1) a)

Dada $f(x) = \begin{cases} \dfrac{\ln(3x+1)}{2x} & \text{si } x > 0 \\ 7x^2 + \boldsymbol{a} & \text{si } x \leq 0 \end{cases}$ hallar $\boldsymbol{a} \in \mathbb{R}$, tal que $f(x)$ sea continua en $x = 0$

b) **Usando definición** determinar si existe $f'(0)$, en el punto 1) a), para el valor hallado de $\boldsymbol{a}$.

2) a) ¿La recta tangente a la curva definida implícitamente por: $x^2 . y^2 - e^{(y-2)} = 3$ en el punto $(x_0;2)$, con $x_0 > 0$ tiene pendiente $m = \frac{8}{3}$ ? Justificar rta.

b) Determinar las ecuaciones de las rectas tangente y normal a la curva C : $\begin{cases} x = t - 1 \\ y = 2t^2 + 3 \end{cases}$

en el punto (-1 , 3); con $t \in \mathbb{R}$.

3) a) Dada f: $D_f \rightarrow \mathbb{R}$ / $f(x) = \dfrac{mx^2 + 5}{x + n}$ calcular los valores reales de "m" y "n" para que la gráfica de f admita como asíntota a la recta: $y - 3x + 1 = 0$.

Para los valores hallados de "m" y "n" determinar las ecuaciones de otras asíntotas si existen, a la curva de la función dada.

b) Calcular sin usar Regla de L'Hospital: $\lim\limits_{x \to 0} \left( \dfrac{x + x^2}{|x|} \right)$

4) Dada: $D_f \rightarrow \mathbb{R}$ / $f(x) = 3x + \dfrac{3x}{(x-1)}$ determinar: dominio; asíntotas; intervalos de crecimiento/decrecimient coordenadas de los puntos extremos máximos/mínimos (si existen); intervalos de concavidad/convexida puntos de inflexión (si existen).

5) Hallar **k** real, tal que: $\lim\limits_{x \to \infty} \left( \dfrac{x^3 + 5x - 3}{x^2 + 5\,k\,x^3 + 2} \right) = \lim\limits_{x \to +\infty} \left\{ \left( \dfrac{x^2 + 3}{x^2 + 1} \right)^{(3x + 5)} \right\}$

2) a) $x^2 \cdot y^2 - e^{(y-2)} = 3$ en el punto $(x_0, 2)$ con $x_0 > 0$ $f'_{(x_0,2)} = \frac{8}{3}$ ? $\rightarrow f(x_0) = 2$

$$f(x_0) = x_0^2 \cdot 2^2 - e^{(2-2)} = 3$$

$$4x_0^2 = 4$$

$$x_0^2 = 1$$

$$|x_0| = 1 \quad \boxed{x_0 = 1} \quad x_0 = -1$$

$$f'(x,y) = 2x \cdot y^2 + x^2 \cdot 2y \cdot y' - e^{(y-2)} \cdot y' = 0$$

$$f'(1,2) = 2 \cdot 1 \cdot 2^2 + 1^2 \cdot 2 \cdot 2 \cdot y' - e^{(2-2)} \cdot y'$$

$$8 + 4y' - y' = 0$$

$$4y' - y' = -8$$

$$y'(4-1) = -8$$

$$\boxed{y' = -\frac{8}{3}} \longrightarrow \text{FALSO}$$

b) hallar $y_t$ e $y_n$ de $C: \begin{cases} x = t-1 \\ y = 2t^2+3 \end{cases}$ en el pto $(-1,3)$, $t \in \mathbb{R}$

$$C: \begin{cases} x = t-1 \\ y = 2t^2+3 \end{cases} \Rightarrow C': \begin{cases} x' = 1 \\ y' = 4t \end{cases} = \frac{y'(t)}{x'(t)} = \frac{4t}{1} = 4t = 0 = C'(-1,3)$$

$$C_{(-1,3)} = \begin{cases} -1 = t-1 \longrightarrow t = 0 \\ 3 = 2 \cdot t^2 + 3 \longrightarrow t = 0 \end{cases}$$

$$x+1 = t \Rightarrow y = 2(x+1)^2 + 3 \rightarrow y = 2x^2 + 4x + 5$$

$$y_t = f'(-1)(x+1) + f(-1)$$

C.A.
$f(-1) = 2 \cdot 1 - 4 + 5 = 3$

$$y_t = 0\,(x+1) + 3 \Rightarrow y_t = 3 \rightarrow \text{recta horizontal}$$

$\Rightarrow y_n = -1$, pues es $\perp$ a $y_t$.

09/05/2023

1)a)

$$f(x)=\begin{cases} \frac{\ln(3x+1)}{2x} & \text{si } x>0 \\ 7x^2+a & \text{si } x\leq 0 \end{cases}$$

hallar $k \in \mathbb{R}$ / $f(x)$ sea continua en $x=0$.

i) $f(0)=7.0^2+a \rightarrow f(0)=a$

ii) $\lim_{x\to 0}(f(x))$

$\lim_{x\to 0^-}(7x^2+a)=7.0^2+a=a=li$

$\lim_{x\to 0^+}\left(\frac{\ln(3x+1)}{2.x}\right) \overset{L'H}{=} \lim_{x\to 0^+}\left(\frac{\frac{1}{3x+1}.3}{2}\right)=\lim_{x\to 0^+}\left(\frac{3}{6x+2}\right)=\frac{3}{2}=ld$

$\frac{0}{0}$

$\Rightarrow li=ld$ para q $\exists \lim_{x\to 0}(f(x)) \Rightarrow \boxed{a=\frac{3}{2}} \Rightarrow \lim_{x\to 0}(f(x))=\frac{3}{2}$

iii) $f(0)=\lim_{x\to 0}(f(x)) \Longrightarrow \boxed{a=\frac{3}{2}}$

b) def. de derivada. buscar $f'(0)$

$f'(0^-)=\lim_{x\to 0^-}\left(\frac{7x^2+\cancel{\frac{3}{2}}-\cancel{\frac{3}{2}}}{x-0}\right)=\lim_{x\to 0^-}\left(\frac{\frac{7x^2}{x^2}}{\frac{x}{x^2}}\right)=\lim_{x\to 0^-}\left(\frac{7}{\frac{1}{x}}\right)=\frac{7}{\left(\frac{1}{0}\right)}=0$

$\infty$

$f'(0^+)=\lim_{x\to 0^+}\left(\frac{\frac{\ln(3x+1)}{2x}-\frac{3}{2}}{x}\right)=\lim_{x\to 0^+}\left(\frac{\frac{\ln(3x+1)-3x}{2x}}{x}\right)=\lim_{x\to 0^+}\left(\frac{\ln(3x+1)-3x}{2x^2}\right)$

$\overset{L'H}{=}\lim_{x\to 0^+}\left(\frac{\frac{1}{3x+1}.3-3}{4x}\right)=\lim_{x\to 0^+}\left(\frac{\frac{3-3(3x+1)}{3x+1}}{4x}\right)=\lim_{x\to 0^+}\left(\frac{-9x}{4x}\right)\overset{L'H}{=}$

$=\lim_{x\to 0^+}\left(\frac{-9}{4}\right)=-\frac{9}{4}=f'(0^+)$

$\Rightarrow f'(0^-)\neq f'(0^+) \Rightarrow \nexists f'(0)$

b) SIN L'H: $\lim_{x \to 0} \left( \frac{x + x^2}{|x|} \right)$

$$f(x) \begin{cases} \frac{x+x^2}{x} & \text{si } x \geq 0 \\ \frac{x+x^2}{(-x)} & \text{si } x < 0 \end{cases}$$

$\frac{0}{0}$

$$\lim_{x \to 0^-} \left( \frac{x + x^2}{-x} \right) = \lim_{x \to 0^-} \left( \frac{x(x+1)}{-x} \right) = \lim_{x \to 0^-} \left( -(x+1) \right) = -1 = li$$

$$\lim_{x \to 0^+} \left( \frac{x + x^2}{x} \right) = \lim_{x \to 0^+} \left( \frac{\cancel{x}(x+1)}{\cancel{x}} \right) = \lim_{x \to 0^+} (x+1) = 1 = ld$$

- $li \neq ld \Rightarrow \nexists \lim_{x \to 0} (f(x))$

a) $f: D_f \to \mathbb{R} \,/\, f(x) = 3x + \frac{3x}{(x-1)} = \frac{3x^2}{x-1}$

• Dom: $\mathbb{R} - \{1\}$

• as.

vertical:

$$\lim_{x\to 1}\left(3x + \frac{3x}{(x-1)}\right) = 3.1 + \frac{3.1}{0} = 3 + \infty = \infty_{//}$$

$\Rightarrow \exists$ as. vertical en $x = 1_{//}$

horizontal:

$$\lim_{x\to\infty}\left(3x + \frac{3x}{(x-1)}\right) = \lim_{x\to\infty}\left(\frac{3x^2 \cancel{-3x} \cancel{+3x}}{(x-1)}\right) = \lim_{x\to\infty}\left(\underbrace{\frac{3x^2}{x-1}}_{\frac{\infty}{\infty}}\right) = \lim_{x\to\infty}\left(\frac{\frac{3x^2}{\cancel{x^2}}}{\frac{\cancel{x}}{x\cancel{x}} - \frac{1}{x^2}}\right) =$$

$$= \lim_{x\to\infty}\left(\frac{3}{\frac{1}{x} - \frac{1}{x^2}}\right) = \frac{3}{\frac{1}{\infty} - \frac{1}{\infty}} = \frac{3}{0} = \infty_{//} \Rightarrow \nexists \text{ as horizontal}$$

oblicua: $y = mx + b$

$$\text{"m"} = \lim_{x\to\infty}\left(\frac{\frac{3x^2}{x-1}}{x}\right) = \lim_{x\to\infty}\left(\frac{3x^2}{x^2-x}\right) = \lim_{x\to 0}\left(\frac{\frac{3\cancel{x^2}}{\cancel{x^2}}}{\frac{\cancel{x^2}}{\cancel{x^2}} - \frac{\cancel{x}}{x^2}}\right) = \lim_{x\to 0}\left(\frac{3}{1-\frac{1}{x}}\right) =$$

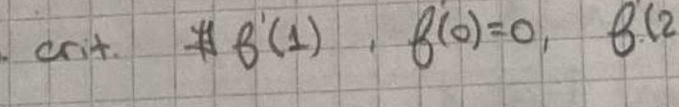
$$= \frac{3}{1 - \frac{1}{\infty}} = \frac{3}{1-0} = 3_{//} = m$$

$$\text{"b"} = \lim_{x\to\infty}\left(\cancel{3x} + \frac{3x}{x-1} - \cancel{3x}\right) = \lim_{x\to\infty}\left(\frac{\frac{3\cancel{x}}{\cancel{x}}}{\frac{\cancel{x}}{\cancel{x}} - \frac{1}{x}}\right) = \lim_{x\to\infty}\left(\frac{3}{1-\frac{1}{x}}\right) = \frac{3}{1-0} = 3_{/} = b$$

$\Rightarrow \exists$ as. oblicua: $y = 3x + 3$

$$\cdot\, f'(x) = \frac{6x.(x-1) - 3x^2.1}{(x-1)^2} = \frac{6x^2 - 6x - 3x^2}{(x-1)^2} = \frac{3x^2 - 6x}{(x-1)^2}$$

• ptos. crit. $\nexists f'(1)$, $f'(0) = 0$, $f'(2) = 0$




$$f'(-1) = \frac{3.(-1)^2 - 6.(-1)}{(-1-1)^2} = \frac{+}{+} = +$$

$$f'(1/2) = \frac{3.(1/2)^2 - 6.(1/2)}{(1/2-1)^2} = \frac{-}{+} = -$$

$$f'(3/2) = \frac{3.(3/2)^2 - 6.(3/2)}{(3/2-1)^2} = \frac{-}{+} = -$$

$$f'(3) = \frac{3.3^2 - 6.3}{(3-1)^2} = \frac{+}{+} = +$$

f crece en: $(-\infty, 0) \cup (2, +\infty)$

f decrece en: $(0, 1) \cup (1, 2)$

• Máx / mín

máx: $(0, f(0)) \rightarrow (0,0)$ //

mín: $(2, f(2)) \rightarrow \left(2, \frac{3 \cdot 2^2}{2-1}\right) \rightarrow (2, 12)$ //

• $f''(x)$: $\quad f'(x) = \frac{3x^2 - 6x}{(x-1)^2}$

$$f''(x) = \frac{(6x-6).(x-1)^2 - (3x^2-6x)(2.(x-1))}{(x-1)^4}$$

$$f''(x) = \frac{(6x-6).(x^2-2x+1^2) - (3x^2-6x).(2x-2)}{(x-1)^4}$$

$$f''(x) = \frac{6x^3 - 12x^2 + 6x - 6x^2 + 12x - 6 - (6x^3 - 6x^2 - 12x^2 + 12x}{(x-1)^4}$$

$$f''(x) = \frac{\cancel{6x^3} \cancel{-18x^2} \overset{6x}{\cancel{+18x}} - 6 \cancel{-6x^3} \cancel{+6x^2} \cancel{+12x^2} \cancel{-12x}}{(x-1)^4}$$

$$f''(x) = \frac{6x-6}{(x-1)^4}$$

$$f''(x) = \frac{6(x-1)}{(x-1)^4} = \frac{6}{(x-1)^3}$$

• $\nexists f''(x_0)$ ó $f''(x_0) = 0$:

$\nexists f''(1)$

• conc/conv.:

$\ominus \quad 1 \quad \oplus$

$f''(0) = \frac{6}{(-1)^3} = \ominus \qquad f''(2) = \frac{6}{1^3} = \oplus$

f es cóncava hacia arriba en: $(1, +\infty)$

" " " " abajo en: $(-\infty, 1)$

• $\nexists$ puntos de inflexión

09/05/2023

3)a) $f: D_f \to \mathbb{R} \,/\, f(x) = \dfrac{mx^2+5}{x+n}$ hallar m y n / as. $y = 3x - 1$

Datos: $\begin{cases} \lim\limits_{x\to\infty} \left(\dfrac{f(x)}{x}\right) = 3 \longrightarrow \text{pendiente} \\ \lim\limits_{x\to\infty} \left(f(x) - 3x\right) = -1 \end{cases}$

soluc.

$$\bullet \lim_{x\to\infty}\left(\frac{\frac{mx^2+5}{x+n}}{x}\right) = \lim_{x\to\infty}\left(\frac{mx^2+5}{x^2+nx}\right) \xrightarrow{\frac{\to\infty}{\to\infty}} \lim_{x\to\infty}\left(\frac{\frac{mx^2}{x^2}+\frac{5}{x^2}}{\frac{x^2}{x^2}+\frac{nx}{x^2}}\right) = \lim_{x\to\infty}\left(\frac{m+\frac{5}{x^2}}{1+\frac{n}{x^2}}\right) =$$

$$= \frac{m+0}{1+0} = \boxed{m \overset{\text{Por dato}}{=} 3}$$

$$\bullet \lim_{x\to\infty}\left(\frac{3x^2+5}{x+n} - 3x\right) = \lim_{x\to\infty}\left(\frac{3x^2+5-3x(x+n)}{x+n}\right) = \lim_{x\to\infty}\left(\frac{\cancel{3x^2}+5-\cancel{3x^2}-3nx}{x+n}\right) =$$

$$\overset{L'H}{=} \lim_{x\to\infty}\left(\frac{-3n}{1}\right) = -3n \overset{\text{Por dato}}{=} 1 \Rightarrow \boxed{n = -\frac{1}{3}}$$

admite otras asintotas?

$$f(x) = \frac{3x^2+5}{x-1/3} = \frac{3x^2+5}{\frac{3x-1}{3}} = \frac{9x^2+15}{3x-1} \,//$$

$f: D_f \to \mathbb{R} \,/\, f(x) = \dfrac{9x^2+15}{3x-1}$

$D_f: \mathbb{R} - \{1/3\}$

as. vertical?

$$\lim_{x\to 1/3}\left(\frac{9x^2+15}{3x-1}\right) = \frac{1+15}{0} = \frac{16}{0} = \infty \Rightarrow \exists \text{ as. vertical}$$

as. horizontal?

$$\lim_{x\to\infty}\left(\frac{9x^2+15}{3x-1}\right) = \lim_{x\to\infty}\left(\frac{\frac{9x^2}{x^2}+\frac{15}{x^2}}{\frac{3x}{x^2}-\frac{1}{x^2}}\right) = \frac{9}{0} = \infty \Rightarrow \nexists \text{ as. horizontal}$$

$\nearrow \frac{\infty}{\infty}$ $\nearrow 1^{(\infty)}$

09/05/2023

5) hallar $k \in \mathbb{R}$ / $\lim_{x \to \infty} \left( \frac{x^3 + 5x - 3}{x^2 + 5kx^3 + 2} \right) = \lim_{x \to +\infty} \left( \left( \frac{x^2 + 3}{x^2 + 1} \right)^{(3x+5)} \right)$

$$\lim_{x \to \infty} \left( \frac{\frac{x^3}{x^3} + \frac{5x}{x^3} - \frac{3}{x^3}}{\frac{x^2}{x^3} + \frac{5kx^3}{x^3} + \frac{2}{x^3}} \right) = \lim_{x \to +\infty} \left( \left( 1 + \frac{x^2+3}{x^2+1} - 1 \right)^{(3x+5)} \right)$$

$$\lim_{x \to \infty} \left( \frac{1 + \frac{5}{x^2} - \frac{3}{x^3}}{\frac{1}{x} + 5k + \frac{2}{x^3}} \right) = \lim_{x \to +\infty} \left( \left( 1 + \frac{\cancel{x^2} + 3 - \cancel{x^2} - 1}{x^2+1} \right)^{(3x+5)} \right)$$

$$\left( \frac{1 + \frac{5}{\infty} - \frac{3}{\infty}}{\frac{1}{\infty} + 5k + \frac{2}{\infty}} \right) = \lim_{x \to +\infty} \left( \left( 1 + \frac{1}{\frac{x^2+1}{2}} \right)^{(3x+5)\left(\frac{x^2+1}{2}\right)\left(\frac{2}{x^2+1}\right)} \right)$$

$$\left( \frac{1 + 0 - 0}{0 + 5k + 0} \right) = \lim_{x \to +\infty} \left( \left( 1 + \frac{1}{\frac{x^2+1}{2}} \right)^{\left(\frac{x^2+1}{2}\right)} \right)^{\lim_{x \to +\infty} \left( \frac{6x+10}{x^2+1} \right)}$$

$$\frac{1}{5k} = e^{\lim_{x \to +\infty} \left( \frac{\frac{6x}{x^2} + \frac{10}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} \right)}$$

$$\frac{1}{5k} = e^{\left( \frac{0 + 0}{1 + 0} \right)}$$

$$\frac{1}{5k} = e^0 = 1$$

$$\boxed{k = \frac{1}{5}}$$

$\Rightarrow$ para que los límites sean iguales, $k = \frac{1}{5}$ //