AM II

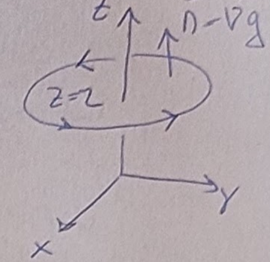
2° PARCIAL

16/12/2022

P1) $\text{rot}\,\bar{f} = (x,\ x^2-2x,\ -z)$

$$C \begin{cases} z = 3-x^2-y^2 \\ z = 2x^2+2y^2 \end{cases} \rightarrow \begin{cases} 2x^2+2y^2 = 3-x^2-y^2 \\ z = 2x^2+2y^2 \end{cases} \rightarrow \begin{cases} x^2+y^2 = 1 \\ z_{(x,y)} = 2 \end{cases} \cdot \ \text{No } g$$

$$\oint_{\partial S} \bar{f}\, d\bar{\lambda} = \iint_{S_g} \text{rot}\,\bar{f}\, d\bar{\sigma} = \iint_{x^2+y^2\le 1} (x,\ x^2-2x,\ -\overset{2}{z})\cdot(0,0,1)\,dx\,dy = \left| \begin{array}{l} g(x,y,z) = z-2 \\ \bar{\nabla} g = (0,0,1) \end{array} \right.$$



$$= \iint_{x^2+y^2\le 1} -2 = -2\ \text{área}\,(S_{xy}) = \boxed{-2\pi}$$

P2) $\bar{f}(x,y,z) = (12x+2yz,\ 6y+2xz,\ 2xy) \qquad D_{\bar{f}} = \mathbb{R}^3$ simplemente conexo

$f'_{1y} = 2z = f'_{2x}$ ✓ $\qquad f'_{1z} = 2y = f'_{3x}$ ✓ $\qquad f'_{2z} = 2x = f'_{3y}$ ✓

$\Rightarrow \bar{f}$ conservativo

$$\phi = \int_0^1 \bar{f}(tx,ty,tz)\cdot(x,y,z)\,dt + k = \int_0^1 (12tx+2t^2yz,\ 6ty+2t^2xz,\ 2t^2xy)(x,y,z)\,dt + k =$$

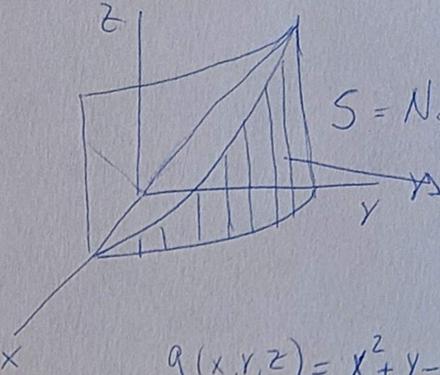
$$= \int_0^1 \left(12t\,x^2 + \cancel{2t^2xy}z + 6t\,y^2 + \overset{4}{\cancel{2}}t^2xyz + 2t^2xyz\right) dt + k =$$

$$\phi = \left[6t^2x^2 + \cancel{\tfrac{4}{3}t^3xyz} + 3t^2y^2 + \overset{2}{\cancel{\tfrac{2}{3}}}t^3xyz\right]_0^1 + k = 6x^2 + 2xyz + 3y^2 + k$$

$$\phi_{(1,a,a)} - \phi(-a,a,1) = 6 + 2a^2 + \cancel{3a^2} + \cancel{k} - \left(6a^2 - 2a^2 + \cancel{3a^2} + \cancel{k}\right) =$$

$$= \boxed{6-2a^2} \rightarrow 2a^2 = 6 \rightarrow a^2 = 3 \rightarrow \boxed{a = \pm\sqrt{3}}$$

P3) $\bar{f}(x,y,z) = (x, 2y, x-z)$



$S = N \circ g$

$\bar{n} = \bar{\nabla} g$

$$\begin{cases} S: \; y = 4 - x^2 \; (x,z) \\ 0 \le z \le y \Rightarrow 0 \le z \le 4 - x^2 \\ x \ge 0 \qquad x^2 \le 4 \\ y \ge 0 \qquad 0 \le x \le 2 \end{cases}$$

$g(x,y,z) = x^2 + y - 4 = 0$

$\bar{\nabla} g = (2x, 1, 0)$

$$\iint_{S_{\bar{n}}} \bar{f}\, d\bar{r} = \iint_{S_{xz}} (x, 2(4-x^2), x-z)\frac{(2x,1,0)}{|1|}\, dx\, dz =$$

$$= \iint_{S_{xz}} (\cancel{2x^2} + 8 - \cancel{2x^2})\, dx\, dz = 8\int_0^2 (4-x^2)\, dx = 8\left[4x - \frac{x^3}{3}\right]_0^2 = 8\left(8 - \frac{8}{3}\right) = 64 \cdot \frac{2}{3} = \boxed{\frac{128}{3}}$$

P4) $H \begin{cases} x^2 + z^2 \le 32 \\ z \ge \sqrt{x^2 + 2y^2} \\ 1^\circ \text{ oct} \end{cases}$ $\quad \delta(x,y,z) = kz$

$\sqrt{x^2+2y^2} \le z \le \sqrt{32 - x^2}$

$x^2 + 2y^2 \le 32 - x^2$

$x^2 + y^2 \le 16$

$$\text{masa}(H) = \iint_{H_{xy}} \int_{\sqrt{x^2+2y^2}}^{\sqrt{32-x^2}} kz\, dz\, dx\, dy = k\iint_{H_{xy}} \left[\frac{z^2}{2}\right]_{\sqrt{x^2+2y^2}}^{\sqrt{32-x^2}} dx\, dy =$$

$$= \frac{k}{2}\iint_{H_{xy}} (32 - x^2 - x^2 - 2y^2)\, dx\, dy = \frac{k}{2}\iint_{H_{xy}} (32 - 2x^2 - 2y^2)\, dx\, dy =$$

$$= \frac{k}{2}\int_0^4\int_0^{\pi/2} (32\rho - 2\rho^3)\, d\varphi\, d\rho = \frac{k}{2}\,\frac{\pi}{2}\left[16\rho^2 - \frac{\rho^4}{2}\right]_0^4 = \frac{k\pi}{\cancel{4}}\left(16 - \frac{16}{2}\right)\cancel{16} =$$

$$= \boxed{32 k\pi}$$