

Probabilistic Algorithms

Homework 3

P1. (100 points)

Implement in MATLAB the algorithm SIMPLEX for the linear programming problem

$$\min L(\mathbf{d}) = \mathbf{c}^T \mathbf{d} + \vartheta = \sum_{i=1}^n c_i d_i + \vartheta, \text{ under the constraints}$$

$$\begin{cases} \mathbf{A}\mathbf{d} & \leq \mathbf{b} \\ d_i & \geq 0, i = 1..n \end{cases}$$

$$\text{where } \mathbf{A} = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \in R^{m \times n}, \mathbf{d} = \begin{pmatrix} d_1 \\ \vdots \\ d_n \end{pmatrix} \in R^n, \mathbf{c} = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} \in R^n, \mathbf{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} \in R^m, b_i > 0, i = 1..m.$$

The MATLAB function

function `x = simplex(c, A, b)`

will receive as input parameters the vector **c**, the array **A** and the vector **b**, and will return the optimal solution **d*** in the vector **x**.

Remark: you may test your implemented function using the numerical example for the LP problem detailed in the course support.

P2 (50 points bonus)

Implement a MATLAB function

function `[n, c] = parse(s)`

which receives as input a string **s** corresponding to a linear constraint and returns two outputs, the number of decision variables (**n**) and the vector of corresponding coefficients (**c**). Examples:

1. for the input '4X1 - 5X2 + X3 < 12', the function returns $n = 3$ and $c = [4, -5, 1, 12]$;
2. for the input 'X1 - 7X3 + 2X5 > 3', the function returns $n = 5$ and $c = [1, 0, -7, 0, 2, 3]$;

You may consider any useful hypothesis about the format of the string **s** (e.g., all variables are expressed as a space character, followed by an integer number (including 1), followed by capital letter 'X', followed by the index of the variable (integer number), followed by a space character;).