01_counting

Sunday, January 5, 2020 11:13 PM

01_counting

CS109: Probability for Computer Scientists

David Varodayan

January 6, 2020

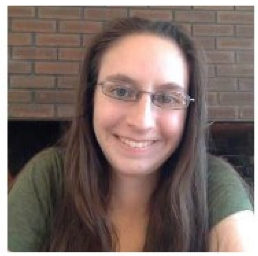
Slides adapted from those written by Lisa Yan

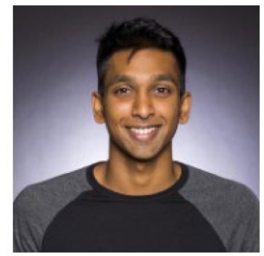
Welcome to CS109!

David Varodayan







What about you?

Today's plan

Course Mechanics

Why you should take CS109

Counting!

Course mechanics (light version)

- For more info, read the Administrivia handout
- Course website:

  http://cs109.stanford.edu/

Prerequisites

CS106B/X

Programming
Recursion
Hash tables
Binary trees

MATH 51/CME 100

Multivariate differentiation
Multivariate integration
Basic facility with linear algebra (vectors)

CS103
(co-requisite OK)

Proofs (induction)
Set theory
Math maturity

Important!

we will review

Staff contact

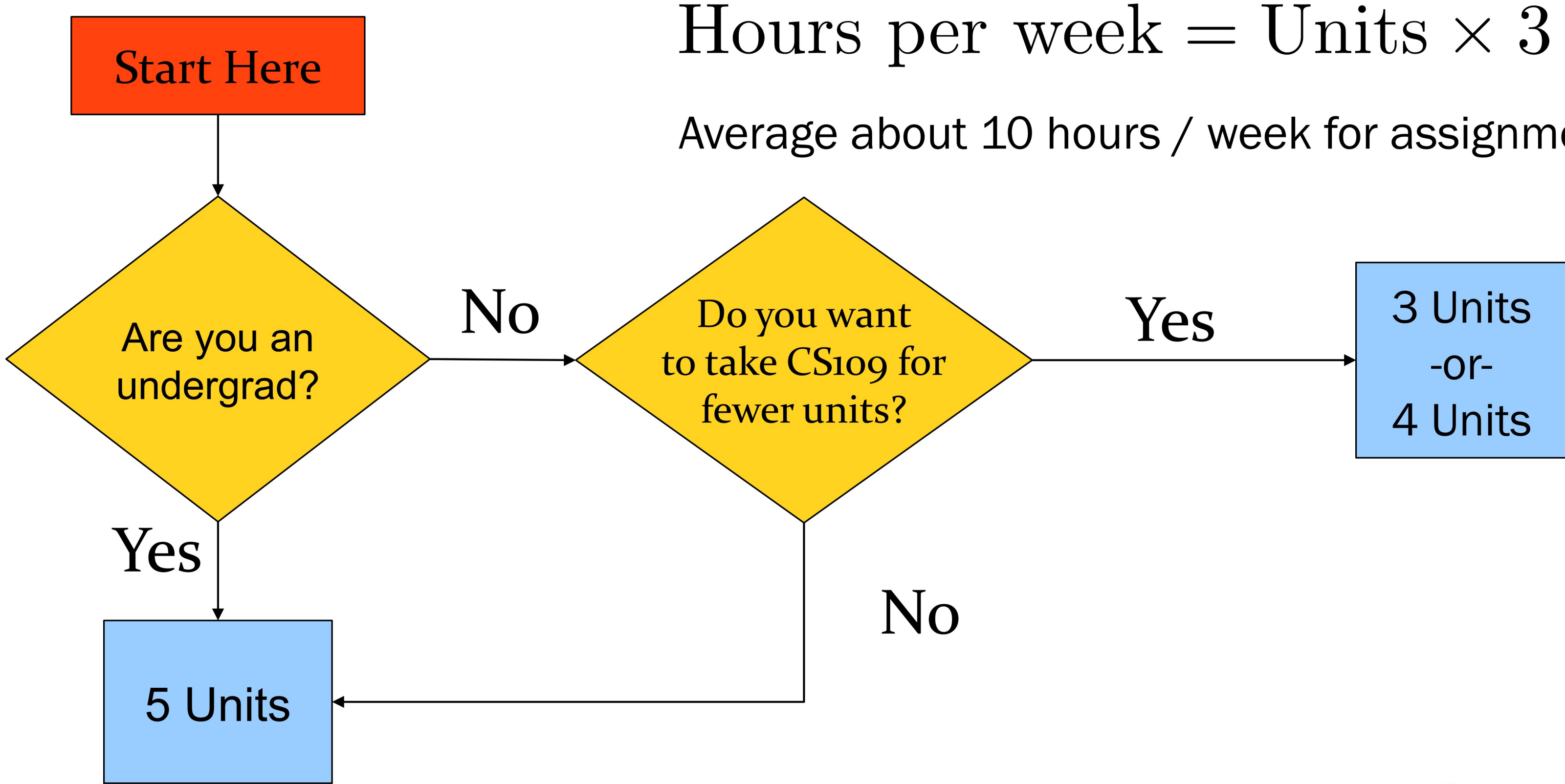
- Piazza
- Email cs109@cs.stanford.edu
- Working office hours
- Contact David for course level issues, extensions, etc.

varodayan@stanford.edu

How many units should I take?

$$\text{Hours per week} = \text{Units} \times 3$$

Average about 10 hours / week for assignments



Where you learn

- Lectures (not videotaped)
- Lecture notes (on website)
- Textbook readings (optional)
- Discussion Section → starting week 2
- Problem Sets

Class breakdown

45%	**6 Problem Sets**
20%	**Midterm** Monday, February 10, 7:00–9:00pm
30%	**Final** Wednesday, March 18, 3:30–6:30pm
5%	**Section Participation**

Problem Sets

Late Days:

2

(class days)
(for Problem Sets only)

require

python 3

Review session
(time/location TBA)

CS193Q: Introduction
to Python Programming

Stanford Honor Code

Permitted

- Talk to the course staff
- Talk with classmates (cite collaboration)
- Look up general material online

NOT permitted:

- Copy answers:
  from classmates
  from former students
  from previous quarters
- Copy answers from the internet
  Besides, these are usually incorrect

Questions on logistics?

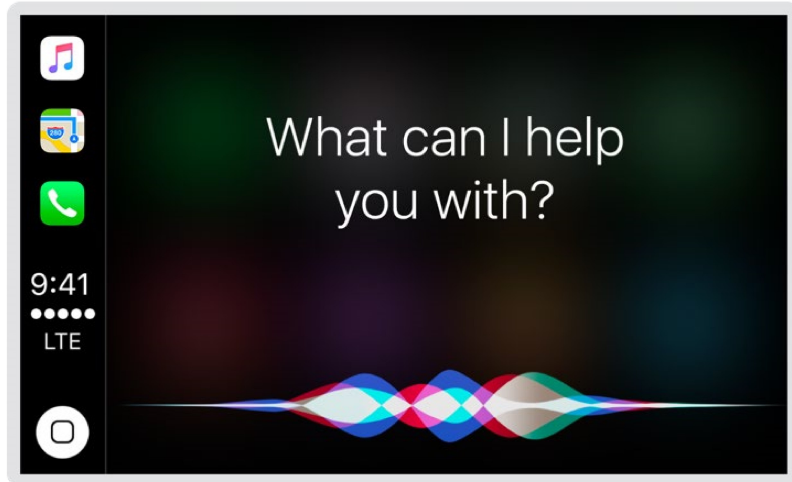
Today's plan

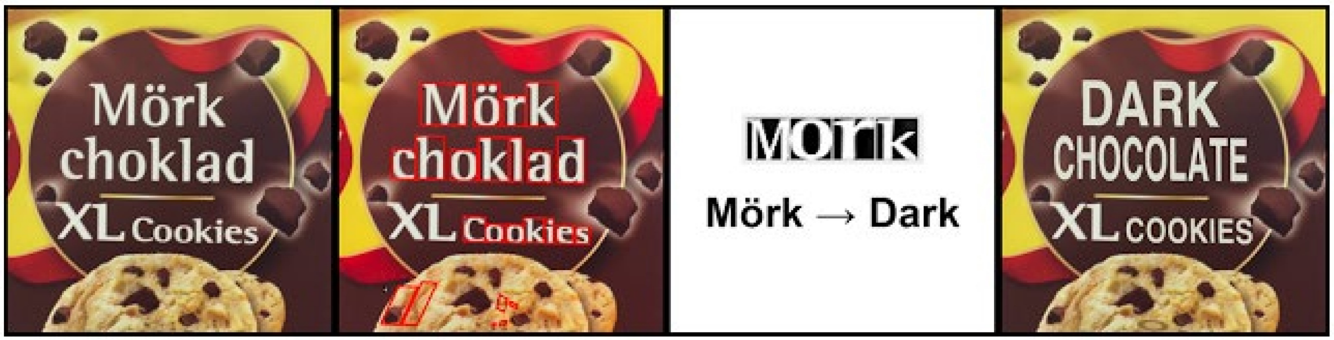
Course Mechanics

Why you should take CS109 → for the love of learning

Counting!

Machine Learning





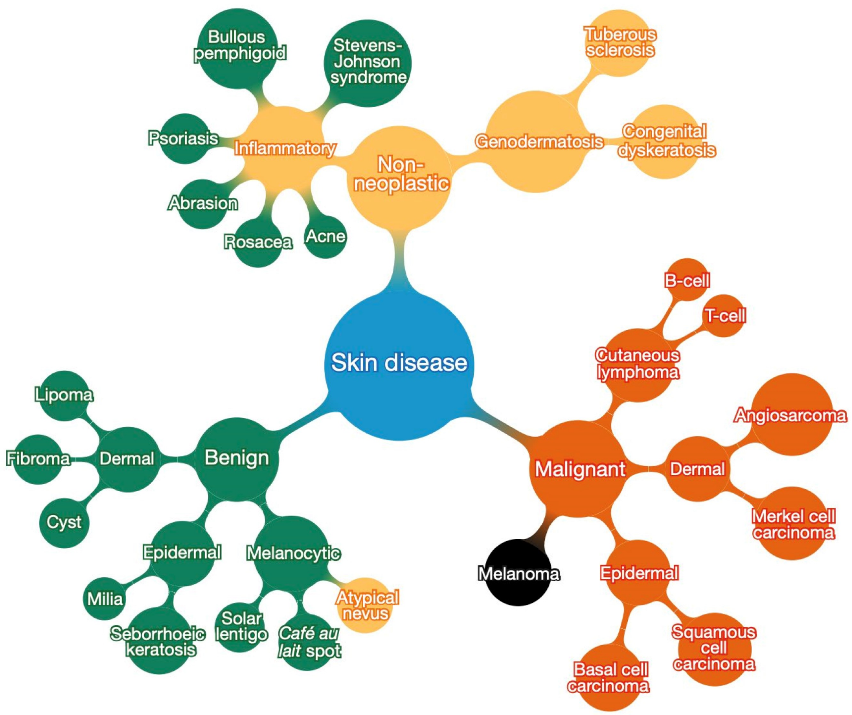
Probability

Computation

What is Machine Learning?

Data

Machine learning for good



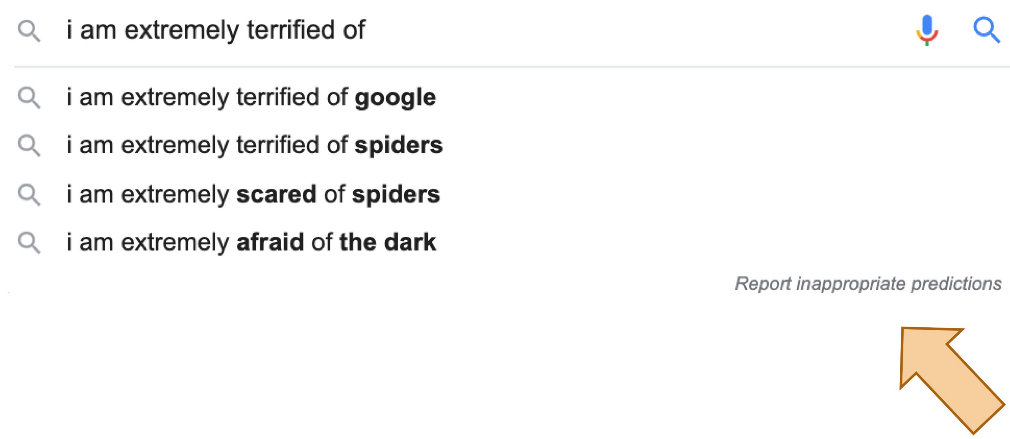
A machine learning algorithm performs **better than** the best dermatologists

Developed in 2017 at Stanford

Esteva, Andre, et al. "Dermatologist-level classification of skin cancer with deep neural networks." *Nature* 542.7639 (2017): 115-118.

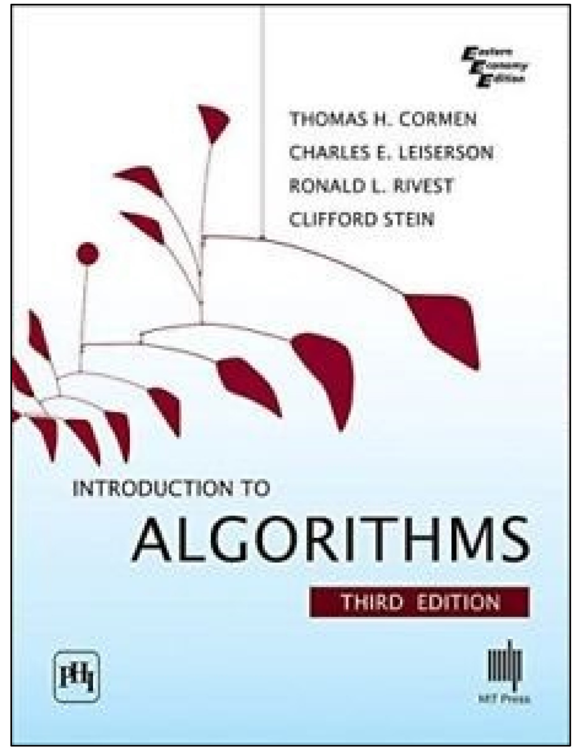
Machine learning not good

Algorithms of Oppression,
Safiya Umoja Noble. 2018



Probability is more than just machine learning

Clinical Trials



CS 161

Probability is not always intuitive

A patient takes a Zika test that returns positive.

What is the probability that they have the Zika virus?

- 0.8% of people have the virus
- Test has 90% positive rate for people with the virus
- Test has 7% positive rate for people without the virus

90%? 50%? 7-9%? 9%

Today's plan

Course Mechanics

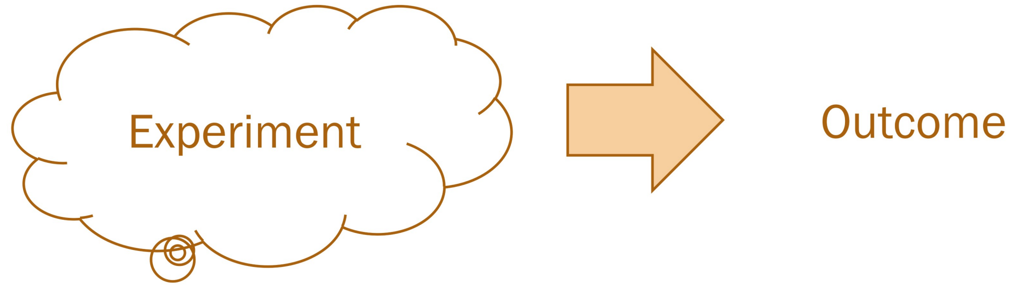
Why you should take CS109

Counting!

01: Counting

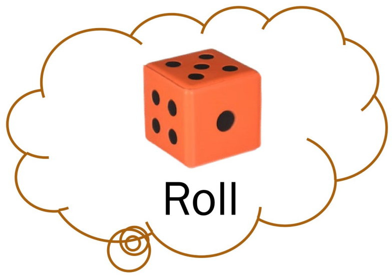
What is Counting?

An experiment in probability:



Counting: How many possible outcomes can occur from performing this experiment?

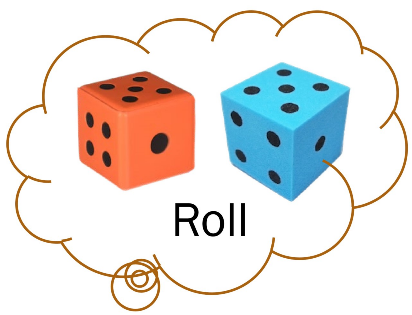
What is Counting?



6

{1, 2, 3, 4, 5, 6}

Outcomes



36

{(1, 1) , (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1) , (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1) , (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1) , (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1) , (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1) , (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Sum Rule of Counting

Cardinality of A

If the outcome of an experiment can be either from

Set A, where $|A| = m$,

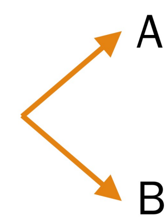
or Set B, where $|B| = n$,

where $A \cap B = \emptyset$,

Then the number of outcomes of the experiment is

$|A| + |B| = m + n$.

One experiment



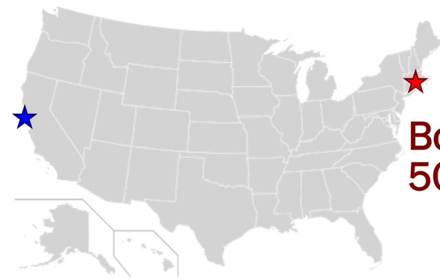
Video streaming application

A $\quad A \cap B = \emptyset$

B $\quad |A| + |B| = m + n$

Your application has distributed servers in 2 locations.

If a server request is sent to the application, how large is the set of servers it can get routed to?



San Jose
100 servers

Boston
50 servers

A

B

$A \cap B \neq \emptyset$

$|A| + |B| = 150$

Product Rule of Counting

If an experiment has two parts, where

The first part's outcomes are from Set A, where $|A| = m$, and

The second part's outcomes are from Set B, where $|B| = n$,

Then the number of outcomes of the experiment is

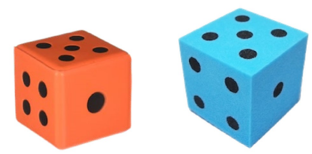
$$|A||B| = mn.$$

Two-step experiment

$\longrightarrow$ A $\longrightarrow$ B

Dice

How many possible outcomes are there from rolling two 6-sided dice?



$|A| = 6$ $|B| = 6$

$|A||B| = 36$

Inclusion-Exclusion Principle

If the outcome of an experiment can be either from

Set A or set B,

where A and B may overlap,

Then the total number of outcomes of the experiment is

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

One experiment

A

B only

Sum Rule of Counting:
A special case

Transmitting bytes over a network

A

B only

Inclusion-Exclusion Principle
$|A \cup B| = |A| + |B| - |A \cap B|$

An 8-bit string is sent over a network.

- The receiver only accepts strings that either start with 01 or end with 10.

`01001100`

$\underbrace{}_{\text{byte (8 bits)}}$

byte (8 bits)

How many 8-bit strings will the receiver accept?

Define
A : 8-bit strings starting with 01
B : 8-bit strings ending with 10

0 1 ? ? ? ? ? ?

? ? ? ? ? ? 1 0

$A \cap B$? 0 1 ? ? ? ? 1 0

$|A| = 2^6 = 64$

$|B| = 2^6 = 64$

$|A \cap B| = 2^4 = 16$

$|A \cup B| = 64 + 64 - 16 = 112$

General Principle of Counting

If an experiment has r steps, such that

Step i has n_i outcomes for all $i = 1, \dots, r$,

Then the number of outcomes of the experiment is

$$n_1 \times n_2 \times \cdots \times n_r = \boxed{\prod_{i=1}^{r} n_i \, .}$$

Multi-step experiment

$\rightarrow$ 1 $\rightarrow$ 2 $\rightarrow$ ...

Product Rule of Counting: A special case

License plates

→ 1 → 2 → …

General Principle of Counting
$n_1 \times n_2 \times \cdots \times n_r = \prod_{i=1}^{r} n_i$

How many CA license plates are possible if…

(pre-1982)

$26 \times 26 \times 26 \times 10 \times 10 \times 10$
$= 17\ 576\ 000$

(present day)

175 760 000

Floors and ceilings

Floor function

$$\lfloor x \rfloor$$

The largest integer $\leq x$

Ceiling function

$$\lceil x \rceil$$

The smallest integer $\geq x$

Check it out:

$\lfloor 1/2 \rfloor = 0$ $\quad \lfloor 2.9 \rfloor = 2$ $\quad \lfloor 8.0 \rfloor = 8$ $\quad \lfloor -1/2 \rfloor = -1$

$\lceil 1/2 \rceil = 1$ $\quad \lceil 2.9 \rceil = 3$ $\quad \lceil 8.0 \rceil = 8$ $\quad \lceil -1/2 \rceil = 0$

Pigeonhole Principle

For positive integers m and n,

if m objects are placed in n buckets,

then at least one bucket must contain at least $\lceil m/n \rceil$ objects.

Example:

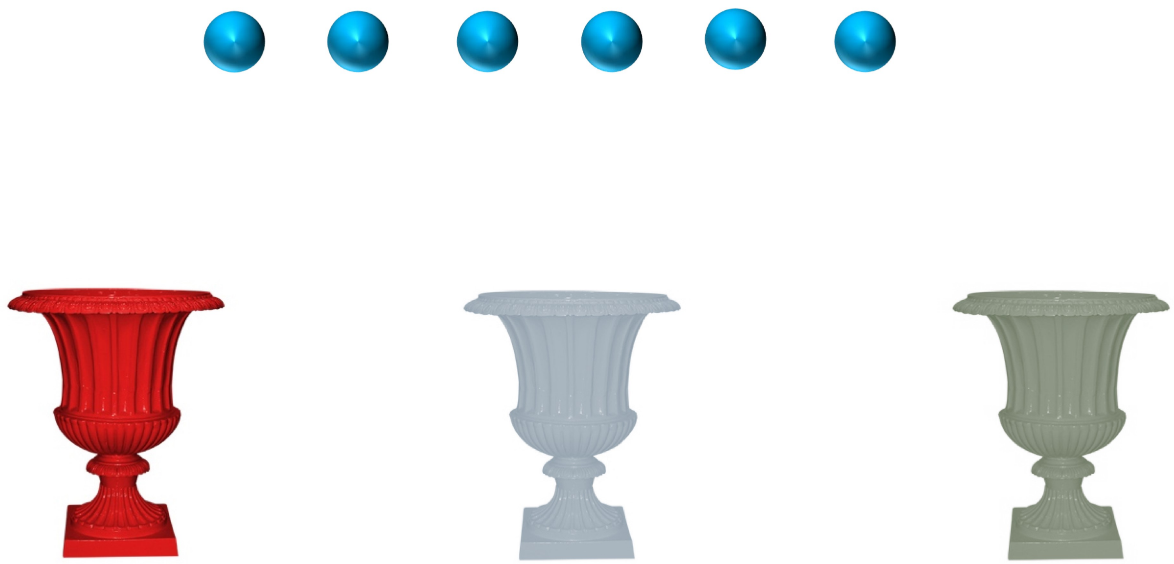
m objects = 10 pigeons

n buckets = 9 pigeonholes

At least one pigeonhole must contain $\lceil m/n \rceil = 2$ pigeons.

Balls and urns

≥ 1 bucket must contain at least $\lceil m/n \rceil$ objects



m balls

n urns (buckets)

~~Balls and urns~~ Hash tables and strings

≥ 1 bucket must contain at least $\lceil m/n \rceil$ objects

Consider a hash table with 100 buckets.

950 strings are hashed and added to the table.

$$n = 100$$
$$m = 950$$

1. Is it guaranteed that at least one bucket contains *at least* 10 entries? $\left\lceil \frac{950}{100} \right\rceil = 10$ Yes
2. Is it guaranteed that at least one bucket contains **at least** 11 entries? No
3. Is it possible to have a bucket with ***no entries***? Yes

Takeaways from this lecture

Inclusion-Exclusion Principle (generalized Sum Rule)

If the outcome of an experiment can be either from Set A **or** set B, where A and B may overlap, then the total number of outcomes of the experiment is

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

One-step experiment

A

B exclusive

General Principle of Counting (generalized Product Rule)

If an experiment has r **steps**, such that step i has n_i outcomes for all $i = 1, \dots, r$, then the total number of outcomes of the experiment is

$$n_1 \times n_2 \times \cdots \times n_r = \prod_{i=1}^{r} n_i \,.$$

Multi-step experiment

$\rightarrow 1 \rightarrow 2 \rightarrow \dots$

Unique 6-digit passcodes

How many unique 6-digit passcodes are possible?

__ __ __ __ __ __

$10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$

Unique 6-digit passcodes with six smudges

How many unique 6-digit passcodes are possible if a phone password uses each of six distinct numbers?

$\_\ \_\ \_\ \_\ \_\ \_$

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$

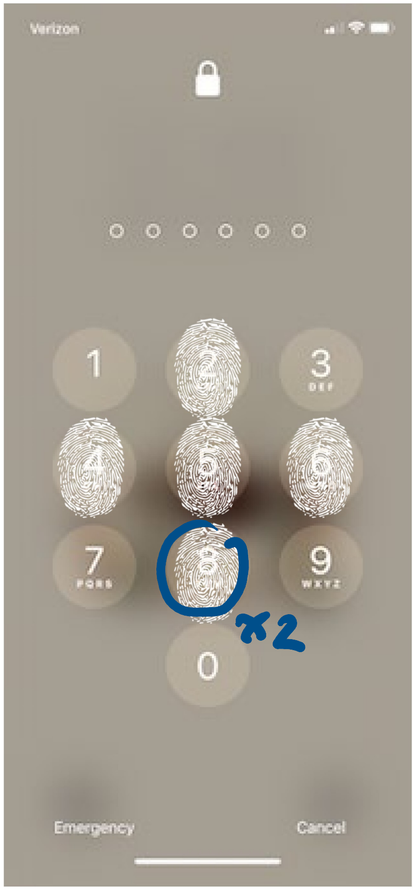
Permutations

A permutation is an ordered arrangement of distinct objects.

The number of unique orderings (permutations) of n distinct objects is

$$\boxed{n!} = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1.$$

Unique 6-digit passcodes with five smudges



How many unique 6-digit passcodes are possible if a phone password uses each of **five** distinct numbers?

<u>Steps</u>:

1. Choose digit to repeat   5
2. Create passcode $\longrightarrow \frac{6!}{2}$

$$5 \times \frac{6!}{2} = 1800$$

2 smudges

$2^6 - 2 = 62$

remove the ways that leave only one smudge

3 smudges

$3^6 - 3 - 3 \cdot 62 = 540$

remove the ways that leave exactly 2 smudges

4 smudges

1 digit 3x

$4 \dfrac{6!}{3!} = 480$

2 digits 2x

$6 \dfrac{6!}{2 \cdot 2} = 1080$

$\underbrace{480 \qquad 1080}_{1560}$