P1) Siendo $f \in C^1$, $f'(\bar{A},(-0.6;0.8)) = -2$ y $f'(\bar{A},(0.8;0.6)) = 1$. **Hallar** $f'(\bar{A},(0.3;-0.4))$.

Indicar las direcciones en que la derivada direccional es nula en el punto $\bar{A}$.

como f es clase C^1 $\rightarrow$ f es diferenciable $\rightarrow$ $\underline{f'(\bar{A};\bar{v}) = \nabla f(\bar{A})\cdot\bar{v}}$

- $f'(\bar{A};(-0{,}6;0{,}8)) = -2 \longrightarrow \nabla f(\bar{A})\cdot(-0{,}6;0{,}8) = -2$
- $f'(\bar{A};(0{,}8;0{,}6)) = 1 \rightarrow \nabla f(\bar{A})\cdot(0{,}8;0{,}6) = 1$

$$\nabla f(\bar{A}) = (\underbrace{f'_x(\bar{A})}_{a};\underbrace{f'_y(\bar{A})}_{b})$$

$$\begin{cases} (a;b)\cdot(-0{,}6;0{,}8) = -2 \rightarrow -0{,}6\,a + b\,0{,}8 = -2 \\ (a;b)\cdot(0{,}8;0{,}6) = 1 \longrightarrow 0{,}8a + 0{,}6b = 1 \end{cases} \rightarrow a = 2 \wedge b = -1$$

$$f'_x(\bar{A}) = 2 \qquad f'_y(\bar{A}) = -1$$

$$\nabla f(\bar{A}) = (2;-1)$$

$$f'(\bar{A};(0{,}3;-0{,}4)) = \nabla f(\bar{A})\cdot(0{,}3;-0{,}4) = (2;-1)\cdot(0{,}3;-0{,}4) = 0{,}6 + 0{,}4 \boxed{= 1}$$

las derivadas direccionales nulas

$$\breve{v}_{nula\ en\ X_0} = \frac{\left(-\frac{\partial f}{\partial y}(X_0)\,,\,\frac{\partial f}{\partial x}(X_0)\right)}{\|\nabla f(X_0)\|} \quad \text{o} \quad \breve{v}_{nula\ en\ X_0} = \frac{\left(\frac{\partial f}{\partial y}(X_0)\,,\,-\frac{\partial f}{\partial x}(X_0)\right)}{\|\nabla f(X_0)\|}$$

$$\|\nabla f(A)\| = \sqrt{4+1} = \sqrt{5}$$

$$\boxed{\breve{v}_{nula_1} = \frac{(1;2)}{\sqrt{5}} \qquad \breve{v}_{nula_2} = \frac{(-1;-2)}{\sqrt{5}}}$$

P2) Siendo $\bar{g}(x,y)=(xy+1, xy-x, xy-1)$, $\bar{\nabla} f(7,3,5)=(3,-2,1)$ y $f \in C^1$. **Calcular** la derivada direccional máxima de $h(x,y)=f(\bar{g}(x,y))$ en (3,2). **Indicar** la dirección.

$$\bar{g}(\bar{x}) = (\overbrace{xy+1}^{g_1}; \overbrace{xy-x}^{g_2}; \overbrace{xy-1}^{g_3})$$

$$D\bar{g}(\bar{x}) = \begin{pmatrix} \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} \\ \frac{\partial g_3}{\partial x} & \frac{\partial g_3}{\partial y} \end{pmatrix} = \begin{pmatrix} y & x \\ y-1 & x \\ y & x \end{pmatrix}$$

$$D\bar{g}(3;2) = \begin{pmatrix} 2 & 3 \\ 1 & 3 \\ 2 & 3 \end{pmatrix}$$

$$Dh(\bar{x}) = \underbrace{Df(\bar{g}(\bar{x}))}_{Df(\bar{x})} \cdot D\bar{g}(\bar{x}) \rightarrow Dh(3;2) = (3;-2;1) \cdot \begin{pmatrix} 2 & 3 \\ 1 & 3 \\ 2 & 3 \end{pmatrix}$$

$1 \times 3 \sim 3 \times 2 = 1 \times 2$

$$\rightarrow (6-2\cdot 1+2;\ 9-6+3) = (6;6)$$

$$Dh(3;2) = (6;6)$$

El valor de la derivada maxima $\rightarrow \|Dh(3;2)\| = \sqrt{36+36} = \boxed{\sqrt{72}}$

la dirección es $= \dfrac{Dh(3;2)}{\|Dh(3;2)\|} = \boxed{\dfrac{(6;6)}{\sqrt{72}}}$

P3) Hallar la recta normal a la superficie Σ definida por la ecuación

$x + yz + \ln(x + y^2 - z - 3) - 3 = 0$ en el punto $\bar{A} = (1, 2, z_0)$. Halar la intersección de dicha recta con el planto XZ.

Recta normal: $\bar{x} = \bar{A} + \lambda \nabla F(\bar{A}) \quad \lambda \in \mathbb{R}$ $\qquad \bar{A} = (1;2;z_0)$

Σ: Superficie de nivel $0 \rightarrow F(x;y;z) = x + yz + \ln(x+y^2-z-3) - 3 = 0$

como $\bar{A} \subset \Sigma \rightarrow F(\bar{A}) = 0$

$F(1;2;z_0) = 0 \rightarrow 1 + 2z_0 + \ln(1+4-3-z_0) - 3 = 0$

$z_0 = 1 \rightarrow 1 + 2 + 0 - 3 = 0$ ✓

$\bar{A} = (1;2;1)$

$$\nabla F(\bar{x}) = \left(1 + \frac{1 \cdot 1}{(x+y^2-z-3)};\ z + \frac{1 \cdot 2y}{(x+y^2-z-3)};\ y - \frac{1 \cdot 1}{(x+y^2-z-3)}\right)$$

$$\nabla F(1;2;1) = \left(1 + \frac{1}{1};\ 1 + \frac{4}{1};\ 2 - \frac{1}{1}\right) = (2;5;1)$$

$$\boxed{\text{Recta Normal } (r):\ \bar{x} = (1;2;1) + \lambda(2;5;1)} \rightarrow \begin{cases} x = 1 + 2\lambda \\ y = 2 + 5\lambda \\ z = 1 + \lambda \end{cases}$$

(b) $r \cap$ plano XZ $(y=0)$

$0 = 2 + 5\lambda \rightarrow \lambda = -\frac{2}{5}$

$x = 1 + 2\cdot\left(-\frac{2}{5}\right) = 1/5$

$y = 2 + 5\cdot\left(-\frac{2}{5}\right) = 0$

$z = 1 + \left(-\frac{2}{5}\right) = \frac{3}{5}$

$r \cap y=0 \rightarrow \boxed{P_0 = \left(\frac{1}{5};0;\frac{3}{5}\right)}$

P4) Hallar la solución de la ecuación $x\frac{dy}{dx} - 4y = x^6 e^x$ sujeta a la restricción $y(1) = 1$ $Q(x) = x^5 e^x$ $R(x) = -\frac{4}{x}$

$$xy' - 4y = x^6 e^x \sim y' - 4\frac{y}{x} = \frac{x^6}{x} e^x \sim \overbrace{y' - \frac{4y}{x} = x^5 e^x}$$

$y_h)$ $y' - 4\frac{y}{x} = 0 \sim y' = 4\frac{y}{x} \sim \frac{dy}{dx} = \frac{4y}{x} \sim \int \frac{dy}{y} = \int 4\frac{dx}{x} \sim$

$$\sim \ln|y| = 4\ln|x| + c \sim e^{\ln|y|} = e^{\ln|x|^4 + c} \sim \underline{y = x^4 \cdot k}$$

$y_G)$ $y_p = x^4 \cdot g(x)$

$y_p' = x^4 \cdot g'(x) + 4x^3 g(x)$ $\quad R(x) = -\frac{4}{x}$ $\quad Q(x) = x^5 \cdot e^x$

$$R(x) y_p + y_p' = Q(x)$$

$$-\frac{4}{x} x^4 \cdot g(x) + x^4 g'(x) + 4x^3 g(x) = x^5 e^x \sim -\cancel{4x^3 g(x)} + x^4 g'(x) + \cancel{4x^3 g(x)} = x^5 e^x$$

$$\sim x^4 g'(x) = x^5 e^x \sim g'(x) = xe^x \sim \frac{dg}{dx} = xe^x \sim \int dg = \int xe^x\,dx$$

$$g(x) = \int xe^x\,dx$$

$\int xe^x\,dx$ $\quad u = x$ $\quad v' = e^x$

$u' = 1$ $\quad v = e^x$

$$u \cdot v - \int v \cdot u'\,dx \to xe^x - \int e^x \cdot 1\,dx \sim xe^x - e^x$$

$$g(x) = xe^x - e^x$$

$$y_p = x^4 g(x) \to \underline{y_p = x^5 e^x - x^4 e^x}$$

$y_G = y_h + y_p \to$ $\boxed{y = x^4 \cdot k + x^5 e^x - x^4 e^x}$ Solución gral

$y(1) = 1 \to x = 1 \wedge y = 1$

$$1 = 1k + \cancel{e} - \cancel{e} \Rightarrow \underline{k = 1}$$

$\boxed{\text{Solución particular} \to y = x^4 + x^5 e^x - x^4 e^x}$

T1) **Definir** continuidad de una función escalar de "n" variables.

Determinar si la función $f(x,y)=\begin{cases}\dfrac{y}{x-y} & x\neq y\\ 0 & x=y\end{cases}$ es continua en (0,0)

Toda la teoria esta en resumen es copiar y pegar...

I) $f(0;0)=0$

II) $\lim\limits_{\bar{X}\to\bar{0}} \dfrac{Y}{X-Y}$ ind

• $\underbrace{Y=mX}$

$\lim\limits_{\bar{X}\to\bar{0}} \dfrac{mX}{X-mX} \sim \lim\limits_{X\to 0} \dfrac{\cancel{X}m}{\cancel{X}(1-m)} = \dfrac{m}{1-m}$

como depende de m entonces

No $\nexists$ lim en $(0;0)$

III) I) $\neq$ II) $\to f(X;Y)$ no es continua

T2) **Definir** derivada direccional de una función escalar en $\Re^2$

Calcular (si existen) las $\underbrace{\text{derivadas direccionales}}_{\breve{v}=(a;b)\ /a^2+b^2=1}$ de $f(x,y)=\begin{cases} y^2/x & x\neq 0 \\ 0 & x=0 \end{cases}$ en (0,0)

$$\lim_{h\to 0} \frac{f((0;0)+h(\breve{v}))-f(0;0)}{h} = \lim_{h\to 0} \frac{f(ha;hb)}{h}$$

Lo analizo por los caminos

- Si $\underbrace{h}_{\neq 0}a=0 \to a=0$

$$\lim_{h\to} \frac{0}{h}=0$$

- Si $ha\neq 0 \to a\neq 0$

$$\lim_{h\to} \frac{\frac{h^2b^2}{ha}}{h} \sim \lim_{h\to 0} \frac{\cancel{h^2}b^2}{\cancel{h^2}a} = \frac{b^2}{a}$$

$$\boxed{f'((0;0);(a;b)) = \frac{\partial f(0;0)}{\partial \breve{v}} = \begin{cases} 0 & \text{si } a=0 \\ \frac{b^2}{a} & \text{si } a\neq 0 \end{cases}}$$