Contents Run.sh

```
kotlinc "1".kt -include-runtime -d "1".jar for x in "1".in; do
   echo ARCHIVO: $x
   cat $x
   echo =======
   java -jar "1".jar < x
   echo ========
done | tee -a $1.print
How to run: './run.sh A' o 'bash run.sh A'
```

Estructuras de Datos

Segment Tree

```
class SegmentTree<T>(
   val n: Int,
   val operation: (T, T) -> T,
   val neutral: T
   var size: Int
   var st: MutableList<T>
   init {
        size = 1
        while (size <= n) size *= 2
        st = MutableList(2 * size) { neutral }
   }
    // Inicializa el segmento del árbol con los valores dados
   fun init(arr: List<T>) {
        for (i in 0 until n) {
            st[i + size] = arr[i]
        for (i in size - 1 downTo 1) {
            st[i] = operation(st[2 * i], st[2 * i + 1])
   }
    // Actualiza un valor en la posición p
   fun update(p: Int, value: T) {
   var index = p + size
        st[index] = value
        while (index > 1)
            index /= 2
            st[index] = operation(st[2 * index], st[2 * index + 1])
        }
   }
    // Realiza una consulta sobre el rango [lq, rq)
    fun query(lq: Int, rq: Int): T {
        var left = lq'+ size
        var right = rq + size
        var lres = neutral
        var rres = neutral
        while (left < right) {</pre>
            if (left % 2 == 1) {
                lres = operation(lres, st[left])
                left++
            if (right % 2 == 1) {
                right--
                rres = operation(st[right], rres)
            left /= 2
            right /= 2
        return operation(lres, rres)
}
```

Fenwick Tree

```
class Fenwick<T>(
    val n: Int,
    val operation: (T, T) \rightarrow T,
    val neutral: T){
    var ft: MutableList<T>
```

```
init. {
    ft = MutableList(n + 1) { neutral }
fun update(p: Int, value: T) {
    var index = p
    while (index <= n) {</pre>
        ft[index] = operation(ft[index], value)
        index += index and -index
fun query(r: Int): T {
    var index = r
    var res = neutral
    while (index > 0) {
        res = operation(res, ft[index])
        index -= index and -index
    return res
```

Grafos

BFS

```
fun bfs(
    graph: List<List<Int>>,
    start: Int
) : List<Int> {
    val n = graph.size
    val min_dist = MutableList(n) { -1 }
    val q = mutableListOf<Int>()
    q.add(start)
    min_dist[start] = 0
    var qi = 0
    while (qi < q.size){</pre>
        val u = q[qi]
        ai++
        for (v in graph[u]){
            if (min_dist[v] == -1){
                min_dist[v] = min_dist[u] + 1
                q.add(v)
        }
    return min_dist
```

Dijkstra

```
import java.util.PriorityQueue
fun Dijkstra(
    grafo: List<List<Pair<Int,Int>>>,
    ĭnicio: Int
) : Pair<List<Int>, List<Int>> {
    val n = grafo.size
    var dist = MutableList(n) { Int.MAX_VALUE }
    var padre = MutableList(n) { -1 }
    dist[inicio] = 0
    val pq = PriorityQueue<Pair<Int,Int>>(compareBy { -it.second })
    pq.add(Pair(inicio, θ))
    while (pq.isNotEmpty()){
        val (u, d) = pq.poll()
        if (d > dist[u]) continue
        for ((v, w) in grafo[u]){
            if (dist[u] + w < dist[v]){
    dist[v] = dist[u] + w</pre>
                 padre[v] = u
                 pq.add(Pair(v, dist[v]))
            }
        }
    return Pair(dist, padre)
```

Bellman-Ford

```
fun BellmanFord(
    grafo: List<List<Pair<Int,Int>>>,
    inicio: Int,
largo: Int
) : List<List<Int>> {
```

```
prof[raiz] = 0
   val n = grafo.size
   var dist = MutableList(largo+1) { MutableList(n) { Int.MAX_VALUE} }
                                                                                    for (k in 1 until K){
   dist[0][inicio] = 0
                                                                                         for (u in 0 until n){
   for (k in 0 until largo){
                                                                                             if (padre[k-1][u] != -1){
        for (u in 0 until n){
                                                                                                 padre[k][u] = padre[k-1][padre[k-1][u]]
            for ((v, w) in grafo[u]){
   if( dist[k][u] != Int.MAX_VALUE)
                                                                                         }
                    dist[k+1][v] = min0f(dist[k+1][v], dist[k][u] + w)
                                                                                    }
                                                                                }
        }
                                                                                fun lca(uu: Int, vv: Int) : Int {
    var u = uu
    var v = vv
   return dist
}
                                                                                    if (prof[u] < prof[v]) return lca(v, u)</pre>
  Floyd-Warshall
                                                                                    for (k in K-1 downTo 0){
   if (prof[u] - (1 shl k) >= prof[v]){
fun FloydWarshall(
                                                                                             u = padre[k][u]
    matriz: List<List<Int>>
                                                                                         }
 : MutableList<MutableList<Int>> {
   val n = matriz.size
   var dist = matriz.map{ it.toMutableList() }.toMutableList()
                                                                                    if (u == v) return u
                                                                                     for (k in K-1 downTo 0){
   for (k in 0 until n){
                                                                                         if (padre[k][u] != padre[k][v]){
    u = padre[k][u]
        for (i in 0 until n){
            for (j in 0 until n){
                                                                                             v = padre[k][v]
                dist[i][j] = minOf(dist[i][j], dist[i][k] + dist[k][j])
                                                                                    }
        }
                                                                                     return padre[0][u]
    return dist
                                                                            }
}
  Kruskal
                                                                                    Strings
fun Kruskal(g: List<Triple<Int,Int,Int>>, n : Int) : Pair<Int, List<Int</pre>
                                                                              Bordes (KMP)
   var uf = MutableList(n){i -> i}
   fun find(x : Int) : Int{
                                                                            fun bordes(s: String): List<Int>{
        if (uf[x] == x) return x
                                                                                val n = s.length
        uf[x] = find(uf[x])
                                                                                val b = MutableList(n+1) { -1 }
        return uf[x]
                                                                                var j = -1;
   }
                                                                                for (i in 0 until n){
   fun union(x: Int, y: Int){
                                                                                    while (j \ge 0 \&\& s[i] != s[j]){
        uf[find(x)] = find(y)
                                                                                         j = b[j]
   val aristas = MutableList(n){i -> i}.sortedBy{g[it].third}
                                                                                    b[i+1] = j
   var valor = 0
   var arbol = mutableListOf<Int>()
                                                                                return b
   for (ar in aristas){
        val(u,v,c) = g[ar]
        if (find(u) != find(v)){
                                                                               Función Z
            union(u,v)
            valor += c
                                                                            fun z(s: String): List<Int>{ // z[i] = max k: s[0,k) == s[i,i+k)
                                                                                val n = s.length
            arbol.add(ar)
                                                                                val z = MutableList(n) { 0 }
        }
   }
                                                                                var 1 = 0
                                                                                var r = 0
    return Pair(valor, arbol)
                                                                                for (i in 1 until_n){
}
                                                                                    if (i <= r) z[i] = min0f(r-i+1, z[i-1])</pre>
                                                                                     while (i+z[i] < n \&\& s[z[i]] == s[i+z[i]]) z[i]++
  Ancestro común menor
                                                                                    if (i+z[i]-1 > r){
                                                                                         1 = i
                                                                                         r = i+z[i]-1
   arbol: List<List<Int>>,
                                                                                    }
    raiz: Int
                                                                                return z
   var K: Int
   var padre: MutableList<MutableList<Int>>>
   var prof: MutableList<Int>
                                                                              Manacher
   init. {
        val n = arbol.size
                                                                            fun Manacher(s: String): Pair<List<Int>, List<Int>>{
                                                                                // (d1, d2) = (impares, pares) palindromes
        while ((1 shl K) < n) K++
                                                                                val n = s.length
        padre = MutableList(K) { MutableList(n) { -1 } }
                                                                                val d1 = MutableList(n) { 0 }
        prof = MutableList(n) { -1 }
                                                                                val d2 = MutableList(n) { 0 }
                                                                                var 1 = 0
        fun dfs(u: Int, p: Int){
                                                                                var r = -1
            padre[0][u] = p
                                                                                for (i in 0 until n){
            for (v in arbol[u]){
                                                                                    var k = if (i > r) 1 else min0f(d1[l+r-i], r-i+1)
                if (v == p) continue
                                                                                     while (i-k \ge 0 \&\& i+k < n \&\& s[i-k] == s[i+k]) k++
                prof[v] = prof[u] + 1
                                                                                    d1[i] = k--
                dfs(v, u)
                                                                                    if (i+k > r){
            }
                                                                                         1 = i-k
        }
                                                                                         r = i+k
```

}

dfs(raiz, -1)

```
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```
}
1 = 0
                                                                               for (i in 0 until n) lcp[i] = plcp[sa[i]]
                                                                                return lcp
   r = -1
   for (i in 0 until n){
        var k = if (i > r) 0 else min0f(d2[1+r-i+1], r-i+1)
                                                                             Duval
        while (i-k-1 >= 0 \&\& i+k < n \&\& s[i-k-1] == s[i+k]) k++
        d2[i] = k--
                                                                           // Dada una string $s$ devuelve la Lyndon decomposition en tiempo
        if (i+k > r){
                                                                           // lineal usando el algoritmo de Duval. Factoriza $s$ como
            1 = i - k - 1
                                                                           // $s_1 s_2 \ldots s_k$ con $s_1 \geqq s_2 \geqq \cdots \geqq s_k$
            r = i+k
                                                                            // y tal que $s_i$ es Lyndon, esto es, es su menor rotación.
                                                                           fun Duval(s: String) : List<String>{
                                                                                val n = s.length
   return Pair(d1, d2)
                                                                                var i = 0
}
                                                                                val ans = mutableListOf<String>()
                                                                               while (i < n){
  Suffix Array
                                                                                   var j = i + 1
                                                                                    var k = i
fun RB(x : Int, n : Int, r: List<Int>) : Int{
                                                                                   while (j < n \&\& s[k] <= s[j]){
   if(x < n) return r[x]</pre>
                                                                                        if (s[k] < s[j]) k = i
    else return 0
                                                                                        else k++
}
fun csort(sa: MutableList<Int>, r: MutableList<Int>, k : Int){
   val n = sa.size
                                                                                    while (i <= k){
   var f = MutableList(max0f(255,n)){0}
                                                                                        ans.add(s.substring(i until i+j-k))
   var t = MutableList(n)\{0\}
                                                                                        i += j-k
   for (i in 0 until n) f[RB(i+k,n,r)]++
   var sum = 0
   for (i in 0 until f.size){
                                                                                return ans
        var v = f[i]
f[i] = sum
sum += v
                                                                           // Obtener la mínima rotaciónn de $s$: en la descomposición de
                                                                           // Lyndon de $s^2$ es el último $i<|s|$ con el que empieza una
   for (i in 0 until n){
        t[f[RB(sa[i]+k,n,r)]++] = sa[i]
                                                                             Hashing
   }
    for (i in 0 until n) sa[i] = t[i]
                                                                           const val P: Long = 1777771
}
                                                                           val MOD: List<Long> = listOf(999727999, 1070777777)
                                                                           val PI: List<Long> = listOf(325255434, 10018302)
fun suffix_array(s0: String): List<Int>{
                                                                           class Hashing(c: Char) {
   val s = s0 + '\u0000'
                                                                               val h: MutableList<Long>
   val n = s.length
                                                                                val p: MutableList<Long>
   var rank: Int
                                                                               val pi: MutableList<Long>
   var sa = MutableList(n){it}
                                                                               init {
   var r = MutableList(n){it -> s[it].code}
                                                                                   h = MutableList(PI.size) { i -> c.code * P % MOD[i]}
   var t = MutableList(n){0}
                                                                                   p = MutableList(PI.size) { P }
   var k = 1
                                                                                   pi = PI.toMutableList()
   while (k<n){</pre>
                                                                               }
        csort(sa,r,k)
        csort(sa,r,0)
                                                                                // Agrega un prefijo : H(s1) + H(s2) = H(s2s1)
        t[sa[0]] = 0
                                                                               operator fun plus(h2: Hashing) : Hashing {
        rank = 0
for (i in 1 until n){
                                                                                   val ans = Hashing('a')
                                                                                    for (i in 0 until PI.size){
            if(r[sa[i]] != r[sa[i-1]] || RB(sa[i]+k,n,r) != RB(sa[i-1]+
                                                                                        ans.h[i] = (h[i] * h2.p[i] + h2.h[i]) % MOD[i]
ans.p[i] = p[i] * h2.p[i] % MOD[i]
                 k,n,r)) rank++
            t[sa[i]] = rank
                                                                                        ans.pi[i] = pi[i] * h2.pi[i] % MOD[i]
        for (i in 0 until n) r[i] = t[i]
                                                                                    return ans
        if (r[sa[n-1]]==n-1) break
                                                                                }
                                                                                // Elimina un prefijo
        println("k = $k")
                                                                                operator fun minus(h2: Hashing) : Hashing {
                                                                                    val ans = Hashing('a')
   return sa
                                                                                   for (i in 0 until PI.size){
}
                                                                                        ans.h[i] = (h[i] - h2.h[i] + MOD[i]) % MOD[i] * h2.pi[i] %
                                                                                             MOD[i]
  LCP (Estructura)
                                                                                        ans.p[i] = p[i] * h2.pi[i] % MOD[i]
                                                                                        ans.pi[i] = pi[i] * h2.p[i] % MOD[i]
fun computar_lcp(s0: String, sa: List<Int>): MutableList<Int>{
   val s = s0 + '\u0000'
                                                                                    return ans
   val n = s.length
                                                                               }
   var L = 0
   var lcp = MutableList(n){0}
                                                                                /// O simplemente comparar h1.h == h2.h en vez de h1==h2
   var plcp = MutableList(n){0}
                                                                               override fun equals(h2: Any?): Boolean {
   var phi = MutableList(n){0}
                                                                                   if (h2 !is Hashing) return false
                                                                                    return h == h2.h
   phi[sa[0]] = -1
                                                                               }
   for (i in 1 until n) phi[sa[i]] = sa[i-1]
                                                                           }
   for (i in 0 until n){
        if (phi[i] == -1){
                                                                           fun hash_neutro() : Hashing{
            plcp[i] = 0
                                                                               var ans = Hashing('a')
            continue
                                                                                for (i in 0 until PI.size){
                                                                                   ans.h[i] = 0
        while (s[i+L] == s[phi[i]+L]) L++
                                                                                   ans.p[i] = 1
        plcp[i] = L
                                                                                   ans.pi[i] = 1
        L = max0f(L-1,0)
```

return ans

}

```
}
class StringHasher(s: String){
    val h: MutableList<Hashing>
    init {
        h = MutableList(s.length+1) {
            if (it == 0) hash_neutro()
            else Hashing(s[it-1])
        }
        for (i in 1 until s.length+1){
            h[i] = h[i] + h[i-1]
        }
    }
}
// Hash de s[1,r)
fun hash(1: Int, r: Int) : Hashing {
        return h[r] - h[1]
    }
}
```

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Matemáticas

Identidades

```
\begin{split} C_n &= \frac{2(2n-1)}{n+1} C_{n-1} \\ C_n &= \frac{1}{n+1} \binom{2n}{n} \\ C_n &\sim \frac{4^n}{n^{3/2} \sqrt{\pi}} \\ F_{2n+1} &= F_n^2 + F_{n+1}^2 \\ F_{2n} &= F_{n+1}^2 - F_{n-1}^2 \\ \sum_{i=1}^n F_i &= F_{n+2} - 1 \\ F_{n+i} F_{n+j} &- F_n F_{n+i+j} = (-1)^n F_i F_j \\ \sum_{i=0}^n r^i &= \frac{r^{n+1}-1}{r-1} \\ \sum_{i=1}^n i^2 &= \frac{n \cdot (n+1) \cdot (2n+1)}{6} \\ \sum_{i=1}^n i^3 &= \left(\frac{n \cdot (n+1)}{2}\right)^2 \\ \sum_{i=1}^n i^4 &= \frac{n \cdot (n+1) \cdot (2n+1) \cdot (3n^2 + 3n - 1)}{12} \\ \sum_{i=1}^n i^5 &= \left(\frac{n \cdot (n+1)}{2}\right)^2 \cdot \frac{2n^2 + 2n - 1}{3} \\ \sum_{i=1}^n i \cdot \binom{n-1}{i-1} &= 2^{n-1} \\ \sum_{i=1}^n i \cdot \binom{n-1}{i-1} &= n \cdot 2^{n-1} \\ \text{(M\"obius Inv. Formula) Let} \\ g(n) &= \sum_{d \mid n} g(d) \mu \left(\frac{n}{d}\right) \end{split}
```

Teoremas

(Tutte) A graph, G = (V, E), has a perfect matching if and only if for every subset U of V, the subgraph induced by V - U has at most |U| connected components with an odd number of vertices. Petersens Theorem. Every cubic, bridgeless graph contains a perfect matching. (Dilworth) In any finite partially ordered set, the maximum number of elements in any antichain equals the minimum number of chains in any partition of the set into chains Pick: A=I+B/2-1 (area of polygon, points inside, points on border) Problema de los monstruos: Si los monstruos me sacan X vida y luego me dan Y vida, me conviene primero enfrentar a los Y≥X en orden creciente de X, y luego a los Y<X en orden decresciente de Y. Grafo planar: regiones = ejes - nodos + componentesConexas + 1; Condición: aristas≤3*vertices-6

Convolución rápida (FFT y Karatsuba)

```
import kotlin.math.round
data class Complex(val r: Double, val i: Double){
```

```
operator fun plus(x: Complex) = Complex(r + x.r, i + x.i)
    operator fun minus(x: Complex) = Complex(r - x.r, i - x.i)
    operator fun times(x: Complex) = Complex(r*x.r - i*x.i, r*x.i + i*x
          .r)
    operator fun div(x: Double) = Complex(r/x, i/x)
class FFT(lg0: Int){
    val lg = lg0+1 
val n = 1 shl lg
    val w = MutableList(n+1){Complex(0.0, 0.0)}
        val ang0 = 2.0 * Math.PI / n.toDouble()
        for (i in 0 until n+1){
  val ang = ang0 * i
             w[i] = Complex(Math.cos(ang), Math.sin(ang))
    }
    fun fft(a: List<Complex>, inv: Boolean = false) : List<Complex>{
        val p = MutableList(n) { a[Integer.reverse(it) ushr (32 - lg)]
        var len = 2
        while (len <= n) {</pre>
             val step = n / len
             for (i in 0 until n step len) {
                 for (j in 0 until len / 2) {
                     val u = p[i + j]
val v = p[i + j + len / 2] * if (inv) w[n - j *
                          step] else w[j * step]
                     p[i + j] = u + v
                     p[i + j + len / 2] = u - v
             len *= 2
        }
        if (inv) {
             for (i in 0 until n) {
                 p[i] = p[i] / n.toDouble()
        }
        return p
    fun multiply(a: List<Long>, b: List<Long>) : List<Long>{
        val a_c = a.map { Complex(it.toDouble(), 0.0) } + MutableList(n
              -a.size)\{Complex(0.0, 0.0)\}
        val b_c = b.map { Complex(it.toDouble(), 0.0) } + MutableList(n
              -b.size) {Complex(0.0, 0.0)}
        val fa = fft(a_c)
        val fb = fft(b_c)
        val fc = MutableList(n){fa[it] * fb[it]}
        val c = fft(fc, true)
        return c.map { round(it.r).toLong() }
fun Karatsuba(a : List<Long>, b : List<Long>) : List<Long>{
    val m = maxOf(a.size, b.size)
    val n = 1 shl (32 - Integer.numberOfLeadingZeros(m - 1))
    val aa = a + MutableList(n - a.size) { 0.toLong() }
    val bb = b + MutableList(n - b.size) { 0.toLong() }
    return karatsuba(aa, bb)
fun karatsuba(a: List<Long>, b: List<Long>): List<Long> {
   if (a.size <= 16) { // Reducir el tamaño de la condición base</pre>
        val c = MutableList(2 * a.size - 1) { θL }
        for (i in a.indices) {
             for (j in b.indices) {
   c[i + j] += a[i] * b[j]
        return c
    val n = a.size
    val k = n / 2
    val a0 = a.subList(0, k)
    val a1 = a.subList(k, n)
    val b0 = b.subList(0, k)
    val b1 = b.subList(k, n)
```

val z2 = karatsuba(a1, b1)

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}

}

}

}

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```

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```
inv[i] = mod - (mod / i) * inv[mod.toInt() % i] % mod
    val z0 = karatsuba(a0, b0)
    val a0a1 = List(k) { a0[it] + a1[it] }
val b0b1 = List(k) { b0[it] + b1[it] }
                                                                                                  fi[i] = fi[i-1] * inv[i] % mod
                                                                                        }
    val z1 = karatsuba(a0a1, b0b1)
                                                                                         fun Comb(n: Int, k: Int): Long {
    val result = MutableList(2 * n) { 0L }
                                                                                             if (k < 0 || k > n) return 0
return f[n] * fi[k] % mod * fi[n - k] % mod
    for (i in z0.indices) result[i] += z0[i]
for (i in z2.indices) result[i + n] += z2[i]
    for (i in z1.indices) result[i + k] += z1[i] - z0.get0rElse(i) { 0L
           } - z2.getOrElse(i) { 0L }
                                                                                         fun Perm(n: Int, k: Int): Long {
                                                                                             if (k < 0 | | k > n) return 0
return f[n] * fi[n - k] % mod
    return result
  Criba de Eratostenes
                                                                                        fun PermRepetidos(ns : List<Int>): Long {
                                                                                             var res = f[ns.sum()]
class Criba(n: Int){
                                                                                             for (n in ns) {
    res = res * fi[n] % mod
    var criba = MutableList(n+1){-1}
    init {
         for (i in 2..n){
                                                                                             return res
             if (criba[i] == -1){
                  if (n/i>=i) for (j in i*i until (n+1) step i){
                      if (criba[j] == -1) criba[j] = i
                                                                                      Mobius
             }
        }
                                                                                    fun Mobius(n: Int): List<Int>{
    }
                                                                                         var mobius = MutableList(n+1){1}
                                                                                        mobius[0] = 0
    fun fact(n: Int) : MutableMap<Int,Int> {
                                                                                         for(i in 2..n){
        var res = mutableMapOf<Int,Int>()
                                                                                             if(mobius[i]!=0){
         var x = n
                                                                                                  for(j in (i+i)..n step i);
         while(criba[x] != -1){
                                                                                                      mobius[j] -= mobius[i];
             res[criba[x]] = res.getOrDefault(criba[x], 0) + 1
             x /= criba[x]
                                                                                             }
         if(x != 1) res[x] = res.getOrDefault(x, 0) + 1
                                                                                         return mobius
         return res
    }
}
                                                                                             Geometría
  Potencia binaria
fun binPow(b : Long, e : Long, mod : Long): Long{
                                                                                      Punto
    var res = 1L
    var b = b%mod
    var e = e
                                                                                    import kotlin.math.*
    while (e > 0) {
                                                                                    class pt(x: Double, y: Double): Comparable<pt>{
        if (e % 2 == 1L) res = (res * b) % mod
                                                                                        val x = x
        b = (b * b) \% mod
                                                                                        val y = y
                                                                                        operator fun plus(p: pt) = pt(x + p.x, y + p.y)
                                                                                        operator fun minus(p: pt) = pt(x - p.x, y - p.y) operator fun times(k: Double) = pt(x * k, y * k)
    return res
                                                                                        operator fun div(k: Double) = pt(x / k, y / k)
                                                                                        operator fun times(p: pt) = x * p.x + y * p.y

operator fun rem(p: pt) = x * p.y - y * p.x

fun angle(p: pt) = acos((this * p) / (this.norm() * p.norm()))

fun norm2() = x * x + y * y
interface Ring<T> {
    operator fun T.times(other: T): T
    val one: T
                                                                                        fun norm() = sqrt(norm2())
fun<T> generic_bin_pow(b: T, e : Long, ops : Ring<T>): T{ // c * b^e
                                                                                        fun unit() = if (norm() > 0) this / norm() else pt(0.0, 0.0) fun rot(r: pt) = pt(this % r, this * r)
    var b = b
var e = e
var c = ops.one
                                                                                        fun rot(a: Double) = this.rot(pt(cos(a), sin(a)))
    with(ops){
                                                                                         fun left(p: pt, q: pt) = (q - p).unit() % (this - p).unit() > EPS
        while(e > 0){
             if (e % 2 == 1L) c *= b
                                                                                        operator override fun compareTo(p: pt): Int = when {
             b *= b
                                                                                             abs(this.x - p.x) > EPS \rightarrow this.x.compareTo(p.x)
             e /= 2
                                                                                             else -> this.y.compareTo(p.y)
        }
    return c
                                                                                        override fun equals(other: Any?) = other is pt && abs(x - other.x)<</pre>
                                                                                               EPS && abs(y - other.y)<EPS
                                                                                        override fun toString() = "($x, $y)"
  Combinatoria
                                                                                    val ccw90 = pt(1.0, 0.0)
class Combinatory(n: Int, mod: Long) {
                                                                                    val cw90 = pt(-1.0,0.0)
    var f: LongArray = LongArray(n + 1) { 0 }
    var inv: LongArray = LongArray(n + 1) { 0 }
                                                                                       Segmento
    var fi: LongArray = LongArray(n + 1) { 0 }
    var mod: Long = mod
                                                                                    import kotlin.math.*
    init {
                                                                                    class Segment(val f: pt, val s: pt) {
         for(i in 0..1){
                                                                                        fun length(): Double {
             `f[i] = 1
inv[i] = 1
                                                                                             val dx = f.x - s.x
val dy = f.y - s.y
return sqrt(dx * dx + dy * dy)
             fi[i] = 1
                                                                                        }
        for(i in 2..n){
    f[i] = i * f[i-1]%mod
```

}

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```
fun pc(a: pt, b: pt, o: pt): Double = (a-o) % (b-o)
fun pe(a: pt, b: pt, o: pt): Double = (a-o) * (b-o)
fun intersect(a: Segment, b: Segment): Boolean{
    val fb = 0.compareTo(pc(a.f, a.s, b.f))
    val sb = 0.compareTo(pc(a.f, a.s, b.s))
    val fa = 0.compareTo(pc(b.f, b.s, a.f))
    val sa = 0.compareTo(pc(b.f, b.s, a.s))
    if ((fb * sb < 0) && (fa * sa<0)) return true</pre>
    if ((fb==0 && pe(a.f, a.s, b.f)<=0) || (sb==0 && pe(a.f, a.s,b.s)
          <=0)) return true;
       ((fa==0 && pe(b.f, b.s, a.f)<=0) || (sa==0 && pe(b.f, b.s, a.s)
          <=0)) return true;
    return false
}
fun dist(p: pt, s: Segment): Double{
    val a = abs(pc(s.f,s.s,p))
    val b = hypot(s.f.x - s.s.x, s.f.y - s.s.y)
    val h = a/b
    val c = hypot(b,h)
   val d1 = (s.f-p).norm()
val d2 = (s.s-p).norm()
    if(b<EPS || c<= d1 || c<= d2) return min0f(d1,d2)</pre>
fun dist(a: Segment, b : Segment) : Double{
    if(intersect(a,b)) return 0.0
    return minOf(
        minOf(dist(a.f,b),dist(a.s,b)),
        min(dist(b.f,a),dist(b.s,a))
```

Capsula convexa

```
fun chull(ps: List<pt>) : List<pt>{
   if(ps.size < 3) return ps</pre>
   val p = ps.sorted()
   val ch = mutableListOf<pt>()
   for(pi in p){
        while(ch.size > 1 && ch[ch.size - 1].left(ch[ch.size - 2], pi))
              ch.removeAt(ch.size - 1)
        ch.add(pi)
   ch.removeAt(ch.size - 1)
    val t = ch.size
   for(pi in p.reversed()){
        while(ch.size > t+1 && ch[ch.size - 1].left(ch[ch.size - 2], pi
             )) ch.removeAt(ch.size - 1)
        ch.add(pi)
   ch.removeAt(ch.size - 1)
   return ch
}
```

Tablas y Cotas

```
Primos cercanos a 10^n
```

1000000021 1000000033

```
9941 9949 9967 9973 10007 10009 10037 10039 10061
10067 10069 10079
99961 99971 99989 99991 100003 100019 100043 100049
100057 100069
999959 999961 999979 999983 1000003 1000033 1000037
1000039
9999943 9999971 9999973 9999991 10000019 10000079
10000103 10000121
99999941 99999959 99999971 99999989 100000007 1000000037
100000039 100000049
999999893 999999929 999999937 1000000007 10000000009
```

Cantidad de primos menores que 10^n

```
\pi(10^1) = 4 ; \pi(10^2) = 25 ; \pi(10^3) = 168 ; \pi(10^4) = 1229
; \pi(10^5) = 9592 ; \pi(10^6) = 78.498 ; \pi(10^7) = 664.579 ;
\pi(10^8) = 5.761.455; \pi(10^9) = 50.847.534;
```

```
\pi(10^{10}) = 455.052,511; \pi(10^{11}) = 4.118.054.813;
\pi(10^{12}) = 37.607.912.018
  Divisores
Cantidad de divisores (\sigma_0) para algunos n/\neg \exists n'
n, \sigma_0(n') \geqslant \sigma_0(n)
\sigma_0(60) = 12; \sigma_0(120) = 16; \sigma_0(180) = 18; \sigma_0(240)
= 20 ; \sigma_0(360) = 24 ; \sigma_0(720) = 30 ; \sigma_0(840) = 32
; \sigma_0(1260) = 36 ; \sigma_0(1680) = 40 ; \sigma_0(10080) = 72 ;
\sigma_0(15120) = 80; \sigma_0(50400) = 108; \sigma_0(83160) = 128;
\sigma_0(110880) = 144; \sigma_0(498960) = 200; \sigma_0(554400) = 216
; \sigma_0(1081080) = 256 ; \sigma_0(1441440) = 288 \sigma_0(4324320) =
384 ; \sigma_0(8648640) = 448
Suma de divisores (\sigma_1) para algunos n/\neg \exists n' < n, \sigma_1(n') \geqslant
\sigma_1(n) ; \sigma_1(96) = 252 ; \sigma_1(108) = 280 ; \sigma_1(120) = 360
; \sigma_1(144) = 403 ; \sigma_1(168) = 480 ; \sigma_1(960) = 3048 ;
\sigma_1(1008) = 3224; \sigma_1(1080) = 3600; \sigma_1(1200) = 3844
; \sigma_1(4620) = 16128 ; \sigma_1(4680) = 16380 ; \sigma_1(5040) =
19344 ; \sigma_1(5760) = 19890 ; \sigma_1(8820) = 31122 ; \sigma_1(9240)
= 34560; \sigma_1(10080) = 39312; \sigma_1(10920) = 40320;
\sigma_1(32760) = 131040 ; \sigma_1(35280) = 137826 ; \sigma_1(36960)
= 145152 ; \sigma_1(37800) = 148800 ; \sigma_1(60480) = 243840 ;
\sigma_1(64680) = 246240; \sigma_1(65520) = 270816; \sigma_1(70560)
= 280098 ; \sigma_1(95760) = 386880 ; \sigma_1(98280) = 403200 ;
\sigma_1(100800) = 409448; \sigma_1(491400) = 2083200;
\sigma_1(498960) = 2160576; \sigma_1(514080) = 2177280; \sigma_1(982800)
= 4305280 ; \sigma_1(997920) = 4390848 ; \sigma_1(1048320) = 4464096
; \sigma_1(4979520) = 22189440 ; \sigma_1(4989600) = 22686048 ;
\sigma_1(5045040) = 23154768; \sigma_1(9896040) = 44323200;
\sigma_1(9959040) = 44553600; \sigma_1(9979200) = 45732192
  Factoriales
                        11! = 39.916.800
 0! = 1
 1! = 1
                        12! = 479.001.600 (\in int)
 2! = 2
                        13! = 6.227.020.800
 3! = 6
                        14! = 87.178.291.200
 4! = 24
                        15! = 1.307.674.368.000
                        16! = 20.922.789.888.000
 5! = 120
                        17! = 355.687.428.096.000
 6! = 720
```

Consejos

Debugging

7! = 5.040

8! = 40.320

9! = 362.880

10! = 3.628.800

 ¿Si n = 0 anda? (similar casos borde tipo n=1, n=2, etc)

18! = 6.402.373.705.728.000

19! = 121.645.100.408.832.000

21! = 51.090.942.171.709.400.000

 $20! = 2.432.902.008.176.640.000 \in 11$

¿Si hay puntos alineados anda?

max signed tint = 9.223.372.036.854.775.807

max unsigned tint = 18.446.744.073.709.551.615

- ¿Si es vacío anda?
- ¿Si hay multiejes anda?
- ¿Si no tiene aristas anda?
- ¿Si tiene ciclos anda?
- ¿Si tiene un triángulo anda?
- ¿Los arrays son suficientemente grandes? (siempre denle bastante de más por las dudas, pero tampoco se ceben como para que ya no entre en memoria XD)
- ¿Puede dar integer overflow? (SIEMPRE mirar el integer overflow con MUCHO cuidado)
- ¿Podés dividir por cero en algún caso?

- ¿El caso base está bien hecho y se llega siempre?
- ¿Están bien puestas las cotas iniciales de la binary / inicialización del acumulador máximo/mínimo?
- ¿Estás inicializando bien antes de cada caso?
- ¿Le copiaste el input dos veces en el archivo de entrada (para ver que de igual y bien las dos veces)? [No aplica cuando viene solo una instancia de input]
- ¿Pasa los ejemplos? [No es joda, Leo se quedo afuera de la mundial por esto]

Hitos de prueba

- 45min todas las columnas de la tabla llena
- 2h todos conocen todo
- 3h reunión estratégica
- 4h reunión estratégica