Planning & Scheduling
Assignment 4

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1 Graded Exercise 4.2

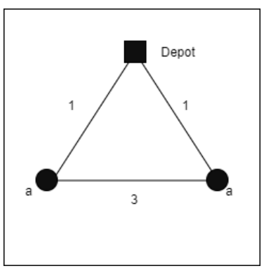

Figure 1

1. In the simple example shown in figure 1, the triangular inequality - which can be formalized as $\forall e = \{u, v\} \in E$ and for every vertex x, we have

$$w(u, v) \leq w(u, x) + w(x, v) \tag{1}$$

does not hold since,

$$w(a, b) = 3 \leq w(0, a) + w(0, b) = 1 + 1 = 2 \tag{2}$$

Specifically, in terms of tours, we have:

- For k=1, $C = (0, a, b, 0)$ where $w(C) = 1 + 3 + 1 + 5$.
- For k=2,
  - $C_1 = (0, a, 0)$ which $w(C_1) = 1 + 1 = 2$
  - $C_2 = (0, b, 0)$ which $w(C_2) = 1 + 1 = 2$

Hence,

$$w(C) = 5 > \sum_{1}^{2} w(C_i) = 4 \tag{3}$$

and thus, since the goal is to find delivery tours of minimum total length $\sum w(C_i), \forall j = \{1, ..., k\}$ we can conclude that for the above example using two tours is better than using a single one.

2. Considering again the above example but this time with $w(e) \in \{1, 2, \}$. We can show that in this case the triangular inequality holds:

- For k=1, $C = (0, a, b, 0)$ where $w(C) = 1 + 2 + 1 = 4$
- For k=2,
  - $C_1 = (0, a, 0)$ which $w(C_1) = 1 + 1 = 2$
  - $C_2 = (0, b, 0)$ which $w(C_2) = 1 + 1 = 2$

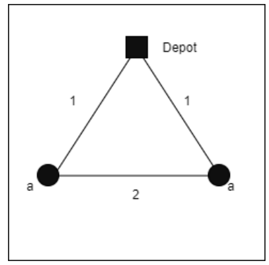

Figure 2

Thus, $w(C) = 4 = \sum_1^2 w(C_i) = 4$ which implies that the solution for k=1 is no worse than the one for k=2.

Thus, for $w(e) \in \{1, 2\} \forall e \in E$ the triangular inequality always holds since for all possible edge-weights sums i.e. 1+1=2, 1+2=3 and 2+2=4, either of them will always exceed or be equal to a single edge weight.

More formally, denoting the three edges e_1, e_2 and e_3 with $w(e_i) \in \{1, 2, \} \forall i = \{1, 2, 3\}$ we can assume that

$$w(e_1) + w(e_2) + w(e_3) \leq 2w(e_1) + 2w(e_3) \tag{4}$$

where the LHS is at most 6 (and at least 3) and the RHS is at most 8 (and at least 4). This can be simplified to

$$w(e_2) \leq w(e_1) + w(e_3) \tag{5}$$

which in the worst case would be $w(e_2) = w(e_1) + w(e_3)$, implying that one tour is equivalent to using two tours.

In general (i.e. for different shapes and more vertices/edges), the above reasoning also applies in the sense that if the triangular inequality holds one is essentially allowed for short cutting between vertices (i.e. customers) and thus the given graph breaks down into "several triangles" for which using one tour is in the worst case equivalent to using more than one.