Hermitian Matrices

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1 Definition

We say a complex square matrix A is **Hermitian** if it is equal to its own **conjugate transpose**, ie $\forall i, j$ we have that:

$$A_{ij} = \overline{A_{ji}}$$

Alternate names for the conjugate transpose of a matrix include the **adjoint** or **transjugate**, and may be referred to by any of the following symbols:

$$A^H \equiv A^* \equiv \overline{A^T}$$

We can view Hermitian matrices as an extension of real symmetric matrices as they share many of the same properties.

2 Equality with Adjoint

3 The Spectral Theorem

We give the Spectral Theorem in the complex case: *If an $n \times n$ matrix A is Hermitian, then there exists a **orthonormal** basis for $\mathbb{C}^n$ consisting of the* *eigenvectors of A such that all corresponding eigenvalues are real.*

This theorem makes many claims, each of which we will prove in turn:

3.1 All eigenvalues of A are real

Let λ be an eigenvalue of A, with corresponding eigenvector v with entries given by $a_k + ib_k$. Then starting with $Av = v$:

$$\begin{aligned} (v^*)^T Av &= (v^*)^T \lambda v \\ &= \lambda (v^*)^T \lambda v \end{aligned}$$