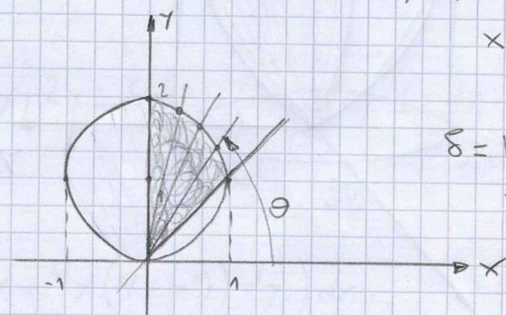
1) Hallar la masa de la lamina plana limitada por $y=x$; $x=0$
$x^2+y^2-2y=0$ si la densidad en c/punto es proporcional a la distancia al eje x



$x^2+(y-1)^2=1$

$\delta = K|y|$ pero como $y \geq 0 \Rightarrow |y| = y$

$$\left.\begin{array}{l} x = \rho\cos\theta \\ y = \rho \operatorname{sen}\theta \end{array}\right\} \rho^{\cancel{2}} = 2\cancel{\rho}\operatorname{sen}\theta$$

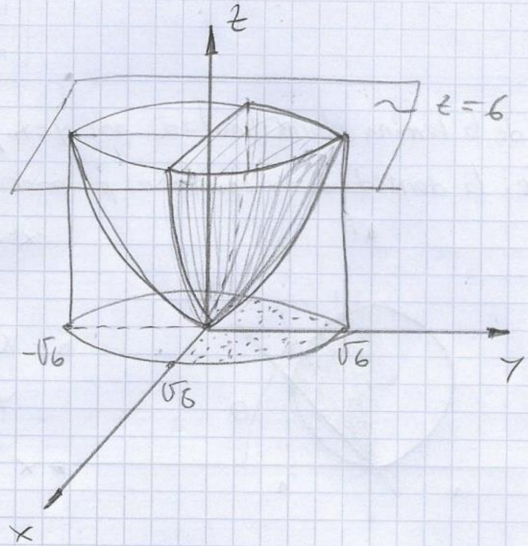
$$M = \int_{\pi/4}^{\pi/2}\left(\int_0^{2\operatorname{sen}\theta} K\cdot\overbrace{\rho\operatorname{sen}\theta}^{y}\,\rho\, d\rho\right)d\theta = \int_{\pi/4}^{\pi/2} K\operatorname{sen}\theta\,\frac{1}{3}\rho^3\Big|_0^{2\operatorname{sen}\theta} d\theta = \frac{K8}{3}\int_{\pi/4}^{\pi/2}\operatorname{sen}^4\theta\, d\theta$$

$$M = \frac{8K}{3}\left[\frac{3}{8}\theta - \frac{\operatorname{sen}2\theta}{4} + \frac{\operatorname{sen}4\theta}{32}\right]_{\pi/4}^{\pi/2} = \frac{8K}{3}\left[\frac{3\pi}{16} - \frac{3\pi}{32} - \left(0 - \frac{1}{4}\right) + 0\right] =$$

$$M = \frac{8k}{3}\left[\frac{3\pi}{32} + \frac{1}{4}\right]$$

2) Calcule el flujo de $\bar{f}(x,y,z) = (3y^6z^2,\ y+xz^2,\ 3x+2y)$ a traves de la superficie frontera del solido determinado por:
$x^2+y^2 \leq 6 \ \wedge\ z \geq x^2+y^2 \ \wedge\ y \geq 0$

2

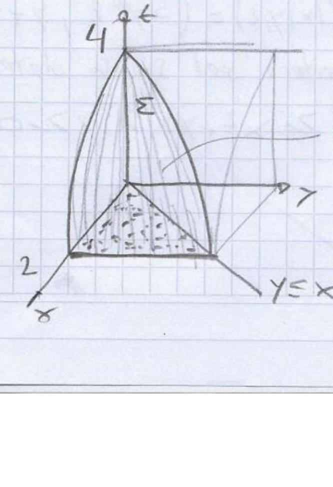


$$\operatorname{div}\bar{f} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 0 + 1 + 0$$

$$\phi_T = \int_0^{\pi} \left(\int_0^{\sqrt{6}} \left(\int_{\rho^2}^{6} \rho \, dz \right) d\rho \right) d\theta = \pi \int_0^{\sqrt{6}} \rho(6-\rho^2)\, d\rho = \pi \left[\frac{6\rho^2}{2} - \frac{\rho^4}{4} \Big|_0^{\sqrt{6}} \right]$$

$$\phi_T = \pi \left[\frac{36}{2} - \frac{36}{4} \right] \Rightarrow \phi_T = 9\pi$$

3) Siendo $f(x,y,z) = (2xy, 2yz, 4yz)$, calcule el flujo a traves de la superficie abierta de ecuación $z = 4 - x^2$ con $y \leq x$ en el 1er octante. Indique graficamente que orientación adopto para Σ



$F(\bar{x}) = x^2 + z - 4$

$\bar{\nabla} F = (2x, 0, 1)$

HOJA N°

FECHA

③

$$\phi = \iint_{\Sigma} \bar{f} \cdot \breve{n}\, d\Sigma \Rightarrow \phi = \iint \bar{f} \cdot \bar{\nabla} F \cdot \frac{dx\,dy}{|F'_z|} \Rightarrow$$

$$\phi = \int_0^2 \int_0^x (2xy, 2yz, 4yz) \cdot (2x, 0, 1)\, \frac{dx\,dy}{1}$$

$$4 - x^2$$

$$\phi = \int_0^2 \left(\int_0^x (4x^2y + 4y(4 - x^2))\, dy \right) dx$$

$$\phi = \int_0^2 \left(\int_0^x (4x^2y + 16y - 4yx^2)\, dy \right) dx$$

$$\phi = \int_0^2 \left(\int_0^x (16y)\, dy \right) dx$$

$$\phi = \int_0^2 \left(\frac{16y^2}{2} \Big|_0^x \right) = \int_0^2 8x^2\, dx$$

$$\phi = \frac{8x^3}{3} \Big|_0^2 \Rightarrow \boxed{\phi = \frac{64}{3}}$$

4) Sea $f(x, y, z) = (xy; y^3; yz)$ y sea C la curva definida por la intersección de las superficies de ecuaciones $z = x + y^2$, con $x = y^2$. Calcular la circulación de f desde $(1; 1; 2)$ hasta $(4; 2; 8)$

NOTA

(4)

parametrizaremos la curva $\bar{\gamma}(t)$ tomando $y=t \Rightarrow$

$x=t^2$; $z=t^2+t^2=2t^2 \Rightarrow \bar{\gamma}(t)=(t^2;t;2t^2)$

$\bar{\gamma}'(t)=(2t;1;4t)$

$f(\bar{\gamma}(t))=(t^2\cdot t;t^3;t\cdot 2t^2)=(t^3;t^3;2t^3)$

$f(\bar{\gamma}(t))\cdot\bar{\gamma}'(t)=(t^3;t^3;2t^3)\cdot(2t;1;4t)=2t^4+t^3+8t^4$

$f(\bar{\gamma}(t))\cdot\bar{\gamma}'(t)=10t^4+t^3$

$$I=\int_1^2 (10t^4+t^3)\,dt=\frac{10}{5}t^5+\frac{t^4}{4}\Big|_1^2=\left(2\cdot 2^5+\frac{2^4}{4}\right)-\left(2\cdot 1^5+\frac{1^4}{4}\right)$$

$$=68-(9/4)=$$

$$\boxed{I=\frac{263}{4}}$$

NOTA