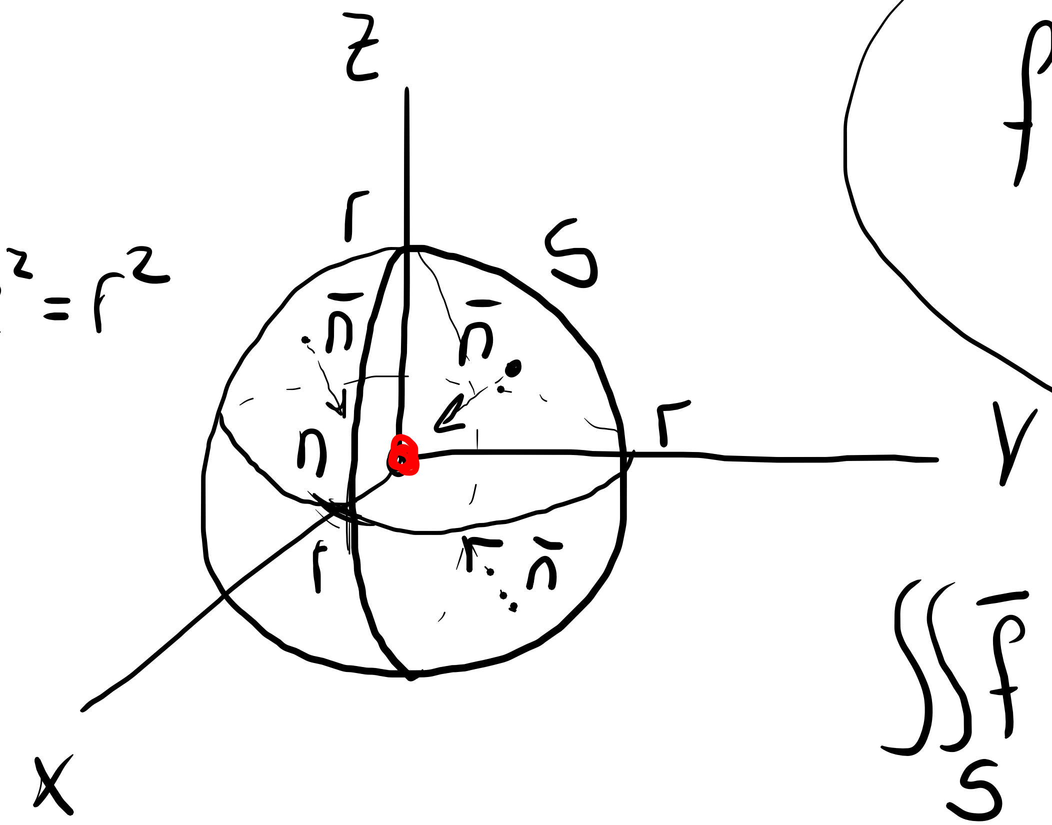
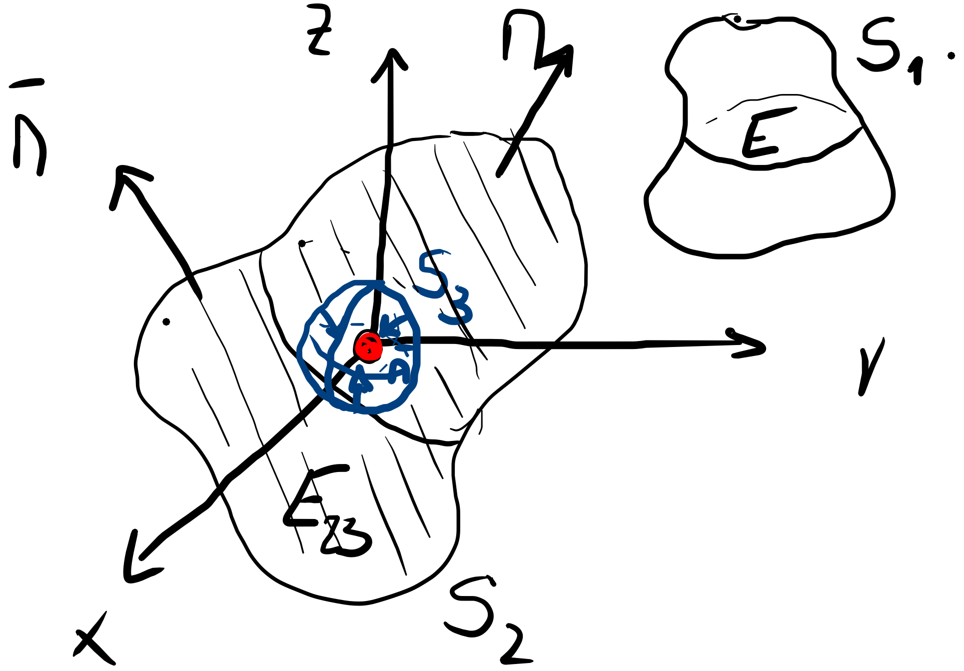
$x^2+y^2+z^2=r^2$



$$\bar f(x,y,z) = (f_1, f_2, f_3) \longrightarrow \operatorname{div} \bar f = 0$$

$$\bar f \in \mathbb{R}^3 - \{(0,0,0)\}$$

$$\iint_S \bar f \, d\bar\sigma = -4k\pi$$



$$\iint_{S_1=\partial E^+} \bar f \, d\bar\sigma = \iiint_E \operatorname{div} \bar f \, dx\,dy\,dz = 0$$

$$\iint_{\partial E_{23}^+} \bar f \, d\bar\sigma = \iiint_{E_{23}} \operatorname{div} \bar f \, dx\,dy\,dz = 0 = \boxed{\underbrace{\iint_{S_2} \bar f \, d\bar\sigma}_{+4k\pi}} + \underbrace{\iint_{S_3} \bar f \, d\bar\sigma}_{-4k\pi}$$

10) Calcule el flujo de $\bar{f}$ a través de S, indicando gráficamente la orientación del versor normal que ha elegido, o bien que se le solicite en cada caso.(&)

c) $\bar{f}(x,y,z)=(xy,\ zx,\ y-xz^2)$ a través del trozo de superficie cilíndrica de ecuación $y=x^3$ con $0\le z\le x+y,\ x+y\le 10$.

$$0 \le z \le x+y \le 10$$

$$\boxed{0 \le z \le x+x^3} \le 10 \quad \rightarrow \quad 0 \le x \le 2$$

$$0 \le x+x^3 \le 10$$

$$S: \ y_{(x,z)} = x^3 \ \longrightarrow \ S = N_0 g \ \rightarrow \ g_{(x,y,z)} = y - x^3 \ \rightarrow \ \bar{\nabla g} = (-3x^2, 1, 0) = \bar{n}$$

$$g(x,y,z) = -y + x^3 \longrightarrow \bar{\nabla g} = (3x^2, -1, 0) = \bar{n}$$

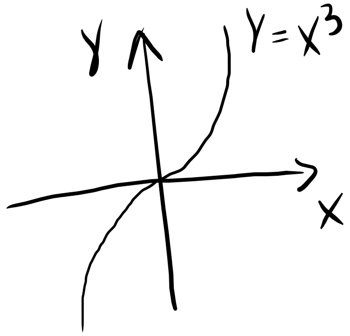
$$\iint_{S_g} \bar{f}\, d\bar{\sigma} = \iint_{S_{xz}} \bar{f}(x, y_{(x,z)}, z) \cdot \frac{\overset{\bar{n}}{\bar{\nabla g}}}{|g'_y|}\, dx\, dz = \int_{x=0}^{2}\int_{z=0}^{x+x^3} (x\cdot x^3,\ zx,\ x^3 - xz^2)\cdot \frac{(-3x^2, 1, 0)}{|1|}\, dz\, dx =$$

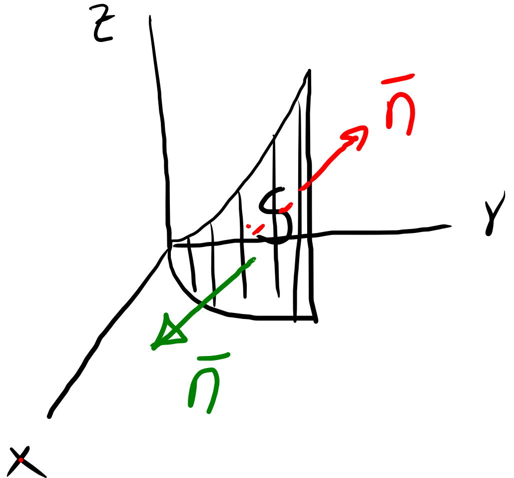
$$= \int_0^2 \int_0^{x+x^3} (-3x^6 + zx)\, dz\, dx = \int_0^2 \left[-3x^6 z + \frac{z^2}{2} x\right]_0^{x+x^3} dx =$$

$$= \int_0^2 \left[-3x^6(x+x^3) + \frac{x}{2}(x^2 + 2x^4 + x^6)\right] dx = \int_0^2 \left[-3x^7 - 3x^9 + \frac{x^3}{2} + x^5 + \frac{x^7}{2}\right] dx =$$

$$= \left[-\frac{3}{8}x^8 - \frac{3}{10}x^{10} + \frac{x^4}{8} + \frac{x^6}{6} + \frac{x^8}{16}\right]_0^2 = 16\left(-6 - \frac{\cancel{192}^{\,96}}{\cancel{10}_{\,5}} + \frac{1}{8} + \frac{2}{3} + 1\right) =$$

$$= \boxed{-\frac{5618}{15}}$$





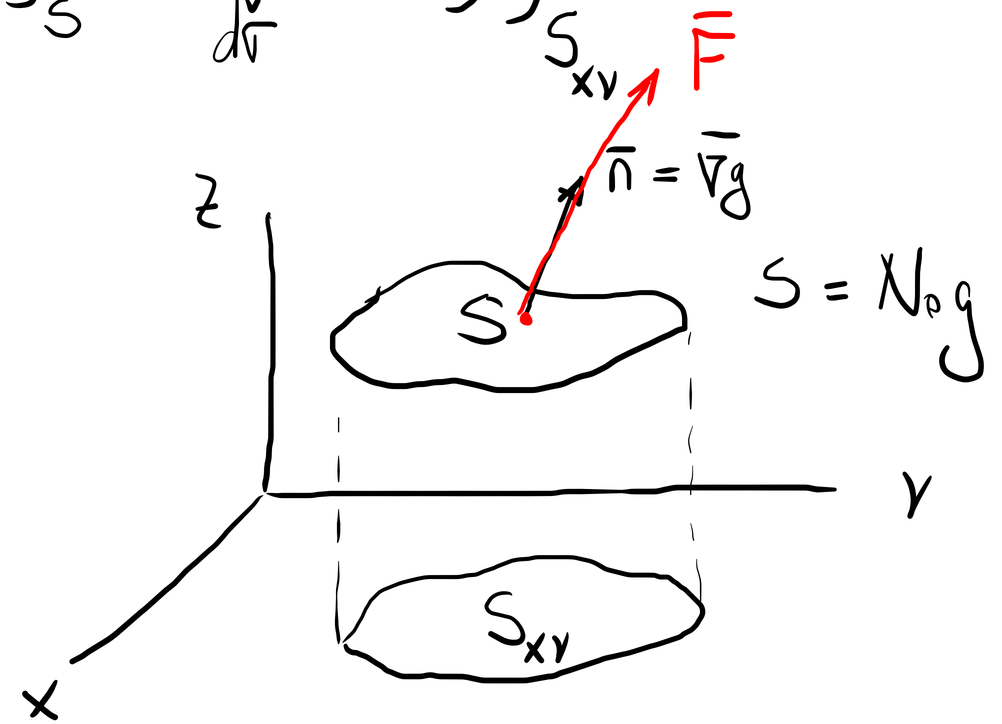
CORRECTO

07) Sea $\overline{F} = k\,\check{n}$ con $k > 0$ constante, demuestre que $\iint_S \overline{F}\cdot\check{n}\,d\sigma = F$ área(S) con $F = \|\overline{F}\|$.(*)

$$\overline{F}_{(x,y,z)} = k\cdot\check{n} = k(n_x, n_y, n_z) \qquad k>0$$

$$\iint_S \overline{F}\,\underbrace{\check{n}\,d\sigma}_{d\check{\sigma}} = \iint_{S_{xy}} k(n_x,n_y,n_z)\cdot\frac{\overline{\nabla}g}{|g'_z|}\,dx\,dy = k\iint_{S_{xy}} \underbrace{(n_x,n_y,n_z)\cdot\frac{(g'_x,g'_y,g'_z)}{\|g'_x,g'_y,g'_z\|}}_{\check{n}\,\cdot\,\check{n}\;=1}\;\underbrace{\frac{\|g'_x,g'_y,g'_z\|}{|g'_z|}\,dx\,dy}_{d\sigma}$$

$$= k\iint_{S_{xy}} d\sigma = k\cdot \text{área}(S) = \boxed{F\cdot\text{área}(S)}$$



$$\overline{F} = k\cdot\check{n}$$

$$\|\overline{F}\| = \underbrace{|k|}_{k}\cdot\underbrace{\|\check{n}\|}_{=1}$$

$$F = \|\overline{F}\| = k$$