Probabilistic Algorithms
Homework 1

P1 (100 points)

Consider the quarter circle of radius 1 inside the square of edge 1 (see the figure).
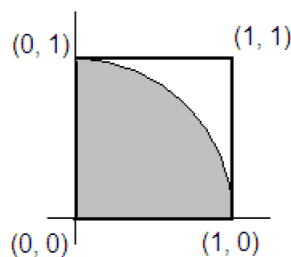

Figure 1: Quarter circle inside unit square

1. Consider the following experiment : select randomly a point inside the unit square (i.e. generate a tuple (x, y) from the uniform distribution on $[0, 1] \times [0, 1]$). What is the probability that such a point, randomly generated, falls inside the quarter circle?

2. Based on the answer for question a), write a MatLab function `pi_estimate(N)` that calculates an estimate of π using N trials of this experiment.

3. For a fixed N, call the `pi_estimate()` function 100 times and calculates, based on the vector of 100 estimates for π, the following values : the minimum estimate (π_{min}), the maximum estimate (π_{max}), the average estimate (π_{mean}), the standard deviation of estimates $(\Delta \pi)$.

4. Fill the following table:

N	π_{min}	π_{max}	π_{mean}	$\Delta \pi$
100				
1000				
10000				
100000				
1000000				
10000000				

P2 (25 points bonus)

Consider a two-sided error Monte-Carlo algorithm which may return two outputs (decisions),

$$\begin{cases} \text{"}yes\text{"}, & P(\text{"}yes\text{"} \text{ is wrong}) = \varepsilon < 1/2 \\ \text{"}no\text{"}, & P(\text{"}no\text{"} \text{ is wrong}) = \varepsilon < 1/2 \end{cases}$$

Suppose that this algorithm is running 10 times on the same input and generate the following sequence of decisions: "yes", "no", "no", "yes", "yes", "yes", "no", "no", "yes", "yes".

1. Propose a rule that allows the user to choose the "right" decision after N trials on the same input. What gives your rule for this particular case $(N = 10)$?

2. If $P(\text{"}yes\text{"}$ is wrong$) \neq P(\text{"}no\text{"}$ is wrong$)$, do you think you must modify the rule ?

P3 (25 points bonus)

Suppose we have a random number generator that, at each call, returns one of the two values $\{a, b\}$ with a probability of 0.5. Propose a method of how to use this generator to get uniform numbers on $(0, 1)$.