2+9y^2 = 36 \quad C_2 = x^2+y^2 = 4$

Hallar $\oint_{C_2^+} f\, d\bar{s}$ sabiendo $\oint_{C_1^+} f\, d\bar{s} = 7\pi$



$$\oint_{C^+} \bar{f}\, d\bar{s} = \iint_D 5\, dA = 5\iint_D dA = 5A(D)$$

$A(D) = ab\pi - r^2\pi = 12\pi - 4\pi = 8\pi$

$5A(D) = 40\pi$

son ambas positivas

$$\oint_{C} \bar{f}\, d\bar{s} = \int_{C_1^+} f\, ds - \int_{C_2^+} f\, d\bar{s}$$

$$\int_{C_2^+} f\, d\bar{s} = \int_{C_1^+} f\, d\bar{s} - \oint_{C^+} f\, d\bar{s} = 7\pi - 40\pi = -33\pi$$

$$\left[\oint_{C_2^+} f\, d\bar{s} = -33\pi\right]$$

NOTA

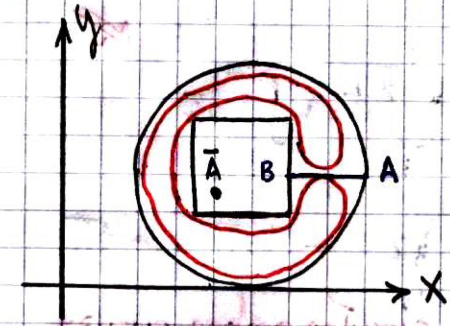
HOJA

FECHA

(9) $\bar{f} = (P, Q) \quad Q'_x - P'_y = 6$

Hallar $\oint_{C_1^+} \bar{f}\, d\bar{s}$ sabiendo que $\oint_{C_2^+} \bar{f}\, d\bar{s} = 12$ $\quad C_1$ circunf $R=8$

C_2 cuadrado lado $= 5$



Como $\bar{A}$ no pertenece al dominio tengo que cerrar el camino para poder aplicar Green

$$\oint_{C^+} \bar{f}\, d\bar{s} = \oint_{C_1^+} \bar{f}\, d\bar{s} + \cancel{\int_{C_{AB}} \bar{f}\, d\bar{s}}_{\boxed{A\to B}} - \oint_{C_2^+} \bar{f}\, d\bar{s} + \cancel{\int_{C_{AB}} \bar{f}\, d\bar{s}}_{\boxed{B\to A}}$$

$$\oint_{C^+} \bar{f}\, ds = \oint_{C_1^+} \bar{f}\, d\bar{s} - \oint_{C_2^+} \bar{f}\, d\bar{s} = \iint_D Q'_x - P'_y\, dA$$

$$\oint_{C_1^+} \bar{f}\, d\bar{s} = \underbrace{6\iint_D dA}_{6\,A(D)} + 12$$

$6\,A(D) \Longrightarrow$ region $\Longrightarrow A(D) = 64\pi - 25$

$6\,A(D) =$

$$\left[\oint_{C_1^+} \bar{f}\, ds = 384\pi - 150 + 12 = 384\pi - 138\right]$$

(12) Por teorema de Stokes

$$\oint_{C^+} \bar{f}\, ds = \iint_\Sigma (\bar{\nabla} \times \bar{f})\, \check{n}\, d\sigma \Longrightarrow$$

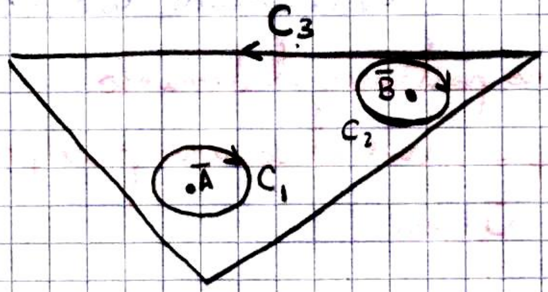
$$\int_{C_1^+} \bar{f}\, d\bar{s} + \int_{C_2^+} \bar{f}\, d\bar{s} + \int_{C_3^+} \bar{f}\, d\bar{s} = \iint_\Sigma (\bar{\nabla} \times \bar{f})\, \check{n}\, d\sigma$$

Como $D\bar{f}$ es simetrica $\Longrightarrow \bar{\nabla} \times \bar{f} = 0$

NOTA

Si $\bar{f}$ tiene matriz jacobiana continua y simétrica fuera de un pto P entonces la circulación $\oint_{C^+} \bar{f}\,d\bar{s}$ del campo alrededor de P tiene siempre el mismo valor independientemente del tamaño y la forma de la curva $\Longrightarrow$

Tener encuenta $\nabla \times \bar{f} = 0$



$$\oint_{C_3^+} \bar{f}\,d\bar{s} = \oint_{C_1^+} \bar{f}\,d\bar{s} + \int_{C_2^+} \bar{f}\,d\bar{s}$$

$$\oint_{C_3} \bar{f}\,d\bar{s} = 12\pi - 16\pi = -4\pi$$

circulan ambas curvas para el mismo lado $\Rightarrow$ se restan.

(14) Para que admita función potencial el campo debe ser conservativo $\Longrightarrow$ si es conservativo $\oint_{C^+} \bar{f}\,d\bar{s} = 0$

Muestro por contraejemplo que no admite función potencial

$C: 4x^2 + y^2 = 1$ (Curva sugerida en la guía)

(16) y (17) No me llevo bien con física.

(18) (a) $\bar{f}(x,y) = \left(\dfrac{y}{(x-1)^2+y^2} ; \dfrac{1-x}{(x-1)^2+y^2}\right)$ Compruebo si es conservativo para una región que contenga a los ptos fuera del dominio

$C: (x-1)^2 + y^2 = 1$

$\gamma(t) = (\cos t\ 1 ; \operatorname{sen} t) \quad 0 \le t \le 2\pi$

$\gamma'(t) = (-\operatorname{sen} t ; \cos t)$

$$\int_{C^+} F\,ds = \int_0^{2\pi} (\operatorname{sen} t ; \cos t)(-\operatorname{sen} t ; \cos t)\,dt = \int_0^{2\pi} -\operatorname{sen}^2 t - \cos^2 t\,dt = -2\pi \neq 0$$

F no es conservativa y la curva no encierra todos los puntos de D $\Rightarrow$ no admite función potencial.

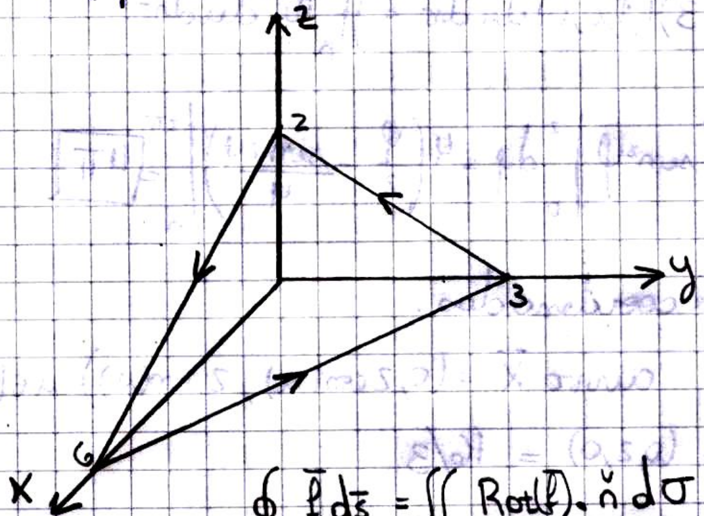
NOTA

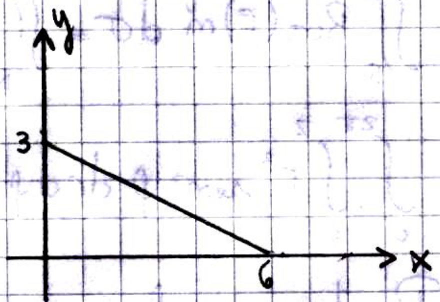
Así se resuelven los otros hay que utilizar una curva que se encuentre fuera del Dom si $\oint_{C^+} \bar{f}\, d\bar{s} = 0$ y $P'_y = Q'_x \Rightarrow$ el campo es conservativo y admite función potencial. (Son todos iguales tengan en claro el concepto y como emplearlo para dar contra ejemplo).

(19) Propiedades enumeradas en cualquier libro decente de análisis.

(20) $\bar{f}(x,y,z) = (x-y,\, x+y\,;\, z-x-y)$

$$\begin{cases} x + 2y + 3z = 6 \\ \text{planos coordenados} \end{cases}$$





$0 \leq x \leq 6$

$0 \leq y \leq -\frac{1}{2}x + 3$

$$\oint_{C^+} \bar{f}\, d\bar{s} = \iint_{\Sigma} \mathrm{Rot}(\bar{f}) \cdot \breve{n}\, d\sigma$$

$$\mathrm{Rot}(F) = \begin{vmatrix} \breve{i} & \breve{j} & \breve{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x-y & x+y & z-x-y \end{vmatrix} = (-1\,;\, 1,\, 2)$$

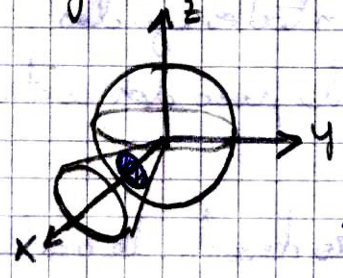
$$T(u,v) = \left(u,\, v,\, 2 - \frac{1}{3}u - \frac{2}{3}v\right)$$

$$\breve{n} = T'_u \times T'_v = \begin{vmatrix} \breve{i} & \breve{j} & \breve{k} \\ 1 & 0 & -1/3 \\ 0 & 1 & -2/3 \end{vmatrix} = (1/3\,;\, 2/3\,;\, 1)$$

$$\int_0^6 \int_0^{-1/2x+3} (-1,1,2)(1/3,2/3,1)\,dy\,dx = \int_0^6 \frac{7}{3}\left(-\frac{1}{2}x+3\right)dx = \left(7x - \frac{7x^2}{12}\right)\Big|_0^6 = \boxed{21}$$

(21) $\bar{F}(x,y,z) = (xy;\ y-x;\ yz^2)$ $\quad\begin{cases} x^2+y^2+z^2 = 8 \\ x = \sqrt{y^2+z^2} \end{cases} \Longrightarrow \begin{cases} y^2+z^2 = 4 \\ x = 2 \end{cases}$

Dibujo estimativo



$$\Downarrow$$

$$\gamma(t) = (2;\ 2\cos t;\ 2\,\text{sen}\,t) \quad 0 \le t \le 2\pi$$

$$\text{Rot}(\bar{F}) = \begin{vmatrix} \breve{i} & \breve{j} & \breve{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & y-x & yz^2 \end{vmatrix} = (z^2;\ 0;\ -1-x)$$

$T(u,v) = (2,u,v)$

$T'_u \times T'_v = (1,0,0) = \breve{n}$

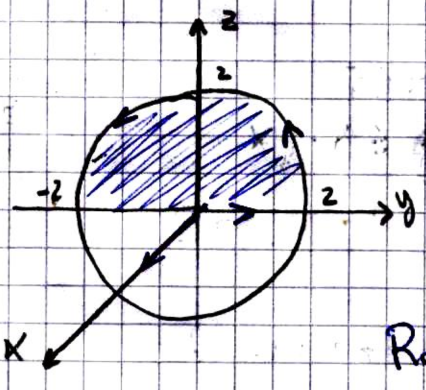
$$\iint_S \text{Rot}(\bar{F})\breve{n}\,d\sigma = \iint_D (v^2, 0, -3)(1,0,0)\,du\,dv = \iint_D v^2\,du\,dv$$

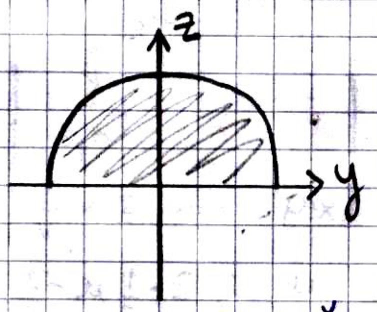
$$\int_0^{2\pi}\int_0^2 r^3\,\text{sen}^2\varphi\,dr\,d\varphi = \int_0^{2\pi} \frac{r^4}{4}\,\text{sen}^2\varphi\Big|_0^2 d\varphi = 4\left(\frac{\varphi}{2} - \frac{\text{sen}\,2\varphi}{4}\right)\Big|_0^{2\pi} = \boxed{4\pi}$$

Perdon cambie mucho de coordenadas.

(22) $\text{rot}(\bar{F}(x,y,z)) = (3,1,2y)$ $\quad$ curva $\bar{X} = (0, 2\cos(u);\ 2\,\text{sen}(u))$ $u \in [0,\pi]$

$\oint$ curv. $(0,-2,0)$ hasta $(0,2,0) = 16/3$





Rot dato $\Longrightarrow$ averiguo $\breve{n}$

$$T'_u \times T'_v = \begin{vmatrix} \breve{i} & \breve{j} & \breve{k} \\ 0 & -2\,\text{sen}\,u & 2\cos u \\ 0 & & \end{vmatrix}$$

no hace falta como proyecto sobre zy $\breve{n} = (1,0,0)$

regla de la mano izq.

HOJA

FECHA

$$\begin{cases} y = r\cos\theta \\ z = r\,\text{sen}\,\theta \\ x = 0 \end{cases} \qquad Df = r \qquad 0 \le r \le 2 \qquad 0 \le \theta \le \pi$$

$$\int_0^{\pi}\int_0^{2} (3, 1, 2r\cos\theta)(1,0,0)\, dr\, d\theta \cdot r = \int_0^{\pi}\int_0^{2} 3r\, dr\, d\theta$$

$$\int_0^{\pi} 6\, d\theta = 6\pi \qquad \oint f\, ds = \boxed{6\pi - \frac{16}{3}}$$ => porque pide circulación de $\underbrace{(0,2,0)\ a\ (0,-2,0)}_{0\ a\ \pi}$

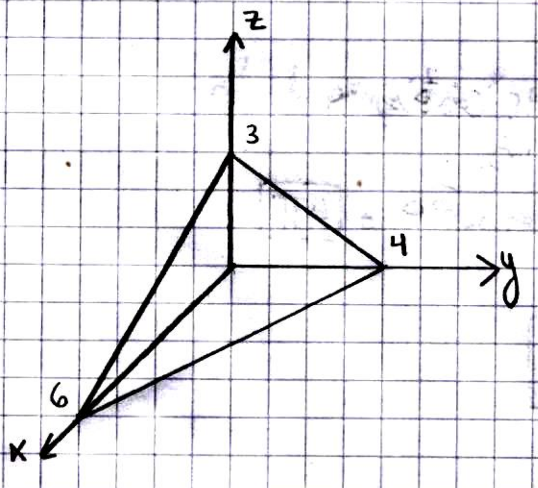
(23) $\bar{f}(x,y,z) = (xy, yz, xz) \quad [0,1]\times[0,2]\times[0,3]$

$$\iint_{\Sigma} \bar{f}\, \breve{n}\, d\sigma = \iiint_V \text{div}\,\bar{f}\, dV \qquad \text{div} f = P'_x + Q'_y + R'_z = y + z + x$$

$$\int_0^1\int_0^2\int_0^3 y+z+x\; dz\, dy\, dx = \int_0^1\int_0^2 yz + \frac{z^2}{2} + xz \Big|_0^3 dy\, dx = \int_0^1\int_0^2 3y + \frac{9}{2} + 3x\; dy\, dx$$

$$\int_0^1 \frac{3}{2}y^2 + \frac{9}{2}y + 3xy\Big|_0^2 dx = \int_0^1 6 + 9 + 6x\; dx = 15x + 3x^2\Big|_0^1 = \boxed{18}$$

(24) $\bar{f}(x,y,z) = (x-y-z;\ y-x-z,\ g(x,y)) \quad 2x+3y+4z \le 12 \quad 1^{\circ}$ octante



$12 - 2x - 3y$

$3 - \frac{1}{2}x - \frac{3}{4}y$

$0 \le x \le 6$

$0 \le y \le 4 - \frac{2}{3}x$

$0 \le z \le 3 - \frac{x}{2} - \frac{3}{4}y$

$$\text{div}\,\bar{f} = 1 + 1 + 0 = 2$$

$$\iiint_V 2\, dV = \iint_{\Sigma} \bar{f}\, \breve{n}\, d\sigma$$

NOTA

$$\int_0^6\int_0^{\frac{12-2x}{3}}\int_0^{\frac{12-2x-3y}{4}} 2\,dz\,dy\,dx = 2\int_0^6\int_0^{\frac{12-2x}{3}} \frac{12-2x-3y}{4}\,dy\,dx$$

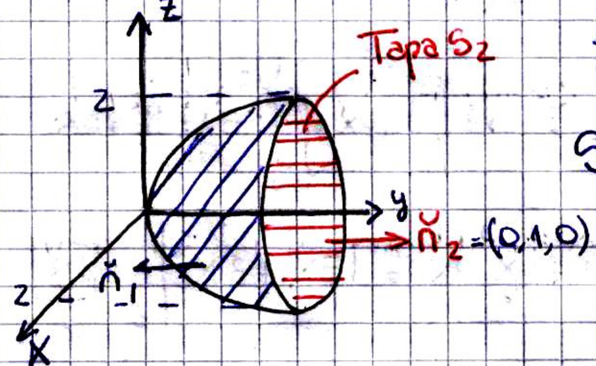
$$\frac{1}{2}\int_0^6\int_0^{\frac{12-2x}{3}} 12-2x-3y\,dy\,dx = \frac{1}{2}\int_0^6 12y-2xy-\frac{3}{2}y^2\Big|_0^{\frac{12-2x}{3}}dx$$

$$\frac{1}{2}\int_0^6 12\frac{(12-2x)}{3}-2x\frac{(12-2x)}{3}-\frac{3}{2}\left(\frac{12-2x}{3}\right)^2 dx = \boxed{24}$$ Hecha con calculadora!

25) $\bar{f}(x,y,z)=(x^2z^2,\ 1+xyz^2,\ 1-xz^3)$ $\quad y=x^2+z^2$ con $y<4$

$\operatorname{div} f = 2xz^2+xz^2-3xz^2=0$

Pero para aplicar div la superficie debe ser cerrada $\Rightarrow$ hay que agregar la tapa



flujo $S_1=0$ pq $\operatorname{div}=0$

$S_2:\begin{cases}\text{Proyecto sobre } xz\\ x=r\cos\theta\\ y=4\\ z=r\,\text{sen}\,\theta\end{cases}$ $\quad 0\le r\le 2\quad 0\le\theta\le 2\pi$

$$\int_0^{2\pi}\int_0^2 (r^4\cos^2\theta\,\text{sen}^2\theta,\ 1+4r^2\cos\theta\,\text{sen}^2\theta,\ 1-r^4\cos\theta\,\text{sen}^3\theta)(0,1,0)\ r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^2 r+4r^3\underbrace{\cos\theta\,\text{sen}^2\theta}_{0}\,dr\,d\theta$$

$$\int_0^{2\pi}\frac{r^2}{2}+r^4\cos\theta\,\text{sen}^2\theta\Big|_0^2=\int_0^{2\pi}2+16\cos\theta\,\text{sen}^2\theta=\boxed{4\pi}$$

$$\left[\text{flujo}_{S_1} f = 4\pi\right]$$

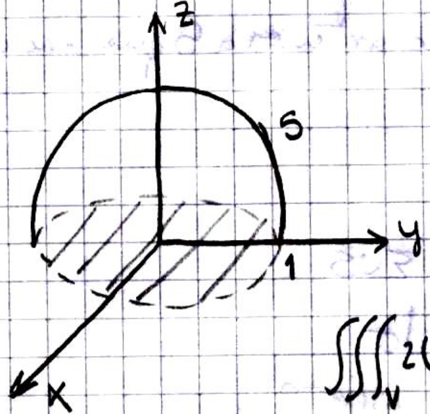
NOTA

HOJA

FECHA

(26) El flujo a traves de una superficie cerrada que contenga al origen es igual a 0.

(27) $\overline{f}(x,y,0) = (x,y,x^2)$ $\quad div(\overline{f}(x,y,z)) = z(1+z)$ $\quad S: z = \sqrt{1-x^2-y^2}$



$$\iint_{\Sigma=\partial V} \overline{f}\, \check{m}^{\text{saliente}}\, d\sigma = \iiint_V div(f)\, dV$$

$$\iint_S f\check{m}\, d\sigma + \iint_{S_{z=0}} f\check{m}\, d\sigma = \iiint_V div(f)\, dV$$

$$\iiint_V z(1+z)\, dV = z\Big[\underbrace{\iiint_V 1\, dV}_{Vol(V)=\frac{2}{3}\pi} + \iiint_V z\, dV\Big] \quad V: \begin{cases} 0 \le \varphi \le 2\pi \\ 0 \le \theta \le \pi/2 \\ 0 \le \rho \le 1 \end{cases}$$

$$\iiint_V z\, dV = \int_0^{2\pi}\int_0^1\int_0^{\pi/2} \rho\cos\theta\, \rho^2 sen\theta\, d\theta\, d\rho\, d\varphi = \int_0^{2\pi}\int_0^1 \rho^3 \frac{sen^2\theta}{2}\Big|_0^{\frac{\pi}{2}} d\rho\, d\varphi$$

$$\int_0^{2\pi} \frac{\rho^4}{8}\Big|_0^1 d\varphi = \frac{1}{8}2\pi = \frac{\pi}{4} \Rightarrow \iiint_V div f\, dV = \frac{\pi}{4} + \frac{2}{3}\pi = \frac{11}{6}\pi$$

Porcion de plano $z=0 \,/\, x^2+y^2 \le 1 = \begin{cases} x = u \\ y = v \\ z = 0 \end{cases}$

$T'_u \times T'_v = (0,0,1)$ entrante no sirve, cambio de signo al final !!!

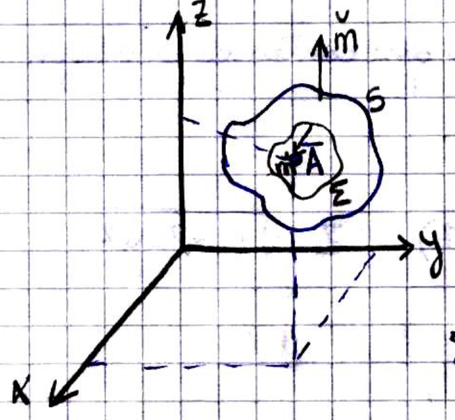
$$\iint_{S_{z=0}} f\check{m}\, d\sigma = -\iint_D (u,v,u^2)(0,0,1)\, du\, dv = -\iint_D u^2\, du\, dv$$

$$-\int_0^1\int_0^{2\pi} r^2\cos^2\varphi\, r\, d\varphi\, dr = -\int_0^{2\pi} r\left(\frac{\varphi}{2} + \frac{sen 2\varphi}{4}\right)\Big|_0^1 dr = -\pi\frac{r^4}{4}\Big|_0^1 = \boxed{-\frac{\pi}{4}}$$

$$\iint_S f\check{m}\, d\sigma - \frac{\pi}{4} = \pi\frac{11}{6} \quad \left[\iint_S f\check{m}\, d\sigma = \frac{25}{12}\pi \text{ con } \check{m} \text{ salientes}\right]$$

NOTA

(28) $\bar{f}: \mathbb{R}^3 - \{\bar{A}\} \to \mathbb{R}^3 / f \in C^1 \quad \operatorname{div} f = 0$



① Si no encierra el pto A

$$\iint_{S=\partial V} f \breve{m}\, d\sigma = \iiint_V \underbrace{\operatorname{div} f}_{0}\, dV = 0$$ el flujo a través de S es 0

② Si encierra el pto A

Σ superficie cerrada anterior a S que encierra al pto A

$$\iint_{\Sigma \cup S} \bar{f} \breve{m}\, d\sigma = \iiint_{V} \overset{0}{\operatorname{div} f}\, dV = 0$$

V → campo limitado por Σ∪S

$$0 = \iint_{\Sigma \cup S} f \underset{\text{saliente}}{\breve{m}}\, d\sigma = \iint_S f \underset{\text{saliente}}{\breve{m}}\, d\sigma + \iint_\Sigma f \underset{\text{entrante}}{\breve{m}}\, d\sigma$$

$$\Rightarrow \iint_S f \underset{\text{saliente}}{\breve{m}}\, d\sigma = -\iint_\Sigma f \underset{\text{entrante}}{\breve{m}}\, d\sigma \Rightarrow \iint_S f \breve{m}\, d\sigma = \underbrace{\iint_\Sigma f \breve{m}\, d\sigma}_{\text{saliente}}$$

Siempre que la superficie encierre al pto el flujo va a ser el mismo porque $\operatorname{div} f = 0$. En base a esto verificar pto 26 creo que no es correcto lo que puse. $\nabla\nabla\nabla$

(29) $E = Kq \dfrac{\bar{r}}{r^3} = \dfrac{Kq}{(\sqrt{x^2+y^2+z^2})^3}(x,y,z)$ $\quad |z| = \sqrt{R^2 - x^2 - y^2}$

$S: x^2+y^2+z^2 = R^2 \longrightarrow G(x,y,z) = R^2$

$$\iint_S E \breve{m}\, d\sigma = \iint_{D_{xy}} E \frac{\nabla G}{|G'_z|}\, dx\,dy$$

$$\iint_{D_{xy}} \frac{Kq}{(\sqrt{x^2+y^2+z^2})^3} \frac{\underset{2x^2+2y^2+2z^2}{(x,y,z)}\,(2x;2y;2z)}{|2z|}\, dx\,dy = \iint_{D_{xy}} \frac{Kq}{(\sqrt{x^2+y^2+z^2})^{1/2}} \frac{z}{z|z|}\, dx\,dy$$

NOTA

Si $z=\sqrt{R^2-x^2-y^2}$ paso a polares

$$\iint_{D_{xy}} \frac{Kq}{R}\,\frac{1}{\sqrt{R^2-x^2-y^2}}\,dx\,dy$$

$$\frac{Kq}{R}\int_0^{2\pi}\int_0^{R}\frac{1}{\sqrt{R^2-r^2}}\,r\,dr\,d\varphi=\frac{Kq}{R}\int_0^{2\pi}-\sqrt{R^2-r^2}\,\Big|_0^R\,d\varphi=\frac{Kq}{\cancel{R}}\cancel{R}\,2\pi=\boxed{2Kq\pi}$$

IMPORTANTE: flujo por $z:\sqrt{R^2-x^2-y^2}$ es la mitad de la esfera pq había quedado un módulo, en este caso se puede multiplicar por dos porque está todo al cuadrado y no cambia los signos

$$\Longrightarrow \iint_S E\,\check{m}\,d\sigma=\boxed{4\pi Kq}$$

(31) $\varphi\in C^2$ armonico $\Longrightarrow \nabla^2\varphi=\mathrm{div}(\nabla\varphi)=0$

$$=\mathrm{div}\left(\frac{\partial\varphi}{\partial x};\frac{\partial\varphi}{\partial y};\frac{\partial\varphi}{\partial z}\right)=\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0$$

$$\bar{f}=\varphi\,\bar{\nabla}\varphi \qquad \oiint_{\partial H} f\,\check{m}\,d\sigma\geq 0$$

$$\oiint_{\partial H} f\,\underset{\text{saliente}}{\check{m}}\,d\sigma=\iiint_H \mathrm{div}f\,dv \qquad \mathrm{div}f=\mathrm{div}\overbrace{\left(\varphi\frac{\partial\varphi}{\partial x},\varphi\frac{\partial\varphi}{\partial y},\varphi\frac{\partial\varphi}{\partial z}\right)}^{f}$$

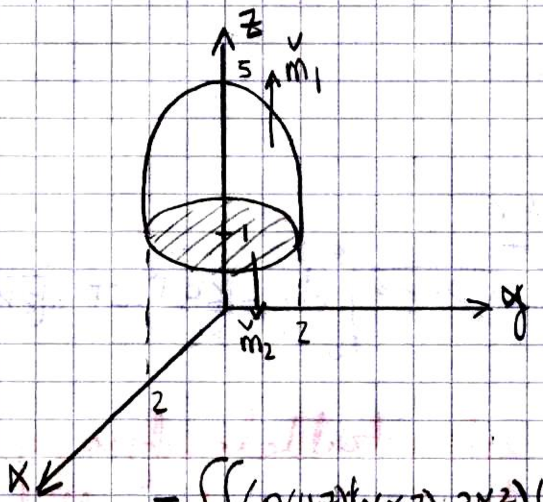
$$\mathrm{div}f=\left(\frac{\partial\varphi}{\partial x}\right)^2+\varphi\frac{\partial^2\varphi}{\partial x^2}+\left(\frac{\partial\varphi}{\partial y}\right)^2+\varphi\frac{\partial^2\varphi}{\partial y^2}+\left(\frac{\partial\varphi}{\partial z}\right)^2+\varphi\frac{\partial^2\varphi}{\partial z^2}$$

$$=\|\bar{\nabla}\varphi\|^2+\varphi\underbrace{\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}\right)}_{0\ (\varphi\ \text{armonico})}$$

$$\mathrm{div}\,f=\|\nabla\varphi\|^2\geq 0$$

Como el argumento de la integral ≥ 0 el resultado de la integral será ≥ 0. Verificado.

(32) $\bar{f}(x,y,z) = (g(y,z); h(x,z); 3x^2) \qquad \operatorname{div} f = 0$



$S_1 = 0$

$S_2 = \quad z = 5 - x^2 - y^2 \qquad z = 1$

$x^2 + y^2 = 4$

$\begin{cases} x = u \\ y = v \\ z = 1 \end{cases} \qquad T'_u \times T'_v = (0,0,1)$

Entrante no sirve cambio signo

$$-\iint (g(y,z), h(x,z), 3x^2)(0,0,1)\,dx\,dy = -\iint 3x^2\,dx\,dy$$

$$-3\int_0^2 \int_0^{2\pi} r^3 \cos^2\theta \, d\theta\, dr = \boxed{-12\pi}$$ con normal saliente.

(34) $f \in C^1 \quad \bar{f}(x,y,z) = (z + x\,g(2xy); y\,g(2xy); zxy - 2z\,g(2xy))$

$\bar{f}(1,1,1) = (3;2;-3) \quad f$ solenoidal $\to \operatorname{div} f = 0$

$$\bar{f}(1,1,1) = (1 + g(2); g(2); 1 - g(2)) = (3;2;3) \iff g(2) = 2$$

$$\operatorname{div} f = \cancel{g(2xy)} + x\,g'(2xy)\,2y + \cancel{g(2xy)} + y\,g'(2xy)\,2x + xy - \cancel{2g(2xy)}$$

$$4xy\,g'(2xy) + xy = 0 \qquad 2xy = t$$

$$2t\,g'(t) + \frac{t}{2} = 0$$

$$g'(t) = -\frac{t}{2}\,\frac{1}{2t} \Rightarrow g(t) = -\frac{1}{4}t + C$$

$$g(2) = -\frac{1}{2} + C = 2 \to C = \frac{5}{2}$$

$$g(t) = -\frac{1}{4}t + \frac{5}{2} \qquad g(2xy) = \frac{-xy}{2} + \frac{5}{2}$$

$$f(x,y,z) = \left(\cancel{z} - \frac{x^2y}{2} + \frac{5}{2}; y\left(\frac{-xy}{2} + \frac{5}{2}\right); \left(\frac{-xy}{2} + \frac{5}{2}\right)(-2z) + zxy\right)$$

$$\left[\bar{f}(x,y,z) = \left(z + \frac{(5x - yx^2)}{2}; \frac{(5y - xy^2)}{2}; 2xyz - 5z\right)\right]$$