14) Verifique si los siguientes campos admiten función potencial; de existir, determínela.

a) $\bar{f}(x,y) = (\underbrace{y-2xy+1}_{f_1}, \underbrace{x+1-x^2}_{f_2})$. ~~c) $\bar{f}(x,y,z) = (z\cos(xz), z, y+x\cos(xz))$~~.

COND NEC. $f'_{2x} \equiv f'_{1y}$

$$\left.\begin{array}{l} f'_{2x} = 1-2x \\ f'_{1y} = 1-2x \end{array}\right\} \text{iguales}$$

$$\left.\begin{array}{l} f'_{2x} = 1-2x \\ f'_{1y} = 1-2x \\ D_{\bar f} = \mathbb{R}^2 \text{ simplemente conexo} \end{array}\right\} \begin{array}{l} \bar f \text{ conservativo, admite función potencial} \\ \exists g : \mathbb{R}^2 \to \mathbb{R} \,/\, \bar\nabla g = \bar f \end{array}$$

$$g(x,y) = \int_0^1 \bar f(tx,ty)\cdot(x,y)\,dt + k = \int_0^1 (ty - 2t^2xy + 1, tx + 1 - t^2x^2)\cdot(x,y)\,dt + k =$$

$$= \int_0^1 (\cancel{txy} - \cancel{2t^2x^2y} + x + 2txy + y - 3t^2x^2y)\,dt + k = \Big[+tx + t^2xy + ty - t^3x^2y\Big]_0^1 + k =$$

$$\boxed{g(x,y) = +x + xy + y - x^2y + k}$$ potencial de $\bar f$

$$\bar\nabla g = (+1 + y - 2xy,\ x + 1 - x^2)$$

14) Verifique si los siguientes campos admiten función potencial; de existir, determínela.

a) $\bar{f}(x,y) = (y-2xy+1, x+1-x^2)$. ~~c) $\bar{f}(x,y,z) = (z\cos(xz), z, y+x\cos(xz))$~~.

14) Verifique si los siguientes campos admiten función potencial; de existir, determínela.

a) ~~$\bar{f}(x,y) = (y - 2xy + 1, x + 1 - x^2)$~~. c) $\bar{f}(x,y,z) = (\underbrace{z\cos(xz)}_{f_1}, \underbrace{z}_{f_2}, \underbrace{y + x\cos(xz)}_{f_3})$.

CONS. NEC

$$\begin{cases} f'_{2x} = f'_{1y} \longrightarrow f'_{2x} = 0 = f'_{1y} \checkmark \\ f'_{3x} = f'_{1z} \longrightarrow f'_{3x} = \cos(xz) + xz(-\operatorname{sen}(xz)) = f'_{1z} = \cos(xz) + xz(-\operatorname{sen}(xz)) \checkmark \\ f'_{2z} = f'_{3y} \longrightarrow f'_{2z} = 1 = f'_{3y} = 1 \checkmark \end{cases}$$

$D_{\bar{f}} = \mathbb{R}^3$ simplemente conexo.

$\Rightarrow \bar{f}$ es conservativo

$$g(x,y,z) = \int_0^1 \bar{f}(tx,ty,tz) \cdot \overline{(x,y,z)}\, dt + k = \dots$$

$$\bar{f} = (f_1, f_2, f_3) = \bar{\nabla} g = (g'_x, g'_y, g'_z) \rightarrow g'_x = f_1, \quad g'_y = f_2, \quad g'_z = f_3$$

$$g = \int z \cdot \cos(xz)\, dx = \underline{\operatorname{sen}(xz) + C_1(y,z)}$$

$$g = \int z\, dy = \underline{zy + C_2(x,z)}$$

$$g = \int (y + x\cos(xz))\, dz = \underline{yz + \operatorname{sen}(xz) + C_3(x,y)}$$

$$\boxed{g(x,y,z) = \operatorname{sen}(xz) + yz + C}$$

VERIFICAMOS $\bar{\nabla} g = (z\cos(xz), z, x\cos(xz) + y) \checkmark$

c) $\bar{f}(x,y,z) = (z\cos(xz), z, y + x\cos(xz))$.

18) Sea $\bar{f} \in C^1 / \bar{f}(x,y) = (x\,y^2\ ,\ y\,g(x))$, determine g de manera que $\bar{f}$ admita función potencial; suponga que $\underline{\bar{f}(2{,}1) = (2{,}6)}$.

$$f'_{2x} = f'_{1y} \rightarrow \cancel{y} \cdot g'(x) = 2x\cancel{y} \rightarrow g(x) = \int 2x\,dx \rightarrow g(x) = x^2 + C$$

$$\bar{f}(2,1) = (2,6) = (2,\ g(2)) \rightarrow g(2) = 6 \longrightarrow g(2) = \underbrace{6 = 2^2 + C}$$

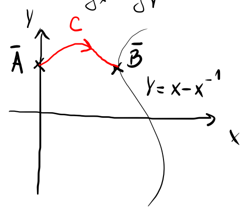
$$C = 2$$

$$\boxed{g(x) = x^2 + 2}$$

17) Sean $\bar{f}$ un campo de gradientes con matriz jacobiana $D\bar{f}(x,y)=\begin{pmatrix}2y-6x & 2x\\ 2x & 0\end{pmatrix}$ y C una curva abierta cualquiera con puntos extremos $\bar{A}$ perteneciente al eje y y $\bar{B}$ perteneciente a la curva de ecuación $y=x-x^{-1}$. Demuestre que $\int_C \bar{f}\cdot d\bar{s}=0$, sabiendo que la gráfica de su función potencial pasa por (1,1,3) con plano tangente de ecuación $z = y+2$.

$$\bar{f} = (\underbrace{f_1}_{g'_x}, \underbrace{f_2}_{g'_y}) \to \begin{matrix} f'_{1x} = 2y-6x & f'_{1y} = 2x \\ f'_{2x} = 2x & f'_{2y} = 0 \end{matrix}$$

iguales



$$\int_{\substack{C\\ \bar{A}\to\bar{B}}} \bar{f}\, d\lambda = g(\bar{B}) - g(\bar{A})$$

$$\left.\begin{matrix} f'_{1x} = g''_{xx} = 2y-6x \to g'_x = \int (2y-6x)\,dx = 2xy - 3x^2 + C_1(y) \\ f'_{1y} = g''_{xy} = \underbrace{2x + C_1'(y) = 2x} \to C_1'(y) = 0 \to C_1 = cte \end{matrix}\right\} g'_x = 2xy - 3x^2 + C_1$$

$$g = \int (2xy - 3x^2 + C_1)\,dx = x^2y - x^3 + C_1x + C_2(y) \leftarrow$$

$$f_2 = g'_y = x^2 + C_2'(y) \to f'_{2x} = g''_{yx} = 2x$$

$$f'_{2y} = g''_{yy} = C_2''(y) = 0 \to C_2'(y) = cte = a$$

$$C_2(y) = ay + b$$

$$\boxed{g(x,y) = x^2y - x^3 + C_1x + ay + b}$$

$g(1,1)=3$ ①

$z = y+2 \to z'_x = 0 = g'_{x(1,1)} = 2xy - 3x^2 + C_1$ ②

$z'_y = 1 = g'_{y(1,1)} = x^2 + a$ ③

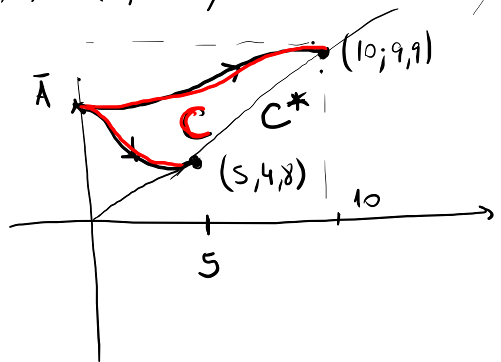
$$\begin{cases} \cancel{1} - \cancel{1} + C_1 + a + b = 3 \to b = 3 - 1 = 2 \\ 2 - 3 + C_1 = 0 \to C_1 = 1 \\ \cancel{1} + a = \cancel{1} \to a = 0 \end{cases}$$

$$\boxed{g(x,y) = x^2y - x^3 + x + 2}$$

17) Sean $\bar{f}$ un campo de gradientes con matriz jacobiana $D\bar{f}(x,y)=\begin{pmatrix}2y-6x & 2x\\ 2x & 0\end{pmatrix}$ y C una curva abierta cualquiera con puntos extremos $\bar{A}$ perteneciente al eje y y $\bar{B}$ perteneciente a la curva de ecuación $y=x-x^{-1}$. Demuestre que $\int_C \bar{f}\cdot d\bar{s}=0$, sabiendo que la gráfica de su función potencial pasa por (1,1,3) con plano tangente de ecuación $z = y+2$.

$$\int_{\substack{C\\ (0,y)\to(x,\,x-x^{-1})}} \bar{f}\, d\lambda = g(x, x-x^{-1}) - g(0,y) = x^2(x-x^{-1}) - x^3 + x + \cancel{2} - \cancel{2} =$$

$$= \cancel{x^3} - \cancel{x} - \cancel{x^3} + \cancel{x} = \boxed{0}$$



$C^*: \; y = x - x^{-1}$

19) Siendo $\bar{f}(x,y,z) = (\underbrace{2xy+z^2}_{f_1}, \underbrace{x^2}_{f_2}, \underbrace{2xz}_{f_3})$, **verifique** que admite función potencial en $\Re^3$ y **calcule** $\int_C \bar{f} \cdot d\bar{s}$ a lo largo de la curva C de ecuación $\bar{X} = (2t + e^{t^3-t}, t^2 - t, 3t)$ con $0 \le t \le 1$ orientada en el sentido que impone la parametrización que se indica.

$$\left.\begin{array}{l} f'_{1y} = 2x = f'_{2x} \checkmark \\ f'_{1z} = 2z = f'_{3x} \checkmark \\ f'_{2z} = 0 = f'_{3y} \checkmark \\ D_{\bar{f}} = \mathbb{R}^3 \text{ simplemente conexo} \end{array}\right\} \text{COND. SUFICIENTE} \quad \bar{f} \text{ conservativo}$$

$$\int_C \bar{f}\, d\bar{\lambda} = g(\bar{B}) - g(\bar{A})$$

$$\bar{A} = \bar{X}_{(t=0)} = (1,0,0) \qquad \bar{B} = \bar{X}_{(t=1)} = (3,0,3)$$

$$g(x,y,z) = \int_0^1 \bar{f}(tx,ty,tz)\cdot(x,y,z)\,dt + k = \int_0^1 (2t^2xy + t^2z^2, t^2x^2, 2t^2xz)(x,y,z)\,dt + k =$$

$$= \int_0^1 (\cancel{2t^2x^2y} + \cancel{t^2xz^2} + 3t^2x^2y + 3t^2xz^2)\,dt + k =$$

$$= \left[t^3x^2y + t^3xz^2\right]_0^1 + k = x^2y + xz^2 + k = g(x,y,z)$$

$$\int_C \bar{f}\, d\bar{\lambda} = g(3,0,3) - g(1,0,0) = 27 + k - k = \boxed{27}$$

11) Dado $\bar{f}(x,y) = (\overbrace{9x^2+2y+y^2}^{f_1}, \overbrace{2x+2xy}^{f_2})$, demuestre que $\bar{f}$ admite función potencial ϕ en $\Re^2$. Suponiendo $\phi(0,0)=2$, analice la existencia de extremos locales de $\phi(x,y)$ clasificándolos y calculándolos.

$$\left.\begin{array}{l} f'_{2x} = 2+2y = f'_{1y} \ \checkmark \quad \text{CONS. NEC.} \\ D_{\bar{f}} = \mathbb{R}^2 \end{array}\right\} \text{CONS. SUF.} \quad \bar{f} \text{ conservativo}$$

$$\phi'_x = f_1 \rightarrow \phi = \int (9x^2+2y+y^2)\,dx = 3x^3+2xy+xy^2+C_{(y)}$$

$$f_2 = \phi'_y = 2x+2xy+C'_{(y)} = 2x+2xy \rightarrow C'_{(y)} = 0 \rightarrow C = \text{cte}$$

$$\phi(x,y) = 3x^3+2xy+xy^2+C \qquad \phi(0,0) = 2 = C$$

$$\phi(x,y) = 3x^3+2xy+xy^2+2$$

$$f_1 = \phi'_x = 9x^2+2y+y^2 = 0$$

$$f_2 = \phi'_y = 2x+2xy = 0 = 2x(1+y) \longrightarrow x=0 \ \vee \ y=-1$$

$x=0$:
$$2y+y^2=0$$
$$y(2+y)=0$$
$$y=0 \ \vee \ y=-2$$

$y=-1$:
$$9x^2-2+1=0$$
$$x^2=\frac{1}{9}$$
$$x=\frac{1}{3} \ \vee \ x=-\frac{1}{3}$$

Ptos críticos $(0,0)$; $(0,-2)$; $\left(\frac{1}{3},-1\right)$; $\left(-\frac{1}{3},-1\right)$

$$Hf = \begin{pmatrix} 18x & 2+2y \\ 2+2y & 2x \end{pmatrix} \rightarrow Hf_{(0,0)} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \rightarrow \det Hf_{(0,0)} = -4 < 0 \Rightarrow (0,0) \text{ pto silla.}$$

$$Hf_{(0,-2)} = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix} \rightarrow \det Hf_{(0,-2)} = -4 < 0 \Rightarrow (0,-2) \text{ pto silla}$$

$$Hf_{\left(\frac{1}{3},-1\right)} = \begin{pmatrix} 6 & 0 \\ 0 & 2/3 \end{pmatrix} \rightarrow \det Hf_{\left(\frac{1}{3},-1\right)} = 4 > 0 \ , \ f''_{xx}\left(\tfrac{1}{3},-1\right) = 6 > \ \Rightarrow$$

f tiene mín. local en $\left(\frac{1}{3},-1\right)$

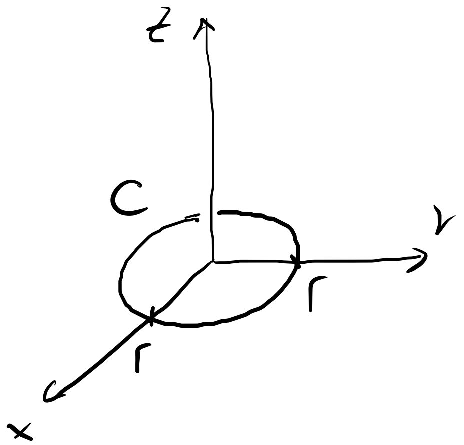
$$Hf_{\left(-\frac{1}{3},-1\right)} = \begin{pmatrix} -6 & 0 \\ 0 & -2/3 \end{pmatrix} \rightarrow \det Hf_{\left(-\frac{1}{3},-1\right)} = 4 > \ , \ f''_{xx}\left(-\tfrac{1}{3},-1\right) < 0 \Rightarrow$$

f tiene un máx local en $\left(-\frac{1}{3},-1\right)$

18) Analice si $\bar{f}$ admite función potencial en su dominio natural; en "c" y "d" suponga $\varphi \in C^1$:

c) $\bar{f}(x,y,z)=(\underbrace{\frac{y}{x^2+y^2}}_{f_1},\underbrace{\frac{-x}{x^2+y^2}}_{f_2},\underbrace{\varphi(z)}_{f_3})$

$$\left.\begin{aligned}
&\left.\begin{aligned}
f'_{1y} &= \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2} \\
f'_{2x} &= \frac{-x^2-y^2+2x^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}
\end{aligned}\right\} \text{iguales. } \checkmark \\
&f'_{1z} = 0 = f'_{3x} \quad \checkmark \\
&f'_{2z} = 0 = f'_{3y} \quad \checkmark
\end{aligned}\right\} \text{COND. NEC.}$$



$D_f = \{(x,y,z) \in \mathbb{R}^3 \,/\, x^2+y^2 \neq 0\}$ no es simplemente conexo

$C: \bar{\lambda}(t) = (r \cdot \cos(t), r \operatorname{sen}(t), 0) \qquad t \in [0, 2\pi]$

$$\oint_C \bar{f}\, d\lambda = \int_0^{2\pi} \bar{f}(\overbrace{r\cdot\cos(t)}^{x}, \overbrace{r \operatorname{sen}(t)}^{y}, \overbrace{0}^{z}) \cdot (-r \operatorname{sen}(t), r \cdot \cos(t), 0)\, dt =$$

c) $\bar{f}(x,y,z)=(\frac{y}{x^2+y^2},\frac{-x}{x^2+y^2},\varphi(z))$

$$= \int_0^{2\pi} \left(\frac{r \operatorname{sen}(t)}{r^2}, \frac{-r\cos(t)}{r^2}, \varphi(0)\right)(-r \operatorname{sen}(t), r\cos(t), 0)\, dt =$$

$$= \int_0^{2\pi} \left(-\operatorname{sen}^2(t) - \cos^2(t)\right) dt = -\int_0^{2\pi} \operatorname{sen}^2(t)\, dt - \int_0^{2\pi} \cos^2(t)\, dt =$$

$$= -\left[\frac{t}{2} - \frac{\operatorname{sen}(2t)}{4}\right]_0^{2\pi} - \left[\frac{t}{2} + \frac{\operatorname{sen}(2t)}{4}\right]_0^{2\pi} = -\pi - \pi = -2\pi \neq 0$$

$\Rightarrow \bar{f}$ no es conservativo

04) Resuelva las siguiente ecuaciones diferenciales totales exactas o convertibles a este tipo.

a) $2xy\,dx+(x^2+\cos y)\,dy=0$. ~~d) $(y^2-y)dx+x\,dy=0$. (*)~~

$$\underbrace{2xy}_{\phi'_x}\,dx+\underbrace{(x^2+\cos(y))}_{\phi'_y}\,dy=\underbrace{0}_{d\phi}$$

$$\phi''_{xy}=\phi''_{yx}$$

$$\left.\begin{aligned}\phi''_{xy}&=2x\\ \phi''_{yx}&=2x\end{aligned}\right\}\text{ iguales.}$$

$$\phi=\int 2xy\,dx=x^2y+C_{(y)}$$

$$\phi'_y=x^2+C'_{(y)}=x^2+\cos(y)$$

$$C'_{(y)}=\cos(y)$$

$$C_{(y)}=\operatorname{sen}(y)+k$$

$$\phi_{(x,y)}=x^2y+\operatorname{sen}(y)+k\longrightarrow\boxed{x^2y+\operatorname{sen}(y)=k}$$

$\bar{f}=(f_1,f_2)$ conservativo $\Rightarrow f'_{1y}=f'_{2x}$

$f_1=\phi'_x\ ,\ f_2=\phi'_y$

$d\phi=\phi'_x\cdot dx+\phi'_y\cdot dy$ si $\phi\in C^2$

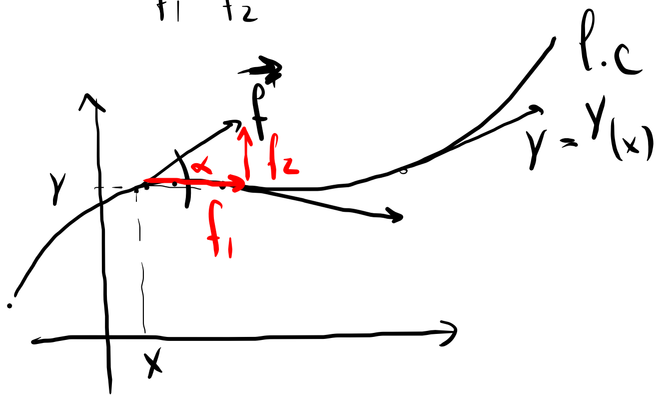
$\Rightarrow\ \boxed{\phi''_{xy}=\phi''_{yx}}$ T. Schwarz.

equivalente. $\boxed{f'_{1y}=f'_{2x}}$

$d\phi=0\iff\boxed{\phi=cte}$

13) Halle y grafique la familia de líneas de campo en los siguientes casos, al graficar recuerde orientar las líneas según el campo en cada punto.

a) $\bar{f}(x,y) = (\underbrace{2y-x}_{f_1}, \underbrace{x}_{f_2})$. e) ~~$\bar{E}$ según TP 11 – item 16.~~



$$\tan(\alpha) = \boxed{\frac{f_2}{f_1} = y'}$$

$$y' = \frac{x}{2y-x} \rightarrow \frac{dy}{dx} = \underbrace{\frac{x}{2y-x}}_{g(x,y)} \rightarrow$$

$$g(\lambda x, \lambda y) = \frac{\lambda x}{2\lambda y - \lambda x} = \frac{x}{2y-x} = g(x,y) \rightarrow$$ ec. dif. homogenea de 1° orden.

$$\boxed{y = u_{(x)} x} \rightarrow y' = u' x + u$$

reemplazo en la ec.

$$u'x + u = \frac{\cancel{x}}{2u\cancel{x} - \cancel{x}} = \frac{1}{2u-1} \rightarrow u'x = \frac{1}{2u-1} - u = \frac{1-2u^2+u}{2u-1} = \frac{du}{dx} x$$

$$\int \frac{dx}{x} = \int \frac{2u-1}{-2u^2+u+1} du = \underbrace{\int \frac{2u \, du}{-2u^2+u+1}}_{\text{TABLA 267}} - \underbrace{\int \frac{du}{-2u^2+u+1}}_{\text{TABLA 266}}$$

$$\frac{-1 \pm \sqrt{1-4\cdot(-2)}}{2(-2)} = \frac{-1 \pm 3}{-4}$$

$$= -\frac{1}{2} \wedge 1$$