1º PARCIAL

31 - 5 - 2023

1) $Dh_{(1,2)} = Df_{(2,1)} \cdot D\bar{g}_{(1,2)} = \begin{pmatrix} 4 & 1 \\ 6 & -2 \end{pmatrix} = \begin{pmatrix} 3 & 5 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 6 & -2 \end{pmatrix} = \begin{pmatrix} 12+30 & 3-10 \end{pmatrix}$

máx $h'_{\check{v}(1,2)} = \sqrt{42^2 + (-7)^2} = \boxed{\sqrt{1813}} \simeq 42,58$

$\check{v} = \left(\frac{42}{\sqrt{1813}}, -\frac{7}{\sqrt{1813}} \right)$

2) $p(2,3) = 9 + 9 - 12 - 4 = 2 = f(2,3)$

$p'_u(2,3) = -2v - 2u = -6 - 4 = -10 = f'_u(2,3)$

$p'_v(2,3) = 2v - 2u = 6 - 4 = 2 = f'_v(2,3)$

$u'_x = 2x$

$v'_x = y$

$u'_y = -2$

$v'_y = x$

$\cancel{f''_{uu}(2,3)}$ $h(2,1) = f(2,3) = 2$

$h'_{x(2,1)} = f'_{u(2,3)} \, u'_{x(2,1)} + f'_{v(2,3)} \, v'_{x(2,1)} = -10 \cdot 4 + 2 \cdot 1 = -38$

$h'_{y(2,1)} = f'_{u(2,3)} \cdot u'_{y(2,1)} + f'_{v(2,3)} \cdot v'_{y(2,1)} = -10 \cdot (-2) + 2 \cdot 2 = 24$

$h(2,01;\, 0,98) \simeq 2 - 38 \cdot 0,01 + 24 \cdot (-0,02) = 2 - 0,38 - 0,48 = \boxed{1,14}$

3) $\bar{\lambda} \begin{cases} x = t \\ y = 5 - t^2 \\ z = 5 - t^2 \end{cases}$ $\quad \bar{\lambda}'(t) = (1, -2t, -2t)$ $\quad \bar{\lambda}(2) = (2, 1, 1)$ $\quad \bar{\lambda}'(2) = (1, -4, -4)$

plano

$\boxed{1(x-2) - 4(y-1) - 4(z-1) = 0}$

cuando $y = 0$ y $z = 0 \rightarrow x - 2 + 4 + 4 = 0 \rightarrow x = -6 \rightarrow \boxed{(-6, 0, 0)}$

4) $\begin{cases} y = kx^3 \\ y' = 3kx^2 \rightarrow k = y'/3x^2 \end{cases} \rightarrow \longrightarrow y = \frac{y'}{3x^2} x^3 \rightarrow 3y = y'x \rightarrow y' = \frac{3y}{x}$ ED 1

$y' = -\frac{x}{3y}$ ED 2 $\rightarrow \int 3y \, dy = \int -x \, dx \rightarrow \frac{3y^2}{2} + \frac{x^2}{2} = \frac{c}{2} \rightarrow \boxed{3y^2 + x^2 = c}$

En $(1,1) \rightarrow k = 1 \rightarrow \boxed{y = x^3}$

$\rightarrow c = 4 \rightarrow \boxed{3y^2 + x^2 = 4}$

$\boxed{\frac{x^2}{3} + y^2 = 19}$

2 T1) si $y = ax^2$ $x \to 0$, $y \to 0$

$$\lim_{x \to 0} f(x, ax^2) = \lim_{x \to 0} \frac{a^2x^4}{x^4 + a^2x^4} = \frac{a^2}{1+a^2} \Rightarrow \nexists \lim_{(x,y)\to(0,0)} f \Rightarrow f \text{ no es cont. en } (0,0)$$

VERDADERO

$\check{v} = (V_x, V_y)$

$$f'_{\check{v}}(0,0) = \lim_{t \to 0} \frac{f(tV_x, tV_y) - f(0,0)}{t} = \lim_{t \to 0} \frac{t^2V_y^2}{t^5V_x^4 + t^3V_y^2} =$$

$$= \lim_{t \to 0} \frac{V_y^2}{t^3V_x^4 + tV_y^2} \quad \text{si } V_y = 0 \to f'_{\check{v}}(0,0) = 0.$$

$$\text{si } V_y \neq 0 \to \nexists f'_{\check{v}}(0,0)$$

FALSO

T2) $f'_{((1,1),(1,3))} = f'_{x(1,1)} \cdot 1 + f'_{y(1,1)} \cdot 3 = 17$

$$f'_{x(1,1)} = 5 \quad \to \quad f'_{y(1,1)} = \frac{17-5}{3} = \boxed{4}$$