

Homework Assignment 4

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1. (a) For the 2-dimensional XOR problem, we select the following four basis vectors:

$$\mathbf{r}^1 = [-1, -1]^\top$$

$$\mathbf{r}^2 = [1, 1]^\top$$

$$\mathbf{r}^3 = [-1, 1]^\top$$

$$\mathbf{r}^4 = [1, -1]^\top.$$

Show that the XOR problem is solved by the radial basis function network with the following weight vector:

$$\mathbf{w} = [1, 1, -1, -1, 0]^\top.$$

2. A nearest-neighbour classifier can be constructed as a radial basis function network by selecting all the input vectors in a training set as basis vectors. In a multi-class classification setting (i.e., there are more than two categories), provide a description on how a weight matrix could be built.

3. Unlike a fixed basis function network, an adaptive basis function network adapts basis vectors so as to maximize the classification accuracy (i.e. to minimize the empirical cost.) In order to do so, we need to be able to compute the gradient of the (logistic regression) distance function with respect to each and every basis vector. Derive this gradient

$$\nabla_{\mathbf{r}^k} D(y^*, M, \phi(\mathbf{x})),$$

assuming that M is a logistic regression classifier and that

$$\phi(\mathbf{x}) = \begin{bmatrix} \exp(-(\mathbf{x} - \mathbf{r}^1)^2) \\ \vdots \\ \exp(-(\mathbf{x} - \mathbf{r}^K)^2) \end{bmatrix}.$$

4. PROGRAMMING ASSIGNMENT