HW4 Q1.1 we know two matched image points

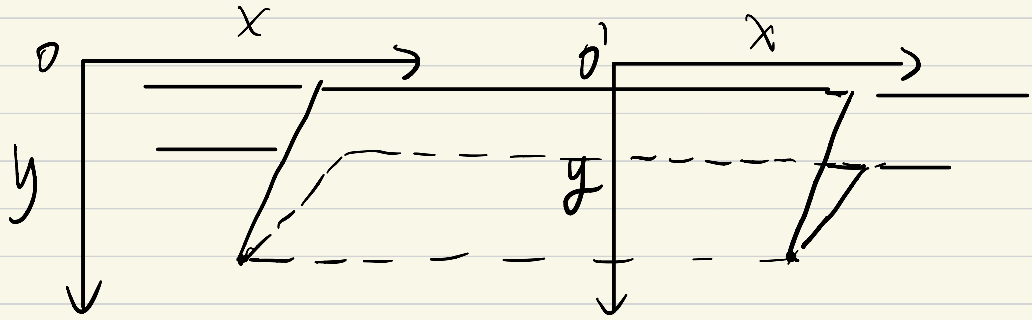
$$x'^T F x = 0 \qquad F = \begin{pmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{pmatrix}$$

$$x' = x = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

so $$(0\ 0\ 1) F \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = f_9 = 0$$

F_{33} element is zero.

Q1.2. because pure traslation along x-axis



because baseline parellel to x-axis two image plane contain x-axis. The intersection of image plane and epipolar plane parellel to x-axis.

assume principal axes pass through the images' origins, respectively

$t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$E = R[t_x]$ $\quad R = I \quad E = [t_x] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

for any point $x = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$

$$l = Ex = \begin{pmatrix} 0 \\ -1 \\ v \end{pmatrix}$$

so $-y + v = 0$

l: $y = v$ parellel to x axis.

Q1.3. rigid motion

from x to x' in the two optical center coordinate system.

$$x' = R_{rel}(x - t_{rel})$$

and x, t_{rel}, x' are coplanar $\Rightarrow$ $(x - t_{rel})^T (t_{rel} \times x) = 0$

$$(x - t_{rel})^T R_{rel}^T = x'^T$$

rotation matrix : $R_{rel} R_{rel}^T = I$ $\Rightarrow$ $(R_{rel}^T)^{-1} = R_{rel}$

$$(x - t_{rel})^T = x'^T R_{rel}$$

$$x'^T R_{rel} (t_{rel} \times x) = 0 \qquad t_{rel} = [t_{rel\,\times}]$$

$$x'^T (R_{rel} [t_{rel\,\times}]) x = 0 \qquad [a_\times] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

$$E = R_{rel} [t_{rel\,\times}]$$

for y y' in the image coordinate

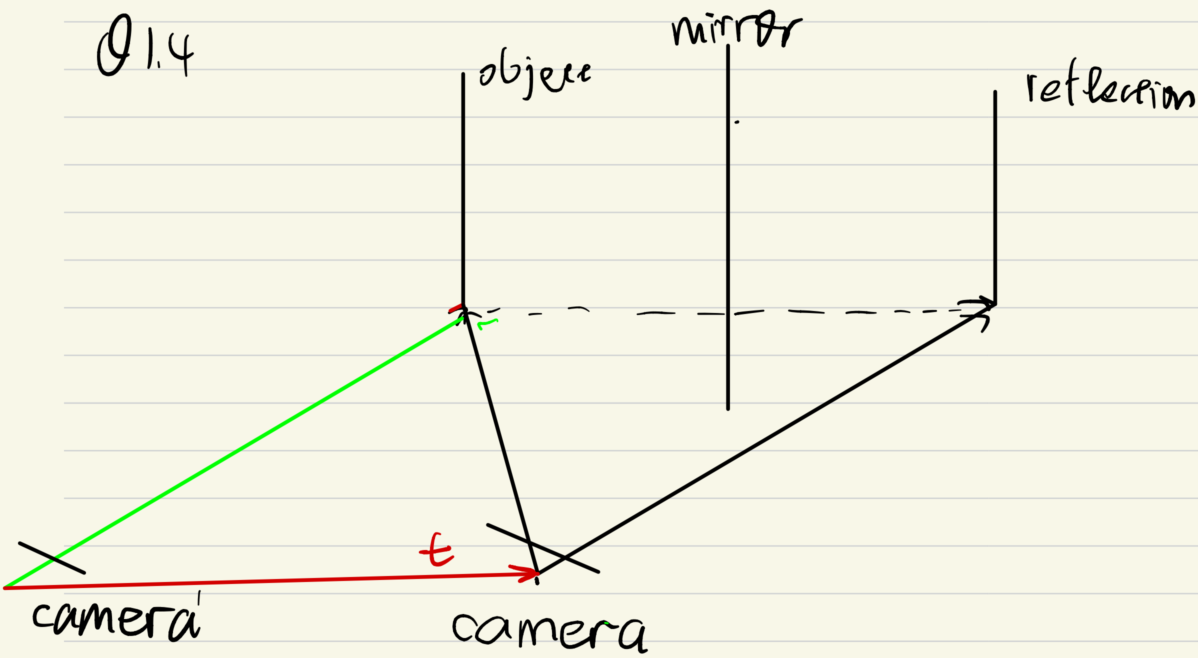
$$y = Kx \qquad y' = Kx'$$

$$x = K^{-1} y \qquad x' = K^{-1} y' \qquad x'^T = y'^T K^{-T}$$

$$y'^T K^{-T} (R_{rel} [t_{rel\,\times}]) K^{-1} y = 0$$

$$F = K^{-T} R_{rel} [t_{rel\,\times}] K^{-1}$$

Q1.4



F: skew-symmetric

$F^T = -F$

this is equivalent to another camera' in the graph obtained by pure translation

$x'^T F x = 0$

every point of the reflection to the camera is equivalent to the point on the object to camera'

as there is no rotation on the camera

$R = I$ $\qquad E^T = -E$

$E = R[t_\times] = [t_\times]$ is skew-symmetric

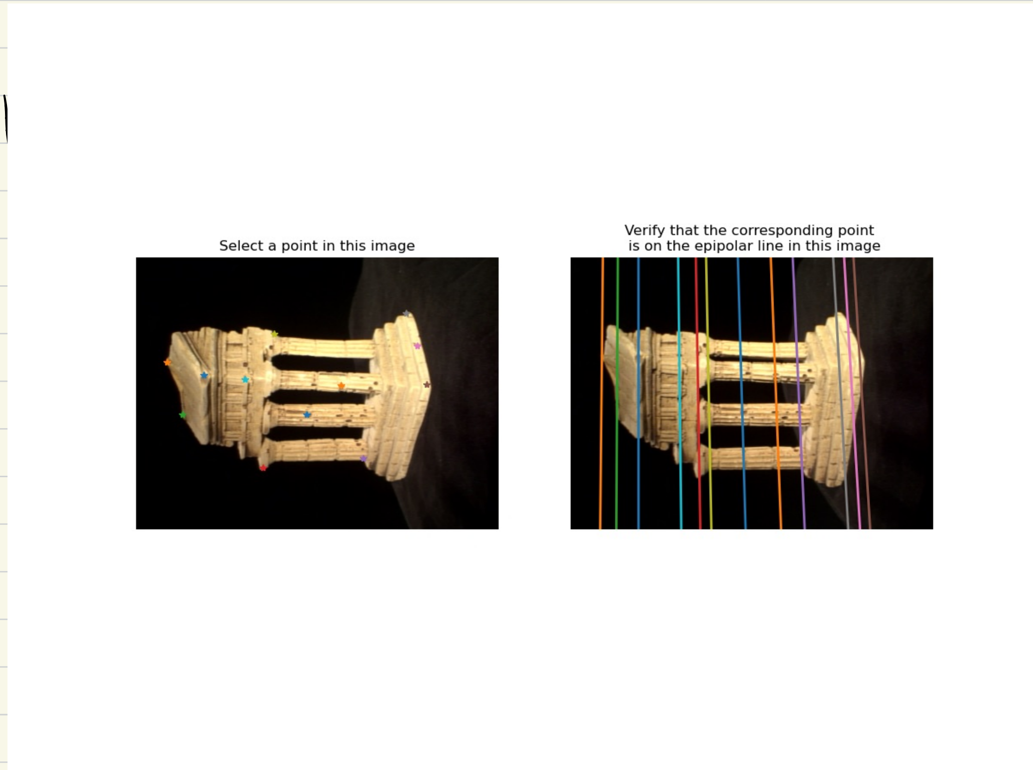
$F = K^{-T} E K^{-1}$ for any x

let $x' = K^{-1}x$

$x^T F x = x^T K^{-T} E K^{-1} x = (K^{-1}x)^T E (K^{-1}x) = x'^T E x' = 0$

thus F is skew-symmetric

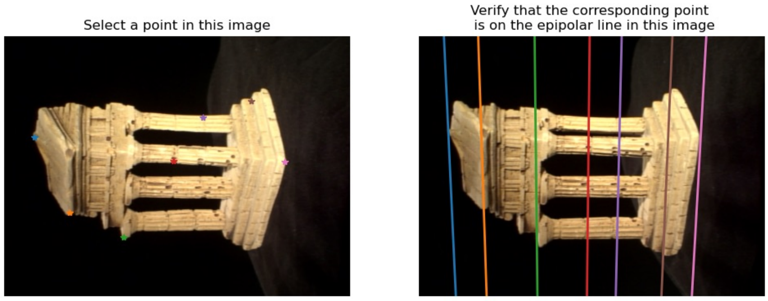
Q2.1



F

```
[[ 2.87029707e-10 -2.98612949e-08  2.16644044e-03]
 [-3.35284538e-07 -7.56035054e-10  3.95804175e-05]
 [-2.08426410e-03 -1.19206648e-06 -8.43847667e-03]]
```

Q2.2



F =

```
[[ 2.22182447e-08 -7.79776117e-08  1.59048178e-03]
 [ 3.47471654e-07 -1.01062468e-08 -8.99183767e-05]
 [-1.66745528e-03  6.90143928e-05  8.01657179e-03]]
```

Q3.1

E =

```
[[ 6.63502510e-04 -6.92775579e-02  3.28277981e+00]
 [-7.77852873e-01 -1.76032995e-03 -9.45591784e-02]
 [-3.29462929e+00 -1.58790884e-02 -1.39003422e-03]]
```

Q3.2

$p_1^i = C_1 P_i \qquad p_2^i = C_2 P_i$

$C_1 : 3\times 4 \qquad C_1 = \begin{pmatrix} r_1^T \\ r_2^T \\ r_3^T \end{pmatrix}$

$z_{1i}\vec{x}_{1i} = C_1 X_i$

$z_{2i}\vec{x}_{2i} = C_2 X_i \qquad X_i = \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$

$$z_{1i}\begin{pmatrix} x_{1i} \\ y_{1i} \\ 1 \end{pmatrix} = \overset{C_1}{\overset{\|}{\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & 1 \end{pmatrix}}}\begin{pmatrix} x_i \\ y_i \\ z_i \\ 1 \end{pmatrix}$$

we have

$$\begin{pmatrix} z_{1i}x_{1i} \\ z_{1i}y_{1i} \\ z_{1i} \end{pmatrix} = \begin{pmatrix} a_{11}x_i + a_{12}y_i + a_{13}z_i + a_{14} \\ a_{21}x_i + a_{22}y_i + a_{23}z_i + a_{24} \\ a_{31}x_i + a_{32}y_i + a_{33}z_i + 1 \end{pmatrix}$$

treat them as dot product

$$z_{1i} = r_3^T \cdot X_i$$

$$(r_3^T X_i)\, y_{1i} = (r_2^T X_i)$$

$$(r_3^T X_i)\, x_{1i} = (r_1^T X_i)$$

thus $\begin{bmatrix} r_3^T y_{1i} - r_2^T \\ r_3^T x_{1i} - r_1^T \end{bmatrix} X_i = 0$

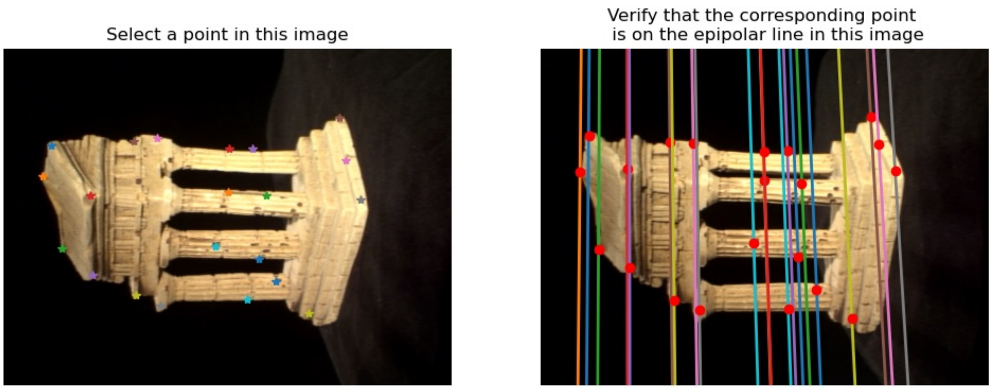
similiarly we have the same equations for

$$z_{2i}\, x_{2i} = C_2 X_i$$

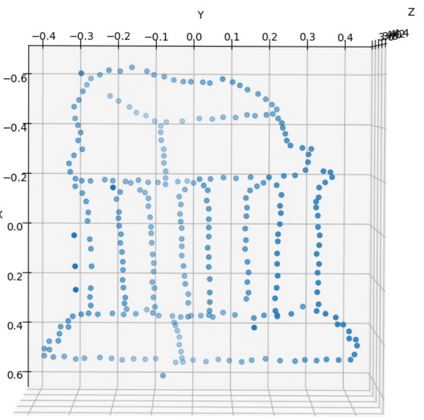
if $C_2 = \begin{pmatrix} r_1'^T \\ r_2'^T \\ r_3'^T \end{pmatrix} \qquad \vec{x}_{2i} = \begin{pmatrix} x_{2i} \\ y_{2i} \\ z_{2i} \end{pmatrix}$

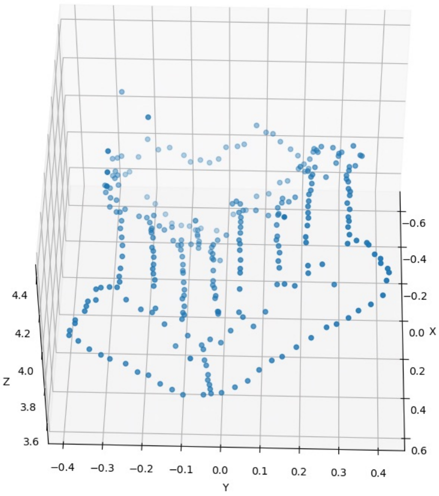
$$\underbrace{\begin{bmatrix} r_3^T y_{1i} - r_2^T \\ r_3^T x_{1i} - r_1^T \\ r_3'^T y_{2i} - r_2'^T \\ r_3'^T x_{2i} - r_1'^T \end{bmatrix}}_{\substack{\| \\ A}} \begin{pmatrix} x_i \\ y_i \\ z_i \\ 1 \end{pmatrix} = 0$$

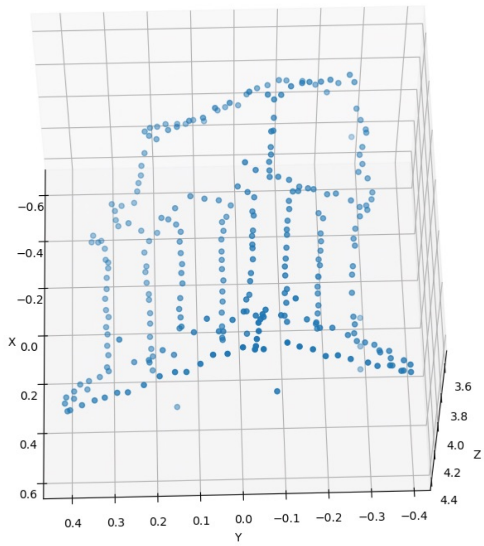
Q4.1

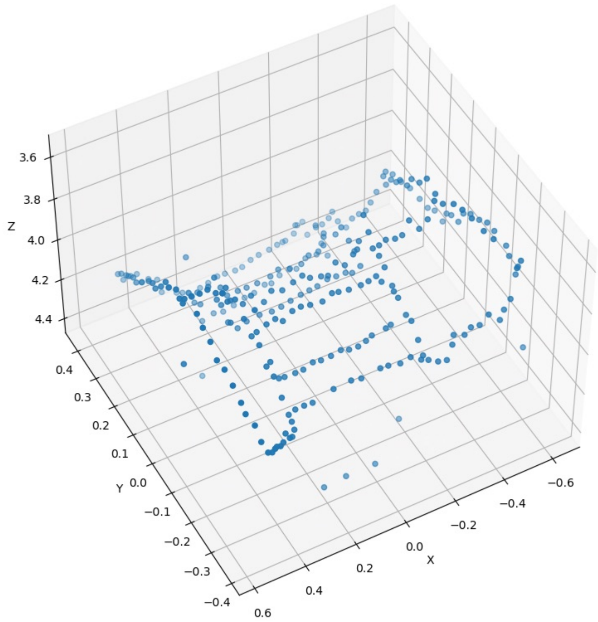


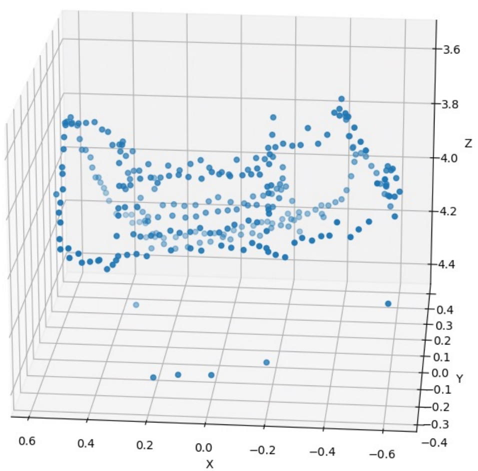
Q4.2



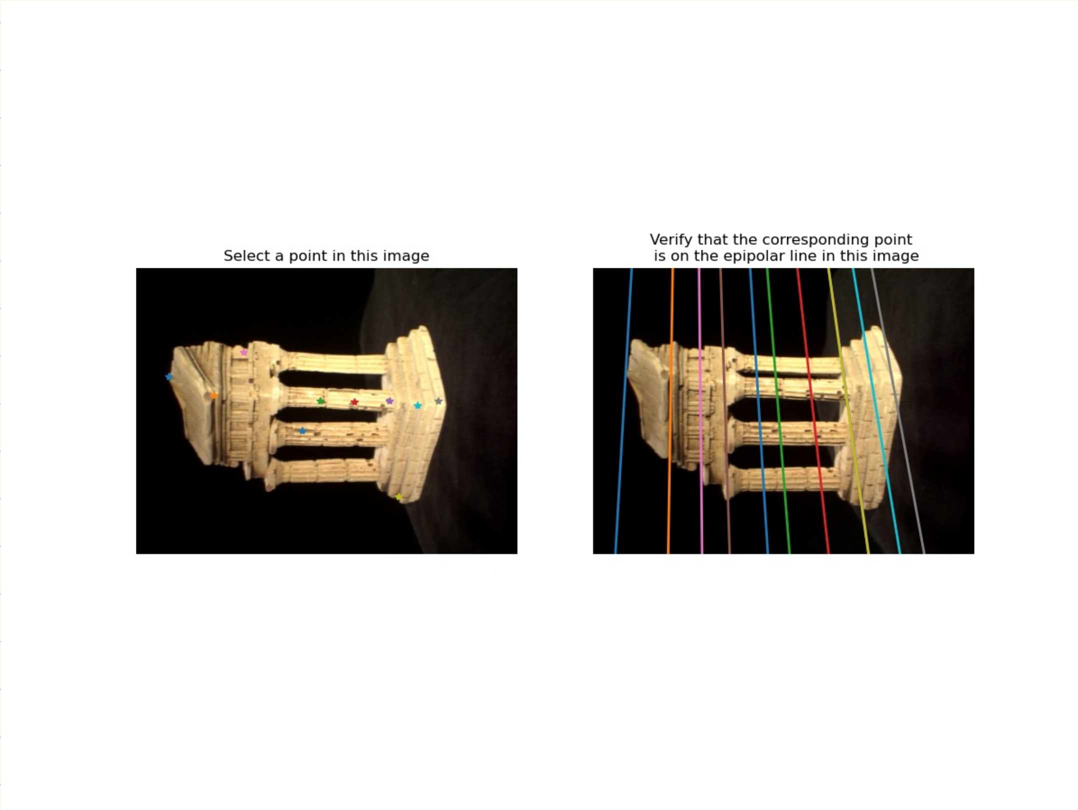




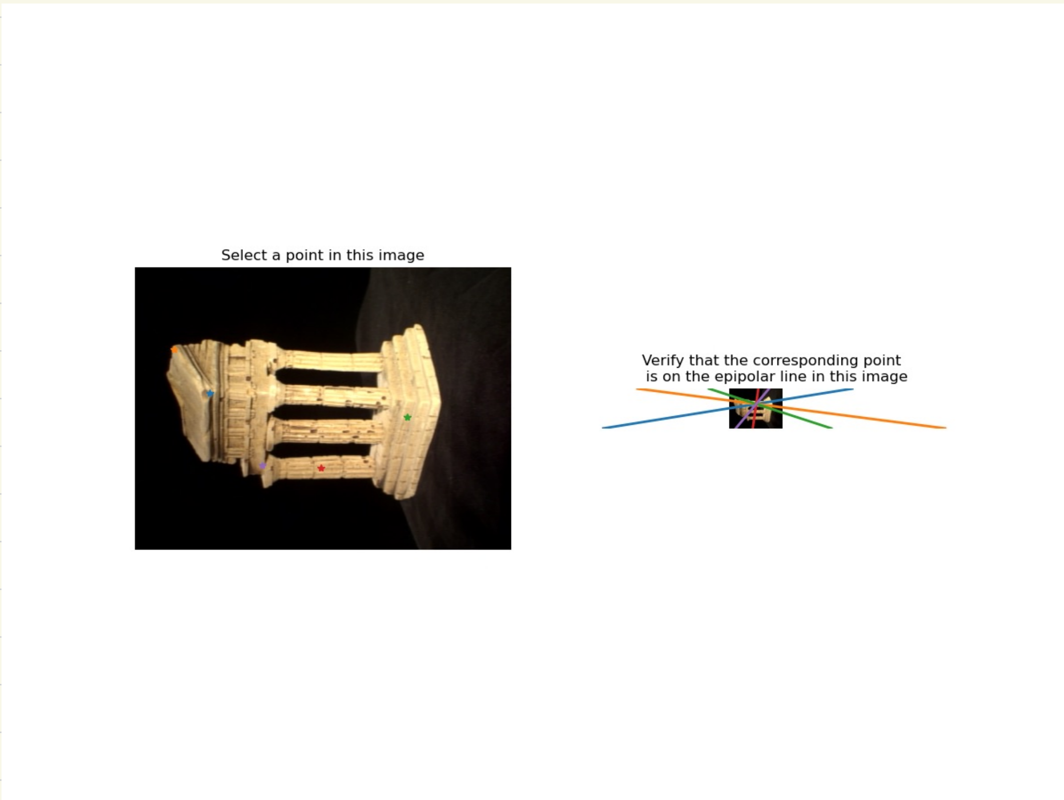




Q.5.1



Ransac F



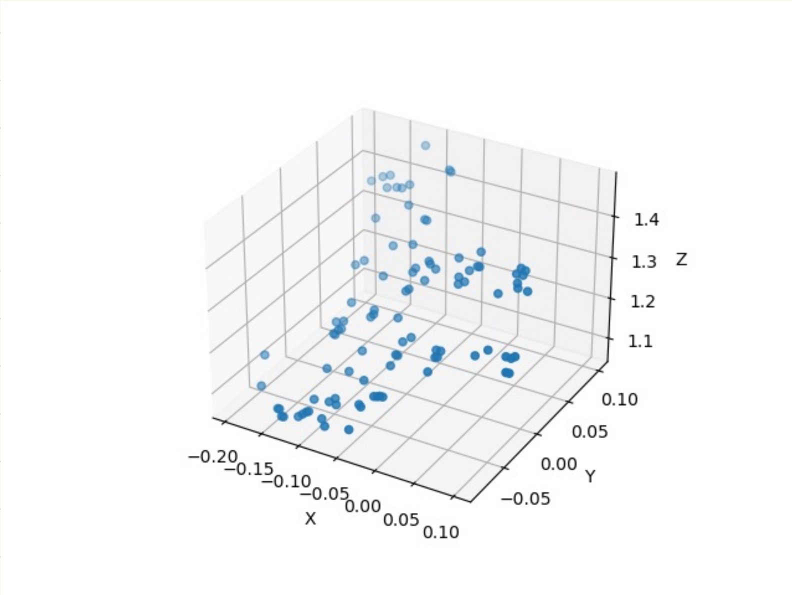
eightpoint F

error matrix: $|x'^T F x|$

threshold: error < 0.005 iterate 2000 times.

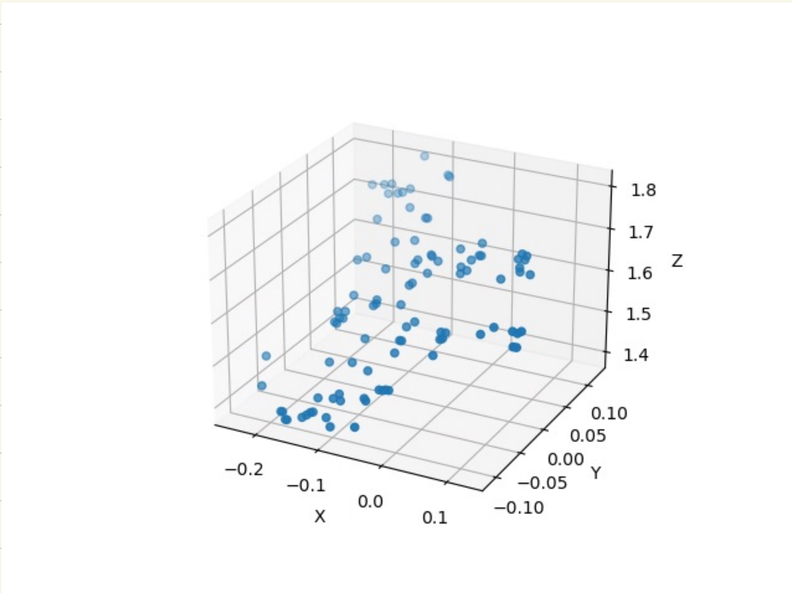
use F8 for all inliers to get the best F.

Q 5.3



without optimization

error: 23628.25273



after optimization

$4.111254773126 \times 10^{-23}$