Multi-stage Planning for Electrifying Transit Bus Systems with Multiformat Charging Facilities
Transit - 96

IDEA 2nd Quarterly Progress Report:

Mathematical Models and Solution Algorithms

1 Problem Description

This project provides a decision support tool to public transit authorities for facilitating the electrification of their transit buses. According to the periodical budget and transit network features, the multi-stage planning framework will provide the decisions at different stages including:

(1) Identify which routes the acquired electric buses should serve;
(2) Identify where to deploy charging facilities (both plug-in at stations and dynamic wireless charging facilities (DWCF) embedded in road pavement);
(3) Identify what should be the right size of onboard battery for a specific route.

The dynamic design of the electric bus system needs to optimize the cost-effectiveness and environmental benefits by balancing short- and long-term system performance.

The mathematical models in this manuscript consider two electric bus operational strategies. One strategy fixes the electric buses to serve on the selected bus lines. The other one allows the rotation of electric buses in all bus lines to ensure each bus line has the opportunity to access service of the electric buses. The strategy fixing the electrification on exact bus lines requires the mathematical model to select the bus lines to be electrified from the bus network first. The strategy considering rotation of electric buses need to electrify the entire bus network.

Both models for the planning problem of the electrification of transit bus system will optimize two objectives. One is the total investment cost for deploying the DWCF. The other one is the greenhouse gas (GHG) emission originated from the energy consumption of the electric buses. The constraints of the model need to ensure the electric buses operate at a certain level of the state of charge (SOC) of the battery. The models are formulated as bi-objective mixed integer programs. We apply the weight sum method to solve this bi-objective problem.

The rest of this manuscript is organized as follows. Section 2 explains the system setting and assumptions for the transit bus systems. Section 3 introduces the data and parameters used in the mathematical models. Section 4 provides the details of two mathematical models for different electric bus operational strategies. Section 4 illustrates the weight sum method for solving bi-objective optimization problems.

2 System Setting and Assumptions

Regular bus system has the following common elements:

1. Each route has a start and an end point of service called base station where electric buses stop for an extended time after a complete service.
2. Each route has multiple stops and is assigned several buses to operate.
3. There are multiple routes for the bus system and some portions of network links and stops are shared by multiple routes.

The following assumptions are made for electrified bus system in this study.

4. Vehicles are operating in closed environment, namely there is no interaction between electric buses and other traffic modes.
5. All the vehicles are the same and vehicles serving the same route have the same battery size and follow the same velocity profile.
6. All the electric buses start their service from base stations with full charged batteries.
7. When a vehicle is operating, there are maximum and minimum allowable energy levels for the battery.

We also know the following facts regarding battery charging and roadway network:

8. The amount of energy charged from power transmitter is proportional to the travel time on the inductive cable.
9. The road slope for each route and travel speed for each electric bus is predetermined.

Thus the settings and assumptions 1-9 compose the electrified bus system as the research objective of this study. Bullet 4 is the assumption not overly restrictive as we can set up bus-only lane along the route or relax the assumption to have EV bus operating in an open environment mixing with other traffic modes by incorporating the uncertainties of travel time and energy consumption as previous literature [1, 2]. According to Bullet 5, it is reasonable to assume that the energy consumption of the same electric buses traveling through the same route is the same since they follow the same speed profile. Bullet 6 is the assumption that can be relaxed by allocating more DWCF along each route if enough idling and charging time cannot be ensured at base stations. Bullet 7 is the assumption based on the battery charging and discharging characteristics within certain range of energy level [3]. Bullet 8 and 9 imply that the same electric buses should receive the same amount of energy from the DWCF and consume the same amount of energy when traveling through the same route deployed with the DWCF.

3. Input Data and Parameters

The input for the mathematical models for planning the electrification of the transit bus system should include the ArcGIS of the transit bus network, the bus service data, the coefficient parameters for estimating the cost and GHG emission, and the technical features of the DWCF.

- ArcGIS of Bus Network: The bus network data should illustrate the roadway network served by the all bus routes and the slope of each road in ArcGIS.
- Bus Service Data: The bus service data indicate the number and frequency of the buses serving their bus routes and the predetermined bus speed to travel through the route.

- Coefficient Parameters: The coefficient parameters are used to calculate the cost of installing inverters and inductive cables for the DWCF and the GHG emission originated from the energy consumption of the electric buses.
- Technical Features Data: The technical features data indicate the energy supply rate and energy loss of the DWCF and the technical features of the electric buses for estimating the energy consumption and the battery degradation.

When the above data are collected, we need to conduct some preprocessing on these data to transfer them to the form that can be used to formulate the mathematical model. To illustrate the finalized data used for the model, we list these data in Table 1.

Table 1 Indices and Parameters

Indices and Sets	
Notation	**Description**
i, j, k	Index of node in the bus network
r	Index of bus route
f	Index of bus frequency
N	Set of nodes in the bus network
A	Set of directed links in the bus network
R	Set of bus lines
F	Set of bus frequency

Parameters		
Notation	**Description**	
l	Length of the link in the bus network	50 meter
c^{inv}	Cost of one inverter	3000 – 5000 $/kWh
c^{cons}	Construction cost per unit length	500 – 1000 $/meter
e^{fix}	Fixed part of energy consumption of the electric bus	
e^{unit}	Unit energy consumption of the electric bus to carry per unit battery	
T	Lifespan of electric bus lines studied	40 year
m	Number of buses in a bus line	
c^{ele}	Electricity rate	0.1 – 0.2 $/kWh
c^{ele}	Battery rate	150 – 300 $/kWh
c^{emi}	Emission conversion factor for electricity	
p	Energy supply rate of the power transmitter	50 – 300 kWh
t	Travel time of the bus travel through a link in the network	
B	Maximal battery size	660 kWh

According to Table 1, the indices and sets are used to represent the transit bus network. The parameter e^{fix} and e^{unit} are used to compute the energy consumption of the electric bus travelling through a specific route. These parameters are estimated by the predetermined bus speed and the technical features of the electric bus. The emission conversion factor c^{emi} is the coefficient to calculate the GHG emission for producing the electricity that is consumed by the electric buses.

4 Mathematical Models

This section explains the two mathematical models for the planning of the electrification of the transit bus systems. The representation of the bus network is presented first to facilitate calculation of the cost for investing the DWCF. Then two objectives capturing the investment cost and GHG emission and the constraints ensuring the SOC of battery are built for the model. Finally, two mathematical models for different electric bus operational strategies are formulated.

4.1 Network Representation

Let $\mathrm{G(N, A)}$ denote the electric bus network where N and A are the sets of nodes and directed links in the network respectively. Each route in the bus system is divided into short links with equal length l. Each link a is represented as node pair $(\mathrm{i,j})$, i.e., $\mathrm{a} = (\mathrm{i,j}) \in \mathrm{A}$, where $\mathrm{i,j} \in \mathrm{N}$ and $\mathrm{i} \neq \mathrm{j}$. Let R denote the set of electric bus lines and $\mathrm{A_r}$ and $\mathrm{N_r}$ denote all the links and corresponding nodes that form the r th line respectively. Besides, we denote the set of intersections in $\mathrm{G(N, A)}$ with more than one incoming links as $N^s, N^s \in N$.

4.2 Objective: Investment Cost of the DWCF

The DWCF apply power transmitter to charge the battery of the electric buses. One power transmitter contains one inverter and inductive cables deployed on a series of adjacent links. Let binary variable $\mathrm{x_{ij}}$ represent if link $(\mathrm{i,j})$ is deployed with an inductive cable, and the total length of the inductive cables can be represented by:

$$l \sum_{\mathrm{(i,j)\in A}} x_{ij}$$

Let $\mathrm{y_i}$ denote if node i is the start node of a series of adjacent links deployed with inductive cables. Then $\sum_{i \in N} y_i$ can be used to represent the total number of inverters in general cases. However, in cases when inductive cables are deployed with the intersection nodes and those nodes are with multiple incoming links, there may be more than one start node for those inductive cables counted although only one inverter is equipped for them as they are adjacent to each other. To account for such situations, let z_i represent if one of the incoming links of intersection $\in N^s$, is allocated with inductive cables. Then the number of inverters can be expressed as follows:

$$\sum_{\mathrm{i\in N}} y_i - \sum_{\mathrm{i\in N^s}} \sum_{\mathrm{(w,i)\in A, w\neq i}} x_{wi} + \sum_{\mathrm{i\in N^s}} z_i$$

Now we can clearly express the number of inverters and the total length of the inductive cables. Hence, we can calculate the total cost for installing these inverters and inductive cables as follows:

$$c^{inv} \left\{ \sum_{\mathrm{i\in N}} y_i - \sum_{\mathrm{i\in N^s}} \sum_{\mathrm{(w,i)\in A, w\neq i}} x_{wi} + \sum_{\mathrm{i\in N^s}} z_i \right\} + c^{cab} l \sum_{\mathrm{(i,j)\in A}} x_{ij}$$

Where c^{inv} denotes the cost of one inverter and c^{cab} denotes the cost of inductive cables per unit length.

4.3 Objective: Cost for Energy Consumption

The cost for energy consumption is calculated by the cost for purchasing the electricity from the power grid to charge the electric buses for the daily operations.

Now we explain the process to estimate the energy consumption of the electric bus travelling through an exact route. Let b_r represent the battery capacity of electric buses on route $r, r \in R$, and let d_{rij} represent the energy consumption of the electric bus traveling through the link (i, j) on route r. The energy consumption d_{rij} can be expressed as:

$$d_{\mathrm{rfij}} = e_{\mathrm{rfij}}^{\mathrm{fix}} + e_{\mathrm{rfij}}^{\mathrm{unit}} b_{\mathrm{r}} \quad \forall (\mathrm{i}, \mathrm{j}) \in \mathrm{A_r}, \forall \mathrm{f} \in \mathrm{F_r}, \forall \mathrm{r} \in \mathrm{R}$$

where $e_{\mathrm{rfij}}^{\mathrm{fix}}$ and $e_{\mathrm{rfij}}^{\mathrm{unit}}$ are predetermined parameters estimated based on the electric bus travel speed, route slope, and technical features of electric buses. The term $e_{\mathrm{rfij}}^{\mathrm{fix}}$ represents the fixed part of energy consumption that is determined after the data preprocessing. The term $e_{\mathrm{rfij}}^{\mathrm{unit}} b_{\mathrm{r}}$ represents the energy consumption for carrying the battery pack and is linearly related to the battery size. For the energy consumption calculation and $\mathrm{c_{rfij}}$ decomposition details, readers can refer to [2].

In this study, our focus is on how battery size change will influence the energy consumption and then leads to the cost for consuming electricity. Let c^{ele} denote the emission conversion factor for electricity. So the total cost arisen by the energy consumption of all electric buses can be represented by the following formula:

$$T \sum_{\mathrm{R}} \sum_{(\mathrm{i,j})} \sum_{f} m_r \, d_{\mathrm{rfij}}$$

where m_{r} represents the number of electric buses on route r. T is the total operation time of electric buses studied.

4.4 Constraint: Certain Level of SOC of Battery

As defined above, there are maximum and minimum allowable SOC for the battery of the electric buses. Let δ_{up} and δ_{low} be the upper and lower limit coefficients for the battery. Then the upper and lower energy level limits are $\delta_{\mathrm{up}} b_{\mathrm{r}}$ and $\delta_{\mathrm{low}} b_{\mathrm{r}}$, respectively. Let $u_{\mathrm{rfi}}, \mathrm{i} \in \mathrm{N}, \mathrm{r} \in \mathrm{R}, \mathrm{f} \in \mathrm{F}$ denote the SOC of the battery of the electric bus at node i of its fth round on route r. It should satisfy the following conditions:

$$u_{\mathrm{rfi}} \leq \delta_{\mathrm{up}} b_{\mathrm{r}} \quad \forall \mathrm{i} \in \mathrm{N_r}, \forall \mathrm{f} \in \mathrm{F_r}, \mathrm{r} \in \mathrm{R}$$

$$u_{\mathrm{rfi}} \geq \delta_{\mathrm{low}} b_{\mathrm{r}} \quad \forall \mathrm{i} \in \mathrm{N_r}, \forall \mathrm{f} \in \mathrm{F_r}, \mathrm{r} \in \mathrm{R}$$

To obtain the SOC of electric bus at node j, let $\mathrm{s_{rfij}}$ represent the amount of energy supply to electric buses on route r to travel through link (i, j) of its fth round, p represent the energy supply rate of the power transmitter after considering the energy loss, and t_{rfij} represent the travel time of electric buses on route r to travel through link (i, j) of its fth round. Then s_{rfij} can be expressed as

$$s_{rfij} = pt_{rfij}x_{ij} \quad \forall (i, j) \in A_r, \forall f \in F_r, r \in R$$

Let o denote the start node for a route and d the end node. The initially fully charged energy level for electric buses that start from base station on route r and round f is u_{rfo}. Then for each node j along route r and round f the energy level u_{rfj} can be obtained:

$$u_{rfj} = u_{rfi} - d_{rfij} + s_{rfij} \quad \forall (i, j) \in A_r, \forall f \in F_r, r \in R$$

Given the bus cycling on the route, the energy level of the bus at the start of a round should be equal to the energy level of the bus at the end of the last round:

$$u_{rfo} = u_{r(f-1)e} \quad \forall f \in F_r, r \in R$$

4.5 Model: Electric Buses for Rotation

The rotation strategy to operate the electric buses in the transit system allows each bus route can be served by the electric buses. So this strategy can be simply referred to as electrifying the entire bus network. Then the model is built to minimize the two objectives by install the DWCF for the entire system. Before we present the details of the model, we first introduce Table 2 to list all the decision variables to be decided in the model.

Table 2 Description of decision variables

Continuous Variable	
Notation	**Description**
u	SOC of the battery of the electric bus
s	Energy supply provided by the wireless charging facility
d	Energy consumption of the electric bus travel through a link in the network
b	Battery size
Binary Variable	
Notation	**Description**
x	Represent if a link is deployed with the inductive cable
y	Represent if a node is the start node of some links deployed with the inductive cable
z	Represent if an intersection node is connected with a link deployed with inductive cable
v	Represent if a bus line is selected to electrify

Based on the above descriptions on parameters and variables, the complete mathematical model to optimize the planning for electrifying the transit bus system considering the rotation operation is presented as follows:

$$\min z_1 = c^{inv}\left\{\sum_{i \in N} y_i - \sum_{i \in N^s} \sum_{(w,i) \in A, w \neq i} x_{wi} + \sum_{i \in N^s} z_i\right\} + c^{cab} l \sum_{(i,j) \in A} x_{ij}$$

$s.t.$

$$y_i \le \sum_{(i,j)\in A} x_{ij}, \qquad \forall i \in N, i \neq j \qquad (1)$$

$$y_i \le 1 - x_{wi}, \qquad \forall i \in N, (w,i) \in A, w \neq i \qquad (2)$$

$$y_i \ \ge x_{ij} - \textstyle\sum_{(w,i)\in A} x_{wi} \qquad \forall i \in N, (i,j) \in A, i \neq j, (w,i) \in A, w \neq i \qquad (3)$$

$$z_i \le \sum_{(w,i)\in A} x_{wi} \qquad \forall i \in N^s, (w,i) \in A, w \neq i \qquad (4)$$

$$z_i \ge x_{wi} \qquad \forall i \in N^s, (w,i) \in A, w \neq i \qquad (5)$$

$$d_{\mathrm{rfij}} = e_{\mathrm{rfij}}^{\mathrm{fix}} + e_{\mathrm{rfij}}^{\mathrm{unit}} b_{\mathrm{r}} \qquad \forall (\mathrm{i},\mathrm{j}) \in \mathrm{A_r}, \forall \mathrm{f} \in \mathrm{F_r}, \forall \mathrm{r} \in \mathrm{R} \qquad (5)$$

$$u_{\mathrm{rfi}} \le \delta_{\mathrm{up}} b_{\mathrm{r}} \qquad \forall \mathrm{i} \in \mathrm{N_r}, \forall \mathrm{f} \in \mathrm{F_r}, \mathrm{r} \in \mathrm{R} \qquad (6)$$

$$u_{\mathrm{rfi}} \ge \delta_{\mathrm{low}} b_{\mathrm{r}} \qquad \forall \mathrm{i} \in \mathrm{N_r}, \forall \mathrm{f} \in \mathrm{F_r}, \mathrm{r} \in \mathrm{R} \qquad (7)$$

$$\mathrm{s_{rfij}} = \mathrm{pt_{rfij}} \mathrm{x_{ij}} \qquad \forall (\mathrm{i},\mathrm{j}) \in \mathrm{A_r}, \forall \mathrm{f} \in \mathrm{F_r}, \mathrm{r} \in \mathrm{R} \qquad (8)$$

$$u_{\mathrm{rf}j} = \ u_{\mathrm{rfi}} - d_{\mathrm{rfij}} + \mathrm{s_{rfij}} \qquad \forall (\mathrm{i},\mathrm{j}) \in \mathrm{A_r}, \forall \mathrm{f} \in \mathrm{F_r}, \mathrm{r} \in \mathrm{R} \qquad (9)$$

$$u_{\mathrm{rf}o} = \ u_{\mathrm{r(f-1)e}} \qquad \forall \mathrm{f} \in \mathrm{F_r}, \mathrm{r} \in \mathrm{R} \qquad (10)$$

$$b_r \le \mathrm{B} \qquad \forall r \in R \qquad (11)$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in A \qquad (12)$$

$$y_i \in \{0,1\} \qquad \forall i \in N \qquad (13)$$

$$z_i \in \{0,1\} \qquad \forall i \in N^s \qquad (14)$$

where constraints (1)-(3) determines if a node is the start node of a series of adjacent links deployed with inductive cables and constraints (4) and (5) determine if one of the incoming links of an intersection nodes is deployed with inductive cables. Constraints (6)-(10) restrict the battery energy level to be always within the allowable range. The maximum and minimum allowable energy levels are set according to the reference [4] which will be beneficial for battery life.

4.6 Model: Fixed Electric Bus Routes

The strategy to conduct the electrification on the fixed the electric bus routes in the transit system need to select the routes for electrification. Different selections of the bus routes for electrification will lead to different DWCF deployment and corresponding different cost and GHG emission. So the selection the electrified routes also need to be optimized. To introduce the decisions on route selection, we introduce a binary variable v_r to indicate if route r is selected to electrify. We note that if route r is not selected, we should remove the GHG emission of this route from the objective of total GHG emission, and also we need to remove the constraints to ensure the energy level of the electric buses on this route. Some mathematical manipulations are applied to ensure that introducing the binary variable can achieve the above goals can still maintain the model to be linear. Then the model considering the route selection for the planning of the electrification of transit bus system can be formulated as follows:

$$\min z_1 = c^{inv}\left\{\sum_{i\in N} y_i - \sum_{i\epsilon N^S}\sum_{(w,i)\in A, w\neq i} x_{wi} + \sum_{i\epsilon N^S} z_i\right\} + c^{cab} l \sum_{(i,j)\epsilon A} x_{ij}$$

$$\min z_2 = T\sum_{R} m_r f_r \sum_{(\mathrm{i,j})} d_{rij}$$

$s.t.$ (1) – (5)

$$v_r = \bar{\mathrm{v}} \qquad \mathrm{r} \in \mathrm{R} \tag{14}$$

$$d_{\mathrm{rij}} = e_{\mathrm{rij}}^{\mathrm{fix}} v_r + e_{\mathrm{rij}}^{\mathrm{unit}} b_{\mathrm{r}} \qquad \mathrm{r} \in \mathrm{R}, \forall (i,j) \in A_r \tag{14}$$

$$u_{\mathrm{ri}} \leq \delta_{\mathrm{up}} b_{\mathrm{r}} \qquad \forall r \in R, j \in N_r \tag{15}$$

$$u_{\mathrm{ri}} \geq \delta_{\mathrm{low}} b_{\mathrm{r}} \qquad \forall r \in R, j \in N_r \tag{16}$$

$$s_{rij} = pt_{rij} x_{ij} \qquad \mathrm{r} \in \mathrm{R}, \forall (i,j) \in A_r \tag{17}$$

$$u_{rj} - u_{ri} + d_{rij} - s_{rij} \geq M(1 - v_r) \qquad \mathrm{r} \in \mathrm{R}, \forall (i,j) \in A_r \tag{18}$$

$$u_{rj} - u_{ri} + d_{rij} - s_{rij} \leq -M(1 - v_r) \qquad \mathrm{r} \in \mathrm{R}, \forall (i,j) \in A_r \tag{19}$$

$$b_r \geq 0 \qquad \forall r \in R \tag{20}$$

$$b_r \leq M v_r \qquad \forall r \in R \tag{21}$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in A \tag{22}$$

$$y_i \in \{0,1\} \qquad \forall i \in N \tag{23}$$

$$z_i \in \{0,1\} \qquad \forall i \in N^s \tag{24}$$

where $\bar{\mathrm{v}}$ is predetermined parameter to indicate the number of bus routes to be electrified. Constraint (14) and (21) can ensure that if route r is not selected, the value of d_{rij} is zero and the corresponding emission is also zero. Constraint (18) and (19) can guarantee the energy level constraint of the battery will not restrict on the route that is not selected.

5. Solution Algorithm for the Proposed Bi-Objective Optimization Model

5.1 Preliminaries

Multi-objective optimization problem

A multi-objective optimization problem can be stated as follows (25):

$$\underset{x \in \mathcal{X}}{\text{minimize}}\, z(x) \coloneqq \{z_1(x), \dots, z_p(x)\} \quad (25)$$

where $\mathcal{X} \subseteq \mathbb{R}^n$ represents the feasible set in the *decision space* and the image $\mathcal{Y}$ of $\mathcal{X}$ under vector-valued function $\mathrm{z} = \{z_1, \dots, z_p\}$ represents the feasible set in the criterion space; i.e., $\mathcal{Y} \coloneqq \mathrm{z}(\mathcal{X}) \coloneqq \{y \in \mathbb{R}^p : y = z(x) \text{ for some } x \in \mathcal{X}\}$. For convenience, we also use the notation $\mathbb{R}^p_{\geqq} \coloneqq \{y \in \mathbb{R}^p : y \geq 0\}$ for the nonnegative orthant of $\mathbb{R}^p$, and $\mathbb{R}^p_{>} \coloneqq \{y \in \mathbb{R}^p : y \geq 0\}$ for the positive orthant of $\mathbb{R}^p$. When $\mathcal{X}$ is defined by a set of affine constraints and $z_1(x), \dots, z_p(x)$ are linear functions, then (25) is a multi-objective linear program (MOLP). When $\mathcal{X} \subseteq \mathbb{Z}^n$, (25) is a multi-objective integer program (MOIP). When $p = 2$, these problems are called a bi-objective linear program (BOLP) and a bi-objective integer program (BOIP) respectively.

Definition1. Non-dominated points

A feasible solution $x' \in \mathcal{X}$ is called efficient or Pareto optimal, if there is no other $x \in \mathcal{X}$ such that $z_k(x) \leq z_k(x')$ for $k = 1, \dots, p$ and $z(x) \neq z(x')$. If x' is efficient, then $z(x')$ is called a non-dominated point. The set of all efficient solutions $x' \in \mathcal{X}$ is denoted by $\mathcal{X}_E$. The set of all non-dominated points $y' = z(x') \in \mathcal{Y}$ for some $x' \in \mathcal{X}_E$ is denoted by $\mathcal{Y}_N$ and referred to as the non-dominated frontier or the efficient frontier.

Definition2. Supported non-dominated point

Let $x' \in \mathcal{X}_E$. If there is a $\lambda \in \mathbb{R}^p$ such that x' is an optimal solution to $\min_{x \in \mathcal{X}} \lambda^T z(x)$, then x' is called a supported efficient solution and $y' = z(x')$ is called a supported non-dominated point.

Definition3. Extreme supported non-dominated points

Another important type of non-dominated points is extreme supported non-dominated (ESN) point. ESN points are the extreme points of the convex hull of all non-dominated points.
Let $\mathcal{Y}^e$ be the set of extreme points of conv($\mathcal{Y}$). A point $y \in \mathcal{Y}$ is called an extreme supported non-dominated point if $y \in \mathcal{Y}^e \cap \mathcal{Y}_N$.

Solution method for MOIP

The set of (feasible) solutions to a MOLP, in the decision space as well as in the criterion space, is convex (assuming that the problem is feasible). Therefore, all efficient solutions to a MOLP are supported, i.e., they can be obtained by optimizing a weighted combination of objective functions. Unfortunately, in general, this is not the case for a MOIP. Therefore, to solve the MOIP is (far) more challenging.

In this work, we apply weighted-sum methods to find all extreme supported non-dominated points for the bi-objective integrated design problem to determine dynamic charging facility location and on-board battery size.

3.2 The Weighted-Sum Method and Algorithm Design

The weighted-sum method (18) finds all extreme supported non-dominated points. It uses the following optimization problem to search a rectangle defined by pints z^1 and z^2 .

$$\min_{x \in \mathcal{X}} \{\lambda_1 z_1(x) + \lambda_2 z_2(x)\}$$

$$\text{subject to } z(x) \in R(\, z^1, z^2)$$

with $\lambda_1 = z_2^1 - z_2^2$ and $\lambda_2 = z_1^2 - z_1^1$, which indicate that the objective function is parallel to the line that connects z^1 and z^2 in the criterion space.

This optimization either returns a new and (possibly extreme) supported non-dominated point, one of z^1 and z^2, or a convex combination of z^1 and z^2. If the optimized result in the criteria space (z_1^{new}, z_2^{new})satisfy the following condition $\lambda_1 z_1^{new} + \lambda_2 z_2^{new} < \lambda_1 z_1^1 + \lambda_2 z_2^1$, the optimum point z^{new} is a newly found non-dominated point which will separate the original rectangle to smaller rectangles to be searched. The pseudo-code for the algorithm design of the bi-objective restoration sequence optimization is illustrated as follows.

Step1. Compute the endpoints z^T and z^B

Step2. Create a list $List.create(L)$;

Step3. Add points z^T and z^B to the list L, $List.add(L, z^T)$, $List.add(L, z^B)$.

Step4. Create a queue P with rectangles to be searched, *PQ.create*(*P*). Add rectangle $R(z^T, z^B)$ to the queue, $PQ.add(P, R(z^T, z^B))$.

Step5. Optimize the weighted sum single objective optimization problem $\min_{x \in \mathcal{X}} \{\lambda_1 z_1(x) + \lambda_2 z_2(x)\}$, if the optimized point in criteria space satisfy the criteria shown below, this point is a newly found ND point which will separate the original rectangle to smaller rectangles to be searched. While the queue is not empty, step 5 will be performed iteratively.

While the queue P is not empty, *not PQ.empty(P)* do

$PQ.pop(P, R(z^1, z^2))$

$x^* \leftarrow \operatorname{argmin}_{x \in \mathcal{X}} (z_2^1 - z_2^2)z_1(x) + (z_1^2 - z_1^1)z_2(x)$

$z \leftarrow z(x^*)$

if $(z_2^1 - z_2^2)z_1 + (z_1^2 - z_1^1)z_2 < (z_2^1 - z_2^2)z_1^1 + (z_1^2 - z_1^1))z_2^1$ then

$List.add(L, z)$

$PQ.add(P, R(z^1, z))$

$PQ.add(P, R(z, z^2))$

Return L

Following the above mentioned pseudo-code, the weighted sum method can be implemented to solve both proposed bi-objective integrated DWCF location and on-board battery size optimization problems.

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