Homework 4

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On my honor as a student, I have neither given nor recieved unauthorized aid on this assignment.

Pre-processing, import libraries and data

In [ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import pymc3 as pm
import seaborn as sns
import theano
import arviz as az
from pymc3 import forestplot, traceplot, plot_posterior
from scipy.stats import multivariate_normal
from itertools import combinations

np.random.seed(6040)

coffee = pd.read_csv('sales-ds6040sum2021.csv')
wine_testing = pd.read_csv('whitewine-testing-ds6040.csv')
wine_training = pd.read_csv('whitewine-training-ds6040-1.csv')
```

Part 1 - Bayesian Hierarchical Modelling

Questions: How does conscientiousness and neuroticism impact the sales of coffee and food, and are coffee and food impacted differently?

Once you control for the personality characteristics of the store managers, what stores should be performing well? (i.e. the rest of the employees might be great, but the store manager might be bringing sales down)

Approach Step 1: Data Exploration

In [ ]:
```
coffee.head()
```

Out[ ]:

	sales	food	con	neur	store
0	1.363821	0	0.216553	1.290224	0
1	-1.119747	1	0.216553	1.290224	0
2	-0.180141	0	0.216553	1.290224	0
3	-2.282334	1	0.216553	1.290224	0
4	0.673304	0	0.216553	1.290224	0

Before I do anything, I use the .head() function to get a preview of the data, and what we can expect. This doesn't give much information right away, but it gives me an overall idea of what the data looks like.

We know our quantitiative variables are:

- sales
- con
- neur

Our categorical variables are:

- food
- store

In [ ]:

```
coffee.groupby(['store', 'food']).mean('sales')
```

Out[ ]:

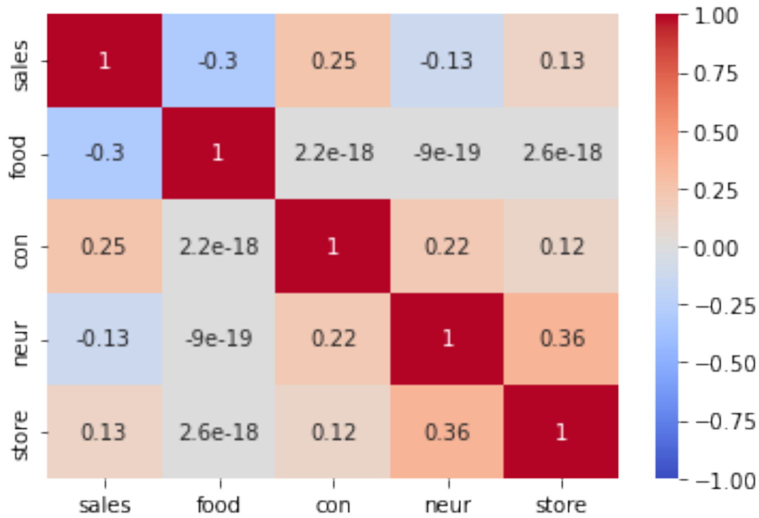
		sales	con	neur
store	**food**			
0	**0**	-0.157253	0.216553	1.290224
	1	-1.442931	0.216553	1.290224
1	**0**	0.250400	-0.135665	-2.276493
	1	-1.178097	-0.135665	-2.276493
2	**0**	0.238989	0.738900	0.093054
	1	-1.284719	0.738900	0.093054
3	**0**	1.124880	-0.579423	-0.457344
	1	-0.386966	-0.579423	-0.457344
4	**0**	1.345538	-0.976866	-0.865172
	1	-0.587611	-0.976866	-0.865172

5	**0**	0.907453	-0.219433	-1.315683
	1	-1.173526	-0.219433	-1.315683
6	**0**	0.469321	-0.130894	-0.874896
	1	-0.214635	-0.130894	-0.874896
7	**0**	0.531118	-2.018870	-1.069713
	1	-2.611578	-2.018870	-1.069713
8	**0**	-0.398823	-0.408856	0.374658
	1	-0.387042	-0.408856	0.374658
9	**0**	-1.385427	-0.568532	0.966242
	1	-3.076568	-0.568532	0.966242
10	**0**	-0.389983	0.410372	0.330495
	1	-0.336451	0.410372	0.330495
11	**0**	-0.805283	-0.506002	-0.114954
	1	-1.693260	-0.506002	-0.114954
12	**0**	1.789909	-0.717169	-0.036987
	1	-1.629684	-0.717169	-0.036987
13	**0**	1.042030	-0.915437	0.420692
	1	0.400225	-0.915437	0.420692
14	**0**	4.055752	0.587351	-0.646036
	1	4.465717	0.587351	-0.646036
15	**0**	1.087828	0.139971	-0.173394
	1	1.409617	0.139971	-0.173394
16	**0**	-0.909599	-0.501073	-0.953021
	1	-2.129063	-0.501073	-0.953021
17	**0**	1.337243	0.325928	0.796620
	1	1.880833	0.325928	0.796620
18	**0**	-0.691490	-0.201251	2.382800
	1	-1.821823	-0.201251	2.382800
19	**0**	-0.248919	0.306285	-0.070093
	1	-1.223730	0.306285	-0.070093

I also want to see the mean sales values for each store, broken down by food offering. The con and neur variables are consistent across each store.

In [ ]:
```
sns.heatmap(coffee.corr(), annot=True, vmin=-1, vmax=1, cmap="coolwarm")
```

Out[ ]:
```
<AxesSubplot:>
```



One of the preliminary visuals I want to generate is a correlation matrix between the variables. One of the key takeaways before we move forward with modeling, is that it appears there is a positive correlation between sales and conscientiousness and a negative correlation between sales and neuroticism. This will be something we continue to explore as we move forward.

Approach Step 2: Data Preparation for Modeling

Before we do any hierachical modeling, we must first prepare the variables properly. Below, I declare each variable as a seperate object to be used in the hierachical model.

In [ ]:
```
stores = len(coffee.store.unique())
store = coffee.store.values
con = coffee.con.values
food = coffee.food.values
neur = coffee.neur.values
sales = coffee.sales.values
```

The first model we run is going to be pretty much identical to what is provided in the companion, the only changes I am making are in the variable names and variable being referenced in the shape of the offset.

Approach Step 3: Run first model

Prior Selection: In all models, I am using a prior of zero. I am chosing this becauseI don't have any information about the patterns in the data at this point. The other reason I am chosing a prior of zero is that it appears that the sales are on either side of 0, so it seems like a good place to estimate.

In [ ]:
```
with pm.Model() as hierarchical_model:

    # Priors for the fixed effects
    # a - overall intercept, level of sales at mean levels of con and neur
    mu_a = pm.Normal('mu_a', mu=0., sigma=2)
    sigma_a = pm.HalfCauchy('sigma_a', beta=1)

    mu_b = pm.Normal('mu_b', mu=0., sigma=1)
    sigma_b = pm.HalfCauchy('sigma_b', beta=1)


    mu_c = pm.Normal('mu_c', mu=0., sigma=1)
    sigma_c = pm.HalfCauchy('sigma_c', beta=0.5)


    mu_d = pm.Normal('mu_d', mu=0., sigma=1)
    sigma_d = pm.HalfCauchy('sigma_d', beta=0.5)


    mu_e = pm.Normal('mu_e', mu=0., sigma=1)
    sigma_e = pm.HalfCauchy('sigma_e', beta=0.5)


    mu_f = pm.Normal('mu_f', mu=0., sigma=1)
    sigma_f = pm.HalfCauchy('sigma_f', beta=0.5)

    a_offset = pm.Normal('a_offset',  mu=0, sd=2, shape=stores)
    a=pm.Deterministic("a", mu_a+a_offset*sigma_a)
    b_offset = pm.Normal('b_offset',  mu=0, sd=2, shape=stores)
    b=pm.Deterministic("b", mu_b+b_offset*sigma_b)

    c= pm.Normal('c',mu=mu_c,sigma=sigma_c)
    d= pm.Normal('d',mu=mu_d,sigma=sigma_d)
    e= pm.Normal('e',mu=mu_e,sigma=sigma_e)
    f= pm.Normal('f',mu=mu_f,sigma=sigma_f)

    sigma_y = pm.HalfCauchy('sigma_y', beta=1)
    y_hat = a[store] + b[store]*coffee.food + c*coffee.con +d*coffee.con*cof
    y_like = pm.Normal('y_like', mu=y_hat, sd = sigma_y, observed = sales)
```

In [ ]:
```
with hierarchical_model:
    hierarchical_trace = pm.sample(1000, n_init=50000, tune=1000, step=pm.NU
```

```
/var/folders/0r/hry4zr3s1m57hjbkgslzxvnr0000gp/T/ipykernel_51520/315265484.p
y:2: FutureWarning: In v4.0, pm.sample will return an `arviz.InferenceData`
object instead of a `MultiTrace` by default. You can pass return_inferenceda
ta=True or return_inferencedata=False to be safe and silence this warning.
  hierarchical_trace = pm.sample(1000, n_init=50000, tune=1000, step=pm.NUTS
(target_accept = 0.99))
Multiprocess sampling (4 chains in 4 jobs)
NUTS: [sigma_y, f, e, d, c, b_offset, a_offset, sigma_f, mu_f, sigma_e, mu_e
, sigma_d, mu_d, sigma_c, mu_c, sigma_b, mu_b, sigma_a, mu_a]
```

100.00% [8000/8000 02:49<00:00

Sampling 4 chains, 150 divergences]

```
/Users/lbassett/opt/anaconda3/lib/python3.9/site-packages/scipy/stats/_conti
nuous_distns.py:624: RuntimeWarning: overflow encountered in _beta_ppf
  return _boost._beta_ppf(q, a, b)
/Users/lbassett/opt/anaconda3/lib/python3.9/site-packages/scipy/stats/_conti
nuous_distns.py:624: RuntimeWarning: overflow encountered in _beta_ppf
  return _boost._beta_ppf(q, a, b)
/Users/lbassett/opt/anaconda3/lib/python3.9/site-packages/scipy/stats/_conti
nuous_distns.py:624: RuntimeWarning: overflow encountered in _beta_ppf
  return _boost._beta_ppf(q, a, b)
/Users/lbassett/opt/anaconda3/lib/python3.9/site-packages/scipy/stats/_conti
nuous_distns.py:624: RuntimeWarning: overflow encountered in _beta_ppf
  return _boost._beta_ppf(q, a, b)
Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000 dr
aws total) took 206 seconds.
There were 53 divergences after tuning. Increase `target_accept` or reparame
terize.
There were 37 divergences after tuning. Increase `target_accept` or reparame
terize.
There were 22 divergences after tuning. Increase `target_accept` or reparame
terize.
There were 38 divergences after tuning. Increase `target_accept` or reparame
terize.
```

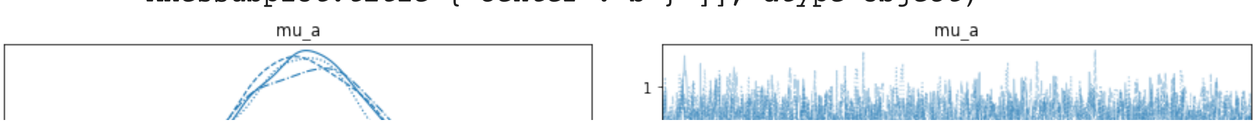
Findings

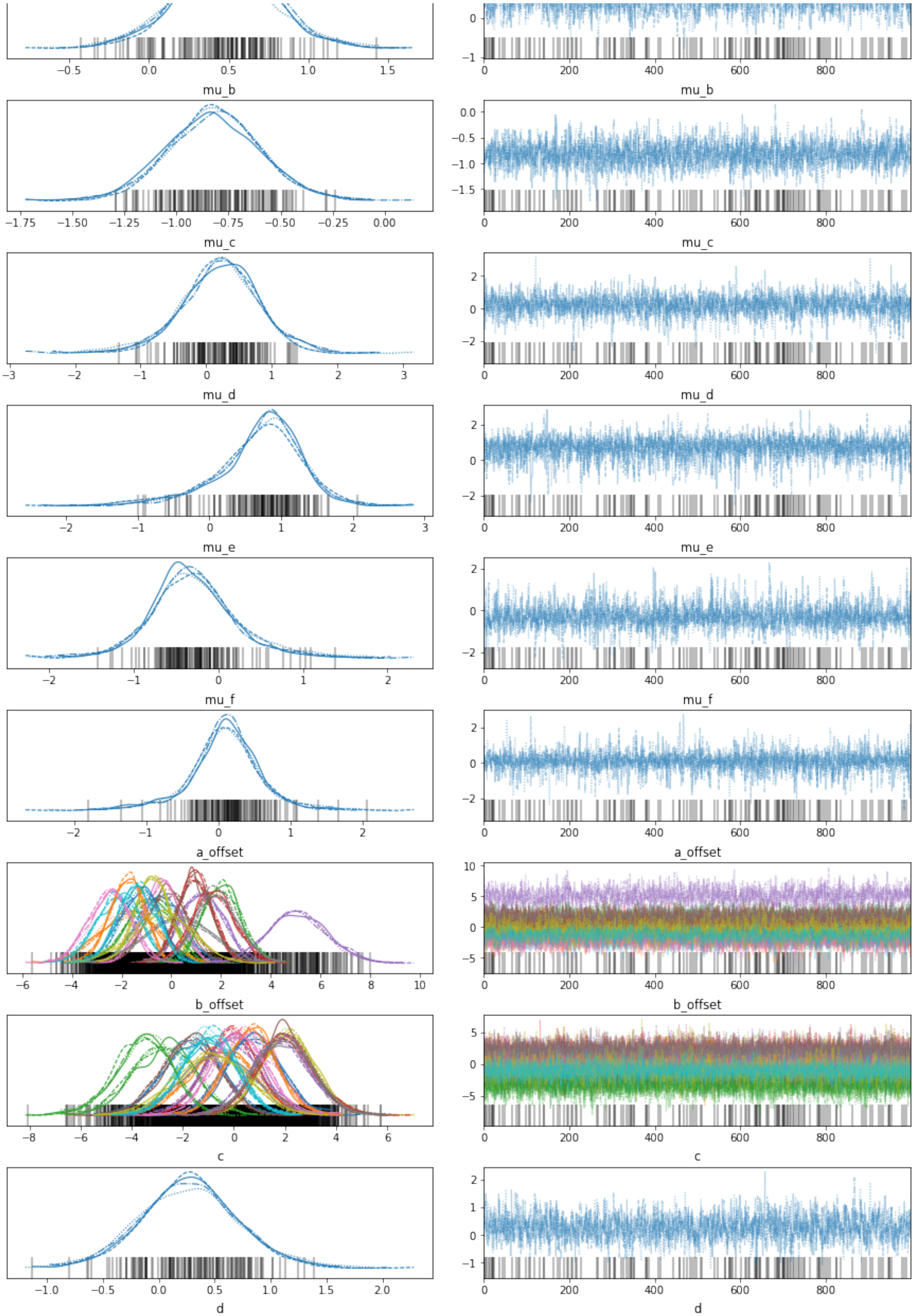
In [ ]:
```
az.plot_trace(hierarchical_trace)
```

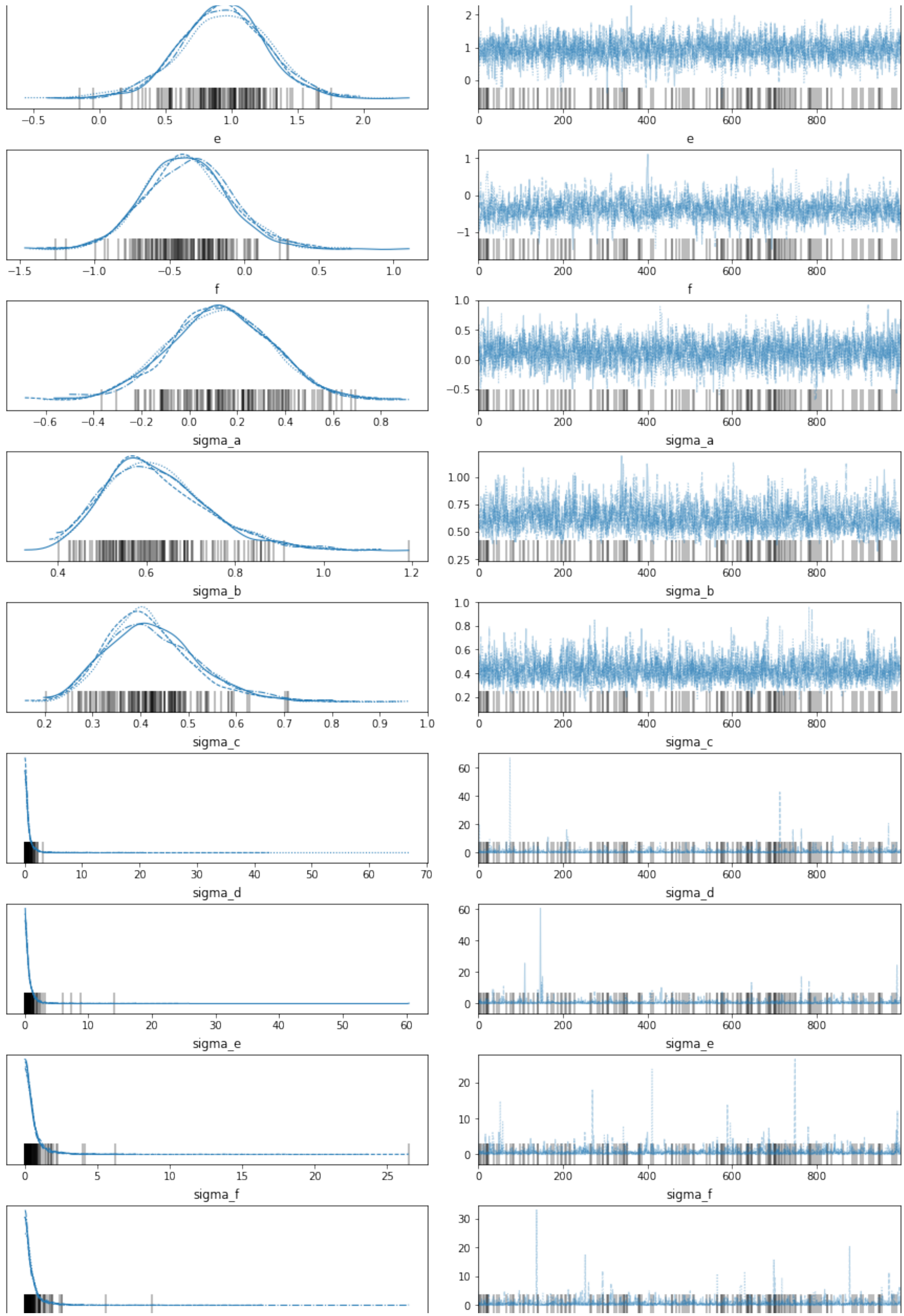
```
/Users/lbassett/opt/anaconda3/lib/python3.9/site-packages/arviz/data/io_pymc
3.py:96: FutureWarning: Using `from_pymc3` without the model will be depreca
ted in a future release. Not using the model will return less accurate and l
ess useful results. Make sure you use the model argument or call from_pymc3
within a model context.
  warnings.warn(
/Users/lbassett/opt/anaconda3/lib/python3.9/site-packages/arviz/plots/tracep
lot.py:197: UserWarning: rcParams['plot.max_subplots'] (20) is smaller than
the number of variables to plot (21), generating only 20 plots
  warnings.warn(
```

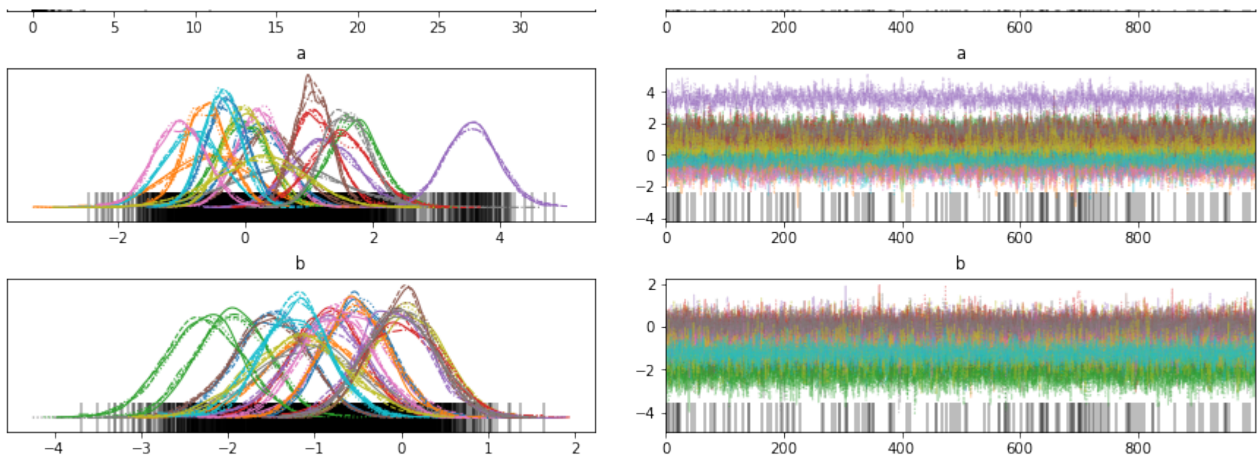
Out[ ]:

```
array([[<AxesSubplot:title={'center':'mu_a'}>,
        <AxesSubplot:title={'center':'mu_a'}>],
       [<AxesSubplot:title={'center':'mu_b'}>,
        <AxesSubplot:title={'center':'mu_b'}>],
       [<AxesSubplot:title={'center':'mu_c'}>,
        <AxesSubplot:title={'center':'mu_c'}>],
       [<AxesSubplot:title={'center':'mu_d'}>,
        <AxesSubplot:title={'center':'mu_d'}>],
       [<AxesSubplot:title={'center':'mu_e'}>,
        <AxesSubplot:title={'center':'mu_e'}>],
       [<AxesSubplot:title={'center':'mu_f'}>,
        <AxesSubplot:title={'center':'mu_f'}>],
       [<AxesSubplot:title={'center':'a_offset'}>,
        <AxesSubplot:title={'center':'a_offset'}>],
       [<AxesSubplot:title={'center':'b_offset'}>,
        <AxesSubplot:title={'center':'b_offset'}>],
       [<AxesSubplot:title={'center':'c'}>,
        <AxesSubplot:title={'center':'c'}>],
       [<AxesSubplot:title={'center':'d'}>,
        <AxesSubplot:title={'center':'d'}>],
       [<AxesSubplot:title={'center':'e'}>,
        <AxesSubplot:title={'center':'e'}>],
       [<AxesSubplot:title={'center':'f'}>,
        <AxesSubplot:title={'center':'f'}>],
       [<AxesSubplot:title={'center':'sigma_a'}>,
        <AxesSubplot:title={'center':'sigma_a'}>],
       [<AxesSubplot:title={'center':'sigma_b'}>,
        <AxesSubplot:title={'center':'sigma_b'}>],
       [<AxesSubplot:title={'center':'sigma_c'}>,
        <AxesSubplot:title={'center':'sigma_c'}>],
       [<AxesSubplot:title={'center':'sigma_d'}>,
        <AxesSubplot:title={'center':'sigma_d'}>],
       [<AxesSubplot:title={'center':'sigma_e'}>,
        <AxesSubplot:title={'center':'sigma_e'}>],
       [<AxesSubplot:title={'center':'sigma_f'}>,
        <AxesSubplot:title={'center':'sigma_f'}>],
       [<AxesSubplot:title={'center':'a'}>,
        <AxesSubplot:title={'center':'a'}>],
       [<AxesSubplot:title={'center':'b'}>,
        <AxesSubplot:title={'center':'b'}>]], dtype=object)
```

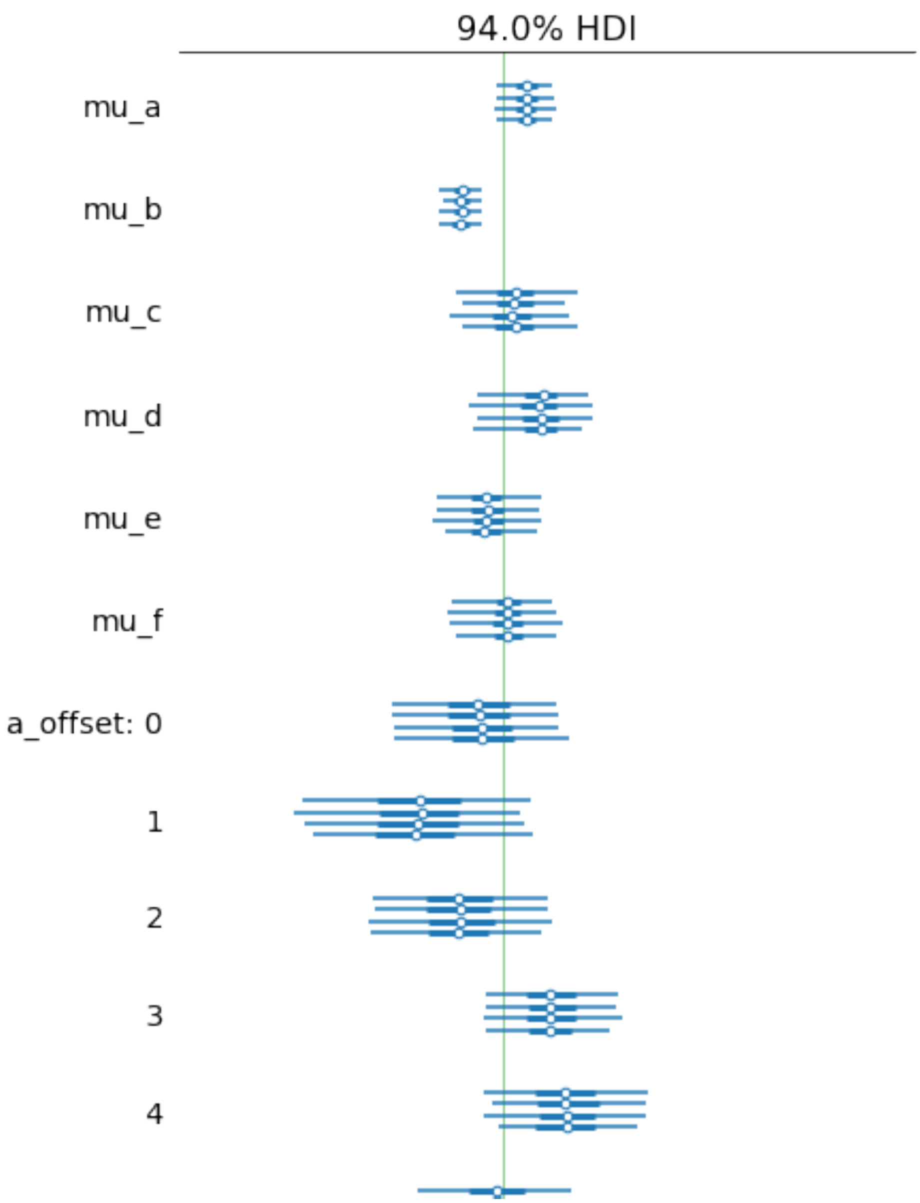


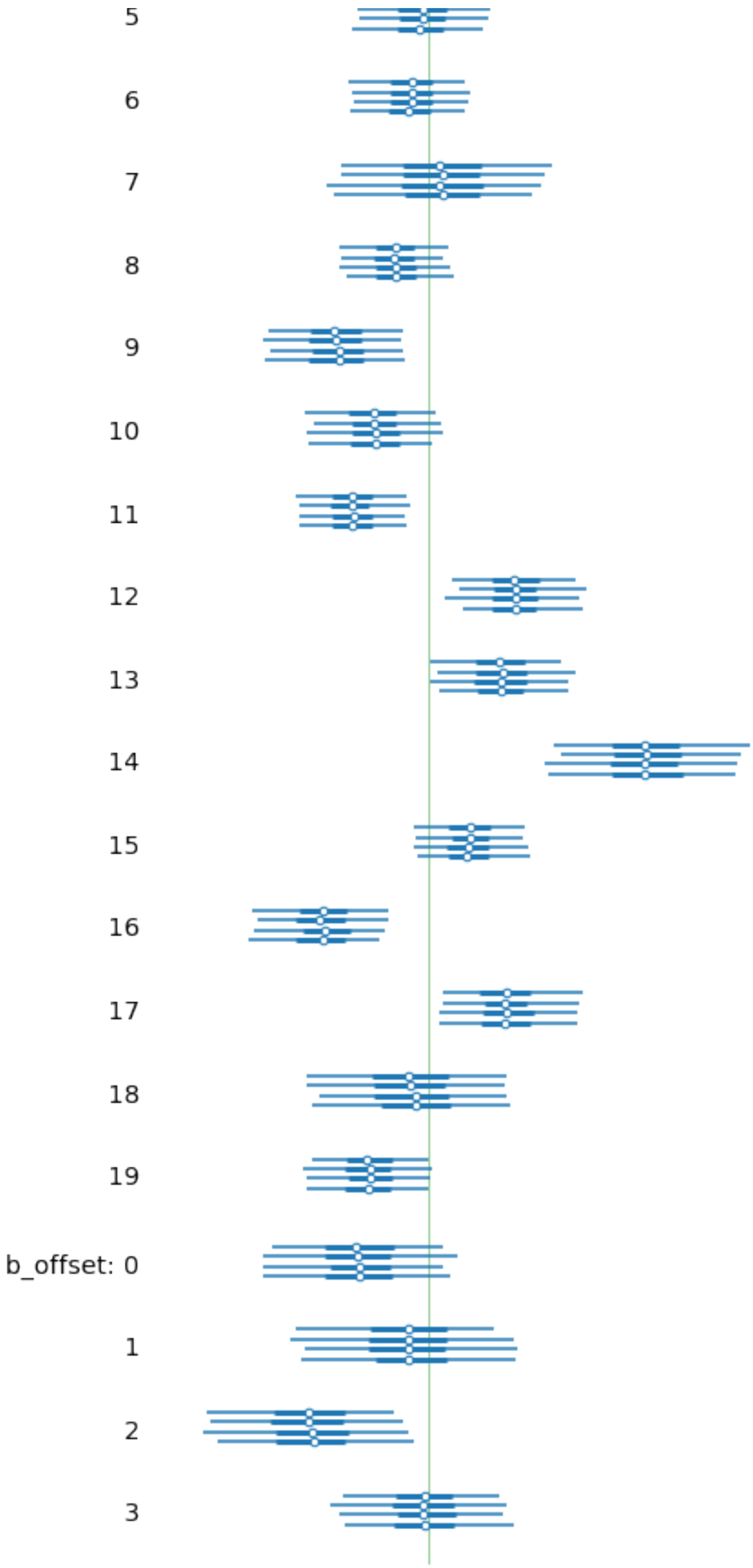

mu_b
mu_b
mu_c
mu_c
mu_d
mu_d
mu_e
mu_e
mu_f
mu_f
a_offset
a_offset
b_offset
b_offset
c
c
d
d


e
e
f
f
sigma_a
sigma_a
sigma_b
sigma_b
sigma_c
sigma_c
sigma_d
sigma_d
sigma_e
sigma_e
sigma_f
sigma_f



In [ ]:
```
#Forest Plots can be quite useful in visualizing the posterior distributions
with hierarchical_model:
    az.plot_forest(hierarchical_trace, rope=(0, 0))
```




5
6
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8
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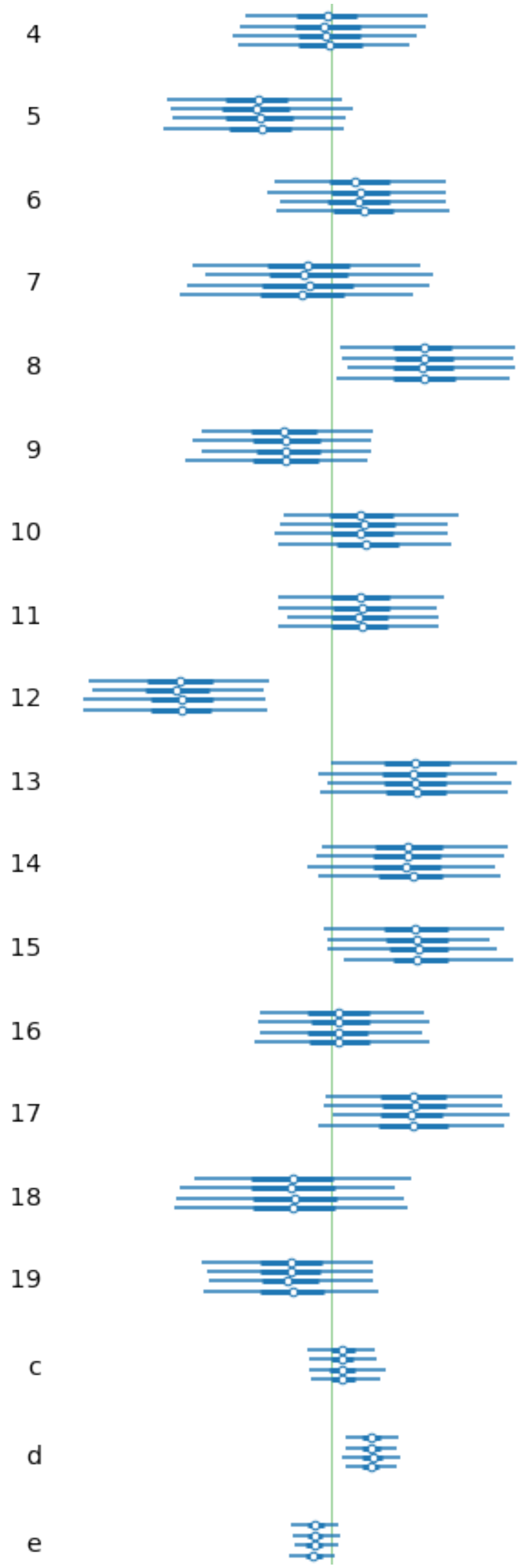
10
11
12
13
14
15
16
17
18
19
b_offset: 0
1
2
3


4
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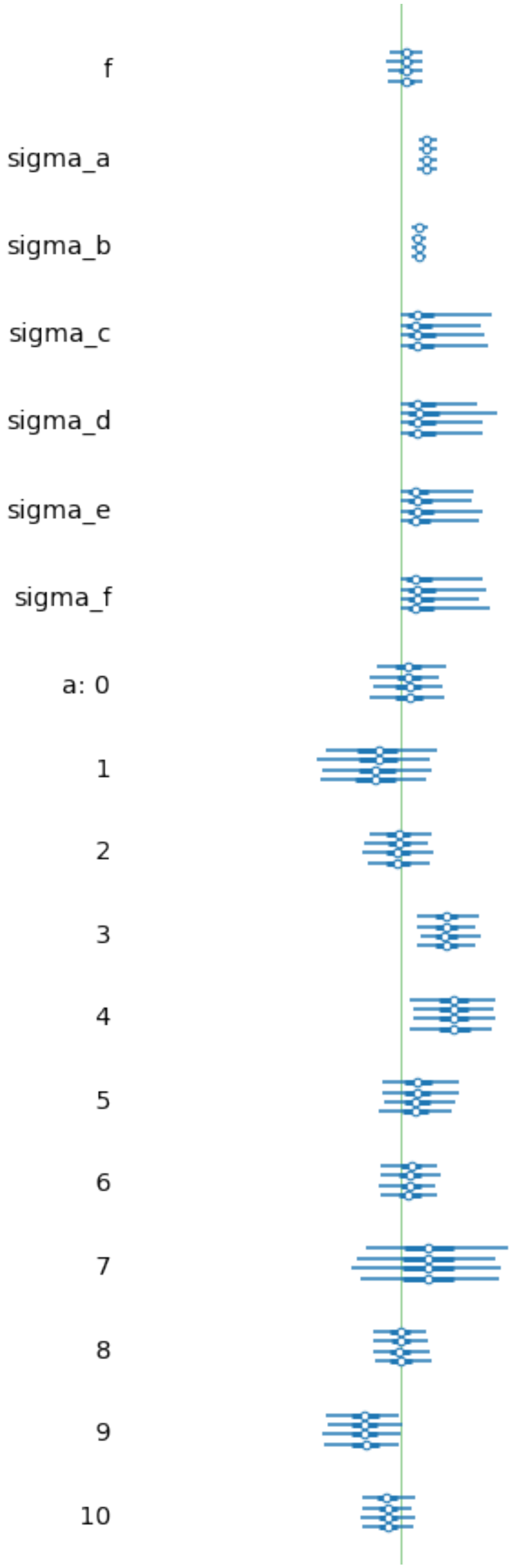
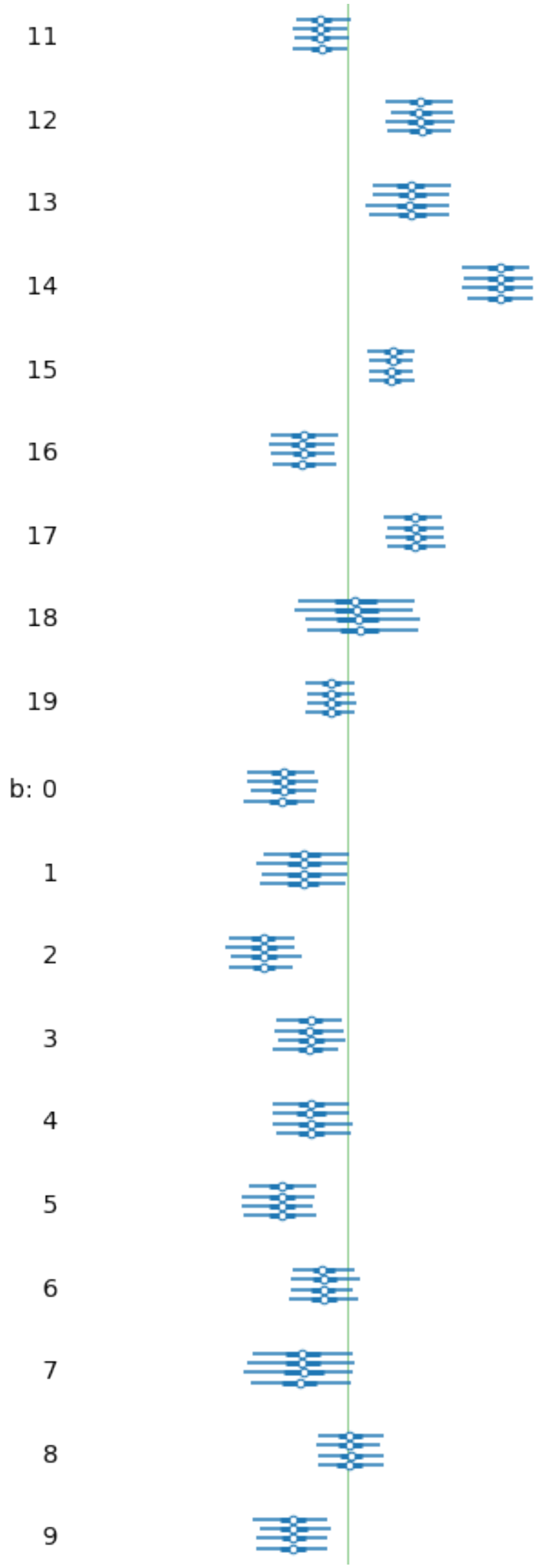
7
8
9
10
11
12
13
14
15
16
17
18
19
c
d
e


f
sigma_a
sigma_b
sigma_c
sigma_d
sigma_e
sigma_f
a: 0
1
2
3
4
5
6
7
8
9
10


11
12
13
14
15
16
17
18
19
b: 0
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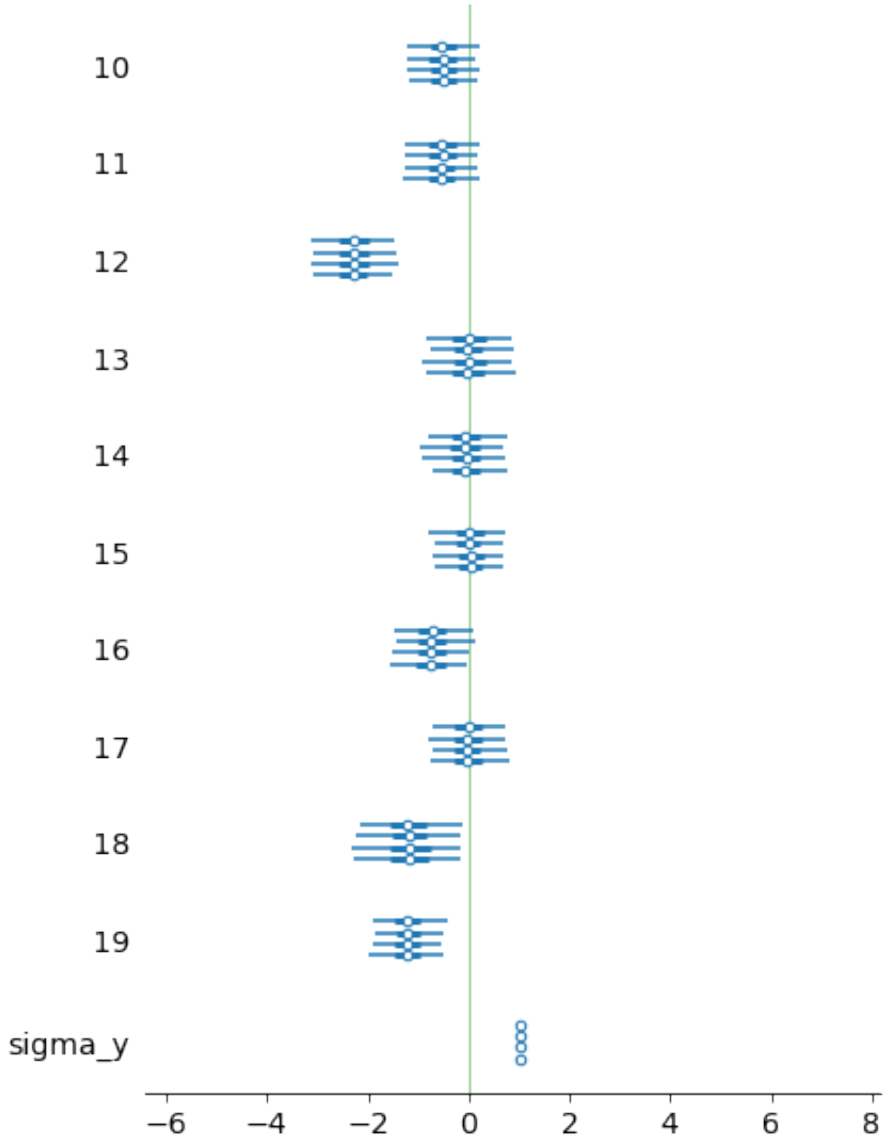


Diagnostic

In [ ]:
```
with hierarchical_model:
    diag = pm.summary(hierarchical_trace)
diag
```

Out[ ]:

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
mu_a	0.476	0.305	-0.164	1.009	0.008	0.006	1299.0	1644.0	1.0
mu_b	-0.817	0.225	-1.253	-0.408	0.005	0.004	1910.0	2029.0	1.0
mu_c	0.206	0.606	-0.899	1.396	0.013	0.010	2110.0	1951.0	1.0
mu_d	0.699	0.601	-0.515	1.780	0.012	0.009	2602.0	2017.0	1.0
mu_e	-0.291	0.531	-1.247	0.810	0.011	0.009	2193.0	2251.0	1.0
...	...	...	...	...	...	...	...	...	...
b[16]	-0.741	0.421	-1.478	0.100	0.007	0.005	4015.0	3184.0	1.0
b[17]	-0.014	0.412	-0.785	0.742	0.006	0.006	4695.0	3502.0	1.0
b[18]	-1.177	0.561	-2.197	-0.081	0.011	0.008	2597.0	2766.0	1.0
b[19]	-1.206	0.378	-1.888	-0.473	0.005	0.004	5361.0	3002.0	1.0
sigma_y	1.014	0.034	0.951	1.078	0.001	0.000	4126.0	2876.0	1.0

97 rows × 9 columns

Summary:

How does conscientiousness and neuroticism impact the sales of coffee and food, and are coffee and food impacted differently?

The behavior of the store manager does impact the sales of coffee and food, and they are impacted differently. C+D refer to concientiousness, and E+F Refer to neuroticism. The forest plot shows a similar story to what we found in the correlation matrix, a conscientious employer has a positive affect on sales, where a neurotic employer has a negative affect on sales.

Once you control for the personality characteristics of the store managers, what stores should be performing well? (i.e. the rest of the employees might be great, but the store manager might be bringing sales down)

Once you control for the behavior of the manager, you are left with the intercept and food sales.

Part 2: Bayesian Model Averaging with Logistic Regression (40 points)

1. First, revisit your HW2 and calculate the misclassification rate and the cross tabs for 3 variable models that used flat priors that performed best on the testing data. You will have 1 model for LDA and 1 model for QDA.

In [ ]:
```
class LDA():
    def __init__(self, dataset, class_var, priors = None):
        n_class = len(dataset[class_var].unique())
        if priors is None:
            priors = np.repeat(1/n_class, n_class)
        self.priors =  np.asarray(priors)
        self.means = dataset.groupby(class_var).mean()
        self.sigma = dataset.cov()
        self.class_var = class_var
        self.training_data = dataset
    def predict_probs(self, data = None):
        if data is None:
            data = self.training_data
        data_temp = data.drop(self.class_var, axis = 1)
        dens_list = []
        col_names = []
        for ind, row in self.means.iterrows():
            col_names.append(ind)
            dens_list.append(multivariate_normal.pdf(data_temp, mean = np.as
        dens_list = pd.DataFrame(np.transpose(np.vstack(dens_list)),columns=
        dens_list = dens_list.mul(self.priors, axis=1)
        dens_list = dens_list.div(dens_list.sum(axis=1), axis=0)
        dens_list['True Class'] = data[self.class_var]
        return dens_list
    def predict_MAP(self, data = None):
        if data is None:
            data = self.training_data
        dens_list = self.predict_probs(data).drop('True Class', axis = 1)
        map_list = dens_list.idxmax(axis = 1)
        maps = {'MAP Class': map_list}
        maps = pd.DataFrame(maps)
        maps['True Class'] = data[self.class_var]
        return maps
    def misclass_rate(self, data = None):
        if data is None:
            data = self.training_data
        maps = self.predict_MAP(data = data)


        maps['Mis_class'] = maps['MAP Class']  == maps['True Class']

        mis_class =  1 - maps['Mis_class'].mean()
```

```
        return mis_class
    def misclass_xtabs(self, data = None):
        if data is None:
            data = self.training_data
        maps = self.predict_MAP(data = data)

        xtabs = pd.crosstab(maps['MAP Class'], maps['True Class'])
        return xtabs
    def misclass_pairplot(self, data = None):
        if data is None:
            data = self.training_data
        maps = self.predict_MAP(data = data)
        temp_dat = data.copy(deep = True)
        temp_dat['Mis-Classified'] = maps['MAP Class']  != maps['True Class'
        plot = sns.pairplot(temp_dat,hue="Mis-Classified", height = 1.5, asp
        return plot
```

In [ ]:

```
class QDA(LDA):
    def __init__(self, dataset, class_var, priors = None):
        n_class = len(dataset[class_var].unique())
        if priors is None:
            priors = np.repeat(1/n_class, n_class)
        self.priors = np.asarray(priors)
        self.means = dataset.groupby(class_var).mean()
        gb = dataset.groupby(class_var)
        self.sigma = {x: gb.get_group(x).cov() for x in gb.groups}
        self.class_var = class_var
        self.training_data = dataset
    def predict_probs(self, data = None):
        if data is None:
            data = self.training_data
        data_temp = data.drop(self.class_var, axis = 1)
        dens_list = []
        col_names = []
        for ind, row in self.means.iterrows():
            col_names.append(ind)
            dens_list.append(multivariate_normal.pdf(data_temp, mean = np.as
        dens_list = pd.DataFrame(np.transpose(np.vstack(dens_list)),columns=
        dens_list = dens_list.mul(self.priors, axis=1)
        dens_list = dens_list.div(dens_list.sum(axis=1), axis=0)
        dens_list['True Class'] = data[self.class_var]
        return dens_list
```

In [ ]:

```
wine_training.wine_quality.value_counts()
wine_training.head()
training_counts = wine_training['wine_quality'].value_counts().to_frame()
total_training = training_counts.sum()[0]
```

Flat Priors: LDA

In [ ]:
```
flat_priors = [1/3, 1/3, 1/3]
LDA_non_informative = LDA(wine_training, 'wine_quality', priors = flat_prior
print('+-----Non-Informative-Priors----+')
print('\t-Training Dataset-')
Non_Inform_Train_MCR = LDA_non_informative.misclass_rate()
print("Mis-Classification Rate:",Non_Inform_Train_MCR)
print("** Cross Tab **")
print(LDA_non_informative.misclass_xtabs())
print('----------------------------------')
print("\t-Testing Dataset-")
Non_Inform_Test_MCR = LDA_non_informative.misclass_rate(wine_testing)
print("Mis-Classification Rate:",Non_Inform_Test_MCR)
print("** Cross Tab **")
print(LDA_non_informative.misclass_xtabs(wine_testing))
```

```
+-----Non-Informative-Priors----+
        -Training Dataset-
Mis-Classification Rate: 0.49496538703587156
** Cross Tab **
True Class   A    C    F
MAP Class
A           69  687   62
C           22  800  202
F            9  591  736
----------------------------------
        -Testing Dataset-
Mis-Classification Rate: 0.48546511627906974
** Cross Tab **
True Class   A    C    F
MAP Class
A           56  348   43
C           15  381  149
F            9  271  448
```

Find Features LDA

In [ ]:
```
#Create List of all possible combinations of 3 features
def find_three_combo(priors):
    combo_of_three_features = [list(x) for x in combinations(wine_training.d

    LDA_train_results = []
    LDA_test_results = []

    #Iterate through all possible combinations of three.
    for combination in combo_of_three_features:
        current_columns = combination + ['wine_quality']
        train_three_feat = wine_training[current_columns]
        test_three_feat = wine_testing[current_columns]
        #print(train_three_feat)
        LDA_3 = LDA(train_three_feat, 'wine_quality', priors=priors)
        train_misclass = LDA_3.misclass_rate()
        test_misclass = LDA_3.misclass_rate(test_three_feat)
        LDA_train_results.append((current_columns, train_misclass))
        LDA_test_results.append((current_columns, test_misclass))

    return(LDA_train_results, LDA_test_results)
```

Print Best Features, LDA

In [ ]:
```
LDA_train, LDA_test = find_three_combo(priors=flat_priors)
train_final = pd.DataFrame(LDA_train, columns = ['Combination', 'Misclass_Ra
test_final = pd.DataFrame(LDA_test, columns = ['Combination', 'Misclass_Rate
train_min = pd.DataFrame(train_final[train_final.Misclass_Rate == train_fina
print("Training:\nThe Combination of Features that Provides the lowest mis-c
print("The corresponding mis-class rate is:" ,train_min['Misclass_Rate'][0])
print()
test_min = pd.DataFrame(test_final[test_final.Misclass_Rate == test_final.Mi
print("Testing:\nThe Combination of Features that Provides the lowest mis-cl
print("The corresponding mis-class rate is:", test_min['Misclass_Rate'][0])
LDA_best_misclass_train = LDA(wine_training[train_min['Combination'][0]], 'w
LDA_best_misclass_test = LDA(wine_testing[test_min['Combination'][0]], 'wine
```

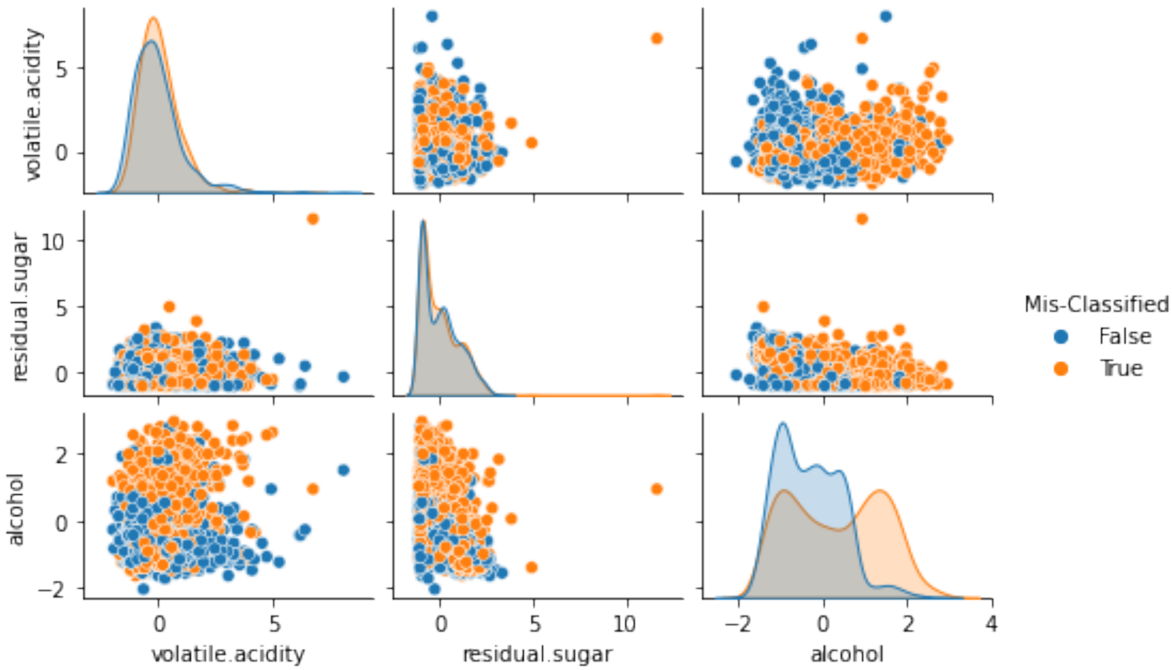
```
Training:
The Combination of Features that Provides the lowest mis-class rate: ['volat
ile.acidity', 'residual.sugar', 'alcohol', 'wine_quality']
The corresponding mis-class rate is: 0.5050346129641283

Testing:
The Combination of Features that Provides the lowest mis-class rate: ['volat
ile.acidity', 'density', 'alcohol', 'wine_quality']
The corresponding mis-class rate is: 0.4877906976744186
```

Graphs:

In [ ]:
```
LDA_best_misclass_train = LDA(wine_training[train_min['Combination'][0]], 'w
LDA_best_misclass_test = LDA(wine_testing[test_min['Combination'][0]], 'wine
LDA_best_misclass_train.misclass_pairplot()
```

Out[ ]: <seaborn.axisgrid.PairGrid at 0x7fe9170da0a0>



Flat Priors, QDA

In [ ]:
```
QDA_non_informative = QDA(wine_training, 'wine_quality', priors = flat_prior
print('+-----Non-Informative-Priors----+')
print('\t-Training Dataset-')
Non_Inform_Train_MCR = QDA_non_informative.misclass_rate()
print("Mis-Classification Rate:",Non_Inform_Train_MCR)
print("** Cross Tab **")
print(QDA_non_informative.misclass_xtabs())
print('---------------------------------')
print("\t-Testing Dataset-")
Non_Inform_Test_MCR = QDA_non_informative.misclass_rate(wine_testing)
print("Mis-Classification Rate:",Non_Inform_Test_MCR)
print("** Cross Tab **")
print(QDA_non_informative.misclass_xtabs(wine_testing))
```

```
+-----Non-Informative-Priors----+
        -Training Dataset-
Mis-Classification Rate: 0.5088105726872247
** Cross Tab **
True Class   A    C    F
MAP Class
A           87  828   90
C            9  799  235
F            4  451  675
----------------------------------
        -Testing Dataset-
Mis-Classification Rate: 0.5186046511627906
** Cross Tab **
True Class   A    C    F
MAP Class
A           58  414   54
C           21  381  197
F            1  205  389
```

Get all features, QDA:

In [ ]:
```
#Create List of all possible combinations of 3 features
def find_three_combo_QDA(priors):
    combo_of_three_features = [list(x) for x in combinations(wine_training.d

    QDA_train_results = []
    QDA_test_results = []

    #Iterate through all possible combinations of three.
    for combination in combo_of_three_features:
        current_columns = combination + ['wine_quality']
        train_three_feat = wine_training[current_columns]
        test_three_feat = wine_testing[current_columns]
        #print(train_three_feat)
        QDA_3 = QDA(train_three_feat, 'wine_quality', priors=priors)
        train_misclass = QDA_3.misclass_rate()
        test_misclass = QDA_3.misclass_rate(test_three_feat)
        QDA_train_results.append((current_columns, train_misclass))
        QDA_test_results.append((current_columns, test_misclass))

    return(QDA_train_results, QDA_test_results)
```

Print Best Features, QDA

In [ ]:
```
QDA_train, QDA_test = find_three_combo_QDA(priors=flat_priors)
train_final = pd.DataFrame(QDA_train, columns = ['Combination', 'Misclass_Ra
test_final = pd.DataFrame(QDA_test, columns = ['Combination', 'Misclass_Rate
train_min = pd.DataFrame(train_final[train_final.Misclass_Rate == train_fina
print("Training:\nThe Combination of Features that Provides the lowest mis-c
print("The corresponding mis-class rate is:" ,train_min['Misclass_Rate'][0])
print()
test_min = pd.DataFrame(test_final[test_final.Misclass_Rate == test_final.Mi
print("Testing:\nThe Combination of Features that Provides the lowest mis-cl
print("The corresponding mis-class rate is:", test_min['Misclass_Rate'][0])
QDA_best_misclass_train = QDA(wine_training[train_min['Combination'][0]], 'w
QDA_best_misclass_test = QDA(wine_testing[test_min['Combination'][0]], 'wine
```

```
Training:
The Combination of Features that Provides the lowest mis-class rate: ['volat
ile.acidity', 'free.sulfur.dioxide', 'sulphates', 'wine_quality']
The corresponding mis-class rate is: 0.4269981120201385

Testing:
The Combination of Features that Provides the lowest mis-class rate: ['volat
ile.acidity', 'free.sulfur.dioxide', 'alcohol', 'wine_quality']
The corresponding mis-class rate is: 0.4511627906976744
```

In [ ]:
```
from mpmath import mp
import numpy as np
import pandas as pd
import statsmodels.api as sm
from statsmodels.tools import add_constant
from itertools import combinations
mp.dps = 50

#This class is based on the BMA class provided by Bill Basener in: https://w
#It has been modified to allow for multinomial regression (logistic regressi
#Specifically, I've hardcoded the model as a 3 category multinomial regressi
class BMA_Wine:

    def __init__(self, y, X, **kwargs):
        # Setup the basic variables.
        self.y = y
        self.X = X
        self.names = list(X.columns)
        self.nRows, self.nCols = np.shape(X)
        self.likelihoods = mp.zeros(self.nCols,1)
        self.likelihoods_all = {}
        self.coefficients_mp = mp.zeros(self.nCols,2)
        self.coefficients = np.zeros((self.nCols, 2))
        self.probabilities = np.zeros(self.nCols)
        # Check the max model size. (Max number of predictor variables to us
        # This can be used to reduce the runtime but not doing an exhaustive
        if 'MaxVars' in kwargs.keys():
            self.MaxVars = kwargs['MaxVars']
        else:
            self.MaxVars = self.nCols
```

```
        # Prepare the priors if they are provided.
        # The priors are provided for the individual regressor variables.
        # The prior for a model is the product of the priors on the variable
        if 'Priors' in kwargs.keys():
            if np.size(kwargs['Priors']) == self.nCols:
                self.Priors = kwargs['Priors']
            else:
                print("WARNING: Provided priors error.  Using equal priors i
                print("The priors should be a numpy array of length equal to
                self.Priors = np.ones(self.nCols)
        else:
            self.Priors = np.ones(self.nCols)
        if 'Verbose' in kwargs.keys():
            self.Verbose = kwargs['Verbose']
        else:
            self.Verbose = False
        if 'RegType' in kwargs.keys():
            self.RegType = kwargs['RegType']
        else:
            self.RegType = 'LS'

    def fit(self):
        # Perform the Bayesian Model Averaging

        # Initialize the sum of the likelihoods for all the models to zero.
        # This will be the 'normalization' denominator in Bayes Theorem.
        likelighood_sum = 0

        # To facilitate iterating through all possible models, we start by i
        # the number of elements in the model.
        max_likelihood = 0
        for num_elements in range(1,self.MaxVars+1):

            if self.Verbose == True:
                print("Computing BMA for models of size: ", num_elements)

            # Make a list of all index sets of models of this size.
            Models_current = list(combinations(list(range(self.nCols)), num_

            # Occam's window - compute the candidate models to use for the n
            # Models_previous: the set of models from the previous iteration
            # Models_next:     the set of candidate models for the next iter
            # Models_current:  the set of models from Models_next that can b
            #                    to a model from Models_previous


            # Iterate through all possible models of the given size.
            for model_index_set in Models_current:

                # Compute the linear regression for this given model.
                model_X = self.X.iloc[:,list(model_index_set)]

                model_regr = sm.MNLogit(self.y, model_X).fit(disp=0)
```

```
                # Compute the likelihood (times the prior) for the model.
                model_likelihood = mp.exp(-model_regr.bic/2)*np.prod(self.Pr

                if self.Verbose == True:
                    pass
                    #print("Model Variables:",model_index_set,"likelihood=",
                self.likelihoods_all[str(model_index_set)] = model_likelihoo

                # Add this likelihood to the running tally of likelihoods.
                likelighood_sum = mp.fadd(likelighood_sum, model_likelihood)
                # Add this likelihood (times the priors) to the running tall
                # of likelihoods for each variable in the model.
                for idx, i in zip(model_index_set, range(num_elements)):
                    self.likelihoods[idx] = mp.fadd(self.likelihoods[idx], m
                    for j in np.arange(model_regr.params.shape[1]):

                        self.coefficients_mp[idx,j] = mp.fadd(self.coefficie
                max_likelihood = np.max([max_likelihood,model_likelihood]) #



        # Divide by the denominator in Bayes theorem to normalize the probab
        # sum to one.
        self.likelighood_sum = likelighood_sum
        for idx in range(self.nCols):
            self.probabilities[idx] = mp.fdiv(self.likelihoods[idx],likeligh
            for j in range(2):
                self.coefficients[idx,j] = mp.fdiv(self.coefficients_mp[idx,

        # Return the new BMA object as an output.
        return self


    def predict_MAP(self,true_class, data):
        data = np.asarray(data)
        result = np.zeros((data.shape[0],3))
        temp = sm.MNLogit(true_class, exog=np.asarray(data))
        result = temp.predict(params = self.coefficients, exog = np.asarray(
        result = pd.DataFrame(result, columns= ["A", "C", "F"])
        res_MAP = result.idxmax(axis=1)
        to_return = pd.DataFrame({'TrueClass':true_class, 'MAP':res_MAP})
        return to_return


    def misclass_rate(self, true_class, data):
        maps = self.predict_MAP(true_class, data)


        maps['Mis_class'] = maps['MAP']  == maps['TrueClass']

        mis_class =  1 - maps['Mis_class'].mean()
```

```
            return mis_class
    def misclass_xtabs(self, true_class, data):
        maps = self.predict_MAP(true_class, data)

        xtabs = pd.crosstab(maps['MAP'], maps['TrueClass'])
        return xtabs

    def summary(self):
        # Return the BMA results as a data frame for easy viewing.
        df = pd.DataFrame([self.names, list(self.probabilities), list(self.c
            ["Variable Name", "Probability", "Avg. Coefficient"]).T
        return df
```

In [ ]:

```
x_train = wine_training.drop(['wine_quality'], axis=1)
y_train = wine_training['wine_quality']

x_test = wine_testing.drop(['wine_quality'], axis=1)
y_test = wine_testing['wine_quality']
```

In [ ]:

```
#train BMA
BMA_trained = BMA_Wine(y_train, x_train)
BMA_trained_fit = BMA_trained.fit()
```

In [ ]:

```
BMA_trained_fit.summary()
```

Out[ ]:

	Variable Name	Probability	Avg. Coefficient
0	fixed.acidity	0.022847	[-0.0006496194390206341, 0.0024533915431253664]
1	volatile.acidity	1.0	[-0.5794998505368034, 0.4309859145395248]
2	citric.acid	0.000331	[-3.741362276800298e-06, -5.731854457064218e-07]
3	residual.sugar	0.999999	[0.18762366290320867, -0.2593434619340806]
4	chlorides	0.000929	[-8.167160338182562e-05, -5.18051174984928e-06]
5	free.sulfur.dioxide	0.003381	[-1.960640091869083e-05, -0.00033178107867380945]
6	total.sulfur.dioxide	0.00047	[-1.8551104711316477e-05, 4.0603803131344844e-06]
7	density	0.091857	[-0.016701248492898016, 0.0395765200767332]
8	pH	0.001963	[5.048712329392856e-05, -0.00014545195326272066]
9	sulphates	0.180932	[0.01627749594433858, -0.01928330908543398]
10	alcohol	1.0	[0.5265454955942414, -0.899872769182548]

The BMA gives us variables voliatile acidity, residual sugar, and alcohol as the best variables for the model.

LDA gave us : ['volatile.acidity', 'residual.sugar', 'alcohol', 'wine_quality'] QDA gave us ['volatile.acidity', 'free.sulfur.dioxide', 'sulphates']

So the BMA returned a model that was identical to the one we found with the QDA!

Question 3

In [ ]:
```
BMA_trained_fit.misclass_rate(y_train, x_train)
```

Out[ ]:
```
0.3590308370044053
```

In [ ]:
```
BMA_trained_fit.misclass_xtabs(y_train, x_train)
```

Out[ ]:

TrueClass	A	C	F
MAP			
A	7	107	18
C	78	1251	203
F	15	720	779

In [ ]:
```
# BMA testing misclass rate
BMA_trained_fit.misclass_rate(y_test, x_test)
```

Out[ ]:
```
0.3540697674418605
```

In [ ]:
```
BMA_trained_fit.misclass_xtabs(y_test, x_test)
```

Out[ ]:

TrueClass	A	C	F
MAP			
A	7	47	10
C	61	632	158
F	12	321	472

The Bayesian model has a much better misclassification rate than the LDA or QDA models for the uninformative priors, however, the misclassification rate for LDA and QDA was much better when we used the proprortional priors than what we see here with Bayesian.

I think the Bayesian model is better than the QDA/LDA models for the same prior, but this also shows how picking the correct prior can be very influential in making accurate predictions.