6. Calcule las siguientes integrales indefinidas utilizando integración por sustitución:

a) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$ c) $\int \frac{\ln(x+1)}{(x+1)}\, dx$ e) $\int x\, e^{x^2}\, dx$

b) $\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\, dx$ d) $\int \frac{1}{x \ln x}\, dx$ f) $\int e^x (1 - e^x)^{-1}\, dx$

g) $\int \operatorname{sen}^3 x\, dx$

a) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = \int e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}\, dx$

$= \int e^{u} 2 \cdot du = 2 \cdot \int e^{u}\, du = 2 \cdot e^{u} + c$

$= 2 \cdot e^{\sqrt{x}} + c$

$u = \sqrt{x}$

$du = (x^{1/2})'\, dx = \frac{1}{2} \cdot x^{-1/2}\, dx = \frac{1}{2 \cdot \sqrt{x}}\, dx$

$\Rightarrow du = \frac{1}{2 \cdot \sqrt{x}}\, dx \Rightarrow 2 \cdot du = \frac{1}{\sqrt{x}}\, dx$

b) $\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\, dx$

$u = \sqrt{x}$

$du = \frac{1}{2 \cdot \sqrt{x}}\, dx \Rightarrow 2 \cdot du = \frac{1}{\sqrt{x}}\, dx$

$\int \operatorname{sen}(\sqrt{x}) \cdot \frac{1}{\sqrt{x}}\, dx = \int \operatorname{sen}(u)\, 2 \cdot du = 2 \int \operatorname{sen}(u)\, du = 2 \cdot -\cos(u) + c = -2 \cdot \cos(\sqrt{x}) + c$

c) $\int \frac{\ln(x+1)}{(x+1)}\, dx$

$u = \ln(x+1)$

$du = \frac{1}{x+1}\, dx$

$\int \ln(x+1) \cdot \frac{1}{x+1}\, dx = \int u\, du = \frac{u^2}{2} + c$

$= \frac{[\ln(x+1)]^2}{2} + c$

d) $\int \frac{1}{x \ln x}\, dx$

$u = \ln(x)$

$du = \frac{1}{x}\, dx$

$\int \frac{1}{x \ln(x)}\, dx = \int \frac{1}{\ln(x)} \cdot \frac{1}{x}\, dx = \int \frac{1}{u}\, du$

$= \ln(u) + c = \ln(\ln(x)) + c$

e) $\int x\, e^{x^2}\, dx = \int e^{x^2} . x\, dx = \int e^{u} \frac{du}{2}$

$u = x^2$

$du = 2x\, dx \Rightarrow \frac{du}{2} = x dx$

$= \frac{1}{2} \int e^{u} du = \frac{1}{2} . e^{u} + C = \frac{e^{x^2}}{2} + C$

f) $\int e^{x} (1 - e^{x})^{-1}\, dx = \int (1-e^{x})^{-1} . e^{x} dx$

$u = 1 - e^{x}$

$du = -e^{x} dx \Rightarrow -du = e^{x} dx$

$= \int u^{-1} . -1 . du = -1 \int u^{-1} du = -1 \int \frac{1}{u} du$

$= -\ln(u) + C = -\ln(1 - e^{x}) + C$

g) $\int \operatorname{sen}^3 x\, dx = \int (\operatorname{sen}(x))^3 dx$

$u = \cos(x)$

$du = -\operatorname{sen}(x)\, dx \Rightarrow \frac{du}{-\operatorname{sen}(x)} = dx$

$= \int \operatorname{sen}^2(x) . \operatorname{sen}(x)\, dx = \int \operatorname{sen}^2(x) . \cancel{\operatorname{sen}(x)} . \frac{1}{-\cancel{\operatorname{sen}(x)}} du$

$1 = \operatorname{sen}^2(x) + \cos^2(x) \Rightarrow 1 - \operatorname{sen}^2(x) = \cos^2(x)$

$\Rightarrow -\operatorname{sen}^2(x) = \cos^2(x) - 1$

$= \int -\operatorname{sen}^2(x) du = \int \cos^2(x) - 1\, dx$

$= \int u^2 du - \int 1 du = \int u^2 du - u = \frac{u^3}{3} - u + C$

$= \frac{[\cos(x)]^3}{3} - \cos(x) + C$