1)

$$Vol(H)=\iiint_H dV=\int_0^{2\pi}d\varphi\int_0^2\rho\,d\rho\int_{\rho\cos\varphi+\rho sen\varphi}^{2\rho\cos\varphi+\rho sen\varphi+3}dz=$$

$$=\int_0^{2\pi}d\varphi(\ )\int_0^2\rho(2\rho\cos\varphi+\rho sen\varphi+3-\rho\cos\varphi-\rho sen\varphi)d\rho=$$

$$=\int_0^{2\pi}d\varphi\int_0^2(2\rho^2\cos\varphi+\rho^2 sen\varphi+3\rho-\rho^2\cos\varphi-\rho^2 sen\varphi)d\rho=$$

$$=\int_0^{2\pi}d\varphi\int_0^2(3\rho+\rho^2\cos\varphi)d\rho=\int_0^{2\pi}\left(3\frac{\rho^2}{2}+\frac{\rho^3}{3}\cos\varphi\right)_0^2 d\varphi=$$

$$=\int_0^{2\pi}\left(6+\frac{8}{3}\cos\varphi\right)d\varphi=\left(6\varphi+\frac{8}{3}sen\varphi\right)_0^{2\pi}=12\pi$$

2)

$$\overline{f}(x,y)=(6xy+2y^2+2\ ,\ 3x^2+4xy-2)$$

a)

$Q'_x=6x+4y \qquad P'_y=6x+4y \ \Rightarrow Q'_x=P'_y$

Se cumple la simetría de la $D\overline{f}$ que además es continua en $\mathbb{R}^2$ que es simplemente conexo $\Rightarrow$ $\overline{f}$ es conservativo $\Rightarrow$ $\overline{f}$ admite función potencial

b) $\nabla\phi=\overline{f}$

$$\begin{cases}\phi'_x=6xy+2y^2+2\\ \phi'_y=3x^2+4xy-2\end{cases}\rightarrow\phi(x,y)=3x^2y+2y^2x+2x-2y+c$$

Por dato: $\phi(1,2)=11\rightarrow\phi(1,2)=6+8+2-4+c=11\rightarrow c=-1$

$\Rightarrow \phi(x,y)=3x^2y+2y^2x+2x-2y-1$

c)

$\phi(1,0)=2-1=1$

3)

$$\text{Flujo}=\oiiint_H \operatorname{div}\overline{f}\ dV=\oiiint_H 2y\ dV=2\int_0^2 dx\int_0^{1-\frac{x}{2}} y\,dy\int_0^{3-\frac{3}{2}x-3y} dz=$$

$$=2\int_0^2 dx\int_0^{1-\frac{x}{2}} y.\left(3-\frac{3}{2}x-3y\right)dy=2\int_0^2 dx\int_0^{1-\frac{x}{2}}\left(3y-\frac{3}{2}xy-3y^2\right)dy=\text{"feo"}...$$

Invierto orden de integración:

$$=2\int_0^1 y\,dy\int_0^{2-2y} dx\int_0^{3(1-\frac{1}{2}x-y)} dz=6\int_0^1 y\,dy\int_0^{2-2y}\left(1-\frac{1}{2}x-y\right)dx=$$

$$=6\int_0^1 y\left(x-\frac{x^2}{2}-xy\right)_0^{2-2y} dy=6\int_0^1 y\left(2-2y-\frac{1}{4}(2-2y)^2-y(2-2y)\right)dy=$$

$$=6\int_0^1 y\left(2-2y-\frac{1}{4}(4-8y+4y^2)-2y+2y^2\right)dy=6\int_0^1 y\left(y^2-2y+1\right)dy=$$

$$=6\int_0^1\left(y^3-2y^2+y\right)dy=6\left(\frac{y^4}{4}-2\frac{y^3}{3}+\frac{y^2}{2}\right)_0^1=6.\left(\frac{1}{4}-\frac{2}{3}+\frac{1}{2}\right)=\frac{1}{2}$$

4)

$$y''-2y'+5y=2x$$

$$y_G=y_H+y_P$$

*Calculo y_H:

$$y''-2y'+5y=0$$

$$r^2-2r+5=0$$

$$r_{1,2}=1\pm 2i$$

$$y_H=e^x(A\cos(2x)+B\operatorname{sen}(2x))$$

*Calculo y_P:

$$y_P=ax+b$$

$$y'_P=a$$

$$y''_P=0$$

Reemplazando en E.D:

$$y''-2y'+5y=-2a+5ax+5b=2x$$

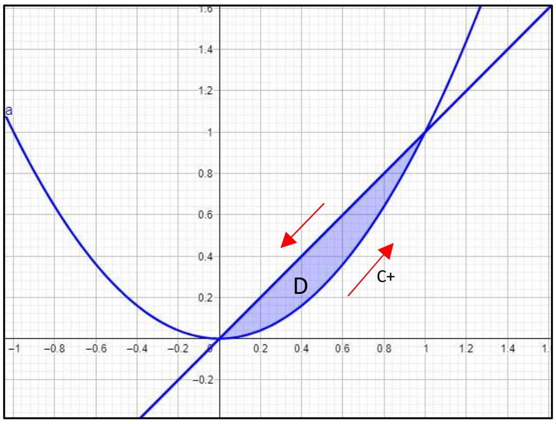
$$\begin{cases}5a=2\\-2a+5b=0\end{cases}\rightarrow a=\frac{2}{5}\quad b=\frac{2a}{5}=\frac{4}{25}$$

$$y_P=\frac{2}{5}x+\frac{4}{25}$$

$$S.G\rightarrow y_G=e^x(A\cos(2x)+B\mathrm{sen}(2x))+\frac{2}{5}x+\frac{4}{25}$$

$$y_G(0)=A+\frac{4}{25}$$

T1)



$$\text{circ.}=\oint_{C^+}\overline{f}.d\overline{s}=\iint_D(Q_x'-P_y')dxdy=\iint_D(3xy-xy)dxdy=\iint_D(2xy)dxdy=$$

$$=2\int_0^1 xdx\int_{x^2}^{x}y\,dy=\int_0^1 x\left(x^2-x^4\right)dx=\left(\frac{x^4}{4}-\frac{x^6}{6}\right)_0^1=\left(\frac{1}{4}-\frac{1}{6}\right)=\frac{1}{12}$$