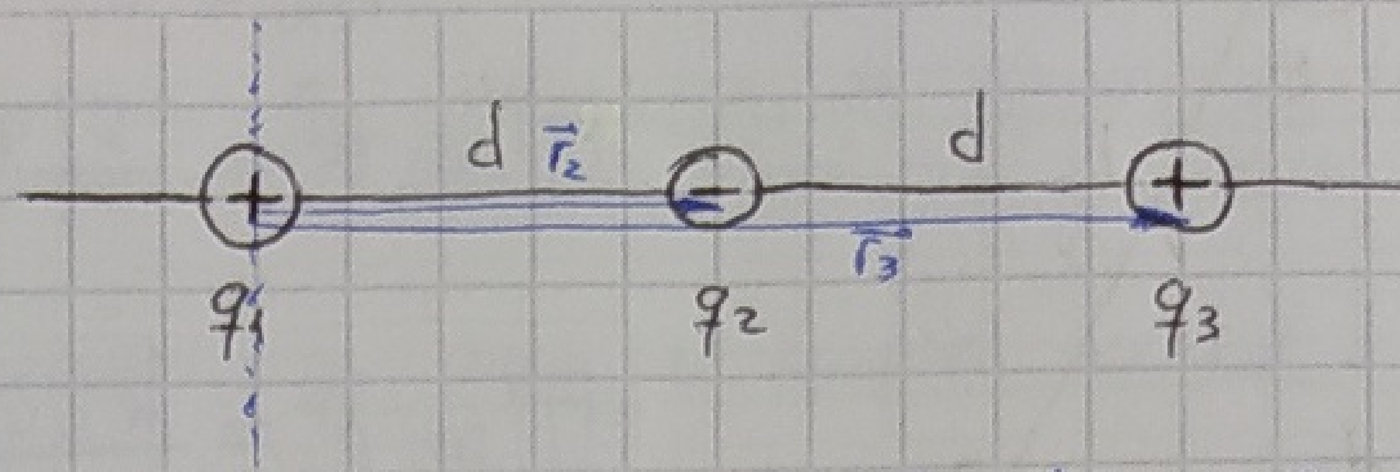
FÍSICA II

ELECTROSTÁTICA
Ley de Coulomb

GUÍA NUEVA
1 - 2 - 3a

1

ⓐ $\vec{F_1} = 0, \quad q_1 = ?$



$$\cancel{\vec{F_1}} = \vec{F_{21}} + \vec{F_{31}}$$
$$0 = \vec{F_{21}} + \vec{F_{31}}$$
$$\vec{F_{21}} = -\vec{F_{31}}$$

$\underline{\vec{r_1} = 0}$ $\quad \underline{\vec{r_2} = d\cdot\check{\imath}}$ $\quad \underline{\vec{r_3} = 2d\cdot\check{\imath}}$

$\vec{r_1} - \vec{r_2} = 0 - d\check{\imath} = -d\cdot\check{\imath}$

$\hookrightarrow |\vec{r_1} - \vec{r_2}|^3 = |-d\cdot\check{\imath}|^3 = d^3$

$\vec{r_1} - \vec{r_3} = 0 - 2d\check{\imath} = -2d\cdot\check{\imath}$

$\hookrightarrow |\vec{r_1} - \vec{r_3}|^3 = |-2d\cdot\check{\imath}|^3 = 8d^3$

$$\frac{\cancel{k}\cdot q_2\cdot\cancel{q_1}\cdot(\vec{r_1}-\vec{r_2})}{|\vec{r_1}-\vec{r_2}|^3} = -\frac{\cancel{k}\cdot q_3\cdot\cancel{q_1}\cdot(\vec{r_1}-\vec{r_3})}{|\vec{r_1}-\vec{r_3}|^3}$$

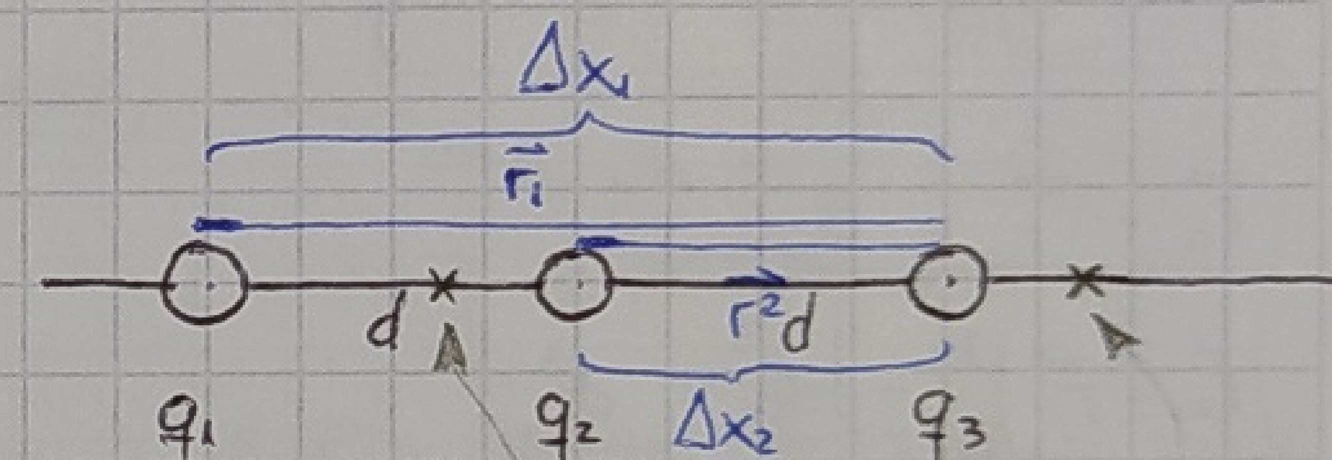
$$\frac{q_2\cdot\cancel{-d\cdot\check{\imath}}}{\cancel{d^3}} = -\frac{q_3\cancel{-2d\check{\imath}}}{8\cdot\cancel{d^3}}$$

$$q_2 = -\frac{1}{4}\cdot q_3$$

$$\boxed{q_3 = -4\cdot q_2}$$

ⓑ $\underbrace{|q_1| = 3\cdot|q_2|}$, $\underbrace{q_3 \text{ en equilibrio}}_{\vec{F_3}=0}$, $\Delta x_2 = ?$

$\left.\begin{matrix} q_1>0 \\ q_2<0 \end{matrix}\right\} q_1 = -3\cdot q_2$



$\left.\begin{matrix} |\vec{r_3}-\vec{r_1}| = \Delta x_1 \\ |\vec{r_3}-\vec{r_2}| = \Delta x_2 \end{matrix}\right\} \Delta x_1 = \Delta x_2 + d$

$$\cancel{\vec{F_3}} = \vec{F_{13}} + \vec{F_{23}}$$
$$0 = \vec{F_{13}} + \vec{F_{23}}$$
$$\vec{F_{13}} = -\vec{F_{23}}$$

$$\frac{\cancel{k}\cdot q_1\cdot\cancel{q_3}\cdot(\vec{r_3}-\vec{r_1})}{|\vec{r_3}-\vec{r_1}|^3} = -\frac{\cancel{k}\cdot q_2\cdot\cancel{q_3}\cdot|\vec{r_3}-\vec{r_2}|}{|\vec{r_3}-\vec{r_2}|^3}$$

$$\frac{-3\cancel{q_2}\cdot\cancel{\Delta x_1}}{\Delta x_1^{\cancel{3}2}} = +\frac{\cancel{q_2}\cdot\cancel{\Delta x_2}}{\Delta x_2^{\cancel{3}2}}$$

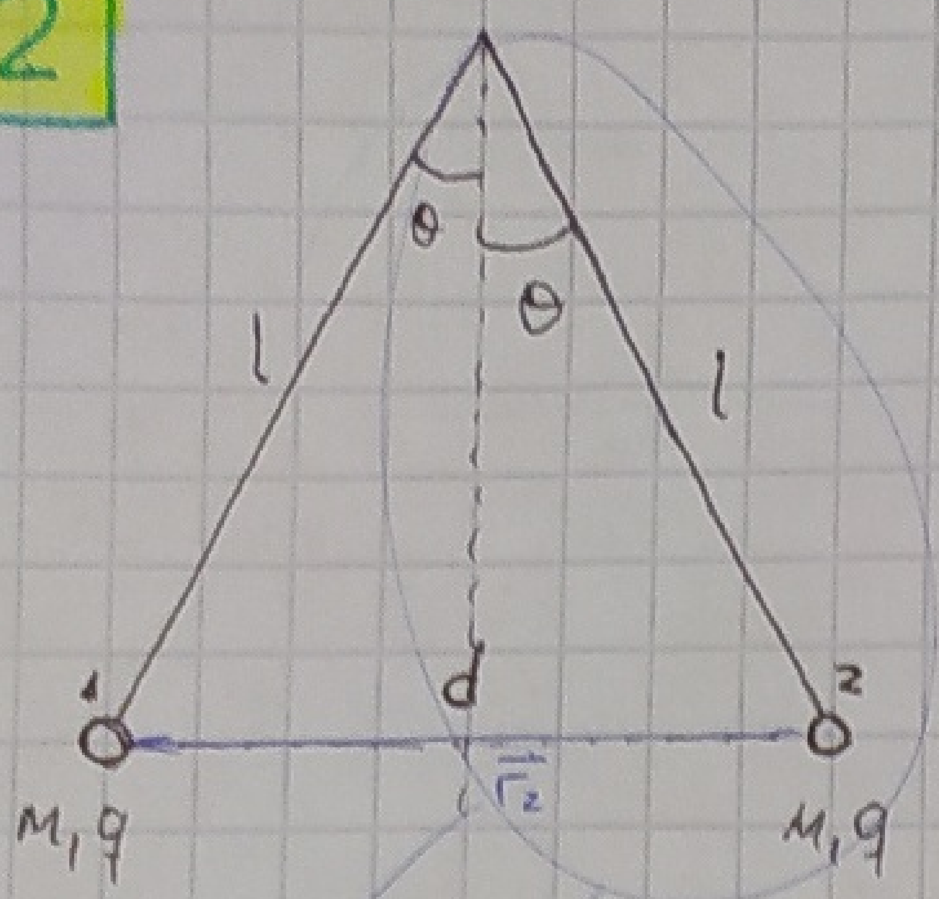
$$3\cdot\Delta x_2^2 = \Delta x_1^2$$
$$3\cdot\Delta x_2^2 = (\Delta x_2 + d)^2$$
$$3\cdot\Delta x_2^2 = \Delta x_2^2 + 2\cdot\Delta x_2\cdot d + d^2$$
$$0 = -2\Delta x_2^2 + 2\Delta x_2 d + d^2 \quad \begin{cases} a = -2 \\ b = 2d \\ c = d^2 \end{cases}$$

$$\boxed{\Delta x_{2_1} = 1{,}36\cdot d}$$
$$\boxed{\Delta x_{2_1} = -0{,}36\cdot d}$$

ⓒ No depende del signo de q_3 (da lo mismo si q_3 es positivo o negativo).

NOTA

2



$$\Sigma \vec{F_x} = 0$$

$$\vec{T_{2x}} = \vec{F_{12}}$$

$$|\vec{T}| \operatorname{sen}\theta = \frac{k \cdot q \cdot q \cdot (\vec{r_2} - \vec{r_1})}{|\vec{r_2} - \vec{r_1}|^3}$$

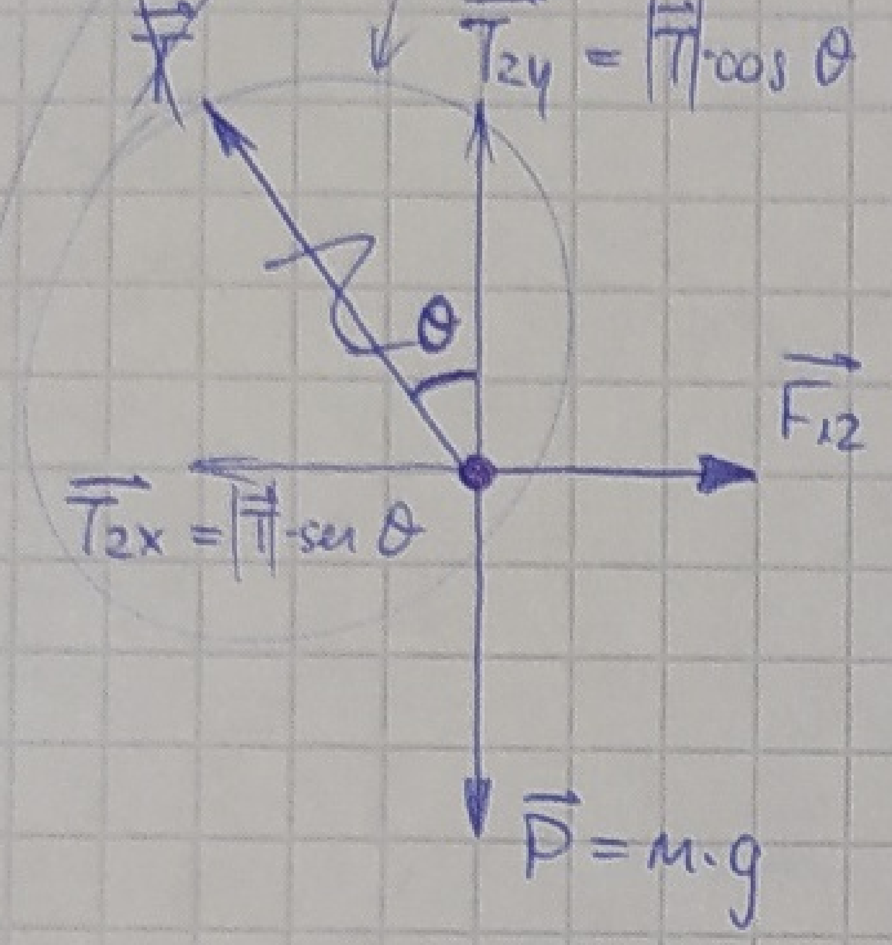
$$\boxed{|\vec{T}| \operatorname{sen}\theta = \frac{k \cdot q^2 \cdot \cancel{d}}{d^{\cancel{3}2}}}$$

$$\boxed{|\vec{T}| \operatorname{sen}\theta = \frac{k \cdot q^2}{d^2}}$$

$$\Sigma \vec{F_y} = 0$$

$$\vec{T_{2y}} = \vec{P}$$

$$\boxed{|\vec{T}| \cdot \cos\theta = m \cdot g}$$



$$\frac{\cancel{|\vec{T}|} \operatorname{sen}\theta}{\cancel{|\vec{T}|} \cos\theta} = \frac{\frac{k \cdot q^2}{d^2}}{m \cdot g}$$

$$\tan\theta = \frac{k \cdot q^2}{d^2 \cdot m \cdot g}$$

$$\tan\theta \cdot d^2 \cdot m \cdot g = k \cdot q^2$$

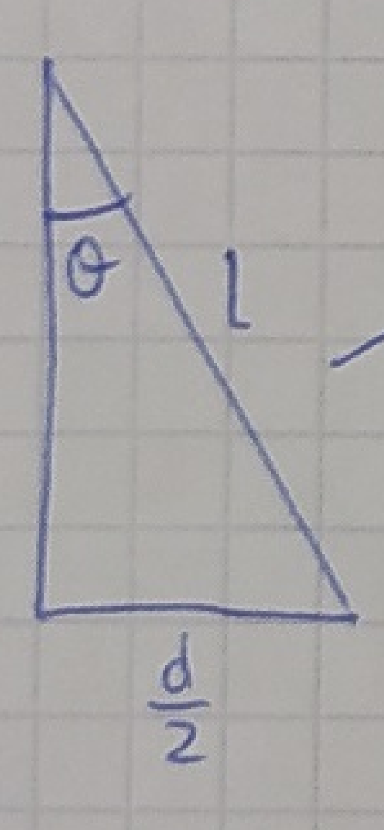
$$q = \sqrt{\frac{\tan\theta \cdot d^2 \cdot m \cdot g}{k}}$$

$$q = d \cdot \sqrt{\frac{\tan\theta \cdot m \cdot g}{k}}$$

$$\boxed{q = 2 \cdot l \cdot \operatorname{sen}\theta \cdot \sqrt{\frac{\tan\theta \cdot m \cdot g}{k}}}$$

$$\underbrace{\vec{r_2} - \vec{r_1} = d}$$

$$\underbrace{|\vec{r_2} - \vec{r_1}|^3 = |d|^3 = d^3}$$



$$\operatorname{sen}\theta = \frac{\frac{d}{2}}{l}$$

$$\underbrace{d = 2 \cdot l \cdot \operatorname{sen}\theta}$$

3

$l_0 + \Delta x$

$q_1 = 4 \cdot 10^{-6}\,C \qquad q_2 = 2 \cdot 10^{-6}\,C$

$\left.\begin{array}{l} l_0 = 0{,}2\,m \\ \Delta x = 0{,}04\,m \end{array}\right\} l_0 + \Delta x = 0{,}24\,m$

F_1 1 Fe $\qquad$ Fe 2 F_2

a)

$r_1 = 0\,m$

$r_2 = l_0 + \Delta x = 0{,}24\,m$

$\vec{r_2} - \vec{r_1} = l_0 + \Delta x$
$= 0{,}24\,m$

$$\Sigma \vec{F} = 0$$

$$Fe = F$$

$$k_{resorte} \cdot \Delta x = \frac{k \cdot q_1 \cdot q_2 \cdot (\vec{r_2} - \vec{r_1})}{|\vec{r_2} - \vec{r_1}|^{\cancel{3}2}}$$

$$k_{resorte} = \frac{k \cdot q_1 \cdot q_2}{\Delta x \cdot |l_0 + \Delta x|^2}$$

$$\boxed{k_{resorte} = \frac{9 \cdot 10^9 \frac{Nm}{C^2} \cdot 4 \cdot 10^{-6}\,C \cdot 2 \cdot 10^{-6}\,C}{0{,}04\,m \cdot |0{,}24\,m|^2}} = \boxed{31{,}25 \frac{N}{m}}$$

NOTA

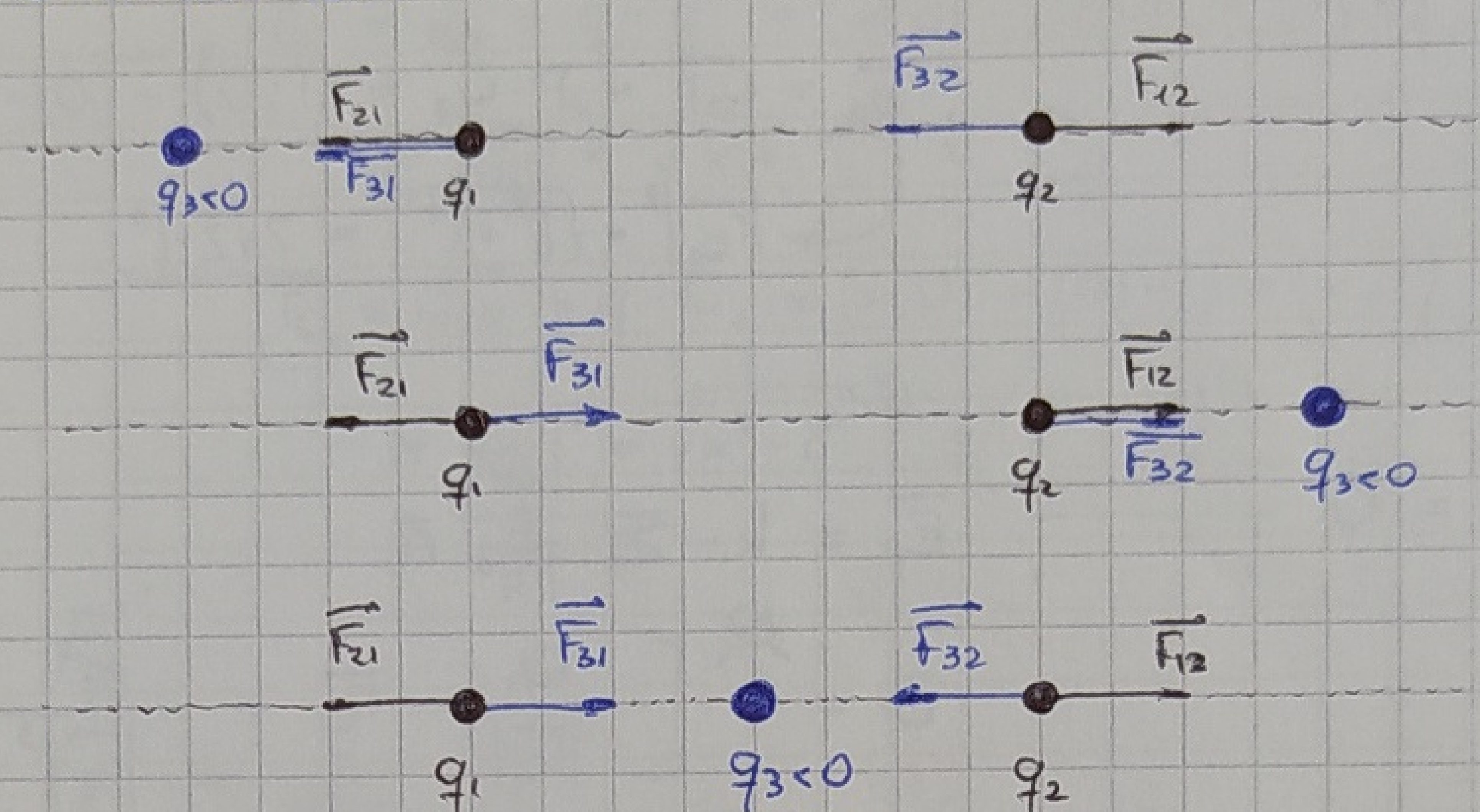
Física II	Electrostática Ley de Coulomb - Campo Eléctrico (puntual)	Guía nueva 3b,c - 4a

(cont.) 3 b c Para lograr el equilibrio, q_3 debe ser de signo negativo, ya que si fuera de signo positivo repelería tanto a q_1 como a q_2.

Como q_1 y q_2 son colineales (1 dimensión), q_3 debe ser también colineal a ellas.

Respecto de la posición relativa, hay tres opciones: a la izquierda de q_1, en el medio o a la derecha de q_2. Veamos qué ocurriría en cada caso:

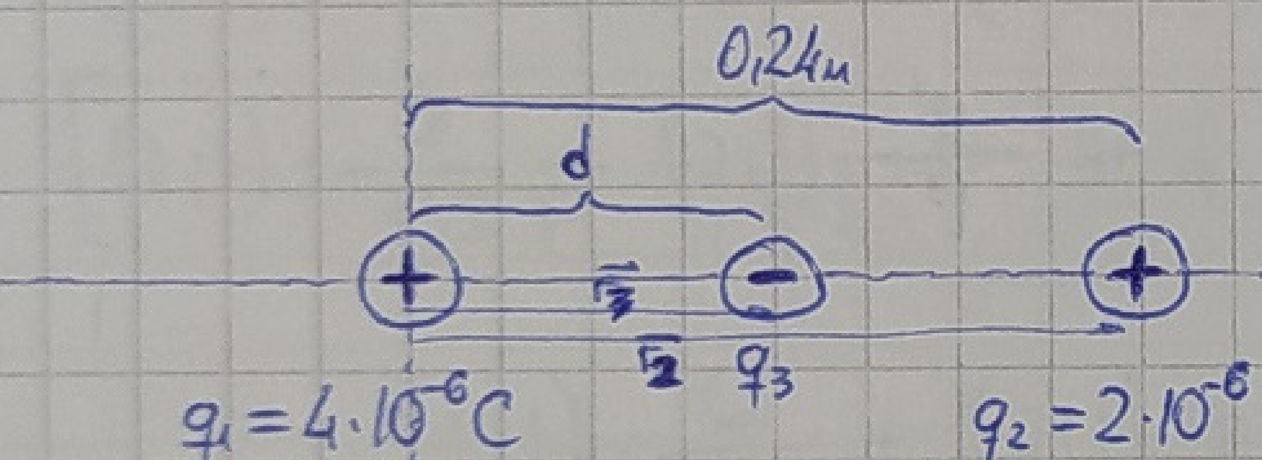
- A la izquierda de q_1, sólo q_2 quedaría en equilibrio (q_1 no):
- A la derecha de q_2, sólo q_1 quedaría en equilibrio (q_2 no):
- En el medio, tanto q_1 como q_2 podrían quedar en equilibrio:



$q_3 < 0$

Por lo tanto q_3 debe estar entre q_1 y q_2 (en el medio).

$0m < d < 0{,}24m$ respecto de q_1



$\vec{r_1} = 0m$

$\vec{r_2} = 0{,}24m$

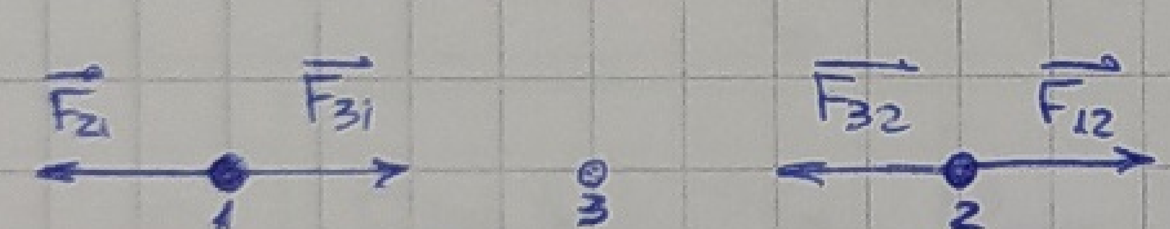
$\vec{r_3} = d$

$\vec{r_1} - \vec{r_2} = 0m - 0{,}24m = -0{,}24m$

$\vec{r_1} - \vec{r_3} = 0m - d = -d$

$\vec{r_2} - \vec{r_3} = 0{,}24m - d$

$\vec{r_2} - \vec{r_1} = 0{,}24m - 0m = 0{,}24m$



$\Sigma \vec{F} = 0$ | $\Sigma \vec{F} = 0$

$\vec{F_{21}} = \vec{F_{31}}$ | $\vec{F_{32}} = \vec{F_{12}}$

$l = -d$

$$\frac{\cancel{k}\cdot q_2\cdot \cancel{q_1}\cdot(\vec{r_1}-\vec{r_2})}{|\vec{r_1}-\vec{r_2}|^3} = \frac{\cancel{k}\cdot q_3\cdot \cancel{q_1}\cdot(\vec{r_1}-\vec{r_3})}{|\vec{r_1}-\vec{r_3}|^3} \qquad \frac{\cancel{k}\cdot q_3\cdot \cancel{q_2}\cdot(\vec{r_2}-\vec{r_3})}{|\vec{r_2}-\vec{r_3}|^3} = \frac{\cancel{k}\cdot q_1\cdot \cancel{q_2}\cdot(\vec{r_2}-\vec{r_1})}{|\vec{r_2}-\vec{r_1}|^3}$$

$$\frac{2\cdot10^{-6}C\cdot(-0{,}24m)}{|-0{,}24m|^3} = \frac{q_3\cdot(-d)}{|-d|^3} \qquad \frac{q_3\cdot\cancel{(0{,}24m-d)}}{|0{,}24m-d|^{\cancel{3}\,2}} = \frac{4\cdot10^{-6}C\cdot\cancel{0{,}24m}}{|0{,}24m|^{\cancel{3}\,2}}$$

$0{,}24m - d > 0$

$$\cancel{+}\frac{1}{28.800}\frac{C}{m^2} = q_3\cdot\frac{\cancel{+d}}{d^{\cancel{3}\,2}} \qquad \frac{q_3}{(0{,}24m-d)^2} = \frac{1}{14400}\frac{C}{m^2}$$

$$\frac{q_3}{d^2} = \frac{1}{28.800}\frac{C}{m^2}$$

$$q_3 = \frac{d^2}{28.800}\frac{C}{m^2}$$

$$\frac{\frac{d^2}{\cancel{2}\,\cancel{28800}}\cancel{\frac{C}{m^2}}}{0{,}0576m^2 - 0{,}48\cdot d + d^2} = \frac{1}{\cancel{14400}}\cancel{\frac{C}{m^2}}$$

$$\frac{1}{2}d^2 = 0{,}0576m^2 - 0{,}48d + d^2$$

$$0 = \frac{1}{2}d^2 - 0{,}48\cdot d + 0{,}0576m^2$$

$$|q_3| = \frac{(0{,}14m)^2}{28.800}\frac{C}{m^2} = 6{,}86\cdot10^{-7}C$$

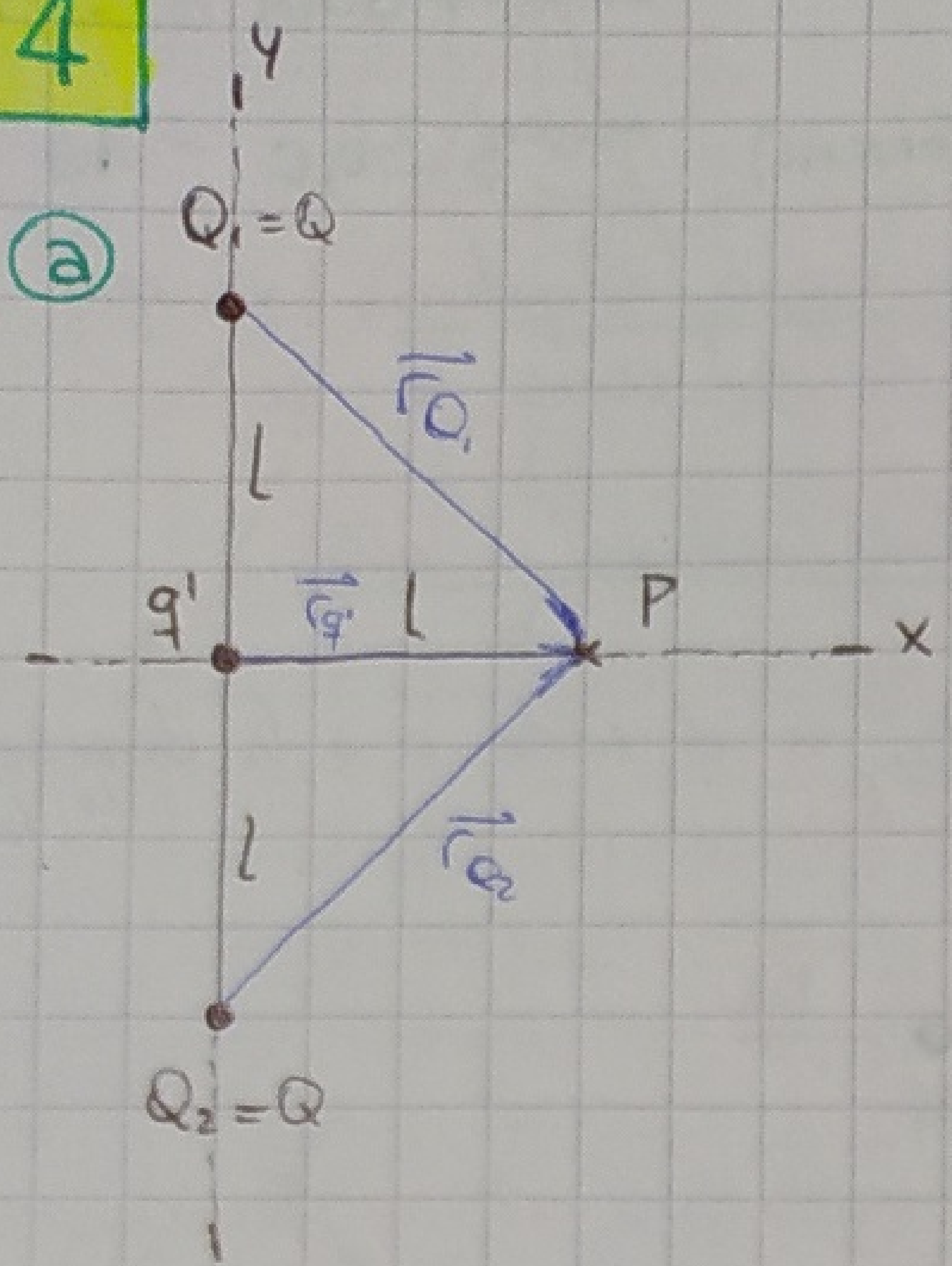
$\to d_1 = \cancel{0{,}82m}$ No cumple con $0m < d < 0{,}24m$

$\to d_2 = 0{,}14m$

NOTA

4

ⓐ



$Q_1: (0; l)$. $\quad q': (0;0)$. $\quad Q_2: (0; -l)$. $\quad P: (l; 0)$.

$\vec{r}_{Q_1} = \overrightarrow{Q_1P} = P - Q_1 = (l;0) - (0;l) = \underline{(l;-l)}$

$\hookrightarrow \underline{|\vec{r}_{Q_1}|^3} = \sqrt{(l)^2 + (-l)^2}^{\,3} = \underline{2\sqrt{2}\, l^3}$

$\vec{r}_{q'} = \overrightarrow{q'P} = P - q' = (l;0) - (0;0) = \underline{(l;0)}$

$\hookrightarrow |\vec{r}_{q'}|^3 = \sqrt{l^2 + 0^2}^{\,3} = \underline{l^3}$

$\vec{r}_{Q_2} = \overrightarrow{Q_2P} = P - Q_2 = (l;0) - (0;-l) = \underline{(l;l)}$

$\hookrightarrow |\vec{r}_{Q_2}|^3 = \sqrt{l^2 + l^2}^{\,3} = \underline{2\sqrt{2} \cdot l^3}$

$\vec{E}_P = \bar{0}$ ——— $\vec{E}_P = k \cdot \sum \frac{q_i}{|\vec{r}_i|^3} \cdot \vec{r}_i$

$$\bar{0} = \overset{\neq 0}{k} \cdot \left[\frac{Q_1}{|\vec{r}_{Q_1}|^3} \cdot \vec{r}_{Q_1} + \frac{q'}{|\vec{r}_{q'}|^3} \cdot \vec{r}_{q'} + \frac{Q_2}{|\vec{r}_{Q_2}|^3} \cdot \vec{r}_{Q_2} \right]$$

$$\bar{0} = \frac{Q}{2\sqrt{2} \cdot l^3} \cdot (l;-l) + \frac{q'}{l^3} \cdot (l;0) + \frac{Q}{2\sqrt{2} \cdot l^3} \cdot (l;l)$$

$$\bar{0} = \overset{\neq 0}{\frac{1}{l^3}} \cdot \left[\frac{Q}{2\sqrt{2}} (l;-l) + q' \cdot (l;0) + \frac{Q}{2\sqrt{2}} \cdot (l;l) \right]$$

$$\bar{0} = \frac{Q}{2\sqrt{2}} \cdot (l;-l) + q' \cdot (l;0) + \frac{Q}{2\sqrt{2}} \cdot (l;l)$$

$$\frac{Q}{2\sqrt{2}} \cdot l + q' \cdot l + \frac{Q}{2\sqrt{2}} \cdot l = 0 \quad \wedge \quad \cancel{-\frac{Q}{2\sqrt{2}} \cdot l} + \cancel{\frac{Q}{2\sqrt{2}} \cdot l} = 0$$

$\underline{0 = 0}$

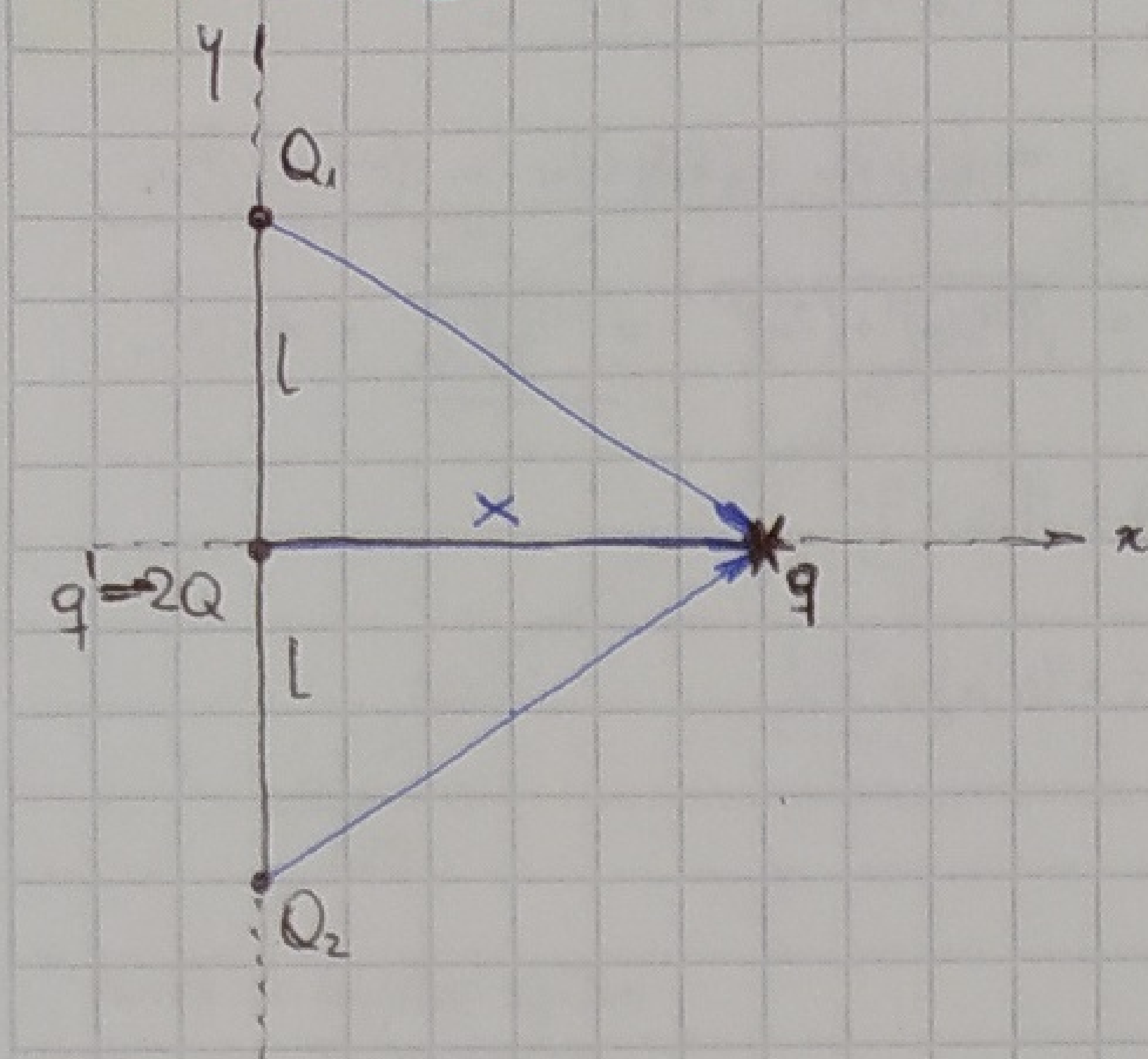
$$\frac{Q}{\sqrt{2}} \cdot l + q' \cdot l = 0$$

$$\left(\frac{Q}{\sqrt{2}} + q' \right) \cdot \overset{\neq 0}{l} = 0$$

$$\frac{Q}{\sqrt{2}} + q' = 0$$

$$\boxed{q' = -\frac{Q}{\sqrt{2}} = -\frac{\sqrt{2}}{2} Q}$$

(cont.) 4 (b)



$Q_1:(0;l)$. $q':(0;0)$ $Q_2:(0;-l)$. $q:(x;0)$, $x>0$.

$\vec{r}_{Q_1} = \overrightarrow{Q_1 q} = q - Q_1 = (x;0)-(0;l) = \underline{(x;-l)}$

$\hookrightarrow |\vec{r}_{Q_1}|^3 = \sqrt{x^2+(-l)^2}^{\,3/2} = \underline{(x^2+l^2)^{3/2}}$

$\vec{r}_{q'} = \overrightarrow{q'q} = q - q' = (x;0)-(0;0) = \underline{(x;0)}$

$\hookrightarrow |\vec{r}_{q'}|^3 = \sqrt{x^2+0^2}^{\,3/2} = \underline{x^3}$

$\vec{r}_{Q_2} = \overrightarrow{Q_2 q} = q - Q_2 = (x;0)-(0;-l) = \underline{(x;l)}$

$\hookrightarrow |\vec{r}_{Q_2}|^3 = \sqrt{x^2+l^2}^{\,3/2} = \underline{(x^2+l^2)^{3/2}}$

$$\vec{F} = q\cdot\vec{E}_q$$

$$\vec{F} = q\cdot\left[k\cdot\sum\frac{q_i}{|\vec{r}_i|^3}\cdot\vec{r}_i\right]$$

$$\vec{F} = q\cdot k\cdot\left[\frac{Q}{(x^2+l^2)^{3/2}}\cdot(x;-l) + \frac{-2Q}{x^3}\cdot(x;0) + \frac{Q}{(x^2+l^2)^{3/2}}\cdot(x;l)\right]$$

$$\vec{F} = q\cdot k\cdot\left[\frac{Q}{(x^2+l^2)^{3/2}}\cdot x - \frac{2Q}{x^{3}}\cdot\cancel{x} + \frac{Q}{(x^2+l^2)^{3/2}}\cdot x\ ;\ \underbrace{\cancel{-\frac{Q}{(x^2+l^2)^{3/2}}\,l} + \cancel{\frac{Q}{(x^2+l^2)^{3/2}}\cdot l}}_{=0}\right]$$

$$\vec{F} = q\cdot k\cdot\left[\frac{2Q}{(x^2+l^2)^{3/2}}x - \frac{2Q}{x^2}\ ;\ 0\right]$$

$$\boxed{\vec{F} = q\cdot k\cdot 2Q\cdot\left[\frac{x}{(x^2+l^2)^{3/2}} - \frac{1}{x^2}\ ;\ 0\right]}$$

$\rightarrow \vec{F}_x = q\cdot k\cdot 2Q\cdot\left(\frac{x}{(x^2+l^2)^{3/2}} - \frac{1}{x^2}\right)\hat{\imath}$

$\rightarrow \vec{F}_y = 0\cdot\hat{\jmath}$

(c) $x_2 > x_1$

$$W = \int_{x_1}^{x_2}\vec{F}\cdot d\bar{x}$$

$$\int\frac{x}{\sqrt{x^2+z^2}^{\,3}}dx = -\frac{1}{\sqrt{x^2+z^2}}+c$$

$$W = q\cdot k\cdot 2Q\cdot\int_{x_1}^{x_2}\left[\frac{x}{(x^2+l^2)^{3/2}} - \frac{1}{x^2}\ ;\ 0\right]d\bar{x}$$

$$\int-\frac{1}{x^2}dx = \int -x^{-2}dx = -(-x^{-1})+c = +\frac{1}{x}+c$$

$$\int\frac{1}{x^2}dx = \int x^{-2}dx$$

$$\frac{x^{-2+1}}{-2+1}+c$$

$$\frac{x^{-1}}{-1}+c$$

$$-\frac{1}{x}$$

$$W = q\cdot k\cdot 2Q\cdot\left[\int_{x_1}^{x_2}\frac{x}{(x^2+l^2)^{3/2}}dx + \int_{x_1}^{x_2}\frac{1}{x^2}dx\ ;\ \underbrace{\int_{x_1}^{x_2}0\,dx}_{=0}\right]$$

$$\boxed{W = q\cdot k\cdot 2Q\cdot\left[-\frac{1}{\sqrt{x^2+l^2}} + \frac{1}{x}\Bigg|_{x_1}^{x_2}\ ;\ 0\right]}$$

$$\left(-\frac{1}{\sqrt{x_2^2+l^2}}+\frac{1}{x_2}\right) - \left(-\frac{1}{\sqrt{x_1^2+l^2}}+\frac{1}{x_1}\right)$$

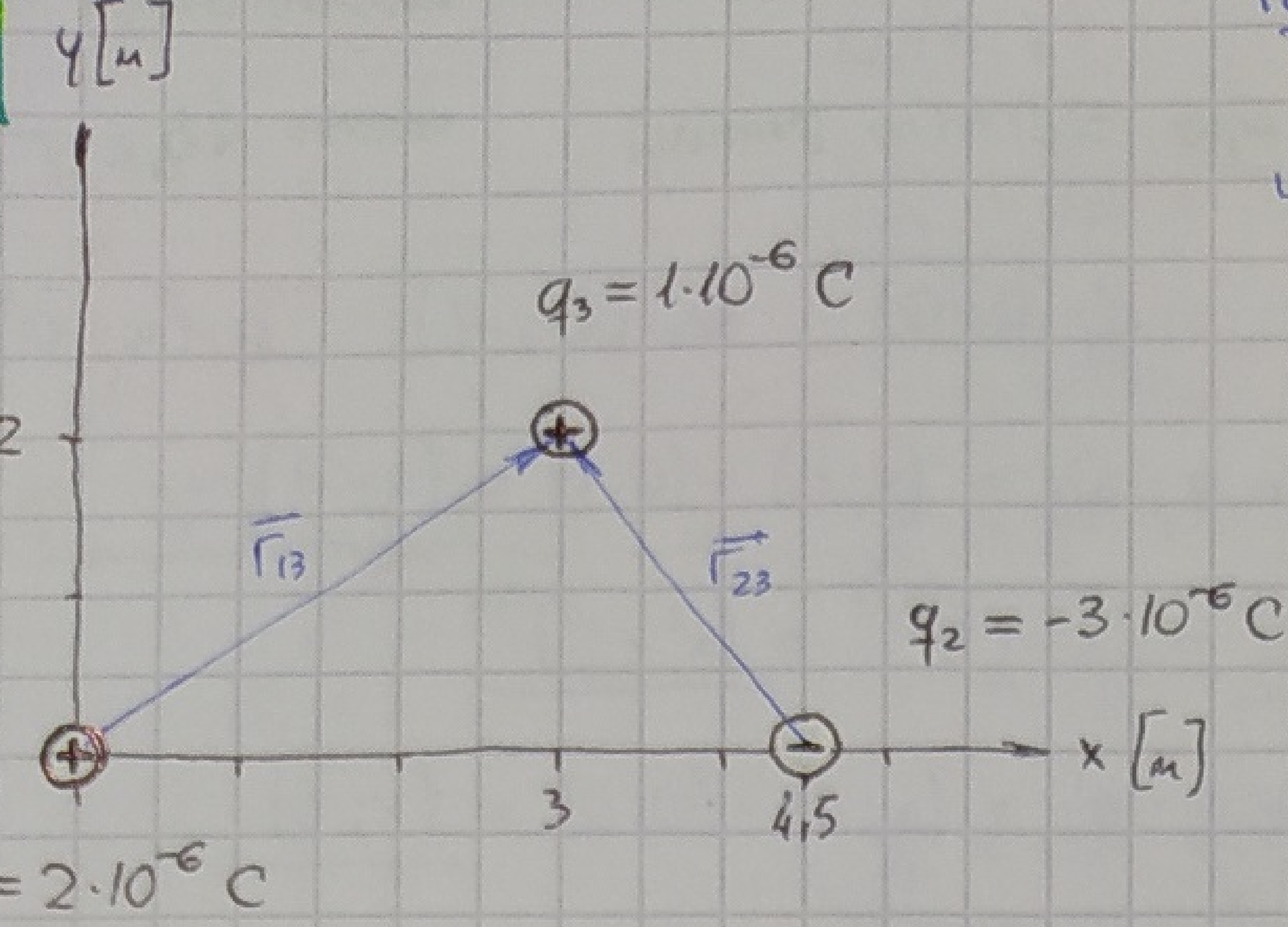
$$-\frac{1}{\sqrt{x_2^2+l^2}}+\frac{1}{x_2}+\frac{1}{\sqrt{x_1^2+l^2}}-\frac{1}{x_1}$$

$$W_x = 2kq'Q\left(-\frac{1}{\sqrt{x_2^2+l^2}}+\frac{1}{x_2}+\frac{1}{\sqrt{x_1^2+l^2}}-\frac{1}{x_1}\right)$$

$$\boxed{W_x = -2kq'Q\left(\frac{1}{\sqrt{x_2^2+l^2}}-\frac{1}{x_2}-\frac{1}{\sqrt{x_1^2+l^2}}+\frac{1}{x_1}\right)}$$

NOTA

5



$r_{q_1}: (0;0)\,m$. $\quad r_{q_2}: (4,5;0)\,m$. $\quad r_{q_3}: (3;2)\,m$.

$\vec{r}_{13} = r_{q_3} - r_{q_1} = (3;2)\,m - (0;0)\,m = (3;2)\,m$

$|\vec{r}_{13}|^3 = \sqrt{(3)^2+(2)^2}^{\,3} = 13^{3/2}\,m$

$\vec{r}_{23} = r_{q_3} - r_{q_2} = (3;2)\,m - (4,5;0)\,m = (-1,5;2)\,m$

$|\vec{r}_{23}|^3 = \sqrt{(-1,5\,m)^2+(2\,m)^2}^{\,3} = \frac{125}{8}\,m = 6,25^{3/2}\,m$

(a)

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$$\vec{F}_3 = \frac{k\cdot q_1\cdot q_3\cdot(\vec{r}_{13})}{|\vec{r}_{13}|^3} + \frac{k\cdot q_2\cdot q_3\cdot(\vec{r}_{23})}{|\vec{r}_{23}|^3}$$

$$\vec{F}_3 = k\cdot q_3\left(\frac{q_1\cdot\vec{r}_{13}}{|\vec{r}_{13}|^3} + \frac{q_2\cdot\vec{r}_{23}}{|\vec{r}_{23}|^3}\right)$$

$$\vec{F}_3 = 9\cdot 10^9\,\frac{N m^2}{C^2}\cdot 1\cdot 10^{-6}\,C\cdot\left[\frac{2\cdot 10^{-6}\,C}{13^{3/2}\,m^4}\cdot(3\,m;2\,m) + \frac{-3\cdot 10^{-6}\,C}{\frac{125}{8}\,m^3}\cdot(-1,5\,m;2\,m)\right]$$

$$\vec{F}_3 = 9\cdot 10^3\,\frac{N\cdot m^2}{C}\cdot\left[\frac{2\cdot 10^{-6}\,C}{13^{3/2}\,\cancel{m}^{2}}\cdot 3\,\cancel{m} + \frac{-3\cdot 10^{-6}\,C}{\frac{125}{8}\,\cancel{m}^{2}}\cdot(-1,5\,\cancel{m})\;;\;\frac{2\cdot 10^{-6}\,C}{13^{3/2}\,\cancel{m}^{2}}\cdot 2\,\cancel{m} + \frac{-3\cdot 10^{-6}\,C}{\frac{125}{8}\,\cancel{m}^{2}}\cdot 2\,\cancel{m}\right]$$

$$\vec{F}_3 = 9\cdot 10^3\,\frac{N m^2}{C}\cdot\left(416\cdot 10^{-9}\,\frac{C}{m^2}\;;-298\cdot 10^{-9}\,\frac{C}{m^2}\right)$$

$$\boxed{\vec{F}_3 = (3,74\cdot 10^{-3}\,;-2,68\cdot 10^{-3})\,N}$$

(b)

$$\vec{F}_3 = q_3\cdot\vec{E}_3$$

$$\vec{E}_3 = \frac{\vec{F}_3}{q_3}$$

$$\vec{E}_3 = \frac{(3,74\cdot 10^{-3}\,N\;;\;-2,68\cdot 10^{-3}\,N)}{1\cdot 10^{-6}\,C}$$

$$\boxed{\vec{E}_3 = (3,74\cdot 10^{3}\;;\;-2,68\cdot 10^{3})\,\frac{N}{C}}$$

FÍSICA II

ELECTROSTÁTICA

Ley de Coulomb – Campo Eléctrico (LINEAL)

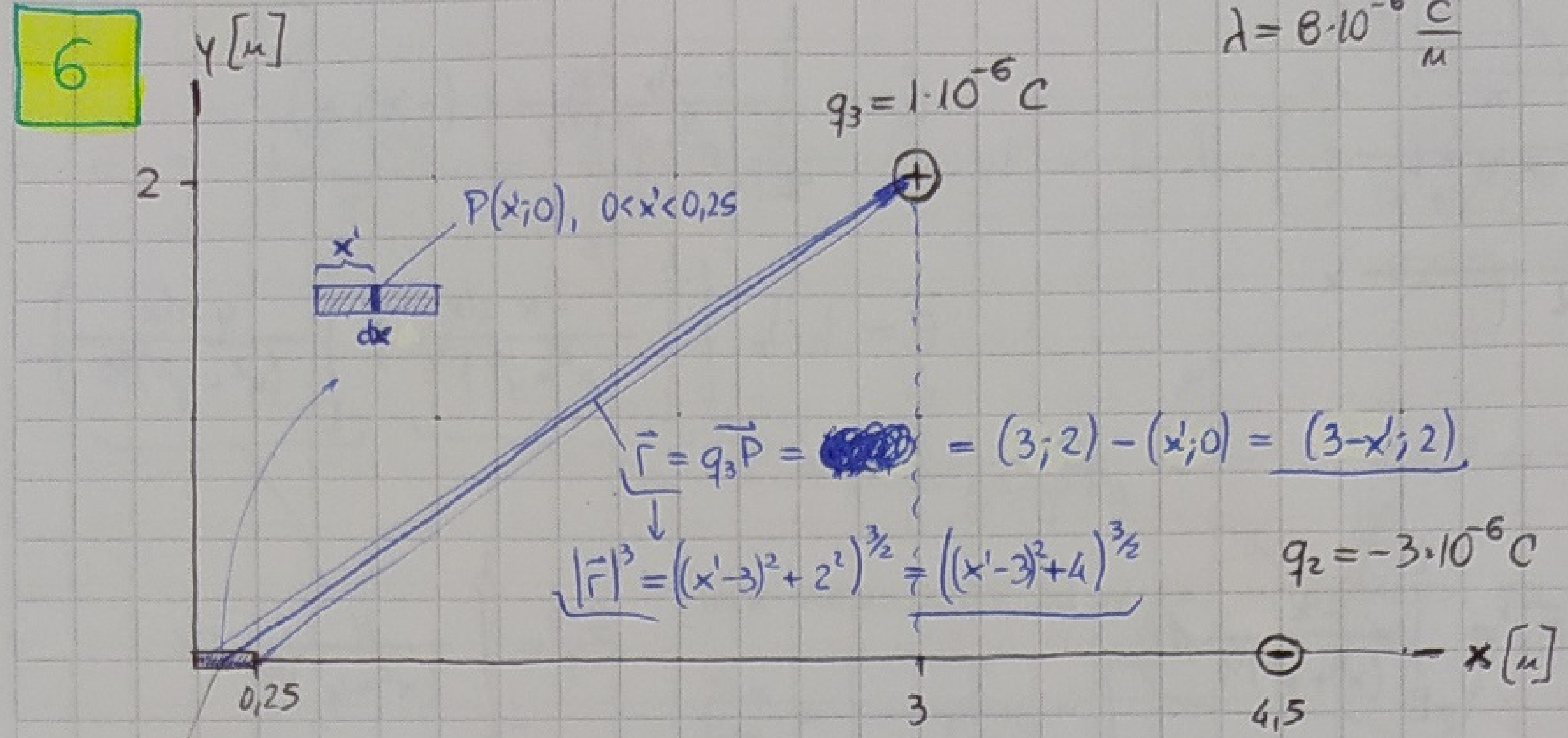
GUÍA NUEVA

6 – 7a,b,(c)

varilla

6

$\lambda = 8\cdot10^{-6}\,\frac{C}{m}$



$\vec{r} = \vec{q_3P} = (3;2)-(x';0) = \underline{(3-x';2)}$

$|\vec{r}|^3 = \left((x'-3)^2+2^2\right)^{3/2} = \left((x'-3)^2+4\right)^{3/2}$

Varilla de longitud 0,25m cargada con $\lambda = 8\cdot10^{-6}\,\frac{C}{m}$

ⓐ $Q = \int_{0m}^{0{,}25m} \lambda\cdot dx$

$Q = \lambda\cdot\int_{0m}^{0{,}25m} dx$

$Q = \lambda\cdot x\Big|_{0m}^{0{,}25m}$

$Q = 8\cdot10^{-6}\,\frac{C}{m}\cdot 0{,}25m$

$\boxed{Q = 2\cdot10^{-6}\,C}$ ✓

ⓑ $\vec{F_3} = \vec{F_{v3}} + \vec{F_{23}}$ — igual al del ejercicio $\boxed{5}$ (las condiciones de q_3 y q_2 son las mismas)

$\boxed{\vec{F_{23}}} = \dfrac{k\cdot q_2\cdot q_3\cdot(\vec{r_{23}})}{|\vec{r_{23}}|^{3}} = 9\cdot10^{9}\,\frac{Nm^2}{C^2}\cdot 1\mu C\cdot(-3\mu C)\cdot\dfrac{(-1{,}5;2)\,m}{\frac{125}{8}\,m}$

$= -0{,}001728\,N\cdot(-1{,}5;2) = \boxed{(2{,}592;-3{,}456)\cdot10^{-3}\,N}$ ✓

$\vec{F_{v3}} = q_3\cdot\vec{E_3}$

$E_3 = \int \dfrac{k\cdot\lambda\cdot dx'\cdot\vec{r}}{|\vec{r}|^3} = k\cdot\lambda\cdot\int_0^{0{,}25}\dfrac{(3-x';2)}{\left((3-x')^2+4\right)^{3/2}}\,dx' = k\cdot\lambda\cdot\left[\int_0^{0{,}25}\dfrac{(3-x')\cdot dx'}{\left((3-x')^2+4\right)^{3/2}};\int_0^{0{,}25}\dfrac{2\cdot dx'}{\left((3-x')^2+4\right)^{3/2}}\right]$

(*) SUSTITUCIÓN DE VARIABLES (**)

(*) $u = (3-x')^2+4 \longrightarrow du = 2(3-x')\cdot(-1) = -2\cdot(3-x')\,dx'$

Si $x'=0 \Rightarrow u = (3-0)^2+4 = 13$

Si $x'=0{,}25 \Rightarrow u = (3-0{,}25)^2+4 = 11{,}5625$

(**) $v = 3-x' \longrightarrow dv = -1\cdot dx'$

Si $x'=0 \longrightarrow v = 3-0 = 3$

Si $x'=0{,}25 \longrightarrow v = 3-0{,}25 = 2{,}75$

$\vec{E_3} = k\cdot\lambda\cdot\left[-\frac{1}{2}\int_{13}^{11{,}5625}\dfrac{-2\cdot(3-x')\,dx}{\left((3-x')^2+4\right)^{3/2}};\ -2\int_3^{2{,}75}\dfrac{-1\cdot dx}{\left((3-x)^2+4\right)^{3/2}}\right]$

- $\int \frac{1}{x^{3/2}}dx = \int x^{-3/2}dx = \frac{x^{-1/2}}{-1/2}+C = \frac{-2}{\sqrt{x}}+C$
- $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}+C$

$\vec{E_3} = k\cdot\lambda\cdot\left[-\frac{1}{2}\int_{13}^{11{,}5625}\frac{du}{u^{3/2}};\ -2\int_3^{2{,}75}\frac{dv}{(v^2+4)^{3/2}}\right] = k\cdot\lambda\cdot\left[-\frac{1}{2}\cdot\left[-\frac{2}{\sqrt{u}}\right]_{13}^{11{,}5625};\ -2\left[\frac{v}{4\sqrt{v^2+4}}\right]_3^{2{,}75}\right]$

$\vec{E_3} = 9\cdot10^{9}\,\frac{Nm^2}{C^2}\cdot 8\cdot10^{-6}\,\frac{C}{m}\cdot(16{,}73;11{,}65)\cdot10^{-3}\,m = \boxed{(1{,}204;0{,}839)\cdot10^{3}\,\frac{N}{C}}$

$\boxed{\vec{F_{v3}}} = q_3\cdot\vec{E_3} = 1\cdot10^{-6}\,C\cdot(1{,}204\cdot10^{3};0{,}839\cdot10^{3})\,\frac{N}{C} = \boxed{(1{,}204;0{,}839)\cdot10^{-3}\,N}$ ✓

$\vec{F_3} = \vec{F_{v3}} + \vec{F_{23}} = (1{,}204;0{,}839)\cdot10^{-3}\,N + (2{,}592;-3{,}456)\cdot10^{-3}\,N$

$\boxed{\vec{F_3} = (3{,}796\cdot10^{-3};-2{,}617\cdot10^{-3})\,N}$

ⓒ Si bien la carga en ambos casos es la misma, la distribución no lo es: en $\boxed{5}$ la distribución de cargas es puntual, mientras que en $\boxed{6}$ es lineal. Por ende, la F y el CE serán distintos.

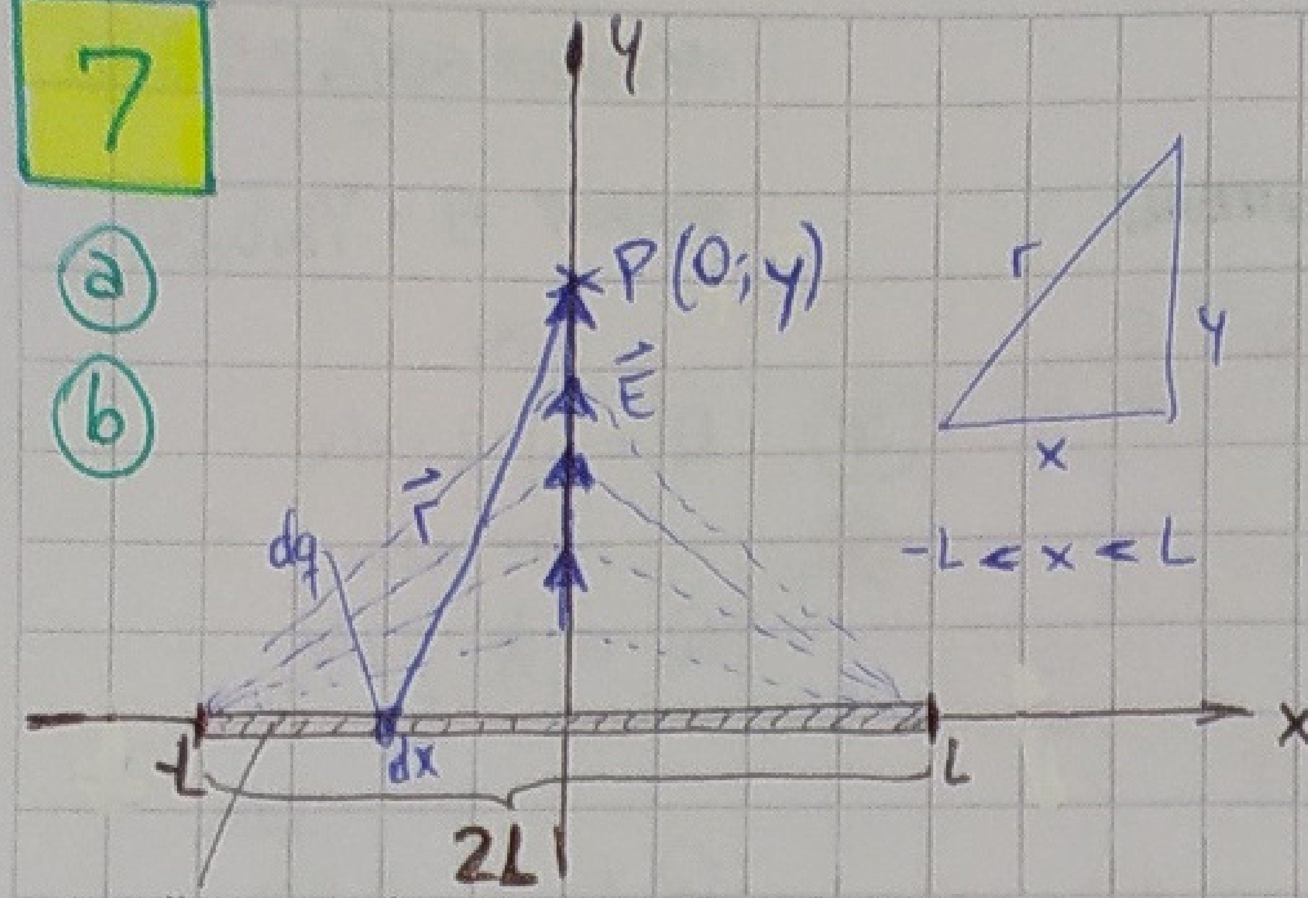
NOTA

varilla

7

(a)

(b)



$-L < x < L$

varilla, con $\lambda = \lambda_0$.

$dq = \lambda \cdot dl = \lambda_0 \cdot dx$

$\vec{r} = \vec{dxP} = P - dx$
$= (0;y) - (x;0)$
$= (-x;y)$

$|\vec{r}|^3 = ((-x)^2 + y^2)^{3/2}$
$= (x^2+y^2)^{3/2}$

C.A.

$$\boxed{\int \frac{x \cdot dx}{(x^2+y^2)^{3/2}}} =$$

$$= -\frac{1}{2}\int \frac{2x \cdot dx}{(x^2+y^2)^{3/2}}$$

SUSTITUCIÓN
$$\begin{cases} u = x^2+y^2 \\ du = 2x \cdot dx \end{cases}$$

$$= -\frac{1}{2}\int \frac{du}{u^{3/2}}$$

$$= -\frac{1}{2}\int u^{-3/2} \cdot du$$

$$= -\frac{1}{2} \cdot \frac{u^{-1/2}}{-\frac{1}{2}} + c$$

$$= \frac{1}{\sqrt{u}} + c$$

$$= \boxed{\frac{1}{\sqrt{x^2+y^2}} + c}$$

$$\vec{E} = \int \frac{k \cdot dq}{|\vec{r}|^3} \cdot \vec{r}$$

$$\vec{E} = \int_{-L}^{L} \frac{k \cdot \lambda_0 \cdot dx}{(x^2+y^2)^{3/2}} \cdot (-x;y)$$

$$\vec{E} = k \cdot \lambda_0 \cdot \left(\int_{-L}^{L} \frac{-x \cdot dx}{(x^2+y^2)^{3/2}} ; \int_{-L}^{L} \frac{y \cdot dx}{(x^2+y^2)^{3/2}} \right)$$

$$\vec{E_x}^* = \int_{-L}^{L} \frac{-x}{(x^2+y^2)^{3/2}} dx$$

$$\vec{E_x}^* = \left[\frac{1}{\sqrt{x^2+y^2}} \right]_{-L}^{L}$$

$$\vec{E_x}^* = \left[\frac{1}{\sqrt{L^2+y^2}} - \frac{1}{\sqrt{(-L)^2+y^2}} \right]$$

$$\vec{E_x}^* = \cancel{\frac{1}{\sqrt{L^2+y^2}}} - \cancel{\frac{1}{\sqrt{L^2+y^2}}}$$

$$\boxed{\vec{E_x}^* = 0}$$

TABLA DE INTEGRALES

$$\int \frac{1}{(x^2+y^2)^{3/2}} dx = \frac{x}{y^2\sqrt{x^2+y^2}} + c$$

$$\vec{E_y}^* = \int_{-L}^{L} \frac{y}{(x^2+y^2)^{3/2}} dx$$

$$\vec{E_y}^* = y \int_{-L}^{L} \frac{1}{(x^2+y^2)^{3/2}} dx$$

$$\vec{E_y}^* = \cancel{y} \cdot \left[\frac{x}{y^{\cancel{2}}\sqrt{x^2+y^2}} \right]_{-L}^{L}$$

$$\vec{E_y}^* = \left[\frac{x}{y \cdot \sqrt{x^2+y^2}} \right]_{-L}^{L}$$

$$\vec{E_y}^* = \left[\frac{L}{y \cdot \sqrt{L^2+y^2}} - \frac{-L}{y \cdot \sqrt{(-L)^2+y^2}} \right]$$

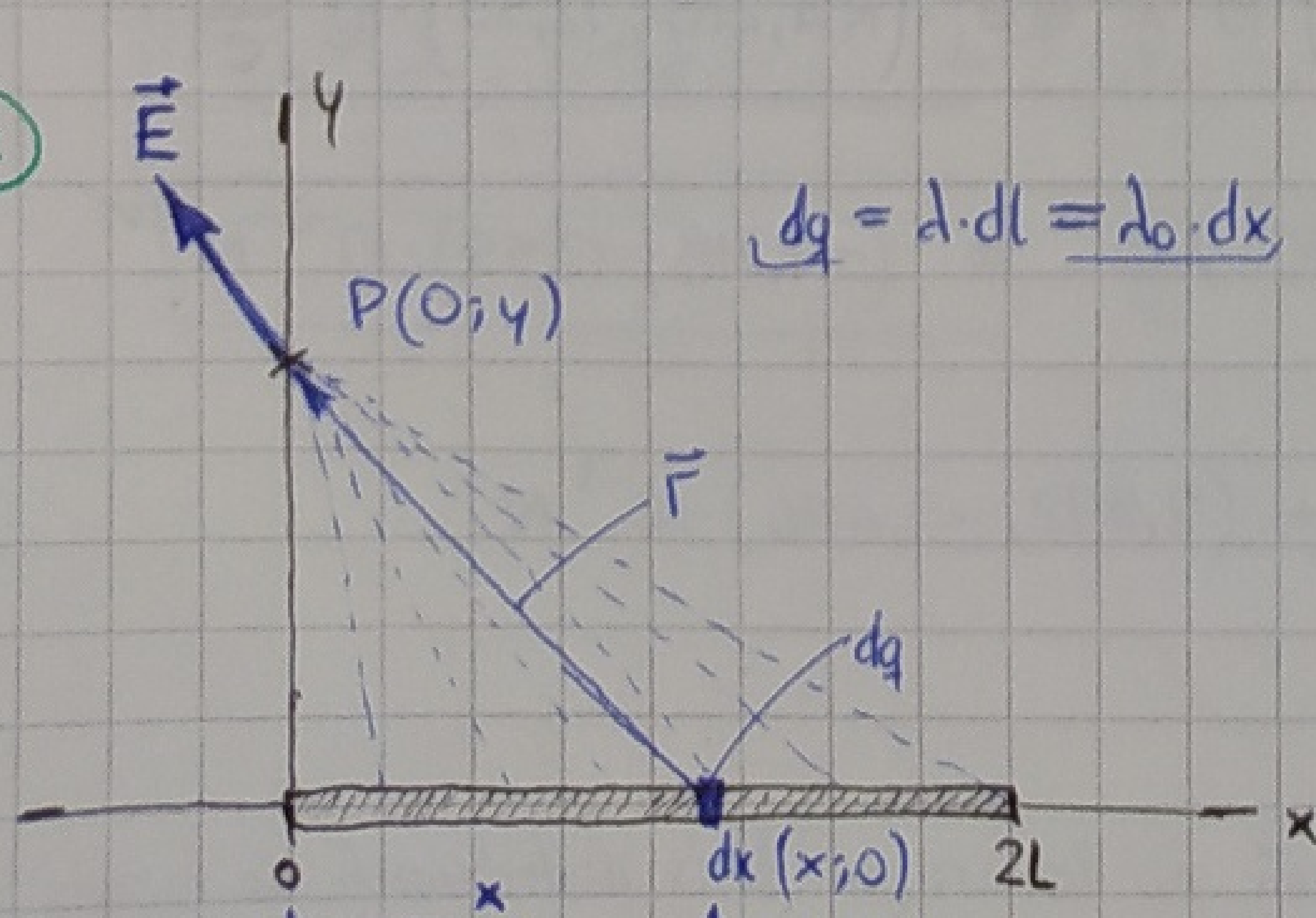
$$\vec{E_y}^* = \left[\frac{L}{y \cdot \sqrt{L^2+y^2}} + \frac{L}{y \cdot \sqrt{L^2+y^2}} \right]$$

$$\boxed{\vec{E_y}^* = \frac{2 \cdot L}{y\sqrt{L^2+y^2}}}$$

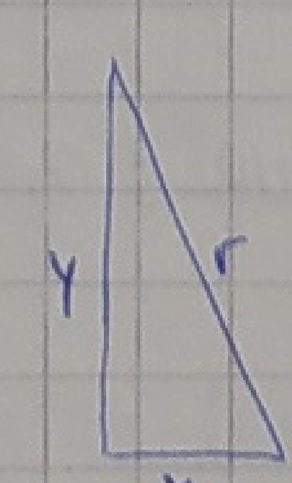
$$\vec{E} = k \cdot \lambda_0 \cdot \left(0 ; \frac{2L}{y\sqrt{L^2+y^2}} \right)$$

$$\boxed{\vec{E} = \left(0 ; k \cdot \lambda_0 \cdot \frac{2 \cdot L}{y\sqrt{L^2+y^2}} \right)}$$

(c)



$dq = \lambda \cdot dl = \lambda_0 \cdot dx$



$0 < x < 2L$

$\vec{r} = \vec{dxP} = P - dx = (0;y) - (x;0) = (-x;y)$

$|\vec{r}|^3 = ((-x)^2+y^2)^{3/2} = (x^2+y^2)^{3/2}$

Cont.) 7 c

$$\vec{E} = \int \frac{k \cdot dq}{|\vec{r}|^3} = \int_0^{2L} \frac{k \cdot \lambda_0 \cdot dx}{(x^2+y^2)^{3/2}} \cdot (-x; y) = k \cdot \lambda_0 \cdot \left(\int_0^{2L} \frac{-x}{(x^2+y^2)^{3/2}} dx ; \int_0^{2L} \frac{y}{(x^2+y^2)^{3/2}} dx \right)$$

$$\vec{E_x}^* = \int_0^{2L} \frac{-x}{(x^2+y^2)^{3/2}} dx \qquad \vec{E_y}^* = y \cdot \int_0^{2L} \frac{1}{(x^2+y^2)^{3/2}} \cdot dx$$

(VER RESOLUCIÓN DE AMBAS INTEGRALES EN EL ÍTEM ANTERIOR)

$$\vec{E_x}^* = \left[\frac{1}{\sqrt{x^2+y^2}} \right|_0^{2L} \qquad \vec{E_y}^* = \cancel{y} \cdot \left[\frac{x}{y^{\cancel{2}} \sqrt{x^2+y^2}} \right|_0^{2L}$$

$$\vec{E_x}^* = \frac{1}{\sqrt{(2L)^2+y^2}} - \frac{1}{\sqrt{0^2+y^2}} \qquad \vec{E_y}^* = \left. \frac{x}{y\sqrt{x^2+y^2}} \right|_0^{2L}$$

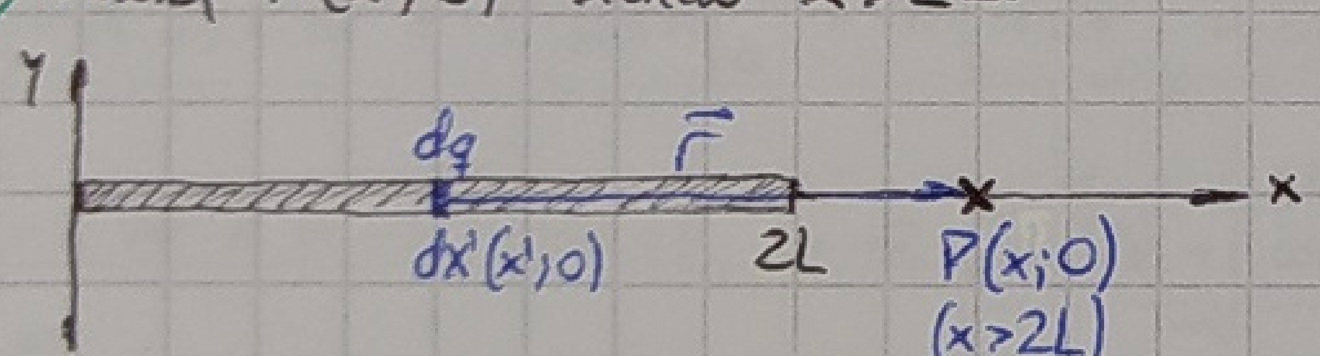
$$\boxed{\vec{E_x}^* = \frac{1}{\sqrt{4L^2+y^2}} - \frac{1}{y}} \qquad \vec{E_y}^* = \frac{2L}{y\sqrt{(2L)^2+y^2}} - \cancel{\frac{0}{y\sqrt{0^2+y^2}}} \;)\neq 0$$

$$\boxed{\vec{E_y}^* = \frac{2 \cdot L}{y\sqrt{4 \cdot L^2+y^2}}}$$

$$\boxed{\vec{E} = k \cdot \lambda_0 \cdot \left(\frac{1}{\sqrt{4L^2+y^2}} - \frac{1}{y} ; \frac{2L}{y\sqrt{4L^2+y^2}} \right)}$$

---

d Ahora, $P(x;0)$ siendo $x > 2L$.



$$\vec{r} = \vec{dxP} = \vec{P} - \vec{dx} = (x;0) - (x';0) = (x-x';0)$$

$$|\vec{r}|^3 = \sqrt{(x-x')^2+0^2}^{\,3} = \sqrt{(x-x')^2}^{\,3} = (x-x')^3$$

$$\vec{E} = \int \frac{k \cdot dq}{|\vec{r}|^3} = \int_0^{2L} \frac{k \cdot \lambda_0 \cdot dx'}{(x-x')^3} \cdot (x-x';0)$$

$$\vec{E} = k \cdot \lambda_0 \cdot \left(\int_0^{2L} \frac{\cancel{(x-x')}}{(x-x')^{\cancel{3}\,2}} dx' ; 0 \right)$$

$$\vec{E} = k \cdot \lambda_0 \cdot \left(\int_0^{2L} \frac{1}{(x-x')^2} dx' ; 0 \right)$$

$$\int \frac{1}{(a-x)^2} \cdot dx = \int (a-x)^{-2} dx \to \begin{array}{l} u = a-x \\ du = -1\,dx = -dx \\ \hookrightarrow dx = -du \end{array}$$

$$\to -\int u^{-2} \cdot du = -\left[\frac{u^{-1}}{-1} + c \right] = \frac{1}{u} + c = \frac{1}{a-x} + c$$

$$\left[\frac{1}{a-x} \right|_0^{2L} = \frac{1}{a-2L} - \left(\frac{1}{a-\cancel{0}} \right) = \frac{1}{a-2L} - \frac{1}{a}$$

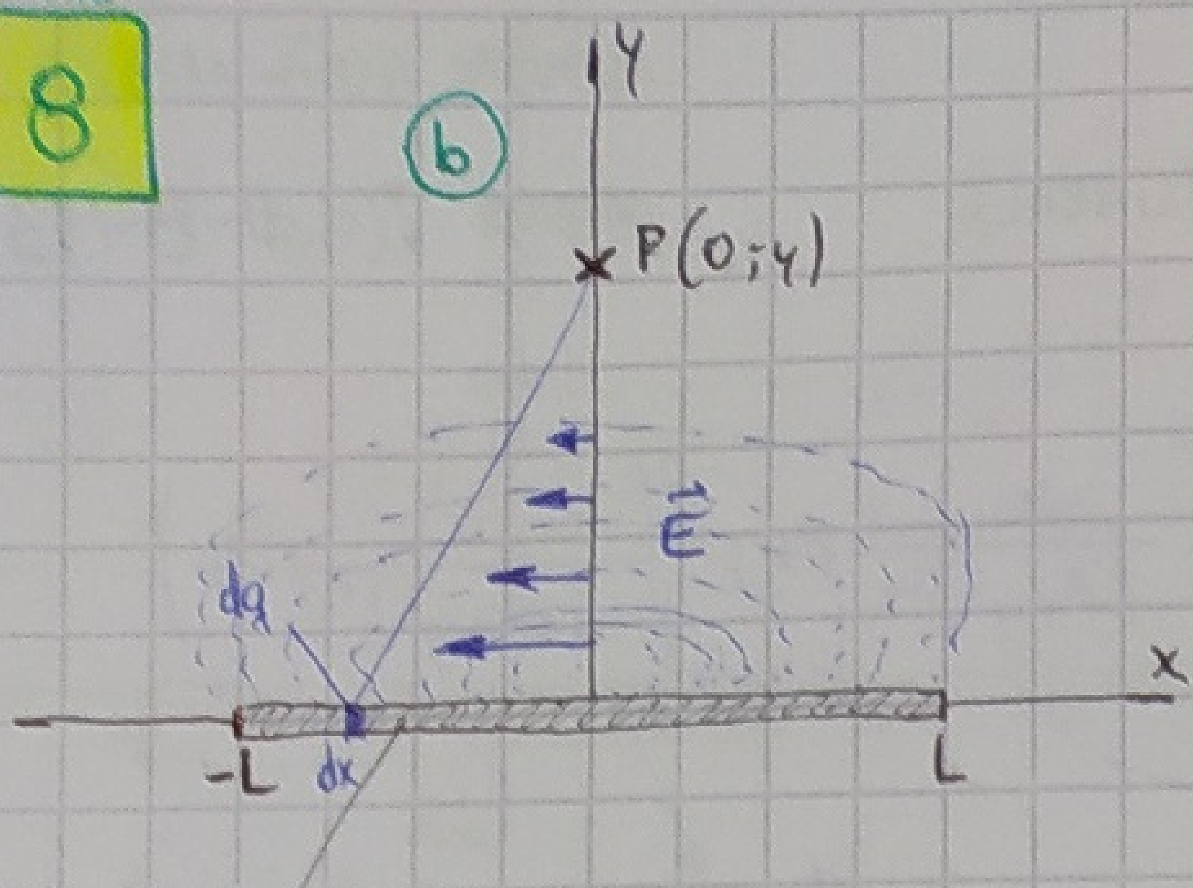
$$\vec{E} = k \cdot \lambda_0 \cdot \left(\frac{1}{x} - \frac{1}{x-2L} ; 0 \right)$$

$$\boxed{\vec{E} = k \cdot \lambda_0 \cdot \left(\frac{1}{a-2L} - \frac{1}{a} ; 0 \right)}$$

NOTA

varilla

8



varilla con carga $\lambda = \lambda_0 \cdot x$, $\lambda_0 > 0$.

a $\boxed{Q} = \int \lambda \cdot dx = \int_{-L}^{L} \lambda_0 \cdot x \cdot dx = \lambda_0 \int_{-L}^{L} x \cdot dx$

$= \lambda_0 \left[\frac{x^2}{2} dx \right|_{-L}^{L} = \lambda_0 \cdot \left[\frac{L^2}{2} - \frac{(-L)^2}{2} \right]$

$= \lambda_0 \cdot \left[\cancel{\frac{L^2}{2} - \frac{L^2}{2}} \right] \boxed{= 0}$

c $\underline{dq = \lambda \cdot dl = \lambda_0 \cdot x \cdot dx}$ $\quad \vec{r} = \overrightarrow{dxP} = P - \vec{dx} = (0;y) - (x;0) = \underline{(-x;y)}$

$\rightarrow |\vec{r}|^3 = ((-x)^2 + y^2)^{3/2} = \underline{(x^2+y^2)^{3/2}}$

$\boxed{\vec{E}} = \int \frac{k \cdot dq}{|\vec{r}|^3} \vec{r} = \int_{-L}^{L} \frac{k \cdot \lambda_0 \cdot x \cdot dx}{(x^2+y^2)^{3/2}} (-x;y) = \boxed{k \cdot \lambda_0 \cdot \int_{-L}^{L} \frac{x \cdot dx}{(x^2+y^2)^{3/2}} \cdot (-x;y)}$

d $\lambda = \lambda_0 \cdot x^2 \longrightarrow \boxed{Q} = \int_{-L}^{L} \lambda_0 \cdot x^2 \cdot dx = \lambda_0 \int_{-L}^{L} x^2 dx = \lambda_0 \left[\frac{x^3}{3} \right|_{-L}^{L} = \lambda_0 \left[\frac{L^3}{3} - \frac{(-L)^3}{3} \right] = \boxed{\lambda_0 \cdot \frac{2}{3} L^3}$

La carga total ya no es nula.

$\underline{dq} = \lambda \cdot dl$
$= \underline{\lambda_0 \cdot x^2 \cdot dx}$

$\boxed{\vec{E}} = \int \frac{k \cdot dq}{|\vec{r}|^3} \cdot \vec{r} = \int_{-L}^{L} \frac{k \cdot \lambda_0 \cdot x^2 \cdot dx}{(x^2+y^2)^{3/2}} = \boxed{k \cdot \lambda_0 \cdot \int_{-L}^{L} \frac{x^2 \cdot dx}{(x^2+y^2)^{3/2}} \cdot (-x;y)}$

$\underline{\vec{r}} = \overrightarrow{dxP} = P - dx$
$= (0;y) - (x;0)$
$= \underline{(-x;y)}$

$\underline{|\vec{r}|^3} = \sqrt{(-x)^2 + y^2}^{\,3}$
$= \underline{(x^2+y^2)^{3/2}}$

13

Anillo:
radio R.
cargado con $\lambda_1 > 0$.

Alambre: longitud 2L.
pasa por el centro del anillo.
cargado con $\lambda_2 = \beta \cdot \lambda_0 \cdot x^2$

$Q_{TOTAL} = 0$

$Q_{TOTAL} = Q_{anillo} + Q_{alambre}$

$0 = \lambda_1 \cdot R \cdot 2\pi + \frac{2}{3} \beta \cdot \lambda_0 \cdot L^3$

$3 \cdot \lambda_1 \cdot R \cdot \cancel{2}\pi = -\cancel{2}\beta \cdot \lambda_0 \cdot L^3$

$\boxed{\lambda_0 = -\frac{3 \cdot \lambda_1 \cdot R \cdot \pi}{\beta \cdot L^3}}$

$\boxed{Q_{anillo}} = \int \lambda \cdot dl$

$= \int_0^{2\pi} \lambda_1 \cdot R \cdot d\alpha$

$= \lambda_1 \cdot R \cdot \int_0^{2\pi}$

$= \lambda_1 \cdot R \cdot \left[\alpha \right|_0^{2\pi}$

$= \lambda_1 \cdot R \cdot (2\pi - 0)$

$\boxed{= \lambda_1 \cdot R \cdot 2\pi}$

$\boxed{Q_{alambre}} = \int \lambda \cdot dl$

$= \int_{-L}^{L} \beta \cdot \lambda_0 \cdot x^2 \cdot dx$

$= \beta \cdot \lambda_0 \cdot \int_{-L}^{L} x^2 \cdot dx$

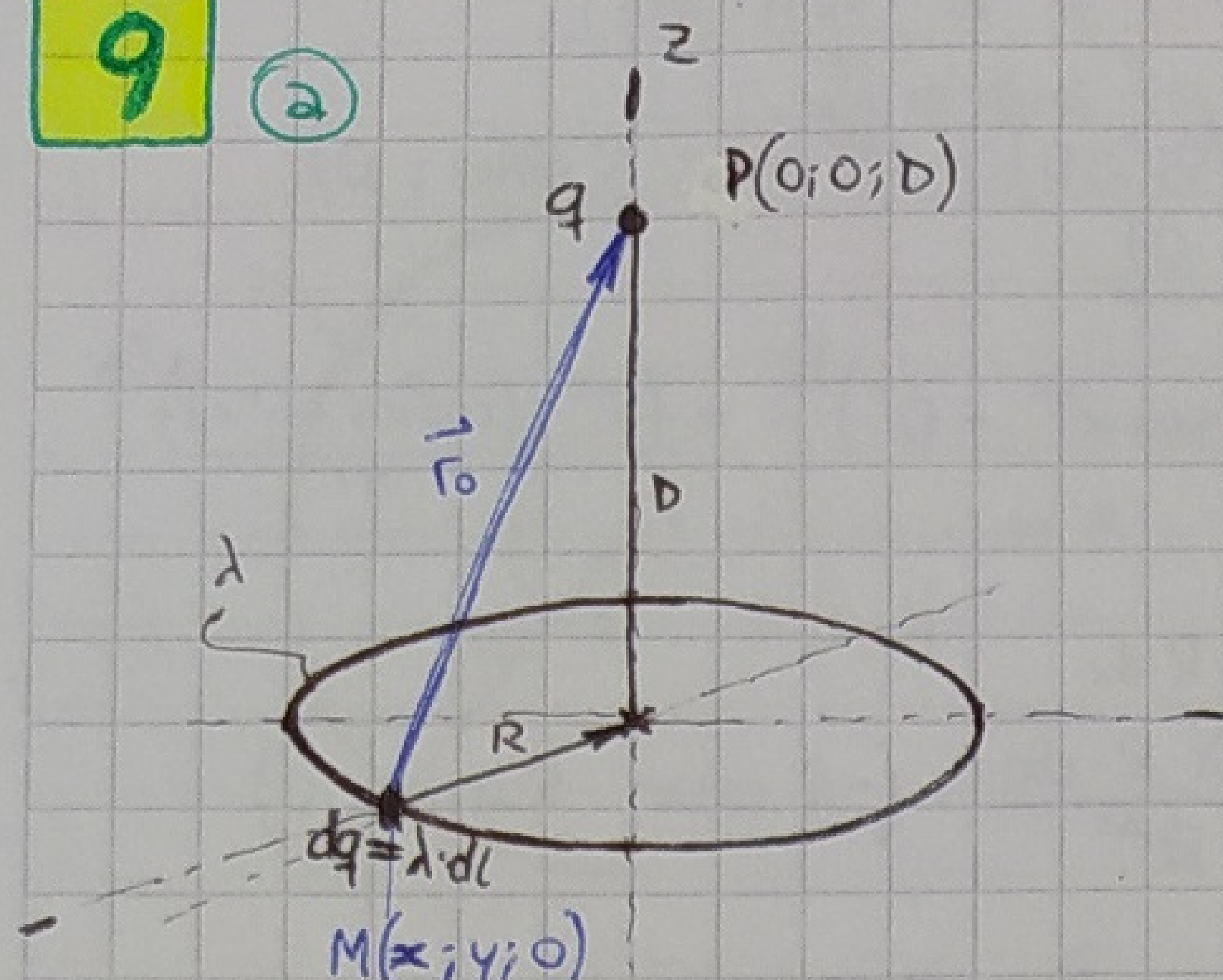
$= \beta \cdot \lambda_0 \cdot \left[\frac{x^3}{3} \right|_{-L}^{L}$

$= \beta \cdot \lambda_0 \cdot \left[\frac{L^3}{3} - \frac{(-L)^3}{3} \right]$

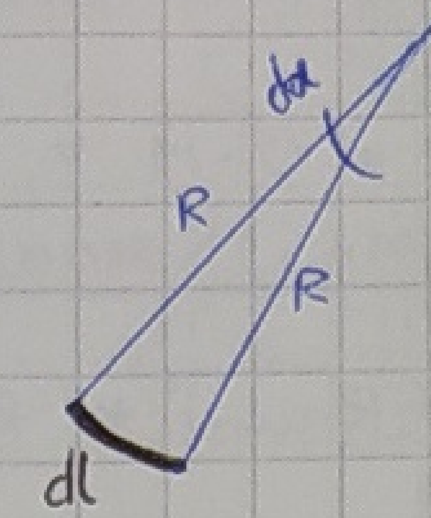
$\boxed{= \frac{2}{3} \beta \cdot \lambda_0 \cdot L^3}$

NOTA

9 ②



$M(x;y;0)$
$-R < x < R$
$-R < y < R$
$x^2+y^2=R^2$

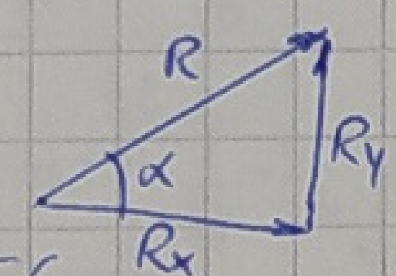


$\text{sen}(d\alpha) = \dfrac{dl}{R} \rightarrow d\alpha = \dfrac{dl}{R} \Rightarrow \underline{dl = R\cdot d\alpha}$, $0 \leq \alpha \leq 2\pi$

Si $d\alpha \to 0 \Rightarrow \text{sen}(d\alpha) = d\alpha$

$$\vec{r}_0 = \overrightarrow{MP} = P-M = (0;0;D)-(x;y;0) = \underline{(-x;-y;D)}$$

$$|\vec{r}_0|^3 = \sqrt{(-x)^2+(-y)^2+D^2}^{\,3} = \underline{(x^2+y^2+D^2)^{3/2}}$$



$M = (R\cdot\cos\alpha;\ R\cdot\text{sen}\,\alpha;\ D)$

$$\begin{cases} \text{sen}\,\alpha = \dfrac{R_y}{R} & \cos\alpha = \dfrac{R_x}{R} \\ \underline{R_y = R\cdot\text{sen}\,\alpha}\ (M_y) & \underline{R_x = R\cdot\cos\alpha}\ (M_x) \end{cases}$$

$$\vec{r}_0 = \overrightarrow{MP} = P-M = (0;0;D)-(R\cos\alpha;\ R\cdot\text{sen}\,\alpha;\ D) = \underline{(-R\cos\alpha;\ -R\,\text{sen}\,\alpha;\ D)}$$

$$|\vec{r}_0|^3 = \left[(-R\cos\alpha)^2+(-R\,\text{sen}\,\alpha)^2+D^2\right]^{3/2} = \left[R^2\underbrace{(\cos^2\alpha+\text{sen}^2\alpha)}_{=1}+D^2\right]^{3/2} = \underline{(R^2+D^2)^{3/2}}$$

$$\vec{E} = \int \frac{k\cdot \overbrace{dq}^{dq=\lambda dl}}{|\vec{r}_0|^3}\cdot\vec{r}_0 = \int \frac{k\cdot\lambda\cdot \overbrace{dl}^{dl=R\cdot d\alpha}}{(x^2+y^2)^{3/2}}\cdot(-x;-y;D) = \int_0^{2\pi}\frac{k\cdot\lambda\cdot R\cdot d\alpha}{(R^2+D^2)^{3/2}}\cdot(-R\cdot\cos\alpha;\ -R\cdot\text{sen}\,\alpha;\ D)$$

$$= \frac{k\cdot\lambda\cdot R}{(R^2+D^2)^{3/2}}\cdot\int_0^{2\pi} d\alpha\cdot(-R\cdot\cos\alpha;\ -R\cdot\text{sen}\,\alpha;\ D) = \frac{k\cdot\lambda\cdot R}{(R^2+D^2)^{3/2}}\cdot\left(\int_0^{2\pi}-R\cdot\cos\alpha\, d\alpha;\ \int_0^{2\pi}-R\cdot\text{sen}\,\alpha\, d\alpha;\ \int_0^{2\pi}D\cdot d\alpha\right)$$

$$\int_0^{2\pi}-R\cdot\cos\alpha\cdot d\alpha = -R\left[\text{sen}\,\alpha\Big|_0^{2\pi}\right] = -R\cdot\underbrace{(\text{sen}\,2\pi-\text{sen}\,0)}_{=0} = \boxed{0}$$

$$\int_0^{2\pi}-R\cdot\text{sen}\,\alpha = -R\left[-\cos\alpha\Big|_0^{2\pi}\right] = -R\cdot\underbrace{(\overbrace{-\cos(2\pi)}^{-1}-(\overbrace{-\cos 0}^{-1}))}_{=0} = \boxed{0}$$

$$\int_0^{2\pi}D\,d\alpha = D\left[\alpha\Big|_0^{2\pi}\right] = D\cdot(2\pi-0) = \boxed{2\pi\cdot D}$$

$$= \frac{k\cdot\lambda\cdot R}{(R^2+D^2)^{3/2}}\cdot(0;\ 0;\ 2\pi D)$$

$$\boxed{\vec{E} = \left(0;\ 0;\ \frac{k\cdot\lambda\cdot R\cdot 2\pi\cdot D}{(R^2+D^2)^{3/2}}\right)}$$

$$\vec{E} = \frac{k\cdot\lambda\cdot R\cdot 2\pi\cdot D}{(R^2+D^2)^{3/2}}\ \breve{k}$$

siendo $k = \dfrac{1}{4\pi\varepsilon_0}$

$$\vec{E} = \frac{\lambda\cdot R\cdot D}{2\cdot\varepsilon_0\cdot(R^2+D^2)^{3/2}}\ \breve{k}$$

$$\Sigma\vec{F}_z = 0$$

$$\vec{P} = \vec{F}_{elz}$$

$$m\cdot g = q\cdot\vec{E}_x$$

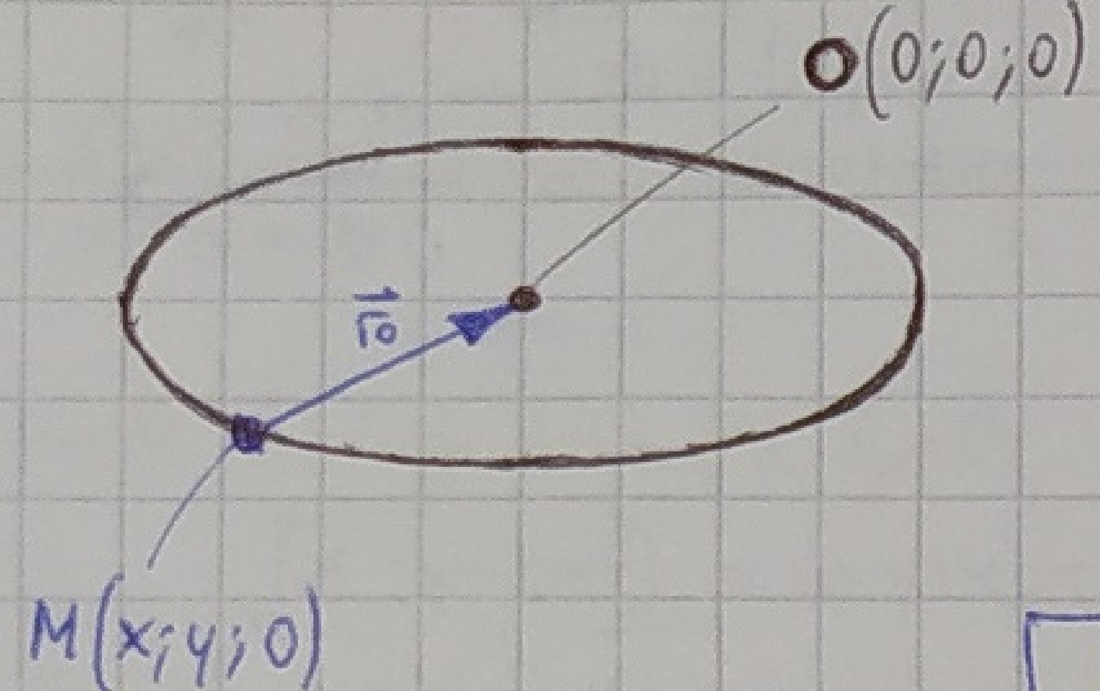
$$m\cdot g = q\cdot\frac{k\cdot\lambda\cdot R\cdot D\cdot 2\pi}{(R^2+D^2)^{3/2}}$$

$$\boxed{\lambda = \frac{m\cdot g\cdot(R^2+D^2)^{3/2}}{q\cdot k\cdot R\cdot D\cdot 2\pi}}$$

NOTA

(b) El campo eléctrico en el centro de la espira ($O(0;0;0)$) será la suma del CE generado por el anillo con carga $\lambda>0$ y del CE generado por la carga $q>0$.

- CE generado por el anillo: (Análogamente a lo resuelto en el ítem (a):)



$$\vec{r_0} = MO = O-M = (0;0;0) - (R\cdot\cos\alpha;\ R\cdot\text{sen}\,\alpha;\ 0)$$
$$= (-R\cdot\cos\alpha;\ -R\cdot\text{sen}\,\alpha;\ 0)$$

$$|\vec{r_0}|^3 = \left((-R\cdot\cos\alpha)^2 + (-R\cdot\text{sen}\,\alpha)^2 + 0^2\right)^{3/2} = \left(R^2\cdot\underbrace{(\cos^2\alpha + \text{sen}^2\alpha)}_{=1}\right)^{3/2}$$
$$= R^3$$

$dq = \lambda\, dl \qquad dl = R\cdot d\alpha$

$$\boxed{\vec{E}_{anillo}} = \int \frac{k\cdot dq}{|\vec{r_0}|^3}\cdot\vec{r_0} = \int_0^{2\pi} \frac{k\cdot\lambda\cdot\cancel{R}\cdot d\alpha}{R^{\cancel{3}2}}\cdot(-R\cdot\cos\alpha;\ -R\cdot\text{sen}\,\alpha;\ 0)$$

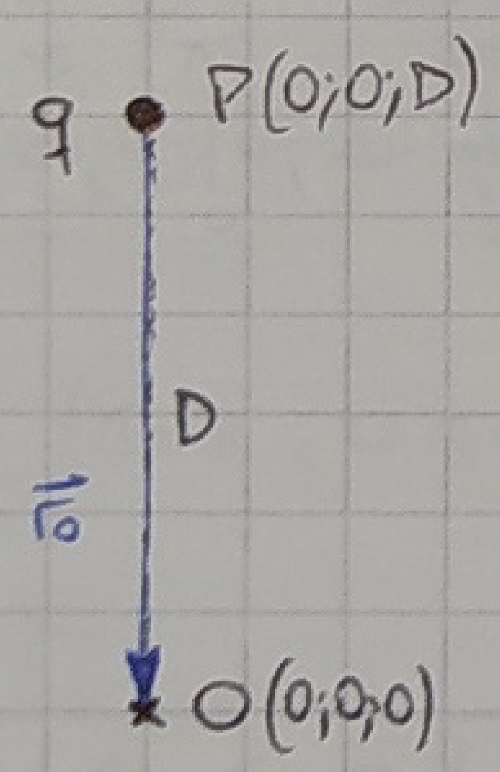
$$= k\cdot\lambda\cdot\left(\int_0^{2\pi}\frac{-\cancel{R}\cdot\cos\alpha}{R^{\cancel{2}}}\cdot d\alpha;\ \int_0^{2\pi}\frac{-\cancel{R}\cdot\text{sen}\,\alpha}{R^{\cancel{2}}}\,d\alpha;\ \underbrace{\cancel{\int_0^{2\pi}\frac{0}{R^2}\cdot d\alpha}}_{=0}\right)$$

$$= k\cdot\lambda\cdot\left(-\frac{1}{R}\cdot\underbrace{\int_0^{2\pi}\cos\alpha\cdot d\alpha}_{=0};\ -\frac{1}{R}\underbrace{\int_0^{2\pi}\text{sen}\,\alpha\cdot d\alpha}_{=0};\ 0\right)$$

ver resolución en el ítem (a).

$$= k\cdot\lambda\cdot(0;\ 0;\ 0)$$
$$\boxed{=(0;0;0)}$$

- CE generado por la carga q:



$$\vec{r_0} = \vec{PO} = O-P = (0;0;0) - (0;0;D) = (0;0;-D)$$

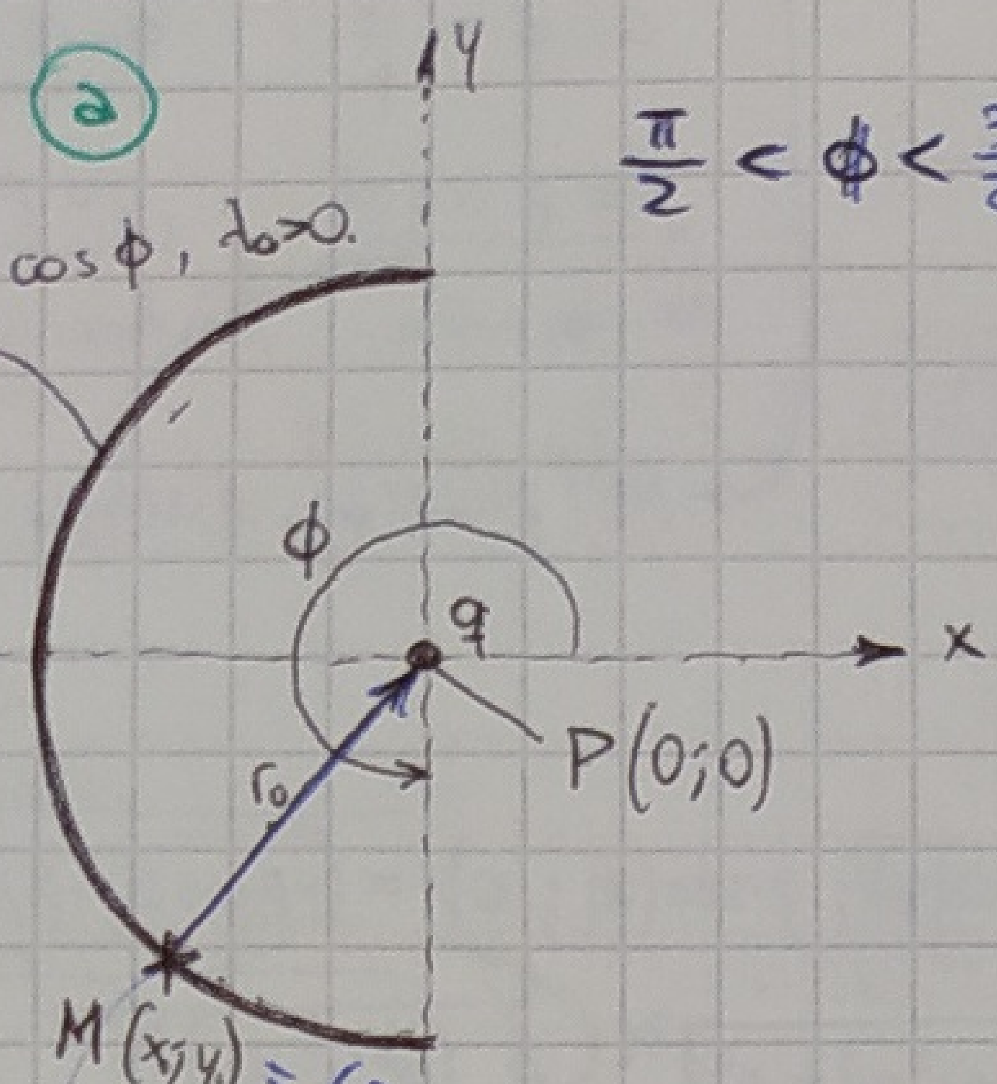
$$|\vec{r_0}|^3 = \left(0^2+0^2+(-D)^2\right)^{3/2} = (D^2)^{3/2} = D^3$$

$$\boxed{\vec{E}_{carga}} = \frac{k\cdot q}{|\vec{r_0}|^3}\cdot\vec{r} = \frac{k\cdot q}{D^3}\cdot(0;0;-D) = \left(0;\ 0;\ -\frac{k\cdot q\cdot\cancel{D}}{D^{\cancel{3}2}}\right)$$
$$\boxed{= \left(0;0;\frac{-k\cdot q}{D^2}\right)}$$

$$\boxed{\vec{E}_{TOTAL}} = \vec{E}_{anillo} + \vec{E}_{carga}$$
$$= (0;0;0) + \left(0;0;-\frac{k\cdot q}{D^2}\right)$$
$$\boxed{= \left(0;\ 0;\ -\frac{k\cdot q}{D^2}\right)}$$

10 ⓐ

$\lambda = \lambda_0 \cdot \cos\phi, \; \lambda_0 > 0.$



$\frac{\pi}{2} < \phi < \frac{3}{2}\pi$

$\vec{r_0} = \vec{MP} = P - M = (0;0) - (R\cdot\cos\phi; R\cdot sen\phi)$
$= (-R\cdot\cos\phi; -R\cdot sen\phi)$

$|\vec{r_0}|^3 = \left[(-R\cdot\cos\phi)^2 + (-R\cdot sen\phi)^2\right]^{3/2} = \left[R^2\cdot\underbrace{(\cos^2\phi + sen^2\phi)}_{=1}\right]^{3/2} = R^3$

$dq = \lambda\cdot dl$

$\lambda = \lambda_0\cdot\cos\theta$

$dl = R\cdot d\phi$

$dq = \lambda_0\cdot\cos\theta\cdot R\cdot d\phi$

$$\boxed{\vec{E}} = \int \frac{k\cdot dq}{|\vec{r_0}|^3}\cdot\vec{r_0} = \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \frac{k\cdot\lambda_0\cdot\cos\phi\cdot R\cdot d\phi}{R^{3/2}}\cdot(-R\cos\phi; -R\cdot sen\phi)$$

$$= k\cdot\lambda_0\cdot\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\frac{\cos\phi\cdot d\phi}{R^2}\cdot(-R\cdot\cos\phi; -R\cdot sen\phi)$$

$$= k\cdot\lambda_0\cdot\left(\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} -\frac{\cancel{R}\cdot\cos^2\phi}{R^{\cancel{2}}}d\phi ; \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} -\frac{\cos\phi\cdot sen\phi\cdot}{R^2}d\phi\right)$$

$$= -\frac{k\cdot\lambda_0}{R}\cdot\left(\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\cos^2\phi\cdot d\phi ; \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\cos\phi\cdot sen\phi\cdot d\phi\right)$$

$$= -\frac{k\cdot\lambda_0}{R}\cdot\left(\frac{\pi}{2}; 0\right)$$

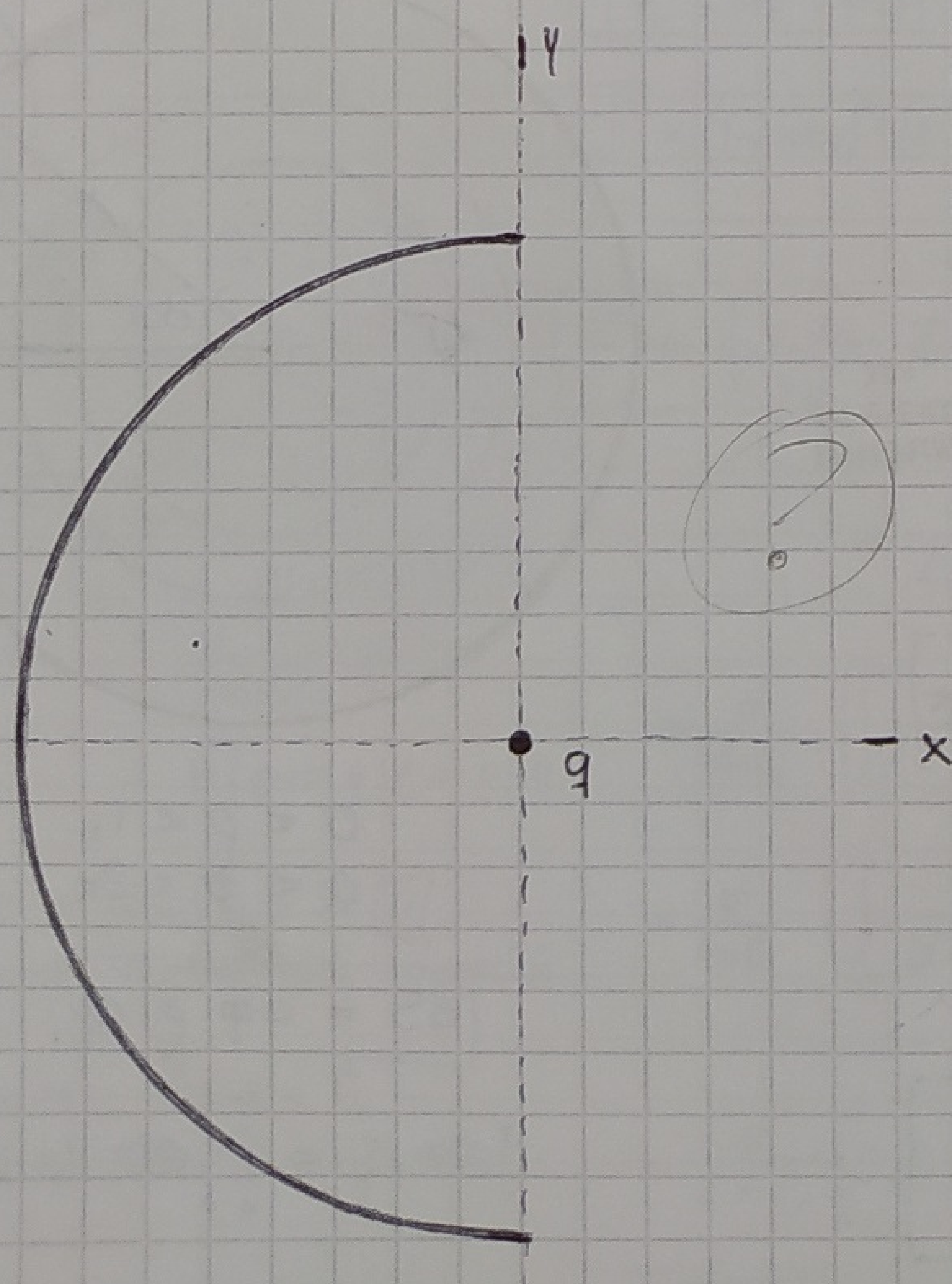
$$\boxed{= \left(-\frac{k\cdot\lambda_0\cdot\pi}{2R}; 0\right)} \longrightarrow \vec{F} = q\cdot\vec{E}$$

$$\vec{F} = q\cdot\left(-\frac{k\cdot\lambda_0\cdot\pi}{2\cdot R}; 0\right)$$

$$\boxed{\vec{F} = \left(-\frac{k\cdot q\cdot\lambda_0\cdot\pi}{2R}; 0\right)}$$

CA
- $\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\cos^2\alpha\cdot d\alpha = \frac{1}{2}(\alpha + sen\alpha\cdot\cos\alpha)\Big|_{\frac{\pi}{2}}^{\frac{3}{2}\pi}$
  $= \frac{3}{4}\pi - \frac{1}{4}\pi = \boxed{\frac{\pi}{2}}$
- $\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\cos\alpha\cdot sen\alpha\cdot d\alpha = sen^2\alpha\Big|_{\frac{\pi}{2}}^{\frac{3}{2}\pi}$
  $= 1 - 1 = \boxed{0}$

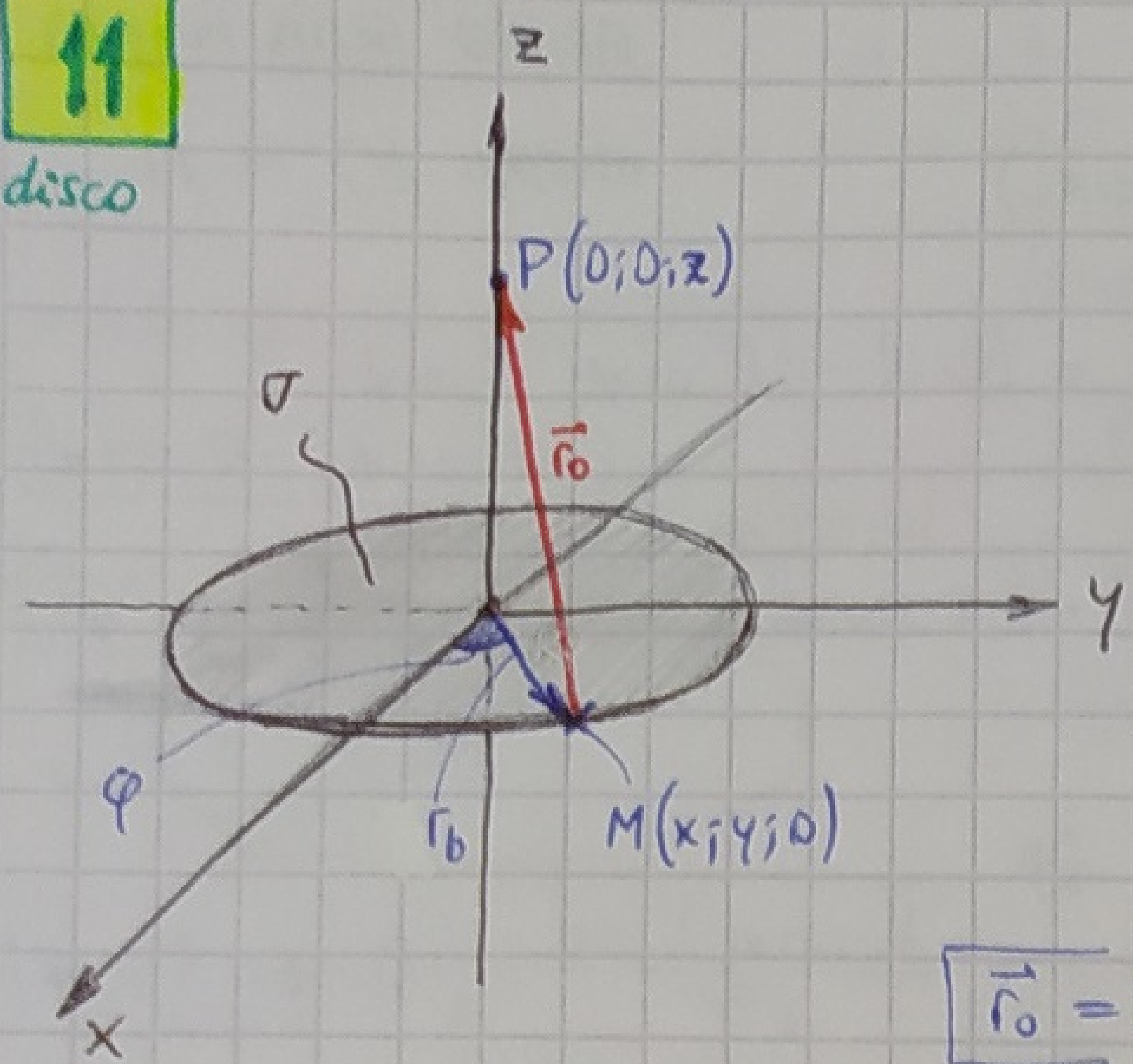
ⓑ

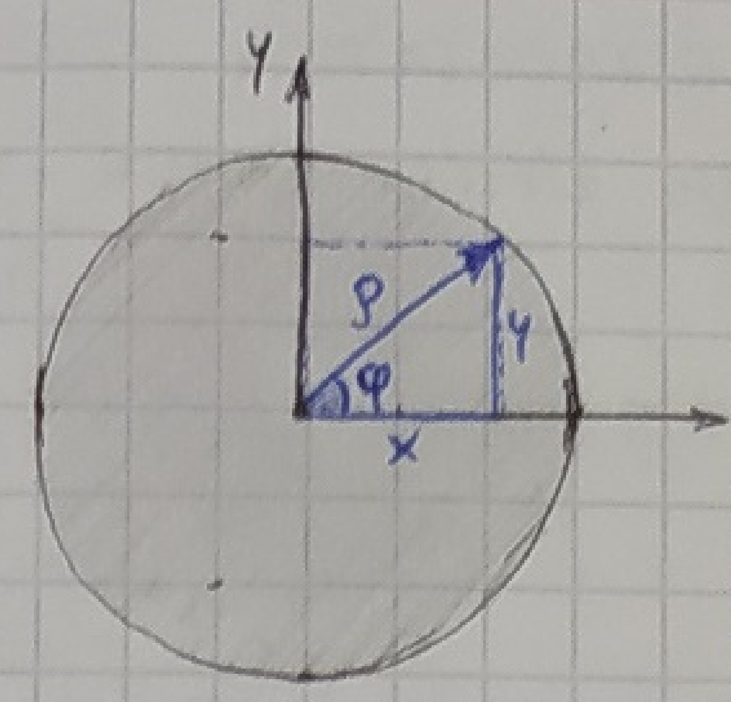


10 - b
Dibuje las líneas de campo en los puntos $(0; 0 < y < R)$ correspondientes a esta distribución.
?

11

disco





M(x;y;0)

$\cos\varphi = \frac{x}{\rho}$ $\quad$ $\operatorname{sen}\varphi = \frac{y}{\rho}$

$x = \rho\cdot\cos(\varphi)$ $\quad$ $y = \rho\cdot\operatorname{sen}(\varphi)$

"todos los radios... para todos los ángulos del disco"

$\to M(\rho\cdot\cos(\varphi);\ \rho\cdot\operatorname{sen}(\varphi);\ 0)$

$0 < \varphi < 2\pi$

$0 < \rho < r_b$

$dq = \sigma\cdot dS$

$$\boxed{\vec{r_0}} = \vec{MP} = P - M = (0;0;z) - (\rho\cdot\cos(\varphi);\ \rho\cdot\operatorname{sen}(\varphi);\ 0) = \boxed{(-\rho\cos(\varphi);\ -\rho\operatorname{sen}(\varphi);\ z)}$$

$$\boxed{|\vec{r_0}|^3} = \left[(-\rho\cdot\cos\varphi)^2 + (-\rho\cdot\operatorname{sen}\varphi)^2 + z^2\right]^{3/2} = \left[\rho^2(\operatorname{sen}^2\varphi + \cos^2\varphi) + z^2\right]^{3/2} = \boxed{(\rho^2+z^2)^{3/2}}$$

$$\vec{E} = \int \frac{k\cdot dq}{|\vec{r_0}|^3}\cdot\vec{r_0} = \int\frac{k\cdot\sigma\cdot dS}{|\vec{r_0}|^3}\cdot\vec{r_0} = k\cdot\sigma\cdot\int_0^{2\pi}\int_0^{r_b}\rho\cdot\frac{(-\rho\cdot\cos(\varphi);\ -\rho\cdot\operatorname{sen}(\varphi);\ z)}{(\rho^2+z^2)^{3/2}}\cdot d\rho\cdot d\varphi$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\cdot\left(\int_0^{2\pi}\int_0^{r_b} -\frac{\rho^2\cdot\cos\varphi}{(\rho^2+z^2)^{3/2}}\,d\rho\, d\varphi;\ \int_0^{2\pi}\int_0^{r_b}\frac{\rho^2\cdot\operatorname{sen}\varphi}{(\rho^2+z^2)^{3/2}}\,d\rho\, d\varphi;\ \int_0^{2\pi}\int_0^{r_b}\frac{\rho\cdot z}{(\rho^2+z^2)^{3/2}}\cdot d\rho\cdot d\varphi\right)$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\cdot\left(\cancel{-\int_0^{r_b}\frac{\rho^2}{(\rho^2+z^2)^{3/2}}\,d\rho\ \boxed{\int_0^{2\pi}\cos\varphi\cdot d\varphi}};\ \cancel{-\int_0^{r_b}\frac{\rho^2}{(\rho^2+z^2)^{3/2}}\,d\rho\ \boxed{\int_0^{2\pi}\operatorname{sen}\varphi\cdot d\varphi}};\ \int_0^{r_b}\frac{\rho\cdot z}{(\rho^2+z^2)^{3/2}}\cdot d\rho\ \boxed{\int_0^{2\pi}d\varphi}\right)$$

$\left[\operatorname{sen}\varphi\right]_0^{2\pi} = \operatorname{sen}(2\pi) - \operatorname{sen}(0) = 0 - 0 = 0$

$\left[-\cos(\varphi)\right]_0^{2\pi} = -\cos(2\pi) - (-\cos(0)) = -1 - (-1) = 0$

$\left[\varphi\right]_0^{2\pi} = 2\pi - 0 = 2\pi$

$$= \frac{\sigma}{4\pi\varepsilon_0}\cdot\left(0;\ 0;\ 2\pi\cdot\int_0^{r_b}\frac{\rho\cdot z}{(\rho^2+z^2)^{3/2}}\,d\rho\right)$$

$$\boxed{\vec{E_x} = 0}$$

$$\boxed{\vec{E_y} = 0}$$

$$\vec{E_z} = \frac{\sigma}{\cancel{4}\cancel{\pi}\cdot\varepsilon_0}\cdot\cancel{2\pi}\, z\ \boxed{\int_0^{r_b}\frac{\rho}{(\rho^2+z^2)^{3/2}}\,d\rho}$$

VER TABLA DE INTEGRALES

$$= \left[-\frac{1}{\sqrt{\rho^2+z^2}}\right]_0^{r_b}$$

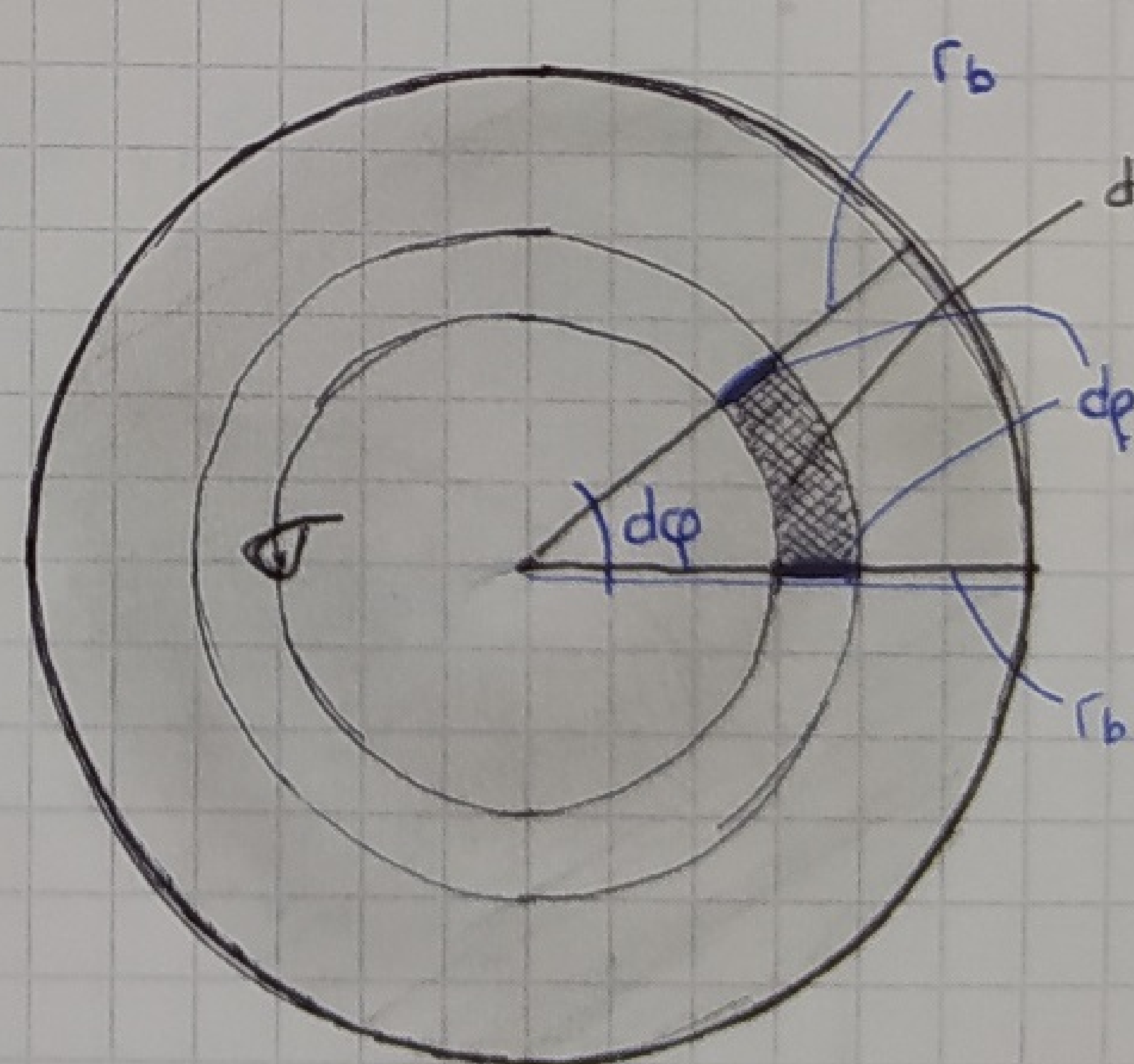
$$= \left[-\frac{1}{\sqrt{r_b^2+z^2}}\right] - \left[-\frac{1}{\sqrt{0^2+z^2}}\right]$$

$$= \frac{1}{z} - \frac{1}{\sqrt{r_b^2+z^2}}$$

$$\vec{E_z} = \frac{\sigma}{2\cdot\varepsilon_0}\cdot z\left(\frac{1}{z} - \frac{1}{\sqrt{r_b^2+z^2}}\right),\quad z\neq 0.$$

$$\boxed{\vec{E_z} = \frac{\sigma}{2\cdot\varepsilon_0}\cdot\left(1 - \frac{z}{\sqrt{r_b^2+z^2}}\right),\quad z\neq 0.}$$

Si $z = 0 \Rightarrow \boxed{\vec{E_z} = 0}$



$0 < \rho < r_b$ RADIO

$0 < \varphi < 2\pi$ ÁNGULO

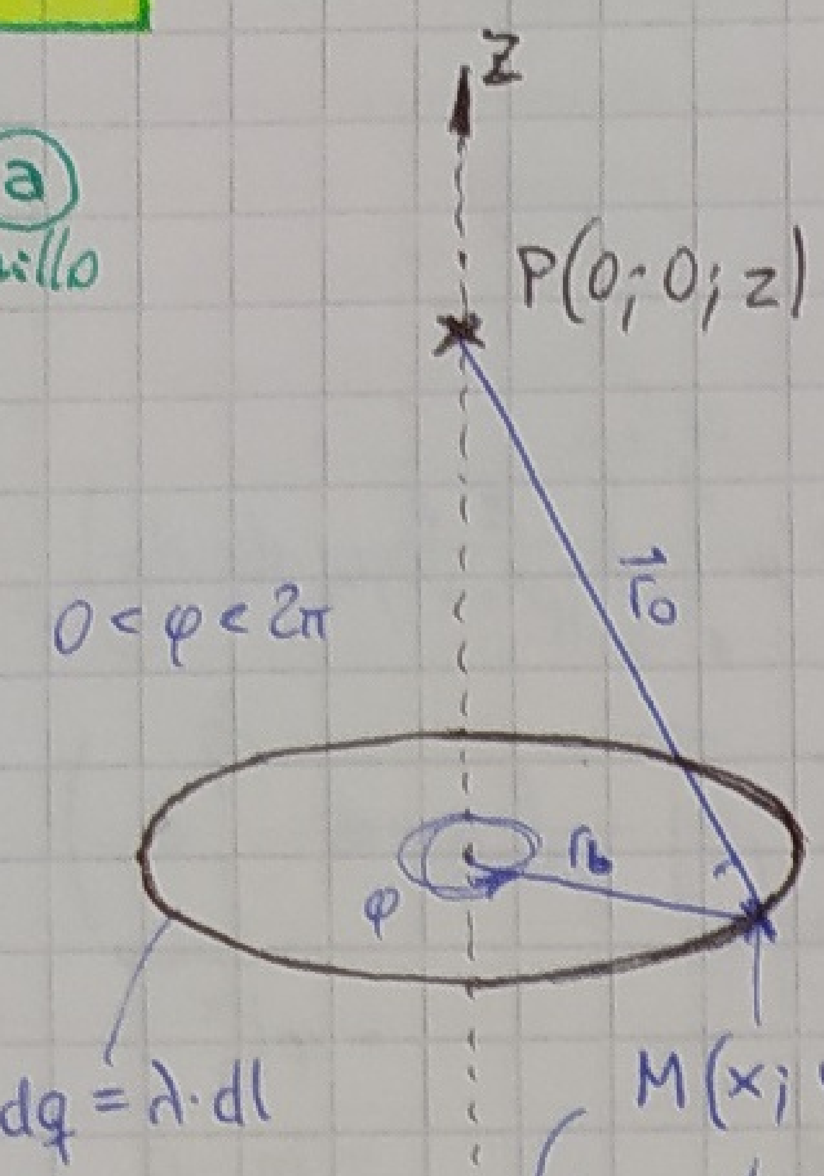
$$\boxed{dS = \underbrace{d\varphi\cdot\rho}_{dl}\cdot d\rho}$$

$$\int dS = \int_0^{2\pi}\int_0^{r_b} d\rho\cdot d\varphi$$

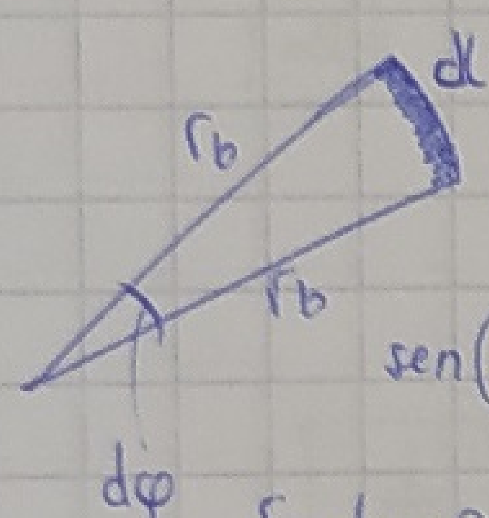
NOTA

12 → 11

a anillo



(en coordenadas cilíndricas
$M(r_b \cdot \cos\varphi; r_b \cdot \text{sen}\,\varphi; 0)$



$\text{sen}(d\varphi) = \frac{dl}{r_b}$

Si $d\varphi = 0 \Rightarrow \text{sen}(d\varphi) \simeq d\varphi$

$d\varphi = \frac{dl}{r_b}$

$\boxed{dl = r_b \cdot d\varphi}$

$$\boxed{\vec{r_0}} = \vec{MP} = P - M = (0; 0; z) - (r_b \cdot \cos\varphi; r_b \cdot \text{sen}\,\varphi; 0) = \boxed{(-r_b \cdot \cos\varphi; -r_b \cdot \text{sen}\,\varphi; z)}$$

$$\boxed{|\vec{r_0}|^3} = \left((-r_b \cdot \cos\varphi)^2 + (-r_b \cdot \text{sen}\,\varphi)^2 + z^2\right)^{3/2} = \left(r_b^2 \cdot (\cos^2\varphi + \text{sen}^2\varphi) + z^2\right)^{3/2} = \boxed{(r_b^2 + z^2)^{3/2}}$$

$dq = \lambda \cdot dl$

$$\boxed{\vec{E}} = \int \frac{k \cdot dq}{|\vec{r_0}|^3} \cdot \vec{r_0} = k \cdot \int \frac{\lambda \cdot dl}{r_0} \cdot \vec{r_0} \qquad dl = r_b \cdot d\varphi$$

$$= k \cdot \lambda \cdot \int_0^{2\pi} \frac{r_b \cdot d\varphi}{(r_b^2 + z^2)^{3/2}} \cdot (-r_b \cdot \cos\varphi; -r_b \cdot \text{sen}\,\varphi; z)$$

$$= k \cdot \lambda \cdot \left(\int_0^{2\pi} -\frac{r_b^2 \cdot \cos\varphi}{(r_b^2 + z^2)^{3/2}} d\varphi ; \int_0^{2\pi} -\frac{r_b^2 \cdot \text{sen}\,\varphi}{(r_b^2 + z^2)^{3/2}} d\varphi ; \int_0^{2\pi} \frac{r_b \cdot z}{(r_b^2 + z^2)^{3/2}} d\varphi \right)$$

$$= k \cdot \lambda \left(\cancel{-\frac{r_b^2}{(r_b^2 + z^2)^{3/2}} \cdot \int_0^{2\pi} \cos\varphi\, d\varphi} ; \cancel{-\frac{r_b^2}{(r_b^2 + z^2)^{3/2}} \cdot \int_0^{2\pi} \text{sen}\,\varphi \cdot d\varphi} ; \frac{r_b \cdot z}{(r_b^2 + z^2)^{3/2}} \cdot \int_0^{2\pi} d\varphi \right)$$

$= [\text{sen}(\varphi)]_0^{2\pi} = \text{sen}(2\pi) - \text{sen}(0) = 0 - 0 = \boxed{0}$

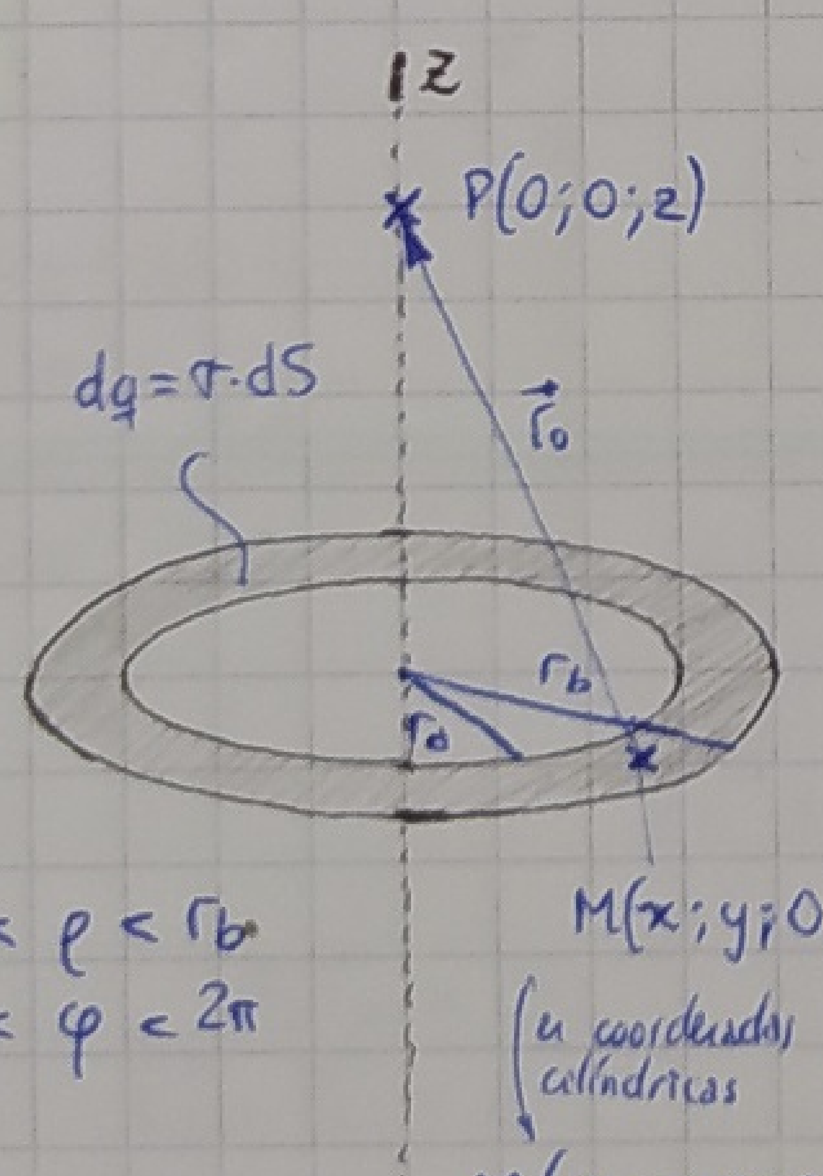
$= [-\cos\varphi]_0^{2\pi} = -\cos(2\pi) - (-\cos(0)) = -1 - (-1) = \boxed{0}$

$= \varphi |_0^{2\pi} = 2\pi - 0 = \boxed{2\pi}$

$$\boxed{= k \cdot \lambda \cdot \left(0; 0; \frac{r_b \cdot z}{(r_b^2 + z^2)^{3/2}} \cdot 2\pi\right)}$$

---

b corona



$r_a < \rho < r_b$

$0 < \varphi < 2\pi$

(en coordenadas cilíndricas
$M(\rho \cdot \cos\varphi; \rho \cdot \text{sen}\,\varphi; 0)$

El dS es el mismo que en 11, sólo cambian los límites de integración de ρ

ANTES: $0 < \rho < r_b$

AHORA: $r_a < \rho < r_b$

$\boxed{dS = d\varphi \cdot \rho \cdot d\rho}$

$$\boxed{\vec{r_0}} = \vec{MP} = P - M = (0; 0; z) - (\rho \cdot \cos\varphi; \rho \cdot \text{sen}\,\varphi; 0) = \boxed{(-\rho \cdot \cos\varphi; -\rho \cdot \text{sen}\,\varphi; z)}$$

$$\boxed{|\vec{r_0}|^3} = \left((-\rho \cdot \cos\varphi)^2 + (\rho \cdot \text{sen}\,\varphi)^2 + z^2\right)^{3/2} = \left(\rho^2 \cdot (\cos^2\varphi + \text{sen}^2\varphi) + z^2\right)^{3/2} = \boxed{(\rho^2 + z^2)^{3/2}}$$

$dq = \sigma \cdot dS$

$$\boxed{\vec{E}} = \int \frac{k \cdot dq}{|\vec{r_0}|^3} \cdot \vec{r_0} = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma \cdot dS}{|\vec{r_0}|^3} \cdot \vec{r_0}$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \cdot \int_0^{2\pi} \int_{r_a}^{r_b} \rho \cdot \frac{(-\rho \cdot \cos(\varphi); -\rho \cdot \text{sen}(\varphi); z)}{(\rho^2 + z^2)^{3/2}} d\rho\, d\varphi$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \cdot \left(\int_0^{2\pi}\int_{r_a}^{r_b} -\frac{\rho^2 \cdot \cos\varphi}{(\rho^2 + z^2)^{3/2}} d\rho\, d\varphi ; \int_0^{2\pi}\int_{r_a}^{r_b} -\frac{\rho^2 \cdot \text{sen}\,\varphi}{(\rho^2 + z^2)^{3/2}} d\rho\, d\varphi ; \int_0^{2\pi}\int_{r_a}^{r_b} \frac{\rho \cdot z}{(\rho^2 + z^2)^{3/2}} d\rho \cdot d\varphi \right)$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \cdot \left(\cancel{\int_{r_a}^{r_b} -\frac{\rho^2}{(\rho^2 + z^2)^{3/2}} d\rho \cdot \int_0^{2\pi} \cos\varphi\, d\varphi} ; \cancel{\int_{r_a}^{r_b} -\frac{\rho^2}{(\rho^2 + z^2)^{3/2}} d\rho \cdot \int_0^{2\pi} \text{sen}\,\varphi \cdot d\varphi} ; z \cdot \int_{r_a}^{r_b} \frac{\rho \cdot d\rho}{(\rho^2 + z^2)^{3/2}} \int_0^{2\pi} d\varphi \right)$$

$= \text{sen}\,\varphi |_0^{2\pi} = \boxed{0}$

$= -\cos(\varphi) |_0^{2\pi} = \boxed{0}$

$= \varphi |_0^{2\pi} = \boxed{2\pi}$

$$z \cdot 2\pi \cdot \int_{r_a}^{r_b} \frac{\rho}{(\rho^2 + z^2)^{3/2}} d\rho = \left[-\frac{1}{\sqrt{\rho^2 + z^2}} \right]_{r_a}^{r_b} = \boxed{-\frac{1}{\sqrt{r_b^2 + z^2}} + \frac{1}{\sqrt{r_a^2 + z^2}}}$$

$$= \frac{\sigma}{4 \cdot \pi \cdot \varepsilon_0} \cdot \left(0; 0; 2\pi \cdot z \cdot \left(\frac{1}{\sqrt{r_a^2 + z^2}} - \frac{1}{\sqrt{r_b^2 + z^2}}\right)\right)$$

$$\boxed{= \frac{\sigma}{2 \cdot \varepsilon_0} \cdot \left(0; 0; z \cdot \left(\frac{1}{\sqrt{r_a^2 + z^2}} - \frac{1}{\sqrt{r_b^2 + z^2}}\right)\right)}$$

NOTA

(c) El dS es el mismo que antes: $dS = d\varphi \cdot \rho \cdot d\rho$, pero cambian los límites de integración del radio: ÁNGULO: $0 < \varphi < 2\pi$ RADIO: $0 < \rho < +\infty$

plano infinito

$$\boxed{\vec{E}} = \int \frac{k \cdot dq}{|\vec{r_0}|^3} \cdot \vec{r_0} = k \int \frac{\sigma \cdot dS}{|\vec{r_0}|^3} \cdot \vec{r_0} = k \cdot \sigma \cdot \int \frac{dS}{|\vec{r_0}|^3} \cdot \vec{r_0}$$

$$= \frac{\sigma}{4\pi \cdot \varepsilon_0} \cdot \int_0^{2\pi} \int_0^{+\infty} \frac{d\varphi \cdot \rho \cdot d\rho}{(\rho^2 + z^2)^{3/2}} \cdot \left(-\rho \cdot \cos(\varphi) ; -\rho \cdot \text{sen}(\varphi) ; z \right)$$

$$= \frac{\sigma}{4\pi \varepsilon_0} \cdot \left(\int_0^{2\pi} \int_0^{+\infty} - \frac{\rho^2 \cdot \cos(\varphi)}{(\rho^2 + z^2)^{3/2}} \, d\varphi \, d\rho ; \int_0^{2\pi} \int_0^{+\infty} - \frac{\rho^2 \cdot \text{sen}(\varphi)}{(\rho^2 + z^2)^{3/2}} \, d\varphi \cdot d\rho ; \int_0^{2\pi} \int_0^{+\infty} \frac{z \cdot \rho}{(\rho^2 + z^2)^{3/2}} \, d\varphi \, d\rho \right)$$

$$= \frac{\sigma}{4\pi \varepsilon_0} \cdot \left(\cancel{\int_0^{+\infty} - \frac{\rho^2}{(\rho^2 + z^2)^{3/2}} d\rho} \cdot \underbrace{\int_0^{2\pi} \cos\varphi \cdot d\varphi} ; \cancel{\int_0^{+\infty} - \frac{\rho^2}{(\rho^2 + z^2)^{3/2}} d\rho} \cdot \underbrace{\int_0^{2\pi} \text{sen}\,\varphi \cdot d\varphi} ; z \cdot \int_0^{+\infty} \frac{\rho}{(\rho^2 + z^2)^{3/2}} d\rho \cdot \underbrace{\int_0^{2\pi} d\varphi} \right)$$

$$= \left[\text{sen}\,\varphi \right]_0^{2\pi} = \text{sen}(2\pi) - (\text{sen}(0)) = 0 - (0) = \boxed{0}$$

$$= \left[-\cos\varphi \right]_0^{2\pi} = -\cos(2\pi) - (-\cos(0)) = -1 - (-1) = \boxed{0}$$

$$= \varphi \Big|_0^{2\pi} = 2\pi - 0 = \boxed{2\pi}$$

$$= \frac{\sigma}{4\pi \varepsilon_0} \cdot \left(0 ; 0 ; 2\pi \cdot z \underbrace{\int_0^{+\infty} \frac{\rho}{(\rho^2 + z^2)^{3/2}} d\rho} \right)$$

$$\left[- \frac{1}{\sqrt{\rho^2 + z^2}} \right]_0^{+\infty} = \underbrace{\left[- \frac{1}{\sqrt{(+\infty)^2 + z^2}} \right]}_{\to 0} - \underbrace{\left[- \frac{1}{\sqrt{0^2 + z^2}} \right]}_{-\frac{1}{z}} = \boxed{\frac{1}{z}}$$

$$= \frac{\sigma}{4\pi \varepsilon_0} \cdot \left(0 ; 0 ; 2\pi \cdot \cancel{z} \cdot \frac{1}{\cancel{z}} \right)$$

$$= \left(0 ; 0 ; \frac{\sigma}{\cancel{4\pi} \varepsilon_0} \cdot \cancel{2\pi} \right)$$

$$\boxed{= \left(0 ; 0 ; \frac{\sigma}{2 \varepsilon_0} \right)}$$

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~~plano infinito~~ CE de un plano infinito

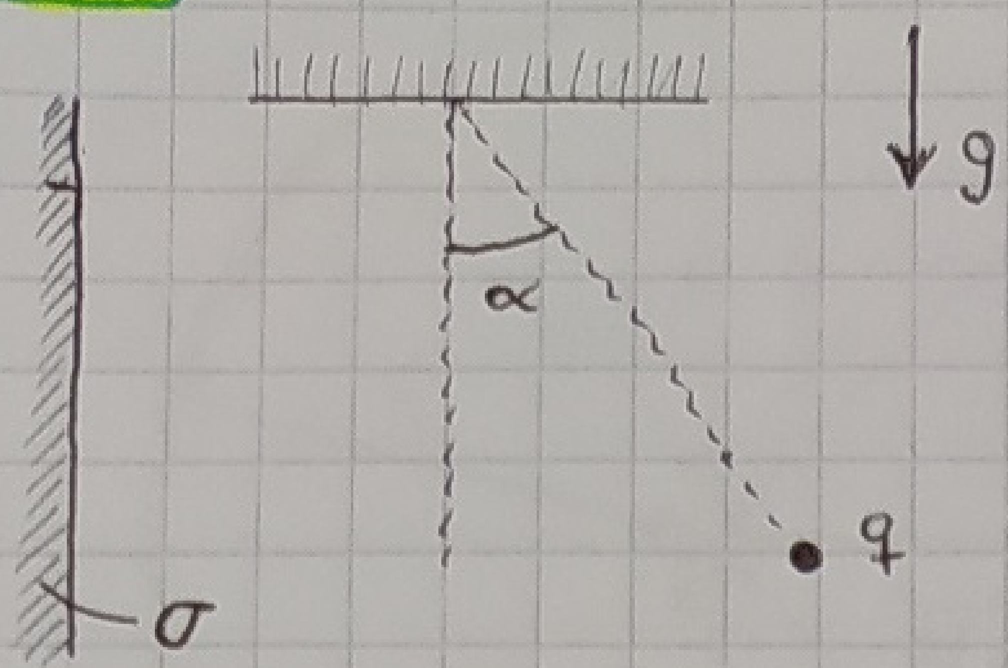
$$\vec{F_e} = m \cdot \vec{a}$$

$$q \cdot \vec{E} = m \cdot \vec{a}$$

$$q \cdot \frac{\sigma}{2 \cdot \varepsilon_0} = m \cdot \vec{a}$$

$$\boxed{\vec{a} = \frac{q \cdot \sigma}{2 \cdot \varepsilon_0 \cdot m}}$$

15

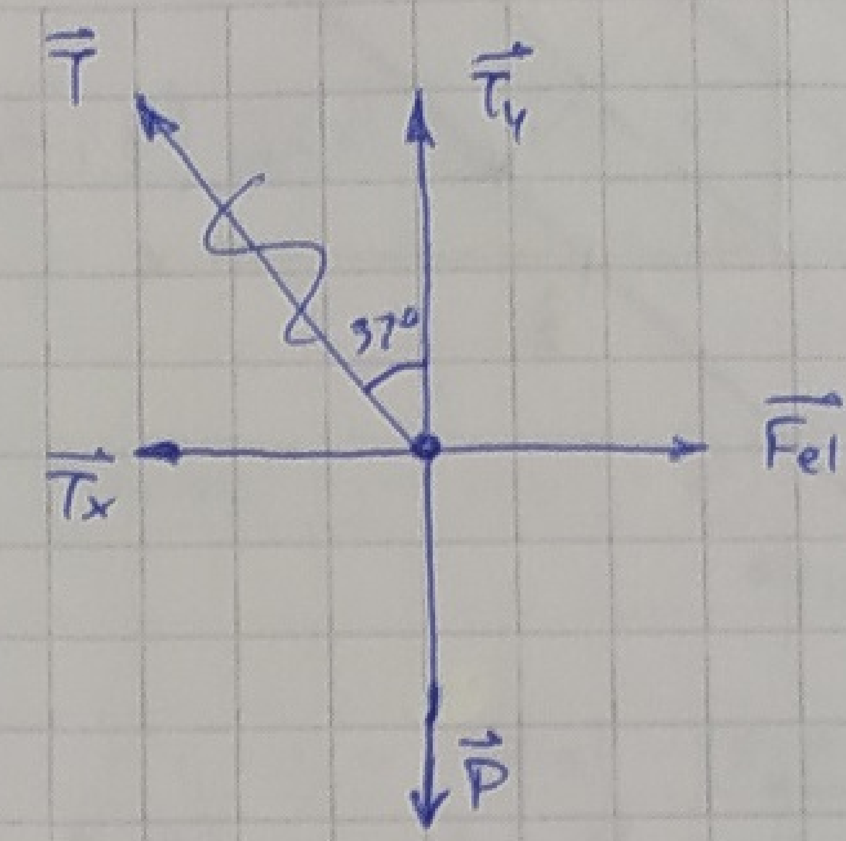


$m = 20g = 0{,}02\,kg$

$\alpha = 37°$

$\sigma = 8{,}85\ \frac{\mu C}{m^2}$

$q = ?$



$$\sum \vec{F} = 0$$

$\sum \vec{F_x} = 0$

$\vec{F_{el}} - \vec{T_x} = 0$

$\vec{F_{el}} = \vec{T_x}$

$q \cdot \vec{E} = \vec{T} \cdot sen(37°)$

$q \cdot \frac{\sigma}{2 \cdot \varepsilon_0} = T \cdot sen(37°)$

$q = \frac{T \cdot sen(37°) \cdot 2 \cdot \varepsilon_0}{\sigma}$

$q = \frac{0{,}25 N \cdot sen(37°) \cdot 2 \cdot 8{,}85 \cdot 10^{-12} \frac{C^2}{Nm^2}}{8{,}85 \frac{\mu C}{m^2}}$

$\boxed{q = 300 \cdot 10^{-9}\, C}$

$\sum \vec{F_y} = 0$

$\vec{T_y} - \vec{P} = 0$

$\vec{T_y} = \vec{P}$

$\vec{T} \cdot cos(37°) = m \cdot g$

$\vec{T} = \frac{m \cdot g}{cos(37°)}$

$\vec{T} = \frac{0{,}2 kg \cdot 10 m/s^2}{cos(37°)}$

$\boxed{\vec{T} = 0{,}25 N}$

16

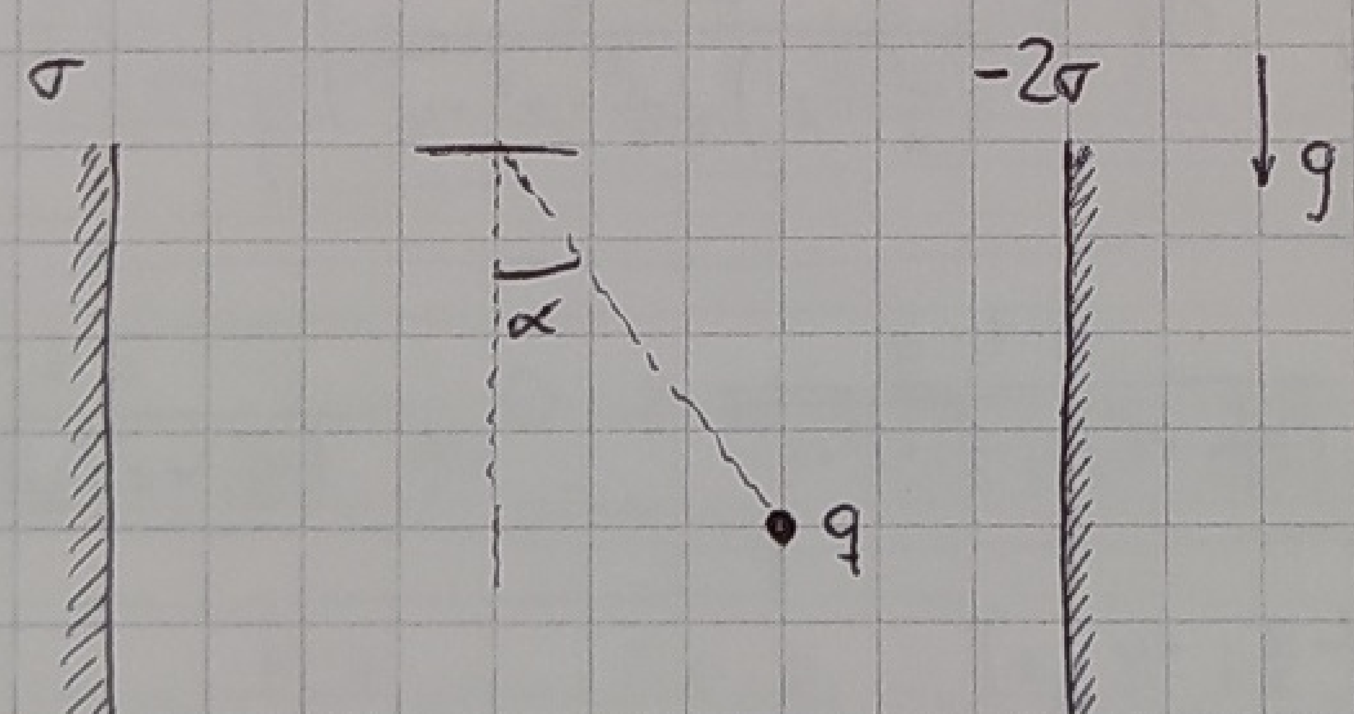
ⓐ

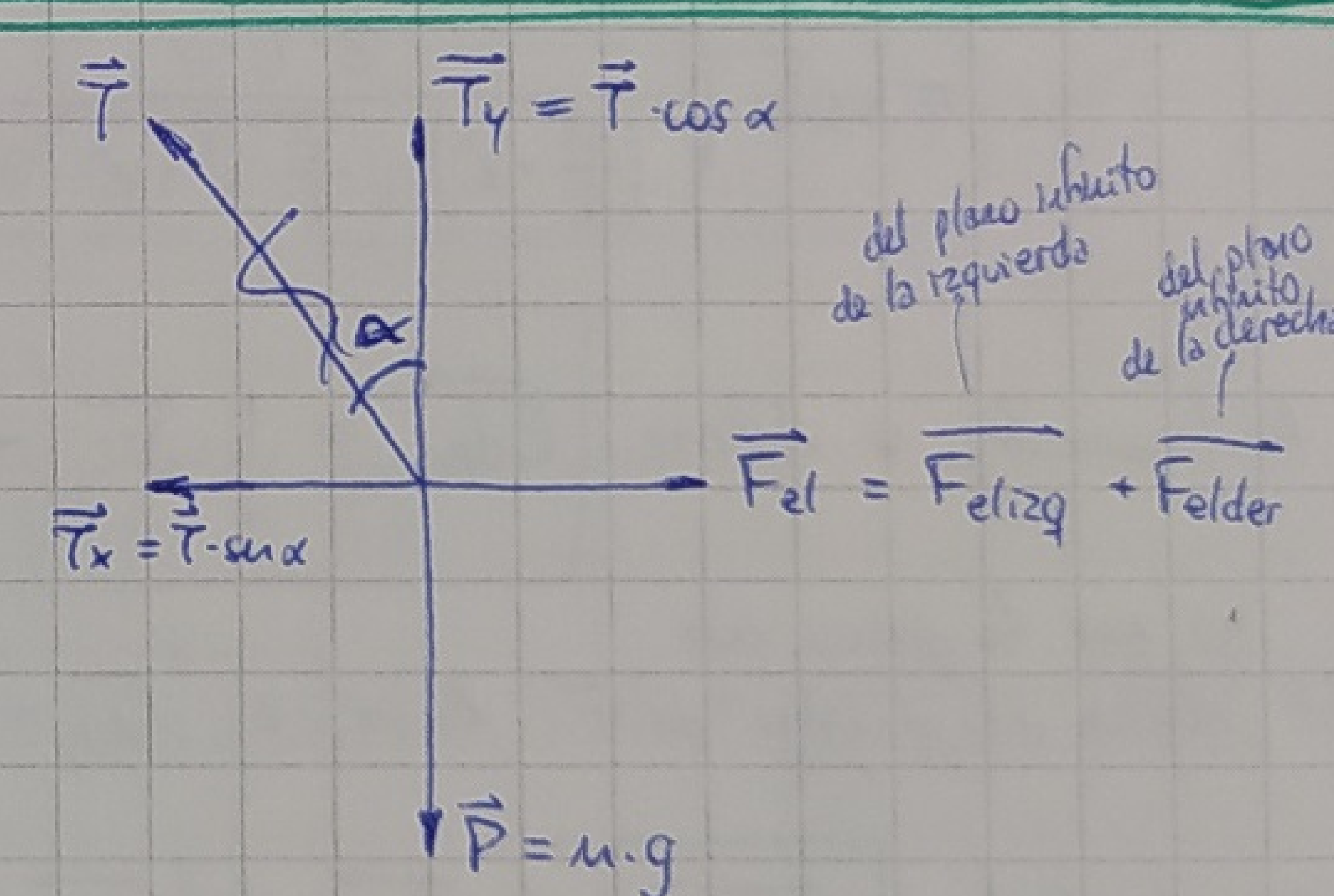
$m = 20g = 0{,}02\,kg$

$d = 25cm = 0{,}25m$

$\sigma = 8{,}85 \cdot 10^{-6}\ C/m^2$

$q = 3 \cdot 10^{-7}\, C$





$$\sum \vec{F} = 0$$

$\sum \vec{F_x} = 0$

$\vec{F_{el}} - \vec{T_x} = 0$

$\vec{F_{elizq}} + \vec{F_{elder}} = \vec{T_x}$

$q \cdot \vec{E_{izq}} + q \cdot \vec{E_{der}} = \vec{T} \cdot sen(\alpha)$

$q \cdot \frac{\sigma}{2 \cdot \varepsilon_0} + q \cdot \frac{2 \cdot \sigma}{2 \cdot \varepsilon_0} = \vec{T}\, sen\,\alpha$

(I) $\boxed{\frac{3 \cdot q \cdot \sigma}{2 \cdot \varepsilon_0} = \vec{T} \cdot sen(\alpha)}$

$\sum \vec{F_y} = 0$

$\vec{T_y} - \vec{P} = 0$

(II) $\boxed{\vec{T} \cdot cos\,\alpha = m \cdot g}$

ⓑ Estando el sistema en equilibrio, el ángulo es independiente de la distancia entre las placas.

$\frac{(I)}{(II)}$ $\frac{\frac{3 \cdot q \cdot \sigma}{2 \cdot \varepsilon_0}}{m \cdot g} = \frac{\vec{T} \cdot sen(\alpha)}{\vec{T} \cdot cos(\alpha)}$

$\frac{3 \cdot q \cdot \sigma}{2 \cdot \varepsilon_0 \cdot m \cdot g} = \vec{T} \cdot tan(\alpha)$

$\alpha = arctan\left(\frac{3 \cdot q \cdot \sigma}{2 \cdot \varepsilon_0 \cdot m \cdot g}\right)$

$\alpha = arctan\left(\frac{3 \cdot 3 \cdot 10^{-7} C \cdot 8{,}85 \cdot 10^{-6} C/m^2}{2 \cdot 8{,}85 \cdot 10^{-12} \frac{C^2}{Nm^2} \cdot 0{,}02 kg \cdot 10 m/s^2}\right)$

$\boxed{\alpha \simeq 66°}$

NOTA

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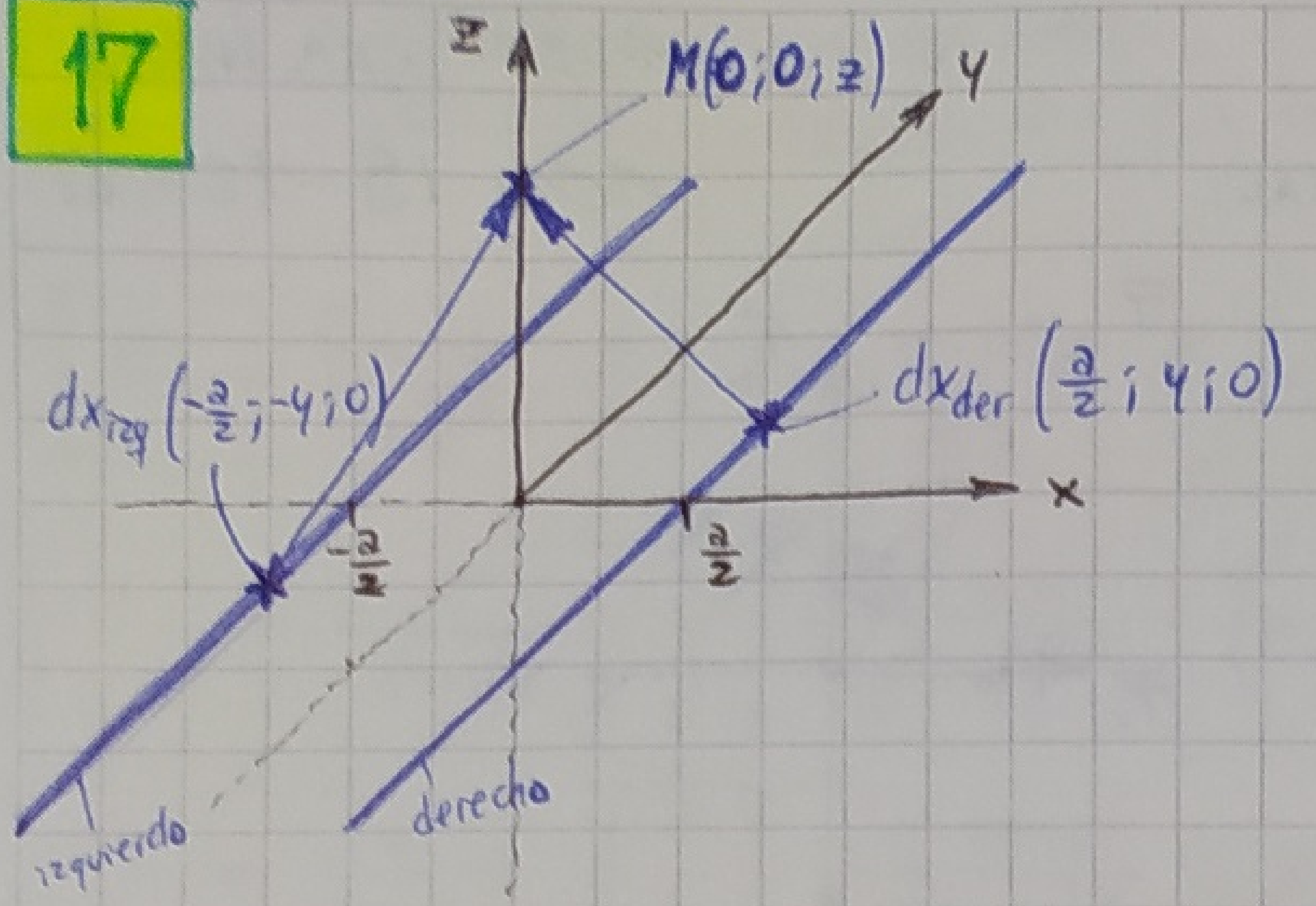

ⓐ Como los hilos están situados simétricamente respecto del eje z, es justamente la componente z del $\vec{E}$ quien sobrevive (las demás se anulan).

ⓒ $\boxed{\vec{r}_{der}} = \overrightarrow{dx_{der}M} = M - dx_{der} = (0;0;z) - \left(\frac{a}{2};y;0\right) = \boxed{\left(-\frac{a}{2};-y;z\right)}$

$\boxed{|\vec{r}_{der}|^3} = \sqrt{\left(-\frac{a}{2}\right)^2 + (-y)^2 + z^2}^{\,3} = \boxed{\left(\frac{a^2}{4} + y^2 + z^2\right)^{3/2}}$

$\boxed{\vec{r}_{izq}} = \overrightarrow{dx_{izq}M} = M - dx_{izq} = (0;0;z) - \left(-\frac{a}{2};-y;0\right) = \boxed{\left(\frac{a}{2};y;z\right)}$

$\boxed{|\vec{r}_{izq}|^3} = \sqrt{\left(\frac{a}{2}\right)^2 + y^2 + z^2}^{\,3} = \boxed{\left(\frac{a^2}{4} + y^2 + z^2\right)^{3/2}}$

$$\boxed{\vec{E}_{izq}} = \int \frac{k \cdot dq}{|\vec{r}_{izq}|^3} \cdot \vec{r}_{izq} = \int_{-L}^{L} \frac{k \cdot \lambda \cdot dy}{\left(\frac{a^2}{4}+y^2+z^2\right)^{3/2}} \cdot \left(\frac{a}{2};y;z\right) = k \cdot \lambda \cdot \int_{-L}^{L} \frac{dy}{\left(\frac{a^2}{4}+y^2+z^2\right)^{3/2}} \cdot \left(\frac{a}{2};y;z\right)$$

$$= k \cdot \lambda \cdot \left(\frac{a}{2} \cdot \int_{-L}^{L} \frac{dy}{\left(\frac{a^2}{4}+z^2+y^2\right)^{3/2}} ; \int_{-L}^{L} \frac{y \cdot dy}{\left(\frac{a^2}{4}+z^2+y^2\right)^{3/2}} ; z \cdot \int_{-L}^{L} \frac{dy}{\left(\frac{a^2}{4}+z^2+y^2\right)^{3/2}}\right) \longrightarrow \text{VER TABLA DE INTEGRALES} < \begin{matrix}196\\197\end{matrix}$$

$$= k \cdot \lambda \cdot \left(\frac{a}{2} \cdot \underbrace{\left[\frac{y}{\left(\frac{a^2}{4}+z^2\right) \cdot \sqrt{\frac{a^2}{4}+z^2+y^2}}\right]_{-L}^{L}}_{= \frac{2 \cdot L}{(\sim)\sqrt{\sim}}} ; \underbrace{\cancel{\left[\frac{-1}{\sqrt{\frac{a^2}{4}+z^2+y^2}}\right]_{-L}^{L}}}_{= -\frac{1}{\sim} - \left(-\frac{1}{\sim}\right) = 0} ; z \cdot \underbrace{\left[\frac{y}{\left(\frac{a^2}{4}+z^2\right) \cdot \sqrt{\frac{a^2}{4}+z^2+y^2}}\right]_{-L}^{L}}_{= \frac{2 \cdot L}{(\sim)\sqrt{\sim}}}\right)$$

$$= k \cdot \lambda \cdot \boxed{\left(\frac{2 \cdot L \cdot a}{\left(\frac{a^2}{4}+z^2\right)\sqrt{\frac{a^2}{4}+z^2+y^2}} ; 0 ; \frac{2 \cdot L \cdot z}{\left(\frac{a^2}{4}+z^2\right) \cdot \sqrt{\frac{a^2}{4}+z^2+y^2}}\right)}$$

Análogamente, se obtiene $\boxed{\vec{E}_{der} = k \cdot \lambda \cdot \left(-\frac{2 \cdot L \cdot a}{\left(\frac{a^2}{4}+z^2\right)\sqrt{\frac{a^2}{4}+z^2+y^2}} ; 0 ; \frac{2 \cdot L \cdot z}{\left(\frac{a^2}{4}+z^2\right)\sqrt{\frac{a^2}{4}+z^2+y^2}}\right)}$

ya que lo único que cambia es el valor de $\vec{r}$. Ahora, es $\vec{r}_{der} = \left(-\frac{a}{2};-y;z\right)$.

Por lo tanto: $\boxed{\vec{E}_{TOTAL}} = \vec{E}_{izq} + \vec{E}_{der} = \boxed{\left(0;0;\frac{4 \cdot L \cdot z}{\left(\frac{a^2}{4}+z^2\right)\sqrt{\frac{a^2}{4}+z^2+y^2}}\right)}$

ⓑ En el origen de coordenadas $\begin{pmatrix}x=0\\y=0\\z=0\end{pmatrix}$, $\boxed{\vec{E} = \vec{0}}$ ya que $E_z = 0$.

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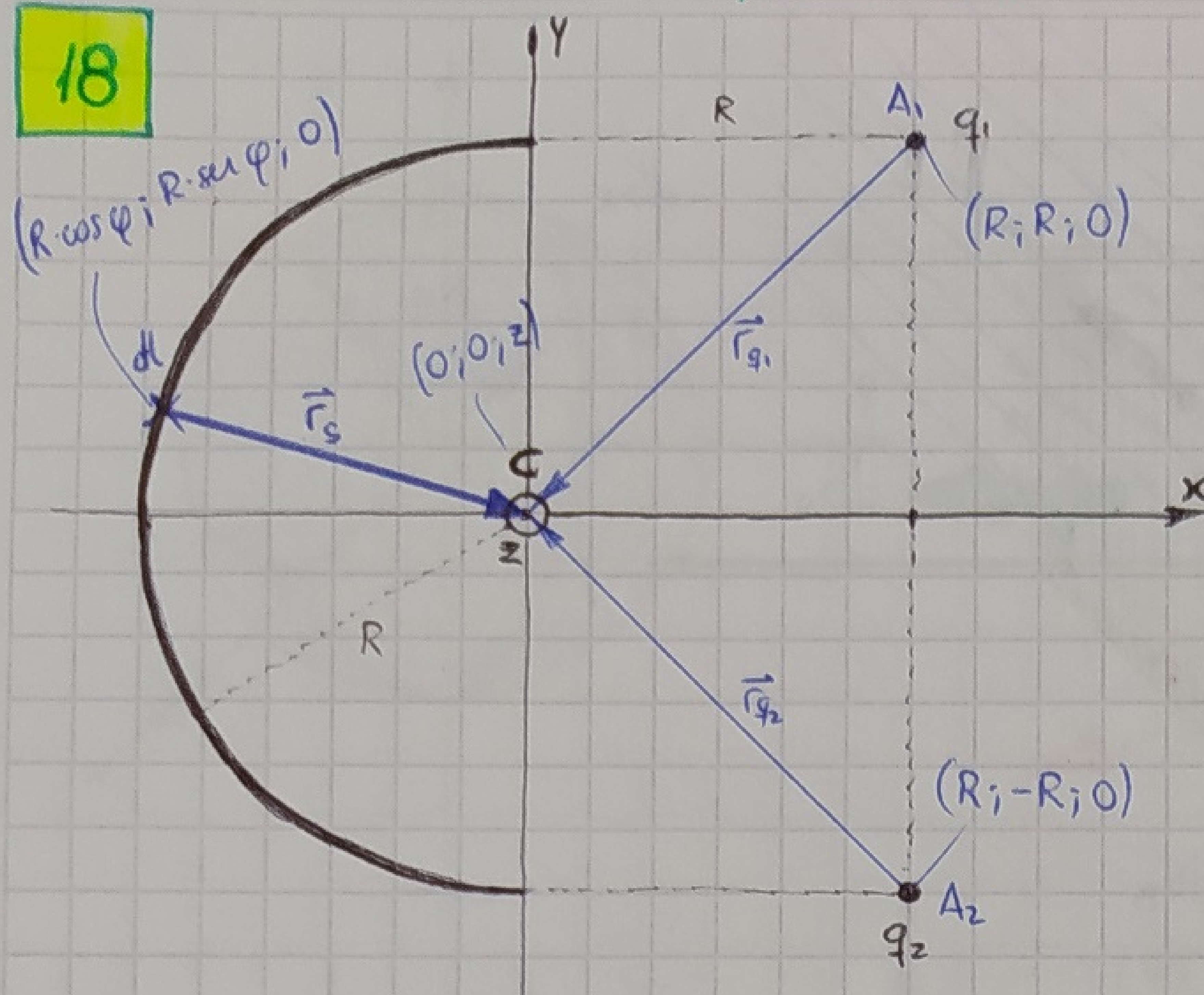

$$\boxed{\vec{r_s} = \vec{dl\,C} = C - dl = (0;0;z) - (R\cdot\cos\varphi; R\,sen\,\varphi; 0) = (-R\cdot\cos\varphi; -R\cdot sen\,\varphi; z)}$$

$$\boxed{|\vec{r_s}|^3 = \left((-R\cos\varphi)^2 + (-R\cdot sen\,\varphi)^2 + z^2\right)^{3/2} = (R^2+z^2)^{3/2}}$$

$$\boxed{\vec{r_{q_1}} = \vec{A_1C} = C - A_1 = (0;0;z) - (R;R;0) = (-R;-R;z)}$$

$$\boxed{|\vec{r_{q_1}}|^3 = \left((-R)^2 + (-R)^2 + z^2\right)^{3/2} = (2R^2+z^2)^{3/2}}$$

$$\boxed{\vec{r_{q_2}} = \vec{A_2C} = C - A_2 = (0;0;z) - (R;-R;0) = (-R;R;z)}$$

$$\boxed{|\vec{r_{q_2}}|^3 = \left((-R)^2 + R^2 + z^2\right)^{3/2} = (2R^2+z^2)^{3/2}}$$

$$\boxed{\vec{E_{semicirc}}} = \int \frac{k\cdot dq}{|\vec{r_s}|^3}\cdot\vec{r_s} = \int \frac{k\cdot\lambda_0\cdot \overbrace{dl}^{R\cdot d\varphi}}{(R^2+z^2)^{3/2}}\cdot(-R\cos\varphi; -R\cdot sen\,\varphi; z)$$

$$= k\cdot\lambda_0\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \frac{R\cdot\varphi}{(R^2+z^2)^{3/2}}\cdot(-R\cos\varphi; -R\cdot sen\,\varphi; z)$$

$$= k\cdot\lambda_0\left(\frac{-R^2}{(R^2+z^2)^{3/2}}\boxed{\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\cos\varphi\cdot d\varphi}\ ;\ \cancel{\frac{-R^2}{(R^2+z^2)^{3/2}}\boxed{\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} sen\,\varphi\cdot d\varphi}}\ ;\ \frac{R\cdot z}{(R^2+z^2)^{3/2}}\boxed{\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} d\varphi}\right)$$

$= sen\,\varphi\Big|_{\pi}^{\frac{3}{2}\pi} = -1-(+1) = \boxed{-2}$; $= -\cos\varphi\Big|_{\frac{\pi}{2}}^{\frac{3}{2}\pi} = 0$; $= \varphi\Big|_{\pi/2}^{3/2\pi} = \frac{3}{2}\pi - \frac{\pi}{2} = \boxed{\pi}$

$$= k\cdot\lambda_0\left(\frac{2R^2}{(R^2+z^2)^{3/2}}\ ;\ 0\ ;\ \frac{\pi\cdot R\cdot z}{(R^2+z^2)^{3/2}}\right) = \boxed{\frac{k\cdot\lambda_0}{(R^2+z^2)^{3/2}}\cdot(2R^2; 0; \pi R\cdot z)}\ ✓$$

$$\boxed{\vec{E_{q_1}}} = \frac{k\cdot q_1}{|\vec{r_{q_1}}|^3}\cdot\vec{r_{q_1}} = \boxed{\frac{k\cdot q}{(2R^2+z^2)^{3/2}}\cdot(-R;-R;z)}$$

$$\boxed{\vec{E_{q_2}}} = \frac{k\cdot q_2}{|\vec{r_{q_2}}|^3}\cdot\vec{r_{q_2}} = \boxed{\frac{k\cdot q}{(2R^2+z^2)^{3/2}}\cdot(-R;R;z)}$$

$$\boxed{\vec{E_q}} = \vec{E_{q_1}} + \vec{E_{q_2}}$$

$$= \frac{k\cdot q}{(2R^2+z^2)^{3/2}}\cdot(-R;-R;z) + \frac{k\cdot q}{(2R^2+z^2)^{3/2}}\cdot(-R;R;z)$$

$$= \frac{k\cdot q}{(2R^2+z^2)^{3/2}}\cdot(-R-R; \cancel{-R+R}; z+z)$$

$$= \boxed{\frac{2\cdot k\cdot q}{(2R^2+z^2)^{3/2}}\cdot(-R; 0; z)}\ ✓$$

$$\boxed{\vec{E_{TOTAL}} = \vec{E_{semicirc}} + \vec{E_q}}\ ✓$$

Dada la existencia de las dos cargas puntuales, el aro no se halla en equilibrio.

Descartada la componente **z** de la fuerza que actúa sobre el aro (todas las cargas actúan en el plano xy) y también la componente **y** (se anulan entre sí), sólo la componente en **x** no es nula.

Esto significa que las cargas puntuales "empujan" (al tener el mismo signo que el aro de circunferencia) al aro hacia la izquierda, es decir, hacia los valores negativos de **x**.

Por lo tanto, para evitar que el aro se desplace, es necesario aplicar una fuerza en la dirección del eje x con sentido hacia la derecha, es decir, hacia los valores positivos de **x**.

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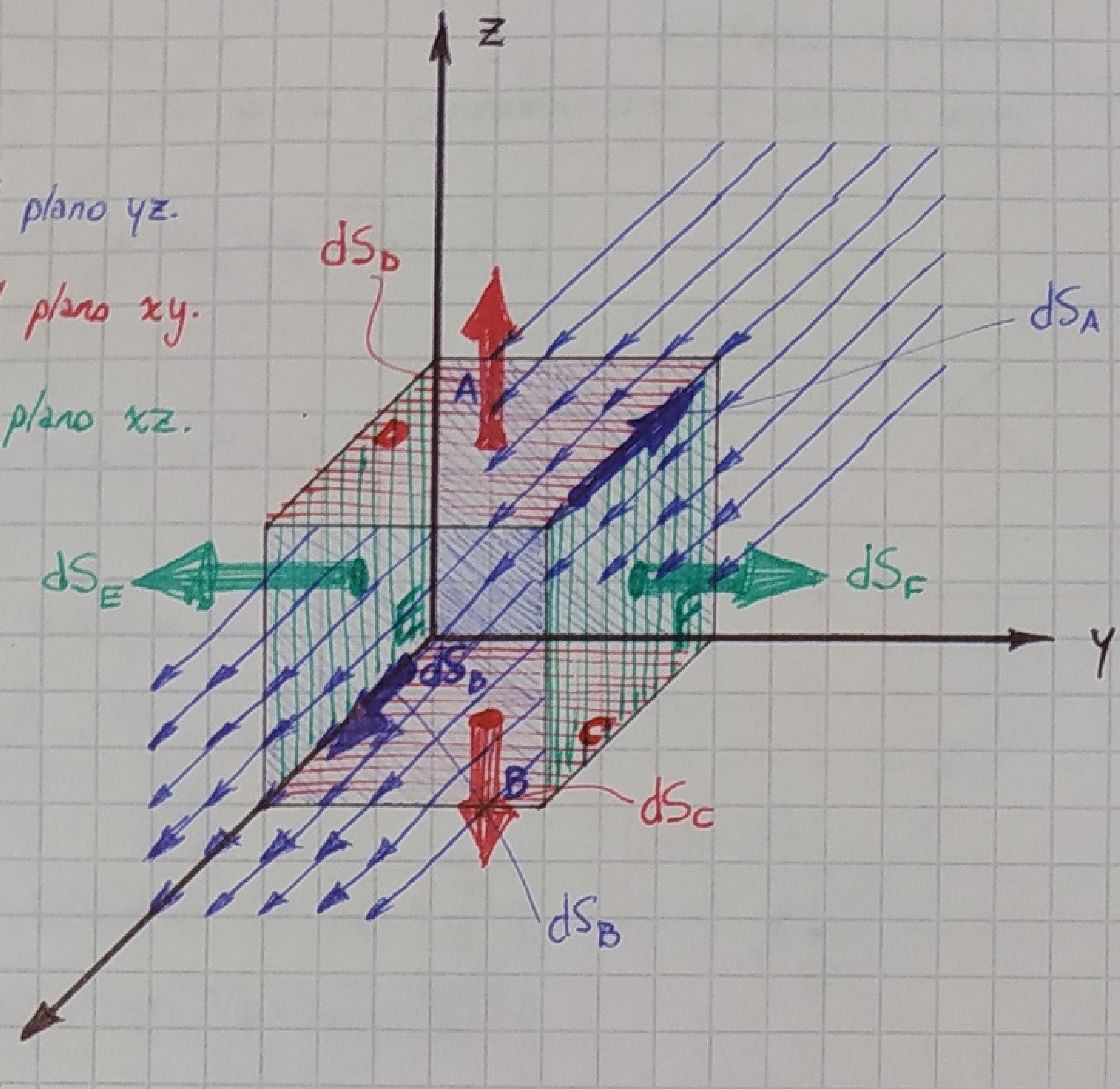
A, B } caras paralelas al plano yz.

C, D } caras paralelas al plano xy.

E, F } caras paralelas al plano xz.



$$\oint_{SG} \vec{E} \cdot \vec{dS} = \frac{Q_{neta}}{\varepsilon_0}$$

$$Q_{neta} = \boxed{\oint_{SG} \vec{E} \cdot \vec{dS}} \cdot \varepsilon_0$$

$$= \int_A |\vec{E_x}| \cdot |\vec{dS_A}| \cdot \cos\overbrace{(\vec{E_x} \wedge \vec{dS_A})}^{\overset{= \cos(180°) = -1}{180°}} + \int_B |\vec{E_x}| \cdot |\vec{dS_B}| \cdot \cos\overbrace{(\vec{E_x} \wedge \vec{dS_B})}^{\overset{= \cos(0°) = 1}{0°}} + \cancel{\int_C |\vec{E_z}| \cdot |\vec{dS_C}| \cos\overbrace{(\vec{E_z} \wedge \vec{dS_C})}^{\overset{= \cos(90°) = 0}{90°}}}$$

$$+ \cancel{\int_D |\vec{E_z}| \cdot |\vec{dS_D}| \cdot \cos\underbrace{(\vec{E_z} \wedge \vec{dS_D})}_{\underset{= \cos(90°) = 0}{90°}}} + \cancel{\int_E |\vec{E_y}| \cdot |\vec{dS_E}| \cos\underbrace{(\vec{E_y} \wedge \vec{dS_E})}_{\underset{= \cos(90°) = 0}{90°}}} + \cancel{\int_F |\vec{E_y}| \cdot |\vec{dS_F}| \cos\underbrace{(\vec{E_y} \wedge \vec{dS_F})}_{\underset{= \cos(90°) = 0}{90°}}}$$

$$= -\int_A |\vec{E_x}| \cdot |\vec{dS_A}| + \int_B |\vec{E_x}| \cdot |\vec{dS_B}| = \int_B |\vec{E_x}| \cdot |\vec{dS_B}| - \int_A |\vec{E_x}| \cdot |\vec{dS_A}|$$

$$= |\vec{E_x}| \cdot \int_B |\vec{dS_B}| - |\vec{E_x}| \cdot \int_A |\vec{dS_A}|$$

$$= |\vec{E_x}| \cdot S_B - |\vec{E_x}| \cdot S_A$$

$$= 560 \frac{N}{C} \cdot (25m)^2 - 410 \frac{N}{C} \cdot (25m)^2$$

$$= \boxed{93.750 \frac{Nm^2}{C}}$$

$$Q_{neta} = 93.750 \frac{\cancel{Nm^2}}{\cancel{C}} \cdot 8{,}85 \cdot 10^{-12} \frac{C^2}{\cancel{Nm^2}}$$

$$\boxed{Q_{neta} = 829{,}6875 \ nC}$$

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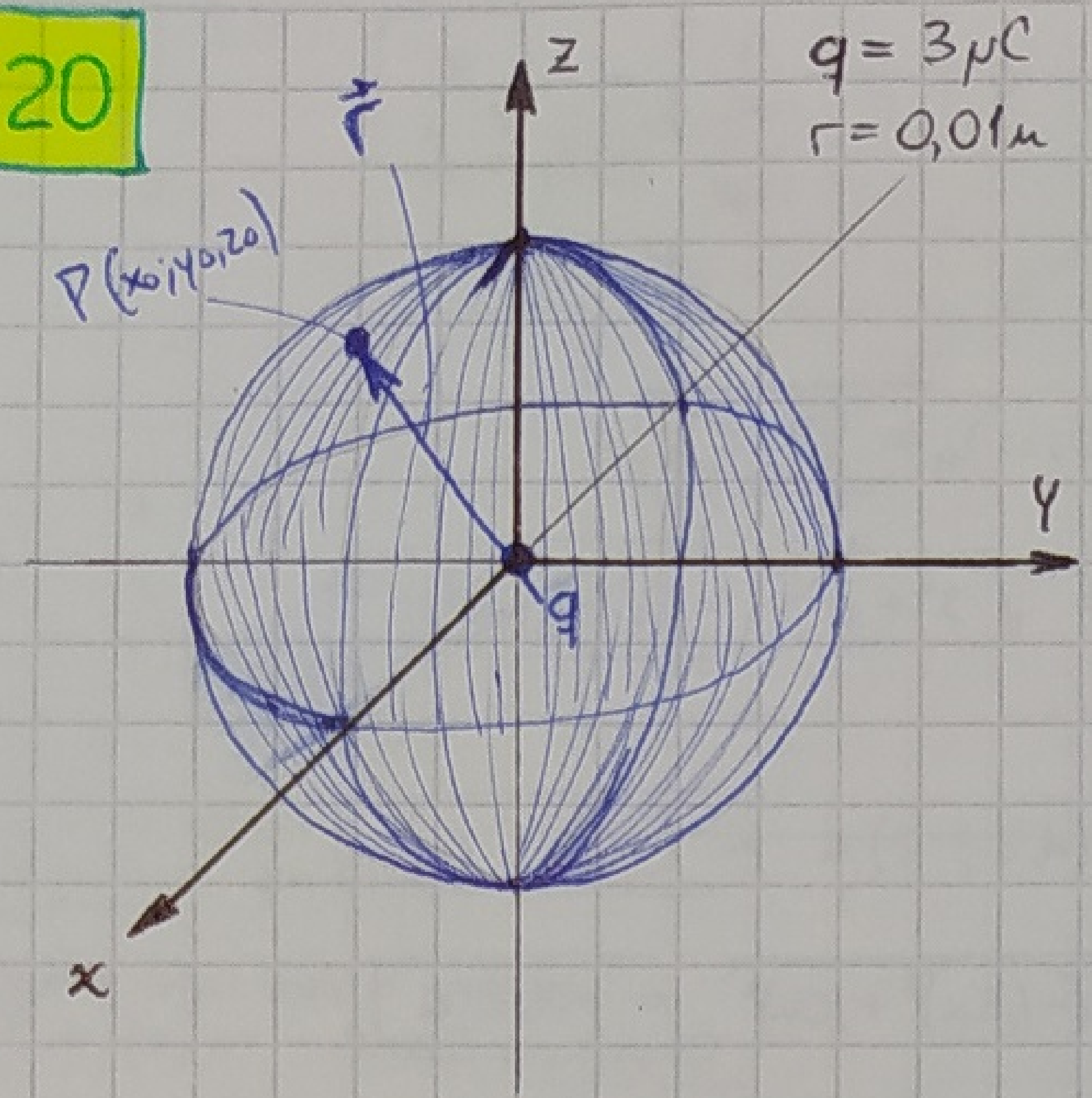
$q = 3\mu C$
$r = 0{,}01 m$



$\vec{r} = r \cdot \breve{e}$ ← versor con la misma dirección y el mismo sentido que $\vec{r}$.

$$|\vec{r}|^3 = |r \cdot \breve{e}|^3 = |r|^3 \cdot \underbrace{|\breve{e}|^3}_{=1} = r^3$$

ⓐ
$$\boxed{\vec{E} = \frac{1}{4\cdot\pi\cdot\varepsilon_0} \cdot \frac{q}{|\vec{r}|^3} \cdot \vec{r}}$$
$$= \frac{1}{4\cdot\pi\cdot\varepsilon_0} \cdot \frac{q}{r^{\cancel{3}2}} \cdot \cancel{r} \cdot \breve{e}$$
$$= \frac{1}{4\cdot\pi\cdot\varepsilon_0} \cdot \frac{q}{r^2} \cdot \breve{e}$$
$$= \frac{1}{4\cdot\pi\cdot 8{,}85\cdot10^{-12}\frac{C^2}{N m^2}} \cdot \frac{3\cdot10^{-6}C}{0{,}01 m} \cdot \breve{e}$$
$$\boxed{= 2{,}76\cdot10^6 \frac{N}{C}}$$ ✓

ⓑ
$$\phi = \frac{Q_{neta}}{\varepsilon_0} = \frac{q}{\varepsilon_0} = \frac{3\cdot10^{-6}C}{8{,}85\cdot10^{-12}\frac{C^2}{N m^2}} = \boxed{339\cdot10^3 \frac{N m^2}{C}}$$ ✓

ⓒ
$$\phi = \frac{Q_{neta}}{\varepsilon_0} = \frac{q}{\varepsilon_0} = \frac{3\cdot10^{-6}C}{8{,}85\cdot10^{-12}\frac{C^2}{N m^2}} = \boxed{339\cdot10^3 \frac{N m^2}{C}}$$ ✓

ⓓ₁ El flujo no depende del área de la superficie que encierra la carga, sino de la carga neta encerrada por dicha superficie.
Como la carga neta sigue siendo la misma, el flujo no varía. ✓

ⓓ₂ El flujo es directamente proporcional al valor de la carga neta encerrada por la superficie.
Como la carga se duplica, también se duplicará el flujo. ✓

ⓔ Como la carga se mantiene dentro de la superficie cerrada, el flujo no varía. ✓

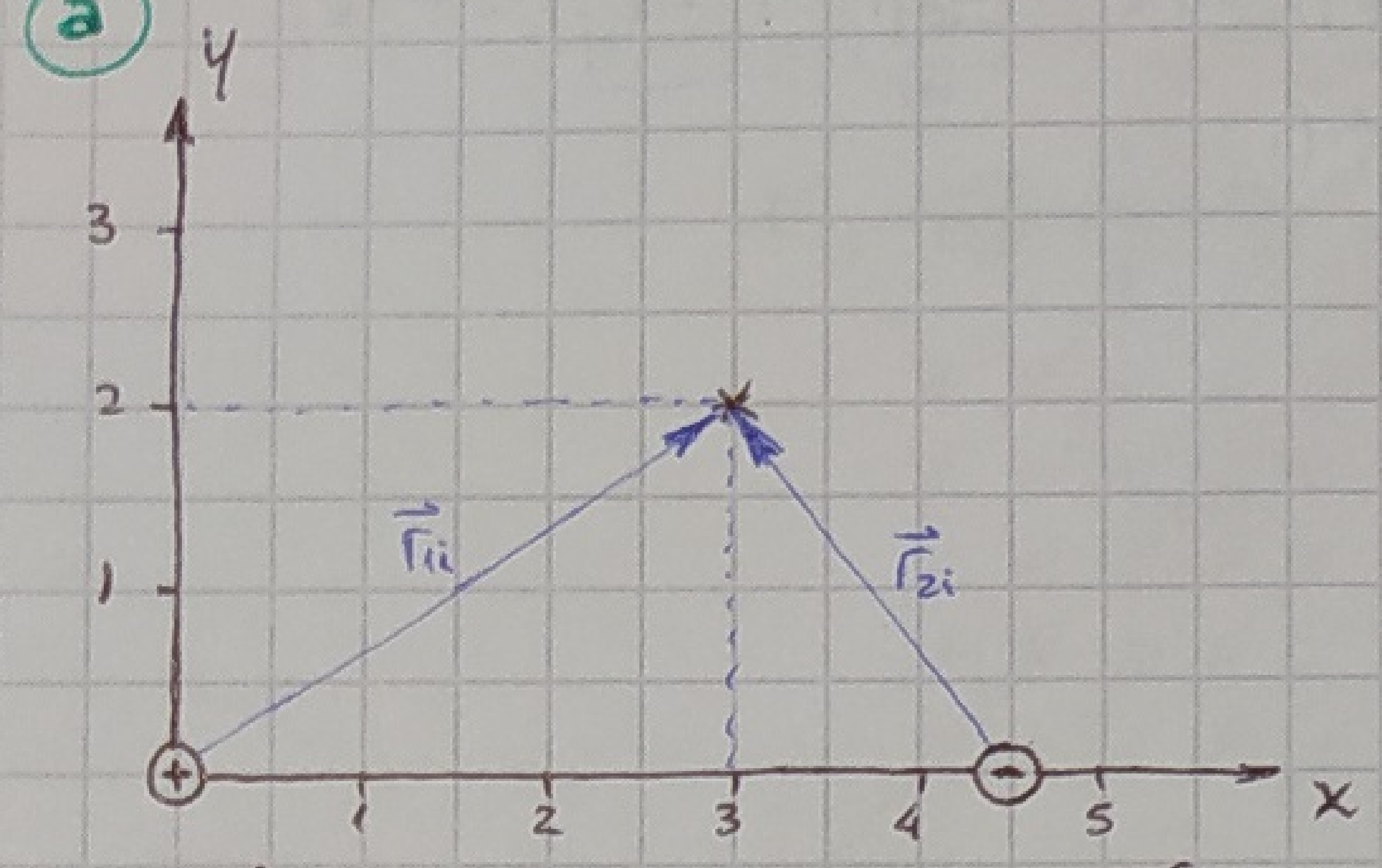
ⓕ Si la carga es desplazada del centro de simetría, el campo eléctrico varía. Esto se debe a que el $\vec{E}$ depende de la distancia entre el punto (sobre donde se genera el CE) y la carga (que genera el CE).

21 → 5

Sabiendo que el $\vec{E}$ en una carga puntual es $\vec{E} = \frac{k \cdot q}{|\vec{r}|^3} \cdot \vec{r} = \frac{k \cdot q}{|\vec{r}|^2} \cdot \breve{r}$,

entonces: $\boxed{\Delta V_{\infty, P} = -\int_{r_\infty} \vec{E} \cdot \vec{dl} = \int_{r_\infty}^{r} \frac{k \cdot q}{r^2} \cdot \breve{r} \cdot \vec{dl} = k \cdot q \cdot \left[\frac{1}{r} - \underbrace{\frac{1}{\infty}}_{\to 0}\right] = \frac{k \cdot q}{r}}$

a)



$q_1 = 2 \cdot 10^{-6} C$ $\qquad$ $q_2 = -3 \cdot 10^{-6} C$

$\boxed{\vec{r_{1i}} = (3;2)_m - (0;0)_m = (3;2)_m}$

$\boxed{|\vec{r_{1i}}| = \sqrt{3^2 + 2^2} = \sqrt{13}\ m}$

$\boxed{\vec{r_{2i}} = (3;2)_m - (4,5;0)_m = (-1,5;0)_m}$

$\boxed{|\vec{r_{2i}}| = \sqrt{(-1,5m)^2 + (0m)^2} = 2,5\ m}$

$\boxed{V_{q_1,(3;2)} = \frac{k \cdot q_1}{|\vec{r_{1i}}|} = \frac{9 \cdot 10^9 \frac{C^2}{Nm^2} \cdot 2 \cdot 10^{-6} C}{\sqrt{13}\ m} = 4992,3\ V}$

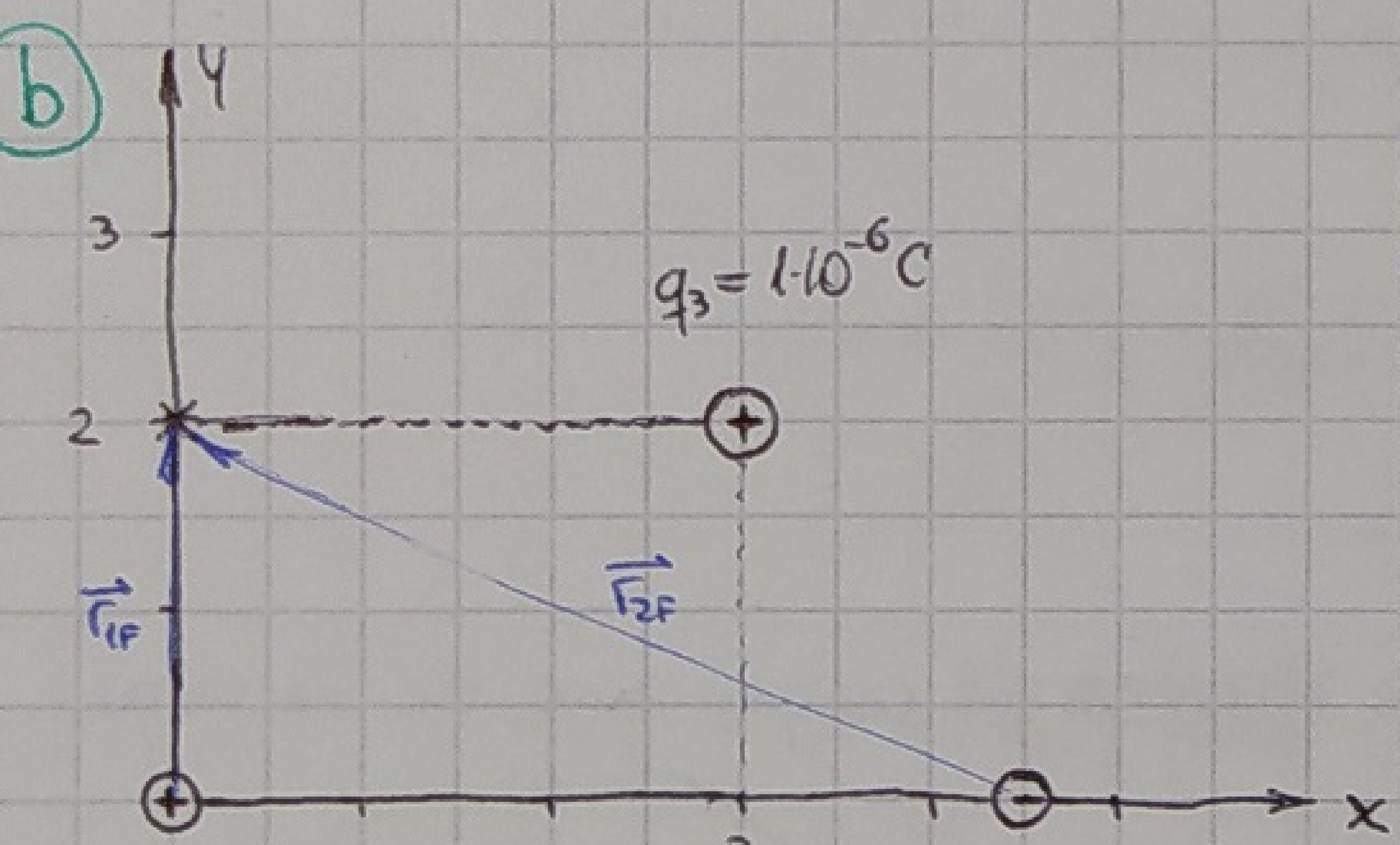
$\boxed{V_{q_2,(3;2)} = \frac{k \cdot q_2}{|\vec{r_{2i}}|} = \frac{9 \cdot 10^9 \frac{C^2}{Nm^2} \cdot (-3 \cdot 10^{-6} C)}{2,5 m} = -10.800\ V}$

$\boxed{V_{(3;2)} = V_{q_1(3;2)} + V_{q_2(3;2)}}$

$= 4992,3 V - 10800 V$

$\boxed{= -5.807,7\ V}$

b)



$q_1 = 2 \cdot 10^{-6} C$ $\qquad$ $q_2 = -3 \cdot 10^{-6} C$

$\boxed{\vec{r_{1F}} = (0;2)_m - (0;0)_m = (0;2)_m}$

$\boxed{|\vec{r_{1F}}| = \sqrt{(0m)^2 + (2m)^2} = 2\ m}$

$\boxed{\vec{r_{2F}} = (0;2)_m - (4,5;0)_m = (-4,5;2)_m}$

$\boxed{|\vec{r_{2F}}| = \sqrt{(-4,5m)^2 + (2m)^2} = \sqrt{24,25}\ m}$

$\boxed{V_{q_1,(0;2)} = \frac{k \cdot q_1}{|\vec{r_{1F}}|} = \frac{9 \cdot 10^9 \frac{C^2}{Nm^2} \cdot 2 \cdot 10^{-6} C}{2 m} = 9000\ V}$

$\boxed{V_{q_2,(0;2)} = \frac{k \cdot q_2}{|\vec{r_{2F}}|} = \frac{9 \cdot 10^9 \frac{C^2}{Nm^2} \cdot (-3 \cdot 10^{-6} C)}{\sqrt{24,25}\ m} = -5482,87\ V}$

$\boxed{V_{(0;2)} = V_{q_1(0;2)} + V_{q_2(0;2)}}$

$= 9000 V - 5482,87 V$

$\boxed{= 3517,13\ V}$

$\boxed{W_{Fel} = q_3 \cdot \Delta V_{(3;2) \to (0;2)} = 1 \cdot 10^{-6} C \cdot 9324,26 V = 9,32 \cdot 10^{-3}\ J}$

$V_{(0;2)} - V_{(3;2)} = 3517,13 V - (-5807,7 V) = 9324,26\ V$

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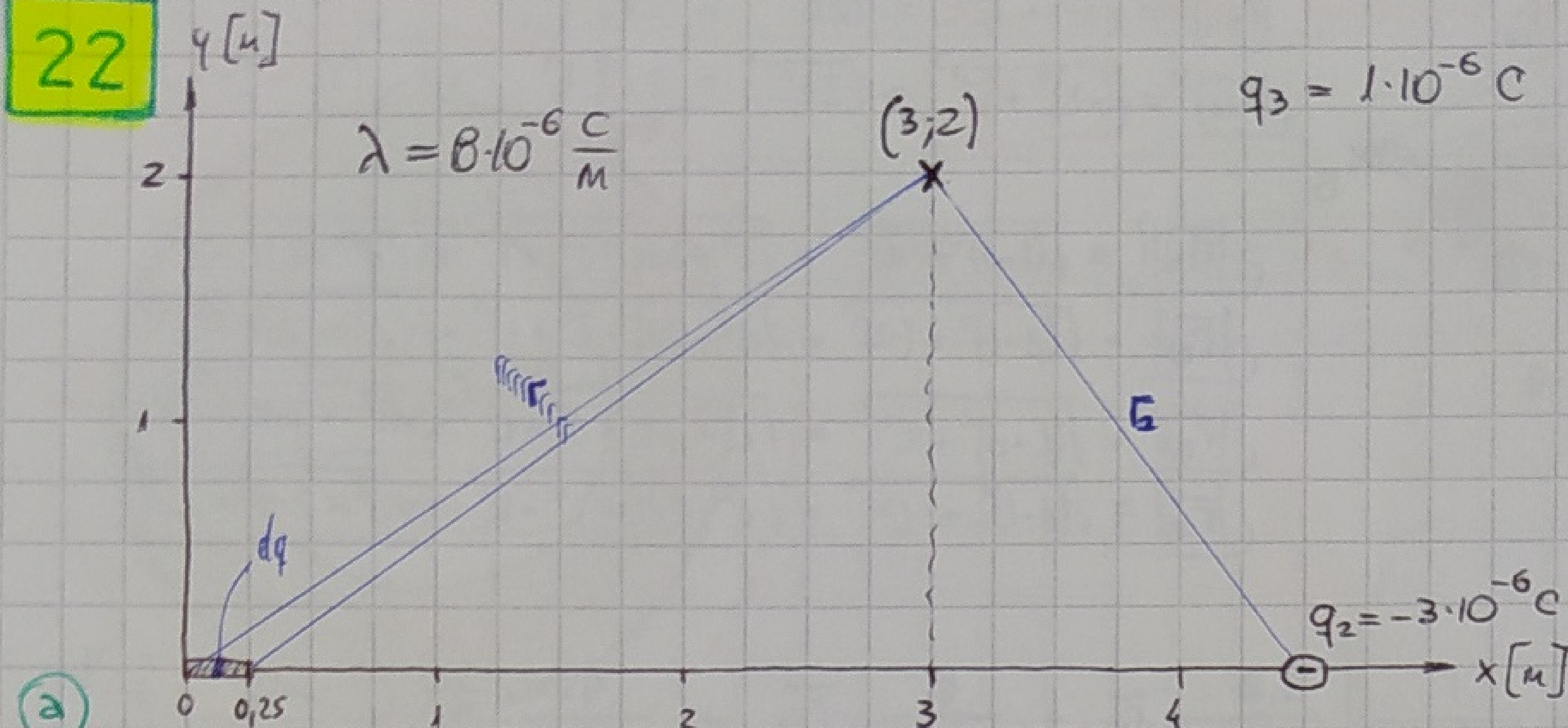


a)

desde el origen hasta el punto (3;2)

$\vec{r} = (3;2) - (0;0) = (3;2)$

desde el origen hasta cada dq

$\vec{r}\,' = (x';0) - (0;0) = x';0$

$\vec{r} - \vec{r}\,' = (3;2) - (x';0) = (3-x';2)$

$\lambda = \frac{dq}{dl} \rightarrow dq = \lambda\cdot dl \Rightarrow dq = \lambda\cdot dx'$

$$V_{varilla,\infty;(3;2)} = \int_{\Omega} \frac{k\cdot dq}{|\vec{r}-\vec{r}\,'|}$$

$$V_{varilla} = \int_0^{0,25} \frac{k\cdot\lambda\cdot dx'}{\sqrt{(3-x') + 2^2}}$$

$$V_{varilla} = k\cdot\lambda\cdot\int_0^{0,25} \frac{1}{\sqrt{(3-x')^2+4}}\,dx'$$

$u = 3-x'$
$du = -dx' \rightarrow dx' = -du$
$x' \rightarrow 0,25 \Rightarrow u \rightarrow 2,75$
$x' \rightarrow 0 \Rightarrow u \rightarrow 3$

$$V_{varilla} = k\cdot\lambda\cdot -\int_3^{2,75} \frac{1}{\sqrt{u^2+4}}\,du$$

$$V_{varilla} = -k\cdot\lambda\cdot\left[\ln\left(u+\sqrt{u^2+4}\right)\right]_3^{2,75}$$

aplicando propiedad del logaritmo

$$V_{varilla} = -9\cdot 10^9\,\frac{Nm^2}{C^2}\cdot 8\cdot 10^{-6}\,\frac{C}{m}\cdot \ln\left(\frac{2,75+\sqrt{2,75^2+4}}{3+\sqrt{3^2+4}}\right)$$

$$\boxed{V_{varilla} = 5140,7\,V}$$

b)

$$W = -q_3\,\Delta V_{\infty;(3;2)}$$

$$\Delta V_{\infty;(3;2)} = V_{(3;2)} - \overset{=0}{V_\infty} = V_{(3;2)}$$

$$V_{(3;2)} = V_{varilla\,(3;2)} + V_{q_2\,(3;2)}$$

$$V_{q_2(3;2)} = \frac{k\cdot q_2}{r_2} \qquad r_2 = \sqrt{1,5^2+2^2} = 2,5\,m$$

$$V_{q_2(3;2)} = \frac{9\cdot 10^9\,\frac{Nm^2}{C^2}\cdot -3\cdot 10^{-6}\,C}{2,5\,m}$$

$$\boxed{V_{q_2(3;2)} = -10.800\,V}$$

$$\Delta V_{\infty;(3;2)} = V_{(3;2)} = V_{varilla\,(3;2)} + V_{q_2(3;2)}$$

$$\Delta V_{\infty;(3;2)} = 5140,7\,V + (-10800\,V)$$

$$\boxed{\Delta V_{\infty;(3;2)} = -5659,3\,V}$$

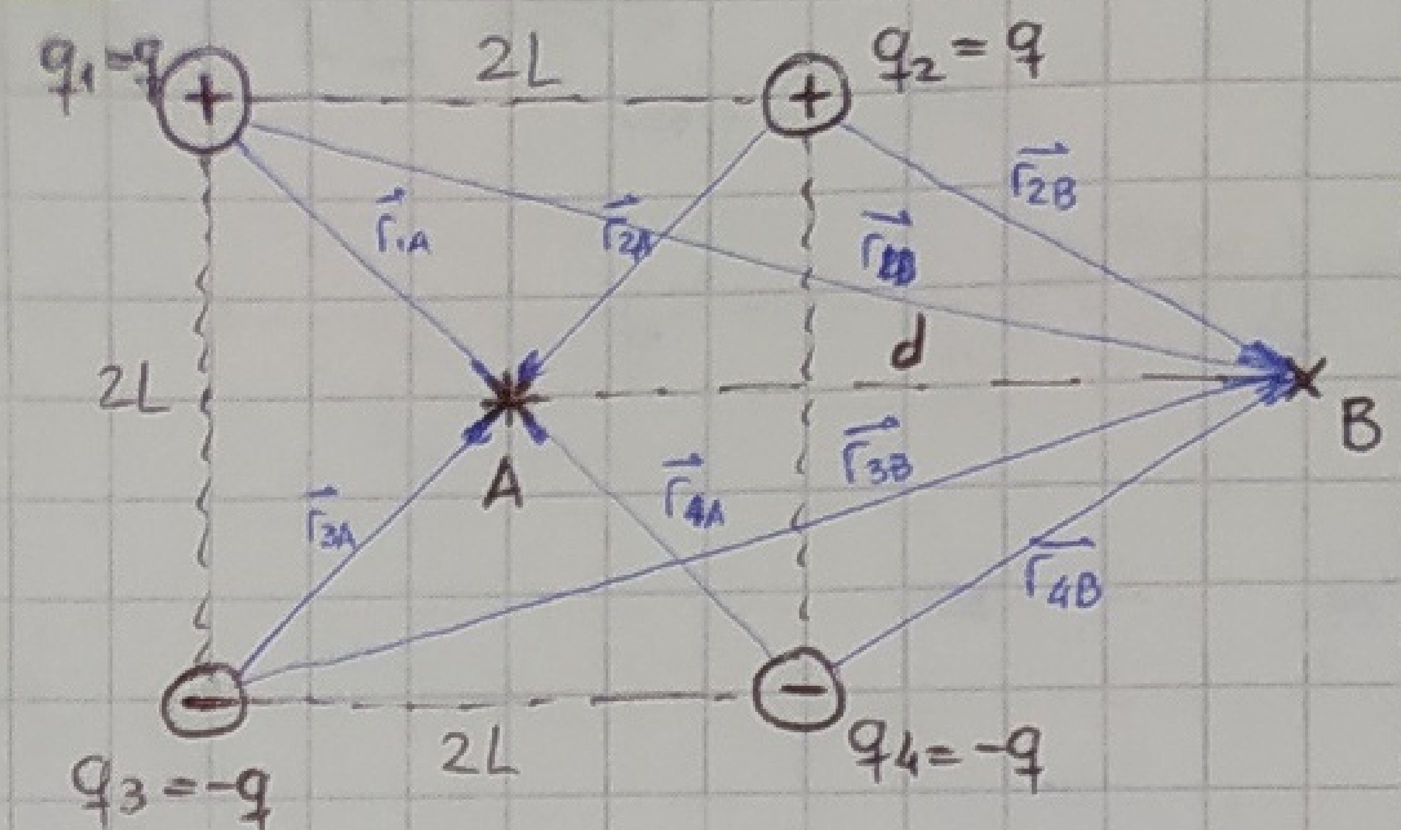
$$W = -q_3\cdot\Delta V_{\infty;(3;2)}$$

$$W = -1\cdot 10^{-6}\,C\cdot(-5659,3\,V)$$

$$\boxed{W = 5,6593\cdot 10^{-3}\,V}$$

$\boxed{23}$

$dist_{A,B} = d$



$$|\vec{r_{1A}}|^3 = \sqrt{L^2 + (-L)^2}^3 = \sqrt{2L^2}^3 = 2\sqrt{2}L^3$$
$$|\vec{r_{2A}}|^3 = \sqrt{(-L)^2 + (-L)^2}^3 = \sqrt{2L^2}^3 = 2\sqrt{2}L^3$$
$$|\vec{r_{3A}}|^3 = \sqrt{L^2 + L^2}^3 = \sqrt{2L^2}^3 = 2\sqrt{2}L^3$$
$$|\vec{r_{4A}}|^3 = \sqrt{(-L)^2 + L^2}^3 = \sqrt{2L^2}^3 = 2\sqrt{2}L^3$$

$\vec{r_{1A}} = (L; -L)$

$\vec{r_{2A}} = (-L; -L)$

$\vec{r_{3A}} = (L; L)$

$\vec{r_{4A}} = (-L; L)$

$$|\vec{r_{1B}}| = \sqrt{(L+d)^2 + (-L)^2} = \sqrt{L^2 + 2dL + d^2 + L^2} = \sqrt{d^2 + 2dL + 2L^2}$$
$$|\vec{r_{2B}}| = \sqrt{(d-L)^2 + (-L)^2} = \sqrt{d^2 - 2dL + L^2 + L^2} = \sqrt{d^2 - 2dL + 2L^2}$$
$$|\vec{r_{3B}}| = \sqrt{(L+d)^2 + L^2} = \sqrt{L^2 + 2dL + d^2 + L^2} = \sqrt{d^2 + 2dL + 2L^2}$$
$$|\vec{r_{4B}}| = \sqrt{(d-L)^2 + L^2} = \sqrt{d^2 - 2dL + L^2 + L^2} = \sqrt{d^2 - 2dL + 2L^2}$$

a

$$\boxed{\vec{E_A}} = \frac{1}{4\pi\varepsilon_0} \cdot \sum \frac{q_0}{|\vec{r}|^3} \cdot \vec{r} = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1}{|\vec{r_{1A}}|^3} \cdot \vec{r_{1A}} + \frac{q_2}{|\vec{r_{2A}}|^3} \cdot \vec{r_{2A}} + \frac{q_3}{|\vec{r_{3A}}|^3} \cdot \vec{r_{3A}} + \frac{q_4}{|\vec{r_{4A}}|^3} \cdot \vec{r_{4A}} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q}{2\sqrt{2}L^3} \cdot (L; -L) + \frac{q}{2\sqrt{2}L^3} \cdot (-L; -L) + \frac{-q}{2\sqrt{2}L^3} \cdot (L; L) + \frac{-q}{2\sqrt{2}L^3} \cdot (-L; L) \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{2\sqrt{2}L^3} \cdot \left[(\cancel{qL}; -qL) + (\cancel{-qL}; -qL) + (\cancel{-qL}; -qL) + (\cancel{qL}; -qL) \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{\sqrt{2}}{2 \cdot 2 \cdot L^3} (0; -4 \cdot q \cdot L) = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{\cancel{4} \cdot \sqrt{2} \cdot \cancel{L}}{\cancel{4} \cdot L^{\cancel{3}2}} \cdot (0; q) = \boxed{-\frac{1}{4\pi\varepsilon_0} \frac{\sqrt{2}}{L^2} (0; q)}$$

b

$$\boxed{W_{AB}} = q \cdot \Delta V_{AB}$$
$$= Q \cdot \underbrace{(V_B - V_A)}_{=0-0=0}$$
$$\boxed{= 0}$$

$$\boxed{V_B} = k \cdot \sum \frac{q_i}{|\vec{r_{iB}}|} = k \cdot \left[\frac{q}{|\vec{r_{1B}}|} + \frac{q}{|\vec{r_{2B}}|} + \frac{-q}{|\vec{r_{3B}}|} + \frac{-q}{|\vec{r_{4B}}|} \right]$$
$$= k \cdot \left[\cancel{\frac{q}{\sqrt{d^2 + 2d^2 + L^2}}} + \cancel{\frac{q}{\sqrt{d^2 - 2dL + L^2}}} - \cancel{\frac{q}{\sqrt{d^2 + 2dL + L^2}}} - \cancel{\frac{q}{\sqrt{d^2 - 2dL + L^2}}} \right] \boxed{= 0}$$

$$\boxed{V_A} = k \cdot \sum \frac{q_i}{|\vec{r_{iA}}|} = k \cdot \left[\frac{q}{|\vec{r_{1A}}|} + \frac{q}{|\vec{r_{2A}}|} + \frac{-q}{|\vec{r_{3A}}|} + \frac{-q}{|\vec{r_{4A}}|} \right]$$
$$= k \cdot \left[\cancel{\frac{q}{\sqrt{2L^2}}} + \cancel{\frac{q}{\sqrt{2L^2}}} - \cancel{\frac{q}{\sqrt{2L^2}}} - \cancel{\frac{q}{\sqrt{2L^2}}} \right] \boxed{= 0}$$

FÍSICA II	ELECTROSTÁTICA Campo Eléctrico - Potencial Electrostático	GUÍA NUEVA 24 - 27

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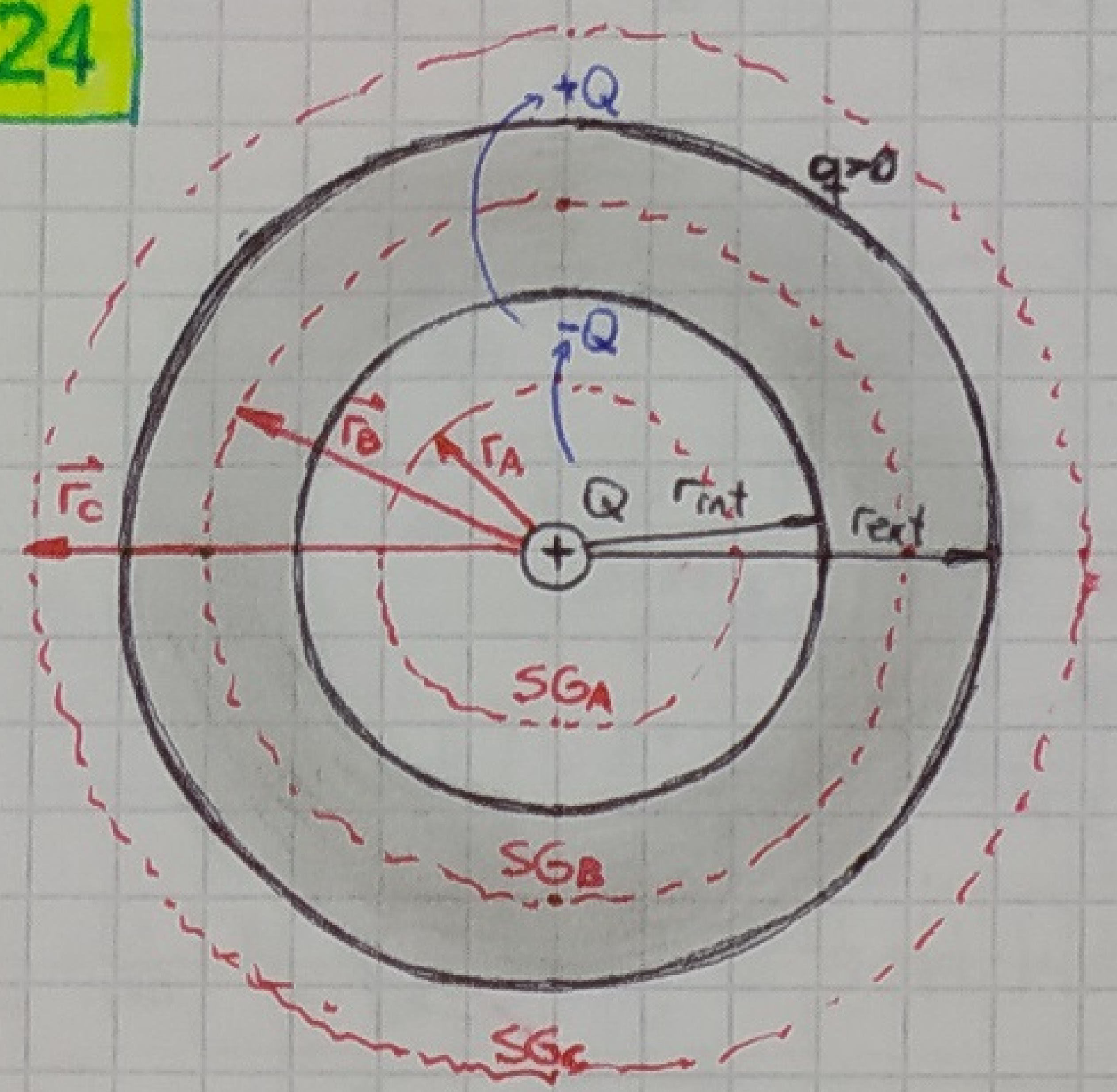

SG_A ⟶ esfera gaussiana de radio r_A.
SG_B ⟶ " " " " r_B.
SG_C ⟶ " " " " r_C.

$0 < r_A < r_{int} < r_B < r_{ext} < r_C$.

Para $\overset{r_C}{r} > r_{ext}$: $\oint_{SG_C} \vec{E}_c \cdot \vec{dS_c} = \dfrac{q_{neta}}{\varepsilon_0}$

$$\oint_{SG_C} |\vec{E}_c| \cdot |\vec{dS_c}| \cdot \underbrace{\cancel{\cos(\overset{=0}{\vec{E}_c \wedge \vec{dS_c}})}}_{=1} = \frac{\cancel{Q} - \cancel{Q} + Q + q}{\varepsilon_0}$$

$$|\vec{E}_c| \cdot \oint_{SG_C} |\vec{dS_c}| = \frac{Q+q}{\varepsilon_0}$$

$$|\vec{E}_c| \cdot S_c = \frac{Q+q}{\varepsilon_0}$$

$$|\vec{E}_c| \cdot 4\pi r^2 = \frac{Q+q}{\varepsilon_0}$$

$$\boxed{|\vec{E}_c| = \frac{Q+q}{4\pi\varepsilon_0 \cdot r^2}}$$

r_{int} r_{ext} r

$$V_{\infty,r} = V_{(r)} - \overset{=0}{V_\infty} = -\int_{\infty}^{r} \vec{E} \cdot \vec{dl} = \int_{r}^{\infty} \vec{E} \cdot \vec{dl}$$

$$V_{(r)} = \int_{r}^{\infty} \vec{E}_c \cdot \vec{dl} = \int_{r}^{\infty} |\vec{E}_c| \cdot |\vec{dl}| \cdot \underbrace{\cancel{\cos(\overset{=0}{\vec{E}_c \wedge \vec{dl}})}}_{=1}$$

$$V_{(r)} = \frac{Q+q}{4\pi\varepsilon_0} \cdot \int_{r}^{\infty} \frac{1}{r^2}\, dr$$

$$\left[-\frac{1}{r}\right]_r^{\infty} = \left(\cancel{-\frac{1}{\infty}}\right) - \left(-\frac{1}{r}\right) = \frac{1}{r}$$

$$\boxed{V_{(r)} = \frac{Q+q}{4\pi\varepsilon_0 \cdot r}}$$

Para $r_{int} < \overset{r_B}{r} < r_{ext}$: $\oint_{SG_B} \vec{E}_B \cdot \vec{dS_B} = \dfrac{q_{neta}}{\varepsilon_0}$

$$\oint_{SG_B} |\vec{E}_B| \cdot |\vec{dS_B}| \cdot \underbrace{\cancel{\cos(\overset{=0}{\vec{E}_B \wedge \vec{dS_B}})}}_{=1} = \frac{\overset{=0}{\cancel{Q - Q}}}{\varepsilon_0}$$

$$|\vec{E}_B| \cdot \underbrace{\oint |\vec{dS_B}|}_{\neq 0} = 0$$

$$\boxed{|\vec{E}_B| = 0}$$

r_{int} r r_{ext}

$$V_{\infty,r} = V_{(r)} - V_\infty = -\int_{\infty}^{r} \vec{E} \cdot \vec{dl} = \int_{r}^{\infty} \vec{E} \cdot \vec{dl}$$

$$V_{(r)} = \cancel{\int_{r}^{r_{ext}} \underset{=0}{\vec{E}_B} \cdot \vec{dl}} + \int_{r_{ext}}^{\infty} \vec{E}_c \cdot \vec{dl}$$

$$V_{(r)} = \int_{r_{ext}}^{\infty} |\vec{E}_c| \cdot |\vec{dl}| \cdot \underbrace{\cancel{\cos(\overset{=0}{\vec{E}_c \wedge \vec{dl}})}}_{=1}$$

$$V_{(r)} = \frac{Q+q}{4\pi\varepsilon_0} \cdot \int_{r_{ext}}^{\infty} \frac{1}{r^2} \cdot dr$$

$$\left(-\frac{1}{r}\right)_{r_{ext}}^{\infty} = \left(\underset{=0}{\cancel{-\frac{1}{\infty}}}\right) - \left(-\frac{1}{r_{ext}}\right) = \frac{1}{r_{ext}}$$

$$\boxed{V_{(r)} = \frac{Q+q}{4\pi\varepsilon_0 \cdot r_{ext}}}$$

Para $r < r_{int}$: $\oint_{SG_A} \vec{E_A} \cdot \vec{dS_A} = \frac{q_{neta}}{\varepsilon_0}$

$$\oint_{SG_A} |\vec{E_A}| \cdot |\vec{dS_A}| \cdot \underbrace{\cos(\overbrace{\vec{E_A} \wedge \vec{dl}}^{=0})}_{=1} = \frac{Q}{\varepsilon_0}$$

$$|\vec{E_A}| \cdot \oint_{SG_A} |\vec{dS_A}| = \frac{Q}{\varepsilon_0}$$

$$|\vec{E_A}| \cdot S_A = \frac{Q}{\varepsilon_0}$$

$$|\vec{E_A}| \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{\vec{E_A} = \frac{Q}{4\pi\varepsilon_0 \cdot r^2}}$$

$$V_{\infty,r} = V_{(r)} - \overbrace{V_\infty}^{=0} = -\int_\infty^r \vec{E} \cdot \vec{dl} = \int_r^\infty \vec{E} \cdot \vec{dl}$$

$$V_{(r)} = \int_r^{r_{int}} \vec{E_A} \cdot \vec{dl} + \cancel{\int_{r_{int}}^{r_{ext}} \underbrace{\vec{E_B}}_{=0} \cdot \vec{dl}} + \int_{r_{ext}}^\infty \vec{E_C} \cdot \vec{dl}$$

$$V_{(r)} = \int_r^{r_{int}} \frac{Q}{4\pi\varepsilon_0 \cdot r^2} \breve{e_r} \cdot \breve{r} dr + \int_{r_{ext}}^\infty \frac{Q+q}{4\pi\varepsilon_0 \cdot r^2} \breve{e_r}$$

$$V_{(r)} = \frac{Q}{4\pi\varepsilon_0} \cdot \underbrace{\int_r^{r_{int}} \frac{1}{r^2} dr}_{} + \frac{Q+q}{4\pi\varepsilon_0} \cdot \underbrace{\int_{r_{ext}}^\infty \frac{1}{r^2} dr}_{}$$

$$= \left(-\frac{1}{r}\right) = \left(-\frac{1}{r_{int}}\right) - \left(-\frac{1}{r}\right) = \frac{1}{r} - \frac{1}{r_{int}} \qquad = \left(-\frac{1}{r}\right)\Big|_{r_{ext}}^{\infty} = \cancel{\left(-\frac{1}{\infty}\right)}_{=0} - \left(-\frac{1}{r_{ext}}\right) = \frac{1}{r_{ext}}$$

$$\boxed{V_{(r)} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_{int}}\right) + \frac{Q+q}{4\pi\varepsilon_0 \cdot r_{ext}}}$$

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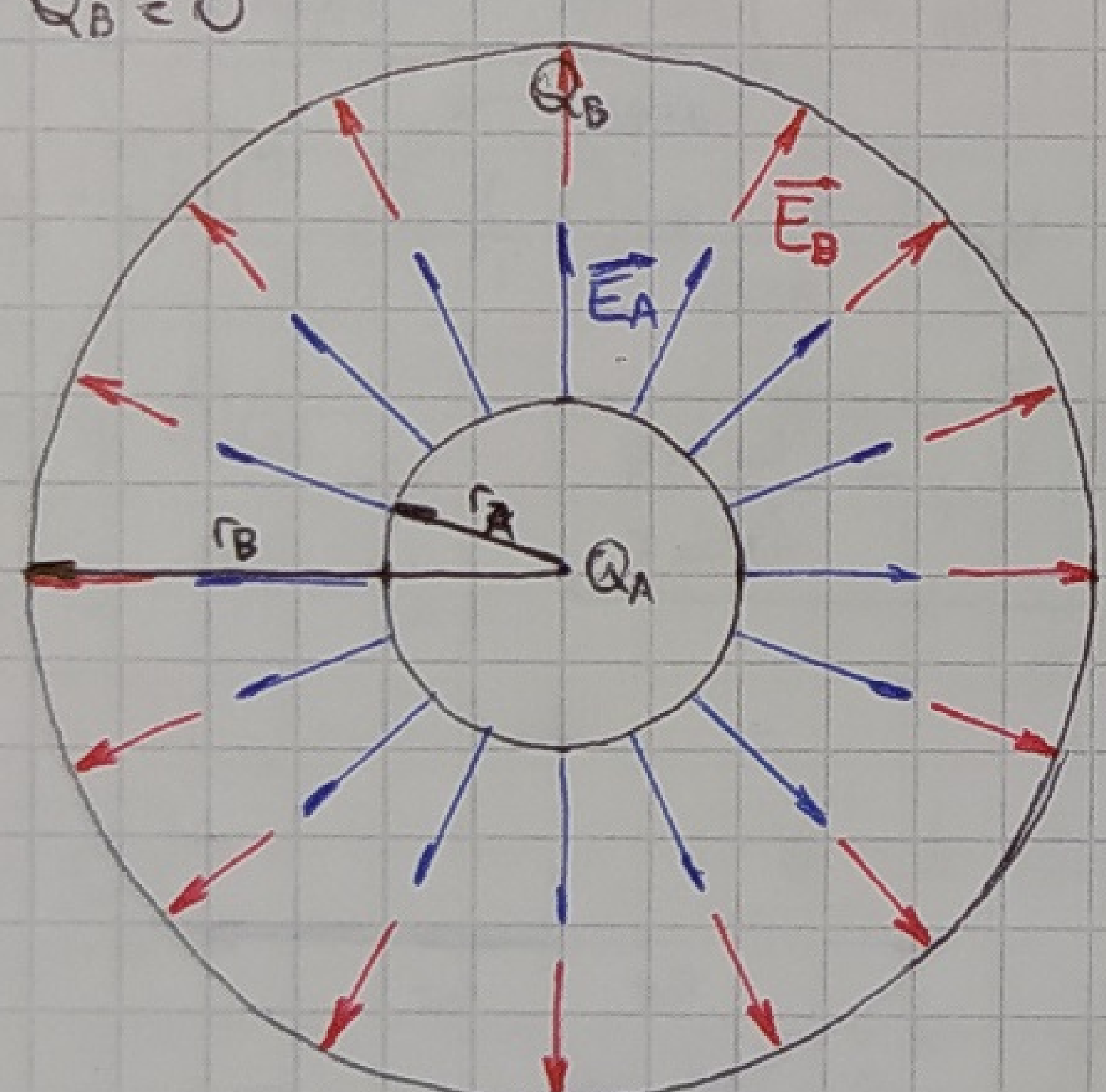
$Q_A > 0$

$Q_B < 0$



a) En la región interna ($r_A < r < r_B$), el campo eléctrico tiene dirección radial y sentido "hacia afuera".

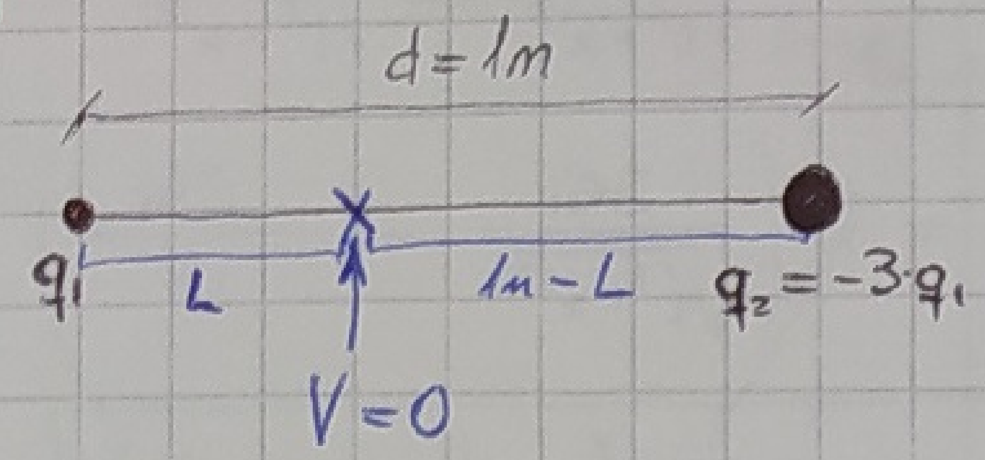
b) Como la carga q es negativa, F tiene sentido opuesto al del campo eléctrico. Por lo tanto, si bien es radial, la fuerza eléctrica sobre la carga q tiene sentido "hacia adentro".

c) Como el gradiente de potencial es siempre opuesto al vector de campo eléctrico, (es radial) pero tiene sentido "hacia adentro".

d) Para transportar la carga q negativa en contra de la fuerza eléctrica, se realizará un trabajo negativo.

FÍSICA II | ELECTROSTÁTICA | GUÍA NUEVA | 25 - 26 - 28

25



$r_1 = L$ $\qquad$ $r_2 = 1m - L$

$$V = \sum \frac{k \cdot q_i}{r_i} = k \cdot \sum \frac{q_i}{r_i}$$

$$0 =$$

$$\frac{\cancel{q_1}}{|L|} = + \frac{3\cancel{q_1}}{|1m - L|}$$

$$|1m - L| = 3 \cdot |L|$$

$$|1m - L| = 3 \cdot L$$

$$L = 0{,}25m \qquad L = -0{,}5m$$

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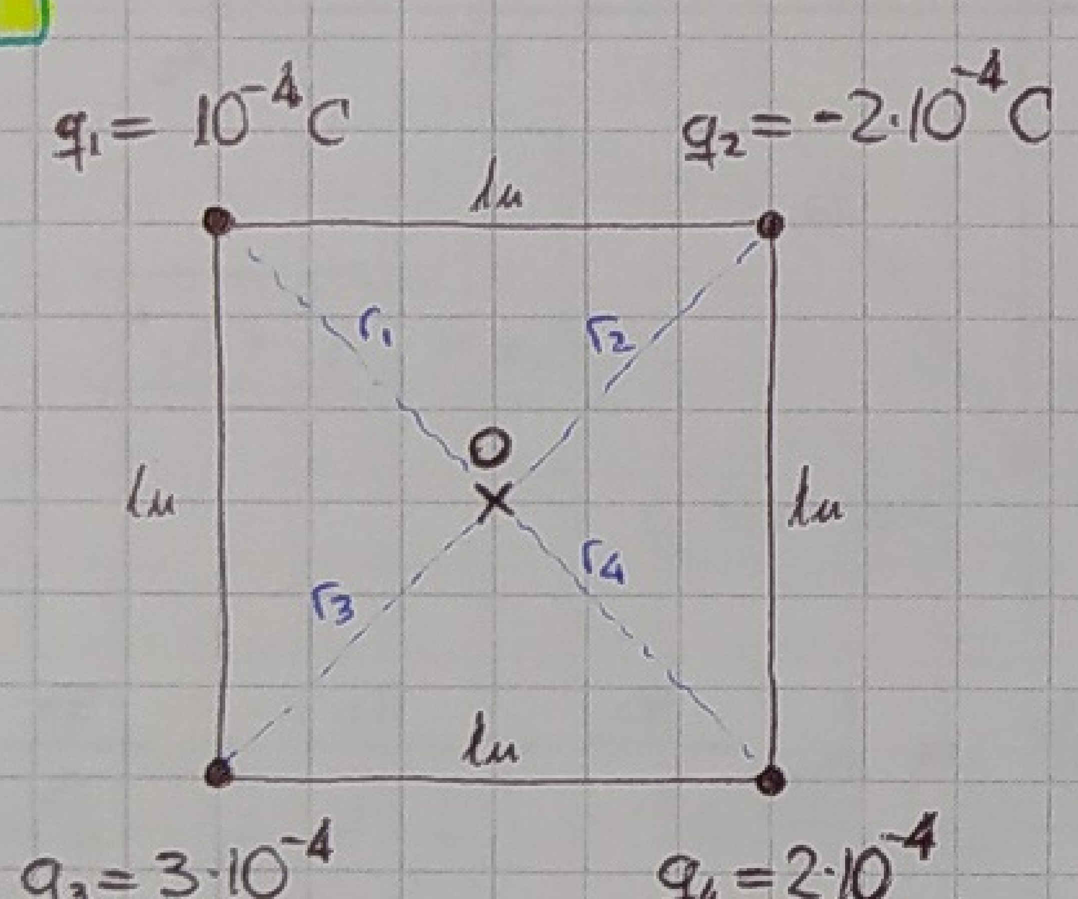

$$r_1 = r_2 = r_3 = r_4 = \sqrt{(0{,}5m)^2 + (0{,}5m)^2} = \frac{\sqrt{2}}{2} m$$

ⓐ $$V_0 = \sum \frac{k \cdot q_i}{r_i} = k \cdot \sum \frac{q_i}{r_i}$$

$$V_0 = k \cdot \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \frac{q_4}{r_4} \right)$$

$$V_0 = 9 \cdot 10^9 \frac{Nm^2}{C^2} \cdot \left(\frac{10^{-4} C}{\frac{\sqrt{2}}{2} m} + \cancel{\frac{-2 \cdot 10^{-4} C}{\frac{\sqrt{2}}{2} m}} + \frac{3 \cdot 10^{-4} C}{\frac{\sqrt{2}}{2} m} + \cancel{\frac{2 \cdot 10^{-4} C}{\frac{\sqrt{2}}{2} m}} \right)$$

$$\boxed{V_0 = 5.091168{,}825 \cdot V}$$

ⓑ El intercambio de posiciones de las cargas no modifica el valor del potencial en el punto O (centro del cuadrado).
Manteniendo constantes los valores de las cargas, el potencial depende únicamente de la distancia al centro de cada carga. Siendo éstas también las mismas, el potencial es el mismo.

ⓒ El valor hallado en ⓐ no vale para todo punto del cuadrado, dado que para otros puntos el valor de la distancia al centro de cada carga será distinto.

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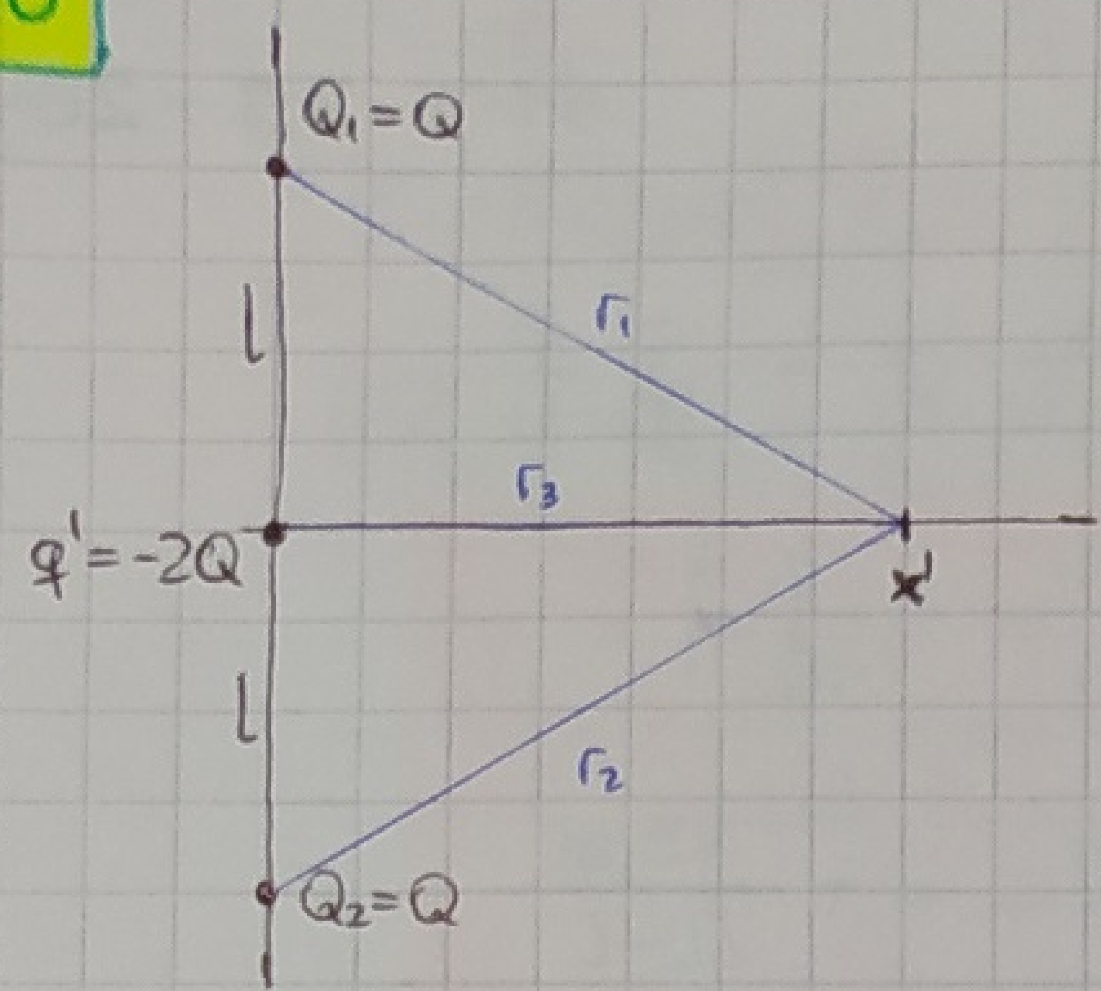


$$r_1 = \sqrt{l^2 + x'^2}$$

$$r_2 = \sqrt{l^2 + x'^2}$$

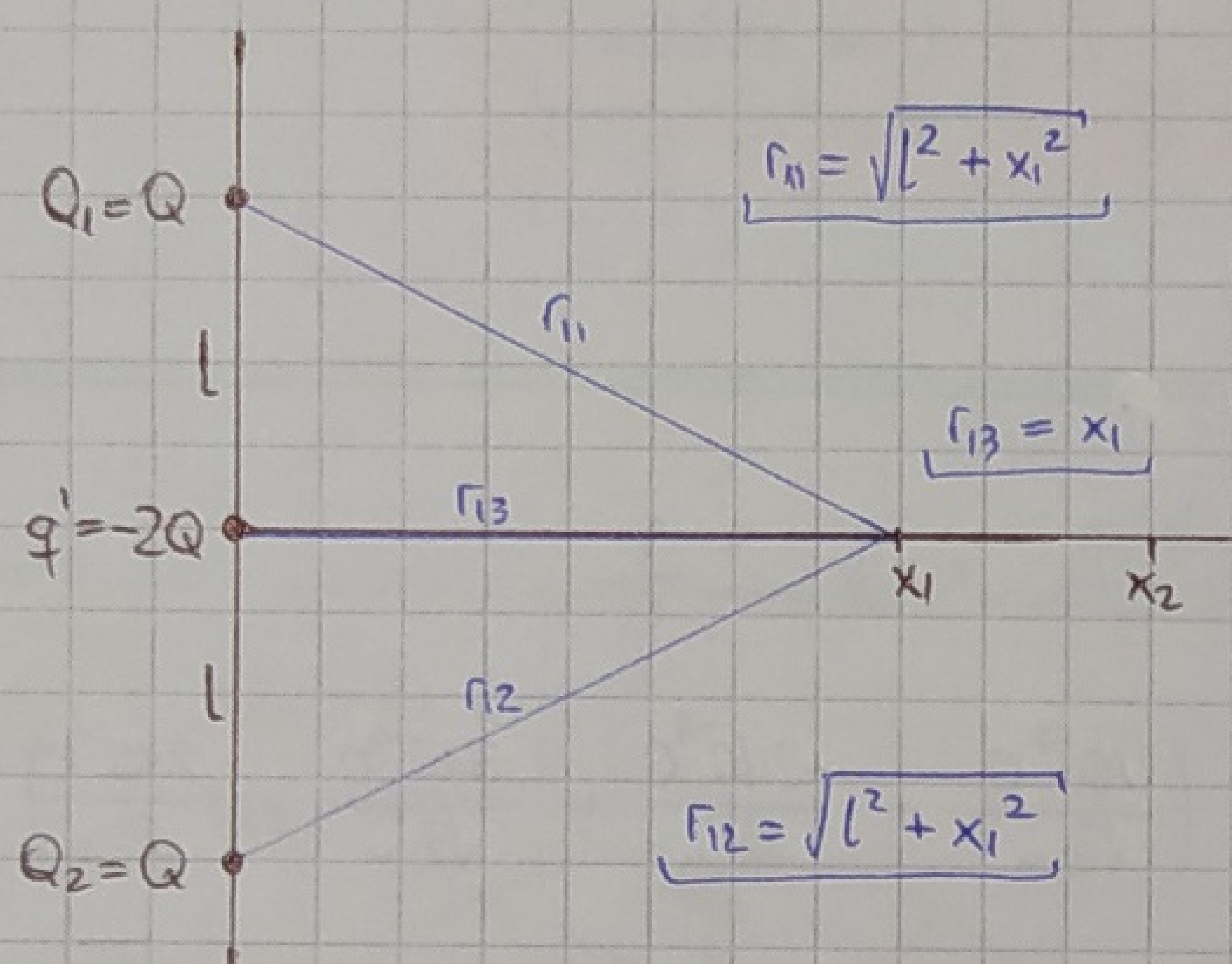
$$r_3 = x'$$

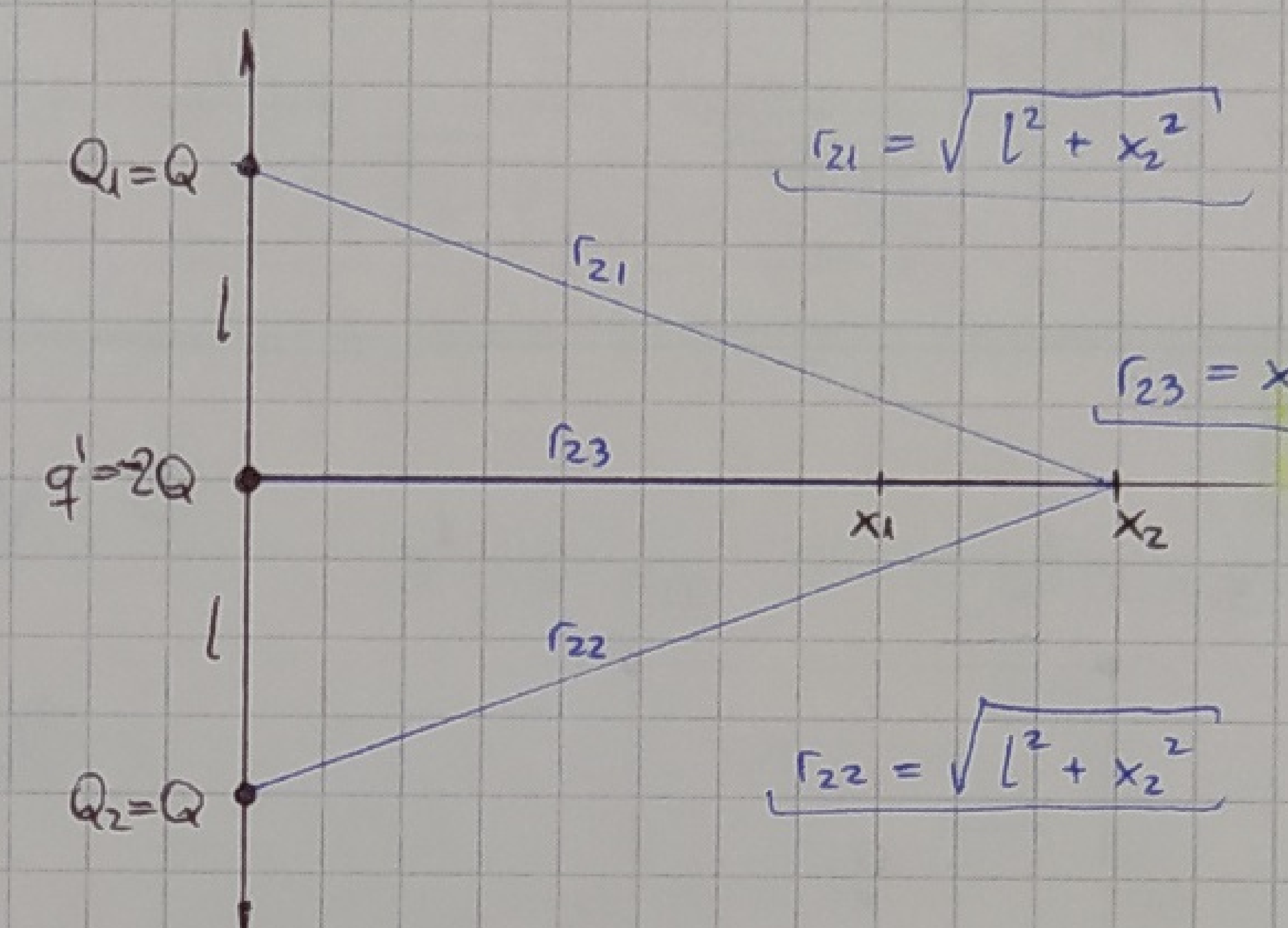
$$V_{x'} = \sum \frac{k \cdot q_i}{r_i} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

$$V_{x'} = k\left(\frac{Q}{\sqrt{l^2+x'^2}} + \frac{Q}{\sqrt{l^2+x'^2}} + \frac{-2Q}{x'}\right)$$

$$V_{x'} = k\left(\frac{2Q}{\sqrt{l^2+x'^2}} - \frac{2Q}{x'}\right)$$

$$\boxed{V_{x'} = k \cdot 2Q\left(\frac{1}{\sqrt{l^2+x'^2}} - \frac{1}{x'}\right)}$$





$$V_{x_1} = \sum \frac{k \cdot q_i}{r_i} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

$$V_{x_1} = k\left(\frac{Q}{\sqrt{l^2+x_1^2}} + \frac{Q}{\sqrt{l^2+x_1^2}} + \frac{-2Q}{x_1}\right)$$

$$V_{x_1} = k \cdot \left(\frac{2Q}{\sqrt{l^2+x_1^2}} - \frac{2Q}{x_1}\right)$$

$$\boxed{V_{x_1} = 2kQ \cdot \left(\frac{1}{\sqrt{l^2+x_1^2}} - \frac{1}{x_1}\right)}$$

$$V_{x_2} = \sum \frac{k q_i}{r_i} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

$$V_{x_2} = k\left(\frac{Q}{\sqrt{l^2+x_2^2}} + \frac{Q}{\sqrt{l^2+x_2^2}} + \frac{-2Q}{x_2}\right)$$

$$V_{x_2} = k\left(\frac{2Q}{\sqrt{l^2+x_2^2}} - \frac{2Q}{x_2}\right)$$

$$\boxed{V_{x_2} = 2kQ\left(\frac{1}{\sqrt{l^2+x_2^2}} - \frac{1}{x_2}\right)}$$

$$W = q \cdot \Delta V \Rightarrow W_{x_1 x_2} = q' \cdot \Delta V_{x_1, x_2}$$

$$W_{x_1,x_2} = q' \cdot \left[\left(2kQ\left(\frac{1}{\sqrt{l^2+x_2^2}} - \frac{1}{x_2}\right)\right) - \left(2kQ\left(\frac{1}{\sqrt{l^2+x_1^2}} - \frac{1}{x_1}\right)\right)\right]$$

$$W_{x_1,x_2} = q' \cdot \left[2kQ \cdot \left(\frac{1}{\sqrt{l^2+x_2^2}} - \frac{1}{x_2} - \frac{1}{\sqrt{l^2+x_1^2}} + \frac{1}{x_1}\right)\right]$$

Tiene la misma magnitud que el trabajo realizado por la fuerza eléctrica (ej. 4) pero de signo opuesto.

$$\boxed{W_{x_1,x_2} = 2kq'Q\left(\frac{1}{\sqrt{l^2+x_2^2}} - \frac{1}{x_2} - \frac{1}{\sqrt{l^2+x_1^2}} + \frac{1}{x_1}\right)}$$

Física II

Electrostática

Campo Eléctrico - Fuerza eléctrica - Potencial - Trabajo

Guía Nueva 29 - 30

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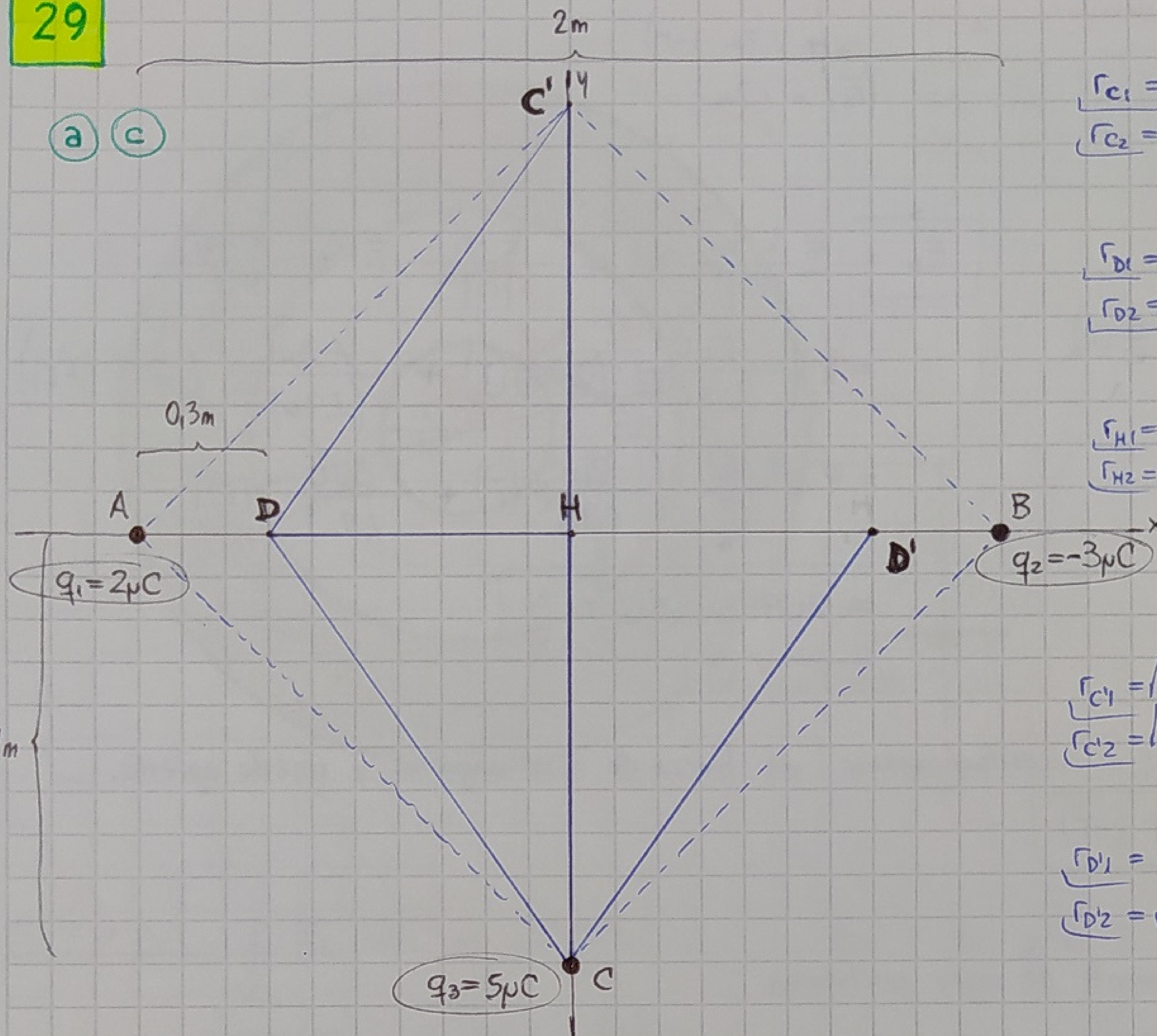
a) c)



$r_{C1} = \sqrt{(1m)^2 + (1m)^2} = \sqrt{2}\,m$

$r_{C2} = \sqrt{(1m)^2 + (1m)^2} = \sqrt{2}\,m$

$r_{D1} = \sqrt{(0{,}3m)^2 + (0m)^2} = 0{,}3\,m$

$r_{D2} = \sqrt{(1{,}7m)^2 + (0m)^2} = 1{,}7\,m$

$r_{H1} = \sqrt{(1m)^2 + (0m)^2} = 1m$

$r_{H2} = \sqrt{(1m)^2 + (0m)^2} = 1m$

$r_{C'1} = \sqrt{(1m)^2 + (1m)^2} = \sqrt{2}\,m$

$r_{C'2} = \sqrt{(1m)^2 + (1m)^2} = \sqrt{2}\,m$

$r_{D'1} = \sqrt{(1{,}7m)^2 + (0m)^2} = 1{,}7m$

$r_{D'2} = \sqrt{(0{,}3m)^2 + (0m)^2} = 0{,}3m$

$$\boxed{V_C} = k \cdot \left(\frac{q_1}{r_{C1}} + \frac{q_2}{r_{C2}}\right) = 9\cdot 10^9 \frac{Nm^2}{C^2} \cdot \left(\frac{2\mu C}{\sqrt{2}m} + \frac{-3\mu C}{\sqrt{2}m}\right) = \boxed{-6.363{,}96\,V}$$ A

$$\boxed{V_D} = k \cdot \left(\frac{q_1}{r_{D1}} + \frac{q_2}{r_{D2}}\right) = 9\cdot 10^9 \frac{Nm^2}{C^2} \cdot \left(\frac{2\mu C}{0{,}3m} + \frac{-3\mu C}{1{,}7m}\right) = \boxed{44.117{,}65\,V}$$ B

$$\boxed{V_H} = k \cdot \left(\frac{q_1}{r_{H1}} + \frac{q_2}{r_{H2}}\right) = 9\cdot 10^9 \frac{Nm^2}{C^2} \cdot \left(\frac{2\mu C}{1m} + \frac{-3\mu C}{1m}\right) = \boxed{-9000\,V}$$ C

$$\boxed{V_{C'}} = k \cdot \left(\frac{q_1}{r_{C'1}} + \frac{q_2}{r_{C'2}}\right) = 9\cdot 10^9 \frac{Nm^2}{C^2} \cdot \left(\frac{2\mu C}{\sqrt{2}m} + \frac{-3\mu C}{\sqrt{2}m}\right) = \boxed{-6.363{,}96\,V}$$ D

$$\boxed{V_{D'}} = k \cdot \left(\frac{q_1}{r_{D'1}} + \frac{q_2}{r_{D'2}}\right) = 9\cdot 10^9 \frac{Nm^2}{C^2} \cdot \left(\frac{2\mu C}{1{,}7m} + \frac{-3\mu C}{0{,}3m}\right) = \boxed{-79.411{,}76}$$ M

a1) $$\boxed{W_{CD}} = q_3 \cdot \Delta V_{CD} \overset{*}{=} q_3 \cdot (V_D - V_C) = 5\mu C \cdot (44.117{,}65\,V - (-6363{,}96\,V)) = \boxed{0{,}25\,W}$$

a2) $$\boxed{W_{CH-HD}} = q_3 \cdot \Delta V_{CH} + q_3\, \Delta V_{HD} = q_3 \cdot (\Delta V_{CH} + \Delta V_{HD}) = q_3 \cdot \left[(\cancel{V_H} - V_C) + (V_D - \cancel{V_H})\right] = q_3 \cdot (V_D - V_C) \rightarrow * \\ = \boxed{0{,}25\,W}$$

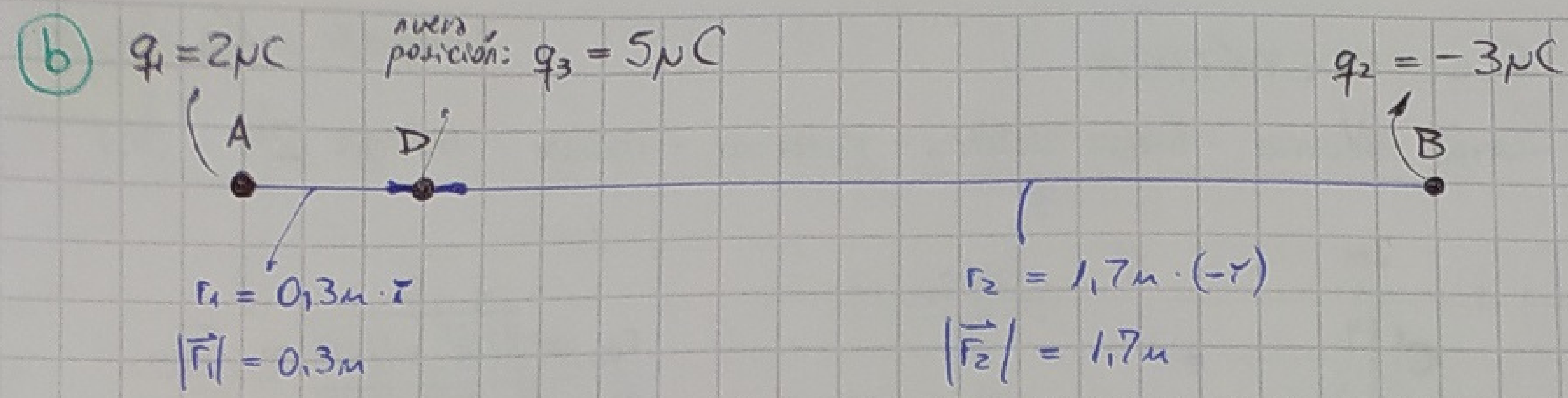
a3) $$\boxed{W_{CC'-C'D}} = q_3 \cdot \Delta V_{CC'} + q_3\, \Delta V_{C'D} = q_3\,(\Delta V_{CC'} + \Delta V_{C'D}) = q_3 \left[(\cancel{V_{C'}} - V_C) + (V_D - \cancel{V_{C'}})\right] = q_3 \cdot (V_D - V_C) \rightarrow * \\ = \boxed{0{,}25\,W}$$

c) $$\boxed{W_{CD'}} = q_3 \cdot \Delta V_{CD'} = q_3\,(V_{D'} - V_C) = 5\mu C \cdot (-79.411{,}76\,V - (-6.363{,}96\,V)) = \boxed{-0{,}36\,W}$$

La intensidad es mayor, pero es de signo negativo. Por lo tanto $|W_{C'D}| > |W_{CD}|$

NOTA

(b)



$$\boxed{\vec{F_D} = q_3 \cdot \vec{E_D}}$$

$$= 5\mu C \cdot 209.342{,}56 \frac{N}{C} (\breve{\imath})$$

$$\boxed{= 1{,}0467\ N\ (\breve{\imath})}$$

$$\boxed{\vec{E_D}} = \sum \frac{k \cdot q_i}{|\vec{r_i}|^3} \cdot \vec{r_i} = \frac{k \cdot q_1}{|\vec{r_1}|^3} \cdot \vec{r_1} + \frac{k \cdot q_2}{|\vec{r_2}|} \cdot \vec{r_2}$$

$$= k \cdot \left(\frac{2\mu C}{(0{,}3m)^{\cancel{3}\,2}} \cdot \cancel{(0{,}3m)} (\breve{\imath}) + \frac{\cancel{-}3\mu C}{(1{,}7m)^{\cancel{3}\,2}} \cdot \cancel{(1{,}7m)} \cdot (\cancel{-}\breve{\imath}) \right)$$

$$= 9 \cdot 10^9 \frac{Nm^2}{C^2} \cdot \left(\frac{2\mu C}{0{,}3m^2} (\breve{\imath}) + \frac{3\mu C}{1{,}7m^2} \cdot (\breve{\imath}) \right)$$

$$\boxed{= 209.342{,}56\ \breve{\imath}\ \frac{N}{C}}$$

Para equilibrar la $\vec{F_D}$ actual, se necesitará aplicar una fuerza de igual magnitud y sentido opuesto.

Es decir, $\boxed{1{,}0467\ N\ (-\breve{\imath})}$.

(d) En ambos casos, únicamente de signo cambiarán.

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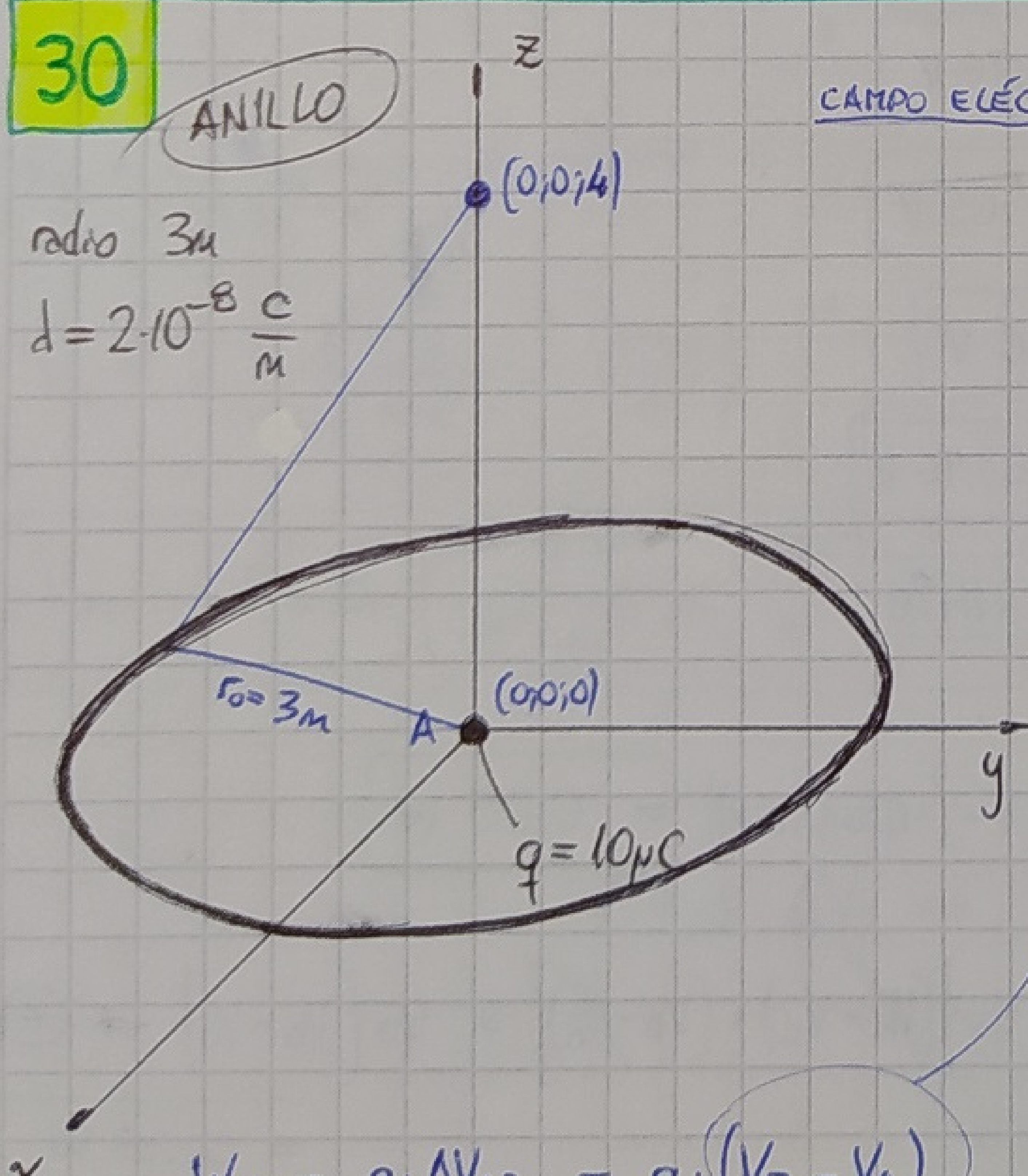

CAMPO ELÉCTRICO (EN EL EJE) GENERADO POR UN ANILLO: (ver ej. $\boxed{9}$)

$$\vec{E} = \frac{\lambda \cdot r_0 \cdot z}{2\varepsilon_0 (r_0^2 + z^2)^{3/2}} \breve{k}$$

$$V_B - V_A = \int \vec{E} \cdot \vec{dl}$$

$$V_B - V_A = \int_0^4 \frac{\lambda \cdot r_0 \cdot z}{2 \cdot \varepsilon_0 (r_0^2 + z^2)^{3/2}} \cdot \breve{k} \cdot \breve{k}\, dz$$

$$V_B - V_A = \frac{\lambda \cdot r_0}{2 \cdot \varepsilon_0} \cdot \boxed{\int_0^4 \frac{z}{(r_0^2 + z^2)^{3/2}}\, dz}$$

$$= \left[-\frac{1}{\sqrt{r_0^2 + z^2}} \right]_0^4$$

$$= \left[-\frac{1}{\sqrt{r_0^2 + 16}} \right] - \left[-\frac{1}{r_0} \right]$$

$$= \frac{1}{r_0} - \frac{1}{\sqrt{r_0^2 + 16}}$$

$$V_B - V_A = \frac{\lambda \cdot r_0}{2 \cdot \varepsilon_0} \cdot \left(\frac{1}{r_0} - \frac{1}{\sqrt{r_0^2 + 16}} \right)$$

$$V_B - V_A = \frac{\lambda}{2 \cdot \varepsilon_0} \cdot \left(1 - \frac{r_0}{\sqrt{r_0^2 + 16}} \right)$$

$$W_{AB} = q \cdot \Delta V_{AB} = q \cdot (V_B - V_A)$$

$$W_{AB} = q \cdot \frac{\lambda}{2\varepsilon_0} \cdot \left(1 - \frac{r_0}{\sqrt{r_0^2 + 16m^2}} \right)$$

$$W_{AB} = 10\mu C \cdot \frac{2 \cdot 10^{-8} \frac{C}{m}}{2 \cdot 8{,}85 \cdot 10^{-12} \frac{C^2}{Nm^2}} \cdot \left(1 - \frac{3m}{\sqrt{(3m)^2 + 16m^2}} \right)$$

$$\boxed{W_{AB} = 4{,}52 \cdot 10^{-3}\ J}$$

FÍSICA II	ELECTROSTÁTICA Campo Eléctrico - Potencial Electrostático	GUÍA NUEVA 30

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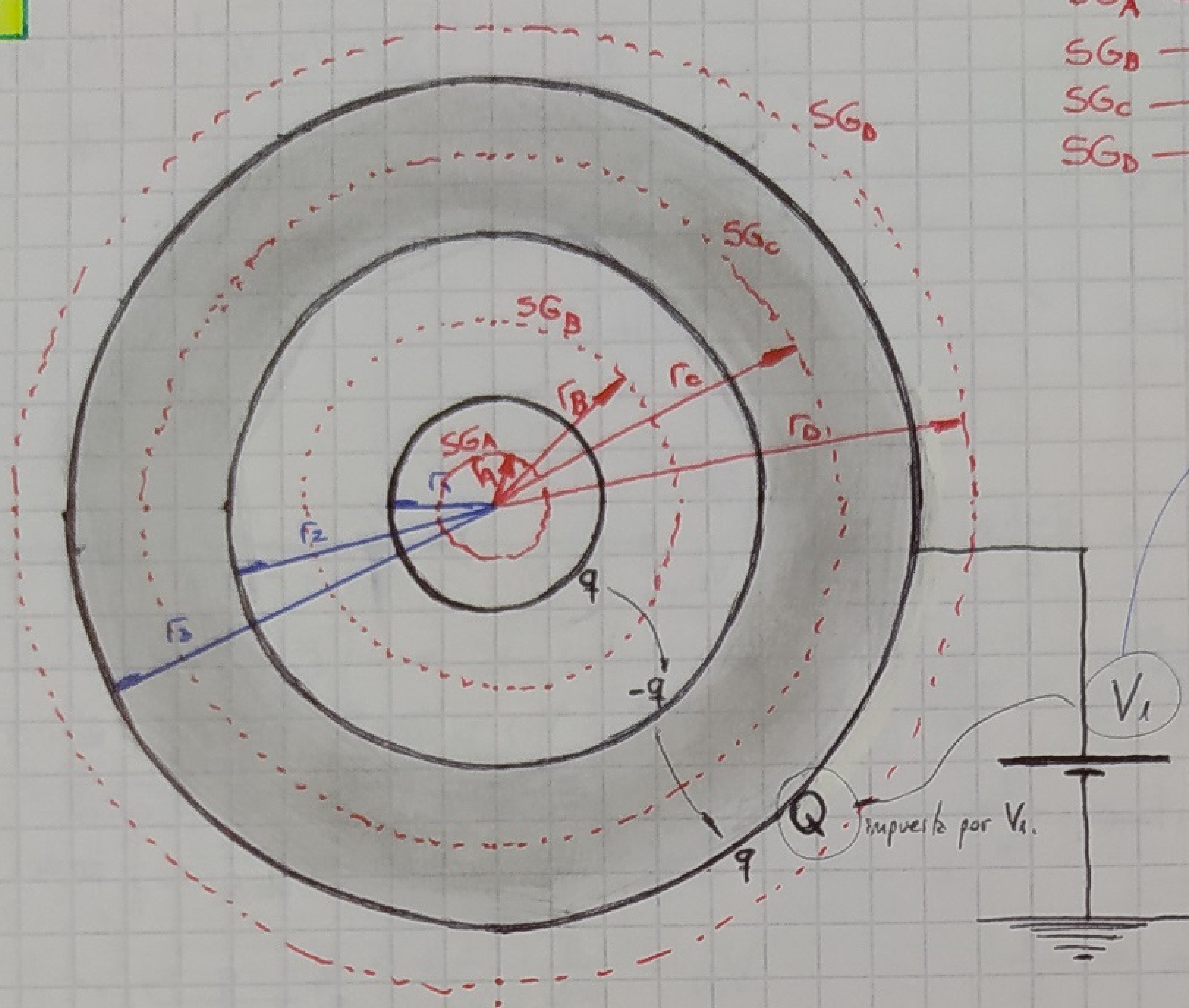
$SG_A \rightarrow$ superficie gaussiana de radio r_A.
$SG_B \rightarrow$ " " " " r_B.
$SG_C \rightarrow$ " " " " r_C.
$SG_D \rightarrow$ " " " " r_D.

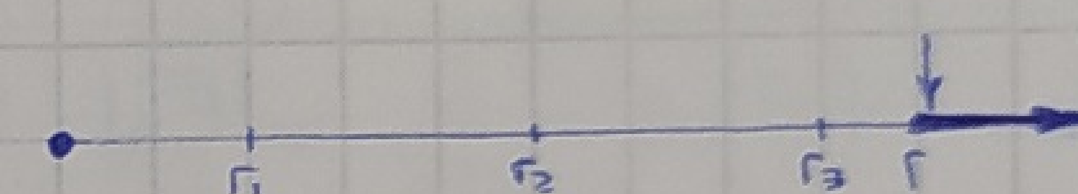


$V_{(r_3)} = V_1$

$\Downarrow$

$V_{(r_2)} = V_1$

Cualquier punto en r_2 está al mismo potencial que cualquier punto en r_3, dado que entre ambos puntos hay un conductor.



Si $r > r_3$ (r_D):

$$\oint_{SG_D} \vec{E_D} \cdot \vec{dS_D} = \frac{Q_{externa}}{\varepsilon_0}$$

$$\oint_{SG_D} |\vec{E_D}| \cdot |\vec{dS_D}| \cdot \cancel{\cos(\widehat{\vec{E_D}, \vec{dS_D}})}^{=1}_{=0} = \frac{Q_{externa}}{\varepsilon_0}$$

$$|\vec{E_D}| \cdot \oint_{SG_D} |\vec{dS_D}| = \frac{Q_{externa}}{\varepsilon_0}$$

$$|\vec{E_D}| \cdot S_D = \frac{Q_{externa}}{\varepsilon_0}$$

$$|\vec{E_D}| \cdot 4\pi r^2 = \frac{Q_{externa}}{\varepsilon_0}$$

$$\boxed{|\vec{E_D}| = \frac{Q_{externa}}{4\pi\varepsilon_0 \cdot r^2}} \ (*)$$

$$V_{\infty,r} = V_{(r)} - \cancel{V_\infty}^{=0} = -\int_\infty^r \vec{E} \cdot \vec{dl} = \int_r^\infty \vec{E} \cdot \vec{dl}$$

$$V_{(r)} = \int_r^\infty \frac{Q_{externa}}{4\pi\varepsilon_0 \cdot r^2} \check{e}_r \cdot \check{r}\, dr$$

$$V_{(r)} = \frac{Q_{externa}}{4\pi\varepsilon_0} \int_r^\infty \frac{1}{r^2} dr$$

$$= \left[-\frac{1}{r}\right]_r^\infty = \left[\cancel{-\frac{1}{\infty}}\right] - \left[-\frac{1}{r}\right] = \boxed{\frac{1}{r}}$$

$$\boxed{V_{(r)} = \frac{Q_{externa}}{4\pi\varepsilon_0 \cdot r}} \ (**)$$

$$\Downarrow$$

$$V_{(r_3)} = \frac{Q_{externa}}{4\pi\varepsilon_0 \cdot r_3}$$

$$V_1 = \frac{Q_{externa}}{4\pi\varepsilon_0 \cdot r_3}$$

$$\boxed{Q_{externa} = V_1 \cdot 4\pi\varepsilon_0 \cdot r_3}$$

De (*): $|\vec{E_D}| = \frac{V_1 \cdot \cancel{4\pi\varepsilon_0} \cdot r_3}{\cancel{4\pi\varepsilon_0} \cdot r^2} \Rightarrow \boxed{\vec{E_D} = \frac{V_1 \cdot r_3}{r^2}}$

De (**): $V_{(r)} = \frac{V_1 \cdot \cancel{4\pi\varepsilon_0} \cdot r_3}{\cancel{4\pi\varepsilon_0} \cdot r} \rightarrow \boxed{V_{(r)} = \frac{V_1 \cdot r_3}{r}}$

Si $r_2 < \overset{r_C}{r} < r_3$:

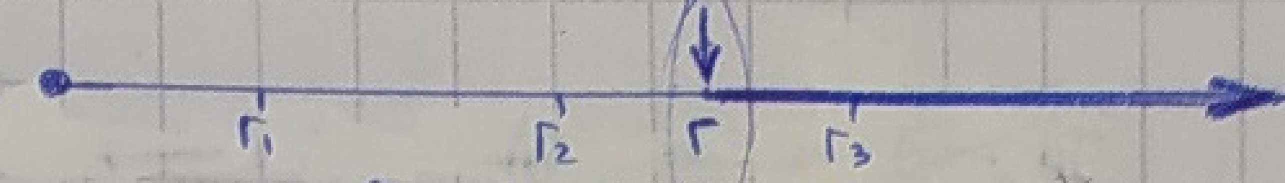
$$\oint_{SG_C} \vec{E}_C \cdot \vec{dS}_C = \frac{q_{neta_C}}{\varepsilon_0}$$

$$\oint_{SG_C} |\vec{E}_C| \cdot |\vec{dS}_C| \cdot \underbrace{\cancel{\cos(\overset{=0}{\vec{E}_D \wedge \vec{dS}_D})}}_{=1} = \frac{0}{\varepsilon_0}$$

$$|\vec{E}_C| \cdot \oint_{SG_C} |\vec{dS}_C| = 0$$

$$|\vec{E}_C| \cdot \underbrace{S_C}_{\neq 0} = 0$$

$$\boxed{|\vec{E}_C| = 0}$$



$$V_{\infty,r} = V(r) - \overset{=0}{\cancel{V_\infty}} = -\int_\infty^r \vec{E} \cdot \vec{dl} = \int_r^\infty \vec{E} \cdot \vec{dl}$$

$$V(r) = \underbrace{\cancel{\int_r^{r_3} |\vec{E}_C| \cdot |\vec{dl}| \cdot \underbrace{\cos(\overset{=0}{\vec{E}_C \wedge \vec{dl}})}_{=1}}}_{=0} + \underbrace{\int_{r_3}^\infty \vec{E}_D \cdot \vec{dl}}_{=V_1}$$

$$\boxed{V(r) = V_1}$$

Si $r_1 < \overset{r_B}{r} < r_2$:

$$\oint_{SG_B} \vec{E}_B \cdot \vec{dS}_B = \frac{q_{neta_B}}{\varepsilon_0}$$

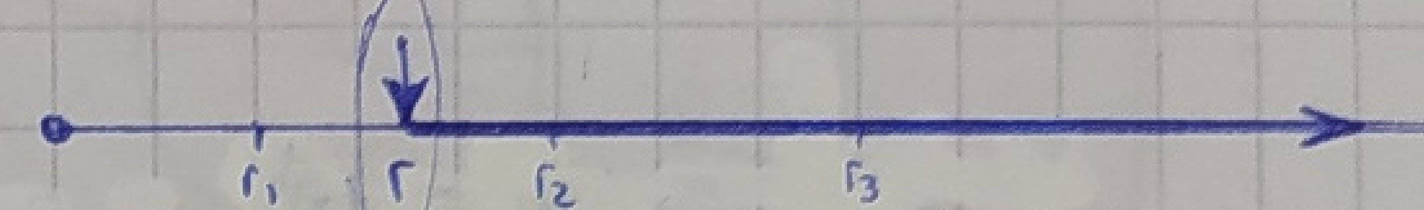
$$\oint_{SG_B} |\vec{E}_B| \cdot |\vec{dS}_B| \cdot \underbrace{\cancel{\cos(\overset{=0}{\vec{E}_B \wedge \vec{dS}_B})}}_{=1} = \frac{q}{\varepsilon_0}$$

$$|\vec{E}_B| \cdot \oint_{SG_B} |\vec{dS}_B| = \frac{q}{\varepsilon_0}$$

$$|\vec{E}_B| \cdot S_B = \frac{q}{\varepsilon_0}$$

$$|\vec{E}_B| \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\boxed{|\vec{E}_B| = \frac{q}{4\pi\varepsilon_0 \cdot r^2}}$$



$$V_{\infty,r} = V(r) - \overset{=0}{\cancel{V_\infty}} = -\int_\infty^r \vec{E} \cdot \vec{dl} = \int_r^\infty \vec{E} \cdot \vec{dl}$$

$$V(r) = \int_r^{r_2} \vec{E}_B \cdot \vec{dl} + \underbrace{\cancel{\int_{r_2}^{r_3} \vec{E}_C \cdot \vec{dl}}}_{=0} + \underbrace{\int_{r_3}^\infty \vec{E}_D \cdot \vec{dl}}_{=V_1}$$

$$V(r) = \frac{q}{4\pi\varepsilon_0} \int_r^{r_2} \frac{1}{r^2} dr + V_1 \qquad \left(-\frac{1}{r}\right)\Big|_r^{r_2} = -\frac{1}{r_2} - \left(-\frac{1}{r}\right) = \frac{1}{r} - \frac{1}{r_2}$$

$$\boxed{V(r) = V_1 + \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_2}\right)}$$

Si $\overset{r_A}{r} < r_1$:

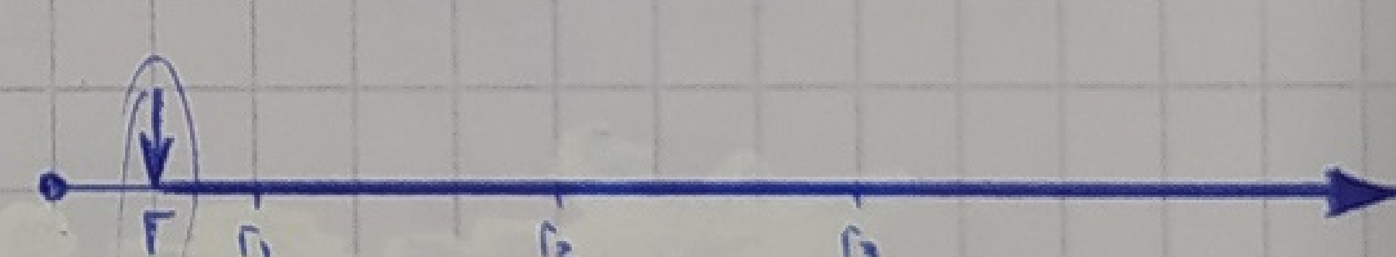
$$\oint_{SG_A} \vec{E}_A \cdot \vec{dS}_A = \frac{q_{neta_A}}{\varepsilon_0}$$

$$\oint_{SG_A} |\vec{E}_A| \cdot |\vec{dS}_A| \cdot \underbrace{\cancel{\cos(\overset{=0}{\vec{E}_A \wedge \vec{dS}_A})}}_{=1} = \frac{0}{\varepsilon_0}$$

$$|\vec{E}_A| \cdot \oint_{SG_A} |\vec{dS}_A| = 0$$

$$|\vec{E}_A| \cdot \underbrace{S_A}_{\neq 0} = 0$$

$$\boxed{|\vec{E}_A| = 0}$$



$$V_{\infty,r} = V(r) - \overset{=0}{\cancel{V_\infty}} = -\int_\infty^r \vec{E} \cdot \vec{dl} = \int_r^\infty \vec{E} \cdot \vec{dl}$$

$$V(r) = \underbrace{\cancel{\int_r^{r_1} \vec{E}_A \cdot \vec{dl}}}_{=0} + \int_{r_1}^{r_2} \vec{E}_B \cdot \vec{dl} + \underbrace{\cancel{\int_{r_2}^{r_3} \vec{E}_C \cdot \vec{dl}}}_{=0} + \underbrace{\int_{r_3}^\infty \vec{E}_D \cdot \vec{dl}}_{=V_1}$$

$$V(r) = \frac{q}{4\pi\varepsilon_0} \int_{r_1}^{r_2} \frac{1}{r^2} dr + V_1 \qquad \left(-\frac{1}{r}\right)\Big|_{r_1}^{r_2} = -\frac{1}{r_2} - \left(-\frac{1}{r_1}\right) = \frac{1}{r_1} - \frac{1}{r_2}$$

$$\boxed{V(r) = V_1 + \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

FÍSICA II

ELECTROSTÁTICA
Campo Eléctrico - Potencial

GUÍA NUEVA
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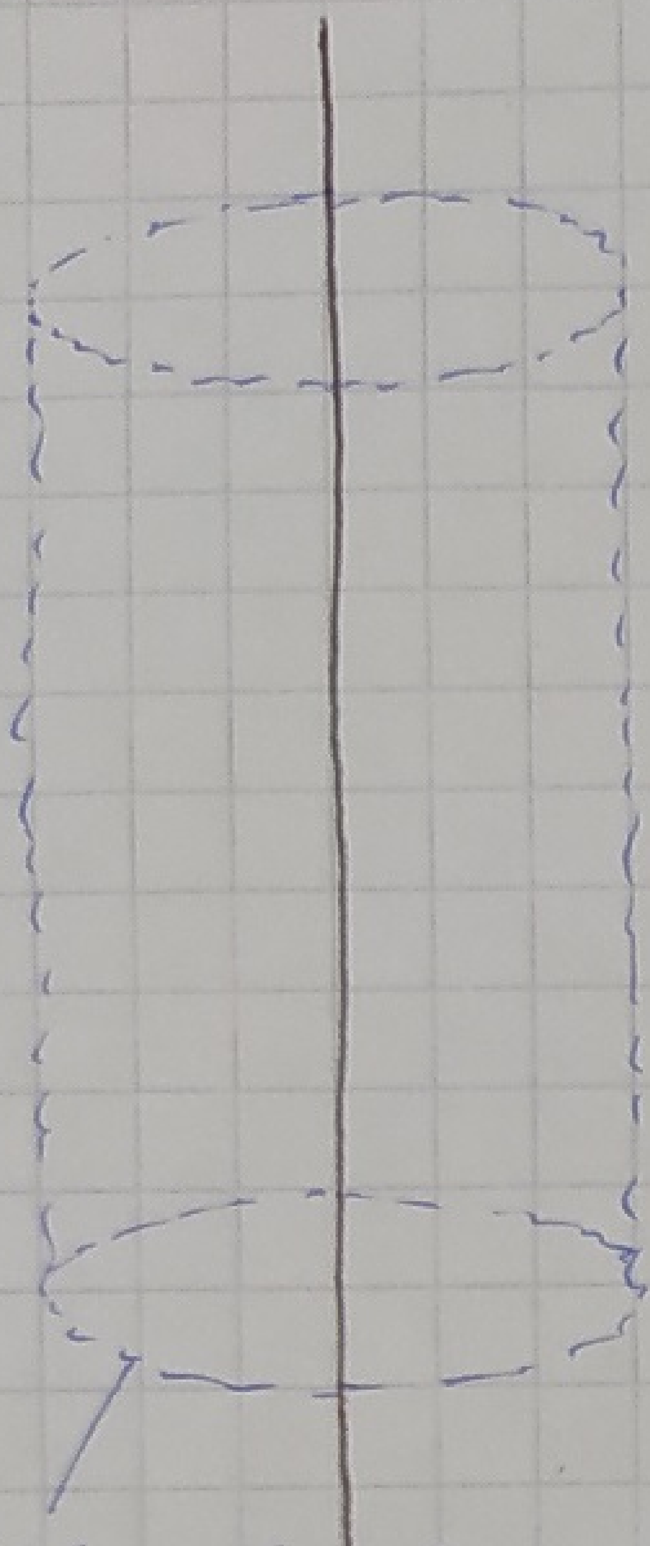
32 Alambre recto infinito con $\lambda > 0$ (longitud L)



SG cilíndrica de radio R

ⓐ
$$\oint_{SG} \vec{E} \cdot \vec{dS} = \frac{Q_{neta}}{\varepsilon_0}$$

$$\oint_{SG} |\vec{E}| \cdot |\vec{dS}| \cdot \underbrace{\cos(\overbrace{\vec{E} \wedge \vec{dS}}^{=0})}_{=1} = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$|\vec{E}| \oint_{SG} |\vec{dS}| = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$|\vec{E}| \cdot S_{\substack{\text{cilindro}\\ \text{sin tapas}}} = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$|\vec{E}| \cdot 2\pi R \cdot \cancel{L} = \frac{\lambda \cdot \cancel{L}}{\varepsilon_0}$$

$$\boxed{|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 \cdot R}}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 \cdot R} \breve{e}_r}$$

ⓑ Para un $r_0 \neq 0$ donde $V(r_0) = 0$:

$$V(r) - \overbrace{V(r_0)}^{=0} = -\int_{r_0}^{r} \vec{E} \cdot \vec{dl}$$

$$V(r) = \int_{r}^{r_0} \vec{E} \cdot \vec{dl}$$

$$V(r) = \int_{r}^{r_0} \frac{\lambda}{2\pi\varepsilon_0 \cdot R} \breve{e}_r \cdot \breve{e}_r \, dR$$

$$V(r) = \frac{\lambda}{2\pi\varepsilon_0} \boxed{\int_{r}^{r_0} \frac{1}{R} dR}$$

$$= \ln(R) \Big|_{r}^{r_0}$$
$$= \ln(r_0) - \ln(r)$$
$$= \ln\left(\frac{r_0}{r}\right)$$

$$\boxed{V(r) = \frac{\lambda}{2\pi\varepsilon_0} \cdot \ln\left(\frac{r_0}{r}\right)}$$

$$V(r) = -\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r}{r_0}\right)$$

ⓒ No, no se puede considerar nulo al potencial en el infinito ya que hay cargas en el infinito.

ⓓ No, para esta distribución de cargas no se puede considerar nulo el potencial sobre el alambre porque sobre el alambre hay cargas.

ⓔ La única simetría que se conserva es la de rotación respecto del eje z.