Course Name: B.Tech.

Semester: 1st

Paper Name: Engineering Mathematics

Paper Code: NBS-102

Time: 3 Hrs. + 20 Minutes per hour extra for V.I. and examinees with writer. Additional 30 Minutes for Mid-Test

Max. Marks: 70

समय – 3 घण्टे + 20 मिनट प्रति घण्टे अतिरिक्त दृष्टिबाधित एवं सहलेखक परीक्षार्थियों के लिए। मिड टेस्ट के लिए 30 मिनट अतिरिक्त

अधिकतम अंक: 70

Instructions:

(i) *The question paper consists of three Sections namely A, B and C. All Sections are compulsory.*

(ii) *Section-A – Each question carries 3 marks. All questions are compulsory.*

(iii) *Section-B – Answer any 5 out of 7 given questions in maximum one hundred fifty (150) words. Each question carries 07 marks.*

(iv) *Section-C – Answer any 2 out of 3 given questions in maximum three hundred (300) words. Each question carries 10 marks.*

(v) *Section-D – All questions are compulsory.*

निर्देश:

(i) प्रश्न-पत्र में तीन खण्ड अ, ब व स हैं। सभी खण्ड अनिवार्य हैं।

(ii) खण्ड-अ में प्रत्येक प्रश्न 03 अंकों का है। सभी प्रश्न अनिवार्य हैं।

(iii) खण्ड-ब में 07 प्रश्नों में से किन्हीं 05 प्रश्नों के उत्तर अधिकतम 150 शब्दों में दीजिये। प्रत्येक प्रश्न 07 अंकों का है।

(iv) खण्ड-स में 03 प्रश्नों में से किन्हीं 02 प्रश्नों के उत्तर अधिकतम 300 शब्दों में दीजिये। प्रत्येक प्रश्न 10 अंकों का है।

(v) खण्ड-द में सभी प्रश्न अनिवार्य हैं।

Section-A (खण्ड-अ)

Objective Questions (वस्तुनिष्ठ प्रश्न)

1. Answer all the following questions: $(3 \times 5 = 15)$

निम्नलिखित सभी प्रश्न अनिवार्य हैं:

(i) The differential equation $Mdx + Ndy$ is exact if:

(A) $\dfrac{\partial M}{\partial N} = \dfrac{\partial N}{\partial y}$

(B) $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

(C) $\dfrac{\partial M}{\partial y} + \dfrac{\partial N}{\partial x} = 0$

(D) $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial M}$

(ii) The value of $\int_0^\infty e^{-3x} x^7 dx$ is:

(A) $\dfrac{\overline{|7}}{3^8}$

(B) $\dfrac{\overline{|8}}{3^8}$

(C) $\dfrac{\overline{|3}}{7^3}$

(D) $\dfrac{\overline{|3}}{8^3}$

(iii) The value of grad r is:

(A) r^2

(B) r

(C) $\bar{r}$

(D) None of these

(iv) The Rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ is:

(A) 3

(B) 2

(C) 1

(D) 4

(v) If $U = \sin^{-1} \dfrac{x}{y} + \tan^{-1} \dfrac{y}{x}$ then the value of $x \dfrac{\partial U}{\partial x} + y \dfrac{\partial U}{\partial y}$ is:

(A) 0

(B) $\dfrac{x}{y}$

(C) $\dfrac{y}{x}$

(D) None of these

Section-B (खण्ड-ब)

Short Answer Questions (लघुउत्तरीय प्रश्न)

2. Answer any five of the following questions in maximum 150 words: (7×5=35)

निम्नलिखित में से किन्हीं पाँच प्रश्नों के उत्तर अधिकतम 150 शब्दों में दीजिये:

(i) If $y = a \cos \log x + b \sin \log x$ then show that $x^2 y_{n+2} + (2n+1) x y_{n+1} + (n^2 + 1) y_n = 0$.

(ii) Solve the following system of equation:

$$2x - 3y + z = 0$$
$$x + 2y - 3z = 0$$
$$4x - y + 2z = 0$$

(iii) Show that $\vec{f} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j}(3xz^2 - y)\vec{k}$ irrotational. Hence find the corresponding scalar function ϕ such that $\vec{f} = \operatorname{grad} \phi$.

(iv) Solve $(D^2 - 4D + 4)y = x^3 e^{2x}$

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(v)   Evaluate $\int_0^\infty \int_x^\infty \dfrac{e^{-y}}{y} dxdy$

(vi)   Show that the system of three vectors $(1, 2, 0)$ $(0, 3, 1)$ and $(-1, 0, 1)$ is L.I. in $V_3(Q)$ where Q is a field of rational numbers.

(vii)   Find the nth differential coefficient of $y = \tan^{-1}\left(\dfrac{x}{a}\right)$.

Section-C (खण्ड-स)

Descriptive Questions (विवरणात्मक प्रश्न)

3.   Answer any two of the following questions in maximum 300 words: (10×2=20)

निम्नलिखित में से किन्हीं दो प्रश्नों के उत्तर अधिकतम 300 शब्दों में दीजिये:

(i)   Find the characteristic equation of the symmetric matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it

is satisfied by A and hence obtain A^{-1}. Express:

(a) $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$

(b) $A^5 - 5A^4 + 3A^3 + 6A^2 - 6A + 2I$ as linear polynomial in A.

(ii)   (a)   Solve $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z} dxdydz$

(b)   Expand $\tan\left(x + \dfrac{\pi}{4}\right)$ as far as the term x^4 and evaluate $\tan 46.5°$ to four significant digits.

(iii)   (a)   Verify Divergence theorem, given that $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ and s is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

(b)   Evaluate by Stoke's theorem $\vec{F} = y^2\vec{i} + x^2\vec{j} - (x+z)\vec{k}$ and c is the boundary of the Triangle with vertices at $(0, 0, 0)$, $(1, 0, 0)$ and $(1, 1, 0)$.

Mid-Test

Section-D (खण्ड-द)

All questions are compulsory and each carry 2 marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न के अंक 2 हैं।

1.   The value of curi grad ϕ is:

(A)   div ϕ

(B)   grad ϕ

(C) ϕ

(D)   None of these

2. The maximum value of the directional derivative of $\phi = xyz^2$ at point $(1, 0, 3)$ is:

   (A) 7

   (B) 8

   (C) 9

   (D) None of these

3. If $y = x^n \log x$ then the value of y_{n+1} is:

   (A) $\dfrac{Ln}{x}$

   (B) $\dfrac{Ln}{x^2}$

   (C) $\dfrac{Ln+1}{x^2}$

   (D) None of these

4. The nth derivative of 2^x is:

   (A) $2^x \log 2$

   (B) $n(\log 2)2^x$

   (C) 2^x

   (D) None of these

5. The value of $\int_0^1 \int_0^{x^2} e^{y/x} dydx$ is:

   (A) $\dfrac{1}{2}$

   (B) $\dfrac{1}{3}$

   (C) $\dfrac{1}{4}$

   (D) None of these

6. The value of $\Gamma 3/2$ is:

   (A) $\sqrt{\pi}$

   (B) $\dfrac{\pi}{2}$

   (C) π

   (D) None of these

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(4)

7. The Rank of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ is:

(A) 1

(B) 2

(C) 3

(D) None of these

8. The Eigen values of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ is:

(A) (2, 2, 5)

(B) (2, 7, 9)

(C) (2, 7, 11)

(D) None of these

9. The degree of the differential equation:

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0 \text{ is:}$$

(A) One

(B) Second

(C) Third

(D) None of these

10. If $\frac{d^2y}{dx^2} + 16y = 0$ then y is:

(A) $y = C_1 \cos 2x + C_2 \sin 2x$

(B) $y = C_1 \cos 4x + C_2 \sin 4x$

(C) $y = 16$

(D) None of these

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