AM II 15/12/2022

P4) $\operatorname{div} \bar{f} = 1 + g''(x) + g'(x) - 2g(x) \equiv 1$

$$g'' + g' - 2g = 0 \Rightarrow m^2 + m - 2 = 0 \Rightarrow m = \frac{-1 \pm \sqrt{1-4(-2)}}{2}$$

$$m = 1 \lor m = -2$$

$$g(x) = C_1 e^x + C_2 e^{-2x}$$

$$g'(x) = C_1 e^x - 2C_2 e^{-2x}$$

$$\bar{f}(0,0,1) = (g'(0), 0, -2g(0)) = (2,0,0) \Rightarrow g(0) = 0 \land g'(0) = 2$$

$$\begin{cases} C_1 + C_2 = 0 \\ C_1 - 2C_2 = 2 \end{cases} \quad \text{RMM} \quad -3C_2 = -2 \Rightarrow C_2 = \frac{2}{3}, \quad C_1 = -\frac{2}{3}$$

$$\boxed{g(x) = -\frac{2}{3} e^x + \frac{2}{3} e^{-2x}}$$

P1) $\delta(x,y,z) = k\sqrt{x^2+y^2}$

$x = \rho\cos(\varphi)$, $y = \rho\,\mathrm{sen}(\varphi)$, $\left|\frac{\partial(x,y)}{\partial(\rho,\varphi)}\right| = \rho$

$$\text{massa}(E) = \iiint_E k\sqrt{x^2+y^2}\, dz\, dx\, dy = \iint_{x^2+y^2\le 4} k\sqrt{x^2+y^2}\; 5\, dx\, dy =$$

with z from $x-3$ to $2+x$.

$$= 5k\int_0^2 \int_0^{2\pi} \rho^2\, d\varphi\, d\rho = 10k\pi \left[\frac{\rho^3}{3}\right]_0^2 = \frac{10}{3} k\pi\, 8 = \boxed{\frac{80}{3} k\pi}$$

P2) $\bar{\lambda}(t) = (t, 9-t^2, 9-t^2) \quad t \in [0,3]$

$$\int_{\bar{A}\to\bar{B}} \bar{f}\, d\bar{\lambda} = -\int_0^3 \left(t(9-t^2), -(9-t^2)^2, (9-t^2)^2\right)(1, -2t, -2t)\, dt =$$

$$= -\int_0^3 (9t - t^3)\, dt = \left[\frac{9}{2}t^2 - \frac{t^4}{4}\right]_3^0 = \frac{81}{4} - \frac{81}{2} = \boxed{-\frac{81}{4}}$$

P3) $x^2+y^2=2 \longrightarrow S \quad \bar{r}(u,v) = (\sqrt{2}\cos(u), \sqrt{2}\,\mathrm{sen}(u), v) \qquad 0 \le u \le 2\pi$

$x^2+y^2+z^2 \le 4 \to z^2 \le 2 \to -\sqrt{2} \le z \le \sqrt{2} \qquad -\sqrt{2} \le v \le \sqrt{2}$

$$\bar{r}'_u \times \bar{r}'_v = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ -\sqrt{2}\,\mathrm{sen}(u) & \sqrt{2}\cos(u) & 0 \\ 0 & 0 & 1 \end{vmatrix} = (\sqrt{2}\cos(u), \sqrt{2}\,\mathrm{sen}(u), 0)$$

$$\iint_S \bar{f}\, d\bar{r} = \iint_{S_{uv}} (\sqrt{2}\cos(u) - v\sqrt{2}\,\mathrm{sen}(u), \sqrt{2}\,\mathrm{sen}(u) + \sqrt{2}\cos(u)\cdot v, \ldots)\cdot(\sqrt{2}\cos(u), \sqrt{2}\,\mathrm{sen}(u), 0)\, du\,dv$$

$$= \iint_{S_{uv}} 2\cos^2(u) - \cancel{2v\,\mathrm{sen}(u)\cos(u)} + \cancel{2v\,\mathrm{sen}(u)\cos(u)} + 2\,\mathrm{sen}^2(u)\; du\,dv$$

$$= 2\int_0^{2\pi}\int_{-\sqrt{2}}^{\sqrt{2}} dv\,du = 2\cdot 2\pi\cdot 2\sqrt{2} = \boxed{8\pi\sqrt{2}}$$

T2) $\phi'_y = x^2 \to \phi = x^2 y + \varphi(x) \to \phi'_x = 2xy + \varphi'(x) = 2xy + 2x\,g'(x^2)$

$\varphi'(x) = 2x\cdot g'(x^2) \to \varphi(x) = \int 2x\,g'(x^2)\,dx = g(x^2) + k$

$\phi(x,y) = x^2 y + g(x^2) + k$

$$\int_{(-2,4)\to(2,5)} \bar{f}\, d\bar{\lambda} = \phi(2,5) - \phi(-2,4) = 20 + g(4) + k - (16 + g(4) + k) = \boxed{4}$$

T1) $$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} = -2 - 1 = -3$$

$$9 = \iint_{D_{xy}} dx\,dy = \iint_{D_{uv}} \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv = 3\,\text{área}(D_{uv}) \to \boxed{\text{área}(D_{uv}) = 3}$$