

$$1) \begin{cases} f'_x(-0,6) + f'_y 0,8 = -2 \\ f'_x 0,8 + f'_y 0,6 = 1 \end{cases}$$

$$f'_x = \frac{\begin{vmatrix} -2 & 0,8 \\ 1 & 0,6 \end{vmatrix}}{\begin{vmatrix} -0,6 & 0,8 \\ 0,8 & 0,6 \end{vmatrix}} = \frac{-2 \cdot 0,6 - 0,8}{-0,6 \cdot 0,6 - 0,8 \cdot 0,8} = \frac{-2}{-0,36 - 0,64} = 2$$

$$f'_{(A, (0,3, -0,4))} = 2 \cdot 0,3 - 1(-0,4) = 0,6 + 0,4 = 1$$

$$f'_y = \frac{\begin{vmatrix} -0,6 & -2 \\ 0,8 & 1 \end{vmatrix}}{\begin{vmatrix} -0,6 & 0,8 \\ 0,8 & 0,6 \end{vmatrix}} = \frac{-0,6 + 1,6}{-1} = -1$$

$$\vec{v} = \left(\frac{1}{\sqrt{5}}, 2 \right) \quad \text{y} \quad \vec{v} = \left(\frac{-1}{\sqrt{5}}, -2 \right)$$

$$2) (x, y) = (3, 2) \rightarrow (u, v, w) = (7, 3, 5)$$

$$D_{\bar{g}} = \begin{pmatrix} y & x \\ y-1 & x \\ y & x \end{pmatrix} \quad D_{g(3,2)} = \begin{pmatrix} 2 & 3 \\ 1 & 3 \\ 2 & 3 \end{pmatrix}$$

$$D_{h(3,2)} = (3 \ -2 \ 1) \begin{pmatrix} 2 & 3 \\ 1 & 3 \\ 2 & 3 \end{pmatrix} = (6-2+2 \quad 9-6+3) = (6 \ 6)$$

$$\max h'_{v(3,2)} = \sqrt{36+36} = \sqrt{72}$$

$$\vec{v} = \frac{(6, 6)}{\sqrt{72}}$$

$$3) g(x, y, z) = x + yz + \ln(x + y^2 - z - 3) - 3 \quad 1 + 2z_0 + \ln(2 - z_0) = 3 \quad z_0 = 1$$

$$\vec{\nabla} g = \left(1 + \frac{1}{x + y^2 - z - 3}, z + \frac{2y}{x + y^2 - z - 3}, y + \frac{-1}{x + y^2 - z - 3} \right)$$

$$\vec{\nabla} g_{(1,2,1)} = (2, 5, 1)$$

$$(x, y, z) = (1, 2, 1) + t(2, 5, 1) \quad t \in \mathbb{R}$$

$$\begin{cases} x = 1 + 2t \\ y = 2 + 5t = 0 \rightarrow t = -2/5 \\ z = 1 + t \end{cases} \quad \begin{aligned} x &= 1 - 4/5 = 1/5 \\ z &= 1 - 2/5 = 3/5 \end{aligned}$$

$$\left(\frac{1}{5}, 0, \frac{3}{5} \right)$$

$$4) \quad y' - \frac{4}{x} y = x^5 e^x$$

$$u'v + u v' - \frac{4}{x} u v = x^5 e^x \rightarrow u'v + u \left(v' - \frac{4}{x} v \right) = x^5 e^x$$

$$\frac{dv}{dx} = \frac{4v}{x} \rightarrow \int \frac{dv}{v} = 4 \int \frac{dx}{x} \rightarrow \ln|v| = 4 \ln|x| + \ln|C_1|$$

$$v = C_1 x^4$$

$$\frac{du}{dx} C_1 x^4 = x^5 e^x \rightarrow \int du = \int \frac{x e^x}{C_1} dx \rightarrow u = \frac{e^x}{C_1} (x-1) + C_2$$

$$y = \left[\frac{e^x}{C_1} (x-1) + C_2 \right] C_1 x^4 \rightarrow \boxed{y = e^x (x^5 - x^4) + C x^4}$$

$$1 = C$$

$$\boxed{y = e^x (x^5 - x^4) + x^4}$$

$$T1) \quad y = \ln x$$

$$\lim_{x \rightarrow 0} \frac{\ln x}{x - \ln x} = \frac{\ln}{1 - \ln} \rightarrow \nexists \lim_{(x,y) \rightarrow (0,0)} f \rightarrow f \text{ not es. cont. in } (0,0)$$

$$T2) \quad \vec{v} = (v_x, v_y)$$

$$\text{si } v_x = 0 \quad f'_{\vec{v}}(0,0) = \boxed{0}$$

$$v_x \neq 0 \quad f'_{\vec{v}}(0,0) = \lim_{t \rightarrow 0} \frac{\frac{t^2 v_y^2}{t v_x}}{t} = \boxed{\frac{v_y^2}{v_x}}$$