2. La variable continua: Unos problemillas

Ejemplo 1

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^2 cx^2\,dx = 1$$

$$\int_0^2 cx^2\,dx = c\int_0^2 x^2\,dx = c\left[\frac{x^3}{3}\right]_0^2 = c\left(\frac{2^3}{3} - \frac{0^3}{3}\right) = c \cdot \frac{8}{3} = \frac{8}{3}c$$

$$\frac{8c}{3} = 1$$

$$c = \frac{3}{8}$$

$$P(0 < x \leq 1) = \int_0^1 f(x)\,dx = \int_0^1 \frac{3}{8}x^2\,dx$$

$$\int_0^1 \frac{3}{8}x^2\,dx = \frac{3}{8}\left[\frac{x^3}{3}\right]_0^1 \simeq \frac{3}{8} \cdot \frac{1}{3} = \frac{3}{24} = \frac{1}{8}$$

$$P(0 < x \leq 1) = \frac{1}{8}$$

Ejemplo 2

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{1}^{\infty} \frac{k}{x^4}\,dx = 1$$

$$\int_{1}^{\infty} \frac{k}{x^4}\,dx = k\int_{1}^{\infty} x^{-4}\,dx$$

$$k\left[\frac{x^{-3}}{-3}\right]_1^{\infty} = k\left[0-\left(-\frac{1}{3}\right)\right] = \frac{k}{3}$$

$$\frac{k}{3} = 1 \Longrightarrow k = 3$$

$$E(x) = \int_1^{\infty} x f(x)\,dx = \int_1^{\infty} x\cdot\frac{3}{x^4}\,dx = 3\int_1^{\infty} x^{-3}\,dx$$

$$= 3\left[\frac{x^{-2}}{-2}\right]_1^{\infty} = 3\left[0-\left(-\frac{1}{2}\right)\right] = \frac{3}{2}$$

$$E(x^2) = \int_1^{\infty} x^2 f(x)\,dx = \int_1^{\infty} x^2\cdot\frac{3}{x^4}\,dx = 3\int_1^{\infty} x^{-2}\,dx$$

$$= 3\left[\frac{x^{-1}}{-1}\right]_1^{\infty} = 3\left[0-(-1)\right] = 3$$

$$Var(x) = E(x^2) - (E(x))^2 = 3 - \left(\frac{3}{2}\right)^2 = 3 - \frac{9}{4} = \frac{12}{4} - \frac{9}{4} = \frac{3}{4}$$

$$P(X>2) = \int_2^{\infty} \frac{3}{x^4}\,dx = 3\left[\frac{x^{-3}}{-3}\right]_2^{\infty} = \frac{1}{8}$$

$$P(X\leq 2) = 1 - P(X>2) = 1 - \frac{1}{8} = \frac{7}{8}$$

$$P(X\leq x) = 1 - \int_x^{\infty} \frac{3}{x^4}\,dx = 1 - \frac{1}{x^3}$$