

Questions

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1 Questions

1.1 Miscellaneous

1. Find all solutions to the equation $5z^5 + 15z^4 + 20z^3 + 15z^2 + 5z + 1 = 0$.
2. Consider the function $f : \mathcal{P}(\mathbb{N}) \rightarrow \mathbb{N}$ defined by:

let p_n be the n th prime number, and let $S = \{n_1, n_2, n_3, \dots\}$ then f is given by:

$$f(S) = p_1^{n_1} * p_2^{n_2} * \dots$$

We claim this implies there is an injection from the powerset of naturals to the naturals and so we provide a counterexample to Cantor's Theorem. Where is the mistake?

1. Show that the Fundamental Theorem of Algebra and the statement that a polynomial of degree n has exactly n complex roots (including multiplicity), are equivalent.
2. Show that if G and H are groups of cardinality p where p is prime, then G and H are isomorphic.
3. Show that there are no solutions to the diophantine equation $a^2 + b^2 = c^2$ if both a and b are odd.

4. Show that if R is a finite integral domain, then it is also a field (Wedderburn's little theorem).
5. Find a closed form for $\sum_{k=1}^n k \cos(kx)$.

1.2 Computational

1. Find the smallest integer n such that $n \ln n > 3^{197}$