Segundo PARCIAL

01/12/2020

1) V o F. Justificar

a) $\sum_{n=1}^{\infty} \frac{n^2 - 3(-1)^n}{2^n}$ es DV (F)

$$\sum_{n=1}^{\infty} \frac{n^2 - 3(-1)^n}{2^n} = \sum_{n=1}^{\infty} \frac{n^2 + 3(-1)^{n+1}}{2^n} =$$

$$= \sum_{n=1}^{\infty} \frac{n^2}{2^n} + \frac{3(-1)^{n+1}}{2^n} = \underbrace{\sum_{n=1}^{\infty} \frac{n^2}{2^n}}_{CV} + 3 \underbrace{\sum_{n=1} \frac{(-1)^{n+1}}{2^n}}_{CV} \text{ es CV}$$

CA

$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ es CV, ya que por criterio de Cauchy:

$$\lim_{n\to\infty} \sqrt[n]{\frac{n^2}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^2}}{2} = \lim_{n\to\infty} \frac{(\overset{\nearrow 1}{\sqrt[n]{n}})^2}{2} =$$

$$= \frac{1}{2} < 1 \quad CV$$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n}$ es CV, ya que por criterio de Leibniz:

- $\lim_{n\to\infty} \frac{1}{2^n} = 0$ ✓

- $n < n+1 \Rightarrow 2^n < 2^{n+1} \Rightarrow \frac{1}{2^n} > \frac{1}{2^{n+1}}$

  $\Rightarrow$ es decreciente ✓

Luego, es CV

b) $\int_0^{1/e} \frac{dx}{x \ln^2 x}$ es CV y su valor representa un área

$$\int_0^{1/e} \frac{dx}{x \ln^2 x} = \lim_{c \to 0^+} \int_c^{1/e} \frac{dx}{x \ln^2 x} =$$

$$= \lim_{c \to 0^+} -\frac{1}{\ln x}\Big|_c^{1/e} = -\lim_{c \to 0^+}\Bigg(\underbrace{\frac{1}{\ln\left(\frac{1}{e}\right)}}_{-1} - \overbrace{\frac{1}{\underbrace{\ln c}_{\to -\infty}}}^{\to 0}\Bigg) =$$

$$= 1 \quad \underbrace{CV}$$

C.A.

$$\int \frac{dx}{x \ln^2 x} = \int \frac{du}{u^2} = \underbrace{-\frac{1}{u}}+c$$

$$= \underbrace{-\frac{1}{\ln x}+c}$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\ln\left(\frac{1}{e}\right) = \overbrace{\ln 1}^{0} - \overbrace{\ln e}^{1} = -1$$

Para ver si su valor representa un área debemos probar que $\frac{1}{x \ln^2 x} > 0$ $\qquad D_f : (0 ; +\infty)$

$\frac{1}{x \ln^2 x} > 0$ cuando $x > 0$ y $\ln^2 x > 0$

$x > 0 \wedge \sqrt{\ln^2 x} > 0$

$x > 0 \wedge |\ln x| > 0$

$x > 0 \wedge [\ln x > 0 \vee \ln x < 0]$

$[x > 1 \vee x < 1]$

$$\Rightarrow \frac{1}{x \ln^2 x} > 0 \quad \forall x \in (0,1) \cup (1;+\infty)$$

Como $(0 ; \frac{1}{e}] \subset (0,1)\cup(1;+\infty) \Rightarrow$ el valor representa un área

Aclaración: $\int_0^{1/e} \frac{dx}{x \ln^2 x}$ es IMPROPIA de 2^{da} especie, ya que:

$$\lim_{x \to 0^+} \frac{1}{x \cdot \ln^2 x} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{\ln^2 x} = \frac{\to \infty}{\to \infty} \underset{\text{B-L'H}}{=}$$

$$= \lim_{x \to 0^+} \frac{-\frac{1}{x^2}}{2\ln x \cdot \frac{1}{x}} = \lim_{x \to 0^+} \frac{-\frac{1}{x}}{2 \ln x} = \frac{\to \infty}{\to \infty} \underset{\text{B-L'H}}{=} \lim_{x \to 0^+} \frac{\frac{1}{x^2}}{\frac{2}{x}} =$$

$$= \lim_{x \to 0^+} \frac{1}{2x} = \underbrace{+\infty}$$

Segundo PARCIAL ②
01/12/2020

② Determine $f(x)$ / $\begin{cases} f'(x) = \dfrac{1}{8+15e^x} \\ f(1) = 0 \end{cases}$

$$f(x) = \int \frac{1}{8+15e^x}\,dx =$$

$$= \int \frac{1}{8+15.u} \cdot \frac{du}{u} =$$

$$= \int \frac{1}{(8+15u)u} \cdot du =$$

$$= \frac{1}{15}\int \frac{1}{(\frac{8}{15}+u)u}\,du =$$

$$= \frac{1}{15}\left[-\frac{15}{8}\int \frac{1}{\frac{8}{15}+u}\,du + \frac{15}{8}\int \frac{1}{u}\,du\right] =$$

$$= -\frac{1}{8}\ln\left|\frac{8}{15}+u\right| + \frac{1}{8}\ln|u| + c$$

$$= \frac{1}{8}\ln|e^x| - \frac{1}{8}\ln\left|\frac{8}{15}+e^x\right| + c$$

$$= \underbrace{\frac{1}{8}\ln\left|\frac{e^x}{e^x+\frac{8}{15}}\right| + c = f(x)}$$

C.A.

$\underline{u = e^x} \Rightarrow du = \overbrace{e^x}^{u}\,dx$

$\underline{\frac{du}{u} = dx}$

$$\frac{1}{(\frac{8}{15}+u).u} = \frac{A}{\frac{8}{15}+u} + \frac{B}{u} =$$

$$= \frac{Au + B(\frac{8}{15}+u)}{(\frac{8}{15}+u)u}$$

$$\Rightarrow 1 = Au + B\left(\frac{8}{15}+u\right)$$

$$= Au + Bu + B.\frac{8}{15}$$

$$1 = (A+B)u + \frac{8}{15}B$$

$\Rightarrow \underline{A+B=0}$ y $\frac{8}{15}B = 1$

$\underline{B = \frac{15}{8}}$

$\Rightarrow \underline{A} = -B = \underline{-\frac{15}{8}}$

$$f(1) = 0 \Rightarrow \frac{1}{8}\ln\left|\frac{e}{e+8/15}\right| + c = 0$$

$$\Rightarrow \frac{1}{8}\left(\overbrace{\ln e}^{1} - \ln(e+8/15)\right) + c = 0$$

$$\Rightarrow \underline{c = \frac{1}{8}\left(\ln\left(e+\frac{8}{15}\right) - 1\right)}$$

Luego, $f(x) = \frac{1}{8}\left[\ln\left|\frac{e^x}{e^x+\frac{8}{15}}\right| + \left(\ln\left(e+\frac{8}{15}\right) - 1\right)\right]$

③ Para $g(x)=\dfrac{x}{\sqrt{1+2x^2}}$ y $h(x)=x$ con $x\in[-2;0]$

a) Dibuje la región limitada por las gráficas de g y h.

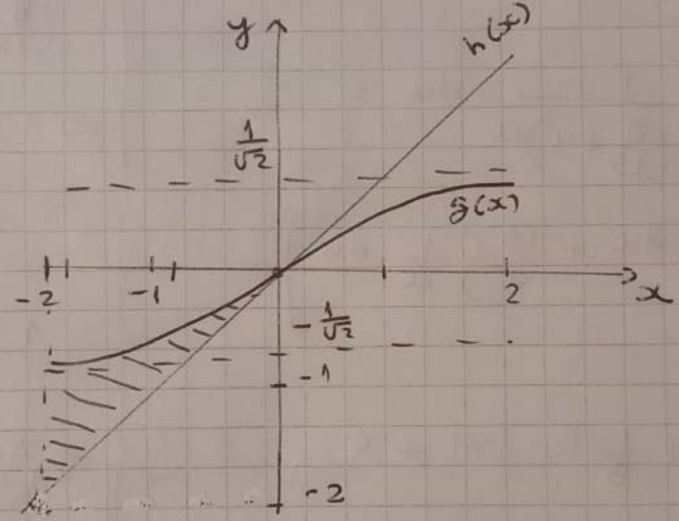
$g(0)=0$ y $h(0)=0$

$$\lim_{x\to-\infty}\frac{x}{\sqrt{1+2x^2}}=\frac{\to\infty}{\to\infty}=\lim_{x\to-\infty}\frac{\cancel{x}}{-\cancel{x}\sqrt{\frac{1}{x^2}+2}}=-\frac{1}{\sqrt2}$$

C.A

$$\sqrt{1+2x^2}=\sqrt{x^2\left(\frac{1}{x^2}+2\right)}=|x|\cdot\sqrt{\frac{1}{x^2}+2}$$

$$\lim_{x\to+\infty}\frac{x}{\sqrt{1+2x^2}}=\frac{\to\infty}{\to\infty}=\lim_{x\to+\infty}\frac{x}{x\sqrt{\frac{1}{x^2}+2}}=\frac{1}{\sqrt2}$$



g es continua por ser cociente de funciones continuas

$\underline{g(x)>0}$ cuando $\dfrac{x}{\sqrt{1+2x^2}}>0 \Rightarrow \underline{x>0}$

$\underline{g(x)<0}$ cuando $\dfrac{x}{\sqrt{1+2x^2}}<0 \Rightarrow \underline{x<0}$

$$g'(x)=\frac{\sqrt{1+2x^2}-x\cdot\frac{1}{\cancel{2}\sqrt{1+2x^2}}\cdot\cancel{4}^{2}x}{\left(\sqrt{1+2x^2}\right)^2}=\frac{\frac{1+2x^2-2x^2}{\sqrt{1+2x^2}}}{1+2x^2}=$$

$$=\frac{1}{(1+2x^2)^{3/2}}>0\ \forall x \Rightarrow g \text{ es estrictamente creciente en } \mathbb{R}$$

(3) b) Calcula el área planteada en a)

$$g(x) = h(x)$$

$$\frac{x}{\sqrt{1+2x^2}} = x \Rightarrow \frac{x}{\sqrt{1+2x^2}} - x = 0 \Rightarrow x\left(\frac{1}{\sqrt{1+2x^2}} - 1\right) = 0$$

$$\Rightarrow \underline{x=0} \vee \frac{1}{\sqrt{1+2x^2}} - 1 = 0$$

$$\frac{1}{\sqrt{1+2x^2}} = 1 \Rightarrow 1 = \sqrt{1+2x^2} \Rightarrow 1^2 = 1+2x^2$$
$$0 = 2x^2 \Rightarrow \underline{x=0}$$

$$|A| = \int_{-2}^{0}\left(\frac{x}{\sqrt{1+2x^2}} - x\right)dx =$$

$$= \int_{-2}^{0} \frac{x}{\sqrt{1+2x^2}}\,dx - \int_{-2}^{0} x\,dx =$$

$$= \frac{1}{2}\sqrt{1+2x^2}\,\Big|_{-2}^{0} - \frac{x^2}{2}\Big|_{-2}^{0} =$$

$$= \frac{1}{2}\left[\sqrt{1+2x^2} - x^2\right]_{-2}^{0} =$$

$$= \frac{1}{2}\left[\underbrace{1 - (3-4)}_{2}\right] = \boxed{1}$$

C. A.

$$u = 1+2x^2$$
$$du = 4x\,dx$$
$$\frac{1}{4}du = x\,dx$$

$$\int \frac{x}{\sqrt{1+2x^2}}\,dx =$$
$$= \int \frac{1}{\sqrt{u}}\frac{du}{4} =$$
$$= \frac{1}{4}\int u^{-1/2}\,du =$$
$$= \frac{1}{2}\sqrt{u} + c$$
$$= \frac{1}{2}\sqrt{1+2x^2} + c$$

Aclaración: $g(x) > h(x)$

$$\frac{x}{\sqrt{1+2x^2}} > x \Rightarrow x\left(\overbrace{\frac{1}{\sqrt{1+2x^2}} - 1}^{<0}\right) > 0 \Rightarrow \underline{x<0}$$

$$x \neq 0$$
$$\Rightarrow 1+2x^2 > 1$$
$$\Rightarrow \frac{1}{\sqrt{1+2x^2}} < 1$$

④ $P_{2,G(x),1}(x) = ?$, $G(x) = \int_0^{x^2} \frac{du}{2+u^2}$

$$P_{2,G(x),1}(x) = G(1) + G'(1)(x-1) + \frac{G''(1)}{2!}(x-1)^2$$

$$G(1) = \int_0^1 \frac{du}{2+u^2} = \frac{\sqrt{2}}{2} \operatorname{arctg}\left(\frac{u}{\sqrt{2}}\right)\Big|_0^1 =$$

$$= \frac{\sqrt{2}}{2}\left[\operatorname{arctg}\left(\frac{1}{\sqrt{2}}\right) - \underbrace{\operatorname{arctg}(0)}_{0}\right]$$

$$= \underline{\frac{\sqrt{2}}{2} \operatorname{arctg}\left(\frac{\sqrt{2}}{2}\right)}$$

C.A.

$$\int \frac{du}{u^2+2} = \frac{1}{2}\int \frac{du}{\frac{u^2}{2}+1} =$$

$$= \frac{1}{2}\int \frac{du}{1+\left(\frac{u}{\sqrt{2}}\right)^2} = \frac{\sqrt{2}}{2}\int \frac{dz}{1+z^2}$$

$$= \frac{\sqrt{2}}{2} \operatorname{arctg}(z) + C = \underline{\frac{\sqrt{2}}{2} \operatorname{arctg}\left(\frac{u}{\sqrt{2}}\right)}$$

$$z = \frac{u}{\sqrt{2}} \Rightarrow dz = \frac{1}{\sqrt{2}} du$$

$$\Rightarrow du = \sqrt{2}\, dz$$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$G'(x) = \frac{1}{2+x^4} \cdot 2x \Rightarrow \underline{G'(1) = \frac{2}{3}}$$

$$G''(x) = \left(\frac{2x}{2+x^4}\right)' = \frac{2(2+x^4) - 2x \cdot 4x^3}{(2+x^4)^2} =$$

$$= \frac{4+2x^4-8x^4}{(2+x^4)^2} = \frac{4-6x^4}{(2+x^4)^2} \Rightarrow \underline{G''(1) = -\frac{2}{9}}$$

Luego, $\underline{P(x) = \frac{\sqrt{2}}{2} \operatorname{arctg}\left(\frac{\sqrt{2}}{2}\right) + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2}$

Segundo PARCIAL

01/12/2020

(5) Determine el intervalo de CV de $\sum_{n=1}^{\infty} \frac{n}{n+1} (-2x)^{n-1}$

Por el criterio de D'Alambert:

$$\lim_{n \to \infty} \left| \frac{\frac{n+1}{n+2} (-2x)^n}{\frac{n}{n+1} (-2x)^{n-1}} \right| = \lim_{n \to \infty} \left| \frac{(n+1)^2}{n(n+2)} \cdot \frac{\cancel{(-2)^n} \, \cancel{x^n}}{\cancel{(-2)^n} \cdot (-2)^{-1} \, \cancel{x^n} \cdot x^{-1}} \right| =$$

$$= \lim_{n \to \infty} \left| \underbrace{\frac{n^2+2n+1}{n^2+2n}}_{\to 1} \cdot -2x \right| = |-2x| = |2x| < 1$$

$$|x| < \frac{1}{2} \Rightarrow \underline{-\frac{1}{2} < x < \frac{1}{2}}$$

Analizo en los extremos:

- $x = -\frac{1}{2}$, $\sum_{n=1}^{\infty} \frac{n}{n+1} \cdot 1^{n-1} = \sum_{n=1}^{\infty} \frac{n}{n+1}$

$$\lim_{n \to \infty} \frac{n}{n+1} = 1 \underset{CN}{\neq} 0 \Rightarrow \sum_{n=1}^{\infty} \frac{n}{n+1} \text{ no es CV}$$

CN condición necesaria

- $x = \frac{1}{2}$, $\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{n}{n+1}$ alternada

Leibniz: $\lim_{n \to \infty} \frac{n}{n+1} = 1 \neq 0 \Rightarrow \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n+1}$ no es C

Luego, el intervalo de CV es $\left(-\frac{1}{2}; \frac{1}{2}\right)$