Final Exam - Fall 2024

Probability Fundamentals

Consider a nine sided die (with sides 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9). You roll this die once. Let:

- A be the event that a given roll yields an even number (this includes 0).
- B be the event that a given roll is greater than or equal to three.
- C be the event that the number appears in the song title "867-5309/Jenny" by Tommy Tutone.

1. Find $\mathrm{P}(A)$, $\mathrm{P}(B)$ and $\mathrm{P}(C)$

2. Find $\mathrm{P}(A \cup B)$

3. Find $\mathrm{P}(B \cap C)$

4. Find $\mathrm{P}(A \cap B^C)$

One ticket will be drawn at random from each of the two boxes below:

$$A: \boxed{\boxed{0}\boxed{1}\ \boxed{2}\ \boxed{3}} \qquad B: \boxed{\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}}$$

5. What is the probability the number drawn from A is greater than the one drawn from B?

6. What is the probability that the number drawn from A is equal to the one drawn from B?

7. What is the probability the number drawn from A is smaller than the one drawn from B?

8. Consider the box $\boxed{\boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{3}\ \boxed{4}}$. I draw two tickets at random **with** replacement. If my first draw is a 2, what is the probability that my second draw is a 3?

9. Consider the box $\boxed{\boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{3}\ \boxed{4}}$. I draw two tickets at random **without** replacement. If my first draw is a 2, what is the probability that my second draw is a 3?

Return to the example of the nine-sided die from earlier in this problem set. Write code in R (and write the code below) to:

10. Simulate rolling the nine-sided die once.

11. Simulate rolling the nine-sided die seven times.

12. Write the line of code which will ensure that you receive the same sample each time you run the code.

Computing Probabilities

Consider picking numbers from the following box.

$$\boxed{\boxed{0}\ \boxed{1}\ \boxed{2}\ \boxed{3}}$$

Let A be the event that the *first pick* yields an even number; B be the event that the *second pick* is greater than or equal to one.

13. Pick two numbers without replacement. Find $\text{P}(B|\text{first pick is } 0)$.

14. Pick two numbers without replacement. Find $\mathrm{P}(B|\text{first pick is 2})$.

15. Pick two numbers with replacement. Find $\mathrm{P}(B|A)$.

Consider a fair, eight-sided die.

16. I roll the die four times. What is the probability that I roll the **same** number on all four rolls?

17. I roll the die twice. What is the probability that the rolls are **different**?

Maganga loves to read books from two genres: mystery and science fiction*. Each time Maganga picks a book to read, they choose independently of previous choices (like a coin toss). That week, they had limited time, so they picked a book to read on only three separate days.

*N.B. Maganga owns no books that are both mystery and science fiction.

Define the events A and B where:

A is the event that Maganga picked a mystery book *more than once* that week;

B is the event that the books Maganga picked that week included *both* a mystery and science fiction book.

18. Find $P(A)$, $P(B)$, $P(A \cup B)$, and $P(B \cup A)$.

19. Are A and B independent?

An American roulette wheel has 38 pockets, of which 18 are red, 18 black, and 2 are green. In each round, the wheel is spun and a white ball lands in one of these 38 pockets.

20. What is the probability of getting at the ball landing in a green pocket at least once in 5 spins of the wheel?

A European roulette wheel has 37 pockets, of which 18 are red, 18 black, *and only 1 green*. The roulette wheel is numbered 0 through 36.

21. Write R code to simulate three spins of this wheel.

22. Now imagine that after each of the three spins, a pocket disappears. Simulate three spins of this magic wheel.

We will now perform our first simulation of the year! For the following questions, consider the European roulette wheel of **Question 7** and ensure your Quarto document will present the same results each time it is rendered. Write your code in the spaces below.

23. Create three vectors: one which contains 100 simulated spins of the European roulette wheel (call this `one_hundred`), one which contains 1,000 such spins (call this `one_thousand`), and another which contains 10,000 such spins (call this `ten_thousand`).

24. Create a new vector that returns `TRUE/FALSE` values for each element in `one_hundred`, where `TRUE` means that the number spun is greater than 18, and save it. Repeat these steps for the `one_thousand` and `ten_thousand` vectors.

25. Find the proportion of numbers spun in each simulation that were greater than 18 (write the code and the proportion). *Hint: how can you take a proportion of a logical vector?*

26. Comment on how the proportions changed with respect to the true probability of spinning a number greater than 18 as the number of spins increased.

27. Suppose A and B are non-empty events such that $P(A) = 0.5$ and $P(B) = 0.7$. What is the smallest and biggest that their union, $P(A \cup B)$, and their intersection, $P(A \cap B)$, can be?

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