AM II

16/12/2022

1º PARCIAL

P1) $\begin{cases} y = cx \\ y' = c \end{cases} \rightarrow y = y'x \rightarrow y' = \dfrac{y}{x} \rightarrow \Big| \; y' = -\dfrac{x}{y} \rightarrow \int y\,dy = -\int x\,dx$

$\dfrac{x^2}{2} + \dfrac{y^2}{2} = c^* \rightarrow x^2 + y^2 = k^2 \rightarrow 4^2 + (-3)^2 = k^2 \rightarrow k = 5 \quad \boxed{x^2+y^2=25}$

P2) $x \ln(z+x-2) + y \cdot e^{yz-6} - 3 = 0 \qquad x=1 \;,\; y=3$

$\ln(z_0-1) + 3 \cdot e^{3z_0-6} - 3 = 0 \qquad z_0 = 2$

$$f'_{x(1,3)} = -\frac{g'_x(1,3,2)}{g'_z(1,3,2)} = -\frac{\ln(z+x-2) + \frac{x}{z+x-2}}{\frac{x}{(z+x-2)} + y^2 \cdot e^{yz-6}}\Bigg|_{(1,3,2)} = -\frac{1}{1+9} = -\frac{1}{10}$$

$$f'_{y(1,3)} = -\frac{g'_y(1,3,2)}{g'_z(1,3,2)} = -\frac{e^{yz-6} + yz e^{yz-6}}{\frac{x}{z+x-2} + y^2 e^{yz-6}}\Bigg|_{(1,3,2)} = -\frac{1+6}{1+9} = -\frac{7}{10}$$

$$f(0{,}99\,;\,3{,}02) \simeq f(1,3) + f'_{x(1,3)} \cdot (-0{,}01) + f'_{y(1,3)} \cdot 0{,}02 =$$

$$\simeq 2 - \frac{1}{10}(-0{,}01) - \frac{7}{10}\,0{,}02 = 2 + 0{,}001 - 0{,}014 = \boxed{1{,}987}$$

P3) $Dh_{(a,b)} = Df_{(\bar{g}(a,b))} \cdot Dg(a,b) = \begin{pmatrix} 2v & 2u \\ 2u & 6v \end{pmatrix}_{(2,1)} \cdot \begin{pmatrix} 3 & 1 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 5 & 4 \end{pmatrix} =$

$= \begin{pmatrix} 6+20 & 2+16 \\ 12+30 & 4+24 \end{pmatrix} = \boxed{\begin{pmatrix} 26 & 18 \\ 42 & 28 \end{pmatrix}}$

P4) $f(x,y) = x^2 y - x^2 + \frac{y^2}{2} - 5y + 1$

$$f'_x = \begin{cases} 2xy - 2x = 0 \rightarrow 2x(y-1) = 0 \rightarrow x = 0 \lor y = 1 \\ x^2 + y - 5 = 0 \end{cases}$$

$f'_y =$

$\rightarrow x = 0 \rightarrow y = 5$

$\rightarrow y = 1 \rightarrow x = 2$

$\rightarrow x = -2$

$\begin{cases} (0,5) \\ (2,1) \\ (-2,1) \end{cases}$ puntos críticos.

$$Hf_{()} = \begin{pmatrix} 2y-2 & 2x \\ 2x & 1 \end{pmatrix} \rightarrow Hf_{(0,5)} = \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow$$ f tiene mínimo en $(0,5)$ y vale $-11,5$

$$Hf_{(2,1)} = \begin{pmatrix} 0 & 4 \\ 4 & 1 \end{pmatrix} \rightarrow$$ f tiene punto silla en $(2,1)$

$$Hf_{(-2,1)} = \begin{pmatrix} 0 & -4 \\ -4 & 1 \end{pmatrix} \rightarrow$$ f tiene punto silla en $(-2,1)$