Q. Is (1) $y' + y = 6$ a linear diff Eq?

① $L(y) = y' + y$

$$L(\alpha u + \beta v) = (\alpha u + \beta v)' + \alpha u + \beta v$$
$$= \alpha u' + \beta v' + \alpha u + \beta v$$
$$= \alpha(u' + u) + \beta(v' + v) = \alpha L(u) + \beta L(v)$$

② the homogeneous form of (1) is

$$y' + y = 0 \quad (2)$$

(we remove any terms that do not depend on y to obtain the homogeneous form)

Let u, v be sol to (2) then

$$u' + u = 0$$
$$v' + v = 0$$

Is $\alpha u + \beta v$ a solution to (2)?

$$(\alpha u + \beta v)' + \alpha u + \beta v = \alpha u' + \alpha u + \beta v' + \beta v$$
$$= \alpha(u' + u) + \beta(v' + v) = 0 + 0 = 0 \quad \checkmark$$