Probability, Statistics & Linear Algebra - Structured Topics

1. Probability and Statistics

1.1 Counting Principles

1. Permutation and Combination
2. Fundamental Counting Principle

1.2 Probability Basics

1. Probability Axioms
2. Sample Space and Events
3. Independent Events
4. Mutually Exclusive Events

1.3 Types of Probability

1. Marginal Probability
2. Conditional Probability
3. Joint Probability
4. Bayes Theorem

1.4 Expectation and Variance

1. Conditional Expectation
2. Conditional Variance

1.5 Descriptive Statistics

1. Mean, Median, Mode
2. Variance and Standard Deviation
3. Correlation and Covariance

1.6 Random Variables

1. Definition and Types (Discrete and Continuous)
2. Discrete Random Variables and Probability Mass Functions (PMF)
3. Continuous Random Variables and Probability Density Function (PDF)

1.7 Discrete Distributions

1. Uniform Distribution
2. Bernoulli Distribution
3. Binomial Distribution

1.8 Continuous Distributions

1. Uniform Distribution
2. Exponential Distribution
3. Poisson Distribution
4. Normal and Standard Normal Distribution
5. t-Distribution
6. Chi-Squared Distribution

1.9 Cumulative Distribution

1. Cumulative Distribution Function (CDF)
2. Conditional PDF

1.10 Theoretical Concepts

1. Central Limit Theorem (CLT)
2. Confidence Interval

1.11 Hypothesis Testing

1. z-Test
2. t-Test
3. Chi-Squared Test

2. Linear Algebra

2.1 Vector Spaces

1. Definition of Vector Space
2. Examples (■■, Polynomial, Function Spaces)
3. Subspaces and their Properties
4. Basis and Dimension

2.2 Linear Dependence and Independence

1. Definitions and Examples
2. Span and Basis Relationship
3. Rank and Independence

2.3 Matrices and Their Properties

1. Matrix Types and Operations
2. Projection Matrix and its Properties
3. Orthogonal Matrix
4. Idempotent Matrix
5. Partitioned Matrix and Applications

2.4 Quadratic Forms

1. Definition and Representation
2. Matrix Form of Quadratic Expressions
3. Positive/Negative/Indefinite Forms

2.5 Systems of Linear Equations

1. Homogeneous and Non-Homogeneous Systems
2. Consistency and Solutions
3. Row Echelon and Reduced Row Echelon Forms
4. Gaussian Elimination

2.6 Eigenvalues and Eigenvectors

1. Characteristic Equation and Computation
2. Diagonalization
3. Spectral Theorem for Symmetric Matrices

2.7 Determinant, Rank, and Nullity

1. Computation and Properties of Determinant
2. Rank and Null Space
3. Rank-Nullity Theorem

2.8 Projections

1. Orthogonal Projection onto Subspace

2 Projection Matrix Derivation
3 Least Squares Approximation

2.9 LU Decomposition

1 Concept of Matrix Factorization
2 Steps of LU Decomposition
3 Applications in Solving Linear Systems