Homework I: Iterative methods for sparse matrices

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Introduction

1 Timing and memory of methods

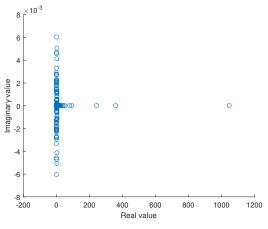
In table 1 the timings of all the methods combined with different preconditioners and the direct method are displayed. There are a few empty spots in the table when using matrix 2. For the preconditioners $\mathrm{ILU}(0)$ and $\mathrm{ILU}(1)$ all the methods can't be executed because the diagonal of the preconditioner multiplied with matrix 2 is zero.

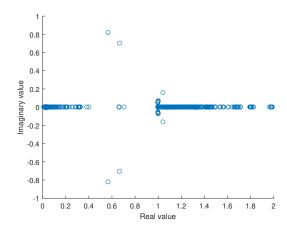
After looking at the longer timings, it can be concluded that generally GM-RES(10) is the fastest method followed by BiCGStab and GMRES(100) is the slowest. The smaller timings don't have a significant difference in them so it's difficult to conclude something from them.

The timings for the direct method aren't so clear. For matrix 3 it is faster, but for matrix 4 and 5, it is a lot slower.

	Matrix 1	Matrix 2	Matrix 3	Matrix 4	Matrix 5
BiCGStab (ILU(0))	0.040769		62.3667	235.744	0.34591
BiCGStab (ILU(1))	2.53761		97.9519	418.621	0.954309
BiCGStab (ARMS)	0.012949	0.229775	48.2617	257.314	3.05756
GMRES(10) (ILU(0))	0.04849		37.008	129.211	0.380574
GMRES(10) (ILU(1))	2.3546		58.8101	242.496	0.952507
GMRES(10) (ARMS)	0.20596	0.2452	28.3702	170.48	3.1586
GMRES(100) (ILU(0))	0.112047		81.9552	398.752	0.341786
GMRES(100) (ILU(1))	2.43374		95.0938	464.788	0.974389
GMRES(100) (ARMS)	0.014858	0.013013	127.325	368.297	3.30781
Direct method	0.021279	0.022208	20.4651	737.353	15.2156

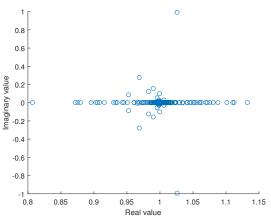
Table 1: Table with the timings for different methods with preconditioners and the direct method, expressed in seconds.





(a) The spectrum of mat01.

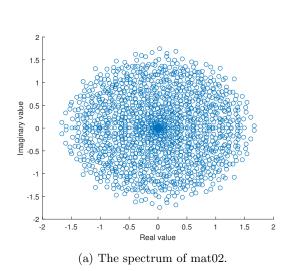
(b) The spectrum of the preconditioner multiplied with mat 01 in which the preconditioner $\mathrm{ILU}(0)$ is used.

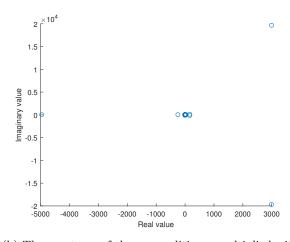


0.15 - 0.15 - 0.05 - 0.05 - 0.05 - 0.15 - 0.15 - 0.15 - 0.2 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 Real value

(d) The spectrum of the preconditioner multiplied with mat01 in which the preconditioner ARMS is used with mat01 in which the preconditioner ILU(1) is used. value k=1 and ARMS_levels=1.

Figure 1: Spectra of mat01 with different preconditioners.





(b) The spectrum of the preconditioner multiplied with mat02 in which the preconditioner ARMS is used with value k=3 and ARMS_levels=4.

Figure 2: Spectra of mat01 with and without a preconditioner.

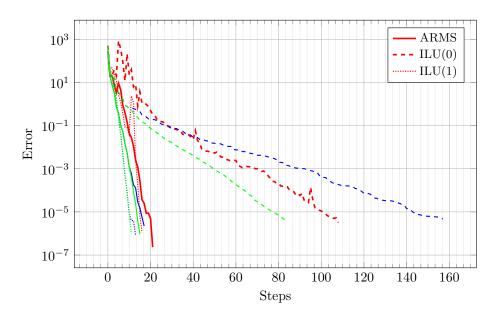


Figure 3: The convergence of matrix mat01 with different methods and preconditioners. Red lines are BiCGStab method, blue lines are GMRES(10) and green ones GMRES(100).

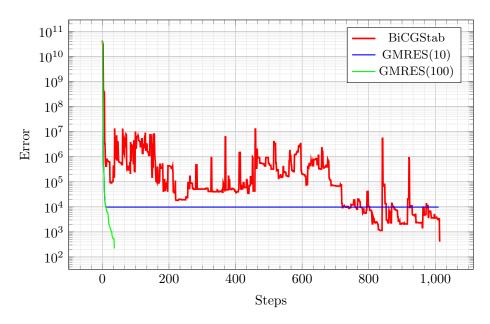


Figure 4: The convergence of matrix mat02 with the ARMS preconditioner.

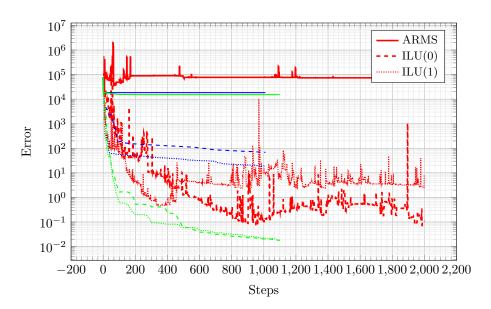


Figure 5: The convergence of matrix mat03 with different methods and preconditioners. Red lines are BiCGStab method, blue lines are GMRES(10) and green ones GMRES(100).

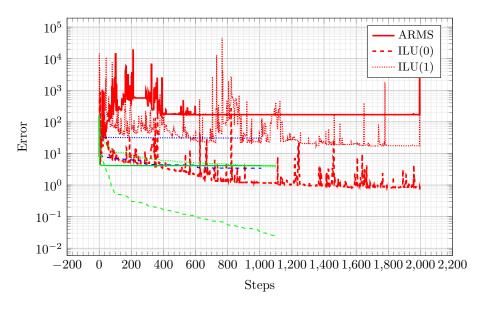


Figure 6: The convergence of matrix mat04 with different methods and preconditioners. Red lines are BiCGStab method, blue lines are GMRES(10) and green ones GMRES(100).

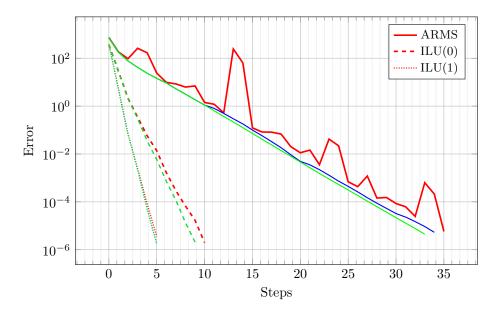


Figure 7: The convergence of matrix mat05 with different methods and preconditioners. Red lines are BiCGStab method, blue lines are GMRES(10) and green ones GMRES(100).

	Matrix 3	Matrix 4
BiCGStab (ILU(0))	2.8	10.2
BiCGStab (ILU(1))	3.7	15.5
BiCGStab (ARMS)	4	14.4
GMRES(10) (ILU(0))	2.8	10.5
GMRES(10) (ILU(1))	3.8	15.8
GMRES(10) (ARMS)	4	14.4
GMRES(100) (ILU(0))	3.9	16.1
GMRES(100) (ILU(1))	4.9	21.3
GMRES(100) (ARMS)	5.1	20.6

Table 2: Table with the memory usage for different methods and preconditioners, expressed in percent of the total memory.

	Matrix 1	Matrix 2	Matrix 3	Matrix 4	Matrix 5
without -ooc	0.021279	0.022208	20.4651		15.2156
with -ooc	0.113117	0.161896	23.5209	737.353	13.2442

Table 3: Table with the timings for the direct method, expressed in seconds.

	Matrix 3	Matrix 4	Matrix 5
without -ooc	11.6		5.3
with -ooc	2.9	61.5	2.1

Table 4: Table with the memory usage for the direct method, expressed in percent of the total memory.