① (b) Derive the procedure for performing Lucas-Kanade algorithm for motion tracking when the motion is known to be affine:

$$u(x,y) = a_1 x + b_1 y + c_1$$

$$v(x,y) = a_2 x + b_2 y + c_2$$

$$I_x u + I_y v \approx -I_t$$

$$I_x (a_1 x + b_1 y + c_1) + I_y (a_2 x + b_2 y + c_2) \approx -I_t$$

$$(I_x x)(a_1) + (I_x y)(b_1) + (I_x)(c_1) + (I_y x)(a_2) + (I_y y)(b_2) + (I_y)(c_2) \approx -I_t$$

$$A \bar{m} = b$$

$$\underset{n\times 6}{\begin{bmatrix} I_{x1}x_1 & I_{x1}y_1 & I_{x1} & I_{y1}x_1 & I_{y1}y_1 & I_{y1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ I_{xn}x_n & I_{xn}y_n & I_{xn} & I_{yn}x_n & I_{yn}y_n & I_{yn} \end{bmatrix}} \underset{6\times 1}{\begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \end{bmatrix}} = \underset{n\times 1}{\begin{bmatrix} -I_{t_1} \\ \vdots \\ -I_{tn} \end{bmatrix}}$$

$$\bar{m} = (A^T A)^{-1} A^T b$$

$$A^T A = \underset{6\times 6}{\begin{bmatrix} \Sigma(I_x x)^2 & \Sigma(I_x x)(I_x y) & \Sigma(I_x x)I_x & \Sigma(I_x x)(I_y x) & \Sigma(I_x x)(I_y y) & \Sigma(I_x x)I_y \\ \Sigma(I_x y)(I_x x) & \Sigma(I_x y)^2 & \Sigma(I_x y)I_x & \Sigma(I_x y)(I_y x) & \Sigma(I_x y)(I_y y) & \Sigma(I_x y)I_y \\ \Sigma I_x(I_x x) & \Sigma I_x(I_x y) & \Sigma I_x^2 & \Sigma I_x(I_y x) & \Sigma I_x(I_y y) & \Sigma I_x I_y \\ \Sigma(I_y x)(I_x x) & \Sigma(I_y x)(I_x y) & \Sigma(I_y x)I_x & \Sigma(I_y x)^2 & \Sigma(I_y x)(I_y y) & \Sigma(I_y x)I_y \\ \Sigma(I_y y)(I_x x) & \Sigma(I_y y)(I_x y) & \Sigma(I_y y)I_x & \Sigma(I_y y)(I_y x) & \Sigma(I_y y)^2 & \Sigma(I_y y)I_y \\ \Sigma I_y(I_x x) & \Sigma I_y(I_x y) & \Sigma I_y I_x & \Sigma I_y(I_y x) & \Sigma I_y(I_y y) & \Sigma I_y^2 \end{bmatrix}}$$

$$A^T b = \underset{6\times 1}{\begin{bmatrix} -\Sigma(I_x x)I_t \\ -\Sigma(I_x y)I_t \\ -\Sigma I_x I_t \\ -\Sigma(I_y x)I_t \\ -\Sigma(I_y y)I_t \\ -\Sigma I_y I_t \end{bmatrix}}$$

$$\bar{m} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \end{bmatrix} = \frac{1}{\det(A^T A)} \left[\operatorname{Adj}(A^T A) \right] A^T b$$