1) $\dfrac{dy}{dx} = \sqrt{y}\,\sqrt{x}$ $\qquad y(1) = 2$

$$\frac{dy}{\sqrt{y}} = \sqrt{x}\,dx$$

$$\int \frac{dy}{\sqrt{y}} = \int \sqrt{x}\,dx$$

$$2\sqrt{y} = \frac{2}{3}\cdot x^{\frac{3}{2}}$$

$$\sqrt{y} = \frac{1}{3}x^{\frac{3}{2}}$$

$$y = \frac{x^3}{9} + C$$

$$2 = \frac{1^3}{9} + C$$

$$2 - \frac{1}{9} = C$$

$$C = \frac{17}{9}$$

$$\boxed{y = \frac{1}{9}x^3 + \frac{17}{9}}$$

2) $G(x) = \int_0^x f(t)\,dt$ $\qquad f(t)\begin{cases} te^t + 1 & t < 1 \\ \frac{\ln t}{t} & t > 1 \end{cases}$

a)

$G(x) = \int_0^x xe^x + 1 \quad dx \quad =$

$\int 1\,dx \to x$

$\int_0^x xe^x \quad dx$ $\qquad u = x \qquad dv = e^x$

$\qquad du = 1 \qquad v = e^x$

$x \cdot e^x - \int_0^x e^x \quad dx$

$xe^x - e^x + x$

$e^x(x-1) + x$

$e^x(x-1) + x \Big|_0^x$

$e^x(x-1) + x - (e^0(0-1) + 0)$

$\qquad 1(-1)$

$e^x(x-1) + x + (+1)$

$G(x) = \begin{cases} e^x(x-1) + x + 1 & x < 1 \end{cases}$

$G(x) = \int_0^1 xe^x + 1 \; dx + \int_1^x \frac{\ln t}{t} \; dt$

$\int_1^x \frac{\ln x}{x} \quad dx$ $\qquad u = \ln(x)$

$du = \frac{1}{x}\,dx$

$x\,du = dx$

$\int_1^x u \; du = \frac{u^2}{2}$

$\int_1^x \frac{u^2}{2}\,du = \frac{\ln^2(x)}{2}\Big|_1^x = \frac{\ln^2(x)}{2} - \overbrace{\frac{\ln^2(1)}{2}}^{0}$

$$G(x) = \begin{cases} e^x (x-1) + x + 1 & x < 1 \\ \dfrac{\ln^2(x)}{2} + 2 & x \geq 1 \end{cases}$$

$\int_0^1 e^x(x-1) + x$

$\underbrace{e(0) + 1 - (e^0(-1) + 0)}$

$0 + 1 + 1 = 2$

B)

$\dfrac{\ln^2(x)}{2}$

$\dfrac{\ln^2(x)}{2} \Big|_1^x$

$\dfrac{\ln^2(x)}{2} - \underbrace{\dfrac{\ln^2(1)}{2}}_{0}$

$G'(x) \neq f(x)$ (B)

$G(x)' = \int_0^x f(x)\, 1 \, dx$

$\int_0^1 e^x(x-1) + x$

$e^1(0) + 1 - (e^0(-1) + 0)$

$e.0 + 1 - 1(-1) + 0$

$1 + 1 = 2$

$G(x)' = x . e^x + 1 \quad x < 1$

$= \dfrac{\ln x}{x} \quad x \geq 1$

$G(x)' = f(x) . 1 - f(0) . 0$

$$G(x)' = \begin{cases} e^x (x-1) + x + 1 \xrightarrow{\text{DERIVADA}} x e^x + 1 \\ \dfrac{\ln^2(x)}{2} + 2 \xrightarrow{\text{DERIVADA}} \dfrac{\ln(x)}{x} \end{cases}$$