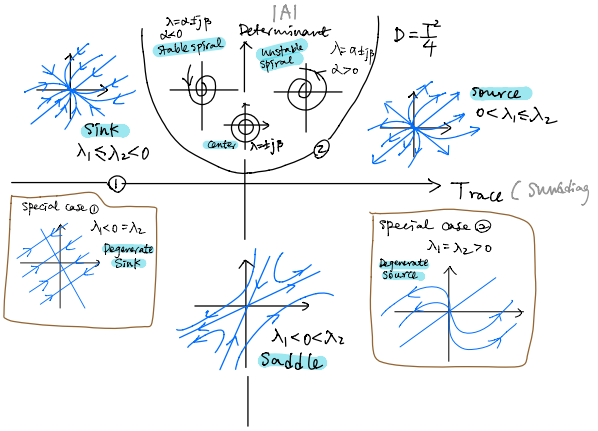
Welcome to 21 260 Section C!



Instructor: Lavender (Yao) Jiang

Office Hours: Tues 4-6 pm, Sun 7:30-8:30 pm, zoom link on Canvas

Goal of recitation: help you learn diffEq better! Review key concepts and practice problems

Please try to participate :)

Today's agenda:

2:45 - 3:30 Concepts Review with examples and practices (10 total)
3:30 - 3:40 Q&A

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1. What is a differential equation?

An equation involving the derivative of an unknown function, solution satisfy the equation
(Comparison with algebraic equation: unknown function VS unknown variables)
e.g.

(i) $y' = 3y$ take $f(x) = e^{3x} \Rightarrow f'(x) = 3e^{3x} = 3f(x)$
$\Rightarrow e^{3x}$ is a solution to (i)

(ii) $x^2 = 3x \Rightarrow x(x-3) = 0 \Rightarrow x = 0$ or $x = 3$ are solutions to (ii)

2. What is the difference between ordinary & partial differential equation? (ODE vs PDE)

ODE has derivative to only 1 variable, PDE has partial derivatives to more than 1 variable
e.g.

$S(t, x, y) : \mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}$ (your sleepiness)

(i) $\frac{dS}{dt} = 2S$ (ii) $\frac{\partial S}{\partial t} = \frac{\partial S}{\partial x} + \frac{\partial S}{\partial y}$

$\nabla_{\vec{b}} \frac{dS}{dt} = \frac{\partial S}{\partial t}$

3. What is the order of a differential equation?

The order is the degree of the highest derivative found in the equation.
e.g.

$f(x) = x'' + x$ $g(m) = m''' + m'' + 1$ $h(t) = t^9 + t^{101} + t$

!! $f \in C^2(\mathbb{R})$, $g \in C^3(\mathbb{R})$, $h \in C^{101}(\mathbb{R})$

C^n: space of functions with n continuous derivatives

4. What makes a differential equation linear?

A differential equation $\mathcal{L}(x, y, \frac{dy}{dx}, \dots, \frac{d^n y}{dx^n}) = 0$ is linear if $\mathcal{L}$ is linear.

Equivalent definition: for any two its solutions g,h, the linear combination of the two solutions are also a solution to the differential equation.

$\alpha g + \beta h$, $\alpha, \beta \in \mathbb{R}$

e.g.

$y' = 3y$ linear?

Linear function $\mathcal{L}$ is linear if $\mathcal{L}: A \to B$, $g, h \in A$, $\mathcal{L}(\alpha g + \beta h) = \alpha \mathcal{L}(g) + \beta \mathcal{L}(h)$

① $\mathcal{L}(y) = y' - 3y$

$\mathcal{L}(\alpha g + \beta h) = (\alpha g + \beta h)' - 3(\alpha g + \beta h)$

$= \alpha g' + \beta h' - 3\alpha g - 3\beta h$

$\alpha \mathcal{L}(g) + \beta \mathcal{L}(h) = \alpha(g' - 3g) + \beta(h' - 3h)$

$= \alpha g' - 3\alpha g + \beta h' - 3\beta h = \mathcal{L}(\alpha g + \beta h)$ ✓

② Let g, h be two solutions for the diff Eq.

then $g' = 3g$ and $h' = 3h$

consider $z := \alpha g + \beta h$, $\alpha, \beta \in \mathbb{R}$

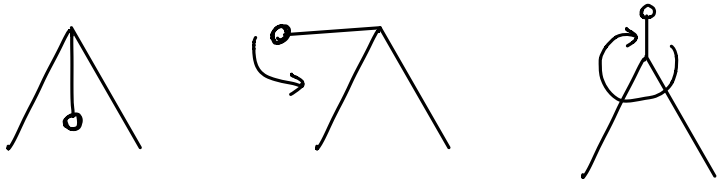
Does z satisfy the diff Eq?

$z' = \alpha g' + \beta h' = 3\alpha g + 3\beta h = 3(\alpha g + \beta h) = 3z$ ✓

5. What is a linear first-order differential equation?

6. What is an IVP (initial value problem)?

A diffEq with given initial condition
(which helps us find a specific solution, rather than a set of possible solutions)
e.g. pendulums



7. What is the general form of a first order differential equation?

$$F(x, y, \frac{dy}{dx}) = 0$$

e.g. $y' = 3y + x \Rightarrow x + 3y - \frac{dy}{dx} = 0 \Rightarrow F(x, y, \frac{dy}{dx}) = x + 3y - \frac{dy}{dx}$

8. What is a separable differential equation?

A 1st order diffEq with the form $\frac{dy}{dx} = f(x,y)$ is said to be separable if $f(x,y) = g(x)\,h(y)$

(we can "factor" or "separate" x and y)

9. How can we solve a separable diffEq?

$$f(x,y) = g(x)\,h(y)$$

$$\frac{1}{h(y)}\frac{dy}{dx} = g(x)$$

$$\frac{d}{dx}\left(\ln(h(y))\right) = g(x)$$

$$\int \frac{d}{dx}\left(\ln(h(y))\right)dx = \int g(x)\,dx + C$$

$$\ln(h(y)) = \int g(x)\,dx + C$$

$$h(y) = e^{\int g(x)dx + C}$$

e.g. ① find solution to the diff Eq

$$\frac{dy}{dx} = x(y-3)$$

$$\frac{1}{y-3}\frac{dy}{dx} = x$$

$$\int \frac{1}{y-3}\frac{dy}{dx}\,dx = \int x\,dx + C_1$$

$$\ln|y-3| = \frac{x^2}{2} + C_1$$

$$|y-3| = e^{\frac{x^2}{2}+C_1} = C_2 e^{\frac{x^2}{2}}$$

$$y-3 = C_2 e^{\frac{x^2}{2}}$$

I.C. $y(0) = 1$

$$1-3 = C_2 e^{0} \Rightarrow \boxed{-2 = C_2}$$

$$y-3 = -2e^{\frac{x^2}{2}}$$

$$y = -2e^{\frac{x^2}{2}} + 3$$

② what if we make ① an IVP?

③ Solve the diff Eq $\frac{dy}{dx} = \frac{3x^2+2}{5y^4+4y^3+2y}$

$$(5y^4+4y^3+2y)\frac{dy}{dx} = 3x^2+2$$

$$\int (5y^4+4y^3+2y)\frac{dy}{dx}\,dx = \int (3x^2+2)\,dx$$

$$y^5+y^4+y^2 = x^3+2x+C$$

"implicit solution"

10. Finding the domain of solutions to IVPs (HW Q4)

① what is the domain of a function?

$\text{Dom}(\sin) = \mathbb{R}$ $\quad \text{Dom}(\arcsin) = [-\frac{\pi}{2}, \frac{\pi}{2}]$

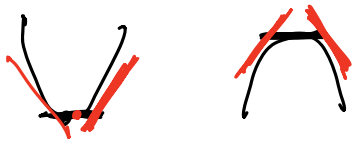
② Suppose you found a function as a solution to a IVP, $g(t) = \frac{1}{3-6t+t^2}$

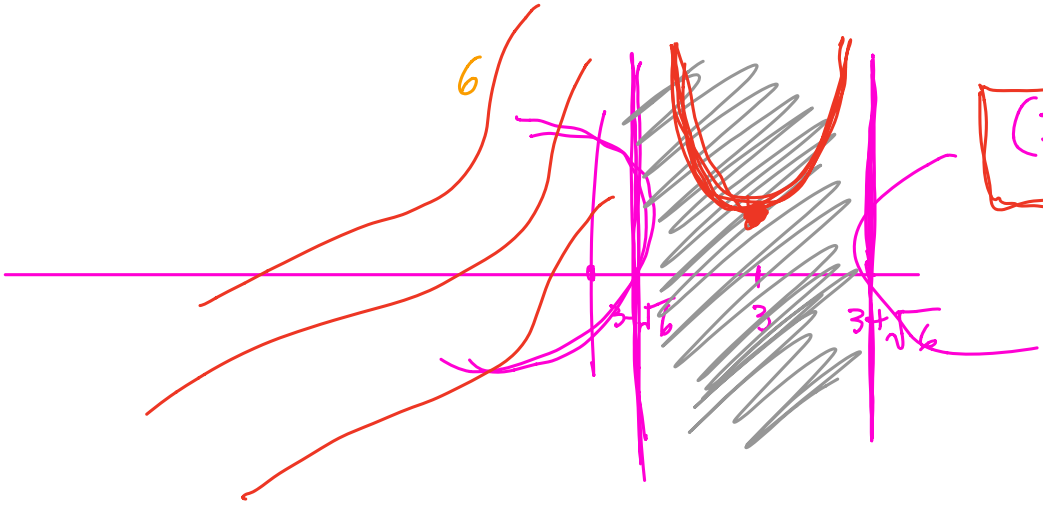
what is the domain of the function?

what is the min of the solution?

t s.t. $g'(t) = 0$

$f(t)$





$(3-\sqrt{6}, 3+\sqrt{6})$