4- Derivabilidad - Recta Tangente y plano normal

1) a) $S_1: y = x^2 \quad S_2. y+z=5 \quad \overline{A} = (2,4,1)$

$$\begin{cases} y = x^2 \\ y+z=5 \end{cases} \qquad \begin{cases} x = t \\ y = t^2 \\ z = 5-t^2 \end{cases} \quad \begin{aligned} f_{(t)} &= (t; t^2; 5-t^2) \Rightarrow \overline{A} = f_{(2)} \\ f'_{(t)} &= (1; 2t; -2t) \Rightarrow f'_{(2)} = (1;4;-4) \end{aligned}$$

$y+z=5$ → Esplano

Ecuac recta tg = $(1;4;-4)\lambda + (2,4,1)$

Ecuac plano normal = $x + 4y - 4z + D = 0$

$x \to 2,\ y \to 4,\ z \to 1 \Rightarrow D = -14 \Rightarrow \boxed{x+4y-4z=14}$

b) $$\begin{cases} z = x^2-y^2 \\ z = x+y \end{cases} \qquad (x+y)(x-y) = (x+y) \qquad x-y=1 \quad x=y+1 \qquad \begin{cases} x = t+1 \\ y = t \\ z = 2t+1 \end{cases} \quad f_{(t)} = (t+1; t; 2t+1) \quad \overline{A} = (2,1,3)$$

$\overline{A} = f_{(1)}$

$f'_{(t)} = (1;1;2) \quad f'_{(1)} = (1,1,2)$

Ecuac recta tg = $(1,1,2)\lambda + (2,1,3)$

Ecuac plano normal = $x + y + 2z + D = 0$

$1 + 1 + 4 + D = 0 \Rightarrow D = -6$

$\boxed{x+y+2z=6}$

c) $$\begin{cases} x^2+y^2+z^2 = 8 \\ z = \sqrt{x^2+y^2} \end{cases} \quad \overline{A} = (0,2,2) \quad \begin{aligned} z^2 &= x^2+y^2 \\ z &= 2 \end{aligned} \quad \begin{aligned} 2x^2+2y^2 &= 8 \\ x^2+y^2 &= 4 \end{aligned} \quad C: \begin{cases} x^2+y^2=4 \\ z=2 \end{cases}$$

$$\begin{cases} x = 2\cos t \\ y = 2\,\text{sen}\,t \\ z = 2 \end{cases} \quad f_{(t)} = (2\cos t; 2\,\text{sen}\,t; 2) \qquad f'_{(\pi/2)} = (-2,0,0)$$

$f'_{(t)} = (-2\,\text{sen}\,t; 2\cos t; 0) \Rightarrow \begin{aligned} 2\cos t &= 0 \\ 2\,\text{sen}\,t &= 2 \end{aligned} \quad t = \pi/2$

Ecuac recta tg = $(-2,0,0)\lambda + (0,2,2)$

Ecuac plano normal = $-2x = 0 \Rightarrow \boxed{x=0}$ No tiene D pq pasa por el origen

. Esta contenida en el plano $z=2$ (es plana)

2) $C = \overline{X} = (u^2; u-2; u+3) \quad P = (9,1,6) \qquad u^2 = 9$

$u-2 = 1 \quad u = 3$

$\overline{X}' = (2u; 1; 1)$

$\overline{X}'_{(3)} = (6;1;1)$ recta tg : $(6;1;1)\lambda + (9,1,6)$

a) $$\begin{cases} x = 6\lambda+9 \\ y = \lambda+1 \\ z = \lambda+6 \end{cases} \Rightarrow \left.\begin{aligned} 6\lambda &= -9 \Rightarrow \lambda = -3/2 \\ \lambda &= -1 \end{aligned}\right\}$$ Absurdo no corta el eje z

reemplazo

Ptos de intersección

$P_1 = (15,2,7) \quad P_2 = (6; 1/2; 11/2)$

b) $z = x - 2y^2$

$(\lambda+6) = (6\lambda+9) - 2(\lambda+1)^2$

$\lambda - 6\lambda = 3 - 2(\lambda^2+2\lambda+1) \Rightarrow -5\lambda - 3 + 2\lambda^2 + 4\lambda + 2 = 0$

$2\lambda^2 - \lambda - 1 = 0$

$(\lambda = 1 \quad \lambda = -1/2$

③ $\overline{X} = (2\cos(t); 2\operatorname{sen}(t), t)$ $t \in [0; 2\pi]$ gráfico hecho en unidad 2 $\Rightarrow$ es como una hélice que aumenta en altura $\Rightarrow$ se ve a simple vista que es alabeada

$t=0 \Rightarrow \overline{X}(0) = (2,0,0)$ (1) formo dos directores y obtengo la normal

$t=\pi/2 \Rightarrow \overline{X}(\pi/2) = (0;2;\pi/2)$ (2) $(1)-(3) = (4;0;-\pi)$

$t=\pi \Rightarrow \overline{X}(\pi) = (-2,0;\pi)$ (3) $(2)-(1) = (-2;2;\pi/2)$ $\breve{m} = \begin{vmatrix} i & j & k \\ 4 & 0 & -\pi \\ -2 & 2 & \pi/2 \end{vmatrix} = (2\pi;0;8)$

Plano π: $2\pi x + 8z = 4\pi$ (USANDO EL PTO (2,0,0))

Sustituyo C en π $4\pi\cos t + 8t = 4\pi \Rightarrow$ como la ecuac no se verifica $\forall t \in [0;2\pi] \Rightarrow$ la curva no está contenida en el plano, es alabeada.

④ a) $f(x,y) = x^4 + 2xy + xy^3 + 1$

$f'_x = 4x^3 + 2y + y^3$ $f'_y = 2x + 3y^2 x$

b) $f(x,y,z) = ye^{2x} + ze^{3y}$

$f'_x = ye^{2x}2$ $f'_y = e^{2x} + ze^{3y}.3$ $f'_z = e^{3y}$

c) $f(x,y) = xe^{x^2+y^2}$

$f'_x = e^{x^2+y^2} + xe^{x^2+y^2}.2x = e^{x^2+y^2}(1+2x^2)$ $f'_y = xe^{x^2+y^2}2y$

Mas de lo mismo derivar y comparar resultados

⑤ a) $f(x,y) = 3x^2 + 2xy$ $\overline{A} = (1,2)$

$$f'_x(1,2) = \lim_{h\to0} \frac{f(1+h;2) - f(1,2)}{h} = \lim_{h\to0} \frac{3(1+h)^2 + 4(1+h) - 7}{h} = \lim_{h\to0} \frac{3h^2 + 6h + \cancel{3} + 4h - \cancel{3}}{h} = \frac{\cancel{h}(\underset{\to 0}{3h}+10)}{\cancel{h}} = \boxed{10}$$

$$f'_y(1,2) = \lim_{h\to0} \frac{f(1;2+h) - f(1,2)}{h} = \lim_{h\to0} \frac{3 + 2(2+h) - 7}{h} = \lim_{h\to0} \frac{2\cancel{h}}{\cancel{h}} = \boxed{2}$$

b) $f(x,y) = \begin{cases} x^2 + 2y & \text{si } x > 1 \\ 2x - y & \text{si } x < 1 \end{cases}$ $\overline{A} = (1,1)$

$$f'_x(1,1) = \lim_{h\to0} \frac{f(1+h;1) - f(1,1)}{h}$$

$$\lim_{h\to0^+} \frac{(1+h)^2 + 2 - 3}{h} = \lim_{h\to0^+} \frac{h^2 + 2h + \cancel{1} - \cancel{1}}{h} = \frac{\cancel{h}(h+2)}{\cancel{h}} = \boxed{2}$$

$$\lim_{h\to0^-} \frac{2(1+h) - 1 - 3}{h} = \frac{2 + 2h - 4}{h} = \boxed{\infty} \quad \therefore \nexists f'_x(1,1)$$

x=1 el lim no se divide en $+y-$

$$f'_y(1,1) = \lim_{h\to0} \frac{f(1;1+h) - f(1,1)}{h} = \lim_{h\to0} \frac{\cancel{1} + 2 + 2h - \cancel{3}}{h} = \frac{2\cancel{h}}{\cancel{h}} = \boxed{2}$$

$h^{1/2} = \sqrt[2]{h^1}$ $\sqrt{h^4} = h^{4/2}$

c) $f(x,y) = \sqrt{x^4 + 2y^2}$ $\overline{A} = (0,0)$

$$f'_x(0,0) = \lim_{h\to0} \frac{f(h;0) - f(0,0)}{h} = \lim_{h\to0} \frac{\sqrt{h^4} - 0}{h} = \frac{h^{\cancel{2}}}{\cancel{h}} = \boxed{0}$$

$$f'_y(0,0) = \lim_{h\to0} \frac{f(0;h) - f(0,0)}{h} = \lim_{h\to0} \frac{\sqrt{2h^2}}{h} = \frac{\sqrt{2}\,\overset{\to 0}{|h|}}{\underset{\to 0}{h}} = \nexists$$

d) $f(x,y) = |x| + |y|$ $\overline{A} = (0,0)$

$$f'_x(0,0) = \lim_{h\to0} \frac{f(h;0) - f(0)}{h} = \lim_{h\to0} \frac{|h| - 0}{h} = \nexists f'_x(0,0)$$ paso lo mismo con $f'_y(0,0)$

6) $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{si } (x,y) \neq (0,0) \\ 0 & \text{si } (x,y) = (0,0) \end{cases}$

a) $y = ax \quad \lim\limits_{x \to 0} \dfrac{x^2 a}{x^2 + a^2 x^2} = \dfrac{a \cancel{x^2}}{\cancel{x^2}(1+a^2)} = \dfrac{a}{1+a^2}$ como quedo en función de $a \Rightarrow \nexists \lim \Rightarrow f$ no es continua en $(0,0)$

b) $f'(\bar{0}, \breve{v}) = \lim\limits_{h \to 0} \dfrac{f(\bar{0} + h\breve{v}) - f(0,0)}{h} = \lim\limits_{h \to 0} \dfrac{f(ha;hb) - 0}{h} = \lim\limits_{h \to 0} \dfrac{\frac{ha.hb}{h^2a^2+h^2b^2}}{h} = \dfrac{\cancel{h^2}ab}{\cancel{h^2}\underbrace{(a^2+b^2)}_{1}} \cdot \dfrac{1}{h}$

$\breve{v} = (a;b)$ con $a^2+b^2=1$

$\lim\limits_{h \to 0} \dfrac{ab}{h} = \begin{cases} 0 & \text{si } ab = 0 \\ \infty & \text{si } ab \neq 0 \end{cases}$ $\Rightarrow f$ es derivable en la dirección de los versores canonicos $(1,0)$, $(-1,0)$, $(0;1)$ y $(0;-1)$ y en este caso el valor de la derivada es 0.

7) $g(x,y) = \begin{cases} \dfrac{x^3 y}{x^6 + y^2} & \text{si } (x,y) \neq (0,0) \\ 0 & \text{si } (x,y) = (0;0) \end{cases}$ Verificamos discontinuidad

$y = x^3 \Rightarrow \lim\limits_{x \to 0} \dfrac{x^6}{2x^6} = \dfrac{1}{2}$

$y = 0 \Rightarrow \lim\limits_{x \to 0} \dfrac{0}{x^6} = \boxed{0}$

$\neq \Rightarrow$ Es discontinua escencial.

$f'(\bar{0}; \breve{v}) = \lim\limits_{h \to 0} \dfrac{f(ha;hb) - f(0,0)}{h} = \lim\limits_{h \to 0} \dfrac{h^3a^3 . hb}{h^6a^6 + h^2b^2} = \dfrac{h^{\cancel{4}\,3} a^3 b}{h^6a^6+h^2b^2} \cdot \dfrac{1}{\cancel{h}} = \dfrac{h^3 a^3 b}{\cancel{h^2}(h^4a^6+b^2)} = \dfrac{0}{b^2} = \boxed{0}$

8) $h: \mathbb{R}^2 \to \mathbb{R} / h(x,y) = x^{1/3} y^{1/3}$

a) $f'_x(0,0) = \lim\limits_{h \to 0} \dfrac{f(h;0) - f(0,0)}{h} = \dfrac{0}{h} = \boxed{0}$ con $f'_y(0,0)$ pasa lo mismo

$\longrightarrow h$ no es 0 sino que tiende a 0

$f'(\bar{0}, \breve{v}) = \lim\limits_{h \to 0} \dfrac{f(ha;hb) - f(0,0)}{h} = \lim\limits_{h \to 0} \dfrac{\cancel{h^{1/3}} a^{1/3} \cancel{h^{1/3}} b^{1/3}}{h^{1/3}} = \dfrac{a^{1/3} b^{1/3}}{h^{1/3}} = \dfrac{ab}{h} = \begin{cases} 0 & ab = 0 \\ \infty & ab \neq 0 \end{cases}$

verifico que f solo es derivable en la dirección de los vectores canonicos

b) $f(\bar{0}) = 0 \quad \lim\limits_{(x,y) \to (0,0)} x^{1/3} y^{1/3} = 0 \Rightarrow \lim\limits_{(x,y) \to (\bar{0})} h(x,y) = f(\bar{0}) \Rightarrow$ la funcion es continua

9) $f: \mathbb{R}^2 \to \mathbb{R} / f(x,y) = \sqrt{4x^2 + y^2} \Rightarrow$ analizo continuidad $f(\bar{0}) = 0 \quad \lim\limits_{(x,y) \to (\bar{0})} \overbrace{\sqrt{4x^2+y^2}}^{\to 0} = \boxed{0}$ Es continua

$f'_x(0,0) = \dfrac{f(h;0) - f(\bar{0})}{h} = \lim\limits_{h \to 0} \dfrac{\sqrt{4h^2} - 0}{h} = \lim\limits_{h \to 0} \dfrac{2|h|}{h} = \begin{cases} \lim\limits_{h \to 0^+} 2\dfrac{h}{h} = \boxed{2} \\ \lim\limits_{h \to 0^-} 2\dfrac{-h}{h} = \boxed{-2} \end{cases} \neq \Rightarrow \nexists \lim \Rightarrow \nexists f'_x(0,0)$

$f'_y(0,0) = \dfrac{f(0;h) - f(\bar{0})}{h} = \lim\limits_{h \to 0} \dfrac{\sqrt{h^2} - 0}{h} = \lim\limits_{h \to 0} \dfrac{|h|}{h} \Rightarrow \nexists \lim \Rightarrow \nexists f'_y(0,0)$

(10) $f(x,y) = 1 - x^2 + 2y^2 \quad \bar{A} = (1,-1) \quad \bar{v} = (3;4) \Rightarrow \breve{v} = \left(\frac{3}{5}; \frac{4}{5}\right)$

$\frac{k}{h} = \frac{4}{3} \Rightarrow k = \frac{4}{3}h \quad f'_{\bar{v}}(1;-1) = \frac{f(1+h;-1+k) - f(1;-1)}{\sqrt{h^2+k^2}} = \lim_{\sqrt{h^2+k^2}\to 0} \frac{1-(1+h)^2+2(-1+k)^2-2}{\sqrt{h^2+k^2}}$

$\lim_{\sqrt{h^2+\left(\frac{4}{3}h\right)^2}\to 0} = \frac{\cancel{1}-h^2-2h-\cancel{1}+2(-1+4/3h)^2-2}{\sqrt{h^2+\left(\frac{4}{3}h\right)^2}} = \lim_{\sqrt{\frac{25}{9}h^2}\to 0} \frac{-h^2-2h+32/9h^2-16/3h+\cancel{2}-\cancel{2}}{\frac{5}{3}h}$

$\lim_{h\to 0} \frac{23/9h^2-22/3h}{5/3h} = \lim_{h\to 0} \frac{\cancel{h}(\overset{\to 0}{23/9h}-22/3)}{\cancel{h}\,5/3} = \frac{-22/3}{5/3} = \boxed{\frac{-22}{5}}$

* fijar mejor concepto

Otra forma

$f'\left((1;-1)\left(\frac{3}{5};\frac{4}{5}\right)\right) = \lim_{h\to 0} \frac{f((1;-1)+h(3/5;4/5)) - f(1;-1)}{h} = \lim_{h\to 0} \frac{f(\overbrace{1+3/5h}^{x};\overbrace{-1+4/5h}^{y}) - f(1;-1)}{h} =$

$\lim_{h\to 0} \frac{1-(1+3/5h)^2+2(-1+4/5h)^2-2}{h} = \lim_{h\to 0} \frac{1-\left(1+\frac{6}{5}h+\frac{9}{25}h^2\right)+2\left(1-\frac{8}{5}h+\frac{64}{25}h^2\right)-2}{h}$

$\lim_{h\to 0} \frac{\cancel{1}-\cancel{1}-6/5h-9/25h^2+\cancel{2}-16/5h+\frac{128}{25}h^2-\cancel{2}}{h} = \frac{\cancel{h}\left(-\frac{22}{5}+\frac{119}{25}h\right)}{\cancel{h}} = \boxed{\frac{-22}{5}}$

(11) (a) $f(x,y) = \begin{cases} \frac{xy-x}{x^2+(y-1)^2} & \text{si } (x,y) \neq (0,1) \\ 0 & \text{si } (x,y) = (0,1) \end{cases} \quad \bar{A} = (0;1)$

$f'((0,1),(a;b)) = \lim_{h\to 0} \frac{f(\overset{x}{ah};\overset{y}{1+bh}) - f(0,1)}{h} = \lim_{h\to 0} \frac{(ah)(1+bh-1)}{(ah)^2+(1+bh-1)^2}$

$\lim_{h\to 0} \frac{(ah)(bh)}{a^2h^2+b^2h^2} = \frac{\cancel{h^2}ab}{\cancel{h^2}\underbrace{(a^2+b^2)}_{1}} = ab$ Es derivable en la dirección de los versores canónicos.

(b) $f(x,y) = \begin{cases} \frac{y^2}{x} & \text{si } (x,y) \neq (0;y) \\ 0 & \text{si } (x;y) = (0;y) \end{cases} \quad \bar{A} = (0;0)$

$f'((0,0);(a;b)) = \lim_{h\to 0} \frac{f'(ah;bh) - f(0;0)}{h} = \lim_{h\to 0} \frac{\frac{h^2b^2}{ha} - 0}{h} = \lim_{h\to 0} \frac{\cancel{h^2}b^2}{\cancel{h}a}\cdot\frac{1}{\cancel{h}} = \begin{cases} \frac{b^2}{a} & \text{si } a \neq 0 \\ \infty & \text{si } a = 0 \end{cases}$

(12) (a) $f(x,y) = \ln(x^2+y^2)$

$f'_x = \frac{1}{x^2+y^2}\,2x$

$f'_y = \frac{2y}{x^2+y^2}$

$1°$ y $2°$ orden Dom $\mathbb{R}^2 - \{\bar{0}\}$

$f'_{xx} = \frac{x(x^2+y^2) - 4x^2}{(x^2+y^2)^2}$

$f'_{yy} = \frac{y(x^2+y^2) - 4y^2}{(x^2+y^2)^2}$

(b) $\bar{F}_{(x;y)} = (x\ln y ; \cancel{y}x)$

$F'_x = (\ln y ; \frac{-y}{x^2})$ $F'_y = (\frac{x}{y} ; \frac{1}{x})$ $F''_{xy} = (\frac{1}{y} ; \frac{-1}{x^2}) = F''_{yx}$ $F''_{xx} = (0 ; \frac{2y}{x^3})$ $F''_{yy} = (-\frac{x}{y^2} ; 0)$

Dom de deriv parciales 1° y 2° orden $/ x \neq 0 \wedge y > 0$

(c) $f_{(x,y)} = \frac{x^2}{x^2+y^2}$ si $(x,y) \neq (0;0)$

$f'_x = \frac{2xy^2}{(x^2+y^2)^2}$

$\to f''_{xx} = \frac{2y^2(x^2+y^2)^2 - 2xy^2 \cdot 2(x^2+y^2)2x}{(x^2+y^2)^4}$

$\to f''_{xy} = \frac{4xy(x^2+y^2)^2 - 2xy^2\, 2(x^2+y^2)2y}{(x^2+y^2)^4}$

$f'_y = \begin{cases} \frac{-2yx^2}{(x^2+y^2)^2} & \text{si } (x;y) \neq (0;0) \\ 0 & \text{si } (x;y) = (0;0) \end{cases}$ f'_x, f''_{xx}, f''_{xy} Dom: $\mathbb{R}^2 - \{\bar{0}\}$; f'_y, f''_{yx}, f''_{yy} Dom $\mathbb{R}^2$

(13) (a) $\bar{X} = (3t^2 ; (2-t) ; 2t^2)$ con $t \geq 0$ $S: x = y^2 + z$

$\bar{X}(t)$ = posicion

$\dot{\bar{X}}(t)$ = velocidad $= (6t ; -1 ; 4t)$

$\ddot{\bar{X}}(t) = (6;0;4)$

Instante en que la particula atraviesa a S (Sustituyo en S x, y, z en terminos de t)

$x = 3t^2$
$y = 2-t$
$z = 2t^2$

$3t^2 = (2-t)^2 + 2t^2 \Rightarrow 0 = 4 - 4t$ $\boxed{t=1}$ *

angulo $\Rightarrow \cos(\hat{\bar{v}\,\bar{a}}) = \frac{(6;-1;4)(6;0;4)}{\|(6;-1;4)\|\|(6;0;4)\|} = \frac{52}{\sqrt{53}\sqrt{52}}$ $\hat{\bar{v}\,\bar{a}} = \arccos\left(\frac{52}{\sqrt{53}\sqrt{52}}\right) = 11° 8' 51''$

(b) $3t^2 - 2t^2 + 2(2-t) = 7 \Rightarrow t^2 - 2t - 3 = 0$ $\begin{cases} \cancel{t=-1} \\ t = 3 \end{cases}$ $t > 0$

Tiempo $t = 3 - 1^* = 2$ segundos.

(18) $\bar{X}_1(t) = (t+1 ; g(t) ; 1+t^2)$ $\bar{X}_2(t) = (2t ; g'(t) ; 2t^2)$ $\bar{X}_1(1) = \bar{X}_2(1) = (2,2,2)$

Los vectores son paralelos cuando son proporcionales

$\bar{X}'_1(t) = k\bar{X}'_2 \Rightarrow (1 ; g'(t) ; 2t) = k(2 ; g''(t) ; 4t)$ $\forall t \geq 0$

$1 = 2k$
$g'(t) = kg''(t)$ $k = \frac{1}{2}$
$2t = 4kt$

$g'(t) = \frac{1}{2}g''(t)$ $w = g'(t)$
$2g'(t) = g''(t)$ $w' = g''(t)$

$2w - w' = 0 \Rightarrow \int 2\,dt = \int \frac{1}{w} w'\,dw \Rightarrow w(t) = Ae^{2t} \Rightarrow g'(t) = Ae^{2t} \Rightarrow g'(1) = Ae^2 = 2$

$g(t) = \int Ae^{2t}dt \Rightarrow g(t) = \frac{A}{2}e^{2t} + C \Rightarrow \bar{X}_{(1)} = (2;2;2) \Rightarrow g(1) = 2$ y $g'(1) = 2$ $A = e^{-2}$

$g(1) = e^{-2}e^2 = 1 + C = 2 \Rightarrow C = 1 \Rightarrow \boxed{g(t) = e^{2t-2} + 1}$

(19) $y = (x-at)^2 + (x+at)^2 \quad a = cte \quad y \quad \frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

$y'_t = 2(x-at).(-a) + 2(x+at)(a)$

$= -2a(x-at) + 2a(x+at) = (-2a)(\cancel{x}-at) - (\cancel{x}+at)) = -2a(-2at) = 4a^2 t$

$y''_{tt} = 4a^2$

$y'_x = 2(x-at) + 2(x+at) = 2x - 2at + 2x + 2at = 4x$

$y'_{xx} = 4$ si multiplico por a^2 verifica $y''_{tt} = a^2 y''_{xx}$

(20) $z = sen(x)\, senh(y)$ satisface $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

$z'_x = senh(y)\cos(x) \qquad z'_y = sen(x)\cosh(y)$

$z''_{xx} = -sen(x)\, senh(y) \qquad z''_{yy} = sen(x)\, senh(y)$

$-sen(x)\, senh(y) + sen(x)\, senh(y) = 0$ Verifica

(21)

Unidad 5 - Diferenciabilidad - Plano Tangente y recta normal

1 a) $\bar{f}(t) = \left(t^3 - 2 ; \frac{t^2-1}{t+1} ; \frac{\cos(t)}{2t-\pi}\right)$

$$\frac{2t(t+1)-(t^2-1)}{(t+1)^2} = \frac{\cancel{2}t^2+2t-\cancel{t^2}+1}{t^2+2t+1} = 1$$

$$D\bar{f}_{(t)} = \begin{pmatrix} 3t^2 \\ 1 \\ \frac{-2\cos(t)}{(2t-\pi)^2} - \frac{\operatorname{sen}(t)}{2t-\pi} \end{pmatrix} \quad W / t \neq -1 \wedge t \neq \pi/2$$

b) $f(x,y) = \frac{xy}{x^2+y^2}$ $\quad Df(x,y) = \left(\frac{y^3-x^2y}{(x^2+y^2)^2} \quad \frac{x^3-xy^2}{(x^2+y^2)^2}\right)$ $\quad W = \mathbb{R}^2 - \{\bar{0}\}$

$f'_x = \frac{y(x^2+y^2)-2x^2y}{(x^2+y^2)^2}$ $\quad f'_y = \frac{x(x^2+y^2)-2y^2x}{(x^2+y^2)^2}$

Todos iguales derivar parcialmente y armar matriz

2) $f(x,y) = \begin{cases} \sqrt{xy} & \text{si } xy \geq 0 \\ x & \text{si } xy < 0 \end{cases}$

$$f'((0;0),(2;-2)) = \frac{f(2h;-1h)-f(0;0)}{h} = \frac{2\cancel{h}-0}{\cancel{h}} = \boxed{2}$$

3) $f(x,y) = \frac{x^2}{y}$

$$f'((0;0);(a;b)) = \frac{f(ah;bh)-f(0;0)}{h} = \frac{a^2h^{\cancel{2}}}{b\cancel{h}} \cdot \frac{1}{\cancel{h}} = \frac{a^2}{b} = \begin{cases} \frac{a^2}{b} & \text{si } b \neq 0 \\ 0 & \text{si } b = 0 \end{cases}$$

$0 \Uparrow$ — $f(x,0) = 0$

No es continua en $0 \Rightarrow$ no es diferenciable

4 a) $f(x,y) = \begin{cases} \frac{x^4}{x+y} & \text{si } x+y \neq 0 \\ 0 & \text{si } x+y = 0 \end{cases}$ $\quad f$ no es continua en $(0,0)$

$$f'((0,0);(a;b)) = \lim_{h\to 0} \frac{f((0,0)+h(a;b))-f(0,0)}{h} = \lim_{h\to 0} \frac{(ah)^4}{(ah+bh)} \cdot \frac{1}{h} = \frac{\cancel{h^2}\, a\overset{\to 0}{h^2}}{\cancel{h^2}(a+b)} = \boxed{0}$$

f es derivable en todas direcciones y su derivada vale 0 pero no es diferenciable ya que no es continua en $(0;0)$

b) $f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & \text{si } (x,y) \neq (0;0) \\ 0 & \text{si } (x,y) = (0,0) \end{cases}$

$$f'((0,0);(a;b)) = \frac{f(ah;bh)-f(0,0)}{h} = \lim_{h\to 0} \frac{a^2bh^3}{(ah)^2+(bh)^2} \cdot \frac{1}{h} = \frac{\cancel{h^3}a^2b}{\cancel{h^3}\underbrace{(a^2+b^2)}_{1}} = a^2b$$

f es derivable en todas direcciones en $(0;0)$ y se verifica $f'(\bar{0};\breve{v}) = a^2b \;\forall \breve{v}$

$f'(\bar{o};\check{v})=0 \iff a^2b=0$ $\begin{cases} a=0 \begin{cases} v_1=(0;1) \\ v_2=(0;-1)\end{cases} \\ b=0 \begin{cases} v_3=(1;0) \\ v_4=(-1;0)\end{cases}\end{cases}$

Existen 4 direcciones de derivada direccional nula de f en P_0, $\therefore$ f no es diferenciable en $(0;0)$

funcion a optimizar: $f'(\bar{o};\check{v})=a^2b$ condicion: $a^2+b^2=1 \Rightarrow a^2=1-b^2 \Rightarrow |a|=\sqrt{1-b^2}$ con $-1\leq b\leq$

Ecuac a maximizar $\varphi(b)=(1-b^2)b$ con $b\in[-1;1]$

$\varphi'(b)=1-3b^2$ ptos criticos $1-3b^2=0 \Rightarrow |b|=\frac{1}{\sqrt{3}}$ $\begin{cases} b_1=\frac{1}{\sqrt{3}} \text{ P.C} \\ b_2=-\frac{1}{\sqrt{3}} \text{ P.C}\end{cases}$

$\varphi(1/\sqrt{3})=\frac{2}{3\sqrt{3}}$ $\Rightarrow$ maximo valor de derivada direccional de f en $(0;0)$ $\Rightarrow v_1=\left(\sqrt{\frac{2}{3}};\frac{1}{\sqrt{3}}\right)$ $v_2=\left(-\sqrt{\frac{2}{3}};\frac{1}{\sqrt{3}}\right)$

$\varphi(-1/\sqrt{3})=\frac{-2}{3\sqrt{3}}$

5) Para que f admita plano tangente $\iff$ f es diferenciable

Se ve a simple vista que f no es continua en el origen $\Rightarrow$ f no es diferenciable $\Rightarrow$ f no admite plano tangente.

6) $z=e^{(x-1)^2}+y^2$ plano Tg horizontal si $f'_x(P_0)=f'_y(P_0)=0$ y $\underline{\text{gradiente}=0}$

$z_T=f(P_0)+\cancel{f'_x(P_0)}^{0}(x-x_0)+\cancel{f'_y(P_0)}^{0}(y-y_0)$

$z_T=e^{(x-1)^2}+y^2$

$f'_x=e^{(x-1)^2}2(x-1)=0 \Rightarrow x=1$

$f'_y=2y=0 \Rightarrow y=0$ $\quad z=1$ Unico pto $=(1,0,1)$

8) a) $f(1{,}96;0{,}96)$ cuando $f(x,y)=\sqrt{23-2x^2-y^2}$ Elijo $P_0=(2;1)$

$f'_x(\bar{x})=\frac{-4x}{2\sqrt{23-2x^2-y^2}}=\frac{-2x}{\sqrt{23-2x^2-y^2}}$

$f'_y(\bar{x})=\frac{-\cancel{2}y}{\cancel{2}\sqrt{23-2x^2-y^2}}$ ambos en $(2;1)$ no se anulan $\Rightarrow f\in C^1\ \forall(x,y)\in E(2;1)$

$f(\underset{x_0+h}{1{,}96};\underset{y_0+k}{0{,}96})\cong f(2;1)+df(2;1)$

$\cong 4+f'_x h+f'_y k$

$\cong 4+(-1)(-0{,}04)+\left(\frac{1}{4}\right)(-0{,}04)$

$\boxed{f(1{,}96;0{,}96)\cong 4{,}05}$

b) $f(0{,}99;1{,}98;1{,}02)$ $f(x,y,z)=xy+\operatorname{sen}(e^{2x-y+3z-3}-1)$

$f(P_0+H)\cong f(P_0)+\underbrace{df(P_0)}_{\nabla f(P_0)\,\bar{H}}$ en $E(P_0)$

Propongo $P_0=(1,2;1)$ $H=(-0{,}01;-0{,}02;0{,}02)$

$f'_x(x,y,z)=y+\cos(e^{2x-y+3z-3}-1)e^{2x-y+3z-3}.2 \Rightarrow f'_x(1,2,1)=4$

$f'_y(x,y,z)=x+\cos(e^{2x-y+3z-3}-1)e^{2x-y+3z-3}(-1) \Rightarrow f'_y(1,2,1)=0$

$f'_z(x,y,z)=\cos(e^{2x-y+3z-3}-1)e^{2x-y+3z-3}.3 \Rightarrow f'_z(1,2,1)=3$

$\nabla f(1,2,1)=(4,0,3)$

$f(0{,}99;1{,}98;1{,}02)=f(1,2,1)+f'_x(1,2,1)(-0{,}01)+f'_y(1,2,1)(-0{,}02)+f'_z(1,2,1)(0{,}02)\cong \boxed{2{,}02}$

10) $\begin{cases} x=u-v^2 \\ y=v^2/u \\ z=u/v \end{cases}$ $yz=\frac{v^2}{u}\frac{u}{v}=v$ $u=yz^2$ $f(y;z)=yz^2-(yz)^2$

$f'_y=z^2-2yz^2 \Rightarrow f'_y(-2,2,1)=-3$

$f'_z=2yz-2zy^2 \Rightarrow f'_z(-2;2;1)=-4$

$x_0=f(2;1)=-2$

$x_T=-3(y-2)-4(z-1)-2=-3y+6-4z+4-2$

$\boxed{x+3y+4z=8}$ plano Tg

recta normal : $(1;3;4)\lambda+(-2;2;1)$

11) $z=x^2-xy^3+x$ pto regular sino se anulan simultaneamente las derivadas parciales

$f'_x=2x-y^3+1$

$f'_y=-3y^2x$ no se anulan sus ptos son regulares Horizontal si gradiente $=0$ (idem Ej 6)

$\longrightarrow y=0 \Rightarrow x=-1/2 \Rightarrow z=-1/4 \lor x=0 \land y=1 \land z=0$

$P_1=(-1/2;0;-1/4)$ $P_2(0,1,0)$

12) $z=\sqrt{9-x^2-y^2}$ en $(1,2,z_0)$ $z_0=f(1,2)=2$ $P_0(1,2,2)$ $z=x^2$

$f'_x=\frac{-\cancel{2}x}{\cancel{2}\sqrt{9-x^2-y^2}} \Rightarrow f'_x(1,2)=-1/2$ $f'_y=\frac{-2y}{2\sqrt{9-x^2-y^2}}=-1$

plano Tg $=-1/2(x-1)-1(y-2)+2$ $z=2=x^2$

$r_0=(-1/2;-1;-1)\lambda+(1,2,2)$ $\begin{cases} x=-1/2\lambda+1 \\ y=-1\lambda+2 \\ z=-\lambda+2 \end{cases}$ $\lambda=2$ $(0;0;0)$

$\lambda=-2$ $(2;4;4)$

(13) (a) $f(x,y) = x^2 - xy^2 \quad \bar{A} = (1,3) \quad f \in C^1(\mathbb{R}^2)$ en particular $\Rightarrow f \in C^1$ en $E(\bar{A}) \therefore f$ es diferenciable en $\bar{A}$

$\bar{\nabla} f(x,y) = (2x - y^2 ; -2xy) \quad \bar{\nabla} f(\bar{A}) = (-7;-6)$

$f'_{MAX}(\bar{A}) = \|\bar{\nabla} f(\bar{A})\| = \sqrt{85} \quad \left(\frac{-7}{\sqrt{85}} ; \frac{-6}{\sqrt{85}}\right) \rightarrow$ dirección del versor

$f'_{MIN}(\bar{A}) = -\|\bar{\nabla} f(\bar{A})\| = -\sqrt{85}$ dirección $= \left(\frac{7}{\sqrt{85}} ; \frac{6}{\sqrt{85}}\right)$

La derivada direc nula de f en $\bar{A}$ se obtiene al derivar f en las direcciones $\breve{v}_1 = \left(\frac{-6}{\sqrt{85}}, \frac{7}{\sqrt{85}}\right)$ y $\breve{v}_2 = \left(\frac{6}{\sqrt{85}} ; \frac{-7}{\sqrt{85}}\right)$ direcciones perpendiculares al gradiente *

(b) $f(x,y,z) = x^2 - yz^3 \quad \bar{A} = (5,2,0)$

$\bar{\nabla} f(x,y,z) = (2x ; -z^3 ; -3z^2 y) \quad \bar{\nabla} f(5,2,0) = (10;0;0)$

$f'_{MAX} = \|\bar{\nabla} f(A)\| = 10 \qquad \breve{v}_1 = (1;0;0)$

$f'_{MIN} = -10 \qquad \breve{v}_2 = (-1,0,0)$

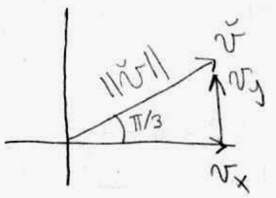
Nula todas las direcciones de la forma $(0,a,b)$ con $\overbrace{a^2+b^2}^{\text{indico que son versores.}} = 1$

(14) $g(x,y) = 3x^4 - xy + y^3 \quad g \in C^1 \Rightarrow g$ es diferenciable, en particular en $(1,2)$

$g'((1,2); \breve{v}) = \bar{\nabla} g(1,2) \cdot \breve{v}$

$\bar{\nabla} g(x,y) = (12x^3 - y ; -x + 3y^2)$

$\bar{\nabla} g(1,2) = (10 ; 11)$



$(10;11)\left(\frac{1}{2} ; \frac{\sqrt{3}}{2}\right) = \boxed{\frac{10 + 11\sqrt{3}}{2}}$

$\frac{v_x}{\|\breve{v}\|} = \cos \pi/3 \qquad v_x = \cos \pi/3 = 1/2$

$v_y = \operatorname{sen} \pi/3 = \frac{\sqrt{3}}{2}$

$\breve{v} = \left(\frac{1}{2} ; \frac{\sqrt{3}}{2}\right)$

(15) $f(x,y) = ax^2y^3 + bx^4 + 4xy$ Hallar a y b / $f'((1,-2), /\!/ y)$

$f \in C^1$ en particular $\forall (x;y) \in E(P_0) \Rightarrow f$ es diferenciable en P_0

$\bar{\nabla} f(x,y) = (2ay^3x + 4bx^3 + 4y ; 3ax^2y^2 + 4x)$ — Dirección paralela al eje y

$\bar{\nabla} f(1,-2) = (-16a + 4b - 8 ; 12a + 4) = K(0;1)$

$-16a + 4b - 8 = 0 \qquad 12a + 4 \neq 0$

$a = \frac{8}{16} - \frac{4b}{16} \qquad a \neq -\frac{1}{3}$

$a = \frac{1}{2} - \frac{1}{4}b$ con $a \neq -\frac{1}{3}$

(16) $\tilde{T}(x,y,z) = e^{2x+y+3z}$

$\overline{\nabla} f_{(x,y,z)} = (e^{2x+y+3z} . 2 ; e^{2x+y+3z} ; e^{2x+y+3z} . 3)$

$\overline{\nabla} f_{(0;0;0)} = (2; 1; 3)$

Dirección $= \dfrac{\nabla f(x,y,z)}{\|\nabla f(x,y,z)\|} = \tilde{v} = \left(\dfrac{2}{\sqrt{14}} ; \dfrac{1}{\sqrt{14}} ; \dfrac{3}{\sqrt{14}}\right)$

(17) $f'(\bar{A} ; (3,4)) = 4$ y $f'(\bar{A}\ (2,7)) = -6$

$\nabla f(\bar{A})(3,4)$

$$\begin{cases} f'_x 3 + f'_y 4 = 4 \\ f'_x 2 + f'_y 7 = 6 \end{cases}$$

$$\begin{array}{r} \cancel{6f'_x + 8f'_y = 8} \\ - \cancel{6f'_x + 21f'_y = 18} \\ \hline -13f'_y = -10 \end{array}$$

$f'_y = -2$ y $f'_x = 4$

ⓐ $\overline{\nabla} f_{(A)}(5,9) =$

$f'_x 5 + f'_y 9 = \boxed{2}$

ⓑ $\|\nabla f_{(A)}\| = 2\sqrt{5}$

ⓒ $f(\bar{A}) = 3 \Rightarrow f(\bar{A} + (\underbrace{0{,}01}_{h} ; \underbrace{-0{,}02}_{K})) = 3 + 4(0{,}01) + 2(+0{,}02) \cong 3{,}08$

(18)

$$\begin{cases} y^2 = x^2 - z^2 \\ z = x \end{cases} \Rightarrow \text{recta normal a la superficie de } z = f(x,y) \text{ en } (1,0,1)$$

$f(0{,}98 ; 0{,}01) \cong Z_T (0{,}98 ; 0{,}01)$

Ecuac Plano Tg en $(1,0,1)$

$Z_T = f(1,0) + f'_x(1,0)(x-1) + f'_y(1,0)(y-0)$

$$\begin{cases} x = t \\ y = 0 \\ z = t \end{cases} \Rightarrow \bar{X} = t(1,0,1) \text{ Direccion normal al grafico de } f \text{ en } \bar{A}$$

Ecuac Plano

$(1,0,1)(x-1 ; y-0 ; z-1) = 0$

$x - 1 + z - 1 = 0$

$x + z = 2$ Ecuac plano Tg $\Rightarrow Z = 2 - x$

$Z_T = (0{,}98 ; 0{,}01) = f(1,0) + f'_x(1,0)(x-1) + f'_y(1,0)(y)$

$= 2 - 0{,}98 \cong 1{,}02$

(19) $\Sigma = ze^{y-2x} - 5 = 0$ en $(1,2,z_0)$

Despejo $z = \dfrac{5}{e^{y-2x}} = 5\,e^{2x-y}$

$f_{(x,y)} = 5\,e^{2x-y}$ < $f'_x = 10\,e^{2x-y}$, $f'_y = -5e^{2x-y}$; $f \in C^1$ en $E(1,2) \Rightarrow f$ es dif $\Rightarrow f$ admite plano Tangente

$f(1,2) = 5$

$z_T = 5 + 10(x-1) - 5(y-2)$

$f(1{,}01\,;\,1{,}97) = 5 + 10(1{,}01-1) - 5(1{,}97-2) \cong 5{,}25$

(20) $z = f(x,y)$ en $(1,2,z_0)$

$2x + 3y + 4z = 1$ direccion $= (1,2) \to (3,4)$ Direccion $(2,2)$

Hallar $f'_{(1,2)}\left(\dfrac{(2;2)}{\sqrt{8}}\right)$

f admite plano Tg en $(1,2,z_0) \Rightarrow f$ es dif en $(1,2,z_0)$

$F'(1,2)\left(\dfrac{1}{\sqrt{2}}; \dfrac{1}{\sqrt{2}}\right) = \nabla f_{(1,2)}\left(\dfrac{1}{\sqrt{2}}; \dfrac{1}{\sqrt{2}}\right)$

$z_T = \dfrac{1}{4} - \dfrac{2}{4}x - \dfrac{3}{4}y \Rightarrow z_T = \dfrac{1}{4} \underbrace{-\dfrac{1}{2}}_{f'_x} x \underbrace{-\dfrac{3}{4}}_{f'_y} y \Rightarrow \left(-\dfrac{1}{2}; -\dfrac{3}{4}\right)\left(\dfrac{1}{\sqrt{2}}; \dfrac{1}{\sqrt{2}}\right) = \dfrac{-5}{4\sqrt{2}} = \boxed{\dfrac{-5\sqrt{2}}{8}}$

6 - Funciones Compuestas e Implicitas

1

ⓐ $\bar{f}(x,y) = (xy\,;\,x-y)$, $\bar{g}(u,v) = (\overset{x}{u^2}\,;\,\overset{y}{v-u})$

$\mathbb{R}^2 \to \mathbb{R}^2$ $\qquad$ $\bar{f}$ $\mathbb{R}^2 \to \mathbb{R}^2$

$$f\circ g \quad \begin{tikzcd} \mathbb{R}^2 \arrow[r, "\bar{g}"] \arrow[rr, bend right, "h"'] & \mathbb{R}^2 \arrow[r, "f"] & \mathbb{R}^2 \end{tikzcd} \qquad h = f\circ \bar{g} = f\,[x(u,v)\,;\,y(u,v)] = (u^2(v-u)\,;\,u^2-v+u)$$

- $Dh_{(1,1)} = Df(g(1,1)).\,Dg(1,1)$

$$Dg = \begin{pmatrix} 2u & 0 \\ -1 & 1 \end{pmatrix} \quad Dg(1,1) = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} \qquad Dh_{(1,1)} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 3 & -1 \end{pmatrix}$$

$$Df = \begin{pmatrix} y & x \\ 1 & -1 \end{pmatrix} \quad Df_{(g(1,1))} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$$

$g\circ f$ Dominio $\mathbb{R}^2$

$Dh_{(1,1)} = Dg(f_{(1,1)})\,Df_{(1,1)}$

$$D(g\circ f)_{(1,1)} = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 0 & -2 \end{pmatrix}$$

ⓑ $f(x,y) = x\sqrt{y}$ $\qquad$ $\bar{g}(u) = (\overset{x}{u}, \overset{y}{2-u})$

$\mathbb{R}^2 \to \mathbb{R}$ $\qquad$ $\mathbb{R} \to \mathbb{R}^2$

$$f\circ g \Rightarrow \begin{tikzcd} \mathbb{R} \arrow[r, "g"] \arrow[rr, bend right, "h"'] & \mathbb{R}^2 \arrow[r, "f"] & \mathbb{R} \end{tikzcd} \qquad h = f(g(u)) = u\sqrt{2-u} \quad \text{Dom}/\ 2-u \geq 0 \Rightarrow 2 \geq u$$

$D(f\circ g)_{(1)} = Df_{(g(1))}\,Dg_{(1)}$

$$Df = \left(\sqrt{y} \quad \frac{x}{2\sqrt{y}}\right) \quad Df_{(g(1))} = \left(1 \quad \frac{1}{2}\right)$$

$$Dg_{(} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad D(f\circ g)_{(1)} = (1 \quad 1/2)\begin{pmatrix} 1 \\ -1 \end{pmatrix} = (1/2)$$

$$g\circ f \Rightarrow \begin{tikzcd} \mathbb{R}^2 \arrow[r, "f"] \arrow[rr, bend right, "h"'] & \mathbb{R} \arrow[r, "\bar{g}"] & \mathbb{R}^2 \end{tikzcd} \qquad h = g(f_{(x,y)}) = (x\sqrt{y}\,;\,2-x\sqrt{y})$$

$D(g\circ f)_{(1)} = Dg(f_{(1,1)})\,Df_{(1,1)}$ $\qquad$ $Df_{(1,1)} = (1 \quad 1/2)$

$$D(g\circ f)_{(1)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}(1 \quad 1/2) = \begin{pmatrix} 1 & 1/2 \\ -1 & -1/2 \end{pmatrix}$$

ⓒ $\mathbb{R}^2 \to \mathbb{R}^2 / \bar{f}(x,y) = (x-y, \sqrt{x+y})$

$\mathbb{R} \to \mathbb{R}^2 / \bar{g}(t) = (2-t; t-3)$

$f \circ g \Rightarrow \underbrace{\mathbb{R} \xrightarrow{g} \mathbb{R}^2 \xrightarrow{f} \mathbb{R}^2}_{h}$ $\quad h(t) = f\circ g(t) = \bar{f}(2-t; t-3) = \left(2-t-(t-3); \sqrt{\underset{=}{2-t+t-3}}\right)$

$\sqrt{-1}$

Absurdo $\nexists \sqrt{-1}$

No queda definida.

$g \circ f \Rightarrow \mathbb{R}^2 \xrightarrow{f} \mathbb{R}^2$

$\mathbb{R} \xrightarrow{g} \mathbb{R}^2$ Tampoco queda definida debido a la dimension de los dominios y codominios.

ⓓ $\mathbb{R}^2 \to \mathbb{R}^3 / \bar{f}(x,y) = (xy^2; y-x; x)$

$\mathbb{R}^3 \to \mathbb{R}^2 / \bar{g}(u,v,w) = (u-v; w\sqrt{1-u})$ $\quad x=-1 \quad y=1$

$f \circ g \Rightarrow \underbrace{\mathbb{R}^3 \xrightarrow{g} \mathbb{R}^2 \xrightarrow{f} \mathbb{R}^3}_{h}$ $\quad h(u,v,w) = f(g(u,v,w)) = \left((u-v)(w\sqrt{1-u})^2; w\sqrt{1-u} - u + v; u-v\right)$

Dom $f\circ g / u \leq 1$

$D(f\circ g)_{(0,1,1)} /$
$Df(g(0,1,1)) \cdot Dg(0,1,1)$ $\begin{pmatrix} y^2 & 2yx \\ -1 & 1 \\ 1 & 0 \end{pmatrix} = Df(g(0,1,1)) = \begin{pmatrix} 1 & -2 \\ -1 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad Dg(0,1,1) = \begin{pmatrix} 1 & -1 & 0 \\ \frac{-w}{2\sqrt{1-u}} & 0 & \sqrt{1-u} \end{pmatrix}$

$D(f\circ g)(0,1,1) = \underset{3\times 2}{\begin{pmatrix} 1 & -2 \\ -1 & 1 \\ 1 & 0 \end{pmatrix}} \underset{2\times 3}{\begin{pmatrix} 1 & -1 & 0 \\ -1/2 & 0 & 1 \end{pmatrix}} = \boxed{\begin{pmatrix} 2 & -1 & -2 \\ -3/2 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}} = \begin{pmatrix} 1 & -1 & 0 \\ -1/2 & 0 & 1 \end{pmatrix}$

$g \circ f$ tambien se puede componer mas de lo mismo.

② $h: D \subset \mathbb{R}^2 \to \mathbb{R} / h(x,y) = x\ln(1-xy)$ definir dos funciones que generen h

$g(u;v) = u\ln(v)$ $\quad$ y $\quad f(x,y) = (x; 1-xy) \Rightarrow g\circ f = h = x\ln(1-xy)$

③ $z = 2uv - 2\sqrt{v-u}$ con $\begin{cases} u = x - y^2 \\ v = x + 2xy - 1 \end{cases}$ resulta $z = h(x,y)$ $\quad 6$

ⓐ $f(u,v) = 2uv - 2\sqrt{v-u}$ $\quad \bar{g}(x,y) = (x-y^2; x+2xy-1)$ $\quad h = 2(x-y^2)(x+2y\cancel{x}-1) - 2\sqrt{(x+2xy-1)-(x-y^2)}$

ⓑ $f'((2,1)(5,5)) = \lim_{h\to 0} \dfrac{f(2+5h; 1+5h) - f(2,1)}{h}$

$(2x-2y^2)(x+2xy-1) - 2\sqrt{x+2xy-1-x+y^2} = 2x^2 + 4x^2y - 2x - 2y^2x - 4xy^3 + 2y^2 - 2\sqrt{2xy-1+y^2}$

$\lim_{h\to 0} \dfrac{2(2+5h)^2 + 4(2+5h)^2(1+5h) - 2(2+5h) - 2(1+5h)^2(2+5h) - 4(2+5h)(1+5h)^3 + 2(1+5h)^2 - 2\sqrt{2(2+5h)(1+5h) - 1 + (1+5h)^2}}{h}$

$= 2(25h^2 + 20h + 4) + 4(25h^2 + 20 + 4)(1+5h) - 4 - 10h - 2(25h^2 + 10h + 1)(2+5h)$

Me aburrí

4) $\omega = u^3 - x v^2$ con $u = x\sqrt{y-x}$

$v = 2x + y^2$

$f'^{\,0\,1}_{(x,y)} = \omega'_u u'_x + \omega'_v v'_x + \omega'_x$

$= 3u^2\left(\sqrt{y-x} + \cancel{\frac{-x}{2\sqrt{y-x}}}\right) + \cancel{(-x2v)}2 + (-v^2)$ con $\begin{cases} u(0;1) = 0 \\ v(0;1) = 1 \end{cases}$

$= 3\,0 \quad + \quad = \boxed{-1}$

5) $f_{(u,v)} = |u-1| - v$ $\quad g(x,y) = (\overbrace{1+x^2}^{u}; \overbrace{2y-1}^{v})$ $\quad h = f \circ g$ es derivable en $(0;0)$

$h = |1+x^2-1| - 2y + 1 = x^2 - 2y + 1$

$\bar\nabla h_{(x,y)} = (2x\,;\,-2)$ $\quad \bar\nabla h_{(0,0)} = (0\,;-2)$ Es derivable en $(0,0)$

6) $x\dfrac{\partial z}{\partial x} = x\left(\dfrac{2x(x^2+y^2) - (x^2-y^2)(2x)}{(x^2+y^2)^2}\right) = \dfrac{\cancel{2x^3} + 2xy^2 - (\cancel{2x^3} - 2xy^2)}{(x^2+y^2)^2} = \dfrac{4x^2y^2}{(x^2+y^2)^2}$

$y\dfrac{\partial z}{\partial x} = y\left(\dfrac{-2y(x^2+y^2) - (x^2-y^2)(2y)}{(x^2+y^2)^2}\right) = \dfrac{-2yx^2 - \cancel{2y^3} - (2yx^2 - \cancel{2y^3})}{(x^2+y^2)^2} \quad -\dfrac{4y^2x^2}{(x^2+y^2)^2}$

$\dfrac{4x^2y^2}{(x^2+y^2)^2} + \left(-\dfrac{4x^2y^2}{(x^2+y^2)^2}\right) = \boxed{0}$

7) $h(x,y) = f_{(2x/y)} - f_{(y/x)}$ con $f \in C^1 \Rightarrow$ f es continua y posee derivadas parciales continuas $\Rightarrow$ f es diferenciable

$h(x,y) = 2\dfrac{x}{y} - \dfrac{y}{x}$ $\qquad x h'_x + y h'_y = 0$

$h'_x = \dfrac{2}{y} + \dfrac{y}{x^2}$ $\qquad h'_y = -2\dfrac{x}{y^2} - \dfrac{1}{x}$

$x h'_x = \dfrac{2x}{y} + \dfrac{y}{x}$ $\qquad y h'_y = -\dfrac{2x}{y} - \dfrac{y}{x}$

$\cancel{\dfrac{2x}{y}} + \cancel{\dfrac{y}{x}} + \left(-\cancel{\dfrac{2x}{y}} - \cancel{\dfrac{y}{x}}\right) = \boxed{0}$ Verifica

8) $f_{(x,y)} = x^2 + y^2 + 1$ y $\bar g(x,y) = (\overbrace{x+y}^{x}; \overbrace{ay}^{y})$ $\quad h = f \circ g$ $\quad h = (x+y)^2 + a^2y^2 + 1$

$= x^2 + 2xy + y^2 + a^2y^2 + 1$

$\bar\nabla h_{(x,y)} = (2(x+y)\,;\,2(x+y) + 2a^2y)$

$\bar\nabla h_{(1,1)} = (4\,;\,4 + 2a^2)$ $\qquad \check v = \dfrac{\bar\nabla h_{(1,1)}}{\|\bar\nabla h_{(1,1)}\|} = \dfrac{(4\,;4+2a^2)}{\sqrt{16 + (4+2a^2)^2}} = \cancel{(5;7)}$

Rehacer

⑧ $f(x,y) = x^2+y^2+1$ $\quad \bar{g}(x,y) = (\overset{x}{x+y}; \overset{y}{ay})$ $\quad h = f\circ\bar{g}$ en $(1,1)$

$\mathbb{R}^2 \to \mathbb{R}$ $\qquad \mathbb{R}^2 \to \mathbb{R}^2$

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "\bar{g}"] \arrow[rr, bend right, "h"'] & \mathbb{R}^2 \arrow[r, "f"] & \mathbb{R} \end{tikzcd}$$

$h(x,y) = (x+y)^2 + (ay)^2 + 1$

$\bar{\nabla} h(x,y) = (2(x+y) \; ; 2(x+y) + 2(ay)a)$

$\bar{\nabla} h(1,1) = (4 \; ; \; 4 + 2a^2)$

$$\left(\frac{5}{\sqrt{74}} ; \frac{7}{\sqrt{74}}\right) = \frac{(4 \; ; 4+2a^2)}{\sqrt{16+(4+2a^2)^2}}$$

$$\frac{5}{\sqrt{74}} = \frac{4}{\sqrt{16+(4+2a^2)^2}} \Rightarrow 5\sqrt{16+(4+2a^2)^2} = 4\sqrt{74}$$

$$25(16+16+16a^2+4a^4) = 16 \cdot 74$$

$\bar{\nabla} h(1,1) = \lambda(5,7)$

$(4 \; ; 4+2a^2) = (\lambda 5 \; ; \lambda 7)$

$\frac{4}{5} = \lambda \qquad 4+2a^2 = \frac{28}{5} \qquad 2a^2 - \frac{8}{5} = 0$

$$\boxed{a = \frac{2\sqrt{5}}{5}} \vee \boxed{a = -\frac{2\sqrt{5}}{5}}$$

⑩ $xy - e^{z-x} = \ln(z)$ $\qquad z = f(x,y)$ en $(1,1)$

① $1 - e^{z-1} = \ln(z)$ $\qquad z=1$ para que verifique ecuac.

② $F \in C^1$ en $(1,1,1)$

③ $f'_z(x,y,z) = -e^{z-x} - \frac{1}{z} = 0$

$f'_z(1,1,1) = -1-1 = -2 \neq 0$

$$f'_x = -\frac{f'_x(1,1,1)}{f'_z(1,1,1)} = -\frac{y+e^{z-x}}{-2} = \frac{-2}{-2} = \boxed{1}$$

$$f'_y = -\frac{f_y(1,1,1)}{f'_z(1,1,1)} = -\frac{x}{-2} = \boxed{\frac{1}{2}}$$

$F(x,y) \cong 1 + 1(x-1) + \frac{1}{2}(y-1)$

⑪ $z^3 + 2xz + yz - x = 0$ $\qquad z = f(x,y)$ en $(1;-2)$

$z^3 + 2z - 2z - 1 = 0 \Rightarrow \boxed{z=1}$ $\qquad f \in C^1$ en $(1,-2,1)$

$f'_z(1,-2,1) = 3z^2 + 2x + y = 3 \neq 0$

$$f'_x = -\frac{f'_x(1,-2,1)}{f'_z(1,-2,1)} = -\frac{2z-1}{3} = \boxed{-\frac{1}{3}}$$

$$f'_y = -\frac{f'_y(1,-2,1)}{f'_z(1,-2;1)} = -\frac{z}{3} = \boxed{-\frac{1}{3}}$$

② $\bar{\nabla} f(1,-2) = \left(-\frac{1}{3} ; -\frac{1}{3}\right)$

⑥ Ecuac plano Tg

$\check{m} = (f'_x ; f'_y ; -1)$

$-\frac{1}{3}x - \frac{1}{3}y - z = D$ $\boxed{\text{Plano Tg} = x + y + 3z = 2}$

$-\frac{1}{3} + \frac{2}{3} - 1 = D = -\frac{2}{3}$ Recta Normal: $(1;1;3)\lambda + (1;-2;1)$

⑫ Hiperboloide una hoja $2x^2 - 2y^2 + z^2 = 1$ y plano Tg $// \pi : z = x - y$

↳ Conjunto de nivel **1**

$f'_x = 4x$

$f'_y = -4y$

$f'_z = 2z$

Plano Tg: $\overline{\nabla} f_{(P_0)} (\overline{X} - P_0) //$ $C = x - y - z = 0$ Conjunto de nivel 0

$\overline{\nabla} C = (1; -1; -1)$

$\overline{\nabla} f = \overline{\nabla} C$ condicion de paralelismo

$4x = 1$ $\qquad x = \frac{1}{2} \qquad x = -\frac{1}{2}$

$-4y = -1 \Rightarrow y = \frac{1}{2} \vee y = -\frac{1}{2}$

$2z = -1$ $\qquad z = -1 \qquad z = 1$

⑬ $x^2 + y^2 + z^2 = 14$ Conjunto de nivel: $\check{m} = \overline{\nabla} h$ $\check{m} = (2x; 2y; 2z)\big|_{(3,1,2)} = (6; 1; 4)$

$$f'\left((3;1,2); \frac{(6;1,4)}{\sqrt{53}}\right) = \lim_{h \to 0} \frac{f(3 + h\frac{6}{\sqrt{53}}; 1 + \frac{h}{\sqrt{53}}; 2 + h\frac{4}{\sqrt{53}}) - f(3;1;2)}{h} = \frac{8(3 + h\frac{6}{\sqrt{53}})^2 - 2(3 + h\frac{6}{\sqrt{53}})(1 + \frac{h}{\sqrt{53}}) - 66}{h}$$

$$\lim_{h \to 0} \frac{8(9 + \frac{36}{\sqrt{53}}h + \cancel{h^2 \frac{36}{53}}) - 2(3 + \frac{3h}{\sqrt{53}} + \frac{6h}{\sqrt{53}} + \cancel{h^2 \frac{6}{53}}) - 66}{h}$$

$$\lim_{h \to 0} \frac{\cancel{72} + \frac{288}{\sqrt{53}}h - \cancel{6} - \frac{18h}{53} - \cancel{66}}{h} = \frac{\cancel{h}\left(\frac{288}{\sqrt{53}} - \frac{18}{\sqrt{53}}\right)}{\cancel{h}} = \frac{270}{\sqrt{53}} \quad \underline{\underline{\text{NO DA}}}$$

⑯ $f : x^2 - y^2 = 12$

$G = z = x + y^2 \Rightarrow x + y^2 - z = 0 \Rightarrow z_0 = 8$

$$\overline{\nabla} f_{(4,2,8)} \times \overline{\nabla} G_{(4,2,8)} = (2x; -2y; 0) \times (1; 2y; -1) = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 8 & -4 & 0 \\ 1 & 4 & -1 \end{vmatrix} = (4; 8; 36) = (1; 2; 9)$$

$\overline{X}_T = (1,2,9)\lambda + (4,2,8)$

$$\begin{cases} x = \lambda + 4 \\ y = 2 + 2\lambda \\ z = 8 + 9\lambda \end{cases}$$

$y = x^2$

$2 + 2\lambda = \lambda^2 + 8\lambda + 16$

$0 = \lambda^2 + 6\lambda + 14$

λ imaginario no existe $\Rightarrow \boxed{\underline{\text{No CORTA}}}$

(17) $z = u + v e^{u-v}$ con $(u,v) = (f(x,y), y^2)$ Hallar $\check{r}$ / $h'((2,1),\check{r}) = 0$ si f queda implícita por $2y - ux - \ln(u) = 0$ $\quad z = h(x,y)$

$\hookrightarrow f(x,y,u)$

$z < \begin{matrix} u \\ v \end{matrix} \bowtie \begin{matrix} x \\ y \end{matrix}$

$v(2,1) = \boxed{1}$

$u(2,1) = f(2,1,u_0) = 2 - 2u_0 - \ln(u_0) = 0 \Rightarrow \boxed{u_0 = 1}$

$h(2,1) = z(1,1) = \boxed{2}$

$h'((2,1),\check{r}) = 0 = \nabla h . \check{r}$

$h'_x = z'_u u'_x + z'_v v'_x$

$h'_x = (1 + v e^{u-v})(-u) + (e^{u-v} + v e^{u-v})(0) \Big|_{(2,1)} = \boxed{-2}$

$h'_y = z'_u u'_y + z'_v v'_y$

$h'_y = (1 + v e^{u-v})(2) + (e^{u-v} - v e^{u-v})(2y) = \boxed{4}$

$(-2;4)(a;b) = 0$

$-2a + 4b = 0 \Rightarrow 4b = 2a \Rightarrow b = \frac{1}{2} a$

$\wedge \quad a^2 + b^2 = 1 \Rightarrow a^2 + \left(\frac{1}{2} a\right)^2 = 1 \Rightarrow a^2 + \frac{1}{4} a^2 = 1 \Rightarrow \frac{5}{4} a^2 = 1 \Rightarrow |a| = \sqrt{\frac{4}{5}}$

$|a| = \frac{2}{\sqrt{5}} \qquad b = \frac{1}{2} \frac{2}{\sqrt{5}} \Rightarrow b = \frac{1}{\sqrt{5}}$

$\check{r}_1 = \left(\frac{2}{\sqrt{5}}; \frac{1}{\sqrt{5}}\right) \quad \check{r} = \left(-\frac{2}{\sqrt{5}}; -\frac{1}{\sqrt{5}}\right)$

(18)

$$\begin{cases} x = x \\ y = x^2 - 3 \\ z = xy - 3x \Rightarrow z = x(x^2-3) - 3x \Rightarrow z = x^3 - 6x \end{cases}$$

$C = (x; x^2-3; x^3-6x) \Rightarrow \nabla C(2;1;-4) = \check{m}_\pi$

$\nabla C = (1; 2x; 3x^2-6) = (1;4;6)$

$x + 4y + 6z = -D$

$(2) + (4)(1) + 6(-4) = -D \qquad D = +18$

$\boxed{\pi: x + 4y + 6z + 18}$

(19) $h = f \circ \bar{g}$ $\quad \bar{g}(\bar{x}) = (e^x, e^{x^2})$ y $f(u,v)$ definido por $y - 1 + \ln(yu,v) = 0$ (4)

$\therefore$ $y = h(x)$ con $h(0) = 1$ satisface $(1+y)y' + (1+2x)y = 0$

$h(x) = f(g(x)) = f(e^x; e^{x^2}) = \ln(e^x . e^{x^2}) = \boxed{x + x^2}$

$f_{(1,1,y_0)} = y_0 - 1 + \ln(y_0) = 0 \Rightarrow y_0 = 1$

$f(u,v) = \ln(u,v)$

$$(1+y)\frac{y'}{y} = 1 + 2x \Rightarrow \int \frac{1}{y}\,dy + \int 1\,dy = \int 1\,dx + \int 2x\,dx$$

$y = h(x) \Rightarrow y = h(0) = 1$ $\qquad \ln|y| + y = x + x^2$

$$\ln|1| + 1 = x + x^2$$

$$\underbrace{x^2 + x = 1}_{h(x)}\ \text{q'}\ h(0) = 1 \Rightarrow \underline{\text{verifica}}$$

$$h(x) = x^2 + x = 1$$

$$h(0) = x^2 + x = \underline{1}$$

(20) $z = f(x,y)$ $\quad x + yz - e^z = 0$ satisface $z z'_x - z'_y = 0$

$$f'_x = -\frac{f'_x}{f'_z} = -\frac{1}{y - e^z} \qquad f'_y = -\frac{f'_y}{f'_z} = -\frac{z}{y - e^z}$$

$$z\left(-\frac{1}{y - e^z}\right) + \left(\frac{z}{y - e^z}\right) = 0 \quad \boxed{\underline{\text{Verifica}}}$$

(21) $h = f \circ \bar{g}$ en $(1;1)$ $\quad f(u,v)$ definido $z - u^2 + v^2 + \ln(v+z) = 0$

$\bar{g}(x,y) = (xy^2, y - x^2)$

$$z < \begin{matrix} u \\ v \end{matrix} \times \begin{matrix} x \\ y \end{matrix}$$

$h'_{MAX} = \|\bar{\nabla} h(P_0)\|$

$h'_x = h'_u\, u'_x + h'_v\, v'_x$ $\qquad h(1;1) = f(g(1;1))\ g(1;1)$

$$g(1,1) = (1;0)$$

$$h'_u = f'_u = -\frac{f'_u}{f'_z} = \frac{-(-2u)}{1 + \frac{1}{v+z}} \qquad f(1,0) = z - 1 + \ln(z) = 0 \Rightarrow \boxed{z = 1}$$

$$\text{en } (1,0,1) = \frac{2}{1 + \frac{1}{1}} = \boxed{1} \qquad h'_v = f'_v = -\frac{2v + \frac{1}{v+z}}{1 + \frac{1}{v+z}} = \boxed{-\frac{1}{2}}$$

$u'_x = y^2$ en $(1,1) = 1$

$v'_x = -2x = \boxed{-2}$

$h'_x(1,1) = 1.1 + \left(-\frac{1}{2}\right)(-2) = \boxed{2}$

$h'_y = h'_u\, u'_y + h'_v\, v'_y$

$u'_y = 2xy$ en $(1,1) = \boxed{2}$

$v'_y = 1$

$h'_y(1,1) = 1.2 + \left(-\frac{1}{2}\right) 2 = \boxed{3/2}$

$\nabla h(1,1) = (2; 3/2) \quad \|\nabla h(1,1)\| = \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \boxed{\frac{5}{2}}$

(22) $w = u^2 v + 3v^2$ con $\begin{cases} u = x+y^2 \\ v = g(x,y) \end{cases}$ $\quad w = h(x,y) \quad h(2{,}98; 2{,}01)$

g definido por $xv + \ln(v+y-2) - 3 = 0 \qquad x=3 \quad y=2$

$3v + \ln(v) - 3 = 0 \quad \boxed{v=1}$

$h(x;y) = (x+y)^2(x+\ln(1+y-2)-3) + 3(x+\ln(y-1)-3)^2$

$h(3;2) = (5)^2(3+\ln(1)-3) + 3(3+\ln(1)-3)^2 = 0$

$h'_x = 2(x+y)\underbrace{(x+\ln(y-1)-3)}_{0} + (x+y)^2(1) + 6\underbrace{(x+\ln(y-1)-3)}_{0} \Big]_{(3;2)} = 25$

$h'_y = 2(x+y)\underbrace{(x+\ln(y-1)-3)}_{0} + (x+y)^2\left(\frac{1}{y-1}\right) + 6\underbrace{(x+\ln(y-1)-3)}_{0}\frac{1}{y-1} \Big]_{(3;2)} = 25$

$h(2{,}98; 2{,}01) = 0 + 25(0{,}02) + 25(-0{,}01) \cong$ **NO DA**

(24) $f \in C^1$ con $\nabla f = (1;-1)$ cte hallar $g/\ h(x) = f(xg(x), g(x))$ g pasa por $(3;1)$

$w(x) = (xg(x), g(x)) \Rightarrow h(x) = f(w(x))$

$h'(x) = Df(w(x)) . Dw(x)$

$(1;-1)\begin{pmatrix} g(x) + xg'(x) \\ g'(x) \end{pmatrix} = g(x) + xg'(x) - g'(x) = 0$ cte

$g(x) + xg'(x) - g'(x) = 0$

$-g(x) = g'(x)(x-1) \Rightarrow \frac{1}{x-1} = -\frac{g'(x)}{g(x)}$

$\int \frac{1}{x-1}\,dx = -\int \frac{dg}{g(x)} = \ln|x-1| = -\ln|g(x)| + c \qquad g(x) = \frac{A}{x-1}$

g en $(3;1) \Rightarrow 1 = \frac{A}{2} \quad A = 2 \Rightarrow \boxed{g(x) = \frac{2}{x-1}}$

(25) $f(u)$ / $y = x\,f(x^2-1)$ solucion de $xy' - y = 2x^3$

$u = x^2 - 1$

$y' = f(x^2-1) + x f'(x^2-1)\,2x = f(x^2-1) + 2x^2 f'(x^2-1)$

$x\left(f(u) + 2x^2 f'(u)\right) - x f(x^2-1) = 2x^3$

$\cancel{x f(u)} + 2x^3 f'(u) - \cancel{x f(u)} = 2x^3$

$f'(u) = 1$

$\int f'(u) = \int 1\, du \Rightarrow \boxed{f(u) = u + C}$

7 - Polinomio de Taylor - Extremos

① $p(x,y) = x^2 - xy + y^3 - 3$ polinomio Taylor de 3° orden en $(1,2)$

ⓐ $f'_x = 2x - y$

$f'_y = -x + 3y^2$

$f''_{xx} = 2$

$f''_{yy} = 6y$

$f''_{xy} = f''_{yx} = -1$

$f'''_{xxx} = 0$

$f'''_{yyy} = 6$

$$P_3(x,y) = f(x,y) + \cancel{f'_x(x-x_0)} + f'_y(y-y_0) + \frac{1}{2}\left[f''_{xx}(x-x_0)^2 + f''_{yy}(y-y_0)^2 + 2f''_{xy}(x-x_0)(y-y_0)\right] + \frac{1}{6}\left[\cancel{f'''_{xxx}(x-x_0)^3} + f'''_{yyy}(y-y_0)^3\right]$$

$$P_3(1,2) = 4 + 11(y-2) + (x-1)^2 + 6(y-2)^2 - (x-1)(y-2) + (y-2)^3$$

ⓑ Si se puede, $f(1,2) = 4$

③ ⓐ $f(x,y) = x - y\sqrt{6-x}$ $\quad \bar{A} = (2,3)$

$f'_x = 1 + y\dfrac{1}{2\sqrt{6-x}}$

$f'_y = -\sqrt{6-x}$

$f''_{xx} = \dfrac{y}{2}\left(+\dfrac{2/4\sqrt{6-x}}{4(6-x)}\right)$

$f''_{yy} = 0$

$f''_{xy} = \dfrac{1}{2\sqrt{6-x}}$

$f''_{yx} = +\dfrac{1}{2\sqrt{6-x}}$

$$P_2(2,3) = -4 + \frac{7}{4}(x-2) - 2(y-3) + \frac{3}{64}(x-2)^2 + \frac{1}{4}(x-2)(y-3)$$

ⓑ $f(x,y) = y\ln(x) + x\ln(y-x)$ $\quad \bar{A} = (1,2)$

$f'_x = y/x + \ln(y-x) + \dfrac{x}{(-x)}$

$f'_y = \ln(x) + x/y$

$f''_{yy} = -(x)\left(\dfrac{-1}{y^2}\right)$

$f''_{xx} = -\dfrac{y}{x^2} + \left(\left(\dfrac{1}{x}\right)(-1)\right)$

$f''_{yx} = \dfrac{1}{x} + \dfrac{1}{y}$

$$P_2(x,y) = - (x-1) + \frac{1}{2}(y-2)$$

Revisar Dsps.

ⓒ $f_{(x,y,z)} = e^{xz} - (z+1)\sqrt{y-x}$ $\quad f_{(1,2,0)} = 0$

$f'_x = e^{xz}.z + (z+1)\frac{1}{2\sqrt{y-x}}$ $\quad f'_{x(1,2,0)} = \frac{1}{2}$

$f'_y = -(z+1)\,\frac{1}{2}\,(y-x)^{-1/2}$ $\quad f'_{y=(1,2,0)} = -\frac{1}{2}$

$f'_z = e^{xz}.x - \sqrt{y-x}$ $\quad f'_{z(1,2,0)} = 0$

$f''_{xx} = z^2 e^{xz} + (z+1)\,\frac{1}{2}\left(+\frac{1}{2}\right)(y-x)^{-3/2}$ $\quad f''_{xx(1,2,0)} = 1/4$

$f''_{xy} = (z+1)\,\frac{1}{2}\left(-\frac{1}{2}\right)(y-x)^{3/2} \Rightarrow f''_{xy(1,2,0)} = -1/4$

$f''_{xz} = e^{xz}xz + e^{xz} + \frac{1}{2\sqrt{y-x}}$ $\quad f''_{xz(1,2,0)} = 3/2$

$f''_{yy} = -\frac{(z+1)}{2}\left(-\frac{1}{2}\right)(y-x)^{-3/2}$ $\quad f''_{yy(1,2,0)} = 1/4$

$f''_{yx} = -\frac{(z+1)}{2}\left(-\frac{1}{2}\right)(y-x)^{-3/2}(-1)$ $\quad f''_{yx}(\bar{A}) = -1/4$

$f''_{yz} = \left(-\frac{1}{2}\right)(y-x)^{-1/2}$ $\quad f''_{yz}(\bar{A}) = -1/2$

$f''_{zz} = x^2 e^{xz}$ $\quad f''_{zz}(\bar{A}) = 1$

$$P_2(\bar{A}) = \cancel{f(\bar{A})}^{\,0} + f'_x(A)(x-x_0) + f'_y(A)(y-y_0) + \cancel{f'_z}^{\,0}(z-z_0) + \frac{1}{2}\Big[f''_{xx}(x-x_0)^2 + f''_{yy}(y-y_0)^2 + f''_{zz}(z-z_0)^2 + 2f''_{xy}(x-x_0)(y-y_0) + 2f''_{xz}(x-x_0)(z-z_0) + 2f''_{yz}(y-y_0)(z-z_0)\Big]$$

$$P_2(A) = \frac{(x-1)}{2} - \frac{(y-2)}{2} + \frac{1}{8}(x-1)^2 + \frac{1}{8}(y-2)^2 + \frac{1}{2}(z-0)^2 - \frac{1}{4}(x-1)(y-2) + \frac{3}{2}(x-1)(z) - \frac{1}{2}(y-2)(z)$$

④ ⓐ $0{,}98^{2{,}01}$ $\quad$ Propongo $P_0 = (1;2)$

$f_{(x,y)} = x^y$ $\quad P_{2(x,y)} = 1 + 2(x-1) + \frac{1}{2}\left[2(x-1)^2 + 2.1(x-1)(y-2)\right]$

$f'_x = y\,x^{y-1}$ $\quad P_2(0{,}98;2{,}01) \cong 1 + 2(0{,}98-1) + (0{,}98-1)^2 + (0{,}98-1)(2{,}01-2) \cong 0{,}9602$

$f'_y = x^y \ln x$

$f''_{xx} = y(y-1)\,x^{y-2}$

$f''_{yy} = \ln^2 x \;\; x^y$

$f''_{xy} = x^{y-1} + y\,x^{y-1}\ln x$

ⓑ $f_{(x,y)} = \sqrt{x} + \sqrt[3]{y}$ $\quad P_0 = (4;8)$

$f'_x = \frac{1}{2\sqrt{x}}$ $\quad P_1(x,y) = 4 + \frac{1}{4}(x-4) + \frac{1}{6}(y-8)$

$f'_y = \frac{1}{3\sqrt[3]{y}}$ $\quad P_1(3{,}99;8{,}06) = 4 + \frac{1}{4}(3{,}99-4) + \frac{1}{6}(8{,}06-8) \simeq 4{,}0075$