6. Calcule las siguientes integrales indefinidas utilizando integración por sustitución:

a) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

b) $\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx$

c) $\int \frac{\ln(x+1)}{(x+1)}\,dx$

d) $\int \frac{1}{x\ln x}\,dx$

e) $\int x\ e^{x^2}\ dx$

f) $\int e^{x}\,(1-\ e^{x})^{-1}\ dx$

g) $\int \operatorname{sen}^3 x\ dx$

a) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}\,dx \Rightarrow du \cdot 2 = \frac{1}{\sqrt{x}}\,dx$

$= \int e^{u} \cdot 2 \cdot du = 2 \cdot \int e^{u}\,du = 2 \cdot e^{u} + C$

$= 2 \cdot e^{\sqrt{x}} + C$ ✓

b) $\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx = \int \operatorname{sen}\sqrt{x} \cdot \frac{1}{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\cdot\sqrt{x}}\,dx \Rightarrow du \cdot 2 = \frac{1}{\sqrt{x}}\,dx$

$= \int \operatorname{sen}(u) \cdot 2 \cdot du = 2\int \operatorname{sen}(u)\,du$

$2 \cdot -\cos(u) + C = -2 \cdot \cos(\sqrt{x}) + C$ ✓

c) $\int \frac{\ln(x+1)}{(x+1)}\,dx = \int \ln(x+1) \cdot \frac{1}{x+1}\,dx$

$u = \ln(x+1)$

$du = \frac{1}{x+1} \cdot 1 = \frac{1}{x+1} \Rightarrow du = \frac{1}{x+1}\,dx$

$= \int u\,du = \frac{u^2}{2} + C = \frac{\ln(x+1)^2}{2} + C$ ✓

d) $\int \frac{1}{x\ln x}\,dx = \int \frac{1}{\ln(x)} \cdot \frac{1}{x}\,dx$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= \int \frac{1}{u} \cdot du = \ln(u) + C = \ln(\ln(x)) + C$ ✓

e) $\int x\ e^{x^2}\ dx = \int e^{x^2} \cdot x \cdot dx$

$u = x^2$

$du = 2x\,dx \Rightarrow \frac{du}{2} = x\,dx$

$= \int e^u \cdot \frac{du}{2} = \frac{1}{2}\int e^u du = \frac{1}{2} \cdot e^u + c$

$= \frac{e^{x^2}}{2} + c$ ✓

f) $\int e^x (1 - e^x)^{-1}\ dx = \int \frac{1}{1-e^x} \cdot e^x\,dx$

$u = 1 - e^x$

$du = -e^x\,dx \Rightarrow -1 \cdot du = e^x dx$

$= \int \frac{1}{u} \cdot -1 \cdot du = -1\int \frac{1}{u} du = -\ln(u)$

$= -\ln(1-e^x)$ ✓

g) $\int \text{sen}^3 x\ dx = \int \text{sen}^2(x) \cdot \text{sen}(x)\ dx$

$u = \cos(x)$

$du = -\text{sen}(x)\,dx \Rightarrow -1 \cdot du = \text{sen}(x) dx$

$\text{sen}^2(x) + \cos^2(x) = 1 \Rightarrow \text{sen}^2(x) = 1 - \cos^2(x)$

$= \int \text{sen}^2(x) \cdot -1\,du = -1\int \text{sen}^2(x)\,du$

$= -1\int 1 - (\cos(x))^2 du = -1\left(\int 1\,du - \int u^2 du\right) = -1\left(1u - \frac{u^3}{3}\right) + c$

$= -u + \frac{u^3}{3} + c = -\cos(x) + \frac{(\cos(x))^3}{3} + c$

✓

7. Calcule las siguientes integrales indefinidas, utilizando integración por partes:

a) $\int x\ \mathrm{e}^x\ \mathrm{d}x$ *d*) $\int x\ \ln(x-1)\ \mathrm{d}x$ *g*) $\int \cos^4 x\ \mathrm{d}x$

b) $\int (1-2x)\ \mathrm{e}^{-2x}\ \mathrm{d}x$ *e*) $\int \mathrm{e}^{-x}\ \mathrm{sen}\, 2x\ \mathrm{d}x$

c) $\int x^2 \cos x\ \mathrm{d}x$ *f*) $\int \cos^2 x\ \mathrm{d}x$

a) $\int \overset{u}{x}.\overset{v}{e^x}\, dx = x.e^x - \int e^x dx$

$u = x \qquad dv = e^x\, dx$

$du = 1dx \qquad v = e^x$

$= x.e^x - e^x + C$

b) $\int (1-2x)\ \mathrm{e}^{-2x}\ \mathrm{d}x$

$u = 1-2x \qquad dv = e^{-2x} dx$

$du = -2\, dx \qquad v = -\frac{1}{2} . e^{-2x}$

$= (1-2x)\left(-\frac{e^{-2x}}{2}\right) - \int \frac{-e^{-2x}}{2} -2.dx$

$\int e^{-2x} dx = \frac{1}{-2} . e^{-2x} + C$

$(1-2x)\left(-\frac{e^{-2x}}{2}\right) - \left(\frac{-1.\cancel{2}}{\cancel{2}} \int e^{-2x}\, dx\right)$

$= (1-2x)\left(-\frac{e^{-2x}}{2}\right) - \left(\frac{-e^{-2x}}{2}\right) \qquad = (1-2x)\left(-\frac{e^{-2x}}{2}\right) + \frac{e^{-2x}}{2}$

$= -\frac{e^{-2x}}{2}\left(-2x + \cancel{1} - \cancel{1}\right) = +\frac{\cancel{2}x.e^{-2x}}{\cancel{2}} = x.e^{-2x} = \frac{x}{e^{2x}} + C$

c) $\int x^2 \cos x\ \mathrm{d}x$

$u = x^2 \qquad dv = \cos(x)\, dx$

$du = 2x\, dx \qquad v = \mathrm{sen}(x)$

$= x^2 . \mathrm{sen}(x) - \int \mathrm{sen}(x).2x\, dx$

$= x^2.\mathrm{sen}(x) - \left(2x.-\cos(x) - \int -\cos.2dx\right)$

$u = 2x \qquad dv = \mathrm{sen}(x)$

$du = 2dx \qquad v = -\cos(x)$

$= x^2.\mathrm{sen}(x) - \left(-2x.\cos(x) - \left(-2.\int \cos(x)\, dx\right)\right.$

$= x^2.\mathrm{sen}(x) - \left(-2x.\cos(x) - \left(-2.\mathrm{sen}(x)\right)\right) = x^2.\mathrm{sen}(x) - \left(-2x.\cos(x) + 2.\mathrm{sen}(x)\right)$

$\downarrow$

$x^2 . sen(x) + 2x . cos(x) - 2 . sen(x)$

d) $\int x \ln(x-1)\ dx$

$u = x-1 \Rightarrow u+1 = x$

$du = dx$

$\int x . Ln(u)\ du = \int (u+1) Ln(u)\ du$

$u' = Ln(u)$ $\quad$ $dv = u+1\ dx$

$du' = \frac{1}{u} du$ $\quad$ $v = \frac{u^2}{2} + u$

$= Ln(u) . \left(\frac{u^2}{2} + u\right) - \int \left(\frac{u^2}{2} + u\right) . \frac{1}{u} du$

$= Ln(u) . \left(\frac{u^2}{2} + u\right) - \int \left(\frac{u^2}{2} + u\right) . \frac{1}{u} du$

$\int \frac{u^2 + 2 . u}{2u} du = \int \frac{\cancel{u}(u+2)}{2 . \cancel{u}} du = \frac{1}{2} \int u + 2\ du = \frac{1}{2}\left(\frac{u^2}{2} + 2u\right)^{+C} = \frac{u^2}{4} + \frac{\cancel{2}u}{\cancel{2}}$

$= \frac{u^2}{4} + u$

$\Rightarrow Ln(u) . \left(\frac{u^2}{2} + u\right) - \int \left(\frac{u^2}{2} + u\right) . \frac{1}{u} du \quad = Ln(u) . \left(\frac{u^2}{2} + u\right) - \left(\frac{u^2}{4} + u\right)$

$Ln(x-1) . \left(\frac{(x-1)^2}{2} + (x-1)\right) - \frac{(x-1)^2}{4} - (x-1)$

$= Ln(x-1) . \left(\frac{(x-1)^2}{2} + (x-1)\right) - \frac{(x-1)^2}{4} - x + 1$

e) $\int e^{-x} \operatorname{sen} 2x \, dx$

$t = 2x \Rightarrow x = \frac{t}{2}$

$dt = 2\,dx \Rightarrow \frac{dt}{2} = dx$

$= \int e^{-t/2} \cdot \operatorname{sen}(t) \, \frac{dt}{2}$

$= \frac{1}{2} \int e^{-t/2} \cdot \operatorname{Sen}(t) \, dt$

$u = \operatorname{sen}(t)$ $\quad dv = e^{-t/2}$

$du = \cos(t)\,dx$ $\quad v = \frac{-1}{2} e^{-t/2}$