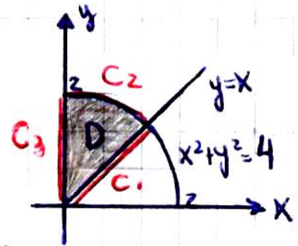
8 - Curvas - Integrales de Linea

② $C = \partial D = C_1 \cup C_2 \cup C_3 \Rightarrow x^2+y^2 \leq 4\ ;\ y \geq x\ ;\ x \geq 0$



$$S = \int_C ds = \int_{C_1} ds + \int_{C_2} ds + \int_{C_3} ds$$

C_1: porcion de recta $y=x$ que une $(0,0)$ con $(\sqrt{2};\sqrt{2})$

$$\begin{cases} x = t \\ y = t \end{cases} \quad \gamma(t) = (t;t) \text{ con } 0 \leq t \leq \sqrt{2} \quad \gamma'(t) = (1;1) \quad \|\gamma'(t)\| = \sqrt{2}$$

$$\int_{C_1} ds = \int_0^{\sqrt{2}} \sqrt{2}\, dt = \sqrt{2}\, t \Big|_0^{\sqrt{2}} = \boxed{2}$$

C_2: arco de circunf $x^2+y^2=4$ que une $(0;2)$ con $(\sqrt{2};\sqrt{2})$

$$\begin{cases} x = 2\cos t \\ y = 2\operatorname{sen} t \end{cases} \quad \gamma(t) = (2\cos t; 2\operatorname{sen} t) \Rightarrow \gamma'(t) = (-2\operatorname{sen} t; 2\cos t) \quad \|\gamma'(t)\| = 2$$

con

$$\int_{C_2} ds = \int_{\pi/4}^{\pi/2} 2\, dt = 2t \Big|_{\pi/4}^{\pi/2} = \pi - \frac{\pi}{2} = \boxed{\frac{\pi}{2}}$$

C_3: porcion de eje y con $0 \leq y \leq 2$

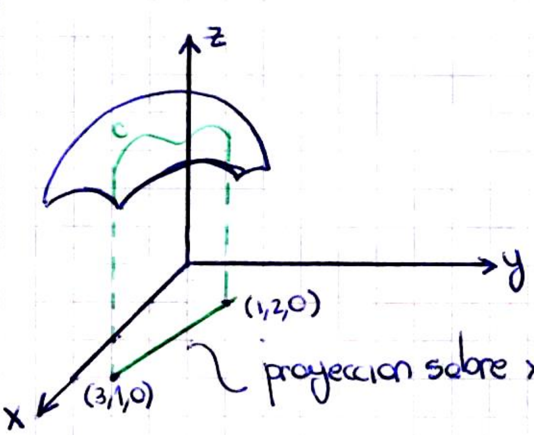
$\gamma(t) = (0;t)$ con $0 \leq t \leq 2$

$$\int_{C_3} ds = \int_0^2 1\, dt = \boxed{2}$$

$$\boxed{\int_C ds = 2 + \frac{\pi}{2} + 2 = 4 + \frac{\pi}{2}}$$

③ $z = x^2 - 4y^2$ desde $(1;2;-15)$ hasta $(3;1;5)$

proyeccion del recorrido sobre plano xy es el segmento de ptos extremos $A=(1,2,0)$ y $B(3;1;0)$



proyeccion sobre xy de C

$$C = \begin{cases} z = x^2 - 4y^2 \\ y = -\frac{1}{2}(x-1) + 2 \end{cases}$$

$dr = (3;1) - (1;2) = (2;-1)$

$r(x,y) = (2;-1)\lambda + (1;2)$

$$\begin{cases} x = 2\lambda + 1 \\ y = -\lambda + 2 \end{cases} \quad \begin{matrix} 3 = 2\lambda + 1 \\ 1 = -\lambda + 2 \quad \lambda = 1 \end{matrix}$$

$$y = -\frac{1}{2}(x-1) + 2$$

$$\begin{cases} x = t \\ y = -\frac{1}{2}t + \frac{5}{2} \\ z = t^2 \end{cases} \quad \text{con } 1 \leq t \leq 3$$

$$-4\left(-\frac{1}{2}t+\frac{5}{2}\right)^2 = t^2 - 4\left(\frac{1}{4}t^2 - \frac{5}{2}t + \frac{25}{4}\right) = \cancel{t^2} - \cancel{t^2} + 10t - 25 = 10t - 25$$

$$\gamma(t) = \left(t; -\frac{t}{2} + \frac{5}{2}; 10t - 25\right) \Rightarrow \gamma'(t) = \left(1; -\frac{1}{2}; 10\right) \Rightarrow \|\gamma'(t)\| = \sqrt{1^2 + \left(-\frac{1}{2}\right)^2 + 10^2} = \frac{9\sqrt{5}}{2}$$

$$\int_C ds = \int_1^3 \frac{9\sqrt{5}}{2}\,dt = \frac{9\sqrt{5}}{2}t\Big|_1^3 = \frac{3}{2}9\sqrt{5} - \frac{1}{2}9\sqrt{5} = \boxed{9\sqrt{5}}$$

5) $\begin{cases} x + y + z = 4 \\ y = 2x \end{cases}$ 1° octante $\quad \delta(x,y,z) = K\sqrt{x^2+y^2}$

$\overline{G} = (\overline{x}, \overline{y}, \overline{z})$

$$\overline{x} = \frac{\int_C x\,\delta(x,y,z)\,ds}{\underbrace{\int_C \delta(x,y,z)\,ds}_{m(c)\ [\text{masa de la curva}]}}$$

Parametrizo C: $\begin{cases} x = t \\ y = 2t \\ z = 4 - 3t \end{cases} \quad 0 \leq t \leq \frac{4}{3}$

$\gamma(t) = (t; 2t; 4 - 3t)$

Integral de la forma $\int_a^b F(\gamma(t))\,\|\gamma'(t)\|\,dt$

Varia en area positiv se saca | |

$$\int_C x\sqrt{x^2+y^2}\,ds = \int_0^{4/3} t\sqrt{t^2+4t^2}\,\|(1;2;-3)\|\,dt = \int_0^{4/3} t\sqrt{5}\,|t|\sqrt{14}$$

$$\sqrt{70}\int_0^{4/3} t^2\,dt = \sqrt{70}\,\frac{t^3}{3}\Big|_0^{4/3} = \boxed{\sqrt{70}\,\frac{64}{81}}$$

m(c) = se calcula una vez para los tres ejes sera la misma masa

$$\int \sqrt{x^2+y^2} = \int_0^{4/3} \sqrt{t^2+4t^2}\,\underbrace{\sqrt{14}}_{\|\gamma'(t)\|} = \int_0^{4/3} \sqrt{5}\,t\sqrt{14} = \sqrt{70}\,\frac{t^2}{2}\Big|_0^{4/3} = \boxed{\sqrt{70}\,\frac{8}{9}}$$

$$\overline{x} = \frac{\cancel{\sqrt{70}}\,\frac{64}{81}}{\cancel{\sqrt{70}}\,\frac{8}{9}} = \boxed{\frac{8}{9}}$$ ⟷ por la parametrizacion $y = 2x$

$$\int_C y\sqrt{x^2+y^2}\,ds = \int_0^{4/3} 2t\sqrt{t^2+4t^2}\,dt\,\sqrt{14} = \sqrt{70}\,\frac{128}{81}$$

$$\overline{y} = \frac{\sqrt{70}\,\frac{128}{81}}{\sqrt{70}\,\frac{8}{9}} = \boxed{\frac{16}{9}}$$

$$\int_C z\sqrt{x^2+y^2}\,ds = \int_0^{4/3} (4-3t)\sqrt{5t^2}\sqrt{14}\,dt = 4\underbrace{\int_0^{4/3} \sqrt{70}\,t\,dt}_{\sqrt{70}\,\frac{8}{9}} - 3\underbrace{\int_0 t\sqrt{70}\,t\,dt}_{\sqrt{70}\,\frac{64}{81}}$$

$$= \sqrt{70}\,\frac{32}{27} \qquad \overline{z} = \frac{\sqrt{70}\,\frac{32}{27}}{\sqrt{70}\,\frac{8}{9}} = \frac{4}{3}$$

$$\boxed{G = \left(\frac{8}{9}; \frac{16}{9}; \frac{4}{3}\right)}$$

⑧ La integral $\int_C \bar{F}\, ds$ representa el trabajo que realiza el campo $\bar{F}$ para mover un objeto a lo largo de la curva C. En el caso particular que $n = 2$

$\bar{F}(x,y) = (P(x,y); Q(x,y))$ $\quad \bar{\gamma}(t) = (x(t); y(t))$ $\quad \wedge \quad x = x(t)$, $\; dx = x'(t)\,dt$

$$\int_C \bar{F}\, ds = \int_a^b \bar{F}(\bar{\gamma}(t))\, \bar{\gamma}'(t)\, dt = \int_a^b (P(x(t); y(t)); Q(x(t); y(t)))\, (x'(t); y'(t))\, dt =$$

$$(\underbrace{x'(t)\,dt}_{dx}; \underbrace{y'(t)\,dt}_{dy})$$

$$\int_C P\,dx + Q\,dy$$

$P \to$ Primer componente del campo $\bar{F}$

$Q \to$ Segunda componente del campo $\bar{F}$

⑨ Sea $\bar{F}$ un campo escalar de clase C^1 en D

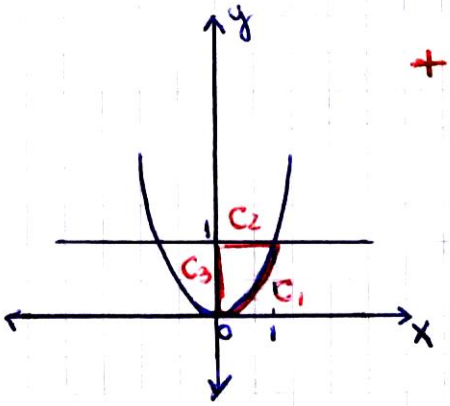
Sea C un arco de curva suave que une el punto A con el punto B parametrizado por $\gamma(t)$ con $a \leq t \leq b$ y tal que $C \subset D \implies \bar{F} = \nabla U$

$$\int_{AB} \bar{F}\, ds = \int_a^b \left[\nabla U(\bar{\gamma}(t))\, \bar{\gamma}'(t)\right] dt = \int_a^b \left[U(\bar{\gamma}(t))\right]' dt = U(\bar{\gamma}(t))\Big|_a^b$$

$$= U(\gamma(b)) - U(\gamma(a)) = U(B) - U(A)$$

Como el valor de la integral es el mismo para cualquier curva C que une A con B diremos que la integral es independiente de la trayectoria utilizada para unir dichos puntos y que $\bar{F} = \nabla U$ es conservativo

⑪ $\bar{f}(x,y) = (y; -x)$ $\quad \partial D : \{ x^2 \leq y \leq 1 \wedge 0 \leq x \leq 1 :$ sentido positivo $\}$



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C_1: $x^2 = y$ que une $(0;0)$ con $(1,1)$

$\begin{cases} x = t \\ y = t^2 \end{cases}$ $\quad \gamma'(t) = (1; 2t)$ con $0 \leq t \leq 1$

$$\int_0^1 (t^2; -t)\,(1; 2t)\, dt$$

reemplazo curva en campo

multiplico derivada curva

$$\int_0^1 (t^2 - 2t^2)\, d = -\frac{t^3}{3}\Big|_0^1 = \boxed{-\frac{1}{3}}$$

C_2: $y=1$ que une $(1,1)$ con $(0;1)$

$$\begin{cases} x=0 \\ y=t \end{cases} \text{ con } 0\leq t\leq 1 \text{ (invierte sentido)}$$

$$-\int_0^1 (t;0)(0;1)\,dt = \boxed{0}$$

C_3 = porcion de eje y / $0\leq y\leq 1$ (invierte sentido)

$\gamma(t)=(0;t)$ $0\leq t\leq 1$ $\qquad = -1$ pq invierte sentido

Circulacion: $-\frac{1}{3}+0-1=\boxed{-\frac{4}{3}}$

⑫ $\bar{f}_{(x,y,z)} = (\overset{P}{x-y};\overset{Q}{x};\overset{R}{yz})$ $\quad C: \begin{cases} z=x-y^2 \\ y=x^2 \end{cases}$ desde $(1,1,0)$ hasta $(-1;1;-2)$

$f\in C'(\mathbb{R}^3) \Rightarrow P'_y=-1 \neq Q'_x=1$ No es conservativo

$$C: \begin{cases} x=t \\ y=t^2 \\ z=t-t^4 \end{cases} \quad \gamma(t)=(t;t^2;t-t^4) \Rightarrow \gamma'(t)=(1;2t;1-4t^3)$$

con $-1\leq t\leq 1$ (Invierte sentido)

$$\int_{-1}^1 (t-t^2;t;t^3-t^6)(1;2t;1-4t^3)\,dt = \int_{-1}^1 t-t^2+2t^2+t^3-4t^6-t^6+4t^9$$

$$\int_{-1}^1 4t^9-5t^6+t^2+t = \frac{4}{10}t^{10}-\frac{5}{7}t^7+\frac{1}{3}t^3+\frac{t^2}{2}\Big|_{-1}^{1} =$$

$$\frac{4}{10}-\frac{5}{7}+\frac{1}{3}+\frac{1}{2}-\left(+\frac{4}{10}+\frac{5}{7}-\frac{1}{3}+\frac{1}{2}\right) = -\frac{16}{21} \text{ como invierte sentido}$$

$$\boxed{=\frac{16}{21}}$$

⑬ $\bar{f}(x,y;z)=(3x;-xz;yz)$

$\gamma(t)=(t-1;t^2;2t)$ con $t\in[1,3]$ Pinicial: $(0;1,2)$ Pfinal: $(2;9;6)$

$$\int_C \bar{f}\,ds = \int_1^3 (3t-3;-(2t^2-2t);2t^3)(1;2t;2)\,dt$$

$$\int_1^3 (3t-3-\cancel{4t^3}+4t^2+\cancel{4t^3})\,dt = \frac{4}{3}t^3+\frac{3}{2}t^2-3t\Big|_1^3 = \boxed{\frac{122}{3}}$$

No se puede asegurar el mismo resultado ya que no es conservativo por ende depende del camino usado.

(14) (a) $\overline{f}(x,y) = (\overset{P}{y - 2xy + 1}\ ;\ \overset{Q}{x + 1 - x^2})$

$\overline{f} \in C^1(\mathbb{R}^2)$ (funciones polinomicas)

$P'_y = (1 - 2x) = Q'_x = 1 - 2x$ $\overline{f}$ es conservativo $\Rightarrow \exists\, U(x,y)$

Busco $U(x,y) \,/\, \overline{\nabla} U = F$

$$\begin{cases} U'_x = y - 2xy + 1 \Rightarrow U(x,y) = \int y - 2xy + 1 \, dx \Rightarrow U(x,y) = yx - yx^2 + x + \alpha(y) \\ U'_y = x + 1 - x^2 = U'_y = x - x^2 + \alpha'(y) \end{cases}$$

$$\alpha'(y) = 1 \Rightarrow \alpha(y) = y + K$$

$U(x,y) = yx - yx^2 + x + y + K$

(b) $\overline{f}(x,y) = (x - y^2 ; y - x^2)$

$\overline{f} \in C^1(\mathbb{R}^2)$ $P'_y = -2y \neq Q'_x = -2x$ No admite funcion potencial

(c) $\overline{f}(x,y,z) = (\overset{P}{z\cos(xz)}\ ;\ \overset{Q}{z}\ ;\ \overset{R}{y + x\cos(xz)})$

$$\left.\begin{array}{l} P'_y = 0 = Q'_x = 0 \\ P'_z = \cos(xz) + z(-\operatorname{sen}(xz) \cdot x) = R'_x = \cos(xz) + x(-\operatorname{sen}(xz) \cdot z) \\ Q'_z = 1 = R'_y = 1 \end{array}\right\} f \text{ es conservativo}$$

Busco $U(x,y,z)$

$$\begin{cases} U'_x = z\cos(xz) \\ U'_y = z \longrightarrow U(\overline{x}) = \int z \, dy = zy + \alpha(x;z) \\ U'_z = y + x\cos(xz) \qquad U'_z = y + \alpha'_z = y + x\cos(xz) \end{cases}$$

$$\alpha'_z = x\cos(xz)$$

$\alpha(x;z) = \int x\cos(xz)\, dz$

$= \int \cos(u)\, du$

$xz = u$
$du = x\, dz$

$\alpha(x;z) = \operatorname{sen}(xz) + B(x)$

$U(x,y,z) = zy + \operatorname{sen}(xz) + B(x)$

$U'_x = \cos(xz) \cdot z + B'(x) \equiv z\cos(xz) \Rightarrow B(x) = 0$

$[U(x,y,z) = yz + \operatorname{sen}(xz) + K]$

(d) $f(x,y,z) = (\overset{P}{2x+y+1}, \overset{Q}{x+z}, \overset{R}{y+2z})$

$$f \in C^1(\mathbb{R}^3) \Rightarrow \left.\begin{array}{l} P'_y = 1 = Q'_x = 1 \\ P'_z = 0 = R'_x = 0 \\ Q'_z = 1 = R'_y = 1 \end{array}\right\} f \text{ es conservativo}$$

Busco $U(x,y,z)$ / $\nabla U = F$

$$\begin{cases} U'_x = 2x+y+1 \\ U'_y = x+z \\ U'_z = y+2z \end{cases}$$

$\rightarrow U(x,y,z) = \int(2x+y+1)dx = x^2+yx+x+\alpha(y;z)$

$U'_y = x + \alpha'_y = x+z$

$\alpha'_y = z \quad \alpha(y;z) = \int z\,dy = yz + B(x)$

$U(x,y,z) = x^2+yx+x+yz+B(x)$

$U'_z = y + B'(x) \qquad B'(x) = 2z \quad B(x) = \int 2z\,dz = z^2+K$

$[U(x,y,z) = x^2+x+yx+yz+z^2+K]$

(16) $f(x,y) = \left(\frac{y}{x^2+y^2}; \frac{-x}{x^2+y^2}\right)$ $\quad$ H f: la matriz jacobiano es continua

$$C: \begin{cases} x = \cos t \\ y = \operatorname{sen} t \end{cases} \quad 0 \le t \le 2\pi$$

$$\int_0^{2\pi} \left(\frac{\operatorname{sen} t}{1}; -\frac{\cos t}{1}\right)(-\operatorname{sen} t; \cos t)\,dt$$

$$\int_0^{2\pi} -\operatorname{sen}^2 t - \cos^2 t = -\int_0^{2\pi} 1 \Rightarrow -t\Big|_0^{2\pi} = -2\pi \neq 0 \text{ Verdadero}$$

No admite funcion potencial en su dominio.

El campo $\bar{F}$ no es conservativo en cualquier region que contenga al $(0;0)$ aun cuando $P'_y = Q'_x$

(17) $Df(x,y) = \begin{array}{cc} P'_x & P'_y \\ \left(\begin{array}{cc} 2y-6x & 2x \\ 2x & 0 \end{array}\right) & \\ Q'_x & Q'_y \end{array}$

B pertenece a $y = x - x^{-1}$ $B = (a; a - a^{-1})$

A pertenece al eje y $A: (0, b)$ con $b \in \mathbb{R}$

La grafica de $U(\overline{x})$ pasa por $(1,1,3)$ con plano Tg de ecuación $\underbrace{z = y + 2}$

$\nabla U(x,y) = f(x,y) = (P;Q)$ $\quad \hookrightarrow U(1,1) = 3$

$U'_x(1,1) = 0 = P(1,1)$

$U'_y(1,1) = 1 = Q(1,1)$

Busco $P(x,y)$

$$\begin{cases} P'_x = 2y - 6x \\ P'_y = 2x \\ P(1,1) = 0 \end{cases} \rightarrow P(x,y) = \int 2x\,dy = 2xy + \alpha(x)$$

$$P'_x = 2y + \alpha'(x) = 2y - 6x$$

$$\alpha'(x) = -6x \Rightarrow \alpha(x) = -\int 6x\,dx = -3x^2 + K_1$$

$$P(x,y) = 2xy - 3x^2 + K_1 \Rightarrow P(1,1) = 0 = 2 - 3 + K_1 \Rightarrow K_1 = 1$$

$$[P(x,y) = 2xy - 3x^2 + 1]$$

Busco $Q(x,y)$

$$\begin{cases} Q'_x = 2x \\ Q'_y = 0 \\ Q(1,1) = 1 \end{cases} \rightarrow Q(x,y) = \alpha(x) \Rightarrow Q'_x = \alpha'(x) = 2x$$

$$\alpha(x) = \int 2x\,dx = x^2 + K_2$$

$$Q(x,y) = x^2 + K_2 = 1 = 1 + K_2 \Rightarrow K_2 = 0$$

$$[Q(x,y) = x^2]$$

$$f(x,y) = (P; Q) = (2xy - 3x^2 + 1; x^2)$$

$$\begin{cases} U'_x = 2xy - 3x^2 + 1 \\ U'_y = x^2 \\ U(1,1) = 3 \end{cases} \Rightarrow U(x,y) = \int x^2\,dy = yx^2 + \alpha(x)$$

$$U'_x = 2xy + \alpha'(x) = 2xy - 3x^2 + 1$$

$$\alpha'(x) = -3x^2 + 1 \Rightarrow \alpha(x) = \int -3x^2 + 1\,dx = -x^3 + x + K_3$$

$$U(x,y) = x^2y - x^3 + x + K_3 \Rightarrow U(1,1) = 3$$

$$[U(x,y) = x^2y - x^3 + x + 2]$$

⑱ $\bar f \in C^1 / \bar f(x,y) = (\overset{P}{xy^2}, \overset{Q}{y\,g(x)})$

$f(2;1) = (2;6)$

$$P'_y = Q'_x \Rightarrow 2xy = y\,g'(x)$$

$$\int g'(x) = \int 2x \Rightarrow g(x) = x^2 + C$$

$$6 = 4 + C \Rightarrow C = 2 \qquad \boxed{g(x) = x^2 + 2}$$

⑲ $\bar f(x,y,z) = (\overset{P}{2xy+z^2}, \overset{Q}{x^2}, \overset{R}{2xz})$ $\quad C: \bar X = (2t + e^{t^3-t};\ t^2-t;\ 3t)$ $\quad 0 \le t \le 1$

$$\bar f \in C^1(\mathbb{R}^3) \Rightarrow \left.\begin{array}{l} P'_y = 2x = Q'_x = 2x \\ P'_z = 2z = R'_x = 2z \\ Q'_z = 0 = R'_y = 0 \end{array}\right\} \bar f \text{ es conservativo} \Rightarrow \text{admite } U$$

$$\begin{cases} U'_x = 2xy + z^2 \\ U'_y = x^2 \\ U'_z = 2xz \end{cases} \Rightarrow U(\bar x) = \int x^2\,dy = x^2 y + \alpha(x;z)$$

$$U'_z = \alpha'_z(x;z) = 2xz$$

$$\alpha(x;z) = \int 2xz\,dz = xz^2 + B(x)$$

$$U(\bar x) = x^2 y + xz^2 + B(x)$$

$$U'_x = 2xy + z^2 + B'(x) = 2xy + z^2 \qquad B(x) = 0$$

$$[U(\bar x) = x^2 y + xz^2] \qquad A = (1;0;0) \quad B = (3;0;3)$$

$$\int_C \bar f\,ds = U(B) - U(A) = 27 - 0 = \boxed{27}$$

⑳ $\bar f(x,y) = (P,Q) \in C^1$ en una región simplemente conexa y abierta

Si $P'_y = Q'_x \Longrightarrow \bar f$ no es conservativa

$\oint \bar f\,ds \neq 0$

Sea una curva cerrada C suave o suave a trozos parametrizada por $\bar g'(t)$ con $t \in [a;b]$

$\bar f(\bar g(t)) = K\,\bar g'(t)$ (proporcionalidad) $\Rightarrow \oint_C \bar f\,ds = \int_a^b \bar f(\bar g(t))\,\bar g'(t)\,dt =$

$\int_a^b K\,\bar g'(t)\,\bar g'(t)\,dt = K \underbrace{\int_a^b \underbrace{\|\bar g'(t)\|^2}_{>0}\,dt}_{>0} \neq 0$ siempre va a ser mayor a 0

porque si $\bar g'(t) = 0$ sería un punto y no una curva. Se demuestra

(21) $\bar{f}(x,y) = (ax; y-ax)$ $\quad C: \bar{X} = (\cos(t); b\,\text{sen}(t)) \wedge t\in[0,2\pi] \wedge a+b=6$

$\bar{X}' : (-\text{sen}\,t;\ b\cos(t))$

$$\int_0^{2\pi} (a\cos(t);\ b\,\text{sen}\,t - a\cos t)(-\text{sen}\,t; b\cos t)\,dt =$$

$$= \int_0^{2\pi} -a\cos(t)\,\text{sen}(t) + b^2\,\text{sen}\,t\cos t - ab\cos^2 t$$

$u = \text{sen}(t)$
$du = \cos(t)$

$$-a\int_0^{2\pi}\cos(t)\,\text{sen}(t) + b^2\int_0^{2\pi}\text{sen}(t)\cos(t) - ab\int_0^{2\pi}\cos^2(t) \Rightarrow \text{por tabla}$$

$$-a\int_0^{2\pi} u\,du + b\int_0^{2\pi} u\,du - \frac{ab}{2}(x + \text{sen}\,x\cos x)\Big|_0^{2\pi}$$

$$-\frac{a}{2}\cancel{\text{sen}^2(t)}\Big|_0^{2\pi} + \frac{b}{2}\cancel{\text{sen}^2(t)}\Big|_0^{2\pi} - \frac{ab}{2}(x + \text{sen}\,x\cos x)\Big|_0^{2\pi}$$

$\boxed{C = -\frac{ab}{2}(2\pi)}$ $\qquad b = 6-a$

$C: -a(6-a)\pi = (-6a+a^2)\pi \qquad C' = (-6+2a)\pi = 0 \quad +6 = 2a \quad a=3$

$C'' : 2\pi \qquad \boxed{a=3 \text{ minimo} \Rightarrow b=3}$

* (4) $\Omega = K|x-1|$

$$C \begin{cases} z = 2y - x^2 \\ y = x \end{cases}$$

$\gamma(t) = (t; t; 2t - t^2)$
$\gamma'(t) = (1; 1; 2-2t)$ $\quad 0\le t\le 1$
$\|\gamma'(t)\| = \sqrt{6-8t+4t^2} = \sqrt{2}\sqrt{3-4t+2t^2}$

$$\int_a^b F(\gamma(t))\,\|\gamma'(t)\|\,dt$$

$$\int_0^1 K|t-1|\sqrt{2}\sqrt{3-4t+2t^2}\,dt$$

$3-4t+2t^2 = u$
$du = 4t-4 = 4(t-1) = \frac{1}{4}du$

$$\frac{K\sqrt{2}}{4}\int_0^1 u^{1/2}\,du = \frac{K\sqrt{2}}{4}\frac{(3-4t+2t^2)^{3/2}}{3/2}\Big|_0^1 = K\left(\frac{\sqrt{2}}{6} - \sqrt{\frac{3}{2}}\right)$$

obs: los sg me quedan al reves

(6) $h.h = \|h\|^2 = K^2$ (cte)

$h'.h + h\,h' = 0$

$h'.h = 0 \Rightarrow h' \perp h$