The Fresnel Relations

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Abstract

This paper is written as the first of four mandatory repports during the course *Experimental Physics II.* In this experiment we will be working with the Fresnel relations, and an comparison of theory and experiment is the primorial purpose.

1 [entry]nty/global/ Introduction

We look at reflection and refraction of a beam of light at the surface of a transparent dielectric. The laws of relection and refraction (*Snell's law*) determine the directions of the reflected and refracted beams. But they are purely geometrical, and no information of intensity nor polarization is given. These are given by the *Fresnel Relations.* This exercise studies the Fresnel relations in detail, and experimental data will be compared with the well known theory of Snell's Law.

2 Theory

In this repport we are working with the Fresnel relations, which describe the amount of s- and p-polarized light transmitted and reflected at the boundary of a dielectric surface. Generally we remind ourself of the relation between the angles of the incoming light θ_i, the reflected light θ_r and refracted (also called transmitted) light θ_t. The angles are measured from the normal of the dielectric surface (see fig. 1). The relation between the incoming and reflected beam is given by the simple relation[1]:

$$\theta_i = \theta_r,$$

The relation between the angles of incidence and refraction is given by Snells' law:

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

where n_1 and n_2 are the refractive indices of the materials at the boundary of the incoming and reflected light beam.

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[1]This is the only solution for the given boundary conditions at the surface and as the light propagates in the same material, and hence has the same index of refraction.

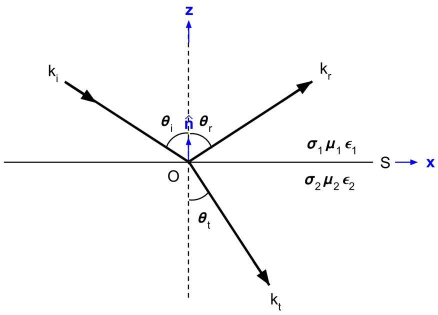

Figure 1: Incomming light beam at angle θ_i to the plane of incidence reflected and refracted at angles θ_r and θ_t respectively. Here k is the orientation of the propagation.

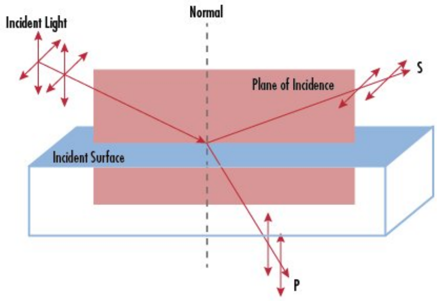

Figure 2: Plane of incidence, spanned out by the normal to the incident surface and the direction vector of the beam.

Polarization

The plane spanned by the normal vector to our dielectric surface $\hat{\mathbf{n}}$ and the wave vector $\mathbf{k}_i$, which is in the direction of the propagation of the incomming light, is called *the plane of incidence* (see fig. 2). The direction of the **E**-field relative to this plane then determines the polarization of the light, if **E** is parallel to the plane of incidence the light is p-polarized and if **E** is perpendicular to the plane of incidence then the light is s-polarized. Light from normal light sources has a mixture of all possible directions of polarization but generally any polarization can be given as a linear combination in a basis of electric field with s- and p-polarization. Using a polarizer one can filter one kind of polarization of a beam.

Fresnel relations

When a beam of light interacts with the surface of a dielectricum a certain percentage of the light will be reflected as well as transmitted. The percentage of light reflected is denoted R and the percentage transmitted is denoted T. As the light is either transmitted or reflected it is natural to conclude that:

$$R + T = 1$$

To make things easier for ourselves we define $R = r^2$ and $T = \frac{n_2 \cos\theta_t}{n_1 \cos\theta_r} t^2$ where $r = \frac{E_1'}{E_1}$ and $t = \frac{E_2}{E_1}$ with E_1 denoting the magnitude of the incoming **E**-field, E_1' denoting the magnitude of the reflected **E**-field and E_2 denoting the magnitude of the transmitted **E**-field. As the direction of the **E**-field can be written in the basis of s- and p-polarization we can define our reflection and transmission indexes for p- and s-polarized light seperately. Our r_p, r_s, t_p and t_s given as functions of θ_r and θ_t are:

$$r_p = \frac{n_2 \cos(\theta_r) - n_1 \cos(\theta_t)}{n_2 \cos(\theta_r) + n_1 \cos(\theta_t)} = \frac{\tan(\theta_r - \theta_t)}{\tan(\theta_r + \theta_t)}$$

$$t_p = \frac{2n_1 \cos(\theta_r)}{n_2 \cos(\theta_r) + n_2 \cos(\theta_t)} = \frac{2\cos(\theta_r)\sin(\theta_t)}{\sin(\theta_r + \theta_t)\cos(\theta_r + \theta_t)}$$

$$r_s = \frac{n_1\cos(\theta_r) - n_2\cos(\theta_t)}{n_1\cos(\theta_r) + n_2\cos(\theta_t)} = -\frac{\sin(\theta_r - \theta_t)}{\sin(\theta_r + \theta_t)}$$

$$t_s = \frac{2n_1\cos(\theta_r)}{n_1\cos(\theta_r) + n_2\cos(\theta_t)} = \frac{2\cos(\theta_r)\sin(\theta_t)}{\sin(\theta_r + \theta_t)}$$

Then we find:

$$\begin{aligned}
R_p &= \frac{\tan(\theta_r - \theta_t)^2}{\tan(\theta_r + \theta_t)^2} \\
T_p &= \frac{\sin(2\theta_r)\sin(2\theta_t)}{\sin^2(\theta_r + \theta_t)\cos^2(\theta_r - \theta_t)} \\
R_s &= \frac{sin^2(\theta_r - \theta_t)}{\sin^2(\theta_r + \theta_t)} \\
T_s &= \frac{\sin(2\theta_r)\sin(2\theta_t)}{\sin^2(\theta_r + \theta_t)}
\end{aligned}$$

3 Experimental Setup

The light source For the beamsize we used a collimating slit of five variable widths.

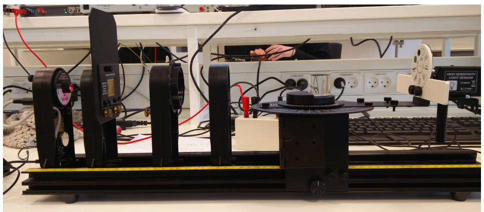

Figure 3: Experimental setup. From left to right: Laser, collimating slit, polarizer, lense, glass, polarizer, photosensor. See logbook for description.

Procedure

Firstly the setup must be carefully aligned in order to maximize the amount of lazer light hitting the detector. We assured that the lazer went as directly as possible through one of the first 3 collimating slits. Afterwards one must assure that the light from the lazer is equally polarized. This is done by leaving the square polarization filter in front of the detector while tuning the polarizer at different angles while observing the intensity of the beam at picoscope. One observes that dependent on the choice of s or p-polarization there is a minima/maxima of the intensity at 0 and 90 degrees respectively. The polarizer is then set to 45 degrees in order to get equally polarized light. There is a slight uncertainty in to the exact 50/50 position of the polarizer as the minima and maxima of the polarized light were quit broad.

We did seperate measurements for transmission and reflection coefficients. To measure the reflection coefficient we measured the beams that were in area between our laser and our dielectric (of course not the incoming lazer beam), and for the transmission we did our measurements on the part of the beam that was refracted to the area between the origin position of the detector and the dielectric. The polarization of the light was changed by tilting the square polarization filter by 90 degrees. The measurement procedure we used were as follows:

Align the outer disk such that the white line on the measurement apparatus is aligned with either 0 or 180 degress. Make sure that the dielectric is placed in the center of the inner desk. Place the inner disk such that 0 degrees aligns with 0 degrees on the outer disk.

Move the inner disk to the desired angular displacement. Move the detector to capture the transmitted or reflected beam respectively. Make sure that the beam of with given polarization is intense enough by tuning the intensity of the detector. The intensity may not be higher than 5 V at 90 degrees for the reflected beam and at 0 degrees for the transmitted beam, as the detector does not tolerate voltages higher than 5 V. The beam can be adjusted up and down on the lazer to hit the detector optimally. Note the angle on the outer disk and the intensity of the beam in picoscope.

Turn off picoscope and measure the background intensity at the given ange of the detector and note it.

Reset experiement to original outer/inner disk configuration before doing a new measurement.

Make sure to measure the intensity at 90 degrees for reflection and 0 degrees for transmission.

4 Data

Snells law

To determine our refractive index of the glass n$_2 wehaveusedalinearfitbetween \sin(\theta_1)$ - our incoming angle - and $\sin(\theta_2)$, where θ_2 denotes our reflected angle. According to snells law the correlation between these quantities are:

$$\frac{n_1}{n_2}\sin(\theta_1) = \sin\theta_2$$

since the refractive index of air is $n_1 \approx 1$ we can determine n_2 from our data as:

$$n_2 = \frac{1}{a}$$

where a is the slope of a linear fit on our data.

On the plot we see our data and a linear fit to our data. One can clearly see that our data does not fit a linear tendency, but forcing the linear fit we can compute the refractive index of our glass from the slope of the fit to $n_2 = 1.7$ wich is 0.2 from the theoretical reflective index of the glass of 1.5. The difference is relatively great, but thos should not surprise us with our data.

Fresnel relations

To compute our transmission and reflection coefficients we have used the relationsship between the measured angle and the maximal intensity of our laser (the measured intensity at 90 degrees for reflection and at 0 degrees for transmission). For any given angle we have:

$$R_(s/p)(\theta) = \frac{I(\theta)}{I(90)} \quad T_(s/p)(\theta) = \frac{I(\theta)}{I(0)}$$

Where the background was substracted from the used intensities.

Air to glass

For this situation we have data for the transmission of s- and p-polarized light, but only we only have data regarding the reflection for our p-polarized light. This is due to the fact that we overranged the laser in order to get a measureble signal at low angles for the s-polarized light, but unfortunately the intensity of the laser signal then became greater than the detectors limit of 5 V at larger angels, thus we were not able to measure our maximal intensity. We intended to measure the s-polarized reflection again but ran out of time.

Looking at our p-polarized reflection we observe that our data generally does not match with the theory, since the theoretical curve does not lie within the error of our data. The error is computed using error proporgation for an uncertainty of 2 degrees on the incoming angle. We can, however, recognize some of the proporties of the theory in our data, such as the low reflection index at small angles with an increase in the percentage of transmitted light at angles above 1.2 radians. Our data actually also seem to fit very well at small angles, it even seems to fit best around the Brewster angle at 56 degrees. Looking at greater angles our data tends towards one faster than theoretically expected.

For our transmission, we see that we had some measurement problems for small angles for our p-polarized light, because it seems that our intensity is almost constant for small angles but follows the theoretical values for greater angles. It it also peculiar that our error for the small angles are small, even though this where our data deviates the most from the theory. Again our errors were computed using the error proporgation law. For the s-polarization we observe a lot of dispertion of our data around our theoretical plot for the small angles, but we seem to be corresponding with when the proper errors are taking into consideration.

Generally our data does not correlate with the theory under the errors we have taken into consideration.

One of the greatest uncertainties in the experiment is reseting the angular measuremt tool. Multiple times during the measurements we experienced that the outer disk rotated with the inner disk when we changed the incoming angle. This has of course contributed to uncertainties on the angle wich we did not take into notice when estimating the uncertainty on the incoming angle. This might also be the one major error that have affected our data so severely that we are not capable of confirming the fresmel relations.

Another thing we noticed during the experiment was that the beam tended to smear for larger angles, which will also create an uncertainty in our intensities which might explain som of the deviances from the theory especially in the reflection-plots. Generally, we can not confirm the Fresnel relations with our data, since our data are not within the acceptable errors in comparison with the theory. This is probably due to our own lack of abilities to perform the measurements precisely and influences from other sources of uncertainties rather than the Fresnel relations not holding for the experiment generally. It is also very unsatisfactory that we were not able to perfrom the measurement for the reflection of s-polarized light and the "glass to air". We can however conclude that our data fits fairly well with the Fresnel relations for smaller angles of reflection and larger transmission angles, thus we can conclude that our data at least confirms the theory in these limits. The general conclusion must be that we need to make a better planned and more structured experiment to fully produce data that can confirm the Fresel Relations.

Logbook

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1 Problem

Research method #1

Optical detector...

Planning

Beamsize: We used a slit of variable width. $1, 2, 3, 4, 5$

Experimental Equipment Available

- Ruler
-
- Collimating slits with 5 slits
- Polarizer with rotational mount
- Polarizer
- Collimating lens
- Rotational mount
- High sensitivity light sensor
- PicoScope

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Critical issues

Intensity of light is half s- and p-polarized. Allignment of detector and laser-beam.

Strategy

Setup

Laboratory setyp

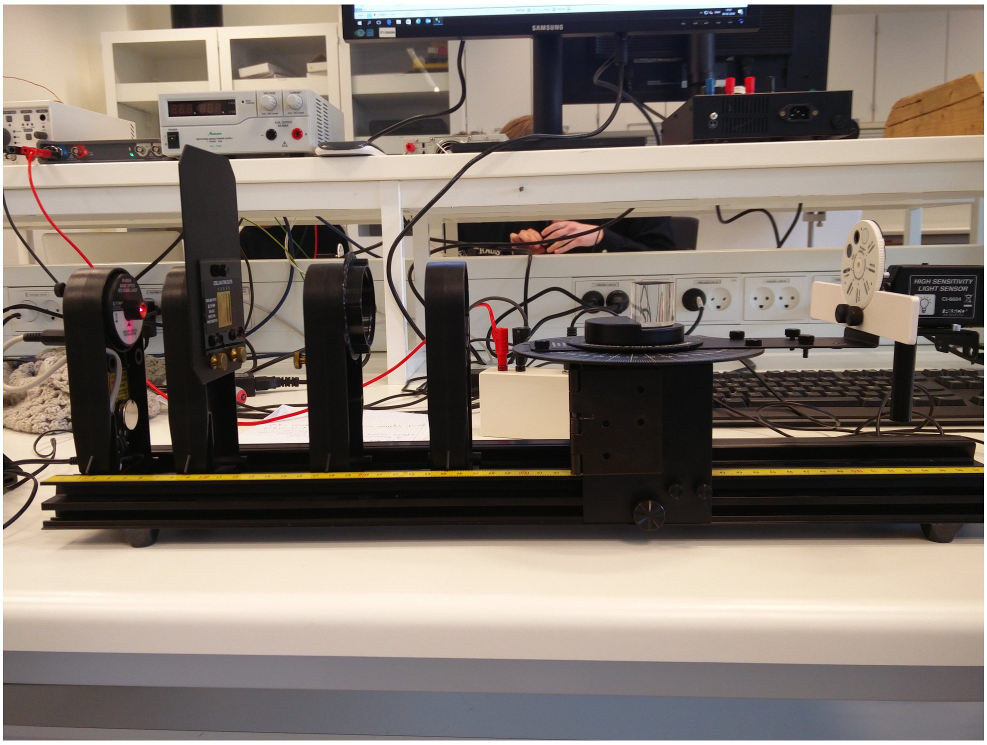

Figure 1: Look at me I am a caption!

Raw data

Fast analysis

Conclusion

Her og der og alle vegne, som du kan se på listing 1

Listing 1: Caption

```
# Preamble
import numpy as np
```

```
import matplotlib.pyplot as plt

# MatploLib koerer TeX
```