Practico 1:

1) a)

$f(x) = \ln(x+1) \Rightarrow f(0) = \ln(1) = 0$

$f'(x) = \frac{1}{x+1} \Rightarrow f'(0) = 1$

$f''(x) = -1 \cdot (x+1)^{-2} \Rightarrow f''(0) = \frac{-1}{1^2} = -1$

$f'''(x) = 2 \cdot (x+1)^{-3} \Rightarrow f'''(0) = \frac{2}{1^3} = 2$

$f''''(x) = -6 \cdot (x+1)^{-4} \Rightarrow f^{(4)}(0) = \frac{-6}{1^4} = -6$

$f^{(5)}(x) = 24 \cdot (x+1)^{-5} \Rightarrow f^{(5)}(0) = \frac{24}{1^5} = 24$

$f(x) = \sum_{k=1}^{n} \frac{(-1)^{k+1} \cancel{(k-1)!}^{\,1}}{\cancel{k!}_{\,k}} x^k + R_n(x)$ $\boxed{f^{(n)}(x) = \frac{(-1)^{n+1} (n-1)!}{(x+1)^n}}$

$\sum_{n=0}^{\infty} \frac{1}{n!} \cdot f^{(n)}(0)(x-0)^n = 0 + \frac{1}{1!} \cdot x^1 + \frac{-1}{2} \cdot x^2 + \frac{\cancel{2}^{\,1}}{\cancel{6}_{\,3}} \cdot x^3$

$f(x) = T_{n,a} + R_{n,a} \Rightarrow |R_{n,a}| = f(x) - T_{n,a}$

$\therefore$ por formula de lagrange para el resto:

$R_{n,a} = \frac{f^{(n+1)}(t)}{(n+1)!} (x-a)^{n+1}$

b) $Ln(1{,}5) = Ln(0{,}5+1) = f(0{,}5) = f\left(\frac{1}{2}\right)$ $\quad 0<t<\frac{1}{2}$

$$\left|Q_n\left(\tfrac{1}{2}\right)\right| < 10^{-10} \Rightarrow \left| \frac{1}{(n+1)!} \cdot \frac{(-1)^{n+2} \cdot n!}{(t+1)^{n+1}} \left(\frac{1}{2}\right)^{n+1} \right| < 10^{-10}$$

$$\Rightarrow \frac{1}{n+1} \frac{1}{(t+1)^{n+1}} \cdot \frac{1}{2^{n+1}} < 10^{-10} \Rightarrow (n+1)(t+1)^{n+1} \cdot 2^{n+1} > 10^{10}$$

tomo $t=0 \Rightarrow (n+1) \cdot 1^{n+1} \cdot 2^{n+1} \Rightarrow n+1 \cdot 2^{n+1}$

(si vale para la t mas chica vale para el resto) ↑

como log es creciente aplico a ambos lados:

$$\log(n+1) + \log(2^{n+1}) > \underset{\log(10^{10})}{10} \Rightarrow \log(n+1) + (n+1) \cdot \log(2) > 10$$

$$\Rightarrow \log(n+1) + \log(2) \cdot n + \log(2) \cdot 1 > 10$$

$$\Rightarrow \frac{\log(n+1)}{\log(2)} + n + 1 > \frac{10}{\log(2)}$$

$\therefore n = 33$ (calculadora)

2) → Reemplazar n en Q_n y resolver inecuación.

Primero hallemos el patron para $f(x) = Ln(x)$

$$f'(x) = \frac{1}{x} = 1 \cdot x^{-1} \Rightarrow f'(1) = 1$$

$$f''(x) = -1 \cdot x^{-2} \Rightarrow f''(1) = -1$$

$$f'''(x) = 2 \cdot x^{-3} \Rightarrow f'''(1) = 2$$

$$f^{(4)}(x) = -6x^{-4} \Rightarrow f^{(4)}(1) = -6$$

$$\therefore f^{(n)}(x) = (-1)^{n+1} \cdot (n-1)! \cdot x^{-n}$$

$$\therefore \sum_{n=0}^{\infty} \frac{(-1)^{n+1} \cdot (n-1)! \cdot x^{-n}}{n!} \cdot (x-1)^n$$

Por el enunciado, sabemos que $n = 1000$

$$\cancel{f(x) = T_{1000} + Q_{1000}}$$

$$\cancel{z}$$

$$|Q_{1000}(z)| < z \Rightarrow \frac{(-1)^{n+2} \cdot (n+1-1)! \, t^{-(n+1)}}{(n+1)!} \cdot (2-1)^{n+1}$$