8)

a)

$$P \vee (Q \vee R) \equiv \underline{(P \vee Q)} \vee (P \vee R)$$

$$\equiv \{\text{Conm. Disyuncion}\}$$

$$P \vee (Q \vee R) \equiv \underline{(Q \vee P) \vee (P \vee R)}$$

$$\equiv \{\text{Asoc. Disyuncion, P:=P; Q:=P; R:=R}\}$$

$$P \vee (Q \vee R) \equiv Q \vee \underline{(P \vee P)} \vee R$$

$$\equiv \{\text{Idempotencia Disyuncion}\}$$

$$P \vee (Q \vee R) \equiv \underline{Q \vee P} \vee R$$

$$\equiv \{\text{Conmut. Disyuncion}\}$$

$$P \vee (Q \vee R) \equiv P \vee (Q \vee R)$$

$$\equiv \{\text{Reflexividad de la equivalencia}\}$$

$$True$$

b)

$$p \vee \underline{True} \equiv True$$

$$\equiv \{\text{Reflexividad equivalencia}\}$$

$$\underline{p \vee (p \equiv p)} \equiv True$$

$$\equiv \{\text{Dist. disyuncion con equivalencia}\}$$

$$\underline{p \vee p} \equiv p \vee p \equiv True$$

$$\equiv \{\text{Idempotencia disyuncion}\}$$

$$\underline{p \equiv p} \equiv True$$

$$\equiv \{\text{Reflexividad equivalencia}\}$$

$$\underline{True \equiv True}$$

$$\equiv \{\text{Reflexividad equivalencia}\}$$

$$True$$

c)

$$\underline{p \vee q \equiv p \vee \neg q} \equiv p$$

$$\equiv \{\text{Dist. disyuncion con equivalencia P:=p; Q:=q; R:=}\neg q\}$$

$$p \vee \underline{(q \equiv \neg q)} \equiv p$$

$$\equiv \{\text{Conmut. equivalencia P:=q; Q:=}\neg q\}$$

$$p \vee \underline{(\neg q \equiv q)} \equiv p$$

$$\equiv \{\text{Def. de negación}\}$$

$$p \vee \neg(\underline{q \equiv q}) \equiv p$$

$$\equiv \{\text{Reflexividad de la equivalencia}\}$$

$$p \vee \underline{\neg True} \equiv p$$

$$\equiv \{\text{Def. False}\}$$

$$\underline{p \vee False} \equiv p$$

$$\equiv \{\text{Elemento neutro de la disyunción}\}$$

$$True$$

9)

a)

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (\underline{p \vee r})$$

$$\equiv \{\text{Conmut. Disyuncion P:=p; Q:=r}\}$$

$$p \vee (q \wedge r) \equiv p \vee q \wedge r \vee p$$

$$p \vee (q \wedge r) \equiv p \vee \underline{(q \wedge r) \vee p}$$

$$\equiv \{\text{Conmut. Disyuncion P:=}(q \wedge r)\text{; Q:=p}\}$$

$$p \vee (q \wedge r) \equiv \underline{p \vee p} \vee (q \wedge r)$$

$$\equiv \{\text{Indempotencia disyunción}\}$$

$$\underline{p \vee (q \wedge r) \equiv p \vee (q \wedge r)}$$

$$\equiv \{\text{Reflexividad de la equivalencia}\}$$

$$True$$

b)

$$\underline{p \wedge p} \equiv p$$

$$\equiv \{\text{Regla dorada P:=p; Q:=p}\}$$

$$p \equiv p \equiv \underline{p \vee p} \equiv p$$

$$\equiv \{\text{Idempotencia disyuncion}\}$$

$$\underline{p \equiv p} \equiv \underline{p \equiv p}$$

$$\equiv \{\text{Reflex. equivalencia}\}$$

$$\underline{True \equiv True}$$

$$\equiv \{\text{Reflex. equivalencia}\}$$

$$True$$

c)

$$p \wedge False \equiv False$$

$$\equiv \{\text{Regla dorada P:=p; Q:=False}\}$$

$$p \equiv False \equiv \underline{p \vee False} \equiv False$$

$$\equiv \{\text{Elem. Neutro Disyuncion}\}$$

$$p \equiv \underline{False \equiv p} \equiv False$$

$$\equiv \{\text{Conmut. Equivalencia P:=False; Q:=p}\}$$

$$\underline{p \equiv p} \equiv \underline{False \equiv False}$$

$$\equiv \{\text{Reflex. equivalencia}\}$$

$$\underline{True \equiv True}$$

$$\equiv \{\text{Reflex. equivalencia}\}$$

$$True$$