

Optimization Algorithms



Mini-Batch Gradient Descent

$$X = \begin{bmatrix} x^{(1)} & x^{(2)} & x^{(3)} & \dots & x^{(m)} \end{bmatrix}$$

(n, m)

$$Y = \begin{bmatrix} y^{(1)} & y^{(2)} & y^{(3)} & \dots & y^{(m)} \end{bmatrix}$$

$(1, m)$

Where X is the vectorized input and Y is their corresponding outputs. But what will happen if we have a large training set, say $m = 5,000,000$?

If we process through all of these training examples in every training iteration, the gradient descent update will take a lot of time.

Instead, we can use a mini batch of the training examples and update the weights based on them.

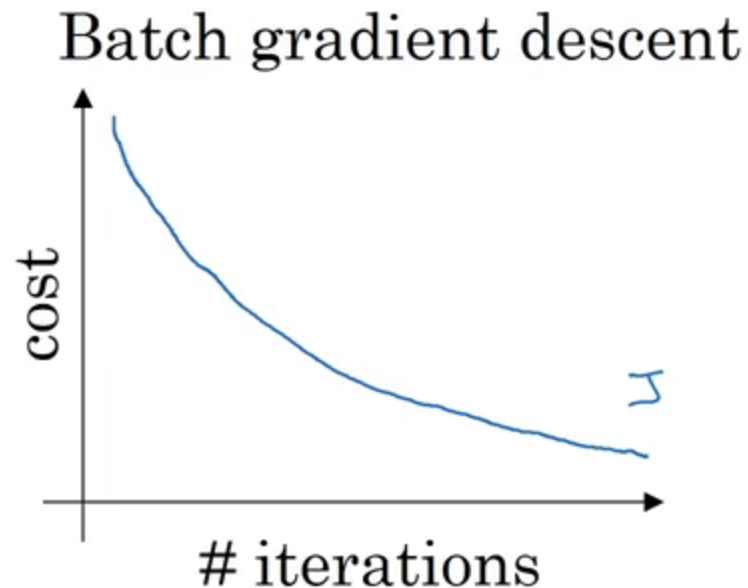
Mini-Batch Gradient

- Suppose we make a mini-batch containing 1000 examples each. This means we have 5000 batches and the training set will look like this:

$$\begin{aligned}
 X_{(n, m)} &= \left[\underbrace{x^{(1)} \ x^{(2)} \ x^{(3)} \ \dots \ x^{(1000)}}_{X_{\{1\}}} \mid \underbrace{x^{(1001)} \ \dots \ x^{(2000)}}_{X_{\{2\}}} \mid \dots \mid \underbrace{\dots \ x^{(m)}}_{X_{\{5,000\}}} \right] \\
 Y_{(1, m)} &= \left[\underbrace{y^{(1)} \ y^{(2)} \ y^{(3)} \ \dots \ y^{(1000)}}_{Y_{\{1\}}} \mid \underbrace{y^{(1001)} \ \dots \ y^{(2000)}}_{Y_{\{2\}}} \mid \dots \mid \underbrace{\dots \ y^{(m)}}_{Y_{\{5,000\}}} \right]
 \end{aligned}$$

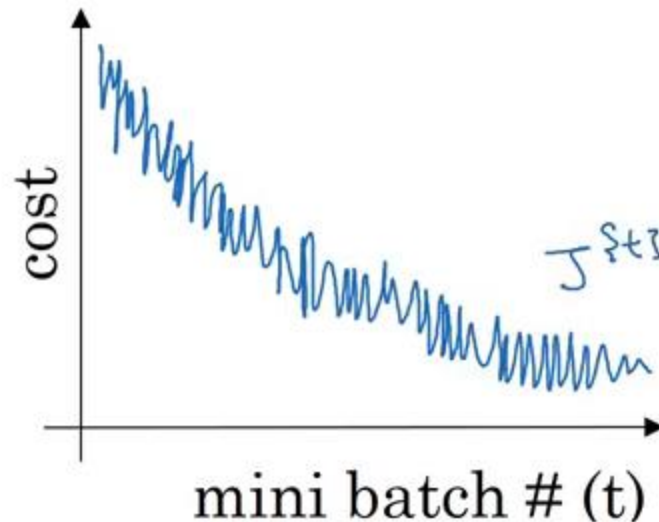
Understanding Mini-Batch Gradient Descent

- In batch gradient descent, our cost function should decrease on every single iteration:



- In the case of mini-batch gradient descent, we only use a specified set of training examples. As a result, the cost function can decrease for some iterations:

Mini-batch gradient descent



How can we choose a mini-batch size?

- **If the mini-batch size = m :**

It is a batch gradient descent where all the training examples are used in each iteration. It takes too much time per iteration.

- **If the mini-batch size = 1:**

It is called stochastic gradient descent, where each training example is its own mini-batch. Since in every iteration we are taking just a single example, it can become extremely noisy and takes much more time to reach the global minima.

- **If the mini-batch size is between 1 to m :**

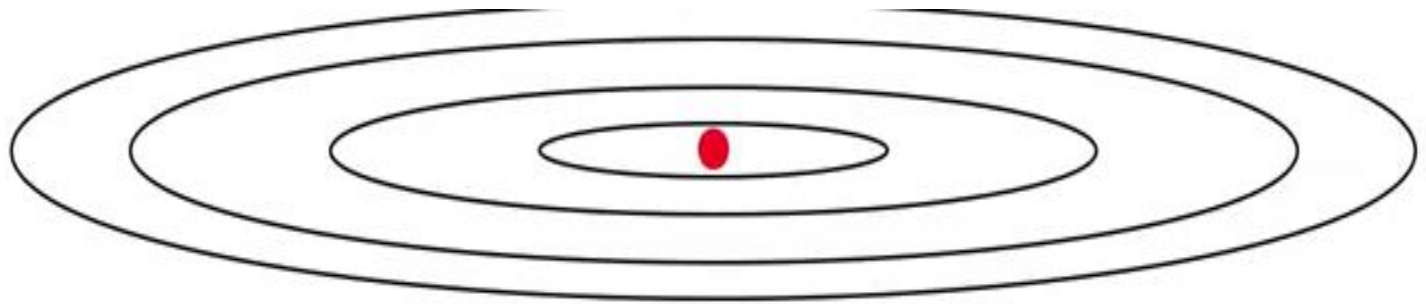
It is mini-batch gradient descent. The size of the mini-batch should not be too large or too small.


Guidelines:

- If the training set is small, we can choose a mini-batch size of $m < 2000$
- For a larger training set, typical mini-batch sizes are: 64, 128, 256, 512
- Make sure that the mini-batch size fits your CPU/GPU memory

Gradient Descent with Momentum

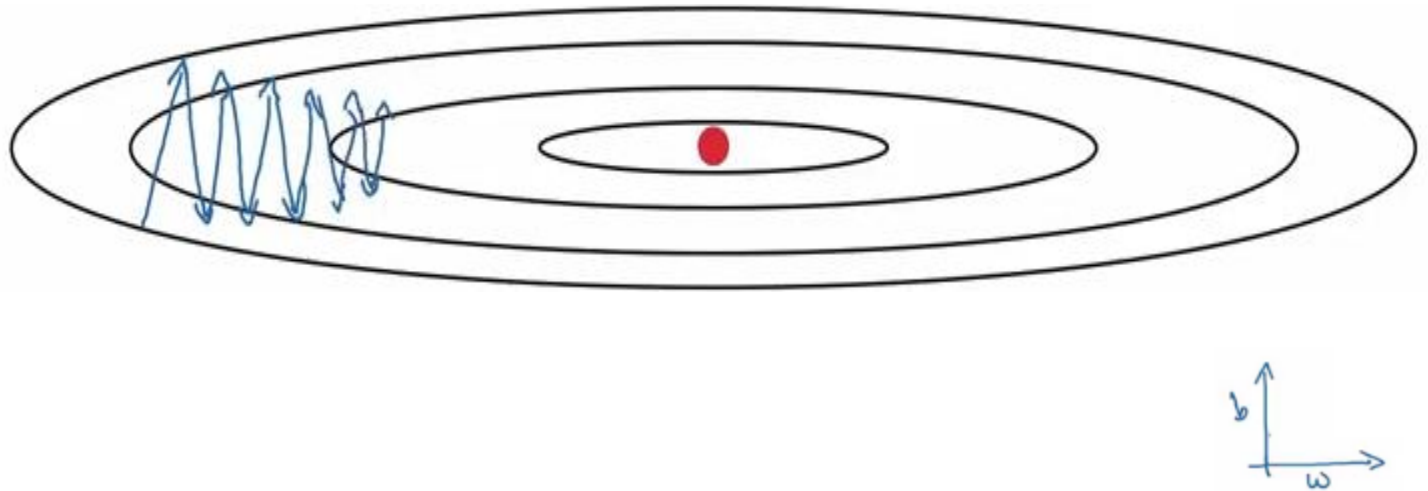
- The underlying idea of gradient descent with momentum is to calculate the exponential weighted average of gradients and use them to update weights. Suppose we have a cost function whose contours look like this:



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- One more way could be to use a larger learning rate. But that could result in large upgrade steps, and we might not reach global minima. Additionally, too small a learning rate makes the gradient descent slower.
 - We want a slower learning in the vertical direction and a faster learning in the horizontal direction which will help us to reach the global minima much faster.

RMSprop

- Consider the example of a simple gradient descent:



Look at the contour shown above and the parameters graph. We want to slow down the learning in b direction, i.e., the vertical direction, and speed up the learning in w direction, i.e., the horizontal direction

Adam

- **Adam optimization algorithm**
- Adam is essentially a combination of momentum and RMSprop.
- There are a range of hyperparameters used in Adam and some of the common ones are:
 - **Learning rate α** : needs to be tuned
 - **Momentum term β_1** : common choice is 0.9
 - **RMSprop term β_2** : common choice is 0.999
 - **ϵ** : 10^{-8}
- Adam helps to train a neural network model much more quickly than the techniques we have seen earlier.

