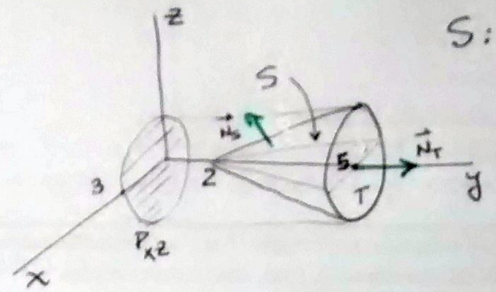
1)



S: $y = 2 + \sqrt{x^2+z^2}$ $\quad y \leq 5$

Superficie cónica, sin "tapa".

Para calcular el flujo de $\vec{F}(x,y,z) = (x, y, z + g(x,y))$ sólo es posible cerrando la superficie y usando el Teorema de Gauss porque la función $g(x,y)$ no se conoce.

$$\operatorname{div}(\vec{F}) = 1 + 1 + 1 = 3$$

$$\therefore \iint\limits_{\substack{S \cup T \\ \text{orientada} \\ \text{hacia} \\ \text{afuera}}} \vec{F} \cdot d\vec{\sigma} = \iiint\limits_{V} \operatorname{div}(\vec{F})\, dx\,dy\,dz$$

$$\therefore \iint\limits_{S} \vec{F} \cdot d\vec{\sigma} = \iiint\limits_{V} 3\, dx\,dy\,dz - \iint\limits_{T} \vec{F} \cdot (0,1,0)\, dx\,dz =$$

$$= \int_0^{2\pi} d\theta \int_0^3 r\,dr \int_{2+r}^{5} 3\,dy - \iint\limits_{P_{xz}} y\Big|_T\, dx\,dz =$$

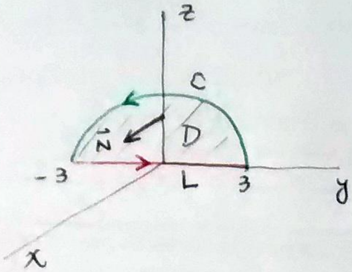
$$= 6\pi \int_0^3 r(3-r)\,dr - \int_0^{2\pi} d\theta \int_0^3 r\,5\,dr$$

$$= 6\pi \left[3\frac{r^2}{2} - \frac{r^3}{3}\right]_0^3 - 10\pi \left[\frac{r^2}{2}\right]_0^3 =$$

$$= 27\pi - 45\pi = \boxed{-18\pi}$$

2)

$x = x$
$y = r\cos\theta$
$z = r\,\text{sen}\,\theta$



Es dato la circulación sobre C. Para calcular la circulación sobre T, consideramos el flujo del rotor a través de D, con normal $(1,0,0)$:

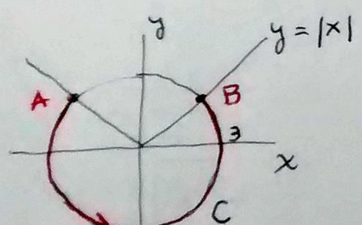
$$\int_{C \cup L} \vec{F}\cdot d\vec{s} = 20 + \int_L \vec{F}\cdot d\vec{s} = \iint_D \nabla\times\vec{F}\cdot(1,0,0)\,dy\,dz =$$

$$= \int_0^{\pi} d\theta \int_0^3 r\,dr\,\left(-z\Big|_D\right) = \int_0^{\pi} d\theta \int_0^3 r(-r\,\text{sen}\,\theta)\,dr$$

$$= \frac{r^3}{3}\Big|_0^3 \cos\theta\Big|_0^{\pi} = -18$$

$$\therefore \int_L \vec{F}\cdot d\vec{s} = -18 - 20 = \boxed{-38}$$

3)



$x^2 + y^2 = 3$
con $y \leq |x|$

$\vec{\gamma}(t) = (\sqrt{3}\cos(t), \sqrt{3}\,\text{sen}(t))$
$t \in [\frac{9}{4}\pi, \frac{9}{4}\pi]$

$A = (-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}})$

$B = (\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}})$

$$\vec{F} = \underbrace{(4x^3y^2e^{x^4}, 2ye^{x^4})}_{\text{conservativo}} + \underbrace{\overset{\vec{g}}{(xy, 0)}}_{\text{no conserv.}}$$

$$\varphi = e^{x^4}y^2 + C$$

$$\int_C \vec{F}\cdot d\vec{s} = \varphi(B) - \varphi(A) + \int_C \vec{g}\cdot d\vec{s} =$$

$$= 0 + \int_{3/4\pi}^{9/4\pi} \overbrace{(3\cos(t)\,\text{sen}(t), 0)}^{\vec{g}} \cdot \overbrace{(-\sqrt{3}\,\text{sen}\,t, \sqrt{3}\cos t)}^{\vec{\gamma}'(t)}$$

$$= -3\sqrt{3}\,\frac{\text{sen}^3 t}{3}\Big|_{3/4\pi}^{9/4\pi} = \boxed{-\frac{\sqrt{6}}{2}}$$

4) Para que $\vec{F}$ sea conservativo, $F'_{1_y} = F'_{2_x}$ necesariamente.

$\therefore \quad g'(x) + 7x = 6x + g''(x) \quad$ con $\quad g(0) = 4$
$g'(0) = 7$

Esto es

$$g'' - g' = x$$

Ec. característica: $\quad \alpha^2 - \alpha = 0$

$$\alpha(\alpha - 1) = 0$$

$$B = \{ e^{0x}, e^{x} \}$$

$$g_H = C_1 + C_2 e^{x}$$

$$g_P = (Ax + B)x \qquad g_P'' = 2A$$

$$g_P' = 2Ax + B$$

$$2A - 2Ax - B = x$$

$$\therefore \quad A = -\tfrac{1}{2}, \quad B = -1$$

$$\therefore \quad g_G = C_1 + C_2 e^{x} - \tfrac{1}{2}x^2 - x$$

$$\left.\begin{array}{l} g(0) = 4 = C_1 + C_2 \\ g'(0) = 7 = C_2 - 1 \end{array}\right\} \begin{array}{l} C_1 = -4 \\ C_2 = 8 \end{array}$$

$$\boxed{\therefore \quad g(x) = -4 + 8e^{x} - \tfrac{1}{2}x^2 - x}$$