The Fresnel Relations

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Abstract

This paper is written as the first of four mandatory repports during the course *Experimental Physics II*. In this experiment we will be working with the Fresnel relations, and an comparison of theory and experiment is the primorial purpose.

1 Introduction

We look at reflection and refraction of a beam of light at the surface of a transparent dielectric. The laws of relection and refraction (*Snell's law*) determine the directions of the reflected and refracted beams. But they are purely geometrical, and no information of intensity nor polarization is given. These are given by the *Fresnel Relations*. This exercise studies the Fresnel relations in detail, and experimental data will be compared with the well known theory of Snell's Law.

2 Theory

In this repport we are working with the Fresnel relations, which describe the amount of s- and p-polarized light transmitted and reflected at the boundary of a dielectric surface. Generally we remind ourself of the relation between the angles of the incoming light θ_i, the reflected light θ_r and refracted (also called transmitted) light θ_t. The angles are measured from the normal of the dielectric surface (see fig. 1). The relation between the incoming and reflected beam is given by the simple relation[1]:

$$\theta_i = \theta_r,$$

The relation between the angles of incidence and refraction is given by Snells' law:

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

where n_1 and n_2 are the refractive indices of the materials at the boundary of the incoming and reflected light beam.

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[1] This is the only solution for the given boundary conditions at the surface and as the light propagates in the same material, and hence has the same index of refraction.

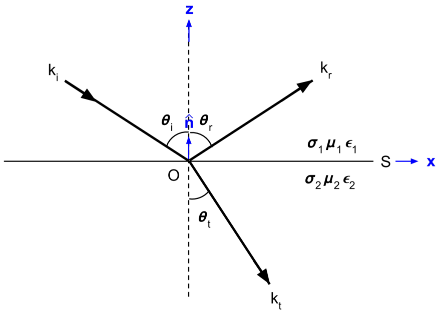

Figure 1: Incomming light beam at angle θ_i to the plane of incidence reflected and refracted at angles θ_r and θ_t respectively. Here k is the orientation of the propagation.

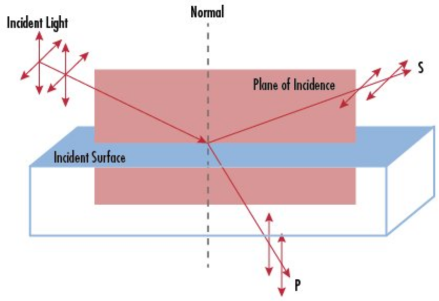

Figure 2: Plane of incidence, spanned out by the normal to the incident surface and the direction vector of the beam.

Polarization

The plane spanned by the normal vector to our dielectric surface $\hat{\mathbf{n}}$ and the wave vector $\mathbf{k}_i$, which is in the direction of the propagation of the incomming light, is called *the plane of incidence* (see fig. 2). The direction of the $\mathbf{E}$-field relative to this plane then determines the polarization of the light, if $\mathbf{E}$ is parallel to the plane of incidence the light is p-polarized and if $\mathbf{E}$ is perpendicular to the plane of incidence then the light is s-polarized. Light from normal light sources has a mixture of all possible directions of polarization but generally any polarization can be given as a linear combination in a basis of electric field with s- and p-polarization. Using a polarizer one can filter one kind of polarization of a beam.

Fresnel relations

When a beam of light interacts with the surface of a dielectricum a certain percentage of the light will be reflected as well as transmitted. The percentage of light reflected is denoted R and the percentage transmitted is denoted T. As the light is either transmitted or reflected it is natural to conclude that:

$$R + T = 1$$

To make things easier for ourselves we define $R = r^2$ and $T = \frac{n_2 \cos\theta_t}{n_1 \cos\theta_r} t^2$ where $r = \frac{E_1'}{E_1}$ and $t = \frac{E_2}{E_1}$ with E_1 denoting the magnitude of the incoming $\mathbf{E}$-field, E_1' denoting the magnitude of the reflected $\mathbf{E}$-field and E_2 denoting the magnitude of the transmitted $\mathbf{E}$-field. As the direction of the $\mathbf{E}$-field can be written in the basis of s- and p-polarization we can define our reflection and transmission indexes for p- and s-polarized light seperately. Our r_p, r_s, t_p and t_s given as functions of θ_r and θ_t are:

$$r_p = \frac{n_2 \cos(\theta_r) - n_1 \cos(\theta_t)}{n_2 \cos(\theta_r) + n_1 \cos(\theta_t)} = \frac{\tan(\theta_r - \theta_t)}{\tan(\theta_r + \theta_t)}$$

$$t_p = \frac{2 n_1 \cos(\theta_r)}{n_2 \cos(\theta_r) + n_2 \cos(\theta_t)} = \frac{2 \cos(\theta_r) \sin(\theta_t)}{\sin(\theta_r + \theta_t) \cos(\theta_r + \theta_t)}$$

$$r_s = \frac{n_1 \cos(\theta_r) - n_2 \cos(\theta_t)}{n_1 \cos(\theta_r) + n_2 \cos(\theta_t)} = -\frac{\sin(\theta_r -}{\sin(\theta_r +}$$

$$t_s = \frac{2 n_1 \cos(\theta_r)}{n_1 \cos(\theta_r) + n_2 \cos(\theta_t)} = \frac{2 \cos(\theta_r) \, \mathrm{s}}{\sin(\theta_r +}$$

Then we find:

$$\begin{aligned}
R_p &= \frac{\tan(\theta_r - \theta_t)^2}{\tan(\theta_r + \theta_t)^2} \\
T_p &= \frac{\sin(2\theta_r)\sin(2\theta_t)}{\sin^2(\theta_r + \theta_t)\cos^2(\theta_r - \theta_t)} \\
R_s &= \frac{sin^2(\theta_r - \theta_t)}{\sin^2(\theta_r + \theta_t)} \\
T_s &= \frac{\sin(2\theta_r)\sin(2\theta_t)}{\sin^2(\theta_r + \theta_t)}
\end{aligned}$$

3 Experimental Setup

The light source For the beamsize we used a collimating slit of five variable widths.

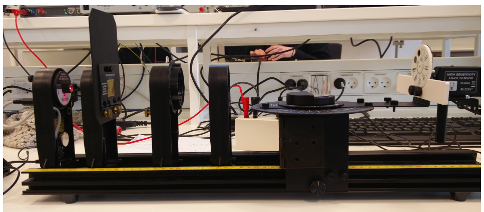

Figure 3: Experimental setup. From left to right: Laser, collimating slit, polarizer, lense, glass, polarizer, photosensor. See logbook for description.

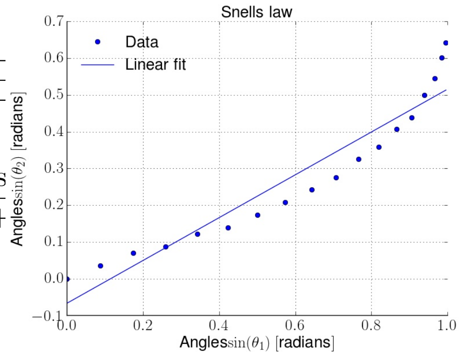

Figure 4: Dat...

4 Data

5 Snell's Law

6 Reflection

p-polarized

s-polarized

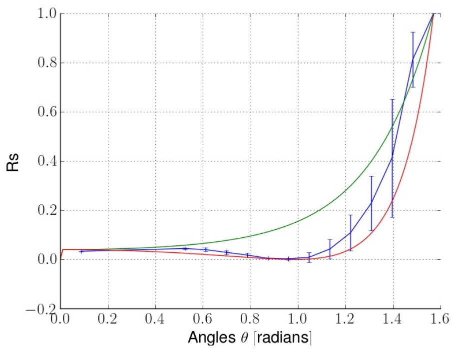

Figure 5: Dat...

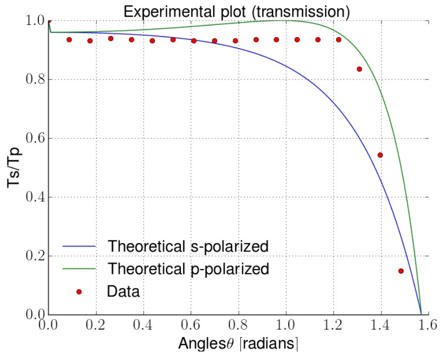

Figure 6: Dat...

7 Transmission

p-polarized

s-polarized

Generally our data does not correlate with the theory under the errors we have taken into consideration.

One of the greatest uncertainties in the experiment is reseting the angular measuremt tool. Multiple times during the measurements we experienced that the outer disk rotated with the inner disk when we changed the incoming angle. This has of course contributed to uncertainties on the angle wich we did not take into notice when estimating the uncertainty on the incoming angle. This might also be the one major error that have affected our data so severely that we are not capable of confirming the fresmel relations.

Logbook

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1 Problem

Research method #1

Optical detector...

Planning

Beamsize: We used a slit of variable width. $1, 2, 3, 4, 5$

Experimental Equipment Available

- Ruler
-
- Collimating slits with 5 slits
- Polarizer with rotational mount
- Polarizer
- Collimating lens
- Rotational mount
- High sensitivity light sensor
- PicoScope

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Critical issues

Intensity of light is half s- and p-polarized. Allignment of detector and laser-beam.

Strategy

Setup

Laboratory setyp

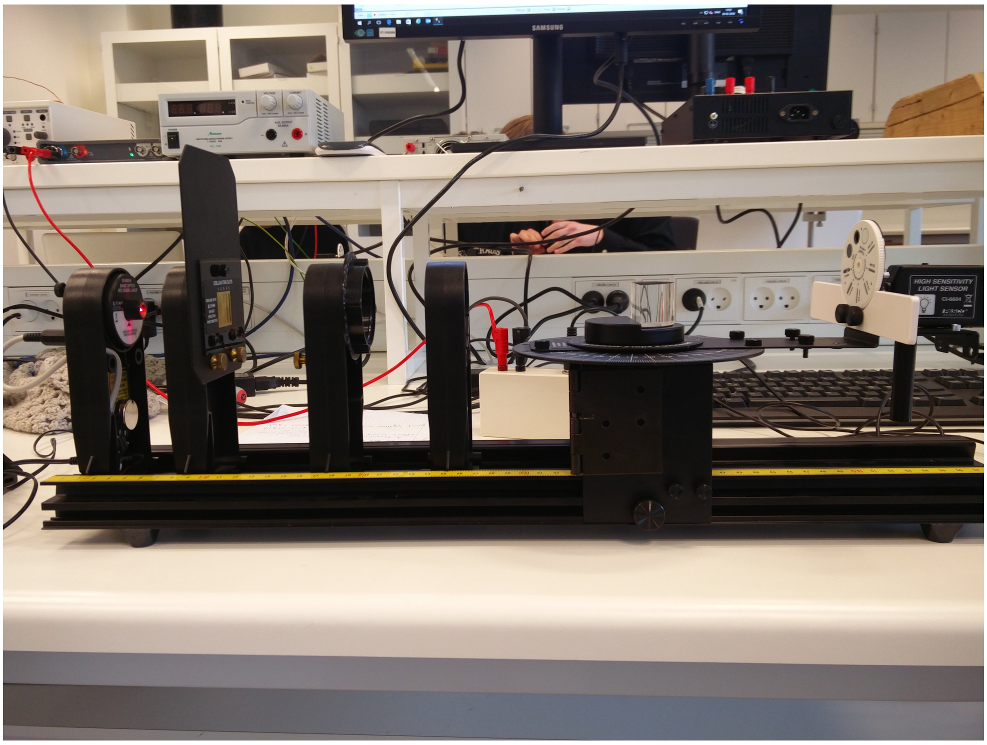

Figure 1: Look at me I am a caption!

Raw data

Fast analysis

Conclusion

Her og der og alle vegne, som du kan se på listing 1

Listing 1: Caption

```
# Preamble
import numpy as np
```

```
import matplotlib.pyplot as plt

# MatploLib koerer TeX
```

Bibliography

Griffiths, David J. *Introduction to Electrodynamics*. Cambridge, 2017. ISBN: 978-1-108-42041-9.

Jensen, Jens Ledet. *Statistik viden fra data*. Aarhus Universitetsforlag, 2012. ISBN: 978-87-7124-0245.