## Contents

```
st[index] = value
                                                                        while (index > 1) {
                                                                            index /= 2
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                                                               1
                                                                            st[index] = operation(st[2 * index], st[2 * index + 1])
   1.1 Segment Tree . . . . . . . . . . . . . . . .
                                                               1
   1.2 Fenwick Tree . . . .
                                                               1
                                                                     // Realiza una consulta sobre el rango [lq, rq)
                                                                    fun query(lq: Int, rq: Int): T {
   var left = lq + size
   var right = rq + size
  Grafos
                                                               1
   2.1 BFS . . . . . . . . . . . .
                                                               1
                                                               1
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                                                                        var lres = neutral
var rres = neutral
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                                                               1
   2.4 Floyd-Warshall . . . . . . .
                                                               1
                                                                        while (left < right) {</pre>
                                                               2
   2.5 Kruskal . . . . . . . . . . . . . . . .
                                                                            if (left % 2 == 1) {
                                                                                lres = operation(lres, st[left])
                                                               2
   2.6 Ancestro común menor . . . .
                                                                               left++
                                                               2
   Strings
                                                                            if (right % 2 == 1) {
   3.1 Bordes (KMP) . . . . . . .
                                                                               right--
                                                               2
                                                                                rres = operation(st[right], rres)
                                                               2
   3.2 Función Z . . . . .
   3.3 Manacher . . . . . . . . .
                                                               2
                                                                            left /= 2
   3.4 Suffix Array . . . . . . .
                                                               2
                                                                            right /= 2
                                                               3
   3.5 LCP (Estructura) . . . . .
                                                                        return operation(lres, rres)
                                                               3
                . . . . . . . . . . . . .
                                                               3 }
   3.7 Hashing
                  . . . . . . . . .
                                                                   Fenwick Tree
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                                                               4
                                                                 class Fenwick<T>(
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                                                               4
                                                                     val n: Int,
   4.3 Convolución rápida (FFT y Karatsuba) .
                                                                     val operation: (T, T) -> T,
                                                               4
                                                                     val neutral: T){
   4.4 Criba de Eratostenes . . . . . . .
                                                               4
                                                                     var ft: MutableList<T>
                                                               5
   4.5 Mobius . . . . . . . .
                                                                     init. {
                                                                        ft = MutableList(n + 1) { neutral }
                                                               5
5 Geometría
                                                               5
   5.1 Punto
                                                                     fun update(p: Int, value: T) {
                                                               5
   var index = p
                                                                        while (index <= n) {</pre>
                                                               5
   5.3 Capsula convexa
                                                                            ft[index] = operation(ft[index], value)
                                                                            index += index and -index
                                                               5
   Tablas y Cotas
                                                                     }
   6.1 Divisores
                                                               6
   6.2 Factoriales . . . .
                                                               6
                                                                     fun query(r: Int): T {
                                                                        var index = r
                                                                        var res = neutral
7 Consejos
                                                               6
                                                                        while (index > 0) {
   7.1 Debugging . . . . .
                                                               6
                                                                            res = operation(res, ft[index])
index -= index and -index
   7.2 Hitos de prueba
                                                                        return res
```

# Estructuras de Datos,

### Segment Tree

```
class SegmentTree<T>(
    val n: Int,
    val operation: (T, T) -> T,
    val neutral: T
    var size: Int
    var st: MutableList<T>
    init {
         size = 1
         while (size <= n) size *= 2
st = MutableList(2 * size) { neutral }</pre>
     // Inicializa el segmento del árbol con los valores dados
    fun init(arr: List<T>) {
         for (i in 0 until n) {
             st[i + size] = arr[i]
         for (i in size - 1 downTo 1) {
    st[i] = operation(st[2 * i], st[2 * i + 1])
    }
```

// Actualiza un valor en la posición p

## Grafos

fun update(p: Int, value: T) { var index = p + size

#### **BFS**

```
fun bfs(
    graph: List<List<Int>>,
    start: Int
) : List<Int> {
    val n = graph.size
    val min_dist = MutableList(n) { -1 }
    val q = mutableListOf<Int>()
    q.add(start)
   min_dist[start] = 0
   var di = 0
    while (qi < q.size){
        val u = q[qi]
        for (v in graph[u]){
            if (min_dist[v] == -1){
                min_dist[v] = min_dist[u] + 1
                q.add(v)
        }
```

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```
return min dist
                                                                               var valor = 0
}
                                                                               var arbol = mutableListOf<Int>()
                                                                               for (ar in aristas){
                                                                                   val (u,v,c) = g[ar]
  Dijkstra
                                                                                   if (find(u) != find(v)){
import java.util.PriorityQueue
                                                                                       union(u,v)
                                                                                       valor += c
fun Dijkstra(
                                                                                       arbol.add(ar)
   grafo: List<List<Pair<Int,Int>>>,
    inicio: Int
                                                                               }
) : Pair<List<Int>, List<Int>> {
                                                                               return Pair(valor, arbol)
   val n = grafo.size
   var dist = MutableList(n) { Int.MAX_VALUE }
   var padre = MutableList(n) { -1 }
                                                                              Ancestro común menor
   dist[inicio] = 0
   val pq = PriorityQueue<Pair<Int,Int>>(compareBy { -it.second })
                                                                           class LCA(
    arbol: List<List<Int>>>,
   pq.add(Pair(inicio, 0))
    while (pq.isNotEmpty()){
                                                                               raiz: Int
       val (u, d) = pq.poll()
                                                                           ){
        if (d > dist[u]) continue
                                                                               var K: Int.
        for ((v, w) in grafo[u]){
                                                                               var padre: MutableList<MutableList<Int>>
            if (dist[u] + w < dist[v]){
    dist[v] = dist[u] + w</pre>
                                                                               var prof: MutableList<Int>
                                                                               init {
                padre[v] = u
                                                                                   val n = arbol.size
                pq.add(Pair(v, dist[v]))
                                                                                   K = 1
                                                                                   while ((1 shl K) < n) K++
                                                                                   padre = MutableList(K) { MutableList(n) { -1 } }
                                                                                   prof = MutableList(n) { -1 }
    return Pair(dist, padre)
}
                                                                                   fun dfs(u: Int, p: Int){
                                                                                       padre[0][u] = p
  Bellman-Ford
                                                                                       for (v in arbol[u]){
                                                                                           if (v == p) continue
                                                                                           prof[v] = prof[u] + 1
fun BellmanFord(
                                                                                           dfs(v, u)
   grafo: List<List<Pair<Int,Int>>>,
    ĭnicio: Int,
                                                                                       }
   largo: Int
                                                                                   }
) : List<List<Int>> {
                                                                                   dfs(raiz, -1)
   val n = grafo.size
                                                                                   prof[raiz] = 0
   var dist = MutableList(largo+1) { MutableList(n) { Int.MAX_VALUE} }
    dist[0][inicio] = 0
                                                                                   for (k in 1 until K){
   for (k in 0 until largo){
                                                                                       for (u in 0 until n){
        for (u in 0 until n){
                                                                                           if (padre[k-1][u] != -1){
            for ((v, w) in grafo[u]){
                                                                                                padre[k][u] = padre[k-1][padre[k-1][u]]
                if( dist[k][u] != Int.MAX_VALUE)
                    dist[k+1][v] = min0f(dist[k+1][v], dist[k][u] + w)
                                                                                       }
                                                                                   }
                                                                               }
    return dist
                                                                               fun lca(uu: Int, vv: Int) : Int { var u = uu \\ var v = vv
}
  Floyd-Warshall
                                                                                   if (prof[u] < prof[v]) return lca(v, u)</pre>
                                                                                   for (k in K-1 downTo 0){
                                                                                       if (prof[u] - (1 shl k) >= prof[v]){
fun FloydWarshall(
   matriz: List<List<Int>>
                                                                                           u = padre[k][u]
) : MutableList<MutableList<Int>> {
   val n = matriz.size
   var dist = matriz.map{ it.toMutableList() }.toMutableList()
                                                                                   if (u == v) return u
   for (k in 0 until n)
                                                                                   for (k in K-1 downTo θ){
        for (i in 0 until n){
                                                                                       if (padre[k][u] != padre[k][v]){
            for (j in 0 until n){
                                                                                           u = padre[k][u]
                dist[i][j] = minOf(dist[i][j], dist[i][k] + dist[k][j])
                                                                                           v = padre[k][v]
                                                                                   return padre[0][u]
    return dist
                                                                               }
                                                                           }
  Kruskal
                                                                                   Strings
fun Kruskal(g: List<Triple<Int,Int,Int>>, n : Int) : Pair<Int, List<Int</pre>
    var uf = MutableList(n){i -> i}
                                                                             Bordes (KMP)
   fun find(x: Int) : Int{
        if (uf[x] == x) return x
                                                                           fun bordes(s: String): List<Int>{
        uf[x] = find(uf[x])
                                                                               val n = s.length
        return uf[x]
                                                                               val b = MutableList(n+1) { -1 }
   }
                                                                               var j = -1;
                                                                               for (i in 0 until n){
    fun union(x: Int, y: Int){
                                                                                   while (j >= 0 && s[i] != s[j]){
        uf[find(x)] = find(y)
                                                                                       j = b[j]
```

j++

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val aristas = MutableList(n){i -> i}.sortedBy{g[it].third}

```
b[i+1] = j
    return b
}
  Función Z
fun z(s: String): List<Int>{ // z[i] = max k: s[0,k) == s[i,i+k)
   val n = s.length
   val z = MutableList(n) { 0 }
   var 1 = 0
   var r = 0
   for (i in 1 until n){
        if (i <= r) z[i] = min0f(r-i+1, z[i-1])</pre>
        while (i+z[i] < n \&\& s[z[i]] == s[i+z[i]]) z[i]++
        if (i+z[i]-1 > r){
            1 = i
            r = i+z[i]-1
        }
    return z
}
  Manacher
fun Manacher(s: String): Pair<List<Int>, List<Int>>{
     / (d1, d2) = (impares, pares) palindromes
    val n = s.length
   val d1 = MutableList(n) { 0 }
   val d2 = MutableList(n) { 0 }
   var 1 = 0
   var r = -1
   for (i in 0 until n){
        var k = if (i > r) 1 else min0f(d1[l+r-i], r-i+1)
        while (i-k \ge 0 \&\& i+k < n \&\& s[i-k] == s[i+k]) k++
        d1[i] = k--
        if (i+k > r){
            1 = i-k
            r = i+k
        }
   1 = 0
   r = -1
   for (i in 0 until n){
        var k = if (i > r) 0 else min0f(d2[1+r-i+1], r-i+1)
        while (i-k-1 >= 0 \&\& i+k < n \&\& s[i-k-1] == s[i+k]) k++
        d2[i] = k--
        if (i+k > r){
            1 = i - k - 1
            r = i+k
        }
   return Pair(d1, d2)
}
  Suffix Array
fun RB(x : Int, n : Int, r: List<Int>) : Int{
   if(x < n) return r[x]</pre>
   else return 0
}
fun csort(sa: MutableList<Int>, r: MutableList<Int>, k : Int){
   val n = sa.size
   var f = MutableList(max0f(255,n)){0}
   var t = MutableList(n){0}
   for (i in 0 until n) f[RB(i+k,n,r)]++
   var sum = 0
   for (i in 0 until f.size){
       var v = f[i]
f[i] = sum
sum += v
   for (i in 0 until n){
        t[f[RB(sa[i]+k,n,r)]++] = sa[i]
   }
    for (i in 0 until n) sa[i] = t[i]
}
fun suffix_array(s0: String): List<Int>{
   val s = s0 + ' \u0000'
    val n = s.length
```

var rank: Int

var sa = MutableList(n){it}

var t = MutableList(n){0}

var r = MutableList(n){it -> s[it].code}

```
var k = 1
    while (k<n){</pre>
        csort(sa,r,k)
        csort(sa,r,0)
        t[sa[0]] = 0
        rank = 0
        for (i in 1 until n){
            if(r[sa[i]] != r[sa[i-1]] || RB(sa[i]+k,n,r) != RB(sa[i-1]+
                 k,n,r)) rank++
            t[sa[i]] = rank
        for (i in 0 until n) r[i] = t[i]
        if (r[sa[n-1]]==n-1) break
        println("k = $k")
    return sa
  LCP (Estructura)
fun computar_lcp(s0: String, sa: List<Int>): MutableList<Int>{
    val s = s0 + ' \u0000'
    val n = s.length
    var L = 0
    var lcp = MutableList(n){0}
    var plcp = MutableList(n){0}
    var phi = MutableList(n){0}
    phi[sa[0]] = -1
    for (i in 1 until n) phi[sa[i]] = sa[i-1]
    for (i in 0 until n){
        if (phi[i] == -1){
            plcp[i] = 0
            continue
        while (s[i+L] == s[phi[i]+L]) L++
        plcp[i] = L
        L = max0f(L-1,0)
    for (i in 0 until n) lcp[i] = plcp[sa[i]]
    return lcp
  Duval
// Dada una string $s$ devuelve la Lyndon decomposition en tiempo
// lineal usando el algoritmo de Duval. Factoriza $s$ como
// s_1 s_2 \cdot s_k \ con s_1 \cdot g_q s_2 \cdot g_q \cdot s_k \
// y tal que $s_i$ es Lyndon, esto es, es su menor rotación.
fun Duval(s: String) : List<String>{
    val n = s.length
    var i = 0
    val ans = mutableListOf<String>()
    while (i < n){
        var j = i + 1
        var k = i
        while (j < n \&\& s[k] <= s[j]){
            if (s[k] < s[j]) k = i
            else k++
        while (i \le k)
            ans.add(s.substring(i until i+j-k))
            i += j-k
        }
    return ans
// Obtener la mínima rotaciónn de $s$: en la descomposición de
// Lyndon de $s^2$ es el último $i<|s|$ con el que empieza una
// Lyndon.
  Hashing
const val P: Long = 1777771
val MOD: List<Long> = listOf(999727999, 1070777777)
val PI: List<Long> = listOf(325255434, 10018302)
class Hashing(c: Char)
    val h: MutableList<Long>
    val p: MutableList<Long>
    val pi: MutableList<Long>
    init {
```

h = MutableList(PI.size) { i -> c.code \* P % MOD[i]}

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```
p = MutableList(PI.size) { P }
        pi = PI.toMutableList()
    // Agrega un prefijo : H(s1) + H(s2) = H(s2s1)
    operator fun plus(h2: Hashing) : Hashing {
        val ans = Hashing('a')
        for (i in 0 until PI.size){
            ans.h[i] = (h[i] * h2.\hat{p}[i] + h2.h[i]) % MOD[i] ans.p[i] = p[i] * h2.p[i] % MOD[i]
            ans.pi[i] = pi[i] * h2.pi[i] % MOD[i]
        return ans
    }
    // Elimina un prefijo
    operator fun minus(h2: Hashing) : Hashing {
        val ans = Hashing('a')
        for (i in 0 until PI.size){
            ans.h[i] = (h[i] - h2.h[i] + MOD[i]) % MOD[i] * h2.pi[i] %
                  MOD[i]
            ans.p[i] = p[i] * h2.pi[i] % MOD[i]
             ans.pi[i] = pi[i] * h2.p[i] % MOD[i]
        return ans
    }
    /// O simplemente comparar h1.h == h2.h en vez de h1==h2
    override fun equals(h2: Any?): Boolean {
        if (h2 !is Hashing) return false
        return h == h2.h
fun hash_neutro() : Hashing{
    var ans = Hashing('a')
    for (i in 0 until PI.size){
        ans.h[i] = 0
ans.p[i] = 1
        ans.pi[i] = 1
    return ans
class StringHasher(s: String){
    val h: MutableList<Hashing>
    init {
        h = MutableList(s.length+1) {
            if (it == 0) hash_neutro()
             else Hashing(s[it-1])
        for (i in 1 until s.length+1){
            h[i] = h[i] + h[i-1]
    }
    // Hash de s[1,r)
    fun hash(1: Int, r: Int) : Hashing {
    return h[r] - h[1]
```

### Matemáticas

### Identidades

}

}

}

$$\begin{split} C_n &= \frac{2(2n-1)}{n+1} C_{n-1} \\ C_n &= \frac{1}{n+1} \binom{2n}{n} \\ C_n &\sim \frac{4^n}{n^{3/2} \sqrt{\pi}} \\ F_{2n+1} &= F_n^2 + F_{n+1}^2 \\ F_{2n} &= F_{n+1}^2 - F_{n-1}^2 \\ \sum_{i=1}^n F_i &= F_{n+2} - 1 \\ F_{n+i} F_{n+j} - F_n F_{n+i+j} &= (-1)^n F_i F_j \\ \sum_{i=0}^n r^i &= \frac{r^{n+1}-1}{r-1} \\ \sum_{i=1}^n i^2 &= \frac{n \cdot (n+1) \cdot (2n+1)}{6} \\ \sum_{i=1}^n i^3 &= \left(\frac{n \cdot (n+1)}{2}\right)^2 \\ \sum_{i=1}^n i^4 &= \frac{n \cdot (n+1) \cdot (2n+1) \cdot (3n^2 + 3n - 1)}{12} \\ \sum_{i=1}^n i^5 &= \left(\frac{n \cdot (n+1)}{2}\right)^2 \cdot \frac{2n^2 + 2n - 1}{3} \end{split}$$

```
\begin{array}{l} \sum_{i=1}^n \binom{n-1}{i-1} = 2^{n-1} \\ \sum_{i=1}^n i \cdot \binom{n-1}{i-1} = n \cdot 2^{n-1} \end{array}
(Möbius Inv. Formula) Let
                                             g(n) = \sum_{d|n} f(d), then
                                              f(n) = \sum_{d|n} g(d)\mu\left(\frac{n}{d}\right)
```

#### Teoremas

(Tutte) A graph, G = (V, E), has a perfect matching if and only if for every subset U of V, the subgraph induced by V - U has at most |U| connected components with an odd number of vertices. Petersens Theorem. Every cubic, bridgeless graph contains a perfect matching. (Dilworth) In any finite partially ordered set, the maximum number of elements in any antichain equals the minimum number of chains in any partition of the set into chains Pick: A=I+B/2-1 (area of polygon, points inside, points on border) Problema de los monstruos: Si los monstruos me sacan X vida y luego me dan Y vida, me conviene primero enfrentar a los Y≥X en orden creciente de X, y luego a los Y<X en orden decresciente de Y. Grafo planar: regiones = ejes - nodos + componentesConexas + 1; Condición: aristas≤3\*vertices-6

### Convolución rápida (FFT y Karatsuba)

```
import kotlin.math.round
data class Complex(val r: Double, val i: Double){
    operator fun plus(x: Complex) = Complex(r + x.r, i + x.i)
    operator fun minus(x: Complex) = Complex(r - x.r, i - x.i)
    operator fun times(x: Complex) = Complex(r*x.r - i*x.i, r*x.i + i*x
          .r)
    operator fun div(x: Double) = Complex(r/x, i/x)
}
class FFT(lg0: Int){
   val lg = lg0+1 
val n = 1 shl lg
    val w = MutableList(n+1){Complex(0.0, 0.0)}
        val ang0 = 2.0 * Math.PI / n.toDouble()
        for (i in 0 until n+1){
            val ang = ang0 * i
            w[i] = Complex(Math.cos(ang), Math.sin(ang))
    }
    fun fft(a: List<Complex>, inv: Boolean = false) : List<Complex>{
        val p = MutableList(n) { a[Integer.reverse(it) ushr (32 - lg)]
        var len = 2
        while (len <= n) \{
            val step = n / len
            for (i in 0 until n step len) {
                for (j in 0 until len / 2) {
                    val u = p[i + j]
val v = p[i + j + len / 2] * if (inv) w[n - j *
                          step] else w[j * step]
                     p[i + j] = u + v
                    p[i + j + len / 2] = u - v
                }
            Îen *= 2
        }
        if (inv) {
            for (i in 0 until n) {
                p[i] = p[i] / n.toDouble()
        }
        return p
```

```
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```

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```

```
fun multiply(a: List<Long>, b: List<Long>) : List<Long>{
                                                                                           if(mobius[i]!=0){
        val a_c = a.map { Complex(it.toDouble(), 0.0) } + MutableList(n
                                                                                               for(j in (i+i)..n step i){
              -a.size){Complex(0.0, 0.0)}
                                                                                                    mobius[j] -= mobius[i];
        val b_c = b.map { Complex(it.toDouble(), 0.0) } + MutableList(n
                                                                                           }
              -b.size){Complex(0.0, 0.0)}
        val fa = fft(a_c)
        val fb = fft(b_c)
                                                                                       return mobius
        val fc = MutableList(n){fa[it] * fb[it]}
        val c = fft(fc, true)
        return c.map { round(it.r).toLong() }
                                                                                           Geometría
    }
}
                                                                                    Punto
fun Karatsuba(a : List<Long>, b : List<Long>) : List<Long>{
    val m = maxOf(a.size, b.size)
                                                                                  import kotlin.math.*
    val n = 1 shl (32 - Integer.numberOfLeadingZeros(m - 1))
                                                                                  class pt(x: Double, y: Double): Comparable<pt>{
    val aa = a + MutableList(n - a.size) { 0.toLong() }
    val bb = b + MutableList(n - b.size) { 0.toLong() }
                                                                                      val x = x
                                                                                      val y = y
    return karatsuba(aa, bb)
                                                                                       operator fun plus(p: pt) = pt(x + p.x, y + p.y)
                                                                                      operator fun minus(p: pt) = pt(x - p.x, y - p.y) operator fun times(k: Double) = pt(x * k, y * k)
fun karatsuba(a: List<Long>, b: List<Long>): List<Long> {
                                                                                      operator fun diw(k: Double) - pt(x / k, y / k)
operator fun div(k: Double) = pt(x / k, y / k)
operator fun times(p: pt) = x * p.x + y * p.y
operator fun rem(p: pt) = x * p.y - y * p.x
fun angle(p: pt) = acos((this * p) / (this.norm() * p.norm()))
fun norm2() = x * x + y * y
fun 2()
    if (a.size <= 16) { // Reducir el tamaño de la condición base
        val c = MutableList(2 * a.size - 1) { 0L }
        for (i in a.indices) {
             for (j in b.indices) {
                 c[i + j] += a[i] * b[j]
                                                                                       fun norm() = sqrt(norm2())
                                                                                       fun unit() = if (norm() > 0) this / norm() else pt(0.0, 0.0)
                                                                                       fun rot(r: pt) = pt(this % r, this * r)
        return c
    }
                                                                                      fun rot(a: Double) = this.rot(pt(cos(a), sin(a)))
                                                                                      fun left(p: pt, q: pt) = (q - p).unit() \% (this - p).unit() > EPS
    val n = a.size
    val k = n / 2
                                                                                      operator override fun compareTo(p: pt): Int = when {
    val a0 = a.subList(0, k)
                                                                                           abs(this.x - p.x) > EPS -> this.x.compareTo(p.x)
    val a1 = a.subList(k, n)
                                                                                           else -> this.y.compareTo(p.y)
    val b0 = b.subList(0, k)
                                                                                      }
    val b1 = b.subList(k, n)
                                                                                      override fun equals(other: Any?) = other is pt && abs(x - other.x)<
    val z2 = karatsuba(a1, b1)
                                                                                            EPS && abs(y - other.y)<EPS
    val z0 = karatsuba(a0, b0)
                                                                                      override fun toString() = "($x, $y)"
    val a0a1 = List(k) { a0[it] + a1[it] }
val b0b1 = List(k) { b0[it] + b1[it] }
                                                                                  val ccw90 = pt(1.0, 0.0)
    val z1 = karatsuba(a0a1, b0b1)
                                                                                  val cw90 = pt(-1.0,0.0)
    val result = MutableList(2 * n) { 0L }
    for (i in z0.indices) result[i] += z0[i]

for (i in z2.indices) result[i + n] += z2[i]

for (i in z1.indices) result[i + k] += z1[i] - z0.getOrElse(i) { 0L import kotlin.math.*
           } - z2.getOrElse(i) { 0L }
                                                                                  class Segment(val f: pt, val s: pt) {
    return result
                                                                                       fun length(): Double {
                                                                                           val dx = f.x - s.x
                                                                                           val dy = f.y - s.y
  Criba de Eratostenes
                                                                                           return sqrt(dx * dx + dy * dy)
                                                                                      }
                                                                                  }
class Criba(n: Int){
    var criba = MutableList(n+1){-1}
                                                                                  fun pc(a: pt, b: pt, o: pt): Double = (a-o) % (b-o)
    init {
                                                                                  fun pe(a: pt, b: pt, o: pt): Double = (a-o) * (b-o)
        for (i in 2..n){
             if (criba[i] == -1){
                                                                                  fun intersect(a: Segment, b: Segment): Boolean{
                 if (n/i>=i) for (j in i*i until (n+1) step i){
                                                                                       val fb = 0.compareTo(pc(a.f, a.s, b.f))
                                                                                      val sb = 0.compareTo(pc(a.f, a.s, b.s))
val fa = 0.compareTo(pc(b.f, b.s, a.f))
                      if (criba[j] == -1) criba[j] = i
             }
                                                                                      val sa = 0.compareTo(pc(b.f, b.s, a.s))
        }
                                                                                       if ((fb * sb < 0) && (fa * sa<0)) return true
    }
                                                                                      if ((fb==0 && pe(a.f, a.s, b.f)<=0) || (sb==0 && pe(a.f, a.s,b.s)
                                                                                             <=0)) return true;
    fun fact(n: Int) : MutableMap<Int,Int> {
        var res = mutableMapOf<Int,Int>()
var x = n
                                                                                       if ((fa==0 && pe(b.f, b.s, a.f)<=0) || (sa==0 && pe(b.f, b.s, a.s)
                                                                                            <=0)) return true;
        while(criba[x] != -1){
                                                                                       return false
             res[criba[x]] = res.getOrDefault(criba[x], 0) + 1
                                                                                  fun dist(p: pt, s: Segment): Double{
                                                                                      val a = abs(pc(s.f,s.s,p))
        if(x != 1) res[x] = res.getOrDefault(x, 0) + 1
                                                                                      val b = hypot(s.f.x - s.s.x, s.f.y - s.s.y)
        return res
                                                                                      val h = a/b
                                                                                      val c = hypot(b,h)
                                                                                      val d1 = (s.f-p).norm()
                                                                                       val d2 = (s.s-p).norm()
  Mobius
                                                                                       if(b<EPS || c<= d1 || c<= d2) return minOf(d1,d2)</pre>
                                                                                       return h
fun Mobius(n: Int): List<Int>{
                                                                                  }
    var mobius = MutableList(n+1){1}
    mobius[0] = 0
                                                                                  fun dist(a: Segment, b : Segment) : Double{
```

if(intersect(a,b)) return 0.0

**for**(i **in** 2..n){

```
(- ejemplo
```

```
Capsula convexa
fun chull(ps: List<pt>) : List<pt>{
   if(ps.size < 3) return ps</pre>
   val p = ps.sorted()
   val ch = mutableListOf<pt>()
   for(pi in p){
       while(ch.size > 1 && ch[ch.size - 1].left(ch[ch.size - 2], pi))
              ch.removeAt(ch.size - 1)
       ch.add(pi)
   ch.removeAt(ch.size - 1)
   val t = ch.size
   for(pi in p.reversed()){
       while(ch.size > t+1 && ch[ch.size - 1].left(ch[ch.size - 2], pi
            )) ch.removeAt(ch.size - 1)
       ch.add(pi)
   ch.removeAt(ch.size - 1)
   return ch
}
```

# Tablas y Cotas

```
Primos cercanos a 10^n
9941 9949 9967 9973 10007 10009 10037 10039 10061
```

return minOf(

}

minOf(dist(a.f,b),dist(a.s,b)),

min(dist(b.f,a),dist(b.s,a))

10067 10069 10079

99961 99971 99989 99991 100003 100019 100043 100049 100057 100069

999959 999961 999979 999983 1000003 1000033 1000037 1000039

9999943 9999971 9999973 9999991 10000019 10000079 10000103 10000121

99999941 99999959 99999971 99999989 100000007 1000000037 100000039 100000049

999999893 999999929 999999937 1000000007 10000000009 1000000021 1000000033

#### Cantidad de primos menores que $10^n$

```
\pi(10^1) = 4; \pi(10^2) = 25; \pi(10^3) = 168; \pi(10^4) = 1229
; \pi(10^5) = 9592 ; \pi(10^6) = 78.498 ; \pi(10^7) = 664.579 ;
\pi(10^8) = 5.761.455 ; \pi(10^9) = 50.847.534 ;
\pi(10^{10}) = 455.052,511; \pi(10^{11}) = 4.118.054.813;
\pi(10^{12}) = 37.607.912.018
```

### Divisores

```
Cantidad de divisores (\sigma_0) para algunos n/\neg \exists n'
n, \sigma_0(n') \geqslant \sigma_0(n)
\sigma_0(60) = 12; \sigma_0(120) = 16; \sigma_0(180) = 18; \sigma_0(240)
= 20 ; \sigma_0(360) = 24 ; \sigma_0(720) = 30 ; \sigma_0(840) = 32
; \sigma_0(1260) = 36 ; \sigma_0(1680) = 40 ; \sigma_0(10080) = 72 ;
\sigma_0(15120) = 80; \sigma_0(50400) = 108; \sigma_0(83160) = 128
\sigma_0(110880) = 144; \sigma_0(498960) = 200; \sigma_0(554400) = 216
; \sigma_0(1081080) = 256 ; \sigma_0(1441440) = 288 \sigma_0(4324320) =
384 ; \sigma_0(8648640) = 448
Suma de divisores (\sigma_1) para algunos n/\neg \exists n' < n, \sigma_1(n') \geqslant n
\sigma_1(n) ; \sigma_1(96) = 252 ; \sigma_1(108) = 280 ; \sigma_1(120) = 360
; \sigma_1(144) = 403 ; \sigma_1(168) = 480 ; \sigma_1(960) = 3048 ;
\sigma_1(1008) = 3224; \sigma_1(1080) = 3600; \sigma_1(1200) = 3844
; \sigma_1(4620) = 16128 ; \sigma_1(4680) = 16380 ; \sigma_1(5040) =
19344 ; \sigma_1(5760) = 19890 ; \sigma_1(8820) = 31122 ; \sigma_1(9240)
= 34560 ; \sigma_1(10080) = 39312 ; \sigma_1(10920) = 40320 ;
\sigma_1(32760) = 131040 ; \sigma_1(35280) = 137826 ; \sigma_1(36960)
= 145152 ; \sigma_1(37800) = 148800 ; \sigma_1(60480) = 243840 ;
\sigma_1(64680) = 246240; \sigma_1(65520) = 270816; \sigma_1(70560)
```

```
= 280098 ; \sigma_1(95760) = 386880 ; \sigma_1(98280) = 403200 ;
\begin{array}{c} \sigma_1(100800) = 409448 \; ; \; \sigma_1(491400) = 2083200 \; ; \\ \sigma_1(498960) = 2160576 \; ; \; \sigma_1(514080) = 2177280 \; ; \; \sigma_1(982800) \\ = 4305280 \; ; \; \sigma_1(997920) = 4390848 \; ; \; \sigma_1(1048320) = 4464096600 \\ ; \; \sigma_1(4979520) = 22189440 \; ; \; \sigma_1(4989600) = 22686048 \; ; \\ \sigma_1(5045040) = 23154768 \; ; \; \sigma_1(9896040) = 44323200 \; ; \\ \sigma_1(9959040) = 44553600 \; ; \; \sigma_1(9979200) = 45732192 \\ \hline \textbf{Factoriales} \\ 0! = 1 & | 11! = 39.916.800 \\ 1! = 1 & | 12! = 479.001.600 \; (\in \text{int}) \\ 2! = 2 & | 13! = 6.227.020.800 \\ 3! = 6 & | 14! = 87.178.291.200 \\ 4! = 24 & | 15! = 1.307.674.368.000 \\ 5! = 120 & | 16! = 20.922.789.888.000 \\ 6! = 720 & | 17! = 355.687.428.096.000 \\ 7! = 5.040 & | 18! = 6.402.373.705.728.000 \\ 8! = 40.320 & | 19! = 121.645.100.408.832.000 \\ \hline \end{array}
  \sigma_1(100800) = 409448; \sigma_1(491400) = 2083200;
                                                                     19! = 121.645.100.408.832.000
      8! = 40.320
                                                                    20! = 2.432.902.008.176.640.000 \in 11
      9! = 362.880
      10! = 3.628.800 | 21! = 51.090.942.171.709.400.000
  max signed tint = 9.223.372.036.854.775.807
```

# Consejos

#### Debugging

• ¿Si n = 0 anda? (similar casos borde tipo n=1, n=2, etc)

max unsigned tint = 18.446.744.073.709.551.615

- ¿Si hay puntos alineados anda?
- ¿Si es vacío anda?
- ¿Si hay multiejes anda?
- ¿Si no tiene aristas anda?
- ¿Si tiene ciclos anda?
- ¿Si tiene un triángulo anda?
- ¿Los arrays son suficientemente grandes? (siempre denle bastante de más por las dudas, pero tampoco se ceben como para que ya no entre en memoria XD)
- ¿Puede dar integer overflow? (SIEMPRE mirar el integer overflow con MUCHO cuidado)
- ¿Podés dividir por cero en algún caso?
- ¿Estás memorizando la recursión bien?
- ¿El caso base está bien hecho y se llega siempre?
- ¿Están bien puestas las cotas iniciales de la binary / inicialización del acumulador máximo/mínimo?
- ¿Estás inicializando bien antes de cada caso?
- ¿Le copiaste el input dos veces en el archivo de entrada (para ver que de igual y bien las dos veces)? [No aplica cuando viene solo una instancia de input]
- ¿Pasa los ejemplos? [No es joda, Leo se quedo afuera de la mundial por esto]

#### Hitos de prueba

- 45min todas las columnas de la tabla llena
- 2h todos conocen todo
- 3h reunión estratégica
- 4h reunión estratégica