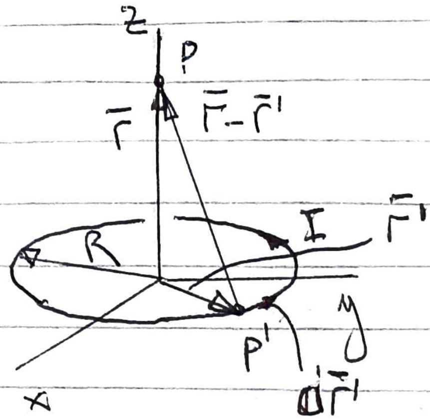
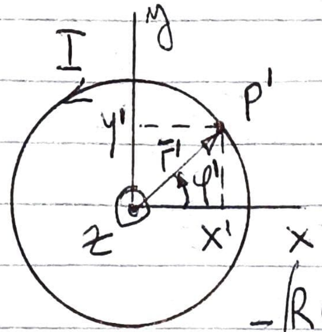
20) a)





$P: \bar{r} = (0,0,z)$

$P': \bar{r}' = (x', y', 0) = (R\cos\varphi', R\,\text{sen}\,\varphi', 0)$

$\bar{r}-\bar{r}' = (-R\cos\varphi', -R\,\text{sen}\,\varphi', z)$

$|\bar{r}-\bar{r}'|^3 = \left[(-R\cos\varphi')^2 + (-R\,\text{sen}\,\varphi')^2 + z^2\right]^{3/2} = (R^2+z^2)^{3/2}$

$d\bar{r}' = (R\cos\varphi', R\,\text{sen}\,\varphi', 0)' \text{ (derivada)} \cdot d\varphi' = (-R\,\text{sen}\,\varphi', R\cos\varphi', 0)\,d\varphi'$

$d\bar{r}' \times (\bar{r}-\bar{r}') = \begin{vmatrix} \breve{e}_x & \breve{e}_y & \breve{e}_z \\ -R\,\text{sen}\,\varphi' & R\cos\varphi' & 0 \\ -R\cos\varphi' & -R\,\text{sen}\,\varphi' & z \end{vmatrix} d\varphi' = (zR\cos\varphi', zR\,\text{sen}\,\varphi', R^2\underbrace{(\cos^2\varphi' + \text{sen}^2\varphi')}_{1})\,d\varphi'$

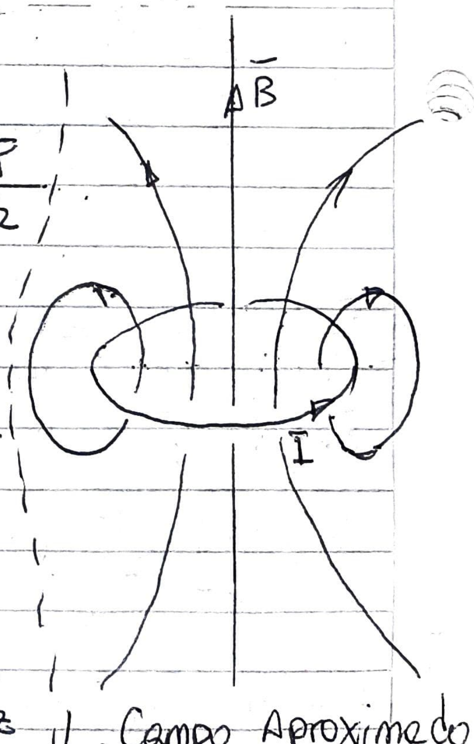
$\bar{B}_{(P)} = \dfrac{\mu_0 I}{4\pi} \displaystyle\int_0^{2\pi} \dfrac{(zR\cos\varphi', zR\,\text{sen}\,\varphi', R^2)\,d\varphi'}{(R^2+z^2)^{3/2}}$

$B_z(P) = \dfrac{\mu_0 I R^2}{4\pi (R^2+z^2)^{3/2}} \displaystyle\int_0^{2\pi} d\varphi' = \dfrac{\mu_0 I R^2}{\cancel{4\pi}_{\,2} (R^2+z^2)^{3/2}} \cancel{2\pi}$

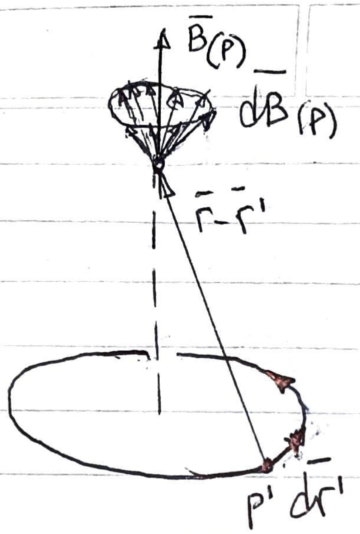
$B_x(P) = \dfrac{\mu_0 I}{4\pi} \dfrac{zR}{(R^2+z^2)^{3/2}} \displaystyle\int_0^{2\pi} \cos\varphi'\,d\varphi' = 0$

$B_y(P) = \dfrac{\mu_0 I}{4\pi} \dfrac{zR}{(R^2+z^2)^{3/2}} \displaystyle\int_0^{2\pi} \text{sen}\,\varphi'\,d\varphi' = 0$

Para el anillo completo $\quad \bar{B} = B_z\,\breve{e}_z = \dfrac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}\,\breve{e}_z$

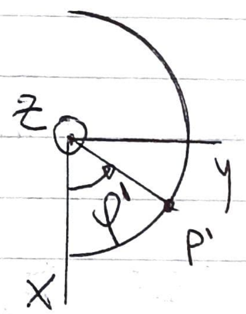


L. Campo Aproximado de una espira con corriente



Al integrar en toda la espira queda sólo la componente en z del campo

b) Para la media espira derecha.



Los expresiones de B_x, B_y y B_z son las mismas del punto a) cambiando los límites de integración por cero y π.

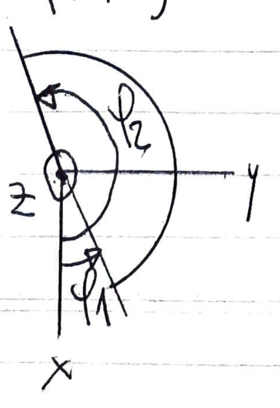
$$B_{x(P)} = \frac{\mu_0 I z R}{4\pi (R^2+z^2)^{3/2}} \underbrace{\int_0^{\pi} \cos\varphi' \, d\varphi'}_{0} = 0$$

$$B_{y(P)} = \frac{\mu_0 I z R}{4\pi (R^2+z^2)^{3/2}} \underbrace{\int_0^{\pi} \sin\varphi' \, d\varphi'}_{+2} =$$

$$B_{z(P)} = \frac{\mu_0 I R^2}{4\pi (R^2+z^2)^{3/2}} \int_0^{\pi} d\varphi' = \frac{\mu_0 I R^2}{4 (R^2+z^2)^{3/2}}$$

$$\bar{B}_{(P)} = \frac{\mu_0 I}{4 (R^2+z^2)^{3/2}} \left(\frac{2Rz}{\pi} \check{e}_y + \frac{R^2}{\pi} \check{e}_z \right)$$

c)



$$\bar{B}_{(P)} = \frac{\mu_0 I}{4\pi} \int_{\varphi_1}^{\varphi_2} \frac{(zR\cos\varphi', zR\sin\varphi', R^2) \, d\varphi'}{(R^2+z^2)^{3/2}}$$

Agregamos d)

Si P está en el origen de coordenadas : del punto a)

$$\bar{B}(0,0,0) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+0^2)^{3/2}} \breve{e}_z = \frac{\mu_0 I}{2R} \breve{e}_z$$ para la espira completa

Para media espira $\bar{B}(0,0,0) = \frac{\mu_0 I}{4R} \breve{e}_z$

" $\frac{1}{4}$ " $\bar{B}(0,0,0) = \frac{\mu_0 I}{8R} \breve{e}_z$