P2) $g'' - 1 = -4g \Rightarrow g'' + 4g = 1 \Rightarrow m^2 + 4 = 0 \quad m = \pm 2i$

$\cancel{y_c = C_1 e^{2x} + C_2 e^{-2x}}$ $\quad y_c = C_1 \cos(2x) + C_2 \operatorname{sen}(2x)$

$y_p = a \quad 4a = 1 \Rightarrow a = \frac{1}{4} \quad y_p = \frac{1}{4}$

$y = C_1 \cos(2x) + C_2 \operatorname{sen}(2x) + \frac{1}{4}$

$y' = -2C_1 \operatorname{sen}(2x) + 2C_2 \cos(2x)$

$y_p' = 0$

$y_p'' = 0$

$C_1 + \frac{1}{4} = 0 \Rightarrow C_1 = -\frac{1}{4}$

$2C_2 = 6 \Rightarrow C_2 = 3$

$f(0,1) = (-4g(0), g'(0) + 1) = (0,7) \Rightarrow g(0) = 0 \quad g'(0) = 6$

$$\boxed{g(x) = -\frac{1}{4}\cos(2x) + 3\operatorname{sen}(2x) + \frac{1}{4}}$$

4) $\bar{\alpha}(t) = (t - t^2, t - t^4) \quad 0 \leq t \leq 1 \qquad \bar{f}(x,y) = (0, x)$

$$\text{área}(D) = \int_0^1 \bar{f}(\alpha_{(t)}) \cdot \bar{\alpha}'(t)\, dt = \int_0^1 (0, t - t^2)(\ldots, 1 - 4t^3)\, dt = \int_0^1 (t - t^2 - 4t^4 + 4t^5)\, dt =$$

$$= \left[\frac{t^2}{2} - \frac{t^3}{3} - \frac{4}{5} t^5 + \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}} t^6 \right]_0^1 = \frac{1}{2} - \frac{1}{3} - \frac{4}{5} + \frac{2}{3} = \frac{\overset{35-34}{15 - 10 - 24 + 20}}{30} =$$

$$= \boxed{\frac{1}{30}}$$

$\{ \bar{T}_{(u,v)} = (x, y, z)$

$\delta(x,y,z) = k\sqrt{x^2+y^2}$

$$x = \rho\cos(\varphi)$$
$$y = \rho\,\mathrm{sen}(\varphi)$$

$$\text{massa}(E) = \iiint_E \int_{x-3}^{2+x} k\sqrt{x^2+y^2}\, dz\, dx\, dy = \iint_{x^2+y^2\leq 4} k\sqrt{x^2+y^2}\; 5\, dx\, dy = \qquad \left|\frac{\partial(x,y)}{\partial(\rho,\varphi)}\right| = \rho$$

$$= 5k\int_0^2 \int_0^{2\pi} \rho^2\, d\varphi\, d\rho = 10k\pi\left[\frac{\rho^3}{3}\right]_0^2 = \frac{10}{3}k\pi\, 8 = \boxed{\frac{80}{3}k\pi}$$

P4) $x^2+y^2=2 \longrightarrow S \quad \overline{r}(u,v) = \left(\sqrt{2}\cos(u), \sqrt{2}\,\mathrm{sen}(u), v\right) \qquad 0 \leq u \leq 2\pi$

$x^2+y^2+z^2 \leq 4 \rightarrow z^2 \leq 2 \rightarrow -\sqrt{2} \leq z \leq \sqrt{2} \qquad -\sqrt{2} \leq v \leq \sqrt{2}$

$$\overline{r}'_u \times \overline{r}'_v = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ -\sqrt{2}\,\mathrm{sen}(u) & \sqrt{2}\cos(u) & 0 \\ 0 & 0 & 1 \end{vmatrix} = \left(\sqrt{2}\cos(u), \sqrt{2}\,\mathrm{sen}(u), 0\right)$$

$$\iint_S \overline{f}\, d\overline{r} = \iint_{S_{uv}} \left(\sqrt{2}\cos(u) - v\sqrt{2}\,\mathrm{sen}(u), \sqrt{2}\,\mathrm{sen}(u) + \sqrt{2}\cos(u)\cdot v, \ldots\right) \cdot \left(\sqrt{2}\cos(u), \sqrt{2}\,\mathrm{sen}(u), 0\right) du\,dv$$

$$= \iint_{S_{uv}} 2\cos^2(u) - \cancel{2v\,\mathrm{sen}(u)\cos(u)} + \cancel{2v\,\mathrm{sen}(u)\cos(u)} + 2\,\mathrm{sen}^2(u)\; du\,dv$$

$$= 2\int_0^{2\pi}\int_{-\sqrt{2}}^{\sqrt{2}} dv\,du = 2\cdot 2\pi \cdot 2\sqrt{2} = \boxed{8\pi\sqrt{2}}$$

T2) $\begin{cases} x = 2u + 2v \\ y = 3u + v \end{cases}$

$$D^* \xrightarrow{\bar{g}} D$$

$$(u,v) \longrightarrow (x,y)$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = 2-6 = -4$$

$$-8 = \iint_D x \, dx\, dy = \iint_{D^*} (2u+2v) \overbrace{|-4|}^{4} \, du\, dv = 4 \iint_{D^*} (2u+2v)\, du\, dv$$

$$\iint_{D^*} (2u+2v) = \frac{-8}{4} = \boxed{-2}$$

FALSO

$$\phi'_y = x^2 \rightarrow \phi = x^2 y + \varphi(x) \rightarrow \phi'_x = 2xy + \varphi'_{(x)} = 2xy + 2x\, g'(x^2)$$

$$\varphi'(x) = 2x \cdot g'(x^2) \rightarrow \varphi_{(x)} = \int 2x\, g'(x^2)\, dx = g(x^2) + k$$

$$\phi_{(x,y)} = x^2 y + g(x^2) + k$$

$$\int_{(-2,4)\rightarrow(2,5)} \bar{f}\, d\bar{\lambda} = \phi_{(2,5)} - \phi_{(-2,4)} = 20 + g(4) + k - \left(16 + g(4) + k\right) = \boxed{4}$$