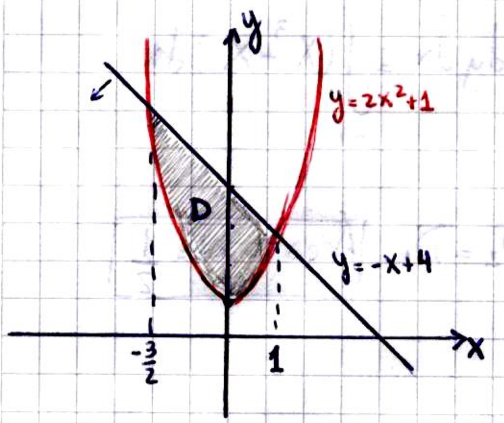
HOJA

FECHA

9 Integrales Multiples

① ⓐ $D = \{(x,y) \in \mathbb{R}^2 / \ y \geq 2x^2+1 \wedge x+y \leq 4\}$



$2x^2+1 = -x+4$

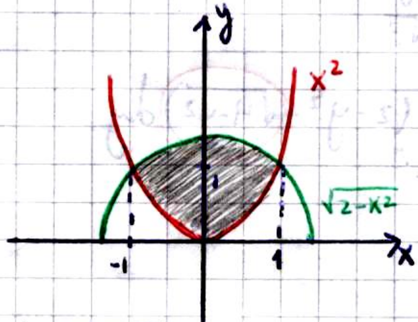
$2x^2+x-3=0$

$$D = \begin{cases} -\frac{3}{2} \leq x \leq 1 \\ 2x^2+1 \leq y \leq -x+4 \end{cases}$$

$$\int_{-\frac{3}{2}}^{1}\int_{2x^2+1}^{-x+4} 1 \, dA = \int_{-\frac{3}{2}}^{1} y \, dx \Big|_{2x^2+1}^{-x+4}$$

$$\int_{-\frac{3}{2}}^{1} (-x+4)-(2x^2+1)\,dx = \int_{-\frac{3}{2}}^{1} (-2x^2-x+3)\,dx = -\frac{2}{3}x^3 - \frac{x^2}{2} + 3x \Big|_{-\frac{3}{2}}^{1} = \boxed{\frac{125}{24}}$$

ⓑ $D: \ x^2 \leq y < \sqrt{2-x^2}$



$x^2 = \sqrt{2-x^2}$

$$D = \begin{cases} -1 \leq x \leq 1 \\ x^2 \leq y \leq \sqrt{2-x^2} \end{cases}$$

$$\int_{-1}^{1}\int_{x^2}^{\sqrt{2-x^2}} 1 \, dy\, dx = \int_{-1}^{1} y \, dx \Big|_{x^2}^{\sqrt{2-x^2}} = \int_{-1}^{1} (\sqrt{2-x^2} - x^2)\,dx$$

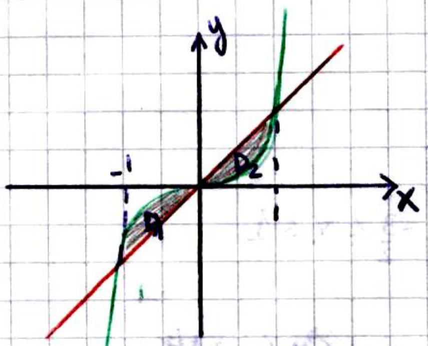
Por tabla

$$= \frac{2}{2}\operatorname{arcsen}\left(\frac{x}{\sqrt{2}}\right) + \frac{\sqrt{2}x}{2}\sqrt{1-\left(\frac{x}{\sqrt{2}}\right)^2} - \frac{x^3}{3}\Big|_{-1}^{1} = \boxed{1,90} \checkmark$$

Comprovando con guía: $\frac{1}{3} + \frac{\pi}{2} \simeq 1,90$

NOTA

(d) D: limitado por las curvas de ecuación $y = x^3$ e $y = x$



$$D_1: \begin{cases} -1 \leq x \leq 0 \\ x \leq y \leq x^3 \end{cases} \qquad D_2 = \begin{cases} 0 \leq x \leq 1 \\ x^3 \leq y \leq x \end{cases}$$

$$\iint_{D_1} 1\, dy\, dx = \int_{-1}^{0}\int_{x}^{x^3} 1\, dy\, dx = \int_{-1}^{0} (x^3 - x)\, dx$$

$$= \frac{x^4}{4} - \frac{x^2}{2} \Big|_{-1}^{0} = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$$

Por simetría $D_2 = D_1$ $\boxed{\text{Área TOTAL} = \frac{1}{2}}$

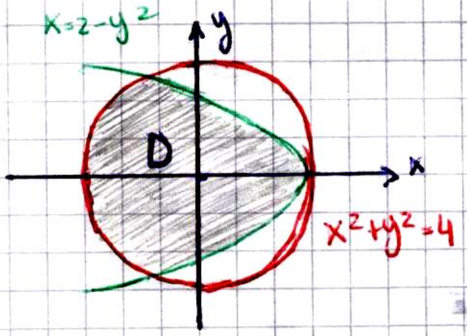
(f) $f_{(x,y)} = (4 - x^2 - y^2;\ 2 - x - y^2)$

$$D = C^+ = \{(x,y) \in \text{Dom } f \,/\, f(x,y) > 0\}$$

$$= \{(x;y) \in \mathbb{R}^2 \,/\, 4 - x^2 - y^2 > 0 \wedge 2 - x - y^2 > 0\}$$

$x^2 + y^2 = 4$

$x = 2 - y^2$



$$D = \begin{cases} -\sqrt{3} \leq y \leq \sqrt{3} \\ -\sqrt{4-y^2} \leq x \leq 2 - y^2 \end{cases}$$

(En este caso conviene usar un tipo 2)

$$\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\sqrt{4-y^2}}^{2-y^2} 1\, dx\, dy = \int_{-\sqrt{3}}^{\sqrt{3}} \left(2 - y^2 + \sqrt{4-y^2}\right) dy$$

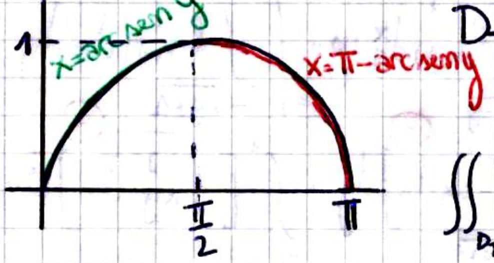
• $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsen\left(\frac{x}{a}\right)$

$$= 2y - \frac{1}{3}y^3 + \frac{y}{2}\sqrt{4 - y^2} + 2\arcsen\left(\frac{y}{2}\right) \Bigg|_{-\sqrt{3}}^{\sqrt{3}} = 2\sqrt{3} - \frac{1}{3}3\sqrt{3} + \frac{\sqrt{3}}{2} + 2\arcsen\left(\frac{\sqrt{3}}{2}\right)$$

$$- \left(-2\sqrt{3} + \frac{3}{3}\sqrt{3} - \frac{\sqrt{3}}{2} + 2\arcsen\left(\frac{\sqrt{3}}{2}\right)\right) = \boxed{3\sqrt{3} + \frac{4}{3}\pi}$$

NOTA

② ⓐ $\iint_D dx\,dy$ D definido por $0 \le y \le \operatorname{sen}(x)$ $0 \le x \le \pi$



$$D_{T_1} = \begin{cases} 0 \le x \le \frac{\pi}{2} \\ 0 \le y \le \operatorname{sen}(x) \end{cases}$$

$$\iint_{D_{T_1}} 1\,dA = \int_0^{\frac{\pi}{2}}\int_0^{\operatorname{sen}(x)} 1\,dy\,dx = \int_0^{\frac{\pi}{2}} \operatorname{sen}(x)\,dx = 1$$

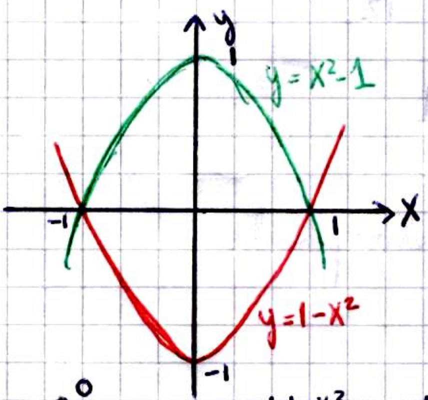
Por simetría multiplico por 2 $\Rightarrow \iint_{D_{T_1}} dA = 2$

$$D_{T_2} = \begin{cases} 0 \le y \le 1 \\ \operatorname{arcsen} y \le x \le \pi - \operatorname{arcsen} y \end{cases}$$

$$\iint_{D_{T_2}} 1\,dA = \int_0^1 \int_{\operatorname{arcsen} y}^{\pi - \operatorname{arcsen} y} 1\,dx\,dy = \int_0^1 \pi - \operatorname{arcsen} y - \operatorname{arcsen} y = \int_0^1 (\pi - 2\operatorname{arcsen} y)\,dy$$

$$\pi y - 2\left(y \operatorname{arcsen} y + \sqrt{1-y^2}\right)\Big|_0^1 = -2 + 2 = 0$$

ⓓ $\iint_D f(x,y)\,dx\,dy$ $\quad x^2-1 \le y \le 1-x^2$ $\quad f(x,y) \begin{cases} xy & \text{si } x \ge 0 \\ -2x & \text{si } x < 0 \end{cases}$



$$D_{T_1} \begin{cases} -1 \le x \le 1 \\ x^2-1 \le y \le 1-x^2 \end{cases}$$

$$\int_{-1}^{1}\int_{x^2-1}^{1-x^2} f(x,y)\,dy\,dx = \int_{-1}^{0}\int_{x^2-1}^{1-x^2} -2x\,dy\,dx + \int_{0}^{1}\int_{x^2-1}^{1-x^2} xy\,dy\,dx$$

$$\int_{-1}^{0} -2xy\,dx\Big|_{x^2-1}^{1-x^2} + \int_0^1 \frac{xy^2}{2}\,dx\Big|_{x^2-1}^{1-x^2} = \int_{-1}^{0} -2x + 2x^3 + 2x^3 - 2x\,dx$$

$x(1-2x^2+x^4) = x - 2x^3 + x^5$

$$+ \int_0^1 \frac{x(1-x^2)^2}{2} - \frac{x(x^2-1)^2}{2} = \int_{-1}^{0} 4x^3 - 4x\,dx + \int_0^1 x(1-x^2)^2$$

¿0?

$$x^4 - 2x^2\Big|_{-1}^{0} + \frac{x^6}{6} - \frac{x^4}{2} + \frac{x^2}{2}\Big|_0^1 = 1 + \frac{1}{6}$$

Cambio orden de integración (y)

$$D_1: \begin{cases} 0 \le y \le 1 \\ -\sqrt{1-y} \le x \le \sqrt{1-y} \end{cases} \qquad D_2: \begin{cases} -1 \le y \le 0 \\ -\sqrt{y+1} \le x \le \sqrt{y+1} \end{cases}$$

$$\iint_{D_{T_2}} f(x,y)\, dA = \iint_{D_1} f(x,y)\, dA + \iint_{D_2} f(x,y)\, dA = \int_0^1 \int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x,y)\, dx\, dy + \int_{-1}^{0} \int_{-\sqrt{y+1}}^{\sqrt{y+1}} f(x,y)\, dx\, dy$$

$$= \int_0^1 \left[\int_{-\sqrt{1-y}}^{0} -2x\, dx \right] dy + \int_0^1 \left[\int_0^{\sqrt{1-y}} xy\, dx \right] dy + \int_{-1}^{0} \left[\int_{-\sqrt{y+1}}^{0} -2x\, dx \right] dy + \int_{-1}^{0} \left[\int_0^{\sqrt{y+1}} xy\, dx \right] dy$$

(1) $\displaystyle \int_0^1 -x^2 \Big|_{-\sqrt{1-y}}^{0} dy = \int_0^1 (1-y)\, dy = y - \frac{y^2}{2} \Big|_0^1 = \frac{1}{2}$

(2) $\displaystyle \int_0^1 y \frac{x^2}{2} \Big|_0^{\sqrt{1-y}} dy = \int_0^1 y \frac{(1-y)}{2}\, dy = \int_0^1 \frac{y - y^2}{2} = \frac{1}{2}\left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_0^1 = \frac{1}{12}$

(3) $\displaystyle \int_{-1}^{0} -x^2 \Big|_{-\sqrt{y+1}}^{0} dy = \int_{1}^{0} y+1\, dy = \frac{y^2}{2} + y \Big|_1^0 = -\frac{3}{2}$

(4) $\displaystyle \int_{-1}^{0} y \frac{x^2}{2} \Big|_0^{\sqrt{y+1}} dy = \frac{1}{2}\int_{-1}^{0} y^2 + y\, dy = \frac{1}{2}\left(\frac{y^3}{3} + \frac{y^2}{2}\right)\Big|_{-1}^{0} = -\frac{1}{12}$

$$\iint_{D_{T_2}} f(x,y)\, dA = \frac{1}{2} + \frac{1}{12} - \frac{3}{2} - \frac{1}{12} = -1$$

2a No me da igual a la guía, si encuentran el error pasenlo

2b Me da $\neq$ Sg

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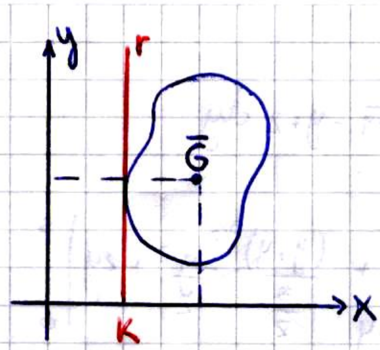
(4)



$\bar{G} = (\bar{x}; \bar{y})$

$$\bar{x} = \frac{\iint_D x\, \delta(x,y)\, dA}{\iint_D \delta(x,y)\, dA} \qquad \bar{y} = \frac{\iint_D y\, \delta(x,y)\, dA}{\iint_D \delta(x,y)\, dA}$$

Momento de inercia: $I = \iint_D (\text{distancia})^2\, \delta(x,y)\, dA$

$$I = \iint_D |\bar{x} - K|^2 \delta(x,y)\, dA = \iint_D (\bar{x} - K)^2 \delta(x,y)\, dA = \iint_D (x^2 - 2Kx + K^2)\, \delta(x,y)\, dA$$

$$I = K^2 \iint_D \delta(x,y)\, dA - 2K \iint_D x\, \delta(x,y)\, dA + \iint_D x^2 \delta(x,y)\, dA$$

Cuadrática cóncava positiva $\Longrightarrow I_{MIN}$ si $K = X_v$ $\qquad X_v = \frac{-b}{2a}$

$$X_v = \frac{\cancel{-}(\cancel{-}2 \iint_D x\, \delta(x,y)\, dA)}{\cancel{2} \iint_D \delta(x,y)\, dA} = \bar{x}$$

Se demuestra que I es min cuando r pasa por $\bar{G}$

(5) a)

$$\int_0^1 \int_0^{\sqrt{1-x^2}} x\, dy\, dx = \int_0^1 xy \Big|_0^{\sqrt{1-x^2}} dx = \int_0^1 x\sqrt{1-x^2}\, dx$$

$u = 1 - x^2$

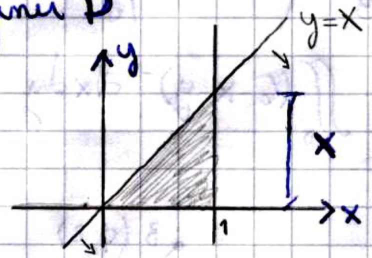
$du = -2x\, dx$

$-\frac{1}{2} du = x\, dx$

$$-\frac{1}{2}\int_0 u^{1/2}\, du = -\frac{1}{2} \frac{u^{3/2}}{\frac{3}{2}} \Big|_0^1 = -\frac{1}{2}\frac{2}{3}(1-x^2)^{3/2}\Big|_0^1 = \boxed{\frac{1}{3}}$$

b) Como e^{x^2} no tiene primitiva hay que redefinir D

$$\int_0^1 \int_y^1 e^{x^2}\, dx\, dy \Longrightarrow y \leq x \leq 1 \quad 0 \leq y \leq 1$$



Cambio el orden

$$\int_0^1 \int_0^x e^{x^2}\, dy\, dx = \int_0^1 e^{x^2} y \Big|_0^x dx$$

$$\int_0^1 x e^{x^2}\, dx = \frac{1}{2}\int_0^1 e^u\, du$$

$x^2 = u$

$\frac{1}{2} du = x$

$$\frac{1}{2} e^{x^2}\Big|_0^1 = \boxed{\frac{e-1}{2}}$$

NOTA

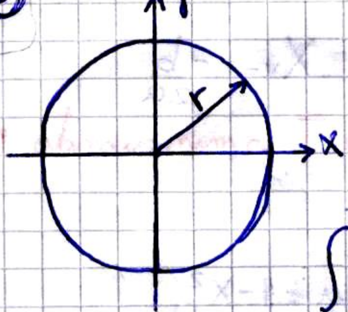
ⓒ $\int_{-4}^{0} dy \int_{-\sqrt{y+4}}^{\sqrt{y+4}} dx + \int_{0}^{5} dy \int_{y-2}^{\sqrt{y+4}} dx$

$\int_{-4}^{0} x \Big|_{-\sqrt{y+4}}^{\sqrt{y+4}} + \int_{0}^{5} x \Big|_{y-2}^{\sqrt{y+4}} = \int_{-4}^{0} 2\sqrt{y+4}\, dy + \int_{0}^{5} \sqrt{y+4} - y + 2\, dy$

$2 \frac{u^{3/2}}{\frac{3}{2}} \Big|_{-4}^{0} + \frac{u^{3/2}}{3/2} - \frac{y^2}{2} + 2y \Big|_{0}^{5} = \frac{2(y+4)^{3/2}}{\frac{3}{2}} \Big|_{-4}^{0} + \frac{(y+4)^{3/2}}{\frac{3}{2}} - \frac{y^2}{2} + 2y \Big|_{0}^{5}$

$= \frac{32}{3} + \frac{31}{2} - \frac{16}{3} = \boxed{\frac{125}{6}}$

* ③



$\delta(x,y) = K|y|$ Coordenadas polares

$m = \iint_D \delta(x,y)\, dA$

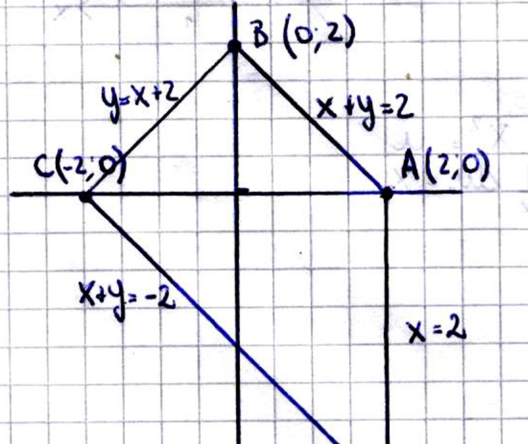
$H: \begin{cases} 0 \le \varphi \le 2\pi \\ 0 \le r \le A \end{cases}$

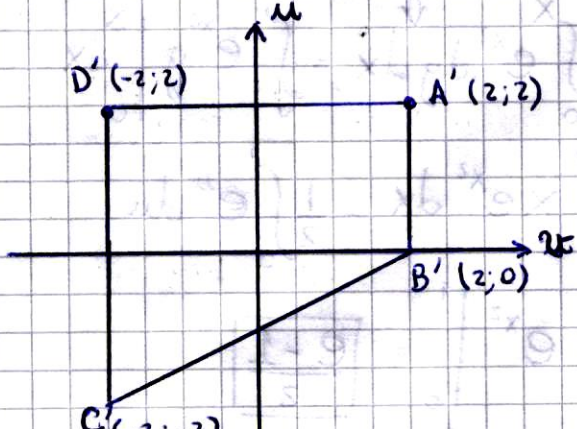
$\int_0^{2\pi} \int_0^{A} K|y|\, r\, dr\, d\varphi = \int_0^{2\pi} \int_0^{A} K\, |r \operatorname{sen}\varphi|\, r\, dr\, d\varphi$

$\int_0^{2\pi} K \frac{r^3}{3} |\operatorname{sen}\varphi| \Big|_0^{A} d\varphi = K \frac{A^3}{3} \left[\int_0^{\pi} \operatorname{sen}\varphi\, d\varphi + \int_{\pi}^{2\pi} -\operatorname{sen}\varphi\, d\varphi \right]$

$K \frac{A^3}{3} \left(-\cos\varphi \Big|_0^{\pi} + \cos\varphi \Big|_{\pi}^{2\pi} \right) = K \frac{A^3}{3} (2+2) = \boxed{\frac{4KA^3}{3}}$

⑥ⓐ $\iint_D (6-x-y)^{-1}\, dx\, dy$ $D: \begin{cases} -2 \le x+y \le 2 \\ y < x+2 \le 4 \end{cases}$ $(x,y) = (uv;\, u-v)$



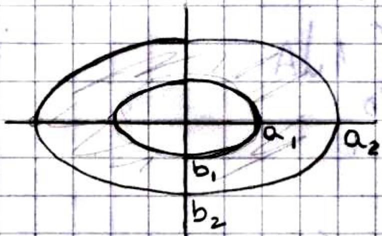


NOTA

$$\int_{-2}^{2}\int_{\frac{u}{2}-1}^{2} (6-u)^{-1}\, dv\, du = \int_{-2}^{2} (6-u)^{-1} v \Big|_{\frac{u}{2}-1}^{2} du = \int_{-2}^{2} \frac{(u-6)+4}{u-6}$$

$$\frac{1}{2} u \Big|_{-2}^{2} + \frac{1}{2} \times 4 \ln(u-6) \Big|_{-2}^{2} = \boxed{2}$$

(b) $1 \le \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 4$ $\quad (x;y) = (a\rho\cos(\varphi); b\rho\,\text{sen}(\varphi))$



$$\frac{\rho^2\cos^2(\varphi)}{a} + \frac{\rho^2\,\text{sen}^2(\varphi)}{b} = \frac{b\rho^2\cos^2(\varphi) + a\rho^2\,\text{sen}^2(\varphi)}{(a+b)}$$

$$= \rho^2 \frac{(a+b)}{(a+b)} = \rho^2 \qquad 1 \le \rho^2 \le 4$$

$$1 \le r \le 2$$

$$\begin{cases} x = a r \cos\varphi \\ y = b r\,\text{sen}\varphi \end{cases} \qquad \int_0^{2\pi}\int_1^2 (r\cos\varphi + \rho\,\text{sen}\varphi)\, abr\, dr\, d\varphi$$

$$\int_0^{2\pi} ab \int_1^2 r^2 (\cos\varphi + \text{sen}\varphi)\, dr\, d\varphi = ab \int_0^{2\pi} \cos\varphi + \text{sen}\varphi\, d\varphi \int_1^2 r^2\, dr$$

$$ab\left(\text{sen}\varphi - \cos\varphi \Big|_0^{2\pi} + \frac{r^3}{3}\Big|_1^2\right) = ab($$

No SALE

ⓔ $\iint_D y(x^2+y^2)^{-1}\,dx\,dy$ $\quad D_{r\varphi}: \begin{cases} \pi/4 \le \varphi \le \frac{3}{4}\pi \\ \frac{2}{\operatorname{sen}\varphi} \le r \le \frac{5}{\operatorname{sen}\varphi} \end{cases}$

$r \operatorname{sen}\varphi = 2 \Rightarrow r = \frac{2}{\operatorname{sen}\varphi}$

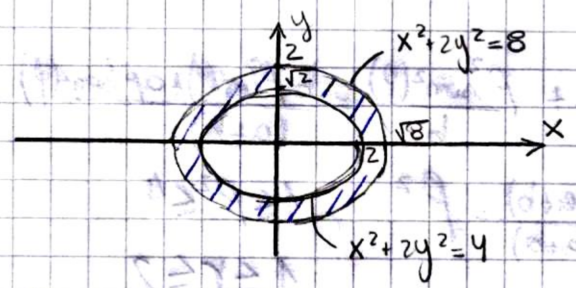
$$\iint_D \frac{y}{x^2+y^2}\,dA = \int_{\pi/4}^{3/4\pi}\int_{\frac{2}{\operatorname{sen}\varphi}}^{\frac{5}{\operatorname{sen}\varphi}} \frac{\cancel{r}\operatorname{sen}\varphi}{\cancel{r^2}}\,\cancel{r}\,dr\,d\varphi = \int_{\pi/4}^{3/4\pi} \operatorname{sen}\varphi\, r \Big|_{\frac{2}{\operatorname{sen}\varphi}}^{\frac{5}{\operatorname{sen}\varphi}} d\varphi$$

$$= \int_{\frac{\pi}{4}}^{\frac{3}{4}\pi} 3\,d\varphi = 3\varphi\Big|_{\pi/4}^{\frac{3}{4}\pi} = \boxed{\frac{3\pi}{2}}$$

⑦ ⓐ $f(x,y) = e^{x^2+2y^2}$ $\quad C_{e^4}$; C_{e^8} Dom f: $\mathbb{R}^2$

$C_{e^4}: \{(x;y)\in\mathbb{R}^2 : e^{x^2+2y^2} = e^4 \Leftrightarrow x^2+2y^2=4$

$e^8 \Leftrightarrow x^2+2y^2 = 8$



$A(D) = \iint_D 1\,dA$

$\begin{cases} x = ar\cos\varphi \\ y = br\operatorname{sen}\varphi \end{cases}$ det $|DT| = abr$ cambio coordenada $\begin{cases} x = r\cos\varphi \\ y = \sqrt{\frac{1}{2}}\,r\operatorname{sen}\varphi \end{cases}$

$D: \begin{cases} 0 \le \varphi \le 2\pi \\ 2 \le r \le 2\sqrt{2} \end{cases}$

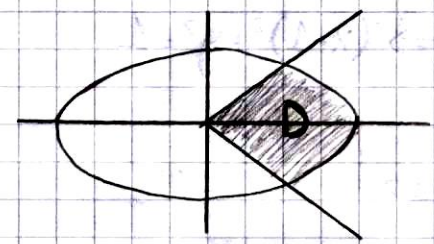
$$A(D) = \int_0^{2\pi}\int_2^{2\sqrt{2}} 1\,1\sqrt{\tfrac{1}{2}}\,r\,dr\,d\varphi = \int_0^{2\pi} \frac{1}{\sqrt{2}}\frac{r^2}{2}\Big|_2^{2\sqrt{2}} d\varphi = \frac{1}{\sqrt{2}}\int_0^{2\pi}(4-2)\,d\varphi$$

$$= \frac{2}{\sqrt{2}}\varphi\Big|_0^{2\pi} = \frac{4}{\sqrt{2}}\pi = \boxed{2\sqrt{2}\pi}$$

Area elipse: $ab\pi$
por si quieren verificar.

NOTA

7b) $f(x;y) = e^{x^2+2y^2}$ $\quad D = \{(x;y) \in \mathbb{R}^2 / x^2+2y^2 \le 4 \wedge x \ge \sqrt{2}|y| \}$



$$\begin{cases} x = 2r\cos\varphi \\ y = \sqrt{2}\, r \operatorname{sen}\varphi \end{cases} \qquad D: \begin{cases} -\pi/4 \le \varphi \le \pi/4 \\ 0 < r < 1 \end{cases}$$

$$\frac{x^2}{4} + \frac{y^2}{2} = 1 \longrightarrow \frac{4r^2\cos^2\varphi}{\cancel{4}} + \frac{2r^2\operatorname{sen}^2\varphi}{\cancel{2}} = 1$$

$$x^2+2y^2 = 4r^2\cos^2\varphi + 4r^2\operatorname{sen}^2\varphi = 4r^2$$

$$\iint_D e^{x^2+2y^2}\, dA = \int_{-\pi/4}^{\pi/4} \int_0^1 e^{4r^2}\, 2\sqrt{2}\, r\, dr\, d\varphi$$

$$\frac{\pi}{2}\, 2\sqrt{2}\, \frac{(e^4-1)}{8} = \boxed{\sqrt{2}\pi\, \frac{(e^4-1)}{8}}$$

8) a) $\iint_D e^{-x^2-y^2}\, dA = \int_{-\infty}^{+\infty} e^{-x^2}\, dx \int_{-\infty}^{+\infty} e^{-y^2}\, dy = \left[\int_{-\infty}^{+\infty} e^{-u^2} \right]^2$

$= \int_0^{2\pi} \int_0^{+\infty} e^{-r^2}\, r\, dr\, d\varphi = \boxed{\sqrt{\pi}}$ pero como elevo al cuadrado $= \boxed{\pi}$

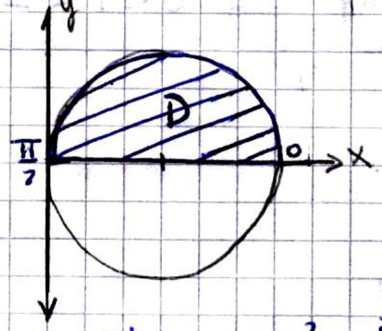
b) $= 2\pi \lim_{R \to +\infty} -\frac{1}{2} e^{-r^2} \Big|_0^R = \boxed{\pi}$

c) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{z^2}{2}}\, dz = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-(z/\sqrt{2})^2} = \frac{\sqrt{2}}{\sqrt{2\pi}} \overbrace{\int_{-\infty}^{+\infty} e^{-u^2}\, du}^{=\sqrt{\pi}} = \sqrt{\pi}\left(\frac{\sqrt{2}}{\sqrt{2}\sqrt{\pi}}\right) = \boxed{1}$

10 a) $\int_0^{\pi/2} d\varphi \int_0^{2\cos(\varphi)} r^3 dr$ $\begin{cases} x = r\cos\varphi \\ y = r\operatorname{sen}\varphi \end{cases}$ $D: \begin{cases} 0 \le \varphi \le \pi/2 \\ 0 \le r \le 2\cos\varphi \end{cases}$

$0 \le r^2 \le 2r\cos\varphi \Rightarrow 0 \le x^2+y^2 \le 2x$

$x^2+y^2 = 2x \Rightarrow x^2-2x+1+y^2 = 1 \Rightarrow (x-1)^2+y^2 = 1$



$D: \begin{cases} 0 \le x \le 2 \\ 0 \le y \le \sqrt{1-(x-1)^2} \end{cases}$

$\cancel{1} - x^2 + 2x \cancel{-1}$

$$\int_0^{\pi/2} d\varphi \int_0^{2\cos(\varphi)} r^3 dr = \int_0^2 \int_0^{\sqrt{1-(x-1)^2}} (x^2+y^2)\,dy\,dx$$

$$\int_0^2 \left(\int_0^{\sqrt{1-(x-1)^2}} x^2\,dy + \int_0^{\sqrt{1-(x-1)^2}} y^2\,dy \right) dx = \int_0^2 \left(x^2 y \Big|_0^{\sqrt{1-(x-1)^2}} + \frac{y^3}{3} \Big|_0^{\sqrt{1-(x-1)^2}} \right) dx$$

$$\int_0^2 x^2\sqrt{1-(x-1)^2} + \int_0^2 \frac{\sqrt{1-(x-1)^2}}{3}\,dx = \boxed{\int_0^2 x^2\sqrt{2x-x^2}\,dx + \int_0^2 \frac{\sqrt{2x-x^2}}{3}\,dx}$$

Resuelvo polares menos cuentas

$$\int_0^{\pi/2} d\varphi \int_0^{2\cos\varphi} r^3 dr = \int_0^{\pi/2} \frac{r^4}{4} \Big|_0^{2\cos\varphi} d\varphi = \int_0^{\pi/2} 4\cos^4\varphi\, d\varphi = \boxed{\frac{3}{4}\pi}$$

• Las integrales simples las resuelvo con Mathcad o calculadora ya que no considero que sea lo elemental del ejercicio

10b $\int_{-\pi/6}^{\pi/3} \int_0^{\sqrt{3}/\cos\varphi} \rho^2 \cos\varphi\, d\rho\, d\varphi$ $D: \begin{cases} -\pi/6 \le \varphi \le \pi/3 \\ 0 \le \rho \le \frac{\sqrt{3}}{\cos\varphi} \end{cases}$

$\operatorname{tg}\varphi = \frac{y}{x}$ $\varphi = \pi/3 \rightarrow \operatorname{tg}\pi/3 = y/x = y = \sqrt{3}x$

$\varphi = -\pi/6 \rightarrow \operatorname{tg}(-\pi/6) = \frac{y}{x} = -\frac{1}{\sqrt{3}} \rightarrow -\frac{1}{\sqrt{3}}x = y$

$\overset{*}{\int_0^{\sqrt{3}}} x\,dx \int_{-\frac{x}{\sqrt{3}}}^{\sqrt{3}x} dy$ * Lo dejo planteado $r = \frac{\sqrt{3}}{\cos\varphi}$ ¿?

* $r\cos\varphi = \sqrt{3} \Rightarrow x = \sqrt{3}$

NOTA

HOJA

FECHA

(11) Sabiendo que $\begin{cases} x = r\cos\varphi \\ y = r\,\text{sen}\,\varphi \\ z = z \end{cases}$ o $\begin{cases} x^2+y^2=r^2 \\ \varphi = \text{arctg}\,(y/x) \\ z = z \end{cases}$

Simplemente es despejar de la integral y queda

$$\int_0^2 dx \int_0^{\sqrt{4+x^2}} dy \int_0^{4-x^2-y^2} \sqrt{x^2+y^2}\, dz$$

Resuelvo en polares

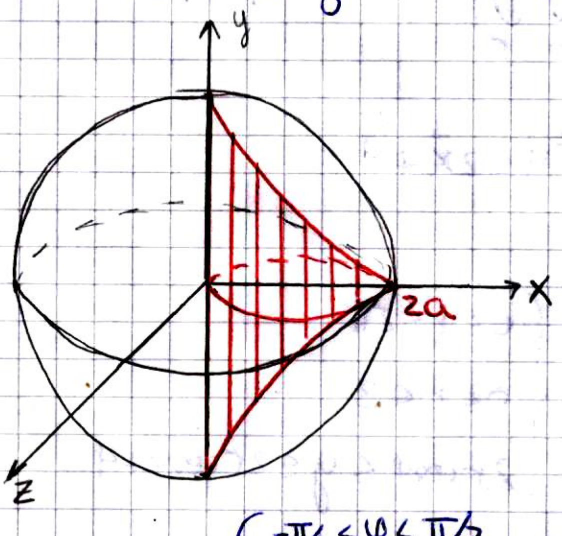
$$\int_0^{\pi/2}\int_0^2\int_0^{4-\rho^2} \rho^2\, dz\, d\rho\, d\varphi = \int_0^{\pi/2}\int_0^2 \rho^2 z\Big|_0^{4-\rho^2} d\rho\, d\varphi = \int_0^{\pi/2}\int_0^2 \rho^2(4-\rho^2)\,d\rho\,d\varphi$$

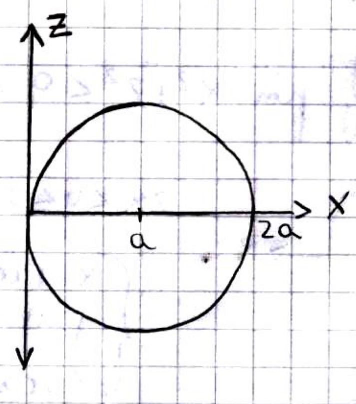
$$\int_0^{\pi/2}\int_0^2 4\rho^2-\rho^4\, d\rho\, d\varphi = \int_0^{2\pi} \frac{4}{3}\rho^3 - \frac{\rho^5}{\sqrt{5}}\Big|_0^2 = \int_0^{2\pi}\frac{64}{15}\,d\varphi = \frac{\pi}{2}\,\frac{64}{15} = \boxed{\frac{32}{15}\pi}$$

(12) Son muchos voy hacer los que creo más "difíciles".

(c)

H definido por $x^2+z^2 \le 2ax$ interior a la esfera de radio $2a$ con centro en el origen de coordenadas / $x^2+y^2+z^2 \le 4a^2$





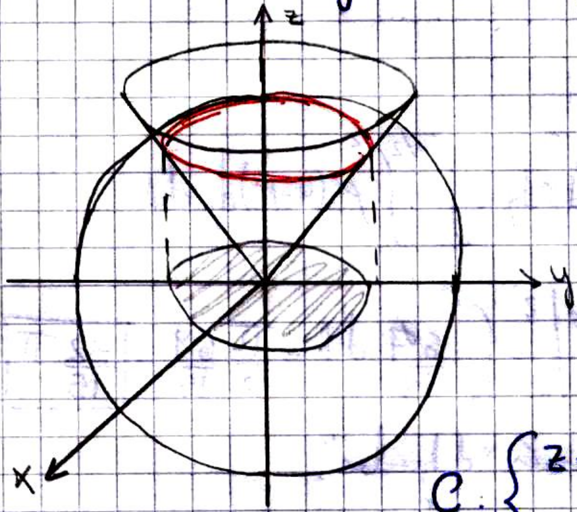
$$H_{r\varphi y} = \begin{cases} -\pi/2 \le \varphi \le \pi/2 \\ 0 \le r \le x^2+z^2 = 2ax \Rightarrow r^2 = 2a\cos\varphi \\ -\sqrt{4a^2-r^2} \le y \le \sqrt{4a^2-r^2} \end{cases}$$

NOTA

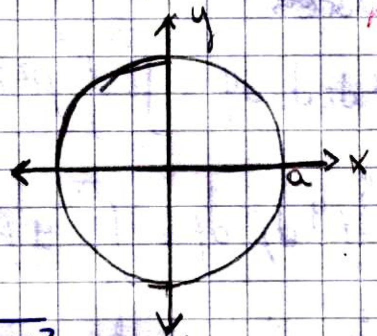
$$Vol(H) = \iiint_H 1\, dV = \int_{-\pi/2}^{\pi/2} \int_0^{2a\cos\varphi} \int_{-\sqrt{4a^2-r^2}}^{\sqrt{4a^2-r^2}} 1\, r\, dy\, dr\, d\varphi$$

$$\int_{-\pi/2}^{\pi/2} \int_0^{2a\cos\varphi} r y \Big|_{-\sqrt{4a^2-r^2}}^{\sqrt{4a^2-r^2}} = \int_{-\pi/2}^{\pi/2} \int_0^{2a\cos\varphi} 2r\sqrt{4a^2-r^2}\, dr\, d\varphi$$

12 d $z \geq \sqrt{x^2+y^2}$; $x^2+y^2+z^2 \leq 2a^2$ con $a>0$



$$\sqrt{x^2+y^2} \leq z \leq \sqrt{2a^2-x^2-y^2}$$



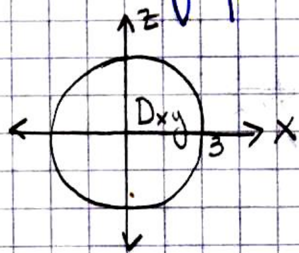
$\varphi = z$
$r = y$
$\cancel{z} = \cancel{x}$

$$C: \begin{cases} z = \sqrt{x^2+y^2} \\ x^2+y^2+z^2 = 2a^2 \end{cases} \Longrightarrow x^2+y^2 = a^2$$

$$H: \begin{cases} 0 \leq \varphi \leq 2\pi \\ 0 \leq r \leq a \\ r \leq z \leq \sqrt{2a^2-r^2} \end{cases}$$

$$Vol(H) = \int_0^{2\pi} \int_0^a \int_r^{\sqrt{2a^2-r^2}} 1\, r\, dz\, dr\, d\varphi$$ queda planteada.

12 f H def por $x^2+z^2 \leq 9$; $y \geq 2x$ $y \leq 2x+4$



$2x \leq y \leq 2x+4$

$$\begin{cases} x = r\cos\varphi \\ y = y \\ z = r\sin\varphi \end{cases}$$

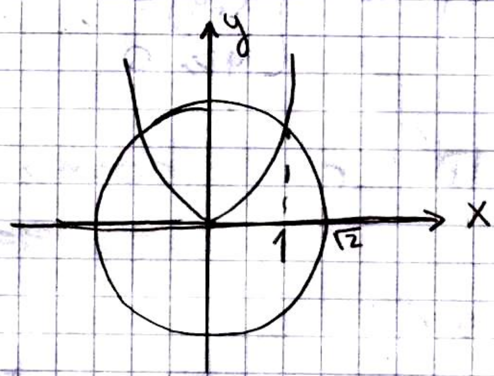
$$H: \begin{cases} 0 \leq \varphi \leq 2\pi \\ 0 \leq r \leq 3 \\ 2r\cos\varphi \leq y \leq 2r\cos\varphi + 4 \end{cases}$$

$$Vol(H) = \iiint_H 1\, dV = \int_0^3 \int_0^{2\pi} \int_{2r\cos\varphi}^{2r\cos\varphi+4} 1\, r\, dy\, dr\, d\varphi$$

NOTA

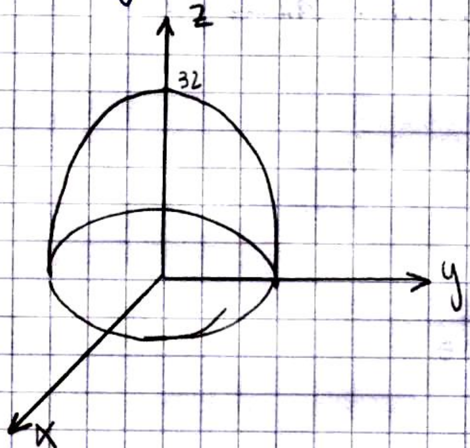
(12) (g) $y \geq x^2$; $x^2+y^2 \leq 2$; $z \geq 0$; $z \leq x$

$$H: \begin{cases} 0 \leq x \leq 1 \\ x^2 \leq y \leq \sqrt{2-x^2} \\ 0 \leq z \leq x \end{cases}$$



$$Vol(H) = \int_0^1 \int_{x^2}^{\sqrt{2-2x^2}} \int_0^x 1 \, dz \, dy \, dx$$

(12 h) $x^2+2y^2+z \leq 32$; $z \geq x^2$



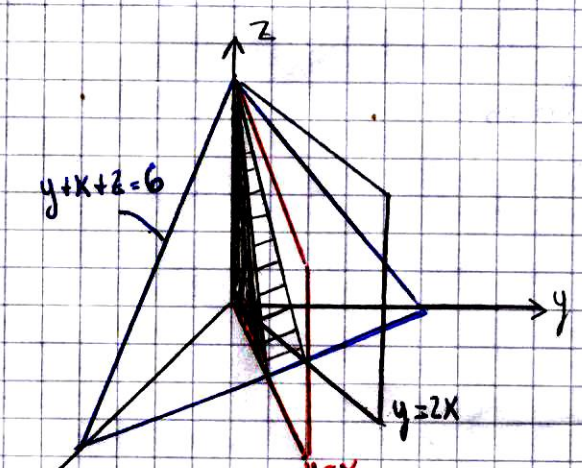
$$\begin{cases} z = 32 - x^2 - 2y^2 \\ z = x^2 \end{cases}$$

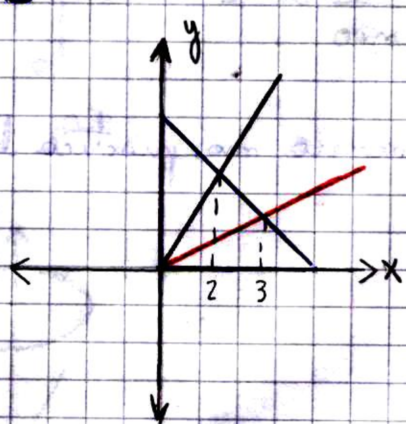
$$x^2 = 32 - x^2 - 2y^2$$
$$x^2 + y^2 = 16$$

$$H: \begin{cases} 0 \leq \varphi < 2\pi \\ 0 < r \leq 4 \\ r^2\cos^2\varphi \leq z \leq 32 - r^2\cos^2\varphi - 2r^2 sen^2\varphi \end{cases}$$

$$Vol(H) = \iiint_H 1 \, dv = \int_0^{2\pi} \int_0^4 \int_{r^2\cos^2\varphi}^{32-r^2\cos^2\varphi - 2r^2 sen^2\varphi} 1 \, r \, dz \, dr \, d\varphi$$

(13) $y = x$ $y = 2x$ $x+y+z=6$; $z=0$





Al final dejo algunos ejercicios que terminaron saliendo:

(9) $\iint \frac{x+4y}{x^2}\,dx\,dy$ D: $\begin{cases} x \geq y \\ x+4y \leq 4 \\ y \geq 0 \end{cases}$ con coordenadas polares.

$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases}$ $|Df| = r$ $\cancel{r}\cos\theta \geq \cancel{r}\,\text{sen}\,\theta$

$\theta = \pi/4$ (único valor donde son iguales)

$r\cos\theta + 4r\,\text{sen}\,\theta \leq 4$

$r\,\text{sen}\,\theta \leq 1 - \frac{r\cos\theta}{4} \longrightarrow r \leq \frac{1}{\text{sen}\,\theta + \frac{\cos\theta}{4}}$

$\begin{cases} 0 \leq r \leq \frac{1}{\text{sen}\,\theta + \frac{\cos\theta}{4}} \\ 0 \leq \theta \leq \pi/4 \end{cases}$

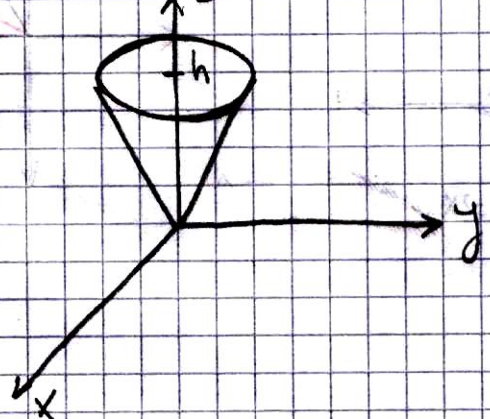
$\iint \frac{\cancel{r}\cos\theta + 4\cancel{r}\,\text{sen}\,\theta}{\cancel{r^2}\cos^2\theta}\,\cancel{r}\,dr\,d\theta$

$\int_0^{\pi/4}\int_0^{\frac{1}{\text{sen}\,\theta + \frac{\cos\theta}{4}}} \frac{\cos\theta + 4\,\text{sen}\,\theta}{\cos^2\theta}\,dr\,d\theta$

$\int_0^{\pi/4} \frac{\cos\theta + 4\,\text{sen}\,\theta}{\cos^2\theta}\,\frac{1}{\text{sen}\,\theta + \frac{\cos\theta}{4}}\,d\theta = \int_0^{\pi/4}\left(\frac{\cancel{\cos\theta + 4\,\text{sen}\,\theta}}{\cos^2\theta}\right)\left(\frac{4}{\cancel{4\,\text{sen}\,\theta + \cos\theta}}\right)dr$

$4\int_0^{\pi/4} \frac{1}{\cos^2\theta}\,dr = 4\,\text{Tg}\,\theta\Big|_0^{\pi/4} = 4(1-0) = \boxed{4}$

(15) La manera más práctica de calcularlo es:



NOTA

HOJA

FECHA

$$z = \sqrt{x^2+y^2} \Rightarrow \begin{cases} x = r\cos\theta\, \text{sen}\,\omega \\ y = r\,\text{sen}\,\theta\, \text{sen}\,\omega \\ z = r\cos\omega \end{cases} \qquad |Df| = r^2\,\text{sen}\,\omega$$

$$h = z = r\cos\omega \Rightarrow r = \frac{h}{\cos\omega} \Rightarrow 0 \le r \le \frac{h}{\cos\omega}$$

$$\int_0^{\omega}\int_0^{2\pi}\int_0^{\frac{h}{\cos\omega}} r^2\,\text{sen}\,\omega\, dr\,d\theta\,d\omega \quad \left(\begin{matrix}\text{Hecha en} \\ \text{MathCAD}\end{matrix}\right) = \left[\frac{\pi h^3}{3}(\tan^2(\omega))\right]$$

17 a $z = 4 - x^2 - y^2$, $z = 8 - 2x^2 - 2y^2$

Densidad distancia al eje $z \Rightarrow K \iiint \sqrt{x^2+y^2}\, dz\,dy\,dx$

$$4 - x^2 - y^2 = 8 - 2x^2 - 2y^2 \Rightarrow x^2 + y^2 = 4$$

$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \\ z = z \end{cases} \qquad |Df| = r \qquad \begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \\ 4 - r^2 \le z \le 8 - 2r^2 \end{cases}$$

$$K\int_0^{2\pi}\int_0^{2}\int_{4-r^2}^{8-2r^2} \sqrt{r^2\cos^2\theta + r^2\,\text{sen}^2\theta}\; r\, dz\,d\theta\,dr = K\int_0^{2\pi}\int_0^{2}\int_{4-r^2}^{8-2r^2} \sqrt{r^2}\, r\, dz\,d\theta\,dr$$

$$= K\int_0^{2\pi}\int_0^{2} r^2(8 - 2r^2 - 4 + r^2)\,dr\,d\theta = K\int_0^{2\pi}\int_0^{2} r^2(4 - r^2)\,dr\,d\theta$$

$$= K\int_0^{2\pi}\int_0^{2} 4r^2 - r^4\, dr\,d\theta = K\int_0^{2\pi} \left.\frac{4}{3}r^3 - \frac{r^5}{5}\right|_0^2 d\theta = K\int_0^{2\pi} \frac{64}{15}\, d\theta = \boxed{K\,\frac{128\pi}{15}}$$

Los dos que siguen son iguales les dejo las densidades

b Densidad $\Rightarrow K \iiint z\, dz\,dy\,dx$

c Densidad $\Rightarrow K \iiint y\, dz\,dy\,dx$

NOTA