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## Estructuras de Datos

### Segment Tree

```
class SegmentTree<T>(  
    val n: Int,  
    val operation: (T, T) -> T,  
    val neutral: T  
) {  
    var size: Int  
    var st: MutableList<T>  
  
    init {  
        size = 1  
        while (size <= n) size *= 2  
        st = MutableList(2 * size) { neutral }  
    }  
  
    // Inicializa el segmento del árbol con los valores dados  
    fun init(arr: List<T>) {  
        for (i in 0 until n) {  
            st[i + size] = arr[i]  
        }  
        for (i in size - 1 downTo 1) {  
            st[i] = operation(st[2 * i], st[2 * i + 1])  
        }  
    }  
  
    // Actualiza un valor en la posición p  
    fun update(p: Int, value: T) {  
        var index = p + size  
        st[index] = value
```

```
        while (index > 1) {  
            index /= 2  
            st[index] = operation(st[2 * index], st[2 * index + 1])  
        }  
    }  
  
    // Realiza una consulta sobre el rango [lq, rq)  
    fun query(lq: Int, rq: Int): T {  
        var left = lq + size  
        var right = rq + size  
        var lres = neutral  
        var rres = neutral  
  
        while (left < right) {  
            if (left % 2 == 1) {  
                lres = operation(lres, st[left])  
                left++  
            }  
            if (right % 2 == 1) {  
                right--  
                rres = operation(st[right], rres)  
            }  
            left /= 2  
            right /= 2  
        }  
        return operation(lres, rres)  
    }  
}
```

### Fenwick Tree

```
class Fenwick<T>(  
    val n: Int,  
    val operation: (T, T) -> T,  
    val neutral: T) {  
    var ft: MutableList<T>  
  
    init {  
        ft = MutableList(n + 1) { neutral }  
    }  
  
    fun update(p: Int, value: T) {  
        var index = p  
        while (index <= n) {  
            ft[index] = operation(ft[index], value)  
            index += index and -index  
        }  
    }  
  
    fun query(r: Int): T {  
        var index = r  
        var res = neutral  
        while (index > 0) {  
            res = operation(res, ft[index])  
            index -= index and -index  
        }  
        return res  
    }  
}
```

## Grafos

### BFS

```
fun bfs(  
    graph: List<List<Int>>,  
    start: Int  
) : List<Int> {  
    val n = graph.size  
    val min_dist = MutableList(n) { -1 }  
    val q = mutableListOf<Int>()  
    q.add(start)  
    min_dist[start] = 0  
    var qi = 0  
    while (qi < q.size) {  
        val u = q[qi]  
        qi++  
        for (v in graph[u]) {  
            if (min_dist[v] == -1) {  
                min_dist[v] = min_dist[u] + 1  
                q.add(v)  
            }  
        }  
    }  
    return min_dist  
}
```

## Dijkstra

```
import java.util.PriorityQueue

fun Dijkstra(
    grafo: List<List<Pair<Int,Int>>>,
    inicio: Int
) : Pair<List<Int>, List<Int>> {
    val n = grafo.size
    var dist = MutableList(n) { Int.MAX_VALUE }
    var padre = MutableList(n) { -1 }
    dist[inicio] = 0
    val pq = PriorityQueue<Pair<Int,Int>>(compareBy { -it.second })
    pq.add(Pair(inicio, 0))
    while (pq.isNotEmpty()){
        val (u, d) = pq.poll()
        if (d > dist[u]) continue
        for ((v, w) in grafo[u]){
            if (dist[u] + w < dist[v]){
                dist[v] = dist[u] + w
                padre[v] = u
                pq.add(Pair(v, dist[v]))
            }
        }
    }
    return Pair(dist, padre)
}
```

## Bellman-Ford

```
fun BellmanFord(
    grafo: List<List<Pair<Int,Int>>>,
    inicio: Int,
    largo: Int
) : List<List<Int>> {
    val n = grafo.size
    var dist = MutableList(largo+1) { MutableList(n) { Int.MAX_VALUE } }
    dist[0][inicio] = 0
    for (k in 0 until largo){
        for (u in 0 until n){
            for ((v, w) in grafo[u]){
                if (dist[k][u] != Int.MAX_VALUE)
                    dist[k+1][v] = minOf(dist[k+1][v], dist[k][u] + w)
            }
        }
    }
    return dist
}
```

## Floyd-Warshall

```
fun FloydWarshall(
    matriz: List<List<Int>>
) : MutableList<MutableList<Int>> {
    val n = matriz.size
    var dist = matriz.map{ it.toMutableList() }.toMutableList()
    for (k in 0 until n){
        for (i in 0 until n){
            for (j in 0 until n){
                dist[i][j] = minOf(dist[i][j], dist[i][k] + dist[k][j])
            }
        }
    }
    return dist
}
```

## Kruskal

```
fun Kruskal(g: List<Triple<Int,Int,Int>>, n : Int) : Pair<Int, List<Int>>{
    >>{
        var uf = MutableList(n){i -> i}
        fun find(x: Int) : Int{
            if (uf[x] == x) return x
            uf[x] = find(uf[x])
            return uf[x]
        }
        fun union(x: Int, y: Int){
            uf[find(x)] = find(y)
        }
        val aristas = MutableList(n){i -> i}.sortedBy{g[it].third}
        var valor = 0
        var arbol = mutableListOf<Int>()
        for (ar in aristas){
            val (u,v,c) = g[ar]
```

```
            if (find(u) != find(v)){
                union(u,v)
                valor += c
                arbol.add(ar)
            }
        }
        return Pair(valor, arbol)
    }
}
```

## Ancestro común menor

```
class LCA(
    arbol: List<List<Int>>,
    raiz: Int
){
    var K: Int
    var padre: MutableList<MutableList<Int>>
    var prof: MutableList<Int>
    init {
        val n = arbol.size
        K = 1
        while ((1 shl K) < n) K++
        padre = MutableList(K) { MutableList(n) { -1 } }
        prof = MutableList(n) { -1 }

        fun dfs(u: Int, p: Int){
            padre[0][u] = p
            for (v in arbol[u]){
                if (v == p) continue
                prof[v] = prof[u] + 1
                dfs(v, u)
            }
        }
        dfs(raiz, -1)
        prof[raiz] = 0

        for (k in 1 until K){
            for (u in 0 until n){
                if (padre[k-1][u] != -1){
                    padre[k][u] = padre[k-1][padre[k-1][u]]
                }
            }
        }

        fun lca(uu: Int, vv: Int) : Int {
            var u = uu
            var v = vv

            if (prof[u] < prof[v]) return lca(v, u)
            for (k in K-1 downTo 0){
                if (prof[u] - (1 shl k) >= prof[v]){
                    u = padre[k][u]
                }
            }
            if (u == v) return u
            for (k in K-1 downTo 0){
                if (padre[k][u] != padre[k][v]){
                    u = padre[k][u]
                    v = padre[k][v]
                }
            }
            return padre[0][u]
        }
    }
}
```

# Strings

## Bordes (KMP)

```
fun bordes(s: String): List<Int>{
    val n = s.length
    val b = MutableList(n+1) { -1 }
    var j = -1
    for (i in 0 until n){
        while (j >= 0 && s[i] != s[j]){
            j = b[j]
        }
        j++
        b[i+1] = j
    }
    return b
}
```

## Función Z

```
fun z(s: String): List<Int>{ // z[i] = max k: s[0,k] == s[i,i+k]
    val n = s.length
    val z = MutableList(n) { 0 }
    var l = 0
    var r = 0
    for (i in 1 until n){
        if (i <= r) z[i] = minOf(r-i+1, z[i-1])
        while (i+z[i] < n && s[z[i]] == s[i+z[i]]) z[i]++
        if (i+z[i]-1 > r){
            l = i
            r = i+z[i]-1
        }
    }
    return z
}
```

## Manacher

```
fun Manacher(s: String): Pair<List<Int>, List<Int>>{
    // (d1, d2) = (impares, pares) palindromes
    val n = s.length
    val d1 = MutableList(n) { 0 }
    val d2 = MutableList(n) { 0 }
    var l = 0
    var r = -1
    for (i in 0 until n){
        var k = if (i > r) 1 else minOf(d1[l+r-i], r-i+1)
        while (i-k >= 0 && i+k < n && s[i-k] == s[i+k]) k++
        d1[i] = k--
        if (i+k > r){
            l = i-k
            r = i+k
        }
    }
    l = 0
    r = -1
    for (i in 0 until n){
        var k = if (i > r) 0 else minOf(d2[l+r-i+1], r-i+1)
        while (i-k-1 >= 0 && i+k < n && s[i-k-1] == s[i+k]) k++
        d2[i] = k--
        if (i+k > r){
            l = i-k-1
            r = i+k
        }
    }
    return Pair(d1, d2)
}
```

## Suffix Array

```
fun RB(x : Int, n : Int, r: List<Int>) : Int{
    if(x < n) return r[x]
    else return 0
}

fun csort(sa: MutableList<Int>, r: MutableList<Int>, k : Int){
    val n = sa.size
    var f = MutableList(maxOf(255,n)){0}
    var t = MutableList(n){0}
    for (i in 0 until n) f[RB(i+k,n,r)]++
    var sum = 0
    for (i in 0 until f.size){
        var v = f[i]
        f[i] = sum
        sum += v
    }
    for (i in 0 until n){
        t[f[RB(sa[i]+k,n,r)]++] = sa[i]
    }
    for (i in 0 until n) sa[i] = t[i]
}

fun suffix_array(s0: String): List<Int>{
    val s = s0 + '\u0000'
    val n = s.length
    var rank: Int
    var sa = MutableList(n){it}
    var r = MutableList(n){it -> s[it].code}
    var t = MutableList(n){0}
    var k = 1
    while (k<n){
        csort(sa,r,k)
        csort(sa,r,0)
        t[sa[0]] = 0
    }
}
```

```
rank = 0
for (i in 1 until n){
    if(r[sa[i]] != r[sa[i-1]] || RB(sa[i]+k,n,r) != RB(sa[i-1]+
        k,n,r)) rank++
    t[sa[i]] = rank
}
for (i in 0 until n) r[i] = t[i]
if (r[sa[n-1]]==n-1) break
k *= 2
println("k = $k")
}
return sa
}
```

## LCP (Estructura)

```
fun computar_lcp(s0: String, sa: List<Int>): MutableList<Int>{
    val s = s0 + '\u0000'
    val n = s.length
    var l = 0
    var lcp = MutableList(n){0}
    var plcp = MutableList(n){0}
    var phi = MutableList(n){0}

    phi[sa[0]] = -1
    for (i in 1 until n) phi[sa[i]] = sa[i-1]
    for (i in 0 until n){
        if (phi[i] == -1){
            plcp[i] = 0
            continue
        }
        while (s[i+l] == s[phi[i]+l]) L++
        plcp[i] = L
        L = maxOf(L-1,0)
    }
    for (i in 0 until n) lcp[i] = plcp[sa[i]]
    return lcp
}
```

## Duval

```
// Dada una string $$ devuelve la Lyndon decomposition en tiempo
// lineal usando el algoritmo de Duval. Factoriza $$ como
// $s_1 s_2 \dots s_k$ con $s_1 \geq s_2 \geq \dots \geq s_k$
// y tal que $s_i$ es Lyndon, esto es, es su menor rotación.
fun Duval(s: String) : List<String>{
    val n = s.length
    var i = 0
    val ans = mutableListOf<String>()
    while (i < n){
        var j = i + 1
        var k = i
        while (j < n && s[k] <= s[j]){
            if (s[k] < s[j]) k = i
            else k++
            j++
        }
        while (i <= k){
            ans.add(s.substring(i until i+j-k))
            i += j-k
        }
    }
    return ans
}

// Obtener la mínima rotación de $$: en la descomposición de
// Lyndon de $s^2$ es el último $i<|s|$ con el que empieza una
// Lyndon.
```

## Hashing

```
const val P: Long = 1777771
val MOD: List<Long> = listOf(999727999, 1070777777)
val PI: List<Long> = listOf(325255434, 10018302)

class Hashing(c: Char) {
    val h: MutableList<Long>
    val p: MutableList<Long>
    val pi: MutableList<Long>
    init {
        h = MutableList(PI.size) { i -> c.code * P % MOD[i] }
        p = MutableList(PI.size) { P }
        pi = PI.toMutableList()
    }

    // Agrega un prefijo : H(s1) + H(s2) = H(s2s1)
    operator fun plus(h2: Hashing) : Hashing {
```

```

val ans = Hashing('a')
for (i in 0 until PI.size){
    ans.h[i] = (h[i] * h2.p[i] + h2.h[i]) % MOD[i]
    ans.p[i] = p[i] * h2.p[i] % MOD[i]
    ans.pi[i] = pi[i] * h2.pi[i] % MOD[i]
}
return ans
}

// Elimina un prefijo
operator fun minus(h2: Hashing) : Hashing {
    val ans = Hashing('a')
    for (i in 0 until PI.size){
        ans.h[i] = (h[i] - h2.h[i] + MOD[i]) % MOD[i] * h2.pi[i] %
            MOD[i]
        ans.p[i] = p[i] * h2.pi[i] % MOD[i]
        ans.pi[i] = pi[i] * h2.p[i] % MOD[i]
    }
    return ans
}

/// 0 simplemente comparar h1.h == h2.h en vez de h1==h2
override fun equals(h2: Any?): Boolean {
    if (h2 !is Hashing) return false
    return h == h2.h
}

}

fun hash_neutro() : Hashing{
    var ans = Hashing('a')
    for (i in 0 until PI.size){
        ans.h[i] = 0
        ans.p[i] = 1
        ans.pi[i] = 1
    }
    return ans
}

class StringHasher(s: String){
    val h: MutableList<Hashing>
    init {
        h = MutableList(s.length+1) {
            if (it == 0) hash_neutro()
            else Hashing(s[it-1])
        }
        for (i in 1 until s.length+1){
            h[i] = h[i] + h[i-1]
        }
    }

    // Hash de s[1,r)
    fun hash(l: Int, r: Int) : Hashing {
        return h[r] - h[l]
    }
}

```

# Matemáticas

## Identidades

$$C_n = \frac{2(2n-1)}{n+1} C_{n-1}$$

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

$$C_n \sim \frac{4^n}{n^{3/2} \sqrt{\pi}}$$

$$F_{2n+1} = F_n^2 + F_{n+1}^2$$

$$F_{2n} = F_{n+1}^2 - F_{n-1}^2$$

$$\sum_{i=1}^n F_i = F_{n+2} - 1$$

$$F_{n+i} F_{n+j} - F_n F_{n+i+j} = (-1)^n F_i F_j$$

$$\sum_{i=0}^n r^i = \frac{r^{n+1} - 1}{r - 1}$$

$$\sum_{i=1}^n i^2 = \frac{n \cdot (n+1) \cdot (2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left( \frac{n \cdot (n+1)}{2} \right)^2$$

$$\sum_{i=1}^n i^4 = \frac{n \cdot (n+1) \cdot (2n+1) \cdot (3n^2 + 3n - 1)}{12}$$

$$\sum_{i=1}^n i^5 = \left( \frac{n \cdot (n+1)}{2} \right)^2 \cdot \frac{2n^2 + 2n - 1}{3}$$

$$\sum_{i=1}^n \binom{n-1}{i-1} = 2^{n-1}$$

$$\sum_{i=1}^n i \cdot \binom{n-1}{i-1} = n \cdot 2^{n-1}$$

(Möbius Inv. Formula) Let

$$g(n) = \sum_{d|n} f(d), \text{ then}$$

$$f(n) = \sum_{d|n} g(d) \mu\left(\frac{n}{d}\right)$$

## Convolución rápida (FFT y Karatsuba)

```

import kotlin.math.round

data class Complex(val r: Double, val i: Double){
    operator fun plus(x: Complex) = Complex(r + x.r, i + x.i)
    operator fun minus(x: Complex) = Complex(r - x.r, i - x.i)
    operator fun times(x: Complex) = Complex(r*x.r - i*x.i, r*x.i + i*x.r)
    operator fun div(x: Double) = Complex(r/x, i/x)
}

class FFT(lg0: Int){
    val lg = lg0+1
    val n = 1 shl lg
    val w = MutableList(n+1){Complex(0.0, 0.0)}

    init {
        val ang0 = 2.0 * Math.PI / n.toDouble()
        for (i in 0 until n+1){
            val ang = ang0 * i
            w[i] = Complex(Math.cos(ang), Math.sin(ang))
        }
    }

    fun fft(a: List<Complex>, inv: Boolean = false) : List<Complex>{
        val p = MutableList(n) { a[Integer.reverse(it) ushr (32 - lg)] }

        var len = 2
        while (len <= n) {
            val step = n / len
            for (i in 0 until n step len) {
                for (j in 0 until len / 2) {
                    val u = p[i + j]
                    val v = p[i + j + len / 2] * if (inv) w[n - j * step] else w[j * step]
                    p[i + j] = u + v
                    p[i + j + len / 2] = u - v
                }
            }
            len *= 2
        }

        if (inv) {
            for (i in 0 until n) {
                p[i] = p[i] / n.toDouble()
            }
        }

        return p
    }

    fun multiply(a: List<Long>, b: List<Long>) : List<Long>{
        val a_c = a.map { Complex(it.toDouble(), 0.0) } + MutableList(n - a.size){Complex(0.0, 0.0)}
        val b_c = b.map { Complex(it.toDouble(), 0.0) } + MutableList(n - b.size){Complex(0.0, 0.0)}
        val fa = fft(a_c)
        val fb = fft(b_c)
        val fc = MutableList(n){fa[it] * fb[it]}
        val c = fft(fc, true)
        return c.map { round(it.r).toLong() }
    }
}

fun Karatsuba(a : List<Long>, b : List<Long>) : List<Long>{
    val m = maxOf(a.size, b.size)
    val n = 1 shl (32 - Integer.numberOfLeadingZeros(m - 1))
    val aa = a + MutableList(n - a.size) { 0.toLong() }
    val bb = b + MutableList(n - b.size) { 0.toLong() }
    return karatsuba(aa, bb)
}

fun karatsuba(a: List<Long>, b: List<Long>): List<Long> {
    if (a.size <= 16) { // Reducir el tamaño de la condición base
        val c = MutableList(2 * a.size - 1) { 0L }
        for (i in a.indices) {

```

```

        for (j in b.indices) {
            c[i + j] += a[i] * b[j]
        }
    }
    return c
}

val n = a.size
val k = n / 2
val a0 = a.subList(0, k)
val a1 = a.subList(k, n)
val b0 = b.subList(0, k)
val b1 = b.subList(k, n)

val z2 = karatsuba(a1, b1)
val z0 = karatsuba(a0, b0)
val a0a1 = List(k) { a0[it] + a1[it] }
val b0b1 = List(k) { b0[it] + b1[it] }
val z1 = karatsuba(a0a1, b0b1)

val result = MutableList(2 * n) { 0L }
for (i in z0.indices) result[i] += z0[i]
for (i in z2.indices) result[i + n] += z2[i]
for (i in z1.indices) result[i + k] += z1[i] - z0.getOrNull(i) { 0L }
    } - z2.getOrNull(i) { 0L }

return result
}

```

## Criba de Eratostenes

```

class Criba(n: Int){
    var criba = MutableList(n+1){-1}
    init {
        for (i in 2..n){
            if (criba[i] == -1){
                if (n/i>=i) for (j in i*i until (n+1) step i){
                    if (criba[j] == -1) criba[j] = i
                }
            }
        }
    }

    fun fact(n: Int) : MutableMap<Int,Int> {
        var res = mutableMapOf<Int,Int>()
        var x = n
        while(criba[x] != -1){
            res[criba[x]] = res.getOrDefault(criba[x], 0) + 1
            x /= criba[x]
        }
        if(x != 1) res[x] = res.getOrDefault(x, 0) + 1
        return res
    }
}

```

# Geometría

## Punto

```

import kotlin.math.*

class pt(x: Double, y: Double): Comparable<pt>{
    val x = x
    val y = y
    operator fun plus(p: pt) = pt(x + p.x, y + p.y)
    operator fun minus(p: pt) = pt(x - p.x, y - p.y)
    operator fun times(k: Double) = pt(x * k, y * k)
    operator fun div(k: Double) = pt(x / k, y / k)
    operator fun times(p: pt) = x * p.x + y * p.y
    operator fun rem(p: pt) = x * p.y - y * p.x
    fun angle(p: pt) = acos((this * p) / (this.norm() * p.norm()))
    fun norm2() = x * x + y * y
    fun norm() = sqrt(norm2())
    fun unit() = if (norm() > 0) this / norm() else pt(0.0, 0.0)
    fun rot(r: pt) = pt(this % r, this * r)
    fun rot(a: Double) = this.rot(pt(cos(a), sin(a)))
    fun left(p: pt, q: pt) = (q - p).unit() % (this - p).unit() > EPS

    operator override fun compareTo(p: pt): Int = when {
        abs(this.x - p.x) > EPS -> this.x.compareTo(p.x)
        else -> this.y.compareTo(p.y)
    }

    override fun equals(other: Any?) = other is pt && abs(x - other.x) <
        EPS && abs(y - other.y) < EPS
    override fun toString() = "($x, $y)"
}

```

```

}

val ccw90 = pt(1.0,0.0)
val cw90 = pt(-1.0,0.0)

Segmento

import kotlin.math.*

class Segment(val f: pt, val s: pt) {
    fun length(): Double {
        val dx = f.x - s.x
        val dy = f.y - s.y
        return sqrt(dx * dx + dy * dy)
    }
}

fun pc(a: pt, b: pt, o: pt): Double = (a-o) % (b-o)
fun pe(a: pt, b: pt, o: pt): Double = (a-o) * (b-o)

fun intersect(a: Segment, b: Segment): Boolean{
    val fb = 0.compareTo(pc(a.f, a.s, b.f))
    val sb = 0.compareTo(pc(a.f, a.s, b.s))
    val fa = 0.compareTo(pc(b.f, b.s, a.f))
    val sa = 0.compareTo(pc(b.f, b.s, a.s))
    if ((fb * sb < 0) && (fa * sa < 0)) return true
    if ((fb==0 && pe(a.f, a.s, b.f)<=0) || (sb==0 && pe(a.f, a.s,b.s)
        <=0)) return true;
    if ((fa==0 && pe(b.f, b.s, a.f)<=0) || (sa==0 && pe(b.f, b.s, a.s)
        <=0)) return true;
    return false
}

fun dist(p: pt, s: Segment): Double{
    val a = abs(pc(s.f,s.s,p))
    val b = hypot(s.f.x - s.s.x, s.f.y - s.s.y)
    val h = a/b
    val c = hypot(b,h)
    val d1 = (s.f-p).norm()
    val d2 = (s.s-p).norm()
    if(b<EPS || c<= d1 || c<= d2) return minOf(d1,d2)
    return h
}

fun dist(a: Segment, b : Segment) : Double{
    if(intersect(a,b)) return 0.0
    return minOf(
        minOf(dist(a.f,b),dist(a.s,b)),
        min(dist(b.f,a),dist(b.s,a))
    )
}

```

## Capsula convexa

```

fun chull(ps: List<pt>) : List<pt>{
    if(ps.size < 3) return ps
    val p = ps.sorted()
    val ch = mutableListOf<pt>()
    for(pi in p){
        while(ch.size > 1 && ch[ch.size - 1].left(ch[ch.size - 2], pi))
            ch.removeAt(ch.size - 1)
        ch.add(pi)
    }
    ch.removeAt(ch.size - 1)
    val t = ch.size
    for(pi in p.reversed()){
        while(ch.size > t+1 && ch[ch.size - 1].left(ch[ch.size - 2], pi)
            )) ch.removeAt(ch.size - 1)
        ch.add(pi)
    }
    ch.removeAt(ch.size - 1)
    return ch
}

```

# Tablas y Cotas

## Primos cercanos a $10^n$

```

9941 9949 9967 9973 10007 10009 10037 10039 10061
10067 10069 10079
99961 99971 99989 99991 100003 100019 100043 100049
100057 100069
999959 999961 999979 999983 1000003 1000033 1000037
1000039

```

9999943 9999971 9999973 9999991 10000019 10000079  
 10000103 10000121  
 99999941 99999959 99999971 99999989 100000007 100000037  
 100000039 100000049  
 999999893 999999929 999999937 1000000007 1000000009  
 1000000021 1000000033

### Cantidad de primos menores que $10^n$

$\pi(10^1) = 4$  ;  $\pi(10^2) = 25$  ;  $\pi(10^3) = 168$  ;  $\pi(10^4) = 1229$   
 ;  $\pi(10^5) = 9592$  ;  $\pi(10^6) = 78.498$  ;  $\pi(10^7) = 664.579$  ;  
 $\pi(10^8) = 5.761.455$  ;  $\pi(10^9) = 50.847.534$  ;  
 $\pi(10^{10}) = 455.052.511$  ;  $\pi(10^{11}) = 4.118.054.813$  ;  
 $\pi(10^{12}) = 37.607.912.018$

### Divisores

Cantidad de divisores ( $\sigma_0$ ) para algunos  $n/\neg\exists n' < n, \sigma_0(n') \geq \sigma_0(n)$

$\sigma_0(60) = 12$  ;  $\sigma_0(120) = 16$  ;  $\sigma_0(180) = 18$  ;  $\sigma_0(240) = 20$  ;  $\sigma_0(360) = 24$  ;  $\sigma_0(720) = 30$  ;  $\sigma_0(840) = 32$  ;  
 $\sigma_0(1260) = 36$  ;  $\sigma_0(1680) = 40$  ;  $\sigma_0(10080) = 72$  ;  
 $\sigma_0(15120) = 80$  ;  $\sigma_0(50400) = 108$  ;  $\sigma_0(83160) = 128$  ;  
 $\sigma_0(110880) = 144$  ;  $\sigma_0(498960) = 200$  ;  $\sigma_0(554400) = 216$  ;  
 $\sigma_0(1081080) = 256$  ;  $\sigma_0(1441440) = 288$  ;  $\sigma_0(4324320) = 384$  ;  
 $\sigma_0(8648640) = 448$

Suma de divisores ( $\sigma_1$ ) para algunos  $n/\neg\exists n' < n, \sigma_1(n') \geq \sigma_1(n)$  ;  
 $\sigma_1(96) = 252$  ;  $\sigma_1(108) = 280$  ;  $\sigma_1(120) = 360$  ;  
 $\sigma_1(144) = 403$  ;  $\sigma_1(168) = 480$  ;  $\sigma_1(960) = 3048$  ;  
 $\sigma_1(1008) = 3224$  ;  $\sigma_1(1080) = 3600$  ;  $\sigma_1(1200) = 3844$  ;  
 $\sigma_1(4620) = 16128$  ;  $\sigma_1(4680) = 16380$  ;  $\sigma_1(5040) = 19344$  ;  
 $\sigma_1(5760) = 19890$  ;  $\sigma_1(8820) = 31122$  ;  $\sigma_1(9240) = 34560$  ;  
 $\sigma_1(10080) = 39312$  ;  $\sigma_1(10920) = 40320$  ;  
 $\sigma_1(32760) = 131040$  ;  $\sigma_1(35280) = 137826$  ;  $\sigma_1(36960) = 145152$  ;  
 $\sigma_1(37800) = 148800$  ;  $\sigma_1(60480) = 243840$  ;  
 $\sigma_1(64680) = 246240$  ;  $\sigma_1(65520) = 270816$  ;  $\sigma_1(70560) = 280098$  ;  
 $\sigma_1(95760) = 386880$  ;  $\sigma_1(98280) = 403200$  ;  
 $\sigma_1(100800) = 409448$  ;  $\sigma_1(491400) = 2083200$  ;  
 $\sigma_1(498960) = 2160576$  ;  $\sigma_1(514080) = 2177280$  ;  $\sigma_1(982800) = 4305280$  ;  
 $\sigma_1(997920) = 4390848$  ;  $\sigma_1(1048320) = 4464096$  ;  
 $\sigma_1(4979520) = 22189440$  ;  $\sigma_1(4989600) = 22686048$  ;  
 $\sigma_1(5045040) = 23154768$  ;  $\sigma_1(9896040) = 44323200$  ;  
 $\sigma_1(9959040) = 44553600$  ;  $\sigma_1(9979200) = 45732192$

### Factoriales

0! = 1	11! = 39.916.800
1! = 1	12! = 479.001.600 (∈ int)
2! = 2	13! = 6.227.020.800
3! = 6	14! = 87.178.291.200
4! = 24	15! = 1.307.674.368.000
5! = 120	16! = 20.922.789.888.000
6! = 720	17! = 355.687.428.096.000
7! = 5.040	18! = 6.402.373.705.728.000
8! = 40.320	19! = 121.645.100.408.832.000
9! = 362.880	20! = 2.432.902.008.176.640.000 ∈ ll
10! = 3.628.800	21! = 51.090.942.171.709.400.000

max signed tint = 9.223.372.036.854.775.807  
 max unsigned tint = 18.446.744.073.709.551.615

## Consejos

### Debugging

- ¿Si  $n = 0$  anda? (similar casos borde tipo  $n=1$ ,  $n=2$ , etc)
- ¿Si hay puntos alineados anda?
- ¿Si es vacío anda?
- ¿Si hay multiejes anda?

- ¿Si no tiene aristas anda?
- ¿Si tiene ciclos anda?
- ¿Si tiene un triángulo anda?
- ¿Los arrays son suficientemente grandes? (siempre denle bastante de más por las dudas, pero tampoco se ceben como para que ya no entre en memoria XD)
- ¿Puede dar integer overflow? (SIEMPRE mirar el integer overflow con MUCHO cuidado)
- ¿Podés dividir por cero en algún caso?
- ¿Estás memorizando la recursión bien?
- ¿El caso base está bien hecho y se llega siempre?
- ¿Están bien puestas las cotas iniciales de la binary / inicialización del acumulador máximo/mínimo?
- ¿Estás inicializando bien antes de cada caso?
- ¿Le copiaste el input dos veces en el archivo de entrada (para ver que de igual y bien las dos veces)? [No aplica cuando viene solo una instancia de input]
- ¿Pasa los ejemplos? [No es joda, Leo se quedo afuera de la mundial por esto]

### Hitos de prueba

- 45min todas las columnas de la tabla llena
- 2h todos conocen todo
- 3h reunión estratégica
- 4h reunión estratégica