Práctico 2

1)

a) Suma todos los elementos de una lista.
b) $\langle \Sigma i : 0 \leq i < \#xs : xs \ !! \ i \rangle$
c) Caso base: xs = []

$$sum.[] = \langle \Sigma i : 0 \leq i < \underline{\#[]} : xs \ !! \ i \rangle$$
$$\equiv \{\text{Def de } \#\}$$
$$sum.[] = \langle \Sigma i : 0 \leq i < 0 : xs \ !! \ i \rangle$$
$$\equiv \{\text{Evaluamos rango}\}$$
$$sum.[] = \langle \Sigma i : False : xs \ !! \ i \rangle$$
$$\equiv \{\text{Rango vacio}\}$$
$$sum.[] = 0$$

Caso inductivo: $xs = y \triangleright ys$

$$HI = \boxed{sum.ys = \langle \Sigma i : 0 \leq i < \#ys : ys \ !! \ i \rangle}$$

$$sum.(y \triangleright ys) = \langle \Sigma i : 0 \leq i < \#(y \triangleright ys) : (y \triangleright ys) \ !! \ i \rangle$$
$$\equiv \{\text{Def de } \#\}$$
$$sum.(y \triangleright ys) = \langle \Sigma i : 0 \leq i < 1 + \#ys : (y \triangleright ys) \ !! \ i \rangle$$
$$\equiv \{\text{Cambio de variable f.x = x+1}\}$$
$$sum.(y \triangleright ys) = \langle \Sigma i : 0 \leq i + 1 < 1 + \#ys : (y \triangleright ys) \ !! \ i + 1 \rangle$$
$$\equiv \{\text{Aritmetica}\}$$
$$sum.(y \triangleright ys) = \langle \Sigma i : i = 0 \vee 1 \leq i + 1 < 1 + \#ys : (y \triangleright ys) \ !! \ i + 1 \rangle$$
$$\equiv \{\text{Aritmetica}\}$$
$$sum.(y \triangleright ys) = \langle \Sigma i : i = 0 \vee 0 \leq i < \#ys : (y \triangleright ys) \ !! \ i + 1 \rangle$$
$$\equiv \{\text{Particion de rango}\}$$
$$sum.(y \triangleright ys) = \langle \Sigma i : i = 0 : (y \triangleright ys) \ !! \ i \rangle + \langle \Sigma i : 0 \leq i < \#ys : (y \triangleright ys) \ !! \ i + 1 \rangle$$
$$\equiv \{\text{Rango unitario}\}$$
$$sum.(y \triangleright ys) = (y \triangleright ys) \ !! \ 0 + \langle \Sigma i : 0 \leq i < \#ys : (y \triangleright ys) \ !! \ i + 1 \rangle$$
$$\equiv \{\text{Def de } \ !! \ \}$$
$$sum.(y \triangleright ys) = y + \langle \Sigma i : 0 \leq i < \#ys : ys \ !! \ i \rangle$$
$$\equiv \{\text{HI}\}$$
$$sum.(y \triangleright ys) = y + sum.ys$$

2)

b) $iga.e.xs = \langle \forall i : 0 \leq i < \#xs : xs \;!!\; i = e \rangle$

Caso Base: xs = [] $iga.e.[] = \langle \forall i : 0 \leq i < \#[] : [] \;!!\; i = e \rangle$

$$\equiv \{\text{Def de } \#\}$$

$iga.e.[] = \langle \forall i : 0 \leq i < 0 : [] \;!!\; i = e \rangle$

$$\equiv \{\text{Evaluamos rango}\}$$

$iga.e.[] = \langle \forall i : False : [] \;!!\; i = e \rangle$

$$\equiv \{\text{Rango vacio}\}$$

iga.e.[] = True

Caso Inductivo: $xs = y \triangleright ys$

$$HI = \boxed{iga.e.ys = \langle \forall i : 0 \leq i < \#ys : ys \;!!\; i = e \rangle}$$

$$iga.e.(y \triangleright ys) = \langle \forall i : 0 \leq i < \#(y \triangleright ys) : (y \triangleright ys) \;!!\; i = e \rangle$$

$$\equiv \{\text{Def de } \#\}$$

$$iga.e.(y \triangleright ys) = \langle \forall i : 0 \leq i < 1 + \#ys : (y \triangleright ys) \;!!\; i = e \rangle$$

$$\equiv \{\text{Aritmetica}\}$$

$$iga.e.(y \triangleright ys) = \langle \forall i : i = 0 \vee 1 \leq i < 1 + \#ys : (y \triangleright ys) \;!!\; i = e \rangle$$

$$\equiv \{\text{Particion de rango}\}$$

$$iga.e.(y \triangleright ys) = \langle \forall i : i = 0 : (y \triangleright ys) \;!!\; i = e \rangle \wedge \langle \forall i : 1 \leq i < 1 + \#ys : (y \triangleright ys) \;!!\; i = e \rangle$$

$$\equiv \{\text{Cambio de variable f.x=x+1}\}$$

$$iga.e.(y \triangleright ys) = \langle \forall i : i = 0 : (y \triangleright ys) \;!!\; i = e \rangle \wedge \langle \forall i : 1 \leq i{+}1 < 1 + \#ys : (y \triangleright ys) \;!!\; i{+}1 = e \rangle$$

$$\equiv \{\text{Aritmetica}\}$$

$$iga.e.(y \triangleright ys) = \langle \forall i : i = 0 : (y \triangleright ys) \;!!\; i = e \rangle \wedge \langle \forall i : 0 \leq i < \#ys : (y \triangleright ys) \;!!\; i{+}1 = e \rangle$$

$$\equiv \{\text{Rango unitario}\}$$

$$iga.e.(y \triangleright ys) = ((y \triangleright ys) \;!!\; 0 = e) \wedge \langle \forall i : 0 \leq i < \#ys : (y \triangleright ys) \;!!\; i + 1 = e \rangle$$

$$\equiv \{\text{Def de } \;!!\; \}$$

$$iga.e.(y \triangleright ys) = (y = e) \wedge \langle \forall i : 0 \leq i < \#ys : ys \;!!\; i = e \rangle$$

$$\equiv \{\text{HI}\}$$

$$iga.e.(y \triangleright ys) = (y = e) \wedge iga.e.ys$$

d) $sumPar.n = \langle \Sigma i : 0 \leq i \leq n \wedge par.i : i\rangle$ Caso base: n = 0

$$sumPar.0 = \langle \Sigma i : 0 \leq i \leq 0 \wedge par.i : i\rangle$$

$$\equiv \{\text{Evaluo rango}\}$$

$$\equiv \{\text{rango unitario y condicion}\}$$

$$(par.i \rightarrow 0 \quad \Box \neg par.i \rightarrow 0)$$

$$\equiv \{\text{Ambos casos comparten resultado}\}$$

$$0$$

Caso inductivo: n = m + 1

$$HI = \boxed{sumPar.m = \langle \Sigma i : 0 \leq i \leq m \wedge par.i : i\rangle}$$

$$sumPar.(m+1) = \langle \Sigma i : 0 \leq i \leq (m+1) \wedge par.i : i\rangle$$

$$\equiv \{\text{Aritmética}\}$$

$$\equiv \{\text{Distributiva de } \wedge \text{ con } \vee\}$$

$$\equiv \{\text{Particion de rango}\}$$

$$\langle \Sigma i : i = m+1 \wedge par.i : i\rangle + \langle \Sigma i : 0 \leq i \leq m \wedge par.i : i\rangle$$

$$\equiv \{\text{Cambio de variable f.x=x+1}\}$$

$$\equiv \{\text{Rango unitario y condicion}\}$$

$$\equiv \{\text{HI}\}$$

$$[par.(m+1) \rightarrow m+1 \mid \neg par.(m+1) \rightarrow 0] + sumPar.m$$

$$\equiv \{\text{Incluyo a la funcion en el analisis por casos}\}$$

$$[par.(m+1) \rightarrow m+1+sumPar.m \mid \neg par.(m+1) \rightarrow 0+sumPar.m]$$

$$\begin{aligned} sumPar.(m+1) = (& \\ & par.(m+1) \rightarrow (m+1) + sumPar.m \\ & \Box \neg par.(m+1) \rightarrow sumPar.m \\ &) \end{aligned}$$

4)

a)

$$sumPot.x.n = \langle \Sigma i : 0 \leq i < n : x^i\rangle$$

Caso base: n=0

$$\langle \Sigma i : 0 \leq i < 0 : x^i \rangle$$

$$\equiv \{\text{Evaluo rango, rango Vacio}\}$$

$$0$$

Caso inductivo: n=k+1

$$HI = \boxed{sumPot.x.k = \langle \Sigma i : 0 \leq i < k : x^i \rangle}$$

$$sumPot.x.(k+1) = \langle \Sigma i : 0 \leq i < (k+1) : x^i \rangle$$

$$\equiv \{\text{Aritmetica}\}$$

$$\langle \Sigma i : 0 \leq i < k \vee i = k : x^i \rangle$$

$$\equiv \{\text{Particion de rango, conmutatividad y rango unitario}\}$$

$$x^k + \langle \Sigma i : 0 \leq i < k : x^i \rangle$$

$$\equiv \{\text{HI}\}$$

$$x^k + sumPot.x.k$$

Ahora derivemos x^k para obtener algo programable

$$exp.x.k = x^k$$

Caso base: k=0

$$exp.x.0 = x^0$$

$$\equiv \{\text{Aritmetica}\}$$

$$exp.x.0 = 1$$

Caso inductivo: k=n+1

$$HI = \boxed{exp.x.n = x^n}$$

$$exp.x.(n+1) = x^{(n+1)}$$

$$\equiv \{\text{Aritmetica}\}$$

$$exp.x.(n+1) = x * x^n$$

$$\equiv \{\text{HI}\}$$

$$exp.x.(n+1) = x * exp.x.n$$

Resultado final:

$$sumPot.x.0 = 0$$

$$sumPot.x.(k+1) = exp.x.k + sumPot.x.k$$

c)

$$cubo.x = x^3$$

Caso base: x=0

$$cubo.x = 0$$

Paso inductivo: x=n+1

$$cubo.(n+1) = (n+1)^3$$

$$n^3 + 3n^2 + 3n + 1$$

$$\equiv \{\text{HI}\}$$

$$cubo.n + 3n^2 + 3n + 1$$

$$\equiv \{\text{Aritmetica y modularizacion}\}$$

$$cubo.n + sumMult.3n.n + sumMult.3.n + 1$$

$$sumMult.x.y = \langle \Sigma i : 1 \leq i \leq y : x \rangle$$

Caso base: y=0

$$sumMult.x.0 = 0$$

Paso inductivo: $y = (n+1)$

$$Hi = sumMult.x.n = \langle \Sigma i : 1 \leq i \leq n : x \rangle$$

$$sumMult.x.(n+1) =$$

$$\langle \Sigma i : 1 \leq i \leq (n+1) : x \rangle$$

$$\equiv \{\text{Aritmetica, particion de rango, rango unitario}\}$$

$$x + \langle \Sigma i : 1 \leq i \leq n : x \rangle$$

$$\equiv \{\text{HI}\}$$

$$x + sumMult.x.n$$

Solucion:

```
cubo.0 = 0
cubo.(n+1) = sumMult.3.(suMult.n.n) + sumMult.3.n + cubo.n
  donde
    sumMult.x.0 = 0
    sumMult.x.(n+1) = x+sumMult.x.n
```

5)

a) iguales [A] -> Bool

$$iguales.xs = \langle \forall i : 0 \leq i < \#xs - 1 : xs \; !! \; i == xs \; !! \; (i+1) \rangle$$

Caso base: $xs = []$

$$iguales.[] =$$
$$\langle \forall i : 0 \leq i < \#[] - 1 : x \; !! \; i == x \; !! \; (i+1) \rangle$$
$$\equiv \{\text{Def } \#\}$$
$$\equiv \{\text{Rango vacio}\}$$
$$True$$

Caso recursivo: $xs = x \triangleright ls$ i) $ls = []$

$$iguales.(x \triangleright []) = \langle \forall i : 0 \leq i < \#(x \triangleright []) - 1 : (x \triangleright []) \; !! \; i == (x \triangleright []) \; !! \; (i+1) \rangle$$
$$\equiv \{\text{Def de } \# \text{ y aritmetica}\}$$
$$iguales.(x \triangleright []) = \langle \forall i : 0 \leq i < 0 : (x \triangleright []) \; !! \; i == (x \triangleright []) \; !! \; (i+1) \rangle$$
$$\equiv \{\text{Rango vacio}\}$$
$$iguales.(x \triangleright []) = True$$

ii) $ls = y \triangleright ys$

$$HI = iguales.(y \triangleright ys) = \langle \forall i : 0 \leq i < \#(y \triangleright ys) - 1 : (y \triangleright ys) \; !! \; i == (y \triangleright ys) \; !! \; (i+1) \rangle$$

$$iguales.(x \triangleright y \triangleright ys) =$$
$$\langle \forall i : 0 \leq i < \#(x \triangleright y \triangleright ys) - 1 : (x \triangleright y \triangleright ys) \; !! \; i == (x \triangleright y \triangleright ys) \; !! \; (i+1) \rangle$$
$$\equiv \{\text{Def de } \# \text{ y aritmetica}\}$$
$$\langle \forall i : 0 \leq i < \#(y \triangleright ys) : (x \triangleright y \triangleright ys) \; !! \; i == (x \triangleright y \triangleright ys) \; !! \; (i+1) \rangle$$
$$\equiv \{\text{Aritmetica}\}$$
$$\equiv \{\text{Particion de rango}\}$$
$$\langle \forall i : i = 0 : (x \triangleright y \triangleright ys) \; !! \; i == (x \triangleright y \triangleright ys) \; !! \; (i+1) \rangle \wedge \langle \forall i : 1 \leq i < \#(y \triangleright ys) : (x \triangleright y \triangleright ys) \; !! \; i == (x \triangleright y \triangleright ys) \; !! \; (i+1)$$
$$\equiv \{\text{Rango unitario y aritmetica}\}$$
$$(x \triangleright y \triangleright ys) \; !! \; 0 == (x \triangleright y \triangleright ys) \; !! \; 1 \wedge \langle \forall i : 1 \leq i < \#(y \triangleright ys) : (x \triangleright y \triangleright ys) \; !! \; i == (x \triangleright y \triangleright ys) \; !! \; (i+1) \rangle$$
$$\equiv \{\text{Def de } !!\}$$
$$x == y \wedge \langle \forall i : 1 \leq i < \#(y \triangleright ys) : (x \triangleright y \triangleright ys) \; !! \; i == (x \triangleright y \triangleright ys) \; !! \; (i+1) \rangle$$
$$\equiv \{\text{Cambio de variable f.x=x+1}\}$$
$$x == y \wedge \langle \forall i : 1 \leq i+1 < \#(y \triangleright ys) : (x \triangleright y \triangleright ys) \; !! \; i+1 == (x \triangleright y \triangleright ys) \; !! \; (i+1+1) \rangle$$
$$\equiv \{\text{Aritmetica}\}$$

$$x == y \wedge \langle \forall i : 0 \leq i < \#(y \triangleright ys) - 1 : (x \triangleright y \triangleright ys) \,!!\, i+1 == (x \triangleright y \triangleright ys) \,!!\, (i+1+1) \rangle$$

$$\equiv \{\text{Def de !!}\}$$

$$x == y \wedge \langle \forall i : 0 \leq i < \#(y \triangleright ys) - 1 : (y \triangleright ys) \,!!\, i == (y \triangleright ys) \,!!\, (i+1) \rangle$$

$$\equiv \{\text{HI}\}$$

$$x == y \wedge iguales.(y \triangleright ys)$$

b) $minimo.xs = \langle Min i : 0 \leq i < \#xs : xs \,!!\, i \rangle$

Caso base: $xs = x \triangleright []$

$$minimo.x \triangleright []$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle Min i : 0 \leq i < \#x \triangleright [] : x \triangleright [] \,!!\, i \rangle$$

$$\equiv \{\text{Def de \#}\}$$

$$\langle Min i : 0 \leq i < 1 : x \triangleright [] \,!!\, i \rangle$$

$$\equiv \{\text{Evaluo rango}\}$$

$$\langle Min i : i = 0 : x \triangleright [] \,!!\, i \rangle$$

$$\equiv \{\text{Rango unitario}\}$$

$$x \triangleright [] \,!!\, 0$$

$$\equiv \{\text{Def de !!}\}$$

$$x$$

Caso recursivo: $xs = (y \triangleright ys)$

$$HI = minimo.ys = \langle Min i : 0 \leq i < \#ys : ys \,!!\, i \rangle$$

$$minimo.(y \triangleright ys)$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle Min i : 0 \leq i < \#(y \triangleright ys) : (y \triangleright ys) \,!!\, i \rangle$$

$$\equiv \{\text{Aritmetica, separacion de rango, rango unitario}\}$$

$$(y \triangleright ys) \,!!\, 0 \; min \; \langle Min \; i : 1 \leq i < \#(y \triangleright ys) : (y \triangleright ys) \,!!\, i \rangle$$

$$\equiv \{\text{Def de !!}\}$$

$$y \; min \; \langle Min \; i : 1 \leq i < \#(y \triangleright ys) : (y \triangleright ys) \,!!\, i \rangle$$

$$\equiv \{\text{Def de \#}\}$$

$$\equiv \{\text{Cambio de variable f.x=x+1}\}$$

$$\equiv \{\text{Aritmetica}\}$$

$$y \ min \ \langle Min \ i : 0 \leq i < \#ys : (y \triangleright ys) \ !! \ i+1 \rangle$$

$$\equiv \{\text{Def de !!}\}$$

$$y \ min \ \langle Min \ i : 0 \leq i < \#ys : ys \ !! \ i \rangle$$

$$\equiv \{\text{HI}\}$$

$$y \ min \ minimo.ys$$

c) $creciente.xs = \langle \forall i : 0 \leq i < \#xs - 1 : xs \ !! \ i < xs \ !! \ i+1 \rangle$

Caso base: $xs = []$

$$creciente.(x \triangleright [])$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle \forall i : 0 \leq i < \#[] - 1 : [] \ !! \ i < [] \ !! \ i+1 \rangle$$

$$\equiv \{\text{Def de \# y evaluo rango}\}$$

$$\equiv \{\text{Rango vacio}\}$$

$$True$$

Caso recursivo: $xs = (x \triangleright ls)$

i) $ls = []$

$$creciente.(x \triangleright [])$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle \forall i : 0 \leq i < \#(x \triangleright []) - 1 : (x \triangleright []) \ !! \ i < (x \triangleright []) \ !! \ i+1 \rangle$$

$$\equiv \{\text{Def de \# y evaluo rango}\}$$

$$\equiv \{\text{Rango vacio}\}$$

$$True$$

ii) $ls = y \triangleright ys$

$HI = creciente.(y : ys) = \langle \forall i : 0 \leq i < \#(y : ys){-}1 : (y : ys) \,!!\, i < (y : ys) \,!!\, i{+}1 \rangle$

$$creciente.(x \triangleright y \triangleright ys)$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle \forall i : 0 \leq i < \#(x : y : ys) - 1 : (x : y : ys) \,!!\, i < (x : y : ys) \,!!\, i + 1 \rangle$$

$$\equiv \{\text{Def de \# y aritmetica}\}$$

$$\langle \forall i : 0 \leq i < (y : ys) : (x : y : ys) \,!!\, i < (x : y : ys) \,!!\, i + 1 \rangle$$

$$\equiv \{\text{Aritmetica, particion de rango, rango unitario}\}$$

$$(x : y : ys) \,!!\, 0 < (x : y : ys) \,!!\, 1 \wedge \langle \forall i : 1 \leq i < (y : ys) : (x : y : ys) \,!!\, i < (x : y : ys) \,!!\, i{+}1 \rangle$$

$$\equiv \{\text{Def de !!}\}$$

$$x < y \wedge \langle \forall i : 1 \leq i < (y : ys) : (x : y : ys) \,!!\, i < (x : y : ys) \,!!\, i + 1 \rangle$$

$$\equiv \{\text{Cambio de variable f.x=x+1}\}$$

$$\equiv \{\text{Aritmetica}\}$$

$$x < y \wedge \langle \forall i : 0 \leq i < (y : ys){-}1 : (x : y : ys) \,!!\, i{+}1 < (x : y : ys) \,!!\, i{+}1{+}1 \rangle$$

$$\equiv \{\text{Def de !!}\}$$

$$x < y \wedge \langle \forall i : 0 \leq i < (y : ys) - 1 : (y : ys) \,!!\, i < (y : ys) \,!!\, i + 1 \rangle$$

$$\equiv \{\text{HI}\}$$

$$x < y \wedge creciente.(y : ys)$$