2° PARCIAL   ANALISIS MATEMATICO II   28/7/2023

P1) $VOL(E) = \iiint_E dx\,dy\,dz = \begin{matrix} x = \rho\cos(\varphi) \\ y = \rho\,\text{sen}(\varphi) \\ z = z \end{matrix} \rightarrow \iint_{E_{\varphi\rho}} \int_{\rho}^{\sqrt{3-2\rho^2}} \rho \, dz\, d\varphi\, d\rho =$

$= \iint_{E_{\varphi\rho}} (\rho\sqrt{3-2\rho^2} - \rho^2)\, d\varphi\, d\rho =$

$\left\{ \begin{array}{l} \rho \leq z \leq \sqrt{3-2\rho^2} \\ \rho^2 \leq 3-2\rho^2 \\ \rho^2 \leq 1 \rightarrow 0 \leq \rho \leq 1 \end{array} \right.$

$= \int_0^1 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\rho\sqrt{3-2\rho^2} - \rho^2)\, d\varphi\, d\rho =$

$= \pi \left[\int_0^1 (\rho\sqrt{3-2\rho^2} - \rho^2)\, d\rho \right] = \pi \left[\frac{(3-2\rho^2)^{3/2}}{-6} - \frac{\rho^3}{3} \right]_0^1 =$

$u = 3-2\rho^2$

$du = -4\rho\, d\rho$

$\int u^{1/2} \frac{du}{-4} = \frac{u^{3/2}}{\frac{3}{\cancel{2}}(\cancel{-4})^{2}}$

$= \pi \left(-\frac{1}{6} - \frac{\cancel{1}^{2}}{\cancel{3}_{6}} - \frac{3\sqrt{3}}{-6} \right) = \boxed{\frac{3\sqrt{3}-3}{6}\pi}$

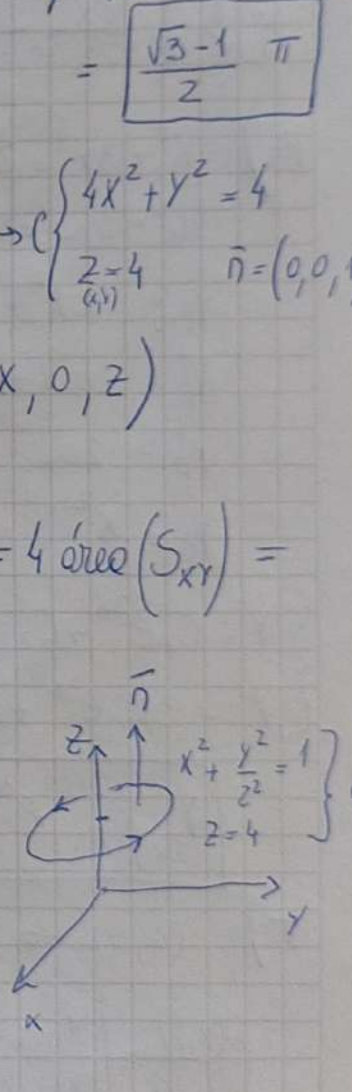
$= \boxed{\frac{\sqrt{3}-1}{2}\pi}$

P2) $C \left\{ \begin{array}{l} z = 4x^2+y^2 \\ z = 8-4x^2-y^2 \end{array} \right. \rightarrow \left\{ \begin{array}{l} z = 4x^2+y^2 \\ z = 8-z \end{array} \right. \rightarrow \begin{array}{l} z = 4x^2+y^2 \\ z = 4 \end{array} \rightarrow C \left\{ \begin{array}{l} 4x^2+y^2 = 4 \\ \underset{(4,y)}{z=4} \end{array} \right. \quad \bar{n} = (0,0,1)$

$\text{rot}\, \bar{f} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & 2xz & xy \end{vmatrix} = (x-2x,\ y-y,\ 2z-z) = (-x, 0, z)$

$\oint_{\partial S_{\bar{n}}^{+}} \bar{f}\, d\bar{\lambda} = \iint_{S_{\bar{n}}} \text{rot}\, \bar{f}\, d\bar{\sigma} = \iint_{S_{xy}} (-x,0,4)(0,0,1)\, dx\, dy = 4\ \text{área}(S_{xy}) =$

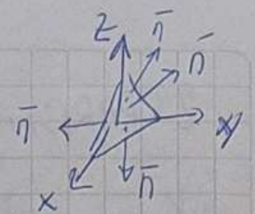
$= 4 \cdot \pi\ 1 \cdot 2 = \boxed{8\pi}$



P3) $\bar{f}(x,y,z) = (x-y-z\,,\ y-x-z\,,\ g(x,y))$

div $\bar{f} = 1+1 = 2$

$\frac{x}{6} + \frac{y}{4} + \frac{z}{3} \leq 1$



$$\iint_{\partial E^+} \bar{f}\, d\sigma = \iiint_E \text{div}\, \bar{f}\, dx\,dy\,dz = 2\int_0^6 \int_0^{\frac{12-2x}{3}} \int_0^{\frac{12-2x-3y}{4}} dz\,dy\,dx =$$

$$= \frac{2}{4}\int_0^6 \int_0^{\frac{12-2x}{3}} (12-2x-3y)\,dy\,dx = \frac{1}{2}\int_0^6 \left[12\cdot y - 2xy - \frac{3}{2}y^2\right]_0^{\frac{12-2x}{3}} dx =$$

$$= \frac{1}{2}\int_0^6 \left(\frac{\cancel{12}^{4}}{\cancel{3}}(12-2x) - \frac{2x}{3}(12-2x) - \frac{\cancel{3}^{1}}{2\cdot \cancel{9}_{3}}(144-48x+4x^2)\right) dx =$$

$$= \frac{1}{2}\int_0^6 \left(48 - 8x - \cancel{8x} + \frac{4}{3}x^2 - \frac{\cancel{48}^{24}}{\cancel{2}} + \cancel{8x} - \frac{2}{3}x^2\right) = \frac{1}{2}\int_0^6 \left(\frac{2}{3}x^2 - 8x + 24\right) dx =$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}x^3}{9} - \cancel{4}^{2}x^2 + \cancel{24}^{12}x\right]_0^6 = \frac{\cancel{6}^{2}\cdot \cancel{36}^{12}}{\cancel{9}_{\cancel{3}_{1}}} - 2\cdot 36 + 72 = 24 - 72 + 72 = \boxed{24}$$

P4) $y'' - 3y' = 2 - 6x$

$m^2 - 3m = 0 \to m(m-3) = 0 \to m_1 = 0\,,\ m_2 = 3 \qquad y_c = C_1 e^{3x} + C_2$

$y_p = ax^2 + bx \quad \to \quad 2a - 6ax - 3b = 2 - 6x$

$y_p' = 2ax + b$

$y_p'' = 2a$

$$\begin{cases} -6a = -6 & \to a = 1 \\ 2a - 3b = 2 & 2 - 2 = 3b \to b = 0 \end{cases}$$

$y_p = x^2$

$$\begin{cases} y = C_1 e^{3x} + C_2 + x^2 & 3 = C_1 + C_2 & C_2 = 3 - C_1 = 2 \\ y' = 3C_1 e^{3x} + 2x & 3 = 3C_1 \to C_1 = 1 \end{cases}$$

$$\boxed{y_{(x)} = e^{3x} + x^2 + 2}$$

T1) $(u,v) \longrightarrow (x,y) \qquad 6 = \iint_D dx\,dy = \iint_{D^*} \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv = 4$ área $(D^*) =$

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = 2 - 6 = -4$

área $(D^*) = \frac{6}{4} = \boxed{\frac{3}{2}}$