SEMESTER END EXAMINATION, APRIL-MAY, 2025

Course Name: - B.Tech (CSE, CE, ECE, EE, ME) **Semester:-** 2^{nd}

Paper Name: - Engineering Mathematics-II **Paper Code:-** NBS-202

Time - 3 Hrs + 20 minutes per hour extra time for V.I. & examinees with writer. **Max Marks-70**

Additional 30 Minutes for Mid-Test.

समय– 3 घण्टे + 20 मिनट प्रति घंटे अतिरिक्त–दृष्टिबाधित एवं सह लेखक परीक्षार्थियों के लिए। **अधिकतम अंक–70**

30 मिनट अतिरिक्त मिड–टेस्ट के लिए।

Instructions:

- The question paper consists of three sections namely A, B, C. All sections are compulsory.
- Section A- Each question carries **3 mark. All questions are compulsory.**
- Section B- Answer any **5 out of 7** given questions. Each question carries **7 marks.**
- Section C- Answer any **2 out of 3** given questions. Each question carries **10 marks.**
- Section D- Each question carries **02 mark**. All questions are compulsory.

निर्देश:

- प्रश्न पत्र में तीन खण्ड अ, ब, व स हैं। सभी खण्ड अनिवार्य हैं।
- खण्ड–अ में प्रत्येक प्रश्न तीन अंक का है। **सभी प्रश्न अनिवार्य हैं।**
- खण्ड–ब में **सात प्रश्नों में से किन्हीं पाँच** प्रश्नों के उत्तर दें। प्रत्येक प्रश्न **सात अंक** का है।
- खण्ड–स में **तीन प्रश्नों में से किन्हीं दो** प्रश्नों के उत्तर दें। प्रत्येक प्रश्न **10 अंक** का है।
- खण्ड–द में प्रत्येक प्रश्न 02 अंक का है। सभी प्रश्न अनिवार्य हैं।

Section – A (खण्ड–अ)

Objective Questions(वस्तुनिष्ठ प्रशन)

1. Answer all the following questions. **5x3 =15**

निम्नलिखित सभी प्रश्न अनिवार्य हैं।

i) The irregular singular point of $(x-1)(x-2)^3\frac{d^2y}{dx^2}+(x-1)^2\frac{dy}{dx}+3(x-1)y=0$ is

a) 0
b) 1
c) 2
d) None of these

X

ii) The inverse Laplace transform of $\frac{e^{-3p}}{p^3}$, is

a) $(t-3)u_3(t)$
b) $(t-3)^2\,u_3(t)$
c) $(t+3)^2\,u_3(t)$
d) $(t+3)u_3(t)$

+3

iii) The complementary function of $r-7s+6t=e^{x+y}$ is:

a) $f_1(y-x)+f_2(y-6x)$
b) $f_1(y+x)+f_2(y-6x)$
c) $f_1(y-x)+f_2(y+6x)$
d) $f_1(y+x)+f_2(y+6x)$

+3

iv) The partial differential equation $y\frac{\partial^2 u}{\partial x^2}+2x\frac{\partial^2 u}{\partial x\partial y}+y\frac{\partial^2 u}{\partial y^2}=0$ is elliptic if

a) $x^2>y^2$
b) $x^2<y^2$
c) $x^2+y^2>1$
d) $x^2+y^2=1$

+3

v) Fourier transform of the function $f(x) = e^{-ax^2}, a > 0$ is

a) $\sqrt{\frac{\pi}{a}}\, e^{-(p^2/2a)}$

b) $\sqrt{\frac{\pi}{a}}\, e^{-(p^2/4a)}$

c) $\sqrt{\frac{\pi}{a}}\, e^{(p^2/4a)}$

d) $\sqrt{\frac{\pi}{a}}\, e^{(p^2/2a)}$

Section - B (खण्ड–ब)

Short Answer Questions (लघुउत्तरीय प्रश्न)

5x7=35

2. Answer any five of the following questions.
निम्नलिखित में से किन्हीं पाँच प्रश्नों के उत्तर दें।

i. वसअम $(D^3 - 3D^2D' - 4DD'^2 + 12D'^3)Z = \sin(y + 2x) + e^{(x+2y)}$

ii. Find $L\{erf\sqrt{t}\}$ and hence prove that $L\{t.\,erf\ 2\sqrt{t}\} = \frac{3p+8}{p^2(p+4)^{3/2}}$.

iii. Find the Fourier transform of $e^{-a|x|}$.

iv. Prove that $(n+1)P_{n+1} = (2n+1)xP_n + nP_{n-1}$

v. Given that $f(x) = x + x^2$ for $-\pi < x < \pi$, find the Fourier expression of $f(x)$.
Deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots$

vi. Prove that $J_{-n}(x) = (-1)^n J_n(x)$, where n is a positive integer.

Or

Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$.

vii. Find the solution of $\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial u}{\partial t}$ for which $u(0,t) = u(l,t) = 0$, $u(x,0) = \sin\frac{\pi x}{l}$ by method of variable separable.

Or

Using Laplace transforms, find the solution of the initial value problem

$$y'' - 4y' + 4y = 64\, sin2t;\ y(0) = 0, y'(0) = 1.$$

Section - C (खण्ड–स)

Descriptive Questions (विवरणात्मक प्रश्न)

3. Answer any two of the following question. 2x10=20
निम्नलिखित में से किन्हीं दो प्रश्नों के उत्तर दें।

i) (a) Find the Laplace transform of $te^{-t}cosht$.

(b) Find the general solution of $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$.

ii) (a) Solve $px + qy = pq$.

(b) Express the polynomial $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre Polynomial.

iii) (a) Find the solution of wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ such that $y = P_0 \cos pt$ (P_0 is a constant), when $x = l$ & $y = 0$ when $x = 0$.

(b) Define the Dirichlet's condition for Fourier series and Fourier series.

SEMESTER END EXAMINATION, APRIL-MAY, 2025

Mid-Test

Semester:- 2^{nd}

Course Name: - B.Tech (CSE, CE, ECE, EE, ME)

Paper Code:- NBS-202

Paper Name: - Engineering Mathematics-II

Max Marks-20

Time - 30 minutes.

2×10=20

All questions are compulsory.
सभी प्रश्न अनिवार्य हैं।

Objective Questions.
बहुविकल्पीय प्रश्न।

1) Degree and order of this equation $\frac{\partial^2 z}{\partial x^2} = (1 + \frac{\partial z}{\partial y})^{2/3}$ is

a) 3, 2
b) 2, 2
c) 2, 3
d) None of these

+2

2) Which of the following represents the steady state behaviour of heat flow

a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = c^2$
c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = c$
d) None of these

+2

3) Solution of the equation $px + qy = z - 3pq$ is

a) $z = ax - by - 3ab$
b) $z = ax + by - 3ab$
c) $z = ax - by + 3ab$
d) $z = ax + by + 3ab$

+2

4) $\frac{1}{D-2D'} e^{2x+y}$ is equal to

a) 2^{2x+y}
b) $\frac{1}{2} x e^{2x+y}$
c) $\frac{1}{2} x^2 e^{2x+y}$
d) $x e^{2x+y}$

+2

5) The integral of this function $\int_0^\infty e^{-3t} . \sin 4t \; dt$ is

a) $\frac{-4}{25}$
b) $\frac{-3}{25}$
c) $\frac{3}{25}$
d) $\frac{4}{25}$

+2

6) The integral of this function $\int_0^\infty t \, e^{-4t} . \sin t \; dt$ is

a) $\frac{4}{279}$
b) $\frac{8}{289}$
c) $\frac{6}{289}$
d) $\frac{8}{279}$

7) The Rodrigue formula for $P_n(x) = k\frac{d^n}{dx^n}(x^2-1)^n$, the Legendre polynomial of degree n is

a) $k = 2^n.n!$
b) $k = \frac{2^n}{n!}$
+2 c) $k = \frac{1}{2^n.n!}$
d) $k = \frac{n!}{2^n}$

8) If $\int_{-1}^{1} P_n(x)\,dx = 2$, then n is

a) 1
b) -1
+2 c) 0
d) None of these

9) The Inverse Fourier sine transform of a function $F(s)$ is

a) $f(x) = \frac{1}{\sqrt{2\pi}}\int_0^\infty F(s).\sin sx\,ds$
b) $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty F(s).\sin sx\,ds$
c) $f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty F(s).\sin sx\,ds$
d) $f(x) = \sqrt{\frac{2}{\pi}}\int_{-\infty}^\infty F(s).\sin sx\,ds$

10) Which one is true recurrence relation for Bessel's function $J_n(x)$

a) $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$
b) $\frac{d}{dx}[x^{-n} J_n(x)] = -x^n J_{n+1}(x)$
c) $\frac{d}{dx}[x^{-n} J_n(x)] = x^{-n} J_{n+1}(x)$
d) $J_n(x) = \frac{x}{2n}[J_{n-1}(x) - J_{n+1}(x)]$

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SEMESTER END EXAMINATION, APRIL-MAY, 2025

Course Name: - B.Tech. Hons (CSE-AIFM, CSE-AIDS) **Semester:-**2^{nd}

Paper Name: - Engineering Mathematics-II **Paper Code:-** NBS-202

Time - 3 Hrs + 20 minutes per hour extra time for V.I. & examinees with writer. **Max Marks-70**

Additional 30 Minutes for Mid-Test.

समय– 3 घण्टे + 20 मिनट प्रति घंटे अतिरिक्त–दृष्टिबाधित एवं सह लेखक परीक्षार्थियों के लिए। अधिकतम अंक–70

30 मिनट अतिरिक्त मिड–टेस्ट के लिए।

Instructions:

- The question paper consists of three sections namely A, B, C. All sections are compulsory.
- Section A- Each question carries **3 mark**. **All questions are compulsory**.
- Section B- Answer any **5 out of 7** given questions. Each question carries **7 marks**.
- Section C- Answer any **2 out of 3** given questions. Each question carries **10 marks**.
- Section D- Each question carries **02 mark**. All questions are compulsory.

Section – A (खण्ड–अ)

Objective Questions(वस्तुनिष्ठ प्रशन)

1. Answer all the following questions. **5x3 =15**

निम्नलिखित सभी प्रश्न अनिवार्य हैं।

i) For the differential equation $(x-1)\frac{d^2y}{dx^2}+(cot\pi x)\frac{dy}{dx}+(cosec^2\pi x)\,y=0$ which of the following statement is true

a) 0 is regular and 1 is irregular

b) 0 is irregular and 1 is regular

c) Both 0 and 1 are regular

d) Both 0 and 1 are irregular

ii) The inverse Laplace transform of $\frac{2s}{2s^2+8}$, is

a) $sin\,2t$

b) $\sinh 2t$

c) $\cosh 2t$

d) $\cos 2t$

iii) The complementary function of $(D^4-a^4)y=0$ is

a) $y=c_1e^{ax}+c_2e^{-ax}$

b) $y=c_1e^{ax}+c_2e^{-ax}+c_3\cos ax+c_4\sin ax$

c) $y=(c_1+x\,c_2)e^{ax}+(c_3+x\,c_4)e^{-ax}$

d) None of these

iv) Fourier transform of a function $f(x)=e^{-\frac{x^2}{2}}$ is

a) $\sqrt{\pi}e^{-(p^2/4)}$

b) $\sqrt{\frac{\pi}{2}}\,e^{-(p^2/4)}$

c) $\sqrt{\frac{\pi}{2}}\,e^{(p^2/4)}$

d) $\sqrt{\pi}e^{(p^2/4)}$

v) Classify the differential equation $\frac{\partial^2u}{\partial t^2}-4\frac{\partial^2u}{\partial x\partial t}+4\frac{\partial^2u}{\partial x^2}$

a) elliptic

b) hyperbolic

c) parabolic

d) None of these

Section – B (खण्ड–ब)

Short Answer Questions (लघुउत्तरीय प्रश्न)

2. Answer any five of the following questions. **5x7=35**

निम्नलिखित में से किन्हीं पाँच प्रश्नों के उत्तर दें।

i) Solve $r - 3s + 2t = e^{2x-y} + e^{x+y} + cos(x + 2y)$.

ii) Find the Laplace transform of

a) $(t-1)^2 u(t-1)$

b) $t\,sin^2 3t$

iii) Find the Fourier transform of e^{-ax^2}.

iv) Prove that $(2n+1)xP_n = (n+1)P_{n+1} - nP_{n-1}$

v) Fourier series for $f(x) = 4 - x^2, -2 \leq x \leq 2$.

vi) Prove that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$, where n is a positive integer.

Or

Solve $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$.

vii) Find the solution of wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ such that $y = P_0 \cos pt$ (P_0 is a constant), when $x = l$ & $y = 0$ when $x = 0$.

Or

Using Laplace transforms, find the solution of the initial value problem

$$y'' - 3y' + 2y = 4t + e^{3t};\ y(0) = 1, y'(0) = -1.$$

Section – C (खण्ड–स)

Descriptive Questions (विवरणात्मक प्रश्न)

3. Answer any two of the following question. **2x10=20**

निम्नलिखित में से किन्हीं दो प्रश्नों के उत्तर दें।

i) (a) Find the Laplace transform of $F(t) = \begin{bmatrix} \cos t\ ,\ 0 < t < \pi \\ 0,\ t > \pi \end{bmatrix}$.

(b) Find the general solution of $(mz - ny)p + (nx - lz)q = ly - mx$.

ii) (a) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x,0) = 6e^{-3x}$.

(b) Express the following polynomial in terms of Legendre Polynomial $1 - 2x + x^2 + 5x^3$.

iii) (a) Find the inverse Laplace transform of $\frac{6}{2p-3} - \frac{3+4p}{9p^2-16} + \frac{8-6p}{16p^2+9}$.

(b) Find the inverse Fourier transform of $f(p) = e^{-|p|y}$.

5x7=35

Mid-Test

Course Name: - B.Tech. Hons (CSE-AIFM, CSE-AIDS) **Semester:-**2^{nd}

Paper Name: - Engineering Mathematics-II **Paper Code:-** NBS-202

Time - 30 minutes. **Max Marks-20**

All questions are compulsory. **2×10=20**

सभी प्रश्न अनिवार्य हैं।

Objective Questions.

बहुविकल्पीय प्रश्न।

1) Order and degree of this equation $\frac{\partial^2 z}{\partial x^2} = (1 + \frac{\partial z}{\partial y})^{2/3}$ is
   a) 3, 2
   b) 2, 2
   c) 2, 3
   d) None of these

2) Two-dimensional wave equation is
   a) $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$
   b) $\frac{\partial^2 u}{\partial z^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$
   c) $\frac{\partial^2 u}{\partial x^2} = c^2\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$
   d) None of these

3) Solution of the equation $px + qy = z + sin\,(pq)$ is
   a) $z = ax - by - sin\,(pq)$
   b) $z = ax + by - sin\,(pq)$
   c) $z = ax - by + sin\,(pq)$
   d) $z = ax + by + sin\,(pq)$

4) $\frac{1}{D-2D'} e^{2x-y}$ is equal to
   a) e^{2x-y}
   b) $\frac{1}{2} x e^{2x-y}$
   c) $\frac{1}{2} x^2 e^{2x-y}$
   d) $\frac{1}{4} e^{2x-y}$

5) Which one is wrong $\int_0^\infty e^{-3t}.\sin 4t\ dt$ is
   a) $L\{erf(\sqrt{t})\} = \frac{1}{s\sqrt{(s+1)}}$
   b) $L\{erf(\sqrt{t})\} = \frac{1}{s\sqrt{(s-1)}}$
   c) $L\{J_0(\sqrt{t})\} = \frac{1}{s} e^{\frac{-1}{4s}}$
   d) $L\{J_0(2\sqrt{t})\} = \frac{1}{s} e^{\frac{-1}{s}}$

6) The integral of this function $\int_0^\infty t\, e^{-4t}.\cos t\ dt$ is
   a) $\frac{15}{279}$
   b) $\frac{15}{289}$
   c) $\frac{17}{289}$
   d) $\frac{17}{279}$

7) The Rodrigue formula for $P_n(x) = \frac{1}{2^n.n!}k$, the Legendre polynomial of degree n is

e) $k = \frac{d^n}{dx^n}(x^2-1)^{n-1}$

f). $k = \frac{d^n}{dx^n}(x^2-1)^n$

g) $k = \frac{d^n}{dx^n}(x^2+1)^n$

h) $k = \frac{d^n}{dx^n}(x^2+1)^{n-1}$

8) For which value of n this result $\int_{-1}^{1} P_n(x)\, dx = 0$, is wrong

a) 1

b) -1

c) 0

d) All above

9) Which one is true recurrence relation for Bessel's function $J_n(x)$

a). $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$

b) $\frac{d}{dx}[x^{-n} J_n(x)] = -x^n J_{n+1}(x)$

c) $\frac{d}{dx}[x^{-n} J_n(x)] = x^{-n} J_{n+1}(x)$

d) $J_n(x) = \frac{x}{2n}[J_{n-1}(x) - J_{n+1}(x)]$

10) The Inverse Fourier cosine transform of a function $F(s)$ is

e) $f(x) = \frac{1}{\sqrt{2\pi}}\int_0^\infty F(s).\cos sx\, ds$

f) $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty F(s).\cos sx\, ds$

g) $f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty F(s).\cos sx\, ds$

h) $f(x) = \sqrt{\frac{2}{\pi}}\int_{-\infty}^\infty F(s).\cos sx\, ds$

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