P1) $\begin{cases} y = kx \\ y' = k \end{cases} \rightarrow \quad y = y'x \quad$ ED1

$\downarrow$

$y = -\dfrac{1}{y'}x \quad$ ED2

$y\dfrac{dy}{dx} = -x \rightarrow \int y\,dy = -\int x\,dx \rightarrow \dfrac{y^2}{2} = -\dfrac{x^2}{2} + C_1 \rightarrow \boxed{x^2+y^2 = C_2}$

$3^2+(-4)^2 = C_2 \rightarrow 25 = C_2 \rightarrow \boxed{x^2+y^2 = 25}$

P2) $z = u\,x\,v^2$ $\quad u = x\sqrt{y}$ $\quad$ tal que $z = h(x,y)$

$$\underbrace{2v + e^{v-2x} - \frac{y}{x} = 1}_{g(x,y,v)} \longrightarrow$$

si $x = 1$, $y = 4$

$u = 2$, $2v + e^{v-2} - 4 = 1$

$v = 2$

$z = 2\cdot 1\cdot 2^2 = 8 = h_{(1,4)}$

$$v'_{x\,(1,4)} = -\left.\frac{g'_x}{g'_v}\right|_{(1,4,2)} = -\frac{-2e^{v-2x} + \frac{y}{x^2}}{2 + e^{v-2x}} = -\frac{-2+4}{2+1} = -\frac{2}{3}$$

$$v'_{y\,(1,4)} = -\left.\frac{g'_y}{g'_v}\right|_{(1,4,2)} = -\frac{-\frac{1}{x}}{2+e^{v-2x}} = -\frac{-1}{3} = \frac{1}{3}$$

$$u'_{x\,(1,4)} = \sqrt{y} = 2$$

$$u'_{y\,(1,4)} = \frac{x}{2\sqrt{y}} = \frac{1}{4}$$

$$z'_{u\,(1,2,2)} = x\,v^2 = 4$$

$$z'_{x\,(1,2,2)} = u\,v^2 = 8$$

$$z'_{v\,(1,2,2)} = 2\,u\,x\,v = 8$$

$$h'_{x\,(1,4)} = z'_u \cdot u'_x + z'_v \cdot v'_x + z'_x$$

$$= 4 \cdot 2 + 8\left(-\frac{2}{3}\right) + 8 = \frac{48-16}{3} = \frac{32}{3}$$

$$h'_{y\,(1,4)} = z'_u \cdot u'_y + z'_v\, v'_y$$

$$= 4 \cdot \frac{1}{4} + 8\,\frac{1}{3} = \frac{3+8}{3} = \frac{11}{3}$$

$$h_{(0{,}99\,;\,4{,}02)} \simeq h_{(1,4)} + h'_{x\,(1,4)}(-0{,}01) + h'_{y\,(1,4)}\,0{,}02 =$$

$$\simeq 8 + \frac{32}{3}(-0{,}01) + \frac{11}{3}\,0{,}02 = 8 - \frac{0{,}1}{3} = \boxed{7{,}9\hat{6}}$$

P3) $z = f(x,y)$ $\qquad \underbrace{xz + yz + \ln(xy + z - 5) - 12}_{g(x,y,z)} = 0$

Si: $x = 1$, $y = 2$,

$z + 2z + \ln(z - 3) - 12 = 0$

$z = 4$

$$z'_x{}_{(1,2)} = -\left.\frac{g'_x}{g'_z}\right|_{(1,2,4)} = -\frac{z + \frac{y}{xy+z-5}}{x + y + \frac{1}{xy+z-5}} = -\frac{4 + \frac{2}{1}}{1+2+1} = -\frac{6}{4} = -\frac{3}{2}$$

$$z'_y{}_{(1,2)} = -\left.\frac{g'_y}{g'_z}\right|_{(1,2,4)} = -\frac{z + \frac{x}{xy+z-5}}{x + y + \frac{1}{xy+z-5}} = -\frac{4 + \frac{1}{1}}{4} = -\frac{5}{4}$$

$$\min f'_{\vec{v}}(1,2) = -\left\|\vec{\nabla} f_{(1,2)}\right\| = -\sqrt{\left(-\frac{3}{2}\right)^2 + \left(-\frac{5}{4}\right)^2} =$$

$$= -\sqrt{\frac{9}{4} + \frac{25}{16}} = -\sqrt{\frac{36+25}{16}} = \boxed{-\frac{\sqrt{61}}{4}}$$

$$\boxed{\vec{v} = \left(\frac{3}{2}, \frac{5}{4}\right) \Big/ \frac{\sqrt{61}}{4}}$$

P4)

$$\begin{cases} x = t+2 \\ y = 2t+5 \\ z = t+1 \end{cases} \qquad \lambda'_{(t)} = (1, 2, 1)$$

$$(t+2)^2 + (\overbrace{2t+5-3}^{2t+2})^2 - (t+1)^2 = 1 \rightarrow \cancel{t^2} + 4t + 4 + 4t^2 + 8t + 4 - \cancel{t^2} - 2t - 1 = 1$$

$$4t^2 + 10t + 6 = 0 \rightarrow 2t^2 + 5t + 3 = 0 \rightarrow t = \frac{-5 \pm \sqrt{25 - 4\cdot 2\cdot 3}}{2\cdot 2} = \frac{-5 \pm 1}{4} = \begin{cases} t_1 = -1 \\ t_2 = -\frac{3}{2} \end{cases}$$

$$(x,y,z)_1 = \boxed{(1, 3, 0)}$$

$$(x,y,z)_2 = \left(-\frac{3}{2}+2,\ \cancel{2}\left(-\frac{3}{\cancel{2}}\right)+5,\ -\frac{3}{2}+1\right) = \boxed{\left(\frac{1}{2}, 2, -\frac{1}{2}\right)}$$

Punto (1) $\quad 1(x-1) + 2(y-3) + 1(z-0) = 0 \rightarrow \boxed{x + 2y + z = 7}$

Punto (2) $\quad \boxed{1\left(x - \frac{1}{2}\right) + 2(y-2) + 1\left(z + \frac{1}{2}\right) = 0}$

T1)

$$D\bar{g}_{(1,2)} = \begin{pmatrix} 2 & 2y \\ 2xy & x^2 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 4 & 1 \end{pmatrix}$$

λ: $x = 1$, $y = 2$ $\rightarrow$ $u = 6$ $v = 2$

$$Df_{(6,2)} = \begin{pmatrix} 24 & 72 \end{pmatrix}$$

$$Dh_{(1,2)} = Df_{(6,2)} \cdot D\bar{g}_{(1,2)} = \begin{pmatrix} 24 & 72 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 48+288 & 96+72 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 336 & 168 \end{pmatrix}}$$

T2) $f(x,y) = x^4 + x^2y^4 + 3 \geq 3 = f(0,0) \quad \forall (x,y) \in E((0,0),\delta)$

f tiene un mínimo local en $(0,0)$ y vale 3