P1) **Calcular** el volumen de la región definida por las siguientes inecuaciones: $x^2+y^2 \leq 4$,
$z \geq x+y$, $z \leq 2x+y+3$

$$x^2+y^2 \leq 4$$

$$x+y \leq z \leq 2x+y+3 \longrightarrow x+\cancel{y} \leq 2x+\cancel{y}+3$$

$$-3 \leq x$$

$$\mathrm{VOL}(E) = \iiint_E dx\,dy\,dz = \iint_{x^2+y^2\leq 4} \int_{x+y}^{2x+y+3} dz \; dx\,dy = \iint_{\underbrace{x^2+y^2}_{\rho^2}\leq 4} \Big[\overbrace{2x+y+3-(x+y)}^{x+3}\Big] dx\,dy =$$

$$= \xrightarrow[\left|\frac{\partial(x,y)}{\partial(\rho,\varphi)}\right| = \rho]{\substack{x=\rho\cos(\varphi)\\ y=\rho\,\mathrm{sen}(\varphi)}} \int_0^2 \int_0^{2\pi} \Big[\rho\cos(\varphi)+3\Big]\rho \; d\varphi\, d\rho = \int_0^2 \Big[\rho^2\,\mathrm{sen}(\varphi) + 3\rho\varphi\Big]_0^{2\pi} d\rho =$$

$$= \int_0^2 3\cdot\rho\cdot 2\pi \; d\rho = 6\pi \left[\frac{\rho^2}{2}\right]_0^2 = \boxed{12\pi}$$

P2) **Verificar** que el campo $\bar{f}(x,y) = (6xy + 2y^2 + 2, 3x^2 + 4xy - 2)$ es conservativo.

Calcular su función potencial sabiendo que vale 11 en (1,2).

Evaluar el potencial en (1,0)

$$\left.\begin{array}{l} \left.\begin{array}{l} f'_{2_x} = 6x + 4y \\ f'_{1_y} = 6x + 4y \end{array}\right\} \text{iguales cond. necesaria} \\ D_{\bar{f}} = \mathbb{R}^2 \text{ simplemente conexo} \end{array}\right\} \text{cond. suficiente} \Rightarrow \bar{f} \text{ es conservativo}$$

$$\phi = \int_0^1 \bar{f}(tx, ty) \cdot (x, y)\, dt + k = \int_0^1 \left(6tx\,ty + 2t^2y^2 + 2,\ 3t^2x^2 + 4tx\,ty - 2\right) \cdot (x, y)\, dt + k =$$

$$= \int_0^1 \left(\overset{9}{\cancel{6}}t^2x^2y + \overset{6}{\cancel{2}}t^2xy^2 + 2x + \cancel{3t^2x^2y} + \cancel{4t^2xy^2} - 2y\right) dt + k =$$

$$\phi_{(x,y)} = \left[3t^3x^2y + 2t^3xy^2 + 2tx - 2ty\right]_0^1 + k = 3x^2y + 2xy^2 + 2x - 2y + k$$

$$\bar{f} = \bar{\nabla}\phi = \left(6xy + 2y^2 + 2,\ 3x^2 + 4xy - 2\right)$$

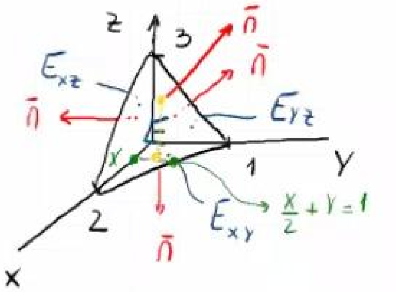
$$\phi_{(1,2)} = 11 = 6 + 8 + 2 - 4 + k \rightarrow \quad k = 11 - 12 = -1$$

$$\boxed{\phi_{(x,y)} = 3x^2y + 2xy^2 + 2x - 2y - 1}$$

$$\phi_{(1,0)} = 2 \cdot 1 - 1 = \boxed{1}$$

P3) Dado el campo $\bar{f}(x,y,z) = (y^3+z^2, y^2, x^2+y^3)$, **calcular** el flujo de $\bar{f}$ a través de la frontera del cuerpo definido por $\frac{x}{2}+y+\frac{z}{3} \leq 1$ en el primer octante.

$$\operatorname{div} \bar{f} = f'_{1x} + f'_{2y} + f'_{3z} = 0 + 2y - 0 = 2y$$



$$\iint_{S=\partial E^+} \bar{f}\, d\bar{\sigma} = \iiint_E \operatorname{div} \bar{f}\, dx\,dy\,dz =$$

$0 \leq x \leq 2$

$0 \leq y \leq 1-\frac{x}{2}$

$0 \leq z \leq (1-\frac{x}{2}-y)3$

$$= \int_0^2 \int_0^{1-\frac{x}{2}} \int_0^{3(1-\frac{x}{2}-y)} 2y\, dz\, dy\, dx = 2\int_0^2 \int_0^{1-\frac{x}{2}} y\, 3\left(1-\frac{x}{2}-y\right) dy\, dx =$$

$$= 6\int_0^2 \left[\frac{y^2}{2} - \frac{x}{2}\frac{y^2}{2} - \frac{y^3}{3}\right]_0^{1-\frac{x}{2}} dx =$$

$$= 6\int_0^2 \left[\frac{1}{2} - \frac{x}{2} + \frac{x^2}{\cancel{4}\,8} - \frac{x}{4}\left(1 - x + \frac{x^2}{4}\right) - \frac{1}{3}\left(1 - 3\frac{x}{2} + 3\frac{x^2}{4} - \frac{x^3}{8}\right)\right] dx =$$

$$= 6\int_0^2 \left(\frac{1}{2} - \cancel{\frac{x}{2}} + \frac{x^2}{8} - \frac{x}{4} + \cancel{\frac{x^2}{4}} - \frac{x^3}{16} - \frac{1}{3} + \cancel{\frac{x}{2}} - \cancel{\frac{x^2}{4}} + \frac{x^3}{24}\right) dx = 6\int_0^2 \left(\frac{-1}{48}x^3 + \frac{x^2}{8} - \frac{x}{4} + \frac{1}{6}\right) dx =$$

$$= 6\left[-\frac{x^4}{48\cdot 4} + \frac{x^3}{24} - \frac{x^2}{8} + \frac{x}{6}\right]_0^2 = 6\left(-\frac{1}{12} + \frac{1}{3} - \frac{1}{2} + \frac{2}{6}\right) = 6\,\frac{\cancel{-1+4-6+4}\;1}{12} = \boxed{\frac{1}{2}}$$

P4) **Hallar** la solución general de la ecuación $y'' - 2y' + 5y = 2x$

Calcular $y(0)$

$$y'' - 2y' + 5y = 2x$$

1°) $y'' - 2y' + 5y = 0 \longrightarrow u^2 - 2u + 5 = 0 \longrightarrow u = \frac{2 \pm \sqrt{4-4\cdot 1\cdot 5}}{2\cdot 1} = \frac{2 \pm \sqrt{-16}}{2}$

$$u = \frac{2 \pm 4i}{2} = \underset{\alpha}{1} \pm \underset{\beta}{2}i$$

$$y_c = e^x\left(C_1 \cos(2x) + C_2 \operatorname{sen}(2x)\right)$$

2°) $y_p = ax + b \longrightarrow 0 - 2a + 5(ax+b) = 2x$

$y_p' = a$ $\qquad -2a + 5ax + 5b = 2x + 0$

$y_p'' = 0$

$$\begin{cases} 5a = 2 & \rightarrow a = \frac{2}{5} \\ -2a + 5b = 0 & \longrightarrow b = 2\cdot\frac{2}{5}\,\frac{1}{5} = \frac{4}{25} \end{cases}$$

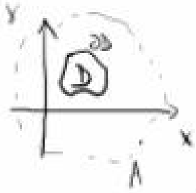
$$y_p = \frac{2}{5}x + \frac{4}{25}$$

3°) $\boxed{y_{(x)} = y_c + y_p = e^x\left(C_1 \cos(2x) + C_2 \operatorname{sen}(2x)\right) + \frac{2}{5}x + \frac{4}{25}}$

4°) $\boxed{y_{(0)} = C_1 + \frac{4}{25}}$

T1) **Enunciar** el Teorema de Green. **Calcular** la circulación de $\bar{f}(x,y)=(xy^2/2,\ 3xy^2/2)$ a lo largo de la curva frontera de la región definida por $x^2 \le y \le x$. **Indicar** el sentido de la circulación adoptado.

HIP: siendo $\bar{f}: A \subseteq \mathbb{R}^2 \to \mathbb{R}^2 \ / \ \bar{f}_{(x,y)} = \left(f_1(x,y),\ f_2(x,y)\right)$



$\bar{f} \in C^1$

$D \subset \mathbb{R}^2 \ / \ \partial D$ curva cerrada simple y regular a trozos

$D \subset A$

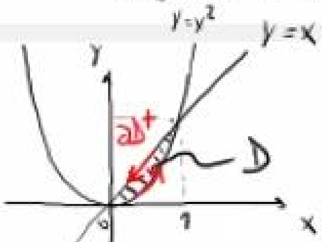
entonces

$$\oint_{\partial D^+} \bar{f}\, d\bar{\lambda} = \iint_D \operatorname{rot} f \, dx\,dy$$

$$\bar{f}_{(x,y)} = \left(\frac{xy^2}{2},\ \frac{3xy^2}{2}\right) \to \operatorname{rot} \bar{f} = f'_{2\,x} - f'_{1\,y} = \frac{3}{2}y^2 - xy$$

$x^2 \le y \le x$

$x^2 \le x \to x^2 - x \le 0$



$$\oint_{\partial D^+} \bar{f}\, d\bar{\lambda} = \int_0^1 \int_{x^2}^{x} \left(\frac{3}{2}y^2 - xy\right) dy\,dx = \int_0^1 \left[\frac{y^3}{2} - x\frac{y^2}{2}\right]_{x^2}^{x} dx =$$

$$= \int_0^1 \left[\cancel{\frac{x^3}{2}} - \cancel{\frac{x^3}{2}} - \left(\frac{x^6}{2} - \frac{x^5}{2}\right)\right] dx = \left[-\frac{x^7}{14} + \frac{x^6}{12}\right]_0^1 =$$

$$= \boxed{-\frac{1}{14} + \frac{1}{12}} =$$

$x \cdot (x-1) \le 0$

$$\begin{cases} x \le 0 \wedge x-1 \ge 0 \\ \vee \\ x \ge 0 \wedge x-1 \le 0 \end{cases} \Rightarrow \begin{cases} \cancel{x \le 0 \wedge x \ge 1} \\ \vee \\ x \ge 0 \wedge x \le 1 \end{cases}$$

T2) **Demostrar** que si y_p es solución de la ecuación $y'' + p(x)\cdot y' + q(x)\cdot y = g(x)$ con $y = y(x)$

entonces $k \cdot y_p$ es solución de la ecuación $y'' + p(x)\cdot y' + q(x)\cdot y = k\cdot g(x)$ ED

$$y_p'' + p(x)\cdot y_p' + q(x)\cdot y_p = g(x)$$

+

$y_{(x)} = k \cdot y_p \longrightarrow$ reemplazo en la E.D.

$y'_{(x)} = k\, y_p'$

$y''_{(x)} = k\, y_p''$

$$k\cdot y_p'' + p(x)\, k\cdot y_p' + q(x)\cdot k\cdot y_p =$$

$$= k\underbrace{\left(y_p'' + p(x)\cdot y_p' + q(x)\cdot y_p\right)}_{\text{POR HIPOTESIS } g(x)} = k\cdot g(x) \quad \text{VERIFICA}$$