$$T(n) = \begin{cases} \Theta(1), & n < 10 \rightarrow \boxed{\text{base case}} \\ \underbrace{T(n/5)}_{\substack{\uparrow \\ \text{rec. call to find median} \\ \text{of medians}}} + \underbrace{T(7n/10)}_{\substack{\uparrow \text{ max} \\ \text{size of} \\ \text{sub-array}}} + \underbrace{\Theta(n)}_{\substack{\uparrow \text{ run-time} \\ \text{of partition}}}, & n \geq 10 \rightarrow \text{recursive case} \end{cases}$$

$i=1$

$$T(n) = T(n/5) + T(7n/10) + \Theta(n)$$

Run time = $O(n) + \Theta(n) = O(n)$
size of array = n

$i=2$

run time = $\Theta(n) + O(n/5) = O(n)$

$$\boxed{T(n/5) = T\left(\frac{1}{5}\cdot\frac{n}{5}\right) + T\left(\frac{1}{5}\cdot\frac{7n}{10}\right) + \frac{\Theta(n)}{5}}$$

size = $n/5$

runtime = $\Theta(n) + O(7n/10) = O(n)$

$$\boxed{T(7n/10) = T\left(\frac{7}{10}\cdot\frac{1}{5}n\right) + \left(\frac{7}{10}\cdot\frac{7}{10}n\right) + \frac{7\Theta(n)}{10}}$$

size = $7n/10$

$+ \Theta(n)$

$i=3$

$\boxed{T(n/25)}$ size = $n/25$

runtime = $O(n)$

$\boxed{T\left(\frac{7n}{50}\right)}$ size = $\frac{7n}{50}$

$\boxed{T(7n/50)}$ size = $7n/50$

$\boxed{T(49n/100)}$ size = $49n/100$

$+ \Theta(n)$

$T(n/25) + 2T\left(\frac{7n}{50}\right) + T\left(\frac{49n}{100}\right)$

$\frac{1}{25} + \frac{14}{50} + \frac{49}{100}$

$4 + 28 + 49$

...

$$i=2 = T\left(\left(\tfrac{1}{5}\right)^2 n\right) + 2T\left(\tfrac{1}{5}\cdot\tfrac{7}{10}n\right) + T\left(\left(\tfrac{7}{10}\right)^2 n\right) + \left(\tfrac{1}{5} + \tfrac{7}{10} + 1\right)\Theta n$$

i generally

adding each time

$$= T\left(\left(\tfrac{1}{5}\right)^i n\right) + iT\left(\tfrac{1}{5}\cdot\tfrac{7}{10}n\right) + T\left(\left(\tfrac{7}{10}\right)^i n\right) + \left(1 + \left(\tfrac{9}{10}\right) + \left(\tfrac{9}{10}\right)^2 + \dots + \left(\tfrac{9}{10}\right)^i\right)\Theta n$$

all work done

$$= \sum_{\cancel{k}=1}^{i-1} a_i = 1 + \left(\tfrac{9}{10}\right)^i + a_{i-1}$$

Because the array decreases by size $\frac{7n}{10}$ each time, and i is the total # of layers before the base case is reached

$\left(\frac{7}{10}\right)^i n = 1 \leftarrow$ (a base case

$\hookrightarrow$ the total decrease each time:

$n = \left(\frac{7}{10}\right)^i \rightsquigarrow \log n = i\log(10/7) \rightsquigarrow \boxed{i = \frac{\log n}{\log(10/7)}}$

$\boxed{\text{number Layers}}$ before base case reached so $n<10 \rightsquigarrow$ base case/when to stop

$$\underset{\text{stops}}{T(n)} = \underbrace{T\left(\left(\tfrac{1}{5}\right)^{\frac{\log n}{\log 5}} n\right)}_{\text{some value of } n} + \underbrace{\underbrace{\frac{\log n}{\log 10/7} T\left(\tfrac{1}{5}\cdot\tfrac{7}{10} n\right)}_{\text{some value of } n} + \underbrace{T\left(\left(\tfrac{7}{10}\right)^{\frac{\log n}{\log 10/7}} n\right)}_{\text{some value of } n}}_{\text{negligible in over all runtime}} + \underbrace{\left[\left(1-\tfrac{9}{10}^{\left(\frac{\log n}{\log 10/7}+1\right)}\right) \Big/ \left(1-\tfrac{9}{10}\right)\right]}_{\text{constant}} \Theta(n)$$

$\Theta(n)$

overall run time will be $\Theta(n)$

Simplified runtime summary

$$\begin{array}{ccccc} & & \Theta(n) & & \\ & \Theta(n/5) & & \Theta(7n/10) & \Theta\left(\frac{49n}{100}\right) \\ \Theta\left(\frac{n}{25}\right) & & \Theta(7n/50) & \Theta(7n/50) & \end{array}$$

so overall the runtime of each rec. call is $\theta(n)$ + some constant number