

Complex Numbers

The set of Complex numbers \mathbb{C} consist of ordered pairs (a, b) , $a, b \in \mathbb{R}$

If $z = (a, b)$, $w = (c, d)$, we say $z = w$ iff $a = c$ AND $b = d$

$$\begin{cases} z + w = (a + c, b + d) \\ z \cdot w = (ac - bd, ad + bc) \\ \theta = (0, 0), 1 = (1, 0) \end{cases}$$

If $z \neq 0$, define $z^{-1} = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$

Then \mathbb{C} with these operations is a field with θ playing the role of 0, 1 the role of 1.

Notation: $i = (0, 1)$, $i^2 = (0, 1)(0, 1) = (-1, 0)$

For $z \in \mathbb{C}$, $z = (a, b)$ and $b = 0$, we call it purely real, and simply write $z = a$

more generally, write $z = a + bi$, $a = \operatorname{Re}(z)$, $b = \operatorname{Im}(z)$