Computer Vision

Module 5-6

① (a) Derive the motion tracking equation from fundamental principles.

⇒ Brightness constancy

$$I(x+\delta x, y+\delta y, t+\Delta t) \approx I(x,y,t) \quad \text{—— ①}$$

⇒ Taylor Series Expansion

$$I(x+\delta x, y+\delta y, t+\Delta t) \approx I(x,y,t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\Delta t \quad \text{—— ②}$$

⇒ Optical Flow Constraint

Substituting ② in ①

$$I(x,y,t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\Delta t \approx I(x,y,t)$$

$$\frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\Delta t \approx 0$$

⇒ Motion Tracking Equation

$I_x = \frac{\partial I}{\partial x}$ (Gradient along x)

$I_y = \frac{\partial I}{\partial y}$ (Gradient along y)

$I_t = \frac{\partial I}{\partial t}$ (Gradient along t)

$$u = \frac{\delta x}{\Delta t}, \quad v = \frac{\delta y}{\Delta t}$$

$$\frac{\partial I}{\partial x}\cdot\frac{\delta x}{\Delta t} + \frac{\partial I}{\partial y}\frac{\delta y}{\Delta t} + \frac{\partial I}{\partial t}\frac{\Delta t}{\Delta t} \simeq 0$$

$$I_x u + I_y v + I_t \simeq 0$$

$$I_x u + I_y v \simeq -I_t$$

$$I_{x_1}u + I_{y_1}v \simeq -I_{t_1}$$

$$I_{x_2}u + I_{y_2}v \simeq -I_{t_2}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$I_{x_n}u + I_{y_n}v \simeq -I_{t_n}$$

$$A\bar{m} = b$$

$$\begin{bmatrix} I_{x_1} & I_{y_1} \\ I_{x_2} & I_{y_2} \\ \vdots & \vdots \\ I_{x_n} & I_{y_n} \end{bmatrix}_{n\times 2} \begin{bmatrix} u \\ v \end{bmatrix}_{2\times 1} = \begin{bmatrix} -I_{t_1} \\ -I_{t_2} \\ \vdots \\ -I_{t_n} \end{bmatrix}_{n\times 1}$$

$$\bar{m} = A^{-1}b$$

$$\bar{m}A^T = A^{-1}A^Tb$$

$$\bar{m} = (A^TA)^{-1}A^Tb$$

$$A^TA = \begin{bmatrix} I_{x_1} & I_{x_2} & \cdots & I_{x_n} \\ I_{y_1} & I_{y_2} & \cdots & I_{y_n} \end{bmatrix}_{2\times n} \begin{bmatrix} I_{x_1} & I_{y_1} \\ I_{x_2} & I_{y_2} \\ \vdots & \vdots \\ I_{x_n} & I_{y_n} \end{bmatrix}_{n\times 2}$$

$$= \begin{bmatrix} \sum_{i=1}^{n} I_{xi}^2 & \sum_{i=1}^{n} I_{xi}I_{yi} \\ \sum_{i=1}^{n} I_{yi}I_{xi} & \sum_{i=1}^{n} I_{yi}^2 \end{bmatrix}_{2\times 2}$$

$$A^T b = \begin{bmatrix} I_{x1} & I_{x2} & \cdots & I_{xn} \\ I_{y1} & I_{y2} & \cdots & I_{yn} \end{bmatrix}_{2\times n} \begin{bmatrix} -I_{t1} \\ -I_{t2} \\ \vdots \\ -I_{tn} \end{bmatrix}_{n\times 1}$$

$$= \begin{bmatrix} \sum_{i=1}^{n} -I_{xi} I_{ti} \\ \sum_{i=1}^{n} -I_{yi} I_{ti} \end{bmatrix}_{1\times 1}$$

$$\overline{m} = \left(\begin{bmatrix} \sum_{i=1}^{n} I_{xi}^2 & \sum_{i=1}^{n} I_{xi} I_{yi} \\ \sum_{i=1}^{n} I_{yi} I_{xi} & \sum_{i=1}^{n} I_{yi}^2 \end{bmatrix} \right)^{-1} \begin{bmatrix} \sum_{i=1}^{n} -I_{xi} I_{ti} \\ \sum_{i=1}^{n} -I_{yi} I_{ti} \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{\det(A^T A)} \begin{bmatrix} \sum_{i=1}^{n} I_{yi}^2 & -\sum_{i=1}^{n} I_{xi} I_{yi} \\ -\sum_{i=1}^{n} I_{yi} I_{xi} & \sum_{i=1}^{n} I_{xi}^2 \end{bmatrix} \begin{bmatrix} -\sum_{i=1}^{n} I_{xi} I_{ti} \\ -\sum_{i=1}^{n} I_{yi} I_{ti} \end{bmatrix}$$