Probabilistic Algorithms Project
Comparing heuristics for TSP

Consider a TSP instance with $N = 411$ cities, where the distance between two cities with coordinates (x_1, y_1) and (x_2, y_2) is given by the Euclidian norm:

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Denote $L(\sigma) = \sum_{i=1}^{N} d(\sigma(i), \sigma(i+1))$ the length of the Hamiltonian cycle represented by σ, where $\sigma = (\sigma(1), \ldots, \sigma(N))$ is a permutation of the set of N cities[1]. The coordinates are stored in the file "TSP_411.txt", where each line has the form "city_index x-coordinate y-coordinate". Conduct a comparison analysis of the performances of various optimisation heuristics applied to this problem, where the list of algorithms includes:

I. Construction heuristics

a. Best insertion;

b. Shortest edge;

c. Saving heuristic;

II. Improvement heuristics

i. Greedy local search using as small move

(a) Swap move,

(b) Translation move,

(c) Inversion move

(d) Mixed move : all the previous moves, each selected with a probability of 1/3.

The total number of moves is set to $10 * N^2$ in each case;

ii. Simulated Annealing algorithm using

(a) Metropolis criterion

(b) Heat bath condition.

[1] You are free to use any type of representation for TSP (permutations, adjacent matrices, incidence matrices) if you consider appropriate for your implementation. In each case, $L(\cdot)$ returns the length of a Hamiltonian cycle.

For each criterion, generate the new configuration using the four variants of small moves described at point [i.]. The initial temperature is set according to the ad-hoc procedure, the cooling schedule is exponential with $T_{new} = 0.95 T_{old}$ and the number of moves at a fixed temperature is set to 1000.

Denote $\mathcal{L}$ the sequence $\{L(\sigma_1^*), ..., L(\sigma_m^*)\}$ of the best solutions generated after calling each implemented algorithm $m = 30$ times [2]. The written report of the project must include:

- For each implemented algorithm, a table showing the minimum of $\mathcal{L}$, the maximum of $\mathcal{L}$, the average of $\mathcal{L}$ and the 95% confidence interval for this average.
- For each implemented algorithm, the plot of the nodes(cities) with the trace of the best solution after $m = 40$ runs (i.e., of $\min(\mathcal{L})$).
- Performance plots:
  - For each local search algorithm, consider

    $$\mathcal{L}(k) = \{L_{(1)}(\sigma_k), L_{(2)}(\sigma_k), ..., L_{(m)}(\sigma_k)\}$$

    where $L_{(i)}(\sigma_k)$ is the best solution after k moves during the i^{th} call. Plot the following graphs: $\min(\mathcal{L}(\sigma_k))$, $\text{mean}(\mathcal{L}(\sigma_k))$ and $\max(\mathcal{L}(\sigma_k))$ vs. number of moves k, with $k = 1..10 * N^2$
  - For each version of simulated annealing algorithm (a total of 8), consider the set $\mathcal{L}_T$ representing the lengths $L(\cdot)$ for all 1000 solutions σ processed at a given temperature T. Plot the following graphs: $\min(\mathcal{L}_T)$, $\text{mean}(\mathcal{L}_T)$ and $\max(\mathcal{L}_T)$ vs. current temperature T (based on a single run).
- Pairwise comparisons of algorithms using two-sample tests. You must compare:
  - Best insertion with Shortest edge;
  - Saving heuristic with Shortest edge;
  - all four variants of local search algorithm;
  - Metropolis SA with Heat bath SA (only for mixed move versions);

NOTE - for the teams with only one member, the list of algorithms will not include Saving heuristic and Local Search algorithms. For the teams with three members, the list of algorithms will include the following Ant Colony Optimisation version:

[2] If your implementation of a particular algorithm takes too much time to run, you may reduce m, but justify, in the written report, your choice.

Step 0. Set the initial pheromone value on each edge: $\tau_{ij} = 1$, $\forall i, j = 1..N$; Put one ant in each node (i.e. $M = N$) and set the tabu list σ_t (the set of visited nodes) of each ant t ($t = 1..M$) as empty. Set the following parameters: $\alpha = 0.5$, $\beta = 0.5$, $\mu = 0.5$, $Q = 100$.

Step 1. Repeat N times

Move: While the M roundtrips are not yet constructed:

(i) For each ant t, select a new node according to the transition probability:

$$p_{ij} = \begin{cases} \frac{\tau_{ij}^{\alpha} * d(i,j)^{-\beta}}{\sum\limits_{k=1..N, k \notin \sigma_t} \tau_{ik}^{\alpha} d(i,k)^{-\beta}}, \text{ if node } j \text{ not yet visited} \\ \\ 0, \text{ if } j \text{ already visited} \end{cases}$$

where i is the last node visited by the ant.

(ii) Add the new node to the tabu list σ_t of the ant.

Update: Update the amounts of pheromones on each edge and clear all tabu lists. Let be $L(\sigma_t^*)$ the length of the roundtrip σ_t^* constructed by ant t in the step **Move**.

(i) For each ant t, calculate $\Delta_{ij}^{t} = \begin{cases} Q/L(\sigma_t^*) \text{ if } ij \text{ is an edge from } \sigma_t^* \\ 0 \text{ if } ij \text{ is not an edge from } \sigma_t^* \end{cases}$

(ii) $\tau_{ij} = \mu * \tau_{ij} + \sum\limits_{t=1..M} \Delta_{ij}^{t}$

(iii) Save the best σ_t^* and empty the tabu list for each ant.