The Limit Superior

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1 Definition

We define the **limit superior** of a sequence a_n as follows:

$$\limsup_{n\to\infty} a_n = \lim_{n\to\infty}\left(\sup_{m\geq n} a_m\right)$$

An equivalent but perhaps more intuitive definition is:

$$\limsup_{n\to\infty} a_n = \sup S$$

Where S is the set of limits of all **convergent** subsequences of a_n. The **limit inferior** is defined similarly:

$$\liminf_{n\to\infty} a_n = \lim_{n\to\infty}\left(\inf_{m\geq n} a_m\right)$$

2 Interpretation

Consider some sequence a_n where the limit superior is not infinite. Then the limit superior b is the smallest real number s.t. $\forall\epsilon > 0$, there exist only a finite n s.t. $a_n > b + \epsilon$.

3 Examples

Define $a_n = \sin n\pi$

4 Questions

Show that a sequence a_n converges iff:

$$\limsup_{n\to\infty} a_n = \liminf_{n\to\infty} a_n$$

Show that if the sequences a_n and b_n are bounded above then:

$$\limsup_{n\to\infty} (a_n + b_n) \leq \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n$$

and find a similar relation for the limit inferior.