PROJECT DESCRIPTION

1 Introduction

The earliest examples of astronomy included charting the "fixed" stars. This was necessarily limited to the brightest stars, which were projected onto the celestial sphere and organized into constellations. Ultimately this led to a physics-based, low resolution, 3D description of the Galaxy. The situation today in cosmology is somewhat similar. We have a splendid two dimensional map of the microwave background and its polarization organized by spherical harmonics and this has led to a working model of the universe in which random primordial potential fluctuations with a well-defined spectrum grow according to deterministic, linear laws to produce contemporary large scale structure in a flat universe that is endowed with a cosmological constant. The traditional goal of the astronomy, to describe the actual disposition of this structure, has hitherto been subsumed into statistical descriptions designed to elucidate the underlying physics, a manifestly successful program. In fact it has been executed so well that it may now be possible to derive the entire 3D structure of our universe within the surface of last scattering. This requires inferring the actual amplitudes and phases of the constituent waves down to some limiting spatial resolution.

This is a proposal to explore how to carry out this program in a series of stages. The first stage is to use the 2D microwave background observations alone to test the internal consistency of the assumptions being made and to recover as much as we can of the 3D potential, velocity and density fields.The second stage is to augment the 2D data with existing 3D measurements from galaxy surveys, gravitational lensing, intermediate Sachs-Wolfe measurements and so on. This should improve the resolution. The third stage is to project the improvement that should come on a decade timescale from future surveys such as LSST, "Stage IV" CMB observations and SKA. The fourth and final stage is to see how far it is possible to go before non-linearity and unmodeled effects prevent further progress. Success in this program will naturally lead to a range of applications, including a more direct reconciliation of local survey data with CMB measurements, an important foundation for Epoch of Reionization studies and the exploration of new cosmological effects.

2 Proposed Research

2.1 Contemporary Cosmology

The last decade has seen remarkable advances in cosmology, spearheaded by increasingly detailed measurements of the cosmic microwave background (CMB) radiation (Hinshaw et al. 2013, Ade et al 2014a, http://www.cosmos.esa.int/web/planck). These accurate measurements have affirmed that a description of the universe with relatively few elements suffices to describe essentially all that is secure in the observations. There is still plenty of room for revisions, retractions and new discoveries but, right now, we have a good working hypothesis that the universe is basically this simple. We start by briefly reviewing this cosmological model, drawing attention to aspects which impinge on the proposal (e.g. Weinberg 2008).

Geometrically, the universe appears to be isotropic and spatially flat, in the sense that its radius of curvature is $\gtrsim 20$ Hubble radii. It is argued to be spatially homogeneous at the same cosmic time. The current universe can be parametrized kinematically by a Hubble constant, a deceleration parameter, and a jerk. There is some tension between Planck's and other measurements of the Hubble constant at the ten percent level (Ade et al. 2014a) . A compromise value of $H_0 = 70\mathrm{km\ s^{-1}\ Mpc^{-1}}$ will be used here but the choice does not matter. The second and third derivatives exhibit acceleration (Riess et al 1998, Permutter et al 1999). The dynamics of the universe is supposed to be governed by Einstein's general relativistic field equations including a cosmological constant that is responsible for this acceleration. There is no widely accepted observation that is inconsistent with this description, although considerable effort has been and will be applied to seeing if general relativity needs to be modified, if the universe is spatially curved or inhomogeneous, if the cosmological constant needs to be replaced by a dark energy component with its own dynamics. If significant evidence appears for any of these, then simple changes will be needed to what follows.

2.1.1 Evolution of Universe

The principal contents of the contemporary universe are (initially) cold dark matter ($\Omega_c = 0.26$), baryonic matter ($\Omega_b = 0.048$), (CMB) radiation ($\Omega_\gamma = 5.4 \times 10^{-5}$), and neutrinos ($\Omega_\nu = 0.0014(\sum m_\nu/100\mathrm{meV})$). The ratio of the neutrinos to the radiation density is derived confidently from theory; the dark matter and baryon densities are measured and not understood. Significant changes in these measurements can be easily accommodated. If these are the only constituents, we have confidence that we understand essentially all of the relevant physics, and, given the contemporary values, we can calculate the evolution of the universe at large given initial conditions defined when its age was ~ 1 ms. At early times, the energy density was dominated by radiation and the matter was dynamically unimportant. However, after $t \sim 50$ kyr, when the scale factor – the size of a region relative to its contemporary size – was $a \sim 0.0003$, the universe became (dark) matter-dominated and the scale factor increased $\propto t^{2/3}$. When $t \sim 380$ kyr, $a \sim 0.0009$, the hydrogen plasma quickly formed atoms, decoupling from the radiation and forming the inside-out, CMB photosphere where the majority of CMB photons we observe today were last scattered with a temperature ~ 2900 K. When $t \sim 600$ Myr, $a \sim 0.1$, the first stars formed and the universe (re)-ionized. This epoch is becoming accessible to observation. Finally when $t \sim 8$ Gyr, $a \sim 0.6$, the cosmological constant came to dominate over the matter, and the universal expansion started to accelerate.

2.1.2 Growth of Fluctuations

The study of the growth of fluctuations, initiated by Lemaître, has been developed to accommodate a wide range of possible world models and help distinguish between them (e.g. Weinberg 2008). This program will continue and assuredly lead to even more accurate models. The simple version is that at some early time the well-coupled radiation-dominated fluid exhibited perturbations described by a set of wave modes. For each mode, there are relative density perturbations in the photons plus neutrinos, dark matter and baryons, as well as a common velocity and a single potential (or metric perturbation), $\Phi_{\mathbf{k}}$, where $\mathbf{k}$ refers to the comoving or contemporary wave vector. If we believe that simple, although unknown, physical principles, established equilibrium on a timescale short compared with the expansion timescale, then it follows that the perturbations are "adiabatic" and the relative values of the amplitudes of these quantities in a given mode are known, such that we only need to specify the starting potential to specify the mode completely. In particular, the observed CMB temperature fluctuations are dictated by the potential at long wavelengths, the velocity at intermediate wavelengths and the density at short wavelengths. However, since all of the quantities are linearly related, we can express the measured perturbation on all scales as a potential perturbation.

These perturbations are just the scalar modes, and they might generally be expected to be accompanied by tensor modes which we shall ignore, despite recent claims that they may have been detected (Ade et al. 2014b).[1] Now, the machinery exists to compute the evolution of the scalar modes given our assumptions. A second (curvature) potential is needed, and the neutrinos and photons have to be treated kinetically, but still, all measurable quantities can be related to $\Phi_{\mathbf{k}}(a)$, with a being the scale factor, until the modes become nonlinear.[2]

None of this has told us about the starting potentials. It has recently been demonstrated, mainly using CMB observations, that the adiabatic hypothesis is quite accurate. Furthermore, the amplitudes of the starting potentials associated with each mode scale $\propto k^{-3/2}$ with random phases and are drawn from a Gaussian distribution so that the potential fluctuations associated with each length scale are scale-independent.[3]

This behavior is consistent with a remarkable early conjecture by Harrison (1970) as elaborated by Zel'dovich (1972). Furthermore, the very existence of long wavelength fluctuations, the flatness and isotropy of the geometry, and the slightly tilted spectrum are all consistent with the simplest version of a much more specific and even bolder conjecture by Guth(1981), Linde(1982) and others, that the universe underwent a period of "inflationary" expansion at much earlier times. Again, we do not understand the underlying physics, but the general outcomes of this mechanism have been corroborated wherever this is possible. However, inflation does not directly impinge on the investigation that we propose.

It is helpful to describe what happens to the potential perturbation as the universe expands. It is simplest to work in the "Newtonian" gauge throughout: the potential associated with a mode remains essentially constant until it "enters the horizon," and its wavelength can have been crossed by light. If the wavelength is short, and the universe is still dominated by radiation, the wave is a modified sound wave which conserves its adiabatic invariant and oscillates with constant velocity amplitude while the potential perturbation decays $\propto a^{-2}$.[4] This continues until matter-dominance. This epoch is absent for waves that enter the horizon after recombination with $k < k_0 \sim 50\text{Gpc}^{-1}$. k_0 can be considered as approximately the wavenumber associated with the first acoustic peak and its aftereffect, Baryon Acoustic Oscillations (BAO). The potential remains roughly constant when matter dominates until the mode becomes nonlinear or until the cosmological constant takes over. The latter causes $\Phi_{\mathbf{k}}$ to decrease by about 20 percent to its value today.

2.2 Stage 1. 3D Gravitational Potential Reconstruction from 2D Surface Data

2.2.1 CMB temperature fluctuations

Most of what we have learned about this universal evolution has come from CMB observations. The conventional way to describe the observations is in terms of spherical harmonics – the generalization of Fourier modes to a sphere – labeled by ℓ and m. Most investigations have focused on measuring the "power" in the temperature fluctuations (including polarization) associated with a given ℓ, obtained by summing products of the coefficients of the harmonic components

[1] Whatever the outcome of the ongoing investigation of this claim, the study proposed here would not be affected by them.

[2] $\Phi_{\mathbf{k}}$ changes relatively little over time for the modes of interest here; it is the accompanying density perturbations – the second spatial derivatives of $\Phi_{\mathbf{k}}$ – that cause the problems.

[3] Actually there is a small tilt in their power spectrum in the sense of there being slightly larger amplitudes at longer wavelengths. This requires a small correction to what follows.

[4] Very short wavelength modes are further "Silk"-damped by radiative diffusion but these will not really concern us.

over m, and comparing it with the predictions of various cosmological models. This program has been wonderfully productive, and has resulted in the world model just outlined. Furthermore these temperature fluctuation amplitudes have been successfully combined with other features of the local universe, such as galaxy counts. One basic assumption that is regularly made is that the particular realization of the universe that we are observing is drawn from a statistical ensemble of universes. When ℓ is large, we have many independent measurements on the associated angular scale, $\sim \pi/\ell$, and so we can measure an rms value for the harmonic component with a small variance. However, when ℓ is small, we have only a few such measurements and the "cosmic" variance is large.

Despite their great value, these statistical measurements inevitably discard information which may be valuable.[5] In this study, only one specific realization of the universe (or, more specifically, a very large universe viewed from one out of many possible vantage points) is observed. It must be of interest to map this in three dimensions as finely and as far as we can for both scientific and popular purposes.

This type of map-making has sometimes been called "cosmography." However, an important distinction should be made which is that, although we are not trying to examine the physics that is exhibited and present at very early times (inflation, baryogenesis, dark matter production and so on) or at late times (cosmological constant, star/black hole formation and so on), we are using the physics that we do understand to the full in making these three dimensional maps, and not simply describing what we observe directly.

Of course local redshift surveys produce maps, although many of them are primarily engaged in minimizing systematic error in statistical measures designed to exhibit or quantify physical effects. Nonetheless, these surveys have been limited in angle and redshift and only a comparatively small fraction of the entire volume bounded by the CMB photosphere with comoving radius $r_{\rm CMB} \sim 14$ Gpc can be investigated. We make this explicit in Figure 1 where we show the interior of the last scattering surface in comoving coordinate space. The long term goal, which this proposal addresses, is to connect the CMB to local surveys and to make an evolving 3D (in other words, a 4D) map of the universe that is valid from before 380kyr to today and out beyond 14 Gpc.

2.2.2 Body Modes

The simple question that motivated this investigation, and which did not seem to have a well-known answer, was how much of the interior of the CMB photosphere could be inferred by CMB observations alone, and with what resolution. This is an example of what is sometimes called holography.[6] In other words we are trying to infer a 3D model – the amplitudes and phases of "body" modes – from information on a 2D surface. At first sight this might seem hopeless, because if one associates ℓ with $kr_{\rm CMB}$, then we are trying to solve for $O((k_{max}r_{\rm CMB})^3)$ Fourier modes using only $O(\ell_{max}^2)$ spherical harmonics. However, if one does not attempt to match the arcminute-scale angular resolution of the Planck all-sky map, then it is, in principle, possible to use all the surface data to infer some of the volumetric structure.

To make this a bit more explicit, note that a single wave labeled by $\mathbf{k}$ contains significant power in the spherical harmonics up to $\ell \sim 2kr_{CMB}$. There are $\sim \ell^2$ complex wave amplitudes, which can in principle help determine $(4\pi/3)(kr/\pi)^3$ Fourier components. This suggests that it should be possible to measure ~ 3000 Fourier modes with characteristic size down to $\pi/k \sim 1.5$ Gpc. Of course this is very coarse resolution compared to our knowledge of the local universe,

[5] This is in the sense that music is far more than a "flicker" power spectrum.

[6] This is not the original meaning of the word.

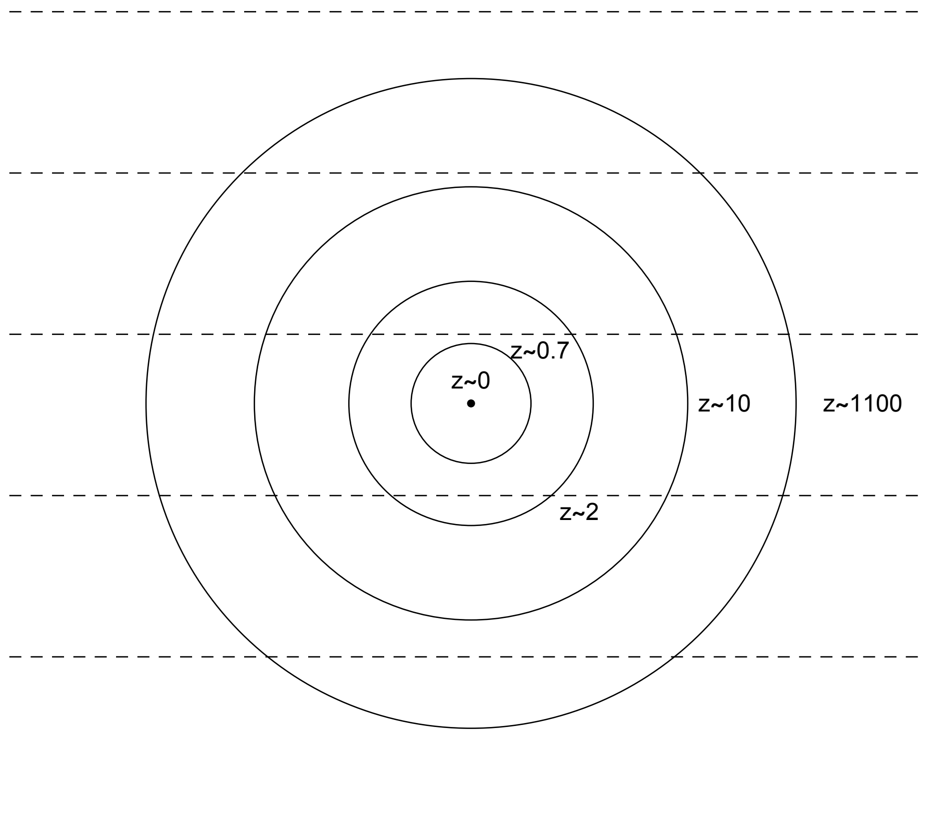

Figure 1: The local universe interior to the CMB photosphere expressed in comoving coordinates. The circles are, in order, redshift $z \sim 0.7$, schematically the limit of present surveys, $z \sim 2$ roughly the effective limit of future surveys, the nominal Epoch of Reionization at $z \sim 10$ and the CMB photosphere at $z \sim 1100$ and a distance $r_{\rm CMB} \sim 14$ Gpc. Also shown as dashed lines are the nodes of a single wave mode with $k \sim 0.9$ Gpc^{-1}. The universe that is in principle directly observable by us (for example using long wavelength gravitational waves) extends only slightly beyond the CMB photosphere.

but such a 3D model would still allow us to explore quite distant regions of space. This spatial resolution is likely to be an overestimate as it makes no allowance for covariance and measurement error, but it does at least motivate us to look at the question in more detail.[7]

Before we do this, we do need to describe the Fourier modes that we are trying to measure. The first point to make is that the CMB fluctuations are observed at a time when the age of the universe was ~ 380 kyr, and any subsequent observations are on our past light cone. However, if we remain in the linear regime and are confident in the machinery for evolving the mode, all of these observations are equivalent and we can regard the mode as known at all times from recombination to today or at lest until it becomes so nonlinear that its dynamics is too complex to describe usefully.

Although the full spectrum of waves can be considered as continuous in $\mathbf{k}$, the fact that our observations are made over a restricted volume means that we can treat the waves as a

[7] To pursue our musical metaphor, different voices and instruments contribute different ranges of frequencies over a total range of roughly ten octaves to a musical performance. We are only listening to the bass range but higher voices and instruments can still contribute to what we hear.

discrete Fourier transform of modes associated with a box in comoving space on which periodic boundary conditions have been imposed. This box should have a side larger than the diameter of the CMB photosphere, $2r_{\rm CMB}$. However, it need not be too large, because very long wavelength modes only contribute significantly to the lowest spherical harmonics, which are automatically discounted. The presence of these modes does degrade our ability to measure body modes with $k > \pi/r_{\rm CMB}$ somewhat. Another point to make is that we must automatically remove the monopolar (zero by definition of a fluctuation), and dipolar (degenerate with motion of the solar system barycenter) components and they cannot be used to measure the body modes.

For a more sophisticated approach, let us simulate the problem by imagining that we have a specific realization of the potential at the time of last scattering in a static universe. We suppose that it can be represented as a sum of Fourier modes $\Phi(\mathbf{x}) = \sum_{\mathbf{k}} \Phi_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}$ and we label all the modes within a sphere in k-space of radius k_{max} using the single index κ. Specifically, this means that the amplitude of each discrete mode with wave vector $\mathbf{k}$ is drawn from a Gaussian distribution of width

$$\sigma_k \propto k^{-3/2}\left(1 + \frac{k_0^2}{k^2}\right) \tag{1}$$

and that the phase is random. This is an adequate approximation to the more detailed evolution of the mode. Then, following convention, we expand this potential in spherical harmonics on the last scattering sphere, which we now assume to have unit radius: $\Phi(\theta,\phi) = \Phi_\lambda Y_\lambda(\theta,\phi)$, where the index λ is shorthand for the combined spherical harmonic indices ℓ, m and the summation convention is assumed. We convert the coefficients Φ_λ into noisy 2-dimensional data, each with an independent measurement error, σ_λ and no covariance.

The next step is to try to recover the 3D body modes from this noisy 2D data. Denote the recovered Fourier coefficients by Φ_κ, where κ is an index labeling all of the modes. The conditional probability that the "data" Φ_λ will be obtained given the "parameters" Φ_κ is then

$$P(\Phi_\lambda|\Phi_\kappa) \propto \prod^{\lambda} e^{-\frac{(\Phi_\lambda - M_{\lambda\kappa}\Phi_\kappa)^2}{2\sigma_Y^2}}, \tag{2}$$

where $M_{\lambda\kappa} = \int d\Omega Y_\lambda^* e^{i\mathbf{k}(\kappa)\cdot\hat{\mathbf{x}}}$. Meanwhile the prior probability for the parameters is

$$P(\Phi_\kappa) \propto \prod^{\kappa} e^{-\frac{\Phi_\kappa^2}{2\sigma_k^2}}. \tag{3}$$

We invoke Bayes' theorem, and maximize minus the logarithm of the posterior density in the usual fashion to solve for Φ_κ:

$$\Phi_\kappa = \left[\frac{\delta_{\kappa\kappa'}}{\sigma_\kappa^2} + \frac{M_{\lambda'\kappa}M_{\lambda'\kappa'}}{\sigma_\lambda^2}\right]^{-1}\frac{M_{\lambda\kappa'}}{\sigma_\kappa^2}\Phi_\lambda \tag{4}$$

that can be solved for the posterior centroid Φ_κ. The derived Fourier components can then be compared with the original input values. Clearly the results depend upon the number of Fourier and spherical harmonic modes that are included and the matrix inversion requires some care.

There is another way to think about this problem. Smooth the potential as defined on the last scattering sphere by convolution or restricting the number of spherical harmonics that are included. Consider the zeros of its 2D gradient which will be isolated maxima, minima and saddle points. Next consider the equipotentials that pass through the saddles - called separatrices - which define a nested set of contours. Now imagine these contours being extended as nested surfaces within the sphere. In addition, continue the zeros as lines inside the sphere. These lines meet at points where the 3D gradient vanishes. The organization and nesting of these points, lines and surfaces defines a specific topology.

However, without additional rules, any solution that respected the topology would be as good as any other.[8] For example, if the potential had satisfied a specific partial differential equation such as Laplace's equation, then the interior solution could have been completely specified by the surface (Dirichlet) conditions. Our invocation of Gaussian priors on the Fourier modes and imposition of the maximum likelihood prescription leads to a specific solution but our confidence in it diminishes as the number of zeros increases or, equivalently, the angular resolution decreases. Finding the optimal way to carry this out in practice, including the polarization data and the results of local surveys, is a major inference problem and this is what this proposal is really about.

At the time of writing, this investigation is still underway but so far, the results are encouraging. As long as the spherical harmonic coefficients are measured relatively accurately, hundreds of Fourier components can be recovered. We propose to continue this investigation and if this finding is confirmed to understand in much more detail this well-posed, albeit somewhat artificial, problem. In particular, we need to understand the optimal number of Fourier components and spherical harmonics to include in the reconstruction for a given CMB map accuracy.

The next step will be to carry out an investigation using simulated temperature data as measured at points on the sky, and finding the optimal procedure to recover the body modes directly from the raw data not from derived quantities where additional complications may arise. Finally, the actual Planck data will be used and the results compared with the simulations if appropriate. The accompanying covariance matrix will indicate how well the underlying assumptions are satisfied, while the posterior predictive residual maps will reveal how well the systematic effects inherent in CMB data have been handled. If the initial indications are realistic then a linear resolution of order 1.5 Gpc should be attainable.

2.2.3 Surface Modes

The above procedure will work at most for long wavelength body modes. There are three reasons why it must fail for short wavelength modes: there are too many to recover, their amplitudes are small and the CMB photosphere actually has a finite width ~ 0.1 Gpc dictated by the actual rate at which the hydrogen recombined. The radiative transfer through the last scattering shell is essential for relating the CMB polarization data to the total temperature fluctuations. It is possible, in principle, to predict the polarization given the temperature fluctuations, and this exercise is likely to be carried out in the next Planck data release. It will also be re-examined in the context of the proposed investigation as it provides an important check on the underlying assumptions.

This suggests an alternative approach to deriving the interior potential map, and this is to use the existing data to continue the potential relatively close to the last scattering surface as has previously been proposed by Yadav & Wandelt (2005) who also considered recovering longer wavelength modes. This approach will work best around the acoustic peak $\ell \sim 200$ and somewhat shorter, where the wavelengths are comparable with the shell thickness. In this calculation the goal will be to recover local Fourier modes by flattening the last scattering surface. We propose to carry put this exercise too using public data and to reconcile this approach with the body

[8] One thing that can be said is that the expectation of the number of stationary points fraction on the sphere observed with a given resolution can be computed for a given prior on the potential Fourier components and this may provide another way to limit non-Gaussianity. Another comment is that this set up provides a natural and possibly fruitful use for the Gauss-Bonnet theorem (e.g. Carroll 2004).

wave reconstruction. It will also teach us how to handle maps where, somewhat unusually, our ability to see detail improves with distance.

An important part of this process is to understand the detailed evolution of the individual Fourier modes through recombination. Although, perfectly adequate and publicly available packages have been developed to carry this out, they seem to lack physical transparency which we will need and so an alternative Monte Carlo approach is under development to help elucidate more clearly how the evolution of a individual mode is expressed as a CMB fluctuation.

2.3 Stage 2. Incorporation of Existing Volumetric Data

2.3.1 CMB Lensing and ISW measurements

An important way to add 3D information is to include CMB lensing (Ade et al. 2014b). A uniform CMB is unchanged by gravitational lensing. However, if there is a gradient in the background temperature, intervening structure will appear as extra power on the scale of intervening large scale structure.[9] The consequences are largest on much smaller scales than those in which we are primarily interested. However there are still integral effects with $\ell \sim 30 - 100$ which are relevant. Furthermore the intense interest in the claim that inflationary B-modes have been detected (Ade et. 2014c) has focused much observational and analytical effort on this region of the spectrum. It is proposed to see if the addition of these measurements will improve the specification of the 3D body modes.

Similar remarks apply to the Integrated Sachs Wolfe effect which is caused by changes in the potential due to the cosmological constant at late times. It is proposed to see if such measurements can also contribute to the specification of large scale structure although here the challenge seems even greater.

2.3.2 Galaxy Surveys and the Local Universe

Most of the use of surveys has been for drawing statistical inferences relating the growth of structure to the CMB emphasizing shorter length scales, notably those associated with BAO and the largest voids ~ 0.1 Gpc. However, these same surveys can also be used to augment the long wavelength CMB data and improve the accuracy and resolution of the resulting 3D map. A good example is the SDSS/BOSS program http://www.sdss.org which covered nearly a third of the sky with over a million redshifts and photometry on galaxies out to $z \sim 0.7$.[10] For our purposes this translates to a comoving volume $\sim 50\mathrm{Gpc}^3$, about 0.005 of the total. Surveys of much rarer quasars and the brightest star forming galaxies which extend to $z \sim 6$ provide much greater volumes over which the potential on Gpc scales can be estimated but with inferior precision.

It is helpful at this point to consider a volume limited-survey of objects out to some radius r. Suppose we have a set of objects, (L^* galaxies, quasars, bright, star-forming galaxies ...) with space density n and we want to measure the amplitude of a given Fourier component with wave vector k of the relative density perturbation associated with this potential $\delta \sim -2k^2\Phi/3a^2H^2$. Now the precision with which the amplitude of a single relative density perturbation Fourier mode can be measured is comparable with the precision with which the fractional density perturbation can be measured in a single region of size equal to the associated length scale. This

[9] More subtle manifestations including those involving polarization are possible (Hu & Okamoto 2002), but this is the main effect.

[10] 21 cm redshift surveys provide an important complement to optical surveys but the survey volumes to date are comparatively modest.

is $\sim k^{3/2}n^{-1/2}$ and must exceed δ. This suggests that the density of such objects must exceed $\sim H_0^4/c^2\Phi k_m ax$ if local surveys can possible connect with the CMB. A slightly more careful calculation indicates that making such a connection with existing survey and CMB data from stage 1 is just possible and so it is worth exploring this further. If, this is achievable, then although the data increment will be small, its value will be much greater because it can act as a phase reference for anchoring the imperfectly specified modes measured by CMB observations.

2.4 Stage 3. Proposed Measurements

2.4.1 Ground-based CMB Telescopes

While there are exciting proposals for future space-based CMB measurements, most attention is currently focused on the next two generations - Stages 3 and 4 - of ground-based CMB telescopes proposed to deliver results in very roughly five and ten years respectively. While these are mostly focused on probing the physics of inflation and neutrinos, they will also improve the measurement of temperature and E-mode polarization on the scales $\ell \lesssim 200$ in which we are primarily interested with signal to noise $\sim 3 \times 10^{-4}$, roughly ten times better than Planck.

2.4.2 Survey Telescopes

Construction has begun on the Large Synoptic Survey Telescope (LSST) http://www.lsst.org/lsst/ which will commence a decade-long survey in 2022. It will survey half the sky (with very strong overlap with the ground-based CMB observations) in six bands detecting about ten billion galaxies out to $z \sim 2$ for L^* galaxies and $z \sim 6$ for bright, star-forming galaxies and quasars. Its primary cosmological goal is to perform a weak lensing survey to see if dark energy (and not just a cosmological constant) is responsible for cosmic acceleration, but it will, in reality, contribute to many more important cosmological measurements. LSST will only provide photometric redshifts, well-calibrated by large spectroscopic surveys such as DESI http://desi.lbl.gov which is projected to measure ~ 20 million redshifts to $z \sim 1$ starting in 2018. As with the CMB, optical weak lensing observations can provide "tomographic" distance information and, in principle, should lead to a better map of the long wavelength perturbations. Another source of new local data will be the Euclid space mission http://www.euclid-ec.org which is scheduled for a 2020 launch and which will carry out weak lensing, baryon acoustic oscillation and redshift space distortion measurements using 1.5 billion galaxies and 50 million redshifts over more than a third of the sky. The proposed WFIRST-AFTA http://wfirst.gsfc.nasa.gov also has an impressive program in observational cosmology. At radio wavelengths, the Canadian High Intensity Mapping Experiment (CHIME) http://chime.phas.ubc.ca will measure BAO out to $z \sim 2.5$ over half the sky.

2.4.3 Epoch of Reionization

There is a large effort underway to probe the Epoch of Reionization, EoR) $6 \lesssim z \lesssim 30$ through hydrogen line measurements. This is an exciting area of discovery as the relevant physics depends upon many factors, notably first star formation and galaxy assembly that are very hard to anticipate.[11] The experiments will probe an ideal range of comoving radius $\sim 8 - 12$ Gpc, interpolating between the CMB photosphere and local surveys, for either contributing to or expanding upon our incorporating our 3D potential map.

[11] JWST, http://www.jwst.nasa.gov scheduled for launch in 2018, will also help indirectly in understanding the universe during this epoch but seems unlikely to provide quantitative measurements of very large scale structure.

On a longer time scale there are ambitious plans to construct an international Square Kilometer Array (SKA) https://www.skatelescope.org. The long term goals include measuring the redshifts of a billion galaxies, performing weak lensing surveys and carrying out more sensitive surveys of the epoch of reionization. It is likely that the SKA capabilities and schedule will become better-defined over the lifetime of this proposed research program.

2.5 Stage 4. Future Possibilities

2.5.1 Limits to Mapping

Although it is proposed to examine the possible roles of many new facilities, it is of interest to consider the limitations to what could be learned about the idiosyncratic structure of our local universe on cosmological length scales with *any* conceivable observing facility operating for say a generational time (cf. Leclerc et al. 2014). This is mostly a question of spatial resolution though it is also partly a question of signal to noise.[12] This cosmic uncertainty principle might be expressible in terms of quite basic principles.

2.5.2 Fundamental Physics and Cosmology

Although this proposal began by making a clear distinction between the physics-driven research largely executed through statistical measurements and mapping our cosmological neighborhood out to $\sim$ 15 Gpc, it is pretty clear that, if it is successful, then the maps may actually be able to contribute to answering more fundamental questions. Here we list a few quick and relatively obvious issues to consider. It should be possible to explore space significantly outside our horizon using the inferred low frequency modes. In practice this will only amount to saying that something egregious - a colliding bubble or brane - has not happened close enough to our past light cone to have crossed it. This might possibly be of interest for some versions the landscape that conclude that the actually number of inflationary e-foldings that happened has to be very close to the minimum required. Another possible benefit is that knowing the gravitational potential allows one to predict the average bulk doppler shift and gravitational redshift that may be measurable for example using observations of Type 1a supernovae. More generally, both the CMB and the survey data, should be improved by insisting on agreement between them. More accurate cosmological parameters should ensue, because we are not doing two separate marginalizations over the phase information at each epoch. Another reward should be helping to handle the vexacious problem of bias by being able to specify the underlying density field. This will, in turn, improve many cosmological tests.

Undoubtedly, the truly ambitious goal for this program which would be transformative, would be to connect individual modes around the first acoustic peak where their amplitude is statistically large, to the corresponding BAO modes measured in large redshift surveys. This would greatly improve both the accuracy (not just the precision) with which cosmological parameters can be measured because it is essentially kinematic and the k's can be calculated for a given set of basic parameters and do not have to be calibrated astronomically. As of this writing, the goal seems out of reach but the possibility is certainly worth further consideration.

[12] An interesting analogy is the measurement of helioseismic modes where the basic spectroscopic job is largely complete and research interest has moved on to understanding the physics of the excitation and damping of the modes.