Lautaro Bachmann

**Ejercicio 1** *(3 pts.)*

(a) *(1.5 Pts.)* Dibuje y calcule el área de la región **encerrada** por los gráficos de las siguientes funciones: $f(x) = (x^2-1)^2$ y $g(x) = 1-x^2$.

Primero veamos las raíces de cada función:

$f(x) = (x^2-1)^2 = ((x-1).(x+1))^2 \Rightarrow$ las raíces de $f(x)$ son 1 y -1

$g(x) = (1-x^2) \Rightarrow 1-x^2 = 0 \Rightarrow 1 = x^2 \Rightarrow \sqrt{1} = x \Rightarrow \pm 1 = x \Rightarrow$ Sus raíces son 1 y -1

Ahora veamos en que punto intersecan con el eje y:

$f(0) = (0^2-1)^2 = (-1)^2 = 1$

$g(0) = (1-0^2) = 1$

Hagamos las tablas de valores:

| x | f(x) |
|---|---|
| $-\frac{3}{4}$ | $49/256$ |
| $-1/2$ | $9/16$ |
| $-1/4$ | $225/256$ |
| $1/4$ | $225/256$ |
| $1/2$ | $9/16$ |
| $3/4$ | $49/256$ |

$f(x) = (x^2-1)^2,$

$f\left(\frac{-3}{4}\right) = f\left(\frac{3}{4}\right) = \left(\frac{(-3)^2}{4^2}-1\right)^2 = \left(\frac{9}{16}-1\right)^2 = \left(\frac{9-16}{16}\right)^2 = \left(\frac{-7}{16}\right)^2 = \frac{49}{256}$

$f\left(\frac{-1}{2}\right) = f\left(\frac{1}{2}\right) = \left(\frac{(-1)^2}{2^2}-1\right) = \left(\frac{1}{4}-1\right)^2 = \frac{(-3)^2}{4^2} = \frac{9}{16}$

$f\left(\frac{-1}{4}\right) = f\left(\frac{1}{4}\right) = \left(\frac{(-1)^2}{4^2}-1\right)^2 = \left(\frac{1}{16}-1\right)^2 = \left(\frac{-15}{16}\right)^2 = \frac{225}{256}$

$$\begin{array}{r} 16 \\ \times 16 \\ \hline 96 \\ 16\phantom{0} \\ \hline 256 \end{array} \qquad \begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ 15\phantom{0} \\ \hline 225 \end{array}$$

| x | g(x) |
|---|---|
| $-\frac{3}{4}$ | $7/16$ |
| $-1/2$ | $3/4$ |
| $-1/4$ | $15/16$ |
| $1/4$ | $15/16$ |
| $1/2$ | $3/4$ |
| $3/4$ | $7/6$ |

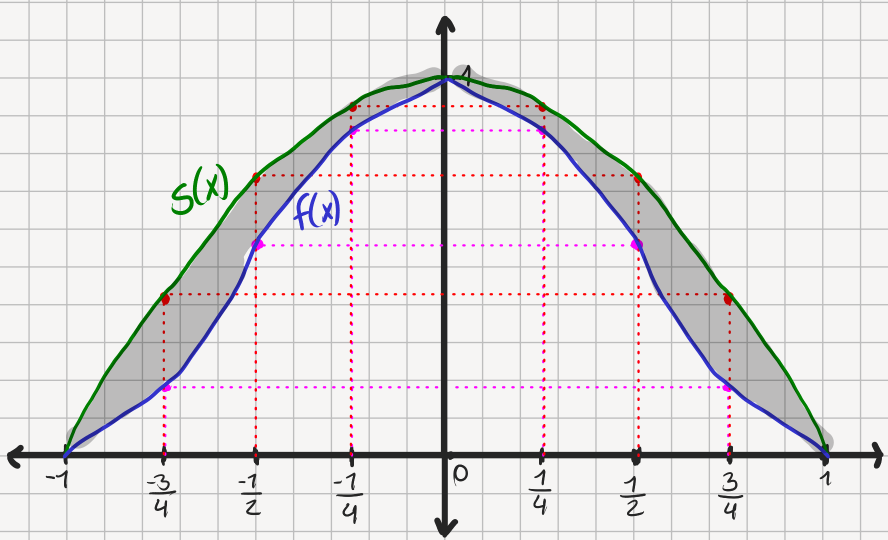
$g(x) = 1-x^2$

$g\left(\frac{-3}{4}\right) = g\left(\frac{3}{4}\right) = 1-\frac{(-3)^2}{4^2} = 1-\frac{9}{16} = \frac{7}{16}$

$g\left(\frac{-1}{2}\right) = g\left(\frac{1}{2}\right) = 1-\frac{(-1)^2}{2^2} = 1-\frac{1}{4} = \frac{3}{4}$

$g\left(\frac{-1}{4}\right) = g\left(\frac{1}{4}\right) = 1-\frac{(-1)^2}{4^2} = 1-\frac{1}{16} = \frac{15}{16}$

Ahora grafiquemos:




∴ teniendo en cuenta el gráfico sabemos que para hallar el área sombreada tenemos que calcular $\int_{-1}^{1} g(x)\,dx - \int_{-1}^{1} f(x)\,dx$, ya que $g(x) \geq f(x)$ en el intervalo $[-1, 1]$

$$\int_{-1}^{1} g(x)\,dx - \int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} 1-x^2\,dx - \int_{-1}^{1} (x^2-1)^2\,dx$$

$$= \frac{4}{3} - \frac{16}{15} = \frac{20-16}{15} = \frac{4}{15}$$

$\therefore$ el area entre $f(x)$ y $g(x)$ es de $\frac{4}{15}$

$$\int_{-1}^{1} 1-x^2\,dx = \int_{-1}^{1} 1\,dx - \int_{-1}^{1} x^2\,dx = x\Big|_{-1}^{1} - \frac{x^3}{3}\Big|_{-1}^{1}$$

$$= 1-(-1) - \frac{1}{3} - \frac{(-1)}{3} = 2 - \frac{2}{3} = \frac{4}{3}$$

$$\int_{-1}^{1} (x^2-1)^2\,dx = \int_{-1}^{1} x^4 - 2x^2 + 1\,dx$$

$$= \int_{-1}^{1} x^4\,dx - \int_{-1}^{1} 2x^2\,dx + \int_{-1}^{1} 1\,dx$$

$$= \frac{x^5}{5}\Big|_{-1}^{1} - 2.\left(\frac{x^3}{3}\Big|_{-1}^{1}\right) + x\Big|_{-1}^{1}$$

$$= \frac{1}{5} - \frac{(-1)}{5} - 2.\left(\frac{1}{3} - \frac{(-1)}{3}\right) + 1-(-1)$$

$$= \frac{2}{5} - 2.\frac{2}{3} + 2 = \frac{2}{5} - \frac{4}{3} + \frac{2}{1} = \frac{6-20+30}{15}$$

$$= \frac{16}{15}$$

(b) *(1.5 Pts.)* Calcule la siguiente integral indefinida $\displaystyle\int \frac{2+3x+x^2}{x(x^2+1)}\,dx.$

$$\frac{x^2+3x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)} = \frac{A(x^2+1) + B.x^2 + Cx}{x(x^2+1)} = \frac{Ax^2 + A + Bx^2 + Cx}{x(x^2+1)}$$

$$= \frac{(A+B).x^2 + A + Cx}{x(x^2+1)} \Rightarrow \frac{x^2+3x+2}{x(x^2+1)} = \frac{(A+B).x^2 + A + Cx}{x(x^2+1)}$$

$$\Rightarrow x^2+3x+2 = (A+B).x^2 + A + Cx$$