SHRI RAMSWAROOP MEMORIAL COLLEGE OF ENGINEERING & MANAGEMENT

B. Tech. [SEM I (DS 1A & ECE 1A)]

TUTORIAL SHEET-1

[Session: 2024-25 (Odd)]

BAS-103: ENGINEERING MATHEMATICS-I

Unit No. & Name:	**I**- MATRICES	**Course Outcome:**	**CO1-** Understand the concept of complex matrices, Eigen values, Eigen vectors and apply the concept of rank to evaluate linear simultaneous equations
Date of Distribution:	17/09/2024	**Name of Faculty:**	Mr. Sachin Singh

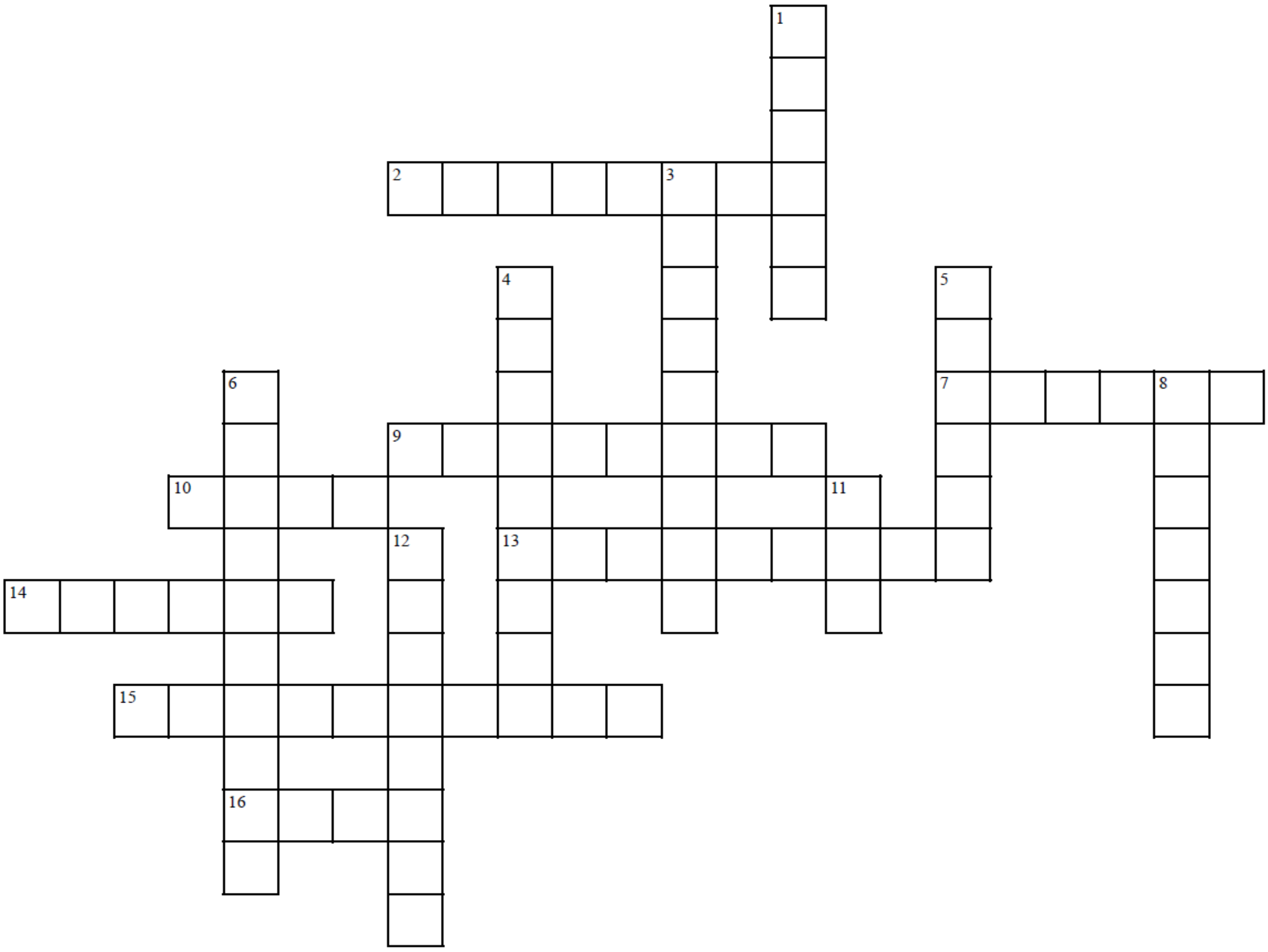
SUBJECTIVE QUESTIONS		**BL**
Elementary transformations, Inverse of a matrix		
Q1)	Find the inverse of the matrix by elementary transformation $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$. **(AKTU, 2021)** $\boldsymbol{Ans.}$: $\boldsymbol{A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}}$	3
Q2)	Find the inverse of the matrix by elementary transformation $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. **(AKTU, 2019)** $\boldsymbol{Ans.}$: $\boldsymbol{A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}}$	3
Rank of matrix		
Q3)	Find the rank of the matrix reducing it to row Echelon form $A = \begin{bmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{bmatrix}$. $\boldsymbol{Ans.: 4}$	3
Q4)	Find the rank of the matrix $A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ by reducing it to normal form. **(AKTU, 2019)** $\boldsymbol{Ans.: 2}$	3
Q5)	Find non-singular matrices P and Q such that PAQ is in normal form where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. **(AKTU, 2021)** $\boldsymbol{Ans.: P \text{ and } Q \text{ depends on elementary operations}}$	3
Solution of system of linear equations		
Q6)	Solve the system of homogenous equations: $x_1 + x_2 + x_3 + x_4 = 0,\ x_1 + 3x_2 + 2x_3 + 4x_4 = 0,\ 2x_1 + x_3 - x_4 = 0.$ **(AKTU, 2024)** $\boldsymbol{Ans.}$: $\boldsymbol{x_1 = a + 2b, x_2 = a, x_3 = -2a - 3b, x_4 = b}$ **where** $\boldsymbol{a}$ **and** $\boldsymbol{b}$ **are arbitrary real number.**	3
Q7)	Test the consistency for the following system of equations and if system is consistent, solve them: $x + y + z = 6,\ x + 2y + 3z = 14,\ x + 4y + 7z = 30.$ **(AKTU, 2023)** $\boldsymbol{Ans.: x = k - 2,\ y = 8 - 2k,\ z = k}$ **where** $\boldsymbol{k}$ **is an arbitrary real number.**	3

Q8)	For what values of λ and μ the following equations: $x+y+z=6, x+2y+5z=10,\ 2x+3y+\lambda z=\mu$ has i) unique solution ii) no solution iii) infinite number of solutions. Also, find the solution for $\lambda=2$ and $\mu=8$. **(AKTU, 2022, 2020)** $\boldsymbol{Ans.: (i)\ \lambda \neq 6\ and\ \mu\ is\ any\ number\ \ (ii)\ \lambda=6\ and\ \mu\neq 16\ \ (iii)\lambda=6\ and\ \mu=16}$ $\boldsymbol{For\ \lambda=2\ and\ \mu=8, x=8, y=-4, z=2}$	3
Characteristic equation, Cayley-Hamilton Theorem and its application		
Q9)	Determine A^{-1}, A^{-2} and A^{-3} if $A=\begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -4 & -3\end{bmatrix}$ using Cayley-Hamilton theorem. **(AKTU, 2024)** $\boldsymbol{Ans.: A^{-1}=\frac{1}{4}\begin{bmatrix}1 & 6 & 6\\ -1 & 6 & 2\\ -1 & -10 & -6\end{bmatrix}}$	3
Q10)	Verify Cayley- Hamilton Theorem for $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$ and hence find A^{-1}. Also express the polynomial $B=A^8-11\,A^7-4\,A^6+\ A^5+A^4-11\,A^3-3\,A^2+2A+I$ as a quadratic polynomial in A and hence, find B. **(AKTU, 2021, 2019)** $\boldsymbol{Ans.: A^{-1}=\begin{bmatrix}1 & -3 & 2\\ -3 & 3 & -1\\ 2 & -1 & 0\end{bmatrix}\ ; B=A^2-2A+I=\begin{bmatrix}13 & 21 & 25\\ 21 & 38 & 46\\ 25 & 46 & 59\end{bmatrix}}$	3
Linear Dependence and Independence of vectors		
Q11)	Show that the vectors $x_1=(1,2,4), x_2=(2,-1,3), x_3=(0,1,2)$ and $x_4=(-3,7,2)$ are linearly dependent. **(AKTU, 2019)**	3
Q12)	Show using a matrix that the set of vectors: $(2,5,2,-3),(3,6,5,2),(4,5,14,14),(5,10,8,4)$ is linearly independent.	3
Eigen values and Eigen vectors		
Q13)	Find eigen values and corresponding eigen vectors for the matrix $\begin{bmatrix}2 & 1 & 1\\ 2 & 3 & 4\\ -1 & -1 & -2\end{bmatrix}$. **(AKTU, 2023)** $\boldsymbol{Ans}$: **Eigen values are 1, -1, 3 & eigen vectors are** $\boldsymbol{k[-1\quad 1\quad 0]^T}$ **,** $\boldsymbol{k[0\quad -1\quad 1]^T, k[-2\quad -3\quad 1]^T}$	3
Q14)	Find eigen values and corresponding eigen vectors for the matrix $\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$. **(AKTU, 2022, 2020)** $\boldsymbol{Ans}$: **Eigen values are -3, -3, 5 & eigen vectors are** $\boldsymbol{[3k_1-2k_2\quad k_2\quad k_1]^T}$ **,** $\boldsymbol{k_3[1\quad 2\quad -1]^T}$	3
Complex Matrices, Hermitian, Skew-Hermitian and Unitary Matrices, Applications to Engineering problems.		
Q15)	Prove that the matrix A= $\frac{1}{3}\begin{bmatrix}-2 & 1 & 2\\ 2 & 2 & 1\\ 1 & -2 & 2\end{bmatrix}$ is orthogonal.	3
Q16)	Define unitary matrix. Show that the matrix $\begin{bmatrix}\alpha+i\gamma & -\beta+i\delta\\ \beta+i\delta & \alpha-i\gamma\end{bmatrix}$ is a unitary matrix if $\alpha^2+\beta^2+\gamma^2+\delta^2=1$. **(AKTU, 2022, 2020)**	3

SUPPLEMENTARY QUESTIONS		BL
Simple Stresses & Strains		
Q1)	Let A be an $n \times n$ matrix. Prove that the constant term of $p_A(x)$ is $(-1)^n \lvert A \rvert$. Use this to show that any singular matrix must have zero as one of its eigen values.	2
Eigen values and Eigen vectors		
Q2)	Prove that the eigen vector corresponding to eigen value λ of matrix A is also an eigen vector of every matrix f(A) where f(x) is any scalar polynomial and the corresponding eigen value for f(A) is f(λ) and in general if g(x) = $\frac{f_1}{f_2}$ and $\lvert f_2(A) \rvert \neq 0$ then g(λ) is an eigen value of g(A) = $f_1(A)\{f_2(A)\}^{-1}$	2
Complex Matrices, Hermitian, Skew-Hermitian and Unitary Matrices, Applications to Engineering problems.		
Q3)	Show that the diagonal elements of a Hermitian matrix are necessarily real.	2
Q4)	Show that the diagonal elements of a skew-Hermitian matrix are either purely imaginary or zero.	2

SHORT-ANSWER TYPE QUESTIONS		BL
Elementary transformations, Inverse of a matrix		
Q1)	Calculate the inverse of matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$.	2
Rank of a matrix		
Q2)	Find the value of b for which the rank of the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$ is 2. **(AKTU, 2021)**	2
Solution of system of linear equations		
Q3)	Show that the system of equations: $x + 3y - 2z = 0, 2x - y + 4z = 0, x - 11y + 14z = 0$ has a non-trivial solution. **(AKTU, 2019)**	2
Characteristic equation, Cayley-Hamilton Theorem and its application		
Q4)	State the Cayley-Hamilton theorem. **(AKTU, 2021)**	1
Q5)	Express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial in A where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.	2
Linear Dependence and Independence of vectors		
Q6)	Show that the system of vectors $X_1 = (1, -1, 1), X_2 = (2, 1, 1)$ and $X_3 = (3, 0, 2)$ are linearly dependent or linearly independent. **(AKTU, 2022)**	2
Q7)	For what value of λ, the vectors $(1, -2, \lambda), (2, -1, 5)$ and $(3, -5, 7\lambda)$ are linearly dependent. **(AKTU, 2020)**	2
Eigen values and Eigen vectors		
Q8)	Find the product and sum of the eigen values for $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$. **(AKTU, 2024)**	2
Q9)	Find the eigen value of the matrix $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ corresponding to the eigen vector $\begin{bmatrix} 51 \\ 51 \end{bmatrix}$. **(AKTU, 2023)**	2
Complex Matrices, Hermitian, Skew-Hermitian and Unitary Matrices, Applications to Engineering problems.		
Q10)	If A is a Hermitian matrix, then show that iA is Skew-Hermitian matrix. **(AKTU, 2023, 2020)**	2
Q11)	Prove that the matrix $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary. **(AKTU, 2021)**	2

CROSSWORD-PUZZLE — BL 1



Across

2. all elements are zero except principal diagonal

7. If rank of A and rank of augmented matrix and elements is equal

9. If A = A’

10. diagonal elements are one only

13. if interchange the rows and column

14. Arrangement of elements in row and column in a rectangular array

15. if A.A’=I

16. All elements are zero

Down

1. diagonal elements are scalar say k

3. If square of A =0

4. if A = Power of A is theta

5. Number of rows and columns are same

6. If rank of A and rank of augmented matrix is equal

8. if A. power of A is theta = I

11. If there are number of columns and one row

12. if modulus of A is zero

REFERENCES

TEXT BOOKS:					
Ref. [ID]	**Authors**	**Book Title**	**Publisher/Press**	**Edition &Year of Publication**	**No. of Books Available in Library**
[T1]	Erwin Kreyszig	Advanced Engineering Mathematics	Wiley Publication India Pvt. Ltd. Delhi	1ST Ed.,2013	29
[T2]	Peter V. O` Neil and Santosh K. Sengar	A text book of Engineering Mathematics	Cengage Learning New Delhi	2nd Ed., 2010	81
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[R1]	B.V. Ramana	Engineering Mathematics	Tata McGraw-Hill New Delhi	4th Edition, 2008	96
[R2]	Shantinarayan & P. K. Mittal	Differential Calculus	S. Chand New Delhi	15TH Ed., 2004	01
[R3]	B.S. Grewal	Higher Engineering Mathematics	Khanna Publication	43rd Ed. , 2015	34
ONLINE/DIGITALREFERENCES:					
Ref. [ID]	**Source Name**	**Source Hyperlink**			
[D1]	Matrices	Engineering Mathematics -II (rcet.org.in)			

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Signature of Faculty:
(With Date) 12.09.2024

Signature of HOD:
(With Date) 14.09.2024