Lautaro Bachmann

Ejercicio 5:

a) $\int \cos(\sqrt{x})\,dx$

$$t = \sqrt{x} = x^{1/2}$$
$$dt = \frac{1}{2}\cdot x^{-1/2}\,dx \Rightarrow dt = \frac{1}{2\cdot\sqrt{x}}\,dx$$
$$\Rightarrow 2\cdot\sqrt{x}\,dt = dx$$

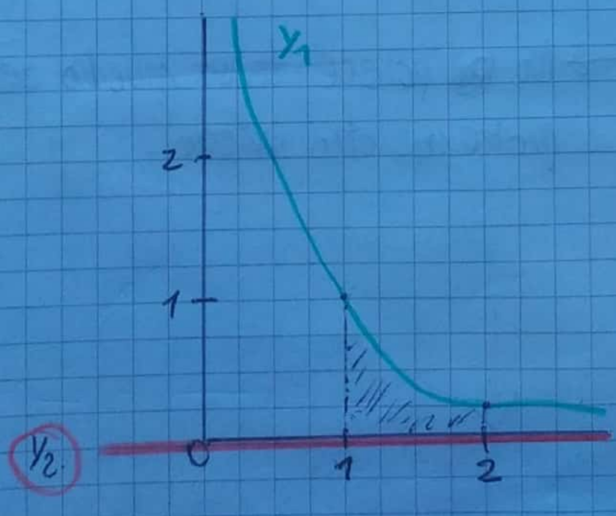
$$= \int \cos(t)\cdot 2\cdot\sqrt{x}\,dt$$
$$= 2\cdot\int \cos(t)\cdot t\,dt$$

$$u = t \qquad dv = \cos(t)\,dt$$
$$du = 1\,dt \qquad v = \operatorname{sen}(t)$$

$$= 2\cdot\left(t\cdot\operatorname{sen}(t) - \int \operatorname{sen}(t)\,dt\right)$$
$$= 2\cdot\left(t\cdot\operatorname{sen}(t) - (-\cos(t))\right) + C$$
$$= 2\cdot\left(t\cdot\operatorname{sen}(t) + \cos(t)\right) + C = 2t\cdot\operatorname{sen}(t) + 2\cdot\cos(t)$$
$$= 2\sqrt{x}\cdot\operatorname{sen}(\sqrt{x}) + 2\cdot\cos(\sqrt{x})$$

Por ende, $\int \cos(\sqrt{x})\,dx = 2\cdot\sqrt{x}\cdot\operatorname{sen}(\sqrt{x}) + 2\cdot\cos(\sqrt{x})$

b) $y_1 = \frac{1}{x^2}$, $y_2 = 0$, $x_1 = 1$, $x_2 = 2$



Escaneado con CamScanner

Primero calculemos el area entre curvas.

$$\int_1^2 y_1 - y_2 \, dx = \int_1^2 \frac{1}{x^2} - 0 \, dx = \int_1^2 \frac{1}{x^2} \, dx$$

$$= \int_1^2 x^{-2} \, dx = \left. \frac{x^{-1}}{-1} \right|_1^2 = \left. -\frac{1}{x} \right|_1^2 = \frac{-1}{2} + \frac{1}{1} = \frac{-1+2}{2}$$

$$= \frac{1}{2}$$

Ahora armemos una ecuacion con el resultado.

$$\frac{1}{2} = 2 \cdot \int_1^k x^{-2} \, dx \Rightarrow \frac{1}{2} = 2 \cdot \left(\left. -\frac{1}{x} \right|_1^k \right)$$

$$\Rightarrow \frac{1}{2} = 2 \cdot \left(-\frac{1}{k} + \frac{1}{1} \right) \Rightarrow \frac{1}{2} = \frac{-2}{k} + 2$$

$$\Rightarrow \frac{1}{2} - \frac{2}{1} = \frac{-2}{k} \Rightarrow k \left(\frac{1-4}{2} \right) = -2$$

$$\Rightarrow k = \frac{-2}{\frac{\frac{-3}{2}}{1}} \Rightarrow k = \frac{+2}{3} \cdot \frac{1}{2} \Rightarrow k = \frac{\cancel{2}^{1}}{\cancel{6}_{3}} \Rightarrow k = \frac{1}{3}$$

Verifiquemos:

$$\frac{1}{2} = 2 \cdot \int_1^{1/3} x^{-2} \, dx$$ → No parece tener mucho sentido probemos otro metodo.

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$$\frac{1}{2} = \int_1^k x^{-2}\,dx + \int_k^2 x^{-2}\,dx$$

$$= \left.\frac{-1}{x}\right|_1^k + \left.\frac{-1}{x}\right|_k^2$$

$$= \left(\frac{-1}{k} - \left(\frac{-1}{1}\right)\right) + \frac{-1}{2} - \left(\frac{-1}{k}\right) = \frac{-1}{k} + 1 + \left(-\frac{1}{2} + \frac{1}{k}\right)$$

$$= \cancel{\frac{-1}{k}} + \cancel{\frac{1}{k}}\ 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{1}{2}$$

Probemos otra cosa:

$$\int_1^k x^{-2}\,dx = \int_k^2 x^{-2} \Rightarrow \left.\frac{-1}{x}\right|_1^k = \left.\frac{-1}{x}\right|_k^2$$

$$\Rightarrow \frac{-1}{k} + 1 = -\frac{1}{2} + \frac{1}{k} \Rightarrow \frac{1}{1} + \frac{1}{2} = \frac{1}{k} + \frac{1}{k}$$

$$\Rightarrow \frac{2+1}{2} = \frac{2}{k} \Rightarrow \frac{3}{2} = \frac{2}{k} \Rightarrow \frac{3}{2}\cdot k = 2$$

$$\Rightarrow k = \frac{\frac{2}{1}}{\frac{3}{2}} \Rightarrow k = \frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}} \Rightarrow k = \frac{1}{3}$$

$$\therefore b = 1 + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}$$

Por ende, el numero b tal que la recta $x = b$ divide dicha region en dos regiones iguales es $b = 4/3$.

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