1) Dar las primitivas de las siguientes funciones:

a) $g(x) = x^3 - 5x$
b) $g(x) = e^{0.3x}$
c) $g(x) = \operatorname{sen}(2x)$
d) $g(x) = 2x\cos(x^2)$
e) $g(x) = x^{3/2}$
f) $g(x) = \sqrt{x+2}$

a) $\int x^3 - 5x\,dx = \int x^3\,dx - \int 5x\,dx = \boxed{\frac{x^4}{4} - 5.\frac{x^2}{2} + C}$

b) $\int e^{0.3x}\,dx = \int e^u \frac{du}{0{,}3} = 0{,}3.\int e^u\,du$

$= 0{,}3.\ e^u + c = \boxed{\frac{3}{10}.\left(\sqrt[10]{e^3}\right)^x + c}$

$u = 0{,}3x$
$du = 0{,}3\,dx$
$\frac{du}{0{,}3} = dx$

d) $\int 2x\cos(x^2)\,dx = \int \cos(x^2)\,2x\,dx$

$= \int \cos(u)\,du = \operatorname{sen}(u) + c = \boxed{\operatorname{sen}(x^2) + c}$

$u = x^2$
$du = 2x\,dx$

f) $\int \sqrt{x+2}\,dx = \int \sqrt{u}\,du = \int u^{\frac{1}{2}}\,du$

$= \frac{u^{3/2}}{\frac{3}{2}} + c = u^{3/2}.\frac{2}{3} + c = \frac{\sqrt{u^3}.2}{3} + c$

$\boxed{= \frac{\sqrt{(x+2)^3}.2}{3} + c}$

$u = x+2$
$du = 1\,dx$
$du = dx$

$\frac{1}{2} + \frac{1}{1} = \frac{1+2}{2} = \frac{3}{2}$

2) Encontrar la primitiva F de $f(x) = \frac{3}{x}$ tal que $F(1) = 5$.

$\int \frac{3}{x}\,dx = \int 3.\frac{1}{x}\,dx = 3.\int \frac{1}{x}\,dx = 3.\ln(x) + c$

$F(1) = 5 \Rightarrow 5 = 3.\ln(1) + c \Rightarrow 5 = 3.0 + c \Rightarrow 5 = c$

$\therefore$ $\boxed{\text{Sol: } 3.\ln(x) + 5}$

3) Encontrar la primitiva F de $f(x) = x + \cos(x)$ que pasa por el punto $(0, 4)$.

$\int x + \cos(x)\,dx = \int x\,dx + \int \cos(x)\,dx = \frac{x^2}{2} + \operatorname{sen}(x) + c$

$F(0) = 4 \Rightarrow 4 = \frac{0^2}{2} + \operatorname{sen}(0) + c \Rightarrow 4 = 0 + 0 + c \Rightarrow 4 = c$

$\therefore$ Sol: $\boxed{F(x) = \frac{x^2}{2} + \operatorname{sen}(x) + 4}$

4) Calcular las derivadas de las siguientes funciones:

a) $f(x) = (33-2x)^{\frac{4}{3}}$
b) $f(x) = e^{2x}$
c) $f(x) = 2^x$
d) $f(x) = \ln(7-x)$
e) $f(x) = \ln(x^2+3x+4)$
f) $f(x) = \ln(e^x + e^{-x})$
g) $f(x) = \ln(\cos(x) + \operatorname{sen}(x))$
h) $f(x) = \dfrac{\cos(x)}{\operatorname{sen}(x)}$

$\frac{4}{3} - \frac{1}{1} = \frac{4-3}{3} = \frac{1}{3}$

a) $g(x) = 33-2x \Rightarrow g'(x) = 0-2 = -2$

$h(x) = x^{4/3} \Rightarrow h'(x) = \frac{4}{3} x^{\frac{1}{3}}$

$f(x) = h(g(x)) \Rightarrow f'(x) = h'(g(x)) \cdot g'(x) = h'(33-2x) \cdot -2$

$= \frac{4}{3} \cdot (33-2x)^{\frac{1}{3}} \cdot -2 = \boxed{\dfrac{-8 \cdot \sqrt[3]{33-2x}}{3}}$ ✓

b) $g(x) = e^x \Rightarrow g'(x) = e^x$

$h(x) = 2x \Rightarrow h'(x) = 2$

$f(x) = g(h(x)) \Rightarrow f'(x) = g'(h(x)) \cdot h'(x) = \boxed{e^{2x} \cdot 2}$

c) $f(x) = 2^x \Rightarrow \boxed{f'(x) = \ln(2) \cdot 2^x}$

h) $f(x) = \cot(x) \Rightarrow \boxed{f'(x) = -\csc^2(x)}$

5) Calcular las siguientes integrales:

a) $\int e^{2x}\,dx$
b) $\int 2^x\,dx$
c) $\int \sqrt[3]{33-2x}\,dx$
d) $\int \dfrac{dx}{7-x}$
e) $\int \dfrac{2x+3}{x^2+3x+4}\,dx$
f) $\int \dfrac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$
g) $\int \dfrac{\cos(x) - \operatorname{sen}(x)}{\cos(x) + \operatorname{sen}(x)}\,dx$
h) $\int \dfrac{1}{\operatorname{sen}^2(x)}\,dx$

a) $\int e^{2x}\,dx = \int e^u \frac{du}{2} = \frac{1}{2}\int e^u\,du$

$\left| \begin{array}{l} u = 2x \\ du = 2\,dx \\ \frac{du}{2} = dx \end{array} \right.$

$= \frac{1}{2} e^u + c = \boxed{\dfrac{e^{2x}}{2} + c}$

b) $\int 2^x\,dx = \boxed{\dfrac{2^x}{\ln 2} + c}$

$\left| \begin{array}{l} u = 33-2x \\ du = 0 - 2\,dx = -2\,dx \\ \frac{du}{-2} = dx \end{array} \right.$

$\frac{1}{3} + \frac{1}{1} = \frac{1+3}{3} = \frac{4}{3}$

c) $\int \sqrt[3]{33-2x}\,dx = \int \sqrt[3]{u}\,\frac{du}{-2} = \frac{-1}{2}\int u^{\frac{1}{3}}\,du$

$= \frac{-1}{2} \cdot \dfrac{u^{4/3}}{\frac{4}{3}} + c = \frac{-1}{2} \cdot \sqrt[3]{u^4} \cdot \frac{3}{4} + c = \boxed{\dfrac{-\sqrt[3]{(33-2x)^4} \cdot 3}{8}}$

d) $\displaystyle\int \frac{dx}{7-x}$

e) $\displaystyle\int \frac{2x+3}{x^2+3x+4}\,dx$

d) $\displaystyle\int \frac{1}{7-x}\,dx = \int \frac{1}{u}\,\frac{du}{-1} = -1\cdot\int \frac{1}{u}\,du$

$u = 7-x$
$du = 0-1\,dx = -1\,dx$
$\frac{du}{-1} = dx$

$-1\cdot \ln|u| + c = -\ln(7-x)+c$

e) $\displaystyle\int \frac{2x+3}{x^2+3x+4}\,dx = \int \frac{1}{u}\,du = \ln u + c$

$u = x^2+3x+4$
$du = 2x+3\,dx$

$= \ln\left(x^2+3x+4\right)+c$