

$$T(n) = \begin{cases} \Theta(1), & n < 10 \rightarrow \text{base case} \\ T(n/5) + T(7n/10) + \Theta(n), & n \geq 10 \rightarrow \text{recursive case} \end{cases}$$

$\uparrow$  rec. call to find median of medians     
  $\uparrow$  max size of sub-array     
 $\uparrow$  run-time of partition

$i=1$

$$T(n) = T(n/5) + T(7n/10) + \Theta(n)$$

Run-time =  $\Theta(n) + \Theta(n) = O(n)$   
size of array =  $n$

$i=2$

$$T(n/5) = T\left(\frac{1}{5} \cdot \frac{n}{5}\right) + T\left(\frac{1}{5} \cdot \frac{7n}{10}\right) + \frac{\Theta(n)}{5}$$

size =  $n/5$

run-time =  $\Theta(n) + O(n/5) = O(n)$

$$T(7n/10) = T\left(\frac{7}{10} \cdot \frac{1}{5}n\right) + T\left(\frac{7}{10} \cdot \frac{7}{10}n\right) + \frac{7\Theta(n)}{10}$$

size =  $7n/10$

run-time =  $\Theta(n) + O(7n/10) = O(n)$

$i=3$

$$T(n/25)$$

size =  $n/25$   
run-time =  $O(n)$

$$T\left(\frac{7n}{50}\right)$$

size =  $7n/50$

$$T(7n/50)$$

size =  $7n/50$

$$T(49n/100)$$

size =  $49n/100$

$$T(n/25) + 2T(7n/50) + T(49n/100)$$

+  $\Theta(n)$

$$i=2 = T\left(\left(\frac{1}{5}\right)^2 n\right) + 2T\left(\frac{1}{5} \cdot \frac{7}{10} n\right) + T\left(\left(\frac{7}{10}\right)^2 n\right) + \left(\frac{1}{5} + \frac{7}{10} + 1\right)\Theta n$$

$i$  generally

$$= T\left(\left(\frac{1}{5}\right)^i n\right) + iT\left(\frac{1}{5} \cdot \frac{7}{10} n\right) + T\left(\left(\frac{7}{10}\right)^i n\right) + (1 + \frac{1}{5} + \frac{7}{10} + \dots + \left(\frac{7}{10}\right)^{i-1})\Theta n$$

all work done

Because the array decreases by size  $\frac{7n}{10}$  each time,

and  $i$  is the total # of layers before the base case is reached

$$\left(\frac{7}{10}\right)^i n = 1 \leftarrow \text{base case}$$

$\hookrightarrow$  the total decrease each time:

$$n = \left(\frac{7}{10}\right)^i \leadsto \log n = i \log(10/7) \leadsto i = \frac{\log n}{\log(10/7)}$$

number of layers

before base case reached

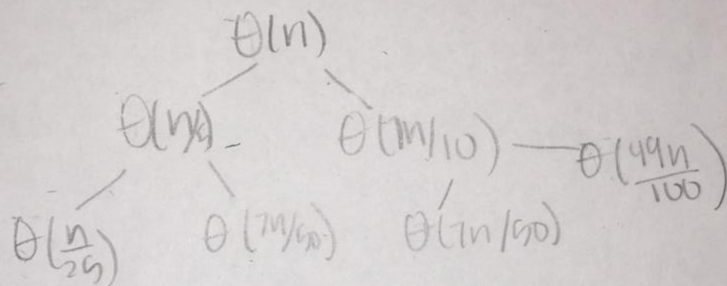
$$= \sum_{k=1}^{i-1} a_k = 1 + \frac{1}{5} + \left(\frac{7}{10}\right) + \dots + \left(\frac{7}{10}\right)^{i-1}$$

SO  $n < 10 \rightarrow$  base case / when to stop

$$T(n) = \underbrace{T\left(\frac{n}{5}\right)}_{\substack{\text{stops} \\ \downarrow \\ \text{some value of } n}} + \underbrace{\frac{\log n}{\log 10} T\left(\frac{1}{5} \cdot \frac{7}{10} n\right)}_{\substack{\text{some value} \\ \text{of } n}} + \underbrace{T\left(\frac{7}{10} \cdot \frac{7}{10} n\right)}_{\substack{\text{negligible in} \\ \text{over all runtime}}} + \underbrace{\left[\frac{(1 - \frac{9}{10}(\log n / \log 10) + 1)}{(1 - \frac{9}{10})}\right] \theta(n)}_{\substack{\text{constant} \# \\ \uparrow \\ \theta(n)}}$$

overall run time will be  $\theta(n)$

Simplified runtime summary



So overall the runtime of each rec. call is  $\theta(n) + \text{some constant number}$