12 - Ecuaciones Diferenciales - 2° parte

1 a

$$y' = \frac{y}{x} + \frac{y^2}{x^2} \quad \text{con } y(1)=1$$

$F(xt, yt) = F(x,y)$

$$\frac{y\cancel{t}}{x\cancel{t}} + \frac{y^2\cancel{t^2}}{x^2\cancel{t^2}}$$ Verifico es homogenea

$$\Longrightarrow y = zx \qquad y' = z'x + z$$

$$z'x + \cancel{z} = \cancel{\frac{zx}{x}} + \frac{z^2\cancel{x^2}}{\cancel{x^2}}$$

$$z'x = z^2 \Longrightarrow \frac{1}{z^2}z' = \frac{1}{x}$$

$$\int \frac{1}{z^2}\,dz = \int \frac{1}{x}\,dx$$

$$-\frac{1}{z} = \ln|x| + C$$

$$-\frac{1}{y/x} = \ln|x| + C \Longrightarrow -\frac{x}{y} = \ln|x| + C$$

$$y = -\frac{x}{\ln|x| + C}$$

$$1 = -\frac{1}{0 + C} \longrightarrow C = -1$$

$$\boxed{SP: \; y\ln|x| - y + x = 0}$$

b $(x^2+y^2)\,dx - 2xy\,dy = 0$

$$(x^2+y^2)\,dx = 2xy\,dy$$

$$\frac{x^2+y^2}{2xy} = y'$$

$$\frac{\cancel{t^2}x^2 + \cancel{t^2}y^2}{2xy\cancel{t^2}}$$ es homogenea.

$$y' = \frac{x^2+y^2}{2xy} \qquad y = zx \qquad y' = z'x + z$$

$$\frac{1+z^2-2z^2}{2z} = \frac{1-z^2}{2z}$$

$$z'x + z = \frac{x^2 + z^2x^2}{2x^2z}$$

$$z'x+z = \frac{x^2(1+z^2)}{2x^2z} \Rightarrow z'x + z = \frac{1+z^2}{2z}$$

$$z'x = \left(\frac{1-z^2}{2z}\right) \Rightarrow z' = \left(\frac{1-z^2}{2z}\right)\frac{1}{x}$$

$$\int \frac{2z}{1-z^2}\,dz = \int \frac{1}{x}\,dx \qquad u = 1-z^2 \quad du = -2z \quad -du = 2z$$

$$-\int \frac{du}{u} = \int \frac{1}{x}\,dx$$

$$-\ln|1-z^2| = \ln|x| + C$$

$$e^{-\ln|1-z^2|} = e^{\ln|x|}e^{C}$$

$$\frac{1}{1-z^2} = xB \Rightarrow 1 = xB - \frac{y^2B}{x}$$

$$\left[SG.\ y^2 = x^2 - \frac{x}{B}\right]$$

c) $\dfrac{dy}{dx} = \dfrac{y + x\cos^2(y/x)}{x}$ con $y(1) = \pi/4$ (Es homogenea)

Si no es de guía no se confien reemplacen !!!

$$y = zx \qquad y' = z'x + z$$

$$z'x + z = \frac{zx + x\cos^2(z)}{\cancel{x}} \Rightarrow z'x + \cancel{z} = \cancel{z} + \cos^2(z)$$

$$\frac{1}{\cos^2(z)}z' = \frac{1}{x} \Rightarrow \int \frac{1}{\cos^2(z)}\,dz = \int \frac{1}{x}\,dx \quad \text{por Tabla}$$

$$\operatorname{tg}(z) = \ln|x| + c$$

$$z = \operatorname{arctg}(\ln|x| + c)$$

$$y = x\operatorname{arctg}(\ln|x| + c)$$

$$\pi/4 = \operatorname{arctg}(c) \Rightarrow c = 1$$

$$\left[SP \Rightarrow y = x\operatorname{arctg}(\ln|x| + 1)\right]$$

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(d) $y' = \dfrac{y}{x-y}$ $\qquad y = zx \quad y' = z'x + z$

$$z'x + z = \frac{zx}{x - zx} \Longrightarrow z'x + z = \frac{z\cancel{x}}{\cancel{x}(1-z)}$$

$$z'x = \frac{z}{1-z} - z = \frac{z-(z-z^2)}{1-z} = \frac{z^2}{1-z}$$

$$z'x = \frac{z^2}{1-z}$$

$$\frac{1-z}{z^2}\, z' = \frac{1}{x} \Longrightarrow \int \frac{1-z}{z^2}\, dz = \int \frac{1}{x}\, dx$$

$$\int z^{-2} - \frac{1}{z}\, dz = \int \frac{1}{x}\, dx$$

$$-\frac{1}{z} - \ln|z| = \ln|x| + C$$

$$-\frac{x}{y} - \ln\left|\frac{y}{x}\right| = \ln|x| + C$$

$$\ln\left(\cancel{x}\frac{y}{\cancel{x}}\right) = \frac{x}{y} + C$$

$$[56 \quad y\ln y + Cy + x = 0]$$

(3) $y' = (x-y-1)(x+y+3)$ $\qquad (x,y) = (u-1; v-2)$

$$\begin{cases} x = u-1 \Longrightarrow x(u) \\ y = v-2 \Longrightarrow y(v(x(u))) \end{cases}$$

$$v'(u) = 1\, y'\, 1 \Longrightarrow v' = y'$$

$$v' = \frac{u-1-v+2-1}{u-1+v-2+3} = \frac{u-v}{u+v} \Longrightarrow \begin{array}{l} v = zu \\ v' = z'u + z \end{array}$$

$$z'u + z = \frac{u - zu}{u + zu} = \frac{1-z}{1+z}$$

$$z'u = \frac{1-2z-z^2}{1+z}$$

$$\int \frac{z+1}{z^2+2z-1}\, dz = \int -\frac{1}{u}\, du$$

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$$\frac{1}{2}\ln(z^2+2z-1) = -\ln|u|+C$$

$$\ln(z^2+2z-1) = -2\ln|u|+C$$

$$z^2+2z-1 = e^{\ln u^{-2}+c}$$

$$z^2+2z-1 = A\,1/u^2$$

$$\frac{v^2}{u^2}+2\frac{v}{u}-1 = A\,1/u^2$$

$$v^2+2uv-u^2 = A \qquad \begin{cases} u = x+1 \\ v = y+2 \end{cases}$$

$$(y+2)^2+2(x+1)(y+2)-(x+1)^2 = A$$

$$y^2+4y+4+2xy+4x+2y+4-(x^2+2x+1) = A$$

$$y^2+4y+4+2xy+4x+2y+4-x^2-2x-1 = A$$

$$\text{SG: } \left[y^2-x^2+2xy+2x+6y = B\right] \qquad A-7=B$$

4 a) $\overbrace{2xy}^{P}\,dx + \overbrace{(x^2+\cos y)}^{Q}\,dy = 0$

$P'_y = 2x \qquad Q'_x = 2x \implies P = U'_x \quad Q = U'_y$

$$\begin{cases} U'_x = 2xy \\ U'_y = x^2+\cos y \end{cases} \implies U(x,y) = \int 2xy\,dx$$

$U(x;y) = yx^2 + \alpha(y)$

$U'_y = x^2 + \alpha'(y)$

$\left[U(x,y) = x^2y + \operatorname{sen} y + C \quad \text{SG}\right] \qquad \alpha'(y) = \cos y \implies \alpha(y) = \operatorname{sen} y + C$

b) $y' = \dfrac{xy^2-1}{1-x^2y} \qquad y(-1) = 1$

$$\underbrace{y'dx}_{dy} = \frac{xy^2-1}{1-x^2y}\,dx$$

$$-\overbrace{(-1+x^2y)}^{Q}\,dy = \overbrace{(xy^2-1)}^{P}\,dx$$

$P'_y = 2xy \qquad Q'_x = +2xy$

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$$\begin{cases} U'_x = xy^2 - 1 \\ U'_y = -1 + x^2y \end{cases}$$

$$U(x,y) = \int xy^2 - 1 \, dx$$

$$U(x,y) = \frac{x^2}{2}y^2 - x + \alpha(y)$$

$$U'_y = x^2y + \alpha'_y$$

$$\alpha(y) = -y$$

$$U(x,y) = \frac{1}{2}x^2y^2 - x - y + C$$

$$1 = \frac{1}{2} 1 + 1 - 1 + C \qquad C = \frac{1}{2}$$

$$[SP: \quad x^2y^2 - 2x - 2y = 1]$$

ⓒ $\underbrace{(6xy - y^3)}_{P}dx + \underbrace{(4y + 3x^2 - 3xy^2)}_{Q}dy = 0$

$$P'_y = 6x - 3y^2 \qquad Q'_x = 6x - 3y^2$$

$$\begin{cases} U'_x = 6xy - y^3 \\ U'_y = 4y + 3x^2 - 3xy^2 \end{cases}$$

$$U(x,y) = \int 6xy - y^3 \, dx$$

$$U(x,y) = 3x^2y - xy^3 + \alpha(y)$$

$$U'_y = 3x^2 - 3xy^2 + \alpha'(y)$$

$$\alpha'(y) = 4y \qquad \alpha(y) = 2y^2 + C$$

$$[U(x,y) = 3x^2y - xy^3 + 2y^2 + C] \text{ SG}$$

ⓓ $\overbrace{(y^2 - y)}^{M}dx + \overbrace{x}^{N}dy = 0$

$$M'_y = 2y - 1 \qquad N'_x = 1$$

$$\frac{N'_x - M'_y}{M} = \frac{1 - 2y + 1}{y^2 - y} = \frac{-2y}{y^2 - y}$$

$$\mu(y) = e^{\int \frac{-2y}{y^2 - y}}$$

$$= e^{-2\ln|y-1| + C}$$

$$\mu(y) = A(y-1)$$

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(f) $\underbrace{y^2 \cos x \, dx}_{M} + \underbrace{(4+5y \operatorname{sen} x)}_{N} dy$

$$\frac{M'_y - N'_x}{N} = \frac{2y\cos x - 5y\cos x}{4+5y \operatorname{sen} x} \quad \nexists\, U(x) = U$$

$$\frac{N'_x - M'_y}{M} = \frac{5y\cancel{\cos x} - 2y\cancel{\cos x}}{y^2 \cancel{\cos x}} = \frac{3}{y} \quad \exists\, U(y) \checkmark$$

$$U(y) = e^{\int 3/y \, dy} = e^{3\ln|y|+C} = Ay^3 \Rightarrow U(y) = y^3$$

$$y^5 \cos x \, dx + (4y^3 + 5y^4 \operatorname{sen} x) dy = 0$$

$$P'_y = 5y^4 \cos x \qquad Q'_x = 5y^4 \cos x \checkmark$$

$$\begin{cases} U'_x = y^5 \cos x \\ U'_y = 4y^3 + 5y^4 \operatorname{sen} x \end{cases}$$

$$U(x,y) = \int y^5 \cos x \, dx$$

$$U(x,y) = y^5 \operatorname{sen} x + \alpha(y)$$

$$U'_y = 5y^4 \cos x + \alpha'(y)$$

$$\alpha'(y) = 4y^3$$

$$\alpha(y) = \int 4y^3 = y^4 + C \qquad \left[U(x,y) = y^5 \operatorname{sen} x + y^4 + C\right] \text{SG.}$$

(6) (a) $y'' + 8y' + 25y = 0$

$$r^2 + 8r + 25 = 0 \Longrightarrow -4 \pm 3i$$

Si $r = a \pm bi \Longrightarrow y_h(x) = C_1 e^{ax} \cos bx + C_2 e^{ax} \operatorname{sen} bx$

$$y = e^{-4x}(A\cos(3x) + B\operatorname{sen}(3x))$$

(b) $y''' - 5y'' + 8y' - 4y - 2 = 0$

$$r^3 - 5r^2 + 8r - 4 = 0 \quad r_1 = 1,\ r_2 = 2,\ r_3 = 2$$

$$y_h = Ae^x + Be^{2x} + Cxe^{2x} - 1/2$$

$$y_p = AX + B \qquad y' = A \quad y'' = 0 \quad y''' = 0$$

$$8A - 4AX - 4B = 2$$

$$4A = 0$$

$$A = 0$$

$$-4B = 2 \qquad B = -1/2 \qquad y_p = -1/2$$

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ⓒ $y'' - 3y' + 2y = xe^x + 2x$

$r^2 - 3r + 2 = 0 \quad r = 2, \; r = 1$

$y_h = Ae^{2x} + Be^x$

$y_p = \cancel{AX+B} \qquad y'_p = A \qquad y''_p = 0$

$-3A + 2AX + 2B = Xe^x + 2X$

$\underbrace{2B - 3A}_{0} + 2AX = X(e^x + 2)$

$2A = e^x + 2$ Les debo la particular

ⓓ $y'' + y = \sec(x)$

$r^2 + 1 = 0 \quad r = \pm i$

$y_h = A\cos x + B\,\text{sen}\,x \qquad y_p = AX + B \qquad y'_p = A \qquad y''_p = 0$

$AX + B = \sec(x)$

⑦ $\bar{f}(x,y) = (\overbrace{y\,g'(x)}^{P}, \overbrace{x^2 - g'(x)}^{Q})$

$P'_y = Q'_x \implies g'(x) = 2X - g''(x)$

$g''(x) + g'(x) = 2X$

$r^2 + r = 0$

$r(r+1) = 0 \quad r = 0, \; r = -1 \qquad g(x)_h = Ae^{-x} + B$

$g_p = AX^2 + BX + C \qquad g'_p = 2AX + B \qquad g''_p = 2A$

$2A + 2AX + B = 2X$

$\begin{cases} 2A = 2 \\ 2A + B = 0 \end{cases} \implies \begin{matrix} A = 1 \\ B = -2 \end{matrix} \qquad g_p = X^2 - 2X$

$g(x) = Ae^{-X} + B + X^2 - 2X$

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