3)

$$prod.([2+3,3] \mathbin{++} [1,0])$$
$$\equiv \{\text{Aritmetica}\}$$
$$prod.([5,3] \mathbin{++} [1,0])$$
$$\equiv \{\text{def} \mathbin{++}\}$$
$$prod.(5 : ([3] \mathbin{++} [1,0]))$$
$$prod.(5 : 3 : ([] \mathbin{++} [1,0]))$$
$$prod.(5 : 3 : [1,0])$$
$$prod.([5,3,1,0])$$
$$\equiv \{\text{Def prod}\}$$
$$5 * 3 * 1 * 0$$
$$\equiv \{\text{Elemento absorbente multiplicacion}\}$$
$$0$$

4) a)

$$\neg\langle \exists j : 0 \leq j < \#ns : \langle \exists i : 0 \leq i < \#ps : \neg((ps \mathbin{!!} i).(ns \mathbin{!!} j))\rangle\rangle$$
$$\equiv \{\text{DeMorgan}\}$$
$$\neg\neg\langle \forall j : 0 \leq j < \#ns : \neg\langle \exists i : 0 \leq i < \#ps : \neg((ps \mathbin{!!} i).(ns \mathbin{!!} j))\rangle\rangle$$
$$\equiv \{\text{DeMorgan}\}$$
$$\neg\neg\langle \forall j : 0 \leq j < \#ns : \neg\neg\langle \forall i : 0 \leq i < \#ps : \neg\neg((ps \mathbin{!!} i).(ns \mathbin{!!} j))\rangle\rangle$$
$$\equiv \{\text{Doble negacion}\}$$
$$\langle \forall j : 0 \leq j < \#ns : \langle \forall i : 0 \leq i < \#ps : ((ps \mathbin{!!} i).(ns \mathbin{!!} j))\rangle\rangle$$

5)

$$f.x = \langle \exists y : 0 \leq y < x : x = y * (y+1)/2\rangle$$

Evaluacion manual:

$$f.5$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle \exists y : 0 \leq y < 5 : 5 = y * (y+1)/2\rangle$$
$$\equiv \{\text{Evaluo rango}\}$$
$$\langle \exists y : y \in \{0,1,2,3,4\} : 5 = y * (y+1)/2\rangle$$
$$\equiv \{\text{Evaluo rango en termino}\}$$

$$5 = 0*(0{+}1)/2 \vee 5 = 1*(1{+}1)/2 \vee 5 = 2*(2{+}1)/2 \vee 5 = 3*(3{+}1)/2 \vee 5 = 4*(4{+}1)/2$$

$$\equiv \{\text{Aritmetica}\}$$

$$5 = 0 \vee 5 = 2/2 \vee 5 = 6/2 \vee 5 = 12/2 \vee 5 = 20/2$$

$$\equiv \{\text{Aritmetica}\}$$

$$False \vee False \vee False \vee False \vee False$$

$$False$$

7)

$f.xs = \langle \forall a, as, bs : xs = (a : as) +\!\!+ (a : bs) : as = bs \rangle$

xs = [1,2,1,7,7,1,3]

$$f.[1, 2, 1, 7, 7, 1, 3]$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle \forall a, as, bs : [1, 2, 1, 7, 7, 1, 3] = (a : as) +\!\!+ (a : bs) : as = bs \rangle$$

as	bs	a	a’
[1,2,1,7,7,1]	[3]	1	3
[1,2,1,7,7]	[1,3]	1	1
[1,2,1,7]	[7,1,3]	1	7
[1,2,1]	[7,7,1,3]	1	7
[1,2]	[1,7,7,1,3]	1	1
[1]	[2,1,7,7,1,3]	1	2

10)

$HI = hGen.xs.n.m = \langle N\ as, bs : xs = as +\!\!+\ bs : n + sum.as = 2 * (\#as + m)\rangle$

$$\langle N\ as, bs : xs = as +\!\!+\ bs : n + sum.(x : as) = 2 * (\#(x : as) + m)\rangle$$

$$\equiv \{\text{Def de sum y } \#\}$$

$$\langle N\ as, bs : xs = as +\!\!+\ bs : n + x + sum.(as) = \underline{2 * (\#as + 1 + m)}\rangle$$

$$\equiv \{\text{Distributividad de la multiplicacion con la suma}\}$$

$$\langle N\ as, bs : xs = as +\!\!+\ bs : n + x + sum.(as) = \underline{2 * (\#as + 1)} + (2 * m)\rangle$$

$$\equiv \{\text{Distributividad de la multiplicacion con la suma}\}$$

$$\langle N\ as, bs : xs = as +\!\!+\ bs : n + x + sum.(as) = 2 * \#as + 2 + 2 * m\rangle$$

$$\equiv \{\text{Resto 2 en ambos lados de la igualdad}\}$$

$$\langle N\ as, bs : xs = as +\!\!+\ bs : n + x - 2 + sum.(as) = \underline{2 * \#as + 2 * m}\rangle$$

$$\equiv \{\text{Distributividad de la multiplicacion con la suma}\}$$

$$\underline{\langle N\ as, bs : xs = as +\!\!+\ bs : n + x - 2 + sum.(as) = 2 * (\#as + m)\rangle}$$

$$\equiv \{\text{HI}\}$$

$$hGen.xs.(n + x - 2).m$$

8)

$quant.n = \langle N\ i : 0 \leq i \leq n : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle$

Caso base: $n = 0$

$$quant.n$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle N\ i : 0 \leq i \leq n : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle$$
$$\equiv \{\text{Evaluo rango, rango vacio}\}$$
$$0$$

Caso inductivo: $n := (n + 1)$

$HI = quant.n = \langle N\ i : 0 \leq i \leq n : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle$

$$quant.(n + 1)$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle N\ i : 0 \leq i \leq (n + 1) : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle$$
$$\equiv \{\text{Aritmetica}\}$$
$$\langle N\ i : 0 \leq i \leq n \vee i = (n + 1) : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle$$
$$\equiv \{\text{Particion de rango}\}$$
$$\langle N\ i : i = (n + 1) : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle +$$
$$\langle N\ i : 0 \leq i \leq n : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle$$
$$\equiv \{\text{Rango unitario}\}$$
$$(\neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle \to 1$$
$$\Box\neg\neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle \to 0$$
$$)$$
$$\langle N\ i : 0 \leq i \leq n : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle$$
$$\equiv \{\text{HI y doble negacion}\}$$
$$(\neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle \to 1$$
$$\Box\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle\rangle \to 0$$
$$)$$
$$quant.n$$
$$\equiv \{\text{Modularizamos}\}$$
$$(\neg comp.k$$

$$\Box comp.k$$

$$)$$

$$quant.n$$

9)

$tieneLargo.xs = \langle \exists i : 0 \leq i < \#xs : \#xs = xs \ !! \ i \rangle$

Caso base: $xs := []$

$$tieneLargo.[]$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle \exists i : 0 \leq i < \#[] : \#[] = [] \ !! \ i \rangle$$

$$\equiv \{\text{Def de \#, evaluo rango, rango vacio}\}$$

$$False$$

Caso inductivo: $xs := (x : xs)$ $HI = tieneLargo.xs = \langle \exists i : 0 \leq i < \#xs : \#xs = xs \ !! \ i \rangle$

$$tieneLargo.(x : xs)$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle \exists i : 0 \leq i < \#(x : xs) : \#(x : xs) = (x : xs) \ !! \ i \rangle$$

$$\equiv \{\text{Def de \#, aritmetica}\}$$

$$\langle \exists i : i = 0 \vee 1 \leq i < \#xs + 1 : \#xs + 1 = (x : xs) \ !! \ i \rangle$$

$$\equiv \{\text{Particion de rango y rango unitario}\}$$

$$(\#xs + 1 = (x : xs) \ !! \ 0) \vee \langle \exists i : 1 \leq i < \#xs + 1 : \#xs + 1 = (x : xs) \ !! \ i \rangle$$

$$\equiv \{\text{Def de !!}\}$$

$$(\#xs + 1 = x) \vee \langle \exists i : 1 \leq i < \#xs + 1 : \#xs + 1 = (x : xs) \ !! \ i \rangle$$

$$\equiv \{\text{Cambio de variable i->i+1, aritmetica}\}$$

$$(\#xs + 1 = x) \vee \langle \exists i : 0 \leq i < \#xs : \#xs + 1 = (x : xs) \ !! \ i + 1 \rangle$$

$$\equiv \{\text{Def de !!}\}$$

$$(\#xs + 1 = x) \vee \langle \exists i : 0 \leq i < \#xs : \#xs + 1 = xs \ !! \ i \rangle$$

No se puede aplicar HI por ende debemos generalizar:

$gTieneLargo.xs.n = \langle \exists i : 0 \leq i < \#xs : \#xs + n = xs \ !! \ i \rangle$

$tieneLargo.xs = gTieneLargo.xs.0$

$$gTieneLargo.xs.0$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle \exists i : 0 \leq i < \#xs : \#xs + 0 = xs \;!!\; i \rangle$$
$$\equiv \{\text{Aritmetica}\}$$
$$\langle \exists i : 0 \leq i < \#xs : \#xs = xs \;!!\; i \rangle$$
$$\equiv \{\text{Especificacion de tieneLargo}\}$$
$$tieneLargo.xs$$

$$\langle \exists i : 0 \leq i < \#xs : \#xs = (xs \;!!\; i) + k \rangle$$
$$\equiv$$
$$\langle \exists i : 0 \leq i < \#xs : \#xs + k = (xs \;!!\; i) \rangle$$