5/11/23

Probability Assignment - 2

1) Given:

Data points = 5.5, 10.5, 13, 22.5, 45, 55

Sample mean, $\bar{x} = 25.25$

Sample standard deviation, $s = 20.2$

Degrees of freedom, $df = 5$ $(n-1)$

sample size, $n = 6$

Null Hypothesis, H_0 : Population mean, $\mu \geq 30$

Alternative Hypothesis, H_a : $\mu < 30$

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$$

$$= \frac{25.25 - 30}{\left(\frac{20.2}{\sqrt{6}}\right)} = \frac{25.25 - 30}{(20.2/2.449)}$$

$$\boxed{t = -0.576}$$

$\alpha = 0.05$

$$\boxed{p\text{-value} = 0.29478}$$

$\because$ p-value < 0.05

$\therefore$ Rejecting the null hypothesis.

The population mean flight delay time for flights departing from the california airport is less than 30 minutes.

2) <u>Given</u>:

$x = 90$

$\mu = 68$

$\sigma = 14$

1 $z = \left(\frac{x-\mu}{\sigma}\right)$

$= \frac{90-68}{14} = 1.57$

Proportion of orders delivered after 90 mins.

$\boxed{P(z > 1.57) = 0.058}$

$\therefore$ 5.8 % of orders are delivered after 90 mins.

2 Z-score for 0.99 is 2.33

$z = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + [z \times \sigma]$

$= 68 + [2.33 \times 14]$

promised delivery time, $\boxed{x = 101.62 \text{ mins.}}$

3) Given:

$\mu = 1.50$

$\sigma = 0.2$

To find the value of d such that the specification cover 95% of the measurements.

$1.50 \pm d$.

For confidence interval 95%, z-score for 2.5th percentile (lower bound) is −1.96 and for 97.5th percentile (upper bound) is 1.96.

For lower bound, $z = \dfrac{(1.50 - d - \mu)}{\sigma}$

$$-1.96 = \frac{1.50 - d - 1.50}{0.2}$$

$$-0.392 = -d \Rightarrow \boxed{d = 0.392}$$

For upper bound, $z = \dfrac{(1.50 + d - \mu)}{\sigma}$

$$1.96 = \frac{1.50 + d - 1.50}{0.2}$$

$$\boxed{d = 0.392}$$

$$\therefore \boxed{d = \pm 0.392}$$

4) <u>Given:</u>

$$\lambda = \frac{1}{100000} = 0.00001$$

$$P(x < 50000) = 1 - e^{-\lambda x}$$

$$= 1 - e^{-(0.00001)(50000)}$$

$$= 1 - e^{-0.5} = 1 - 0.6065$$

$$= 0.3935$$

$$\boxed{P(x < 50000) = 0.3935}$$

The probability that car's transmission will fail during its first 50,000 miles of operation is 0.3935.

5) <u>Given:</u>

$$\alpha = 5000$$

$$\gamma = 0.5$$

① $$P(x < 6000) = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\gamma}}$$

$$= 1 - e^{(-6000/5000)^{0.5}}$$

$$= 0.3297$$

$$\boxed{P(x < 6000) = 0.3297}$$

The probability that a bearing lasts fewer than 6000 hours is 0.3297.

② Mean time to failure $= \alpha \times \Gamma(1 + 1/\beta)$

$$= 5000 \times \Gamma\left(1 + \frac{1}{0.5}\right)$$

$$= 5000 \times [\Gamma(3)]$$

$$= 5000 \times 2 = 10{,}000 \text{ hours}$$

The mean time to failure for the bearings is 10,000 hours.