3 )

$$prod.([2+3,3] \mathbin{+\!\!+} [1,0])$$
$$\equiv \{\text{Aritmetica}\}$$
$$prod.([5,3] \mathbin{+\!\!+} [1,0])$$
$$\equiv \{\text{def} \mathbin{+\!\!+} \}$$
$$prod.(5 : ([3] \mathbin{+\!\!+} [1,0]))$$
$$prod.(5 : 3 : ([] \mathbin{+\!\!+} [1,0]))$$
$$prod.(5 : 3 : [1,0])$$
$$prod.([5,3,1,0])$$
$$\equiv \{\text{Def prod}\}$$
$$5 * 3 * 1 * 0$$
$$\equiv \{\text{Elemento absorbente multiplicacion}\}$$
$$0$$

4 ) a )

$$\neg\langle \exists j : 0 \leq j < \#ns : \langle \exists i : 0 \leq i < \#ps : \neg((ps \mathbin{!!} i).(ns \mathbin{!!} j))\rangle\rangle$$
$$\equiv \{\text{DeMorgan}\}$$
$$\neg\neg\langle \forall j : 0 \leq j < \#ns : \neg\langle \exists i : 0 \leq i < \#ps : \neg((ps \mathbin{!!} i).(ns \mathbin{!!} j))\rangle\rangle$$
$$\equiv \{\text{DeMorgan}\}$$
$$\neg\neg\langle \forall j : 0 \leq j < \#ns : \neg\neg\langle \forall i : 0 \leq i < \#ps : \neg\neg((ps \mathbin{!!} i).(ns \mathbin{!!} j))\rangle\rangle$$
$$\equiv \{\text{Doble negacion}\}$$
$$\langle \forall j : 0 \leq j < \#ns : \langle \forall i : 0 \leq i < \#ps : ((ps \mathbin{!!} i).(ns \mathbin{!!} j))\rangle\rangle$$

5 )

$$f.x = \langle \exists y : 0 \leq y < x : x = y * (y+1)/2\rangle$$

Evaluacion manual:

$$f.5$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle \exists y : 0 \leq y < 5 : 5 = y * (y+1)/2\rangle$$
$$\equiv \{\text{Evaluo rango}\}$$
$$\langle \exists y : y \in \{0,1,2,3,4\} : 5 = y * (y+1)/2\rangle$$
$$\equiv \{\text{Evaluo rango en termino}\}$$

$$5 = 0{*}(0{+}1)/2 \vee 5 = 1{*}(1{+}1)/2 \vee 5 = 2{*}(2{+}1)/2 \vee 5 = 3{*}(3{+}1)/2 \vee 5 = 4{*}(4{+}1)/2$$

$$\equiv \{\text{Aritmetica}\}$$

$$5 = 0 \vee 5 = 2/2 \vee 5 = 6/2 \vee 5 = 12/2 \vee 5 = 20/2$$

$$\equiv \{\text{Aritmetica}\}$$

$$False \vee False \vee False \vee False \vee False$$

$$False$$

7 )

$f.xs = \langle \forall a, as, bs : xs = (a : as) +\!\!+ (a : bs) : as = bs \rangle$

xs = [1,2,1,7,7,1,3]

$$f.[1,2,1,7,7,1,3]$$

$$\equiv \{\text{Especificacion}\}$$

$$\langle \forall a, as, bs : [1,2,1,7,7,1,3] = (a : as) +\!\!+ (a : bs) : as = bs \rangle$$

| as | bs | a | a' |
|---|---|---|---|
| [1,2,1,7,7,1] | [3] | 1 | 3 |
| [1,2,1,7,7] | [1,3] | 1 | 1 |
| [1,2,1,7] | [7,1,3] | 1 | 7 |
| [1,2,1] | [7,7,1,3] | 1 | 7 |
| [1,2] | [1,7,7,1,3] | 1 | 1 |
| [1] | [2,1,7,7,1,3] | 1 | 2 |

10 )

$HI = hGen.xs.n.m = \langle N\ as, bs : xs = as +\!\!+ bs : n + sum.as = 2 * (\#as + m) \rangle$

$$\langle N\ as, bs : xs = as +\!\!+ bs : n + sum.(x : as) = 2 * (\#(x : as) + m) \rangle$$

$$\equiv \{\text{Def de sum y \#}\}$$

$$\langle N\ as, bs : xs = as +\!\!+ bs : n + x + sum.(as) = 2 * (\#as + 1 + m) \rangle$$

$$\equiv \{\text{HI}\}$$

$$hGen.xs.(n + x).(1 + m)$$

8 )

$quant.n = \langle N\ i : 0 \leq i \leq n : \neg \langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i \rangle \rangle$

Caso base: $n = 0$

$$quant.n$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle N\ i : 0 \leq i \leq n : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle$$
$$\equiv \{\text{Evaluo rango, rango vacio}\}$$
$$0$$

Caso inductivo: $n := (n+1)$

$HI = quant.n = \langle N\ i : 0 \leq i \leq n : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle$

$$quant.(n+1)$$
$$\equiv \{\text{Especificacion}\}$$
$$\langle N\ i : 0 \leq i \leq (n+1) : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle$$
$$\equiv \{\text{Aritmetica}\}$$
$$\langle N\ i : 0 \leq i \leq n \vee i = (n+1) : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle$$
$$\equiv \{\text{Particion de rango}\}$$
$$\langle N\ i : i = (n+1) : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle +$$
$$\langle N\ i : 0 \leq i \leq n : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle$$
$$\equiv \{\text{Rango unitario}\}$$
$$(\neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle \rightarrow 1$$
$$\square \neg\neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle \rightarrow 0$$
$$)$$
$$\langle N\ i : 0 \leq i \leq n : \neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle$$
$$\equiv \{\text{HI y doble negacion}\}$$
$$(\neg\langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle \rightarrow 1$$
$$\square \langle \exists x, y : 2 \leq x \leq i \wedge 2 \leq y \leq i : x * y = i\rangle\rangle \rightarrow 0$$
$$)$$
$$quant.n$$
$$\equiv \{\text{Modularizamos}\}$$
$$(\neg comp.k$$
$$\square comp.k$$
$$)$$
$$quant.n$$

9 )

$tieneLargo.xs = \langle \exists i : 0 \leq i < \#xs : \#xs = xs \;!!\; i\rangle$

Caso base: $xs := []$

$$\begin{aligned}
&tieneLargo.[] \\
\equiv &\{\text{Especificacion}\} \\
&\langle \exists i : 0 \leq i < \#[] : \#[] = [] \;!!\; i\rangle \\
\equiv &\{\text{Def de \#, evaluo rango, rango vacio}\} \\
&False
\end{aligned}$$

Caso inductivo: $xs := (x : xs)$ $HI = tieneLargo.xs = \langle \exists i : 0 \leq i < \#xs : \#xs = xs \;!!\; i\rangle$

$$\begin{aligned}
&tieneLargo.(x : xs) \\
\equiv &\{\text{Especificacion}\} \\
&\langle \exists i : 0 \leq i < \#(x : xs) : \#(x : xs) = (x : xs) \;!!\; i\rangle \\
\equiv &\{\text{Def de \#, aritmetica}\} \\
&\langle \exists i : i = 0 \vee 1 \leq i < \#xs + 1 : \#xs + 1 = (x : xs) \;!!\; i\rangle \\
\equiv &\{\text{Particion de rango y rango unitario}\} \\
&(\#xs + 1 = (x : xs) \;!!\; 0) \vee \langle \exists i : 1 \leq i < \#xs + 1 : \#xs + 1 = (x : xs) \;!!\; i\rangle \\
\equiv &\{\text{Def de !!}\} \\
&(\#xs + 1 = x) \vee \langle \exists i : 1 \leq i < \#xs + 1 : \#xs + 1 = (x : xs) \;!!\; i\rangle \\
\equiv &\{\text{Cambio de variable i->i+1, aritmetica}\} \\
&(\#xs + 1 = x) \vee \langle \exists i : 0 \leq i < \#xs : \#xs + 1 = (x : xs) \;!!\; i + 1\rangle \\
\equiv &\{\text{Def de !!}\} \\
&(\#xs + 1 = x) \vee \langle \exists i : 0 \leq i < \#xs : \#xs + 1 = xs \;!!\; i\rangle
\end{aligned}$$

No se puede aplicar HI por ende debemos generalizar:

$gTieneLargo.xs.n = \langle \exists i : 0 \leq i < \#xs : \#xs + n = xs \;!!\; i\rangle$

$tieneLargo.xs = gTieneLargo.xs.0$

$$\begin{aligned}
&gTieneLargo.xs.0 \\
\equiv &\{\text{Especificacion}\} \\
&\langle \exists i : 0 \leq i < \#xs : \#xs + 0 = xs \;!!\; i\rangle \\
\equiv &\{\text{Aritmetica}\} \\
&\langle \exists i : 0 \leq i < \#xs : \#xs = xs \;!!\; i\rangle \\
\equiv &\{\text{Especificacion de tieneLargo}\} \\
&tieneLargo.xs
\end{aligned}$$