d) $f(x) = -\sqrt{x^2 - 4x + 4}$

¿Es inyectiva?

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

$$f(x_1) = f(x_2)$$

$$-\sqrt{x_1^2 - 4x_1 + 4} = -\sqrt{x_2^2 - 4x_2 + 4}$$

$$\frac{-\sqrt{x_1^2 - 4x_1 + 4}}{-1} = \frac{\cancel{-1}\cdot\sqrt{x_2^2 - 4x_2 + 4}}{\cancel{-1}}$$

$$\sqrt{(x_1 - 2)^2} = \sqrt{(x_2 - 2)^2}$$

$$|x_1 - 2| \neq |x_2 - 2|$$

$$x_1 - 2 \vee -(x_1 - 2) \neq x_2 - 2 \vee -(x_2 - 2)$$

Al estar presente el valor absoluto existen varios casos posibles, por ende no siempre serán iguales, por lo tanto, la igualdad no se cumple

17. A partir de los valores conocidos del seno y del coseno de $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ y $\frac{\pi}{2}$, calcule en forma exacta las expresiones que se dan a continuación:

$sen(2t) = 2sen(t)cos(t)$

a) $\operatorname{sen}\frac{2\pi}{3} + \cos\frac{4\pi}{3} + \tan\frac{5\pi}{3}$ *b*) $\operatorname{sen}\frac{5\pi}{6} + \cos\frac{7\pi}{6} + \tan\frac{5\pi}{6}$

a) $sen\left(\frac{2\pi}{3}\right) = sen\left(2 \cdot \frac{\pi}{3}\right) = 2 sen\left(\frac{\pi}{3}\right) \cdot cos\left(\frac{\pi}{3}\right)$

$sen\left(\frac{2\pi}{3}\right) = \cancel{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{\cancel{2}}$

$$\boxed{sen\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}}$$

$cos\left(\frac{4\pi}{3}\right) = cos\left(2 \cdot \frac{2\pi}{3}\right) = cos^2\left(\frac{2\pi}{3}\right) - sen^2\left(\frac{2\pi}{3}\right)$

$\left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$\frac{1}{4} - \frac{3}{4}$

$-\frac{\cancel{2}^{\,1}}{\cancel{4}_{\,2}}$

$-\frac{1}{2}$

$$\boxed{cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}}$$

CA

$cos^2(x) + sen^2(x) = 1$

$cos^2\left(\frac{2\pi}{3}\right) + sen^2\left(\frac{2\pi}{3}\right) = 1$

$cos^2\left(\frac{2\pi}{3}\right) + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$

$cos^2\left(\frac{2\pi}{3}\right) + \frac{3}{4} = 1$

$cos^2\left(\frac{2\pi}{3}\right) = 1 - \frac{3}{4}$ | $\frac{4-3}{4} = \frac{1}{4}$

$cos\left(\frac{2\pi}{3}\right) = \frac{\sqrt{1}}{\sqrt{4}}$

$cos\left(\frac{2\pi}{3}\right) = \frac{1}{2}$

$$\tan\left(\frac{5\pi}{3}\right) = \frac{\operatorname{sen}\left(\frac{5\pi}{3}\right)}{\cos\left(\frac{5\pi}{3}\right)}$$

$$\tan\left(\frac{5\pi}{3}\right) = \frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}}$$

$$\tan\left(\frac{5\pi}{3}\right) = \frac{-\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1}$$

$$\tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$$

$$\operatorname{sen}\frac{2\pi}{3} + \cos\frac{4\pi}{3} + \tan\frac{5\pi}{3}$$

$$\frac{\sqrt{3}}{2} - \frac{1}{2} - \sqrt{3}$$

$$\frac{\sqrt{3}-1}{2} - \frac{\sqrt{3}}{1}$$

$$\frac{1.\sqrt{3}-1-2\sqrt{3}}{2}$$

$$\boxed{\frac{-\sqrt{3}-1}{2}}$$

$$\operatorname{sen}\left(\frac{5\pi}{3}\right) = \operatorname{sen}\left(\pi + \frac{2\pi}{3}\right)$$

$$\Rightarrow \operatorname{sen}(\pi).\cos\left(\frac{2\pi}{3}\right) + \operatorname{sen}\left(\frac{2\pi}{3}\right).\cos(\pi)$$

$$0 \cdot \cos\left(\frac{2\pi}{3}\right) + \frac{\sqrt{3}}{2} \cdot -1$$

$$\boxed{\operatorname{sen}\left(\frac{5\pi}{3}\right) = \frac{-\sqrt{3}}{2}}$$

$$\cos^2(x) + \operatorname{sen}^2(x) = 1$$

$$\cos^2\left(\frac{5\pi}{3}\right) + \operatorname{sen}^2\left(\frac{5\pi}{3}\right) = 1$$

$$\cos^2\left(\frac{5\pi}{3}\right) + \left(\frac{-\sqrt{3}}{2}\right)^2 = 1$$

$$\cos^2\left(\frac{5\pi}{3}\right) + \frac{3}{4} = 1$$

$$\cos^2\left(\frac{5\pi}{3}\right) = 1 - \frac{3}{4}$$

$$\cos\left(\frac{5\pi}{3}\right) = \frac{\sqrt{1}}{\sqrt{4}}$$

$$\boxed{\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}}$$

$e)\ \operatorname{sen} x = \cos(2x)$

$$\operatorname{Sen}(x) = \cos(2x) = \underline{\cos^2(x)} - \operatorname{Sen}^2(x)$$

$$\cos^2(x) + \operatorname{sen}^2(x) = 1 \Rightarrow \cos^2(x) = \underline{1 - \operatorname{Sen}^2(x)}$$

$$\operatorname{Sen}(x) = \cos(2x) = 1 - \operatorname{sen}^2(x) - \operatorname{sen}^2(x)$$

$$\operatorname{Sen}(x) = 1 - 2\operatorname{sen}^2(x)$$

$$+2\operatorname{sen}^2(x) + \operatorname{sen}(x) - 1 = 0$$

Cambio de variable $z = \operatorname{Sen}(x)$

$$2z^2 + z - 1 = 0$$

$a=2,\ b=1,\ c=-1$

$\Delta = b^2 - 4ac$

$1 - 4.2.-1$

$1 + 8$

$\Delta = 9$

$$z_1, z_2 = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow z_1, z_2 = \frac{-1 \pm \sqrt{9}}{2.2}$$

$$= z_1, z_2 = \frac{-1 \pm 3}{4}$$

$$\frac{-1+3}{4} = \frac{\cancel{2}^{1}}{\cancel{4}_{2}} = \frac{1}{2}$$

$$\frac{-1-3}{4} = \frac{-\cancel{4}^{1}}{\cancel{4}_{1}} = -1$$

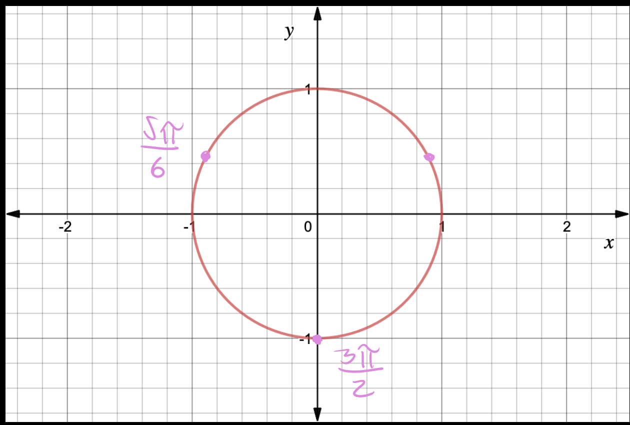
$$z = \frac{1}{2} \qquad\qquad z = -1$$

Cambio de variable

$z = \operatorname{sen}(x)$

$$\operatorname{Sen}(x) = \frac{1}{2} \qquad\qquad \operatorname{Sen}(x) = -1$$

$$x = \frac{\pi}{6} + n.2\pi \ \vee\ x = \frac{5\pi}{6} + n.2\pi \quad \vee \quad \frac{3\pi}{2}$$



$$\text{Solucion} = \left\{ x \in \mathbb{R}^2 \;\middle|\; x = \frac{\pi}{6} + n.2\pi \;\vee\; x = \frac{5\pi}{6} + n.2\pi \;\vee\; \frac{3\pi}{2} + n.2\pi \right\}$$