P1) $\begin{cases} x^2-2x+y^2 \leq 0 \\ z \leq 2+x \\ x+z \geq 2 \end{cases}$ $\begin{cases} x^2-2x+1+y^2 \leq 1 \\ 2-x \leq z \leq 2+x \end{cases}$ $\rightarrow$ $\begin{matrix} (x-1)^2+y^2 \leq 1 \\ 2-x \leq z \leq 2+x \end{matrix}$

$$\text{VOL}(E) = \iint_{E_{xy}} \int_{2-x}^{2+x} dz\,dx\,dy = \iint_{E_{xy}} (\overbrace{2+x-2+x}^{2x})\,dx\,dy \xrightarrow[\left|\frac{\partial(x,y)}{\partial(\rho,\varphi)}\right| = \rho]{\substack{x=\rho\cos(\varphi)+1 \\ y=\rho\,\text{sen}(\varphi)}}$$

$$= 2\int_0^1 \int_0^{2\pi} (\rho\cos(\varphi)+1)\rho\,d\varphi\,d\rho = 2\left[\int_0^1 \rho^2 d\rho \underbrace{\int_0^{2\pi} \cos(\varphi)\,d\varphi}_{=0} + \int_0^1 \rho\,d\rho \int_0^{2\pi} d\varphi\right] =$$

$$= 2\left[\frac{\rho^2}{2}\right]_0^1 2\pi = \cancel{2}\cdot\frac{1}{\cancel{2}}\,2\pi = \boxed{2\pi}$$

P2) $\bar{f}(x,y) = (\underbrace{4xy-2}_{f_1}, \underbrace{2x^2-2y}_{f_2})$

$f'_{1y} = 4x = f'_{2x}$

$D_{\bar{f}} = \mathbb{R}^2$ simplemente conexo $\Bigg\} \Rightarrow \bar{f}$ conservativo

$$\phi = \int_0^1 \bar{f}(tx,ty)\cdot(x,y)\,dt + k = \int_0^1 (4t^2xy-2, 2t^2x^2-2ty)\cdot(x,y)\,dt + k =$$

$$= \int_0^1 (\cancel{4}^{6}t^2x^2y - 2x + \cancel{2}t^2x^2y - 2ty^2)\,dt + k = \left[2t^3x^2y - 2tx - t^2y^2\right]_0^1 + k =$$

$$\phi = 2x^2y - 2x - y^2 + k$$

$$0 = 2 - 2 - 1 + k \rightarrow k = 1$$

$$\boxed{\phi(x,y) = 2x^2y - 2x - y^2 + 1}$$

P3) $\bar{F}(x,y,z) = (y^2, z^2+x^2, x^2)$

$S: \begin{cases} y = x \\ (x,z) \\ x^2+y^2+2z^2 \leq 2 \end{cases}$ $g(x,y,z) = y - x$ $\bar{\nabla} g = (-1, 1, 0)$

$\rightarrow \cancel{2}x^2 + \cancel{2}z^2 \leq \cancel{2}\ 1$

$$\iint_S \bar{F}\, d\bar{\sigma} = \iint_{x^2+z^2\leq 1} (x^2, z^2+x^2, x^2)\frac{(-1,1,0)}{|1|}\, dx\, dz = \iint_{x^2+z^2\leq 1} (-\cancel{x^2}+z^2+\cancel{x^2})\, dx\, dz =$$

$$= \xrightarrow[y = \rho \operatorname{sen}(\varphi)]{x = \rho \cos(\varphi)} \int_0^1 \int_0^{2\pi} \rho^3 \operatorname{sen}^2(\varphi)\, d\varphi\, d\rho = \left[\frac{\rho^4}{4}\right]_0^1 \left[\frac{\varphi}{2} - \frac{\operatorname{sen}(2\varphi)}{4}\right]_0^{2\pi} =$$

$$= \boxed{\frac{1}{4}\pi}$$

P4) $y'' - 4y' + 13y = 26 \qquad y(0) = 1 \qquad y'(0) = 1$

$m^2 - 4m + 13 = 0 \rightarrow m = \frac{4 \pm \sqrt{16 - 4 \cdot 13}}{2} = \frac{4}{2} \pm \frac{\sqrt{-36}}{2} \rightarrow \alpha = 2 \qquad \beta = 3$

$y_c = e^{2x}\left(C_1 \cos(3x) + C_2 \operatorname{sen}(3x)\right)$

$y_p = a \rightarrow 13 \cdot a = 26 \rightarrow a = 2 \rightarrow y_p = 2$

$y_p' = 0$

$y_p'' = 0$

$$\begin{cases} y_{(x)} = e^{2x}\left(C_1 \cos(3x) + C_2 \operatorname{sen}(3x)\right) + 2 \\ y'_{(x)} = 2e^{2x}\left(C_1 \cos(3x) + C_2 \operatorname{sen}(3x)\right) + e^{2x}\left(-3C_1 \operatorname{sen}(3x) + 3C_2 \cos(3x)\right) \end{cases}$$

$$\begin{cases} 1 = C_1 + 2 \\ 1 = 2C_1 + 3C_2 \end{cases} \rightarrow \quad C_1 = -1 \qquad C_2 = \frac{1+2}{3} = 1$$

$$\boxed{y_{(x)} = e^{2x}\left(-\cos(3x) + \operatorname{sen}(3x)\right) + 2}$$