Perspective Viewing

Orthographic Projection

The approach that we have followed up until this point has been to shoot rays that all have the same direction but different origins, mainly the centre (or sample point) within each pixel. These rays are ultimately computing the *projection* of our objects in 3D space onto a flat 2D surface called the *viewing plane*.

If we draw lines that pass through the corners of the viewing plane and are parallel to the direction of the rays, we would end up with an infinite box, which is called the *viewing volume*. Anything within the volume will be projected onto our viewing plane, while everything outside will not be visible. In ray tracing, we do not have to do any special culling of objects that sit outside of our viewing volume, as the rays will never hit them and so they become irrelevant. We can also define a *near* and *far* plane to determine how far back we allow objects to be in our scene. While this process is important for real-time systems, ray tracing does not require them to be there. This current system is called *orthographic projection* and is commonly used in modelling software and CAD systems.

Perspective Projection

The idea we will now explore is to reverse the conditions of the rays: before they all had the same direction, but different origins. We will now have rays that have the same *origin* but different *direction*. Specifically, we are going to define a *centre of projection* which will be the origin for all the rays. The direction will then be determined in relation to the centre of each pixel (or sample point). This new system is called *perspective projection*.

Given that we have now changed the rays, we will now discuss the consequences that this change has. First, the viewing volume is no longer an infinite box. Instead, the viewing volume is now an infinite pyramid, whose peak is the centre of projection. This is called the *viewing frustum*. Similar to before, we can define the near and far planes, though they are not strictly speaking necessary in ray tracing. Another quantity that appears is called the *field of view*, which is defined as the angle subtended at the centre of projection by the top and bottom edges of the window (or viewing plane). The ratio between the width and height of the viewing plane is called the *aspect ratio*.

Properties of Perspective Projection

We will now discuss the properties that the projection has and the effects that it will have on the projected objects.

1. *The perspective projection of an object becomes smaller as the object gets further away from the centre of projection.* This is the main reason why perspective projections are more realistic. If we think about what happens

to objects that are further away, they do indeed seem smaller and they get larger as they move closer to us.

2. *As an object is rotated, its projected width becomes smaller. This is known as foreshortening.* This comes as a consequence of the first property. Imagine a thin rectangular box that is parallel to us. As we rotate the box, one side will move further away from the centre of projection and will therefore appear smaller. Conversely, the side of the box that is moving towards us will be closer and will therefore appear larger.
3. *Perspective projections preserve straight lines.* This is fairly apparent, and is something that is not required by all types of projections. In fact, a type of projection called the *fish-eye* projection (which is used by GoPro cameras for example) does not maintain straight lines.
4. *Sets of parallel lines that are parallel to the view plane remain parallel when projected onto the view plane.* We can see this also as a consequence of properties 1 and 3. A line that is parallel to the view plane will maintain the same distance and will always be the same size. If we add a second line to the mix, we will have two parallel lines.
5. *Sets of parallel lines that are not parallel to the view plane converge to a vanishing point on the view plane.* This also comes as a consequence of properties 1 and 3. Specifically, if the lines aren't parallel with the view plane, there will be points on the lines that will be further away and will be smaller. Since the line must remain straight, any pair of lines not parallel to the view plane will therefore seem to converge at a point, which is known as the *vanishing point*.

Vanishing Points

Before we discuss further about vanishing points, we will define the *ground plane*. Consider a plane that is parallel to the z-axis and therefore perpendicular to our viewing plane. Any ray that is shot from the upper half of the viewing plane will not intersect it, while rays shot from the lower half will. Any ray shot from the centre will "hit" at some point in infinity. These rays are the ones that contribute to the formation of the *horizon*. In this configuration, the horizon will be a horizontal line across the centre of the viewing plane.

The reason to discuss the horizon is that the *vanishing points of horizontal lines are always on the horizon*. This is due to the fact that the vanishing points are at an infinite distance away from us, which coincidentally is the same location of the horizon itself.

We can classify perspective views by the number of vanishing points. Consider two cubes. If we set them side by side, then the lines of the cubes that are not parallel to the viewing plane will converge at the same vanishing point. This system is therefore known as a *one-point* perspective view. Now take the cubes and rotate them so they have no edges aligned with the viewing plane. The edges will now converge on two points, which results in a *two-point* perspective view. Finally, if we align a corner of the cube with the viewing plane, we will end up with a *three-point* perspective view. This leads to the final property:

6. *If a set of parallel lines is parallel to the world coordinate axis, its vanishing point is on the axis and is known as the principal vanishing point.*

In fact, the number of principal vanishing points is related to the number of of world coordinate planes (or axes) that the view plane cuts.

Axis-Aligned Perspective Viewing

The simplest way to implement perspective viewing is to simply move the origins of the rays to a single point, which we will call the *eye*. For the sake of simplicity, we will place the eye on the z-axis, which results in an *axis-aligned perspective*. The new directions of the rays are therefore computed as the point on the pixel minus the eye. The resulting vector must then be normalized and can be used in our calculations.

Processes and Results

Now that we have our viewing system setup, we can now begin to explore what happens when certain quantities are changed. First, we will define d as the distance between the eye and the viewing plane. Let's see what happens when we change it:

- If d decreases, then the eye moves closer to the viewing plane. This results in the field of view *increasing*. The effect that this will have on the projected objects is that they will appear *smaller*. The reasoning behind this is the following: suppose that an object occupies a certain percentage of the field of view. If I move d such that the field of view is doubled, then the percentage that the object occupies will be halved. Notice that the size of the object is constant, and what is changing is the *perceived* or *projected* size of the object.
- If d increases, then the eye moves further away from the viewing plane, causing the field of view to *decrease*. This will result in the size of projected objects to *increase*.

These two properties of d can be used together to emulate a zooming system.

Now let us consider what happens when the *entire* unit is moved together. That is to say, let us hold d constant, and lets move the eye, and the viewing plane together. Since d is constant, the field of view remains the same. As we move closer to the objects, they will appear larger, but something else will happen as well: objects that are further away will start to appear smaller, while objects (or parts of objects) that are closer will be bigger. This is known as *perspective distortion* and is a direct consequence of the properties we discussed earlier. This type of movement is therefore *not* the same as zooming in.

Now do all objects experience distortion? The answer is yes. Do all of them experience it in the same way? No. Objects that have straight lines in them will retain their overall shape. Their dimensions might change, but their shapes will remain. Objects that are curved on the other hand will be distorted more severely by this, to the point where spheres will become ellipsoids.