8) b) $maxLongEq2.e.xs = \langle Max\ bs, cs : xs = bs ++ cs \wedge iga.e.bs : \#bs \rangle$

Caso base: $xs := []$

$$maxLongEq2.e.[]$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle Max\ bs, cs : [] = bs ++ cs \wedge iga.e.bs : \#bs \rangle$$
$$\equiv \{\text{Ya estoy podrido así que: magia negra y metafísica}\}$$
$$0$$

Caso inductivo: $xs := (x : xs)$

$HI = maxLongEq2.e.xs = \langle Max\ bs, cs : xs = bs ++ cs \wedge iga.e.bs : \#bs \rangle$

$$maxLongEq2.e.(x : xs)$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle Max\ bs, cs : (x : xs) = bs ++ cs \wedge iga.e.bs : \#bs \rangle$$
$$\equiv \{\text{Magia negra y metafisica}\}$$
$$\langle Max\ bs, cs : (x : xs) = bs ++ cs \wedge iga.e.bs \wedge bs = [] : \#bs \rangle max maxLongEq2.e.xs$$
$$\equiv \{\text{Eliminacion de variable bs}\}$$
$$\langle Max\ cs : (x : xs) = [] ++ cs \wedge iga.e.bs : \#[] \rangle max maxLongEq2.e.xs$$
$$\equiv \{\text{Termino constante y def de \#}\}$$
$$0\ max\ maxLongEq2.e.xs$$

Resultado final:

```
maxLongEq.e.[] = 0
maxLongEq.e.(x:xs) = maxLongEq2.e.(x:xs) max maxLongEq.e.xs
  where
    maxLongEq2.e.[] = 0
    maxLongEq2.e.(x:xs) = 0 max maxLongEq2.e.xs
```

CORREGIR

9)

a)

Tipo: g :: [Int] -> Int

Descripcion: Funcion que devuelve el mayor numero resultante de sumar dos elementos de una lista.

b) $h.xs = \langle N\ k : 0 \leq k < \#xs : \langle \forall i : 0 \leq i < k : xs\ !!\ i < xs\ !!\ k \rangle \rangle$

Definicion alternativa:

$$h.xs = \langle N\ as, bs : xs = as +\!\!+ bs : \langle \forall i : 0 \leq i < \#as - 1 : as\ !!\ i < as\ !!\ (\#as - 1) \rangle \rangle$$

Tipo: h :: (Ord a) => [a] -> Int

Descripcion: Cantidad de prefijos de xs que cumplen que todos los elementos que no estén en el ultimo lugar del prefijo son menores al elemento en el ultimo lugar del prefijo.

Otra descripcion: Cantidad de prefijos de xs que cumplen que el ultimo elemento del prefijo es el elemento más grande

c) $k.xs = \langle \forall i, j : 0 \leq i \wedge 0 \leq j \wedge i + j = \#xs - 1 : xs\ !!\ i = xs\ !!\ j \rangle$

Tipo: k:: (Eq a) => [a] -> Bool

Descripcion: La lista xs es un palindromo.

d) $l.xs = \langle Max\ p, q : 0 \leq p \leq q < \#xs \wedge \langle \forall i : p \leq i < q : xs\ !!\ i \geq 0 \rangle : q - p \rangle$

Definicion alternativa:

$$l.xs = \langle Max\ as, bs, cs : xs = as +\!\!+ bs +\!\!+ cs \wedge \langle \forall i : 0 \leq i < \#bs : bs\ !!\ i \geq 0 \rangle : \#bs \rangle$$

Tipo: l:: [Int] -> Int

Descripcion: El largo del mayor segmento de xs para el cual se cumpla que todos sus elementos son mayores o iguales a 0

Evaluaciones manuales:

b) Evaluacion manual: $xs = [1, 2, 3, 4]$

$$h.[1, 2, 3, 4]$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle N\ k : 0 \leq k < \#[1, 2, 3, 4] : \langle \forall i : 0 \leq i < k : [1, 2, 3, 4]\ !!\ i < [1, 2, 3, 4]\ !!\ k \rangle \rangle$$

$$\langle N\ k : 0 \leq k < 4 : \langle \forall i : 0 \leq i < k : [1, 2, 3, 4]\ !!\ i < [1, 2, 3, 4]\ !!\ k \rangle \rangle$$

$$\langle N\ k : k \in \{0, 1, 2, 3\} : \langle \forall i : 0 \leq i < k : [1, 2, 3, 4]\ !!\ i < [1, 2, 3, 4]\ !!\ k \rangle \rangle$$

$$\equiv \{\text{Evaluo termino en el rango}\}$$

$$\langle \forall i : 0 \leq i < 0 : [1,2,3,4] \;!!\; i < [1,2,3,4] \;!!\; 0 \rangle +$$

$$\langle \forall i : 0 \leq i < 1 : [1,2,3,4] \;!!\; i < [1,2,3,4] \;!!\; 1 \rangle +$$

$$\langle \forall i : 0 \leq i < 2 : [1,2,3,4] \;!!\; i < [1,2,3,4] \;!!\; 2 \rangle +$$

$$\langle \forall i : 0 \leq i < 3 : [1,2,3,4] \;!!\; i < [1,2,3,4] \;!!\; 3 \rangle +$$

$$\equiv \{\text{Evaluo indexacion}\}$$

$$\langle \forall i : 0 \leq i < 0 : [1,2,3,4] \;!!\; i < 1 \rangle +$$

$$\langle \forall i : 0 \leq i < 1 : [1,2,3,4] \;!!\; i < 2 \rangle +$$

$$\langle \forall i : 0 \leq i < 2 : [1,2,3,4] \;!!\; i < 3 \rangle +$$

$$\langle \forall i : 0 \leq i < 3 : [1,2,3,4] \;!!\; i < 4 \rangle +$$

$$\equiv \{\text{Evaluo Rangos}\}$$

$$\langle \forall i : False : [1,2,3,4] \;!!\; i < 1 \rangle +$$

$$\langle \forall i : i = 0 : [1,2,3,4] \;!!\; i < 2 \rangle +$$

$$\langle \forall i : i \in \{0,1\} : [1,2,3,4] \;!!\; i < 3 \rangle +$$

$$\langle \forall i : i \in \{0,1,2\} : [1,2,3,4] \;!!\; i < 4 \rangle +$$

$$\equiv \{\text{Evaluo rango en termino}\}$$

$$True +$$

$$([1,2,3,4] \;!!\; 0 < 2) +$$

$$([1,2,3,4] \;!!\; 0 < 3) \wedge ([1,2,3,4] \;!!\; 1 < 3)$$

$$([1,2,3,4] \;!!\; 0 < 4) \wedge ([1,2,3,4] \;!!\; 1 < 4) \wedge ([1,2,3,4] \;!!\; 2 < 4)$$

$$\equiv \{\text{Evaluo indexaciones}\}$$

$$True +$$

$$(1 < 2) +$$

$$(1 < 3) \wedge (2 < 3)$$

$$(1 < 4) \wedge (2 < 4) \wedge (3 < 4)$$

$$\equiv \{\text{Logica}\}$$

$$True +$$

$$True +$$

$$True \wedge True$$

$$True \wedge True \wedge True$$

$$\equiv \{\text{Logica}\}$$

$$True + True + True + True$$
$$\equiv \{\text{Cuantificador de conteo}\}$$
$$1 + 1 + 1 + 1$$
$$4$$

$$\langle \forall i : 0 \leq i < k : xs \ !! \ i < xs \ !! \ k \rangle$$
$$\equiv \{\text{k:=3, xs:=[2,4,1,6]}\}$$
$$\langle \forall i : 0 \leq i < 3 : [2,4,1,6] \ !! \ i < xs \ !! \ 3 \rangle$$
$$\langle \forall i : i \in \{0,1,2\} : [2,4,1,6] \ !! \ i < 6 \rangle$$
$$2 < 6 \wedge 4 < 6 \wedge 1 < 6$$
$$True \wedge True \wedge True$$
$$True$$

c) Evaluacion manual: $xs := [1,0,0,1]$

$$k.[1,0,0,1]$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle \forall i,j : 0 \leq i \wedge 0 \leq j \wedge i + j = \#[1,0,0,1] - 1 : [1,0,0,1] \ !! \ i = [1,0,0,1] \ !! \ j \rangle$$
$$\langle \forall i,j : 0 \leq i \wedge 0 \leq j \wedge i + j = 3 : [1,0,0,1] \ !! \ i = [1,0,0,1] \ !! \ j \rangle$$
$$\equiv \{\text{Evaluo rango}\}$$
$$\langle \forall i,j : i,j \in \{(0,3),(1,2),(2,1),(3,0)\} : [1,0,0,1] \ !! \ i = [1,0,0,1] \ !! \ j \rangle$$
$$([1,0,0,1] \ !! \ 0 = [1,0,0,1] \ !! \ 3) \wedge ([1,0,0,1] \ !! \ 1 = [1,0,0,1] \ !! \ 2) \wedge ([1,0,0,1] \ !! \ 2 = [1,0,0,1] \ !! \ 1) \wedge ([1,0,0,1] \ !! \ 3 = [$$
$$\equiv \{\text{Evaluo indexacion}\}$$
$$(1 = 1) \wedge (0 = 0) \wedge (0 = 0) \wedge (1 = 1)$$
$$True \wedge True \wedge True \wedge True$$
$$True$$

10)

$g.xs = \langle Max \ p,q : 0 \leq p < q < \#xs : xs \ !! \ p + xs \ !! \ q \rangle$

Caso base: $xs := []$

$$g.[]$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle Max \ p,q : 0 \leq p < q < \#[] : [] \ !! \ p + [] \ !! \ q \rangle$$
$$\equiv \{\text{Def de } \#\}$$

$$\langle Max\ p,q : 0 \leq p < q < 0 : [] \ !!\ p + [] \ !!\ q \rangle$$
$$\equiv \{\text{Evaluo Rango}\}$$
$$\langle Max\ p,q : False : [] \ !!\ p + [] \ !!\ q \rangle$$
$$\equiv \{\text{Rango vacio}\}$$
$$-\infty$$

Caso inductivo: $xs := (x : xs)$

$HI = g.xs = \langle Max\ p,q : 0 \leq p < q < \#xs : xs \ !!\ p + xs \ !!\ q \rangle$

$$g.(x : xs)$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle Max\ p,q : 0 \leq p < q < \#(x : xs) : (x : xs) \ !!\ p + (x : xs) \ !!\ q \rangle$$
$$\equiv \{\text{Def de } \#\}$$
$$\langle Max\ p,q : 0 \leq p < q < \#xs + 1 : (x : xs) \ !!\ p + (x : xs) \ !!\ q \rangle$$
$$\equiv \{\text{Aritmetica}\}$$
$$\langle Max\ p,q : (p = 0 \wedge p < q < \#xs+1) \vee (1 \leq p < q < \#xs+1) : (x : xs) \ !!\ p + (x : xs) \ !!\ q \rangle$$
$$\equiv \{\text{Particion de rango}\}$$
$$\langle Max\ p,q : (p = 0 \wedge p < q < \#xs + 1) : (x : xs) \ !!\ p + (x : xs) \ !!\ q \rangle\ max$$
$$\langle Max\ p,q : (1 \leq p < q < \#xs + 1) : (x : xs) \ !!\ p + (x : xs) \ !!\ q \rangle$$
$$\equiv \{\text{Llamemos X a la primer expresion cuantificada}\}$$
$$X\ max\ \langle Max\ p,q : (1 \leq p < q < \#xs + 1) : (x : xs) \ !!\ p + (x : xs) \ !!\ q \rangle$$
$$\equiv \{\text{Cambio de variable p->p+1, q->q+1}\}$$
$$X\ max\ \langle Max\ p,q : (1 \leq p+1 < q+1 < \#xs+1) : (x : xs) \ !!\ p+1 + (x : xs) \ !!\ q+1 \rangle$$
$$\equiv \{\text{Aritmetica y def de !!}\}$$
$$X\ max\ \langle Max\ p,q : 0 \leq p < q < \#xs : xs \ !!\ p + xs \ !!\ q \rangle$$
$$\equiv \{\text{HI}\}$$
$$X\ max\ g.xs$$
$$\equiv \{\text{Ahora reemplacemos X por la expresion original}\}$$
$$\langle Max\ p,q : (p = 0 \wedge p < q < \#xs + 1) : (x : xs) \ !!\ p + (x : xs) \ !!\ q \rangle\ max\ g.xs$$
$$\equiv \{\text{Eliminacion de variable}\}$$
$$\langle Max\ q : 0 < q < \#xs + 1 : (x : xs) \ !!\ 0 + (x : xs) \ !!\ q \rangle\ max\ g.xs$$
$$\equiv \{\text{Def de !! y aritmetica}\}$$

$$\langle Max\ q : 1 \leq q < \#xs + 1 : x + (x : xs)\ !!\ q\rangle\ max\ g.xs$$
$$\equiv \{\text{Cambio de variable q->q+1, aritmetica y def de !!}\}$$
$$\langle Max\ q : 0 \leq q < \#xs : x + xs\ !!\ q\rangle\ max\ g.xs$$
$$\equiv \{\text{Distributividad}\}$$
$$(\langle Max\ q : 0 \leq q < \#xs : xs\ !!\ q\rangle + x)\ max\ g.xs$$
$$\equiv \{\text{Modularizamos}\}$$
$$maxLista.xs + x\ max\ g.xs$$

Resultado parcial:

```
g.[] = -infinity
g.(x:xs) = (maxLista.xs + x) `max` g.xs
```

Ahora derivemos maxLista:

$maxLista.xs = \langle Max\ q : 0 \leq q < \#xs : xs\ !!\ q\rangle$

Caso base: $xs := []$

$$maxLista.[]$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle Max\ q : 0 \leq q < \#[] : []\ !!\ q\rangle$$
$$\equiv \{\text{Def de \#, evaluo rango, rango vacio}\}$$
$$-\infty$$

Caso inductivo: $xs := (x : xs)$

$HI = maxLista.xs = \langle Max\ q : 0 \leq q < \#xs : xs\ !!\ q\rangle$

$$maxLista.(x : xs)$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle Max\ q : 0 \leq q < \#(x : xs) : (x : xs)\ !!\ q\rangle$$
$$\equiv \{\text{Def de \#, aritmetica}\}$$
$$\langle Max\ q : q = 0 \vee 1 \leq q < \#xs + 1 : (x : xs)\ !!\ q\rangle$$
$$\equiv \{\text{Particion de rango, rango unitario}\}$$
$$(x : xs)\ !!\ 0\ max\ \langle Max\ q : 1 \leq q < \#xs + 1 : (x : xs)\ !!\ q\rangle$$
$$\equiv \{\text{Cambio de variable q->q+1, aritmetica, def de !!}\}$$
$$x\ max\ \langle Max\ q : 0 \leq q < \#xs : xs\ !!\ q\rangle$$
$$\equiv \{\text{HI}\}$$
$$x\ max\ maxLista.xs$$

Resultado final de la derivacion:

```
g.[] = -infinity
g.(x:xs) = (maxLista.xs + x) `max` g.xs
  where
    maxLista.[] = -infinity
    maxLista.(x:xs) = x max maxLista.xs
```

11)

$f.xs.ys = \langle Min\ i, j : 0 \leq i < \#xs \wedge 0 \leq j < \#ys : |xs\ !!\ i - ys\ !!\ j| \rangle$

Caso base: i) $xs := []$

$$
\begin{gathered}
f.[].ys \\
\equiv \{\text{Especificacion}\} \\
\langle Min\ i, j : 0 \leq i < \#[] \wedge 0 \leq j < \#ys : |[]\ !!\ i - ys\ !!\ j| \rangle \\
\equiv \{\text{Def de \#, evaluo rango}\} \\
\langle Min\ i, j : False \wedge 0 \leq j < \#ys : |[]\ !!\ i - ys\ !!\ j| \rangle \\
\equiv \{\text{Elemento absorbente de la conjuncion}\} \\
\langle Min\ i, j : False : |[]\ !!\ i - ys\ !!\ j| \rangle \\
\equiv \{\text{Rango vacio}\} \\
+\infty
\end{gathered}
$$

ii) $ys := []$

$$
\begin{gathered}
f.xs.[] \\
\equiv \{\text{Especificacion}\} \\
\langle Min\ i, j : 0 \leq i < \#xs \wedge 0 \leq j < \#[] : |xs\ !!\ i - []\ !!\ j| \rangle \\
\equiv \{\text{Def de \#, evaluo rango}\} \\
\equiv \{\text{Elemento absorbente de la conjuncion}\} \\
\equiv \{\text{Rango vacio}\} \\
+\infty
\end{gathered}
$$

Caso inductivo: $xs := (x : xs)$

$HI = f.xs.ys = \langle Min\ i, j : 0 \leq i < \#xs \wedge 0 \leq j < \#ys : |xs\ !!\ i - ys\ !!\ j| \rangle$

$$
\begin{gathered}
f.(x : xs).ys \\
\equiv \{\text{Especificacion}\} \\
\langle Min\ i, j : 0 \leq i < \#(x : xs) \wedge 0 \leq j < \#ys : |(x : xs)\ !!\ i - ys\ !!\ j| \rangle
\end{gathered}
$$

$$\equiv \{\text{Def de \#, aritmetica}\}$$

$$\langle Min\ i,j : (i = 0 \lor 1 \leq i < \#xs + 1) \land 0 \leq j < \#ys : |(x : xs)\ !!\ i - ys\ !!\ j|\rangle$$

$$\equiv \{\text{Distributividad conjuncion disyuncion}\}$$

$$\langle Min\, i,j : (i = 0 \land 0 \leq j < \#ys) \lor (1 \leq i < \#xs + 1 \land 0 \leq j < \#ys) : |(x : xs)\ !!\ i - ys\ !!\ j|\rangle$$

$$\equiv \{\text{Particion de rango}\}$$

$$\langle Min\ i,j : i = 0 \land 0 \leq j < \#ys : |(x : xs)\ !!\ i - ys\ !!\ j|\rangle\ min$$

$$\langle Min\ i,j : 1 \leq i < \#xs + 1 \land 0 \leq j < \#ys : |(x : xs)\ !!\ i - ys\ !!\ j|\rangle$$

$$\equiv \{\text{Llamemos X a la primer expresion cuantificada y trabajemos sobre la segunda expresion}\}$$

$$X\ min\ \langle Min\ i,j : 1 \leq i < \#xs + 1 \land 0 \leq j < \#ys : |(x : xs)\ !!\ i - ys\ !!\ j|\rangle$$

$$\equiv \{\text{Cambio de variable i->i+1, aritmetica y def de !!}\}$$

$$X\ min\ \langle Min\ i,j : 0 \leq i < \#xs \land 0 \leq j < \#ys : |xs\ !!\ i - ys\ !!\ j|\rangle$$

$$\equiv \{\text{HI}\}$$

$$X\ min\ f.xs.ys$$

$$\equiv \{\text{Ahora reemplacemos X por la expresion original}\}$$

$$\langle Min\ i,j : i = 0 \land 0 \leq j < \#ys : |(x : xs)\ !!\ i - ys\ !!\ j|\rangle\ min\ f.xs.ys$$

$$\equiv \{\text{Eliminacion de variable i}\}$$

$$\langle Min\ j : 0 \leq j < \#ys : |(x : xs)\ !!\ 0 - ys\ !!\ j|\rangle\ min\ f.xs.ys$$

$$\equiv \{\text{Def de !!}\}$$

$$\langle Min\ j : 0 \leq j < \#ys : |x - ys\ !!\ j|\rangle\ min\ f.xs.ys$$

$$\equiv \{\text{Modularizacion}\}$$

$$distancia.x.ys\ min\ f.xs.ys$$

Ahora derivemos distancia:

$distancia.x.ys = \langle Min\ j : 0 \leq j < \#ys : |x - ys\ !!\ j|$

Caso base: $ys = []$

$$distancia.x.[]$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle Min\ j : 0 \leq j < \#[] : |x - []\ !!\ j|$$

$$\equiv \{\text{Def de \#, evaluo rango, rango vacio}\}$$

$$+\infty$$

Caso inductivo:

$HI = distancia.x.ys = \langle Min\ j : 0 \leq j < \#ys : |x - ys\ !!\ j|$

$$
\begin{gathered}
distancia.x.(y:ys)\\
\equiv \{\text{Especificacion}\}\\
\langle Min\ j : 0 \leq j < \#(y:ys) : |x - (y:ys)\ !!\ j|\rangle\\
\equiv \{\text{Def de \#, aritmetica, particion de rango}\}\\
\equiv \{\text{Rango unitario}\}\\
|x - (y:ys)\ !!\ 0|\ min\ \langle Min\ j : 1 \leq j < \#ys + 1 : |x - (y:ys)\ !!\ j|\rangle\\
\equiv \{\text{Def de !!}\}\\
|x - y|\ min\ \langle Min\ j : 1 \leq j < \#ys + 1 : |x - (y:ys)\ !!\ j|\rangle\\
\equiv \{\text{Cambio de variable: i->i+1, aritmetica y def de !!}\}\\
|x - y|\ min\ \langle Min\ j : 0 \leq j < \#ys : |x - ys\ !!\ j|\rangle\\
\equiv \{\text{HI}\}\\
|x - y|\ min\ distancia.x.ys
\end{gathered}
$$

Resultado final de la derivacion:

```
f.[].ys = +infinity
f.(x:xs) = distancia.x.ys min f.xs.ys
  where
    distancia.x.[] = +infinity
    distancia.x.(y:ys) = |x-y| min distancia.x.ys
```

12)

$lex.xs.ys = \langle \exists as, bs, c, cs : xs = as +\!\!+ bs \wedge ys = as +\!\!+ (c : cs) : bs = [] \vee bs\ !!\ 0 \prec c\rangle$

Evaluacion manual: $xs := "fa", ys := "fase"$

$$lex."fa"."fase"$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle \exists as, bs, c, cs : "fa" = as ++ bs \wedge "fase" = as ++ (c : cs) : bs = [] \vee bs \;!!\; 0 \prec c \rangle$$
$$\equiv \{\text{Evaluo rango}\}$$

bs	c	Termino	Evaluacion
[]	f	True	True
[]	a	True	True
[]	s	True	True
[]	e	True	True
[a]	f	$a \prec f$	True
[a]	a	$a \prec a$	False
[a]	s	$a \prec s$	True
[a]	e	$a \prec e$	True
[fa]	f	$f \prec f$	False
[fa]	a	$f \prec a$	False
[fa]	s	$f \prec s$	True
[fa]	e	$f \prec e$	False

$$bs \in \{[], a, fa\}, c \in \{f, a, s, e\}$$

Caso base:

$$lex.xs.ys$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle \exists as, bs, c, cs : xs = as +\!\!+\, bs \wedge ys = as +\!\!+\, (c : cs) : bs = [] \vee bs \;!!\; 0 \prec c \rangle$$

Caso inductivo: $HI = lex.xs.ys = \langle \exists as, bs, c, cs : xs = as +\!\!+\, bs \wedge ys = as +\!\!+\, (c : cs) : bs = [] \vee bs \;!!\; 0 \prec c \rangle$

$$lex.xs.ys$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle \exists as, bs, c, cs : xs = as +\!\!+\, bs \wedge ys = as +\!\!+\, (c : cs) : bs = [] \vee bs \;!!\; 0 \prec c \rangle$$