COMP5421 Homework Assignment 2

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1 Q1.5 image with the detected keypoints

Figure 1: Keypoing Detection with edge suppression for chickenbroth_01.jpg

2 Q2.4 image feature matches

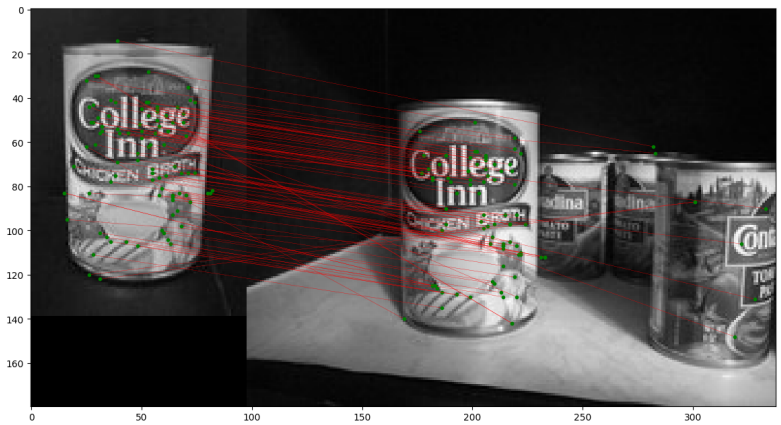

Figure 2: Featrue matches for two chickenbroth images using BRIEF Descriptor

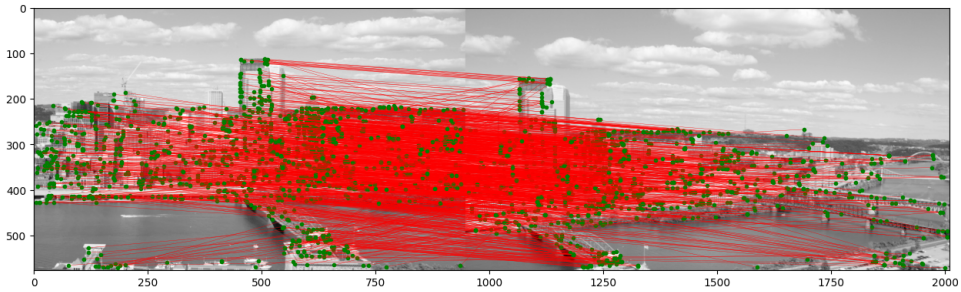

Figure 3: Featrue matches for two incline images using BRIEF Descriptor

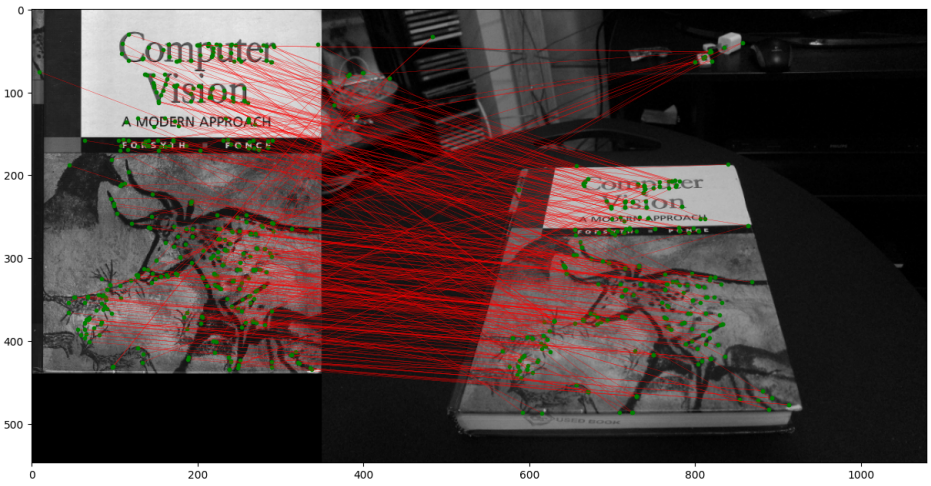

Figure 4: Featrue matches for computer vision text book on desk

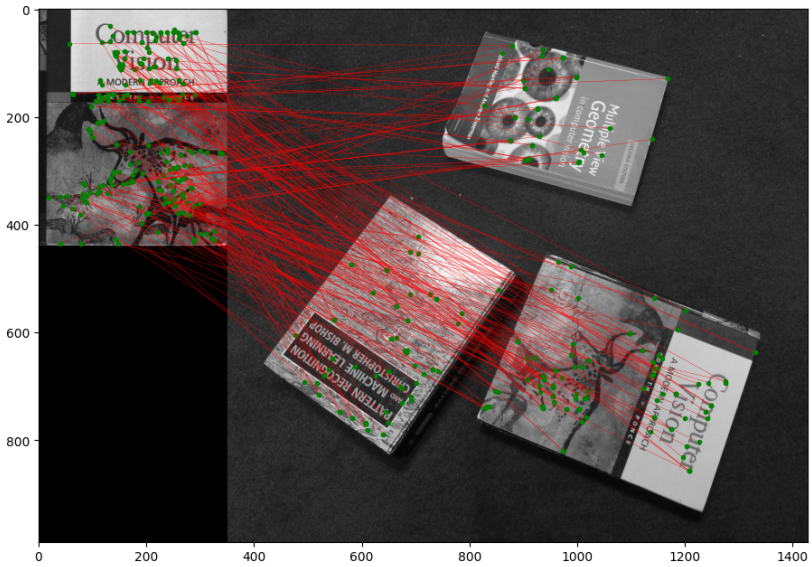

Figure 5: Featrue matches for rotated computer vision text book on floor

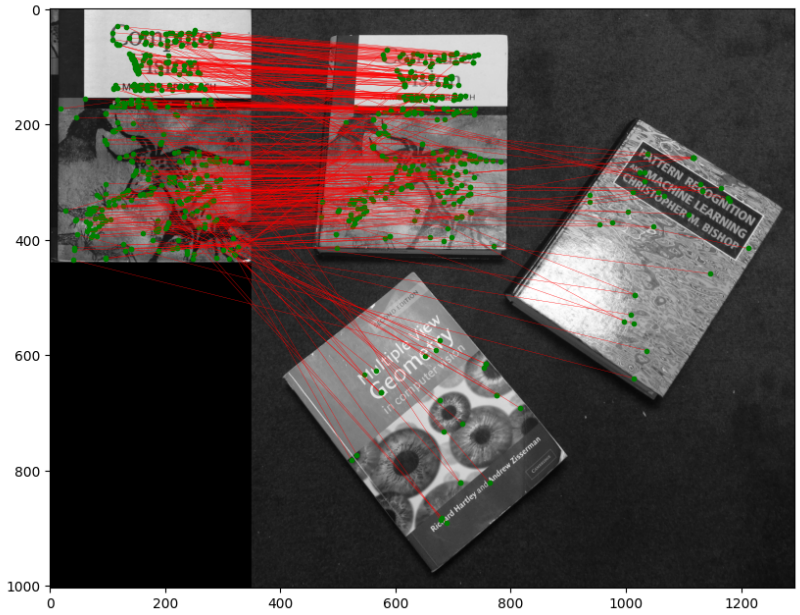

Figure 6: Featrue matches for computer vision text book on floor

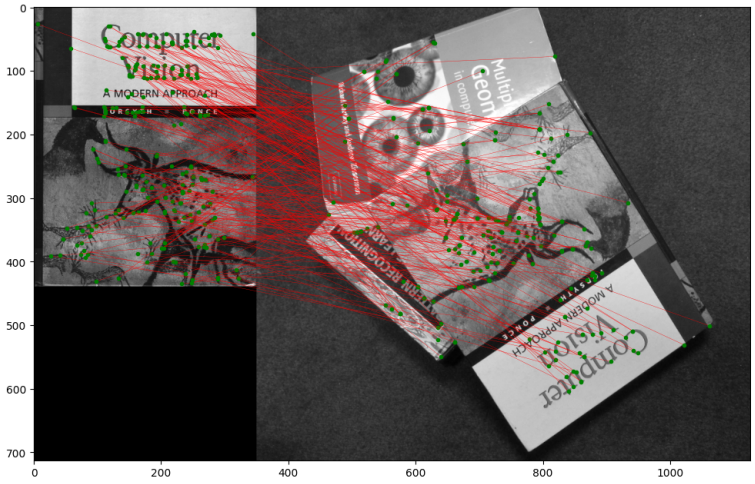

Figure 7: Featrue matches for computer vision text book on a pile of books

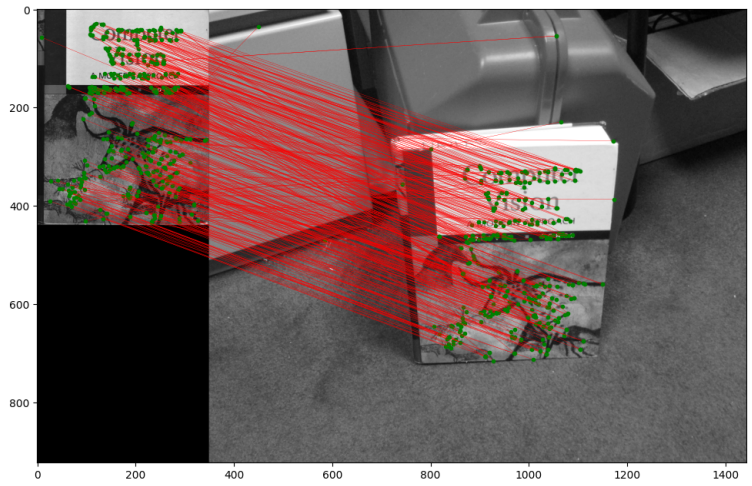

Figure 8: Featrue matches for a standing computer vision text book

When there is rotation, the matches will become worse.
When the background of the image is messy or exists silimar objects, then matches will become worse

3 Q2.5 number of correct feature matches

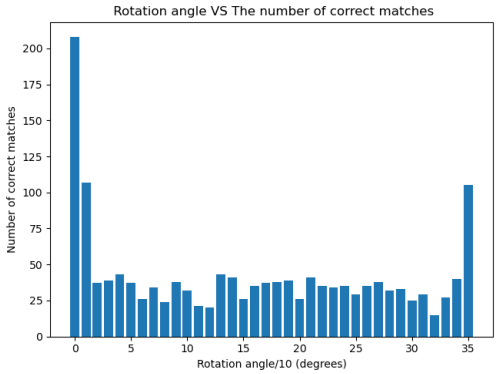

Figure 9: a bar graph showing rotation angle(10 degree each rotation) vs the number of correct matches

When the patch of image is rotated, the descriptor will give different vectors to describe the same two patches with different orientation. Because each time the choice for 256 testing pair is the same and it doesn't change with the rotation

4 Q3.1 find the homography H

(a)

$$\lambda_n \hat{x}_n = H\hat{u}_n \quad for \quad n = 1 : N$$

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$$

$$\hat{x}_n = \begin{bmatrix} x_{n1} \\ x_{n2} \\ 1 \end{bmatrix}, \hat{u}_n = \begin{bmatrix} u_{n1} \\ u_{n2} \\ 1 \end{bmatrix}$$

This is equivalent to

$$\begin{aligned} \lambda_n x_{n1} &= h_1 u_{n1} + h_2 u_{n2} + h_3 \\ \lambda_n x_{n2} &= h_4 u_{n1} + h_5 u_{n2} + h_6 \\ \lambda_n &= h_7 u_{n1} + h_8 u_{n2} + h_9 \end{aligned}$$

eliminate λ_n, Write the equations in matrix form

$$\begin{bmatrix} -u_{n1} & -u_{n2} & -1 & 0 & 0 & 0 & x_{n1}u_{n1} & x_{n1}u_{n2} & x_{n1} \\ 0 & 0 & 0 & -u_{n1} & -u_{n2} & -1 & x_{n2}u_{n1} & x_{n2}u_{n2} & x_{n2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = 0$$

$2N$ equations like this, and this gives matrix A, such that

$$A = \begin{bmatrix} -u_{11} & -u_{12} & -1 & 0 & 0 & 0 & x_{11}u_{11} & x_{11}u_{12} & x_{11} \\ 0 & 0 & 0 & -u_{11} & -u_{12} & -1 & x_{12}u_{11} & x_{12}u_{12} & x_{12} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -u_{N1} & -u_{N2} & -1 & 0 & 0 & 0 & x_{N1}u_{N1} & x_{N1}u_{N2} & x_{N1} \\ 0 & 0 & 0 & -u_{N1} & -u_{N2} & -1 & x_{N2}u_{N1} & x_{N2}u_{N2} & x_{N2} \end{bmatrix}$$

$$A * h = 0$$

(b) 9 elements are in the vector h
(c) 4 points correspondences are required. Because $dofH = 8$, and each correspondence gives 2 equation
(d) consider least square problem:

$$||Ah||_2 = 0$$
$$h^T A^T A h = 0$$

define

$$r(x) = \frac{x^T B x}{x^T x}$$

where $x \in \mathbb{R}^9$ and $B = A^T A$, we want the x to minimize $r(x)$
calculate the partial derivatives of $r(x)$ with respect to the coordinates x_j

$$\frac{\partial r(x)}{\partial x_j} = \frac{\frac{\partial}{\partial x_j} x^T B x}{x^T x} - \frac{x^T B x \frac{\partial}{\partial x_j} x^T x}{(x^T x)^2}$$
$$= \frac{2(Bx)_j}{x^T x} - \frac{(x^T B x) 2 x_j}{(x^T x)^2} = \frac{2}{x^T x}(Bx - r(x)x)_j$$

collect those partial derivatives into a 8-vector,

$$\nabla r(x) = \frac{2}{x^T x}(Bx - r(x)x)$$

consider places s.t. $\nabla r(x) = 0$
then $Bx = r(x)x$, x is the eigenvector
so we find x is the eigenvector of B correspond to the smallest eigenvalue

5 Q6.3 generate Panorama

Figure 10: Panorama figure generated by incline_L.png and incline_R.png

6 Q7.2 Augmented Reality

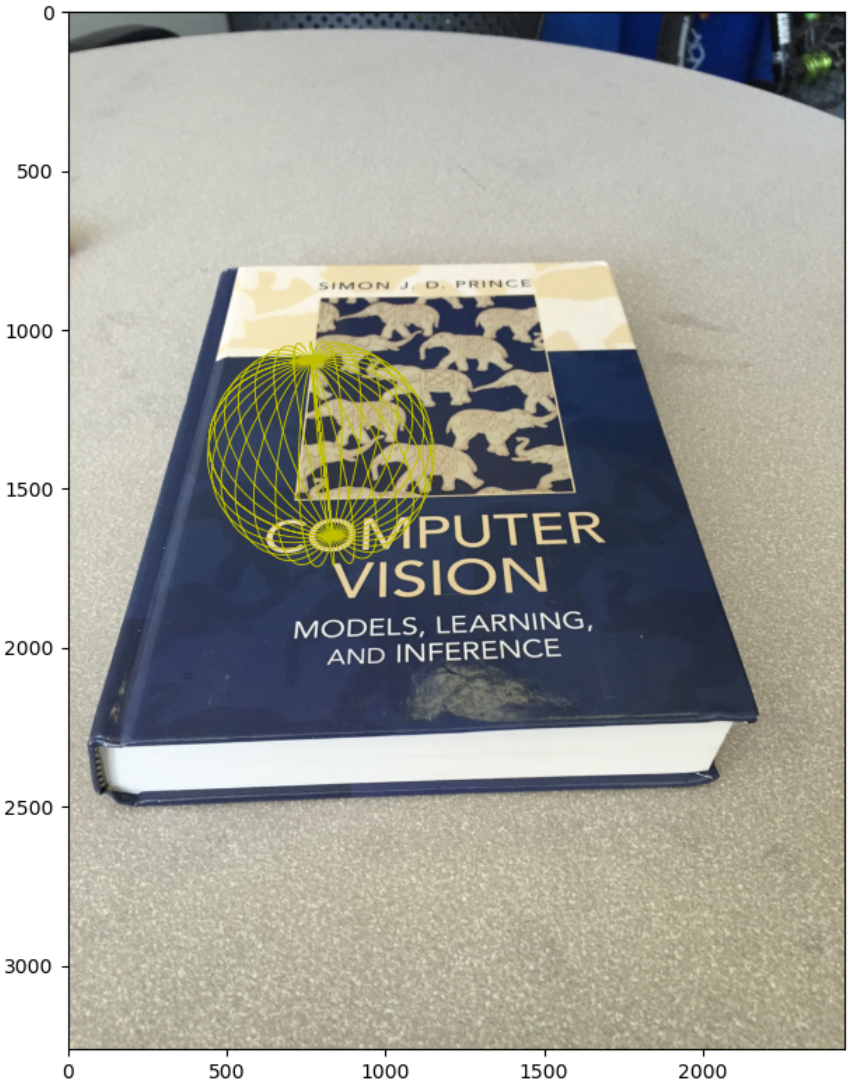

Figure 11: Augmented Reality image, putting a yellow ball on top of the letter O of the computer vision text book