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Iteration 1

$J(m,b) = \frac{1}{n}\sum(y_i - \hat{y}_i)^2$, $\frac{dJ}{dm} = \frac{-2}{n}\sum(y_i - \hat{y}_i)x_i$

$\frac{dJ}{db} = -\frac{2}{n}\sum(y_i - \hat{y}_i)$, $m_{new} = m_{old} - \alpha\left(\frac{dJ}{dm}\right)$

$b_{new} = b_{old} - \alpha\left(\frac{dJ}{db}\right)$

initial $m = -1$, initial $b = 1$,

$\alpha = 0.1$

data points: $\overset{x_1 \; y_1}{(1, 3)}$ $\overset{x_2 \; y_2}{(3, 6)}$

$\hat{y}$ values

$y = mx + b$, therefore $\hat{y}_1 = -1(1) + 1 = 0$, $\hat{y}_1 = 0$

$\hat{y}_2 = -1(3) + 1 = -2$, $\hat{y}_2 = -2$

dm $\qquad$ db

$$\sum (y_i - \hat{y}_i) x_i = 1(3-0) + 3(6-(-2))$$

$$= 3+24 = 27$$

$$\frac{dJ}{dm} = -\frac{2}{n} * \sum (y_i - \hat{y}_i) x_i$$

$$\frac{dJ}{dm} = -\frac{2}{2} * (27) \quad , \quad \frac{dJ}{dm} = -27$$

$$\sum (y_i - \hat{y}_i) = (3-0) + (6-(-2)) = 11$$

$$\frac{dJ}{db} = -\frac{2}{2} \times (11) \quad , \quad \frac{dJ}{db} = -11$$

$$m_{new} = m - \alpha\left(\frac{dJ}{dm}\right) = -1 - 0.1(-27) = -1 + 2.7 = 1.7$$

$$b_{new} = b - \alpha\left(\frac{dJ}{db}\right) = 1 - 0.1(-11) = 1 + 1.1 = 2.1$$

$\therefore \quad m_{new} = 1.7 \,, \quad b_{new} = 2.1$ - Iteration 1

Iteration 2

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$y = mx + b$

from Iteration 1, $m = 1.7$ and $b = 2.1$

$x_1 = 1$

$x_2 = 3$

$\bar{y}_1 = 1.7(1) + 2.1 = 3.8$

$\bar{y}_2 = 1.7(3) + 2.1 = 7.2$

$$\frac{dJ}{dm} = \frac{-2}{n} \sum (y_i - \bar{y}_i) x_i$$

$y_1 = 3, \; y_2 = 6$

$$\sum (y_i - \bar{y}_i) x_i = \cancel{(3-3.8)1 + (6-7.2)}$$

$$= (y_1 - \bar{y}_1)x_1 + (y_2 - \bar{y}_2)x_2$$

$$= (3-3.8)1 + (6-7.2)3$$

$$= -0.8 + (-3.6)$$

$$= -0.8 - 3.6$$

$$= -4.4$$

$$\frac{dJ}{dm} = \frac{-2}{n} \sum (y_i - \bar{y}_i) x_i$$

$$= \frac{-\cancel{2}^{1}}{\cancel{2}} (-4.4) = -1 \times -4.4$$

$$= 4.4$$

$$\frac{dJ}{db} = \frac{-2}{n} \Sigma (y_i - \bar{y}_i)$$

$$\Sigma (y_i - \bar{y}_i) = (y_1 - \bar{y}_1) + (y_2 - \bar{y}_2)$$
$$= (3 - 3.8) + (6 - 7.2)$$
$$= -0.8 + (-1.2)$$
$$= -0.8 - 1.2$$
$$= -2$$

$$\frac{dJ}{db} = \frac{-2}{n} \Sigma (y_1 - \bar{y}_1)$$
$$= \frac{-\cancel{2}^{1}}{\cancel{2}_{1}} (-2)$$
$$= -1 \times -2$$
$$= 2$$

Remember;

$$m_{new} = m_{old} - \alpha \left(\frac{dJ}{dm} \right),$$

$$b_{new} = b_{old} - \alpha \left(\frac{dJ}{db} \right)$$

and $\alpha = 0.1$

$\therefore$

$$m_{new} = 1.7 - 0.1(4.4)$$
$$= 1.7 - 0.44$$
$$= 1.26$$

$$b_{-new} = 2.1 - 0.1(2)$$
$$= 2.1 - 0.2$$
$$= 1.9$$

Date:

Iteration 3

$y = mx + b$, $x_1 = 1$ $x_2 = 3$

$\hat{y}_1 = 1.26(1) + 1.9 = \underline{3.16}$

$\hat{y}_2 = 1.26(3) + 1.9 = 3.78 + 1.9 = \underline{\underline{5.68}}$

$\frac{dJ}{dm} = \frac{-2}{n} \sum (y_1 - \hat{y}_1) x_i$, $y_1 = 3$, $y_2 = 6$

$\sum (y_1 - \hat{y}_1) x_i = 1(3 - 3.16) + 3(6 - 5.68)$

$= -0.16 + 0.96 = \underline{\underline{0.8}}$

$\frac{dJ}{dm} = \frac{-2}{2}(0.8) = -0.8$

$\frac{dJ}{db} = \frac{-2}{n} \sum (y_i - \hat{y}_i)$ $(y_i \; \hat{y}_i)$

$\sum (y_i - \hat{y}_i) = (3 - 3.16) + (6 - 5.68)$

$= -0.16 + 0.32 = 0.16$

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Date:

$$\frac{dJ}{db} = \frac{-2}{2}(0.16) = -0.16$$

$$M_{new} = m_{old} - \alpha\left(\frac{dJ}{dm}\right) = 1.26 - 0.1(-0.8)$$
$$= 1.26 + 0.8$$
$$= \underline{\underline{2.06}}$$

$$b_{new} = b_{old} - \alpha\left(\frac{dJ}{db}\right) = 1.9 - 0.1(-0.16)$$
$$= 1.9$$

$$M_{new} = m_{old} - \alpha\left(\frac{dJ}{db}\right) = 1.26 - 0.1(-0.8)$$
$$= 1.26 + 0.08$$
$$= \underline{\underline{1.34}}$$

$$b_{new} = b_{old} - \alpha\left(\frac{dJ}{db}\right) = 1.9 - 0.1(-0.16)$$
$$= 1.9 + 0.016$$
$$= \underline{\underline{1.916}}$$

Date:

error 1 = $y_1 - \hat{y}_1 = 3 - 3.16 = -0.16$

error 2 = $y_2 - \hat{y}_2 = 6 - 5.68 = 0.32$

$M_{new} = 1.34$

$b_{new} = 1.916$

calculation of errors - initial

$error_i = y_i - \hat{y}_i$, $y_1 = 3$, $y_2 = 6$ initial $m = -1$, initial $b = 1$

$x_1 = 1$ $x_2 = 3$

$y = mx + b$ therefore $\hat{y}_1 = -1(1) + 1 = -1 + 1 = 0$

$\hat{y}_2 = -1(3) + 1 = -3 + 1 = -2$

$\therefore$ $error_1 = y_1 - \hat{y}_1 = 3 - 0 = 3$

$error_2 = y_2 - \hat{y}_2 = 6 - (-2) = 6 + 2 = 8$

Iteration 1

$m = 1.7$ $b = 2.1$, $y = mx + b$, $\hat{y}_1 = 1.7(1) + 2.1 = 3.8$

$\hat{y}_2 = 1.7(3) + 2.1 = 5.1 + 2.1 = 7.2$

$error = y_i - \hat{y}_i$

$\cancel{e_1} =$ $error_1 = 3 - 3.8 = -0.8$

$error_2 = 6 - 7.2 = -1.2$

Iteration 2

$m = 1.26$ $b = 1.9$, $y = mx + b$, $\hat{y}_1 = 1.26(1) + 1.9 = 3.16$

$\hat{y}_2 - 1.26(3) + 1.9 = 3.78 + 1.9 = 5.68$

$error_i = y_i - \hat{y}_i$, $error_1 = 3 - 3.16 = -0.16$

$error_2 = 6 - 5.68 = 0.32$

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Iteration 3

$m = 1.34$ $b = 1.916$, $y = mx + b$, $\hat{y}_1 = 1.34(1) + 1.916 = 3.256$, $\hat{y}_2 = 1.34(3) + 1.916 = 4.02 + 1.916 = 5.936$

$error_i = y_i - \hat{y}_i$

$error_1 = 3 - 3.256 = -0.256$

$error_2 = 6 - 5.936 = 0.064$

	m	b	error 1	error 2
initial	-1	1	3	8
iteration 1	1.7	2.1	-0.8	-1.2
iteration 2	1.26	1.9	-0.16	0.32
iteration 3	1.34	1.916	-0.256	0.064

Trend

The initial errors are large, but as gradient descent progresses, m and ~~b~~ b are updated to reduce the error. ~~towards the~~

As the model gets closer to the best fit line, the error decreases more slowly and updates to m and b become smaller.

This trend shows that gradient descent is working, the error drops after each iteration.