1° PARCIAL ANALISIS MATEMATICO II 28/7/2023

P1) $x \cdot y^2 = k$

$y^2 + x\,2y \cdot y' = 0 \rightarrow y' = -\frac{y^{\cancel{2}}}{2x\cancel{y}} \rightarrow y' = \frac{2x}{y} \rightarrow \frac{dy}{dx} = \frac{2x}{y} \rightarrow$

$\int y\,dy = \int 2x\,dx \rightarrow \boxed{\frac{y^2}{2} = x^2 + C} \qquad \frac{3^2}{2} = 1 + C \rightarrow C = \frac{9}{2} - 1 = \frac{7}{2}$

$\frac{y^2}{2} = x^2 + \frac{7}{2} \rightarrow \boxed{y^2 = 2x^2 + 7}$

P2) $Dh_{(1,1)} = Dg_{(\bar{f}(1,1))} \cdot D\bar{f}_{(1,1)} = \begin{pmatrix} -\frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1+1 & -\frac{1}{2}+2 \end{pmatrix} = \begin{pmatrix} 2 & \frac{3}{2} \end{pmatrix}$

$D\bar{f}_{(1,1)} = \begin{pmatrix} -2x & 1 \\ y^2 & 2xy \end{pmatrix}_{(1,1)} = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$

$(x,y) = (1,1) \rightarrow (u,v) = (0,1) \rightarrow z_0 - 1 + \ln(z_0) = 0 \rightarrow z_0 = 1$

$g'_u(0,1) = -\left.\frac{2u + \frac{1}{u+z}}{1 + \frac{1}{u+z}}\right|_{(0,1,1)} = -\frac{1}{2}$

$g'_v(0,1) = -\left.\frac{-2v}{1 + \frac{1}{u+z}}\right|_{(0,1,1)} = \frac{2}{2} = 1$

$\check{v} = -\frac{(2, 3/2)}{\|(2, 3/2)\|} = -\frac{(4,3)}{\|(4,3)\|} = \boxed{\left(-\frac{4}{5}, -\frac{3}{5}\right)}$

P3) $C \begin{cases} x^2 + y^2 = 25 \\ x^2 + z^2 = 25 \\ y \geq 0 \end{cases} \qquad \begin{cases} x^2 + y^2 = 25 \\ y^2 - z^2 = 0 \\ y \geq 0 \end{cases} \rightarrow \begin{cases} x^2 + y^2 = 25 \\ y = \pm z \\ y \geq 0 \end{cases}$

$\bar{\lambda}(t) = (5\cos(t), 5\,\text{sen}(t), 5\,\text{sen}(t)) \qquad \bar{\lambda}(t_0) = (4,3,3)$

$\bar{\lambda}'(t) = (-5\,\text{sen}(t), 5\cos(t), 5\cos(t)) \qquad \bar{\lambda}'(t_0) = (-3,4,4)$

$\boxed{(x,y,z) = (4,3,3) + \lambda(-3,4,4) \quad \lambda \in \mathbb{R}}$

$\boxed{y = z}$

P4) $f(x,y) = x^2 - y^2 - xy + y + 1$

$$\begin{cases} f'_x = 2x - y = 0 \rightarrow y = 2x \rightarrow \boxed{y = 10} \\ f'_y = -2y - x + 1 = 0 \rightarrow -2 \cdot 2x - x = -1 \rightarrow -5x = -1 \rightarrow \boxed{x = 5} \end{cases}$$ pto crítico

$f''_{xx} = 2 \qquad f''_{xy} = -1 \qquad \det Hf_{(5,10)} = -4 - 1 = -5$

$f''_{yx} = -1 \qquad f''_{yy} = -2 \qquad \boxed{f \text{ no tiene extremos locales en } \mathbb{R}^2}$

T1) $\cos(x)\, y' + \operatorname{sen}(x) \cdot y = 1$

$y = u \cdot v \qquad y' = u'v + uv'$

$\cos(x)\, u'v + \cos(x)\, uv' + \operatorname{sen}(x)\, uv = 1$

$\cos(x)\, u'v + u\left(\cos(x) \cdot v' + \operatorname{sen}(x) \cdot v\right) = 1$

$\frac{dv}{dx} = -\frac{\operatorname{sen}(x)}{\cos(x)} v \rightarrow \int \frac{dv}{v} = -\int \frac{\operatorname{sen}(x)}{\cos(x)} dx \rightarrow \ln|v| = \ln|\cos(x)| + \ln|C_1|$

$v_{(x)} = C_1 \cos(x)$

$\cos(x) \frac{du}{dx} C_1 \cos(x) = 1 \rightarrow \int du = \int \frac{1}{C_1 \cos^2(x)} dx \rightarrow u = \frac{\tan(x)}{C_1} + C_2$

$y_{(x)} = \left(\frac{\tan(x)}{C_1} + C_2\right) C_1 \cos(x) = \boxed{\operatorname{sen}(x) + C\cos(x) = y_{(x)}}$

T2) $f'_{\vec{v}}(0,0) = \lim_{t \to 0} \frac{f(tv_x, tv_y) - f(0,0)}{t}$

si $v_x = 0 \qquad f'_{\vec{v}}(0,0) = \lim_{t \to 0} \frac{0 - 0}{t} = 0$

si $v_x \neq 0 \qquad f'_{\vec{v}}(0,0) = \lim_{t \to 0} \frac{t v_y}{t \cdot t^2 v_x^2} =$ $\xrightarrow{\text{si } v_y = 0} = 0$

$\xrightarrow{\text{si } v_y \neq 0} \nexists f'_{\vec{v}}(0,0)$