Specular Reflection

Quick Review

Before we introduce a new BRDF, let's do a quick recap of what we have. First, let's review the components of the Lambertian BRDF:

BRDF	ρ_{hh}	$f_{r,d}$
Lambertian	$k_d c_d$	$k_d c_d / \pi$

Now, let's look at the L_i for each of the lights that we have previously defined:

Light	L_i
Directional	$f_r(p, l, \omega_o) \star l_s c_l \cos\theta_l$
Point	$f_r(p, l, \omega_o) \star l_s c_l \cos\theta_l$

Note that here the point light has attenuation turned off. Now for the directional light, l is given by the direction of the light itself, whereas for the point light, $l = p_l - p$ where p_l is the location of the light.

Now recall the rendering equation

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{2\pi^+} f_r(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos\theta_i d\omega_i$$

From here, we derived the following expression for diffuse shading:

$$L_o(p, \omega_o) = k_a c_d \star (l_s c_l) + \sum_{j=1}^{n} \frac{k_d c_d}{\pi} \star (l_{s,j} c_{l,j})(n \cdot l_j)$$

Recall that if a surface is determined by more than one BRDF, then the total radiance for that surface is the sum of the BRDFs. With this in mind, we are now ready to begin discussing specular reflection.

We can make materials look shiny by allowing them to reflect light that's focused around the direction of mirror-reflection. This glossy specular reflection results in specular highlights on the surface, which are smeared out reflections of the light sources themselves. If they are bright enough, then they are the colour of the lights, which are usually white.

Modeling

To model specular reflection, we need an expression for the unit vector r in the direction of mirror reflection at the point p given an incoming light direction l

and a normal n. These two vectors and the normal are all in the same plane, called the *plane of incidence*. According to the *law of reflection*, the angle subtended by l and r is equal, so we will use θ_i for both. Note that r is not necessarily the same as the viewing direction ω_o, which also doesn't have to be in the plane of incidence.

To derive r, we note that since l, r, and n are complanar, then r must be a linear combination of l and n:

$$\mathbf{r} = a\mathbf{l} + b\mathbf{n} \tag{1}$$

where a, b are numbers. Since we have two unknowns, we require two equations to determine their values. First notice that the projections of r and l onto n are the same. Taking the dot product of both sides in eq. 1 with n gives

$$\mathbf{r} \cdot \mathbf{n} = a\mathbf{l} \cdot \mathbf{n} + b\mathbf{n} \cdot \mathbf{n}$$

so that

$$(1 - a)\mathbf{l} \cdot \mathbf{n} = b \tag{2}$$

Now consider the unit direction $\mathbf{n}^{\perp}$ that is perpendicular to $\mathbf{n}$ and lies in the plane of incidence. Note that these two vectors form an orthonormal basis in this plane. The projections of r and l onto $\mathbf{n}^{\perp}$ are negatives of each other. Taking the dot product with eq. 1 with $\mathbf{n}^{\perp}$:

$$\mathbf{r} \cdot \mathbf{n}^{\perp} = a\mathbf{l} \cdot \mathbf{n}^{\perp} + b\mathbf{n} \cdot \mathbf{n}^{\perp}$$

leads to

$$a = -1 \tag{3}$$

We can now combine eq. 1, eq. 2 and eq. 3 to obtain the following expression for r:

$$\mathbf{r} = -\mathbf{l} + 2(\mathbf{n} \cdot \mathbf{l})\mathbf{n} \tag{4}$$

Note that this vector is unit by construction. Now that we have the direction of reflection, we need a way to model the way reflected radiance decreases as the angle α between r and ω_o increases.

Phong modelled this with the expression $(\cos\alpha)^e = (\mathbf{r} \cdot \omega_o)^e$ where the *Phong exponent* $e \geq 0$. This number allows us to control the size of the specular highlights. Using Phong's model for specular reflection, we obtain the following for the reflected radiance of a single directional light:

$$L_o(p, \omega_o) = k_s(\mathbf{r} \cdot \omega_o)^e l_s c_l \tag{5}$$

where k_s is the *coefficient of specular reflection*. Now it is important to note that this is an empirical model and is not reciprocal nor does it conserve energy. Fortunately, this can be fixed. One of the problems with eq. 5 is that there is no $\cos\theta_l = \mathbf{n} \cdot \mathbf{l}$ although it is present in the rendering equation. A reciprocal glossy specular BRDF is of the form

$$f_{r,s}(l, \omega_o) = k_s(\mathbf{r} \cdot \omega_o)^e \tag{6}$$

This satisfies the reciprocity condition for all $\mathbf{l}, \omega_o$. With the BRDF in eq. 6, the reflected specular radiance is then

$$L_o(p, \omega_o) = k_s(\mathbf{r} \cdot \omega_o)^e l_s c_l (\mathbf{n} \cdot \mathbf{l}) \tag{7}$$

We can make the BRDF energy conserving by enforcing the condition $k_d + k_s < 1.0$. The reason that although k_d and k_s specify different kinds of reflection, they both come from the same light source. We therefore have the following expression for the sum of the ambient, diffuse, and specular reflection for n point and directional lights:

$$L_o(p, \omega_o) = k_a c_d \star (l_s c_l) + \sum_{j=1}^{n} \frac{k_d c_d}{\pi} \star (l_{s_j} c_{l_j})(\mathbf{n} \cdot \mathbf{l}_j) + \sum_{j=1}^{n} k_s(\mathbf{r}_j \cdot \omega_o)^e (l_{s_j} c_{l_j})(\mathbf{n} \cdot \mathbf{l}_j) \tag{8}$$

An issue with direct illumination is that the angle between $\mathbf{r}$ and ω_o can be greater than $\pi/2$. Because this results in a negative value of $\mathbf{r} \cdot \omega_o$, we must clamp it to positive values in eq. 8

Note that if we want the highlights to have colour, we simply have to multiply $k_s(\mathbf{r}_j \cdot \omega_o)^e$ by c_s.

Viewer Dependence

Based on the formula for diffuse shading, we can see that it varies based only on the angle between the incoming radiance and the surface normal. Therefore, this type of shading is called *viewer-independent* shading. In contrast, if we look at the specular highlights in the real world move around as we look at objects. This is due to the fact that the specular highlight is always centred on the direction of mirror reflection which is defined by the light, the surface, and the viewer. Therefore, specular reflection is *viewer-dependent* shading.

Other Reflection Models

Blinn made the following simple modification to Phong's specular model:

$$f_{r,s}(l, \omega_o) = k_s(\mathbf{n} \cdot \mathbf{h})^e$$

Where $\mathbf{h} = (\mathbf{l} + \omega_o)/|\mathbf{l} + \omega_o|$ is called the *halfway vector* between $\mathbf{l}$ and ω_o. The original motivation for this was efficiency as it does not require the full computation of the reflection vector, and for a directional light with orthographic viewing, $\mathbf{h}$ is constant. That being said, the *Blinn-Phong* model is still widely used today. Note that due to the fact that the angle subtended by $\mathbf{h}$ is not euqal to α, we require larger exponents e in order to match the same effect that we would get from Phong shading.