P1) $\cos(x)\; y' + \text{sen}(x)\; y = 1$ tal que $y_{(x=0)} = 2$

$y = u\, v \qquad y' = u'\, v + u\, v'$

$\cos(x)\left[u'\, v + u\, v'\right] + \text{sen}(x)\; u\; v = 1$

$\cos(x)\; u'\, v + u\underbrace{\left[\cos(x)\; v' + \text{sen}(x)\; v\right]}_{=0} = 1$

$\cos(x)\,\dfrac{dv}{dx} = -\,\text{sen}(x)\; v \rightarrow \displaystyle\int \frac{dv}{v} = -\int \frac{\text{sen}(x)}{\cos(x)}\, dx$

$\rightarrow \ln|v| = \ln|\cos(x)| + \ln|C_1| \rightarrow v_{(x)} = C_1\, \cos(x)$

$s = \cos(x)$

$ds = -\,\text{sen}(x)\, dx$

$\displaystyle\int \frac{1}{s}\, \frac{ds}{(-1)} = -\ln|s|$

$\cos(x)\,\dfrac{du}{dx}\; C_1 \cos(x) = 1 \rightarrow \displaystyle\int du = \int \frac{dx}{C_1 \cos^2(x)} \rightarrow u = \frac{1}{C_1}\tan(x) + C_2$

$\rightarrow y_{(x)} = u\; v = \left(\dfrac{1}{C_1}\tan(x) + C_2\right) C_1\; \cos(x)$

$y_{(x)} = \text{sen}(x) + C_3\, \cos(x)$

$2 = \text{sen}(0) + C_3\, \cos(0)$

$2 = C_3$

$\boxed{y_{(x)} = \text{sen}(x) + 2\, \cos(x)}$

P2) $(x,y,z) = (uv,\ u+v,\ u^2/v) = \overline{\sigma}_{(u,v)}$ en $(3,4,9) = \overline{P}$ $\qquad x+y=19$

$$\overline{n} = \overline{\sigma}'_u \times \overline{\sigma}'_v \Big|_{(3,1)} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ v & 1 & 2u/v \\ u & 1 & -\frac{u^2}{v^2} \end{vmatrix} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 1 & 1 & 6 \\ 3 & 1 & -9 \end{vmatrix} = (-15,\ 27,\ -2)$$

$$\begin{cases} u \cdot v = 3 \\ u+v = 4 \\ \frac{u^2}{v} = 9 \end{cases} \qquad \begin{matrix} u=3 \\ v=1 \end{matrix}$$

recta normal a S en $\overline{P}$

$$(x,y,z) = (3,4,9) + t\,(-15,27,-2) \qquad t \in \mathbb{R} \quad \longrightarrow (3,4,9) + 1\,(-15,27,-2) = \boxed{(-12,\ 31,\ 7)}$$

$$\begin{cases} x = 3 - 15t \\ y = 4 + 27t \\ z = 9 - 2t \end{cases} \qquad \begin{aligned} x+y&=19 \\ 3-15t+4+27t&=19 \\ 12t&=12 \\ t&=1 \end{aligned}$$

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15) Hallar la derivada direccional máxima de la función $h = f \circ \vec{g}$ en el punto $(1,1)$, siendo $\vec{g}(u,v) = (\underbrace{u+v}_{x}, \underbrace{u-v}_{y})$ y $f(x,y)$ se encuentra definida por $\underbrace{z + x^2 - y^2 + \ln(z+x-y)}_{F(x,y,z)} = 3$

$h = f \circ \vec{g}$

$\vec{g}(u,v) = (u+v, u-v)$

$f(x,y) = z + x^2 - y^2 + \ln(z+x-y) = 3$

forma implícita

$\partial(v_x, v_y) \to$ si $\vec{v} = (1,0) \to f'_x$

$\to$ si $\vec{v} = (0,1) \to f'_y$

T. Fun. imp $\to f'_x = \frac{-F'_x}{F'_z} \quad f'_y = \frac{-F'_y}{F'_z}$

$$f'_{\vec{v}}(a_x, a_y) = f'((a_x,a_y),(v_x,v_y)) = \frac{\partial f}{\partial \vec{v}}(a_x,a_y) = \lim_{x \to 0} \frac{f(a_x + t v_x, a_y + t v_y) - f(a_x, a_y)}{t}$$

Por teorema de derivación de funciones compuestas:

$Dh_{(1,1)} = Df_{\vec{g}(1,1)} \cdot D\vec{g}_{(1,1)}$

$Dh = (f'_x \;\; f'_y) \cdot \begin{pmatrix} g'_u \\ g'_v \end{pmatrix} =$

reemplazo (2,0) allá para hallar z

$$f'_x = \frac{-F'_x}{F'_z}\Bigg|_{(2,0,-1)} = \frac{-2x + \frac{1}{z+x-y}}{1 + \frac{1}{z+x-y}}\Bigg|_{(2,0,-1)}$$

$$f'_y = \frac{-F'_y}{F'_z}\Bigg|_{(2,0,-1)} = \frac{-2y + \frac{-1}{z+x-y}}{1 + \frac{1}{z+x-y}}\Bigg|_{(2,0,-1)}$$

DERIVADA DIR MÁXIMA =

$(1,1) \to (2,0) \to -1$

$(u,v) \to (x,y) \to z$

$z < x < u, v$; $z < y < u, v$

$\max h'_{\vec{v}(1,1)} = \|\nabla h\|_{(1,1)}$

acá hago la cuenta: $z_0 + 2^2 - 0 + \ln(z_0 + 2 - 0) = 3$

$\boxed{z_0 = -1}$

analizo en $(2,0,-1)$

$$f'_x\Big|_{(2,0,-1)} = -\frac{4+1}{1+1} = \frac{-5}{2}$$

$$f'_y\Big|_{(2,0,-1)} = -\frac{-1}{2} = \frac{-1}{2}$$

$$D\vec{g} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$Dh = (-5/2 \;\; -1/2) \cdot D\vec{g}_{(1,1)}$

$Dh = (-5/2 \;\; -1/2) \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$Dh = (-5/2 + 1/2 \;\; -5/2 - 1/2)$

$Dh = (-2 \;\; -3)$

$h'_{u(1,1)} \quad h'_{v(1,1)}$

$\sqrt{(-2)^2 + (-3)^2} = \sqrt{13}$

NOTA

P4) $f \in C^2$

$P(x,y) = 5 + x^2 + x(y-1) + 4(y-1)^2$

plano tang a la graf de f en $(0,1,f(0,1))$

$Z = f(0,1) + f'_{x(0,1)}(x-0) + f'_{y(0,1)}(y-1)$

$\boxed{Z = 5}$

$f \cong P$ a $E((0,1),\delta)$ con $\delta \to 0$

$f(0,1) = P(0,1) = 5$

$f'_{x(0,1)} = P'_{x(0,1)} = 2x + y - 1\Big|_{(0,1)} = 0$

$f'_{y(0,1)} = P'_{y(0,1)} = x + 8(y-1)\Big|_{(0,1)} = 0$

$f''_{xx(0,1)} = P''_{xx(0,1)} = 2$

$f''_{xy(0,1)} = f''_{yx(0,1)} = P''_{xy(0,1)} = 1$

$f''_{yy(0,1)} = P''_{yy(0,1)} = 8$

$Hf_{(0,1)} = \begin{pmatrix} 2 & 1 \\ 1 & 8 \end{pmatrix}$

$\Delta_1 = f''_{xx(0,1)} = 2 > 0$

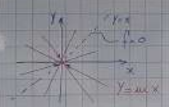
$\Delta_2 = \det Hf_{(0,1)} = 15 > 0$

$\boxed{f \text{ tiene un mínimo local en } (0,1)}$

T1) Siendo $f: A \subseteq \mathbb{R}^n \to \mathbb{R}$

se dice que f es cont en $\bar{a} \in A$ si $\lim\limits_{\bar{x} \to \bar{a}} f(\bar{x}) = f(\bar{a})$

$$f(x,y) = \begin{cases} \dfrac{y}{x-y} & x-y \neq 0 \\ 0 & x-y=0 \end{cases}$$



Siendo $y = mx \quad m \neq 1$

$$\lim_{x\to 0} f(x, mx) = \lim_{x\to 0} \frac{mx}{x - mx} = \lim_{x \to 0} \frac{m}{1-m} = \frac{m}{1-m} \Rightarrow \nexists \lim_{(x,y)\to(0,0)} f(x,y) \Rightarrow f \text{ no es continua en } (0,0)$$

T2) Siendo $f: A \subset \mathbb{R}^2 \to \mathbb{R}$ / A abierto.

f tiene un máximo local en $\bar{a} \in A$ si existe $\delta \in \mathbb{R}^+$ / $f(\bar{a}) \geq f(\bar{x}) \quad \forall\ \bar{x} \in E_{(\bar{a},\delta)}$

$f(x,y) = x^2 + xy - y^2 + y + 1$

$$\begin{cases} f'_x = 2x + y = 0 \rightarrow y = -2x \\ f'_y = x - 2y + 1 = 0 \rightarrow x - 2(-2x) = -1 \end{cases}$$

$x = -\frac{1}{5} \rightarrow y = \frac{2}{5}$

$\left(-\frac{1}{5}, \frac{2}{5}\right)$ punto crítico

$f''_{xx} = 2 \qquad f''_{xy} = 1$

$f''_{yx} = 1 \qquad f''_{yy} = -2$

$\Delta_1 = 2 > 0$

$\Delta_2 = -4 - 1 = -5 < 0$

f tiene un punto silla en $\left(-\frac{1}{5}, \frac{2}{5}\right)$

f no tiene extremos locales en $\mathbb{R}^2$