AI and Deep Learning

Linear Regression & Back-propagation

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Agenda

- Neuron and Regression
- Loss/Error/Cost Function
- Learning and Updating Weights
- Gradient/Slope
- Computation Graph
- Forward Propagation
- Backpropagation

“

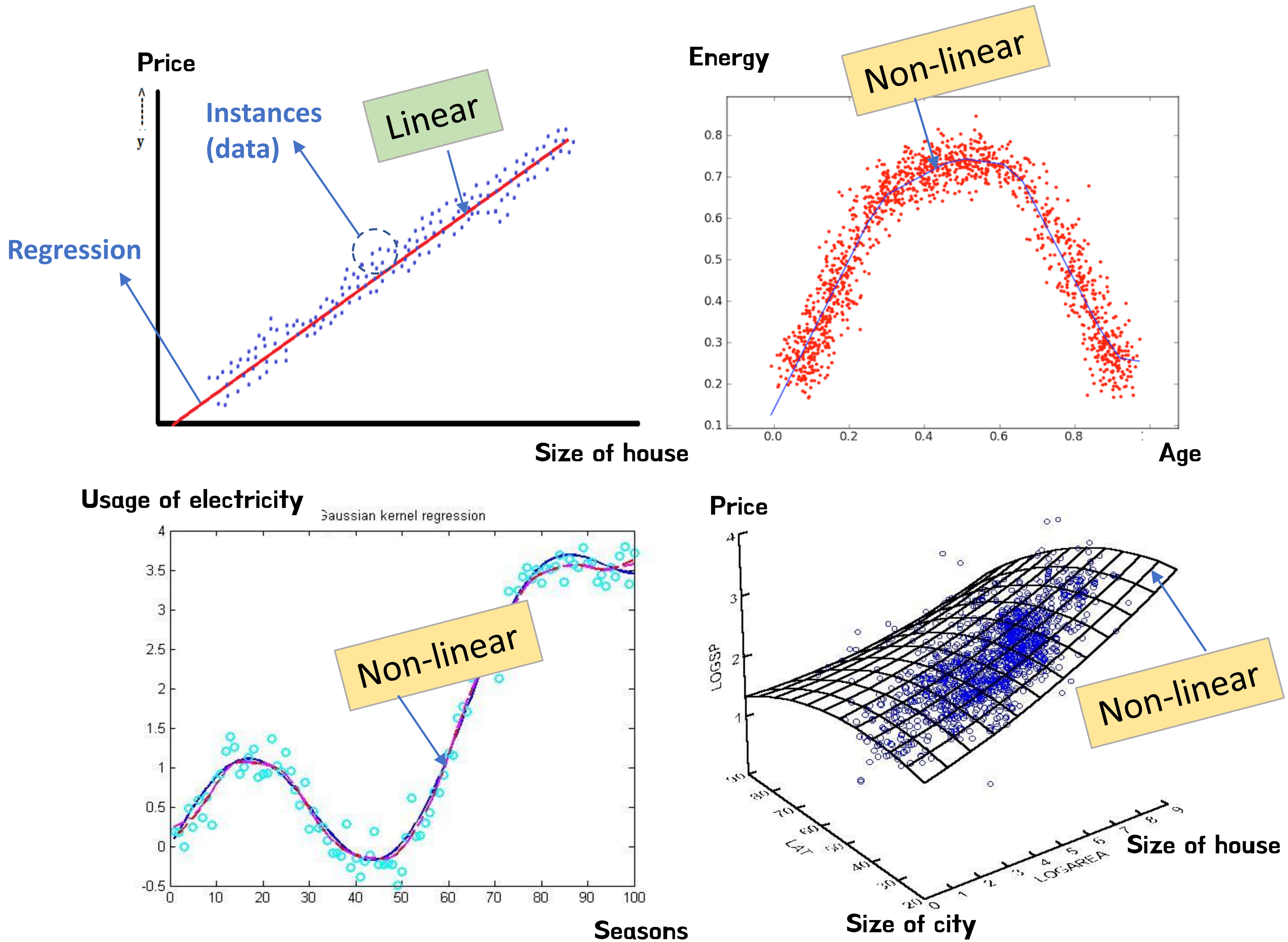
After spending the majority of ocean life, salmons return to their home(river) where they were born.

Regression(회귀)

- To describe a natural phenomena
- A term frequently used in anthropology(인류학) to present a natural tendency

Regression (회귀)

- Statistical measure to determine the relationship between one dependent variable (usually denoted by Y) and a series of other independent variables X.

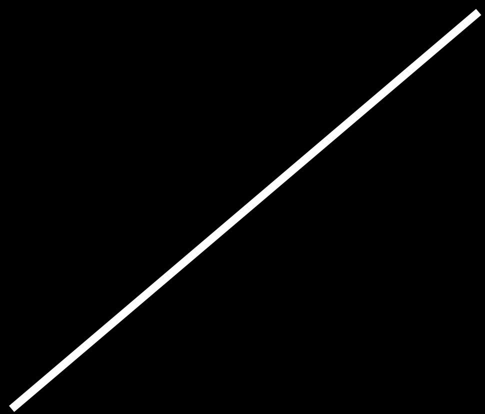


Price
y
Instances
(data)
Linear
Regression
Size of house
Energy
Non-linear
Age
Usage of electricity
aussian kernel regression
Non-linear
Seasons
Price
LOGSP
Non-linear
LAT
LOGAREA
Size of house
Size of city

Linear Regression

The relationship forms linear shape.

ex) wage/hour, price/size of house



Lab

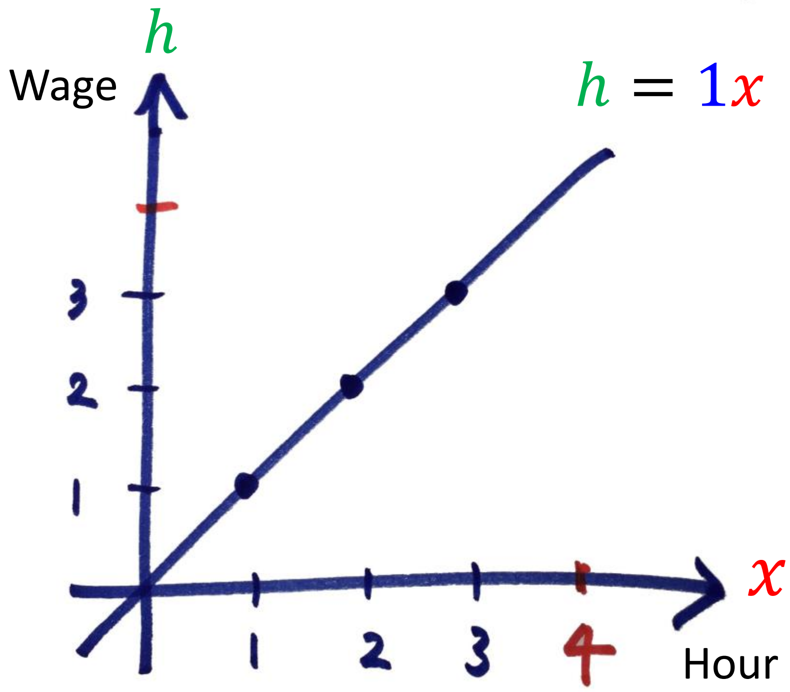
Linear Regression

using desmos

www.desmos.com

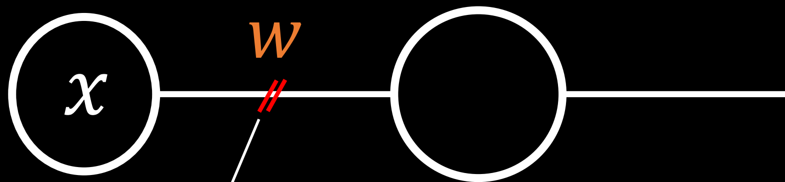
1. Draw a point(data) (1, 1)
2. Add (2, 2), (-1, -1), (-2, -2)
3. $h = x$
4. $h = 2x$
5. $h = wx$ (**rotation**)
6. Move all of the points by adding 1 to y
7. $h = wx + 1$ (**shifting**)
8. $h = wx + b$ (**rotation** and **shifting**)

www.desmos.com

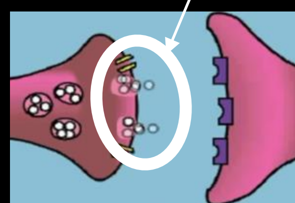


$$h = wx$$

Neuron and regression



$h = wx$



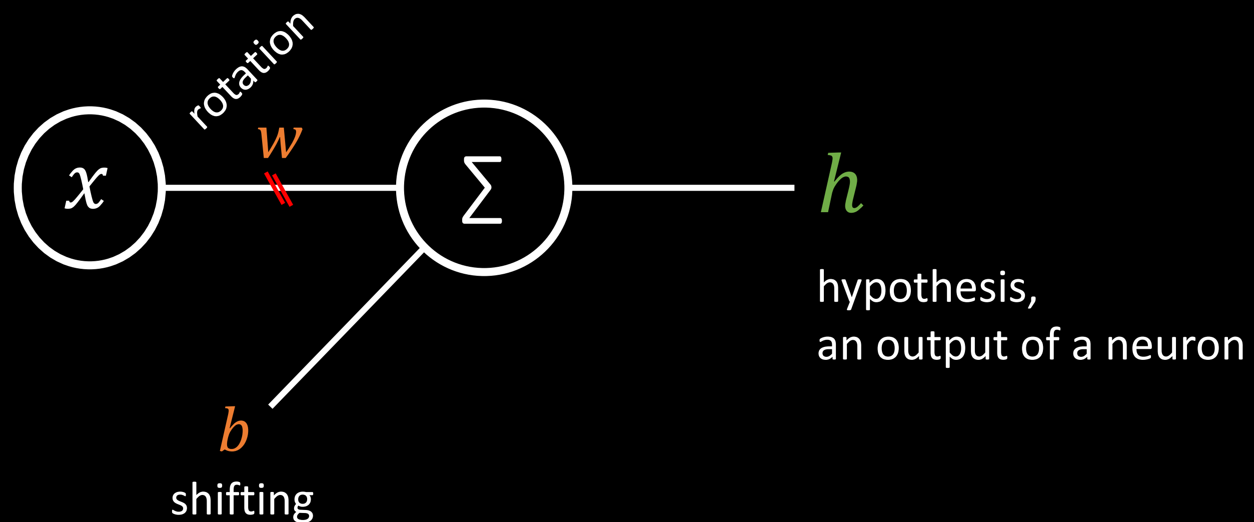
- w : Neuro-transmitter
- if large / if small / if not exists (rotation)
- A neuron represents a linear regression

Hypothesis

$$h = wx$$
$$h = wx + b$$

- An answer by a neuron
- $h$ypothesis : a proposed explanation for a phenomenon (a regression).
- Not proved yet, but it can represents the regression after updating w.

The role of w and b

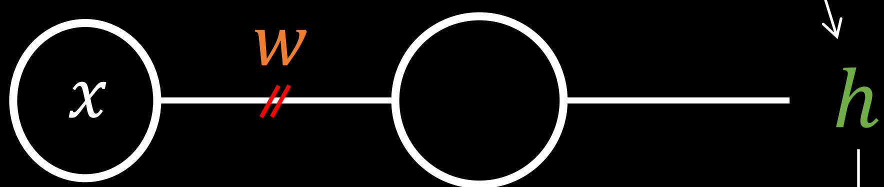


$$h = wx + b$$

How to learn (update w)

- Scolding or blaming the neuron if it is wrong
- The neuron gets stress and automatically updates w to answer well next time so that the error(difference) decreases.
- Designing an 'error(difference) function'.
- The difference between the prediction of a neuron and correct answer
- Error/loss/cost/difference function

Error/difference function



answer by a neuron

ground truth

$$E = |w \cdot x - y|$$

Why absolute?

Error/difference function

The error is the difference between a neuron's answer and it's ground truth.

$$E = |h\text{ypothesis} - y|$$

$$E = |w \cdot x - y|$$

$$E = |w \cdot 1 - 1|$$

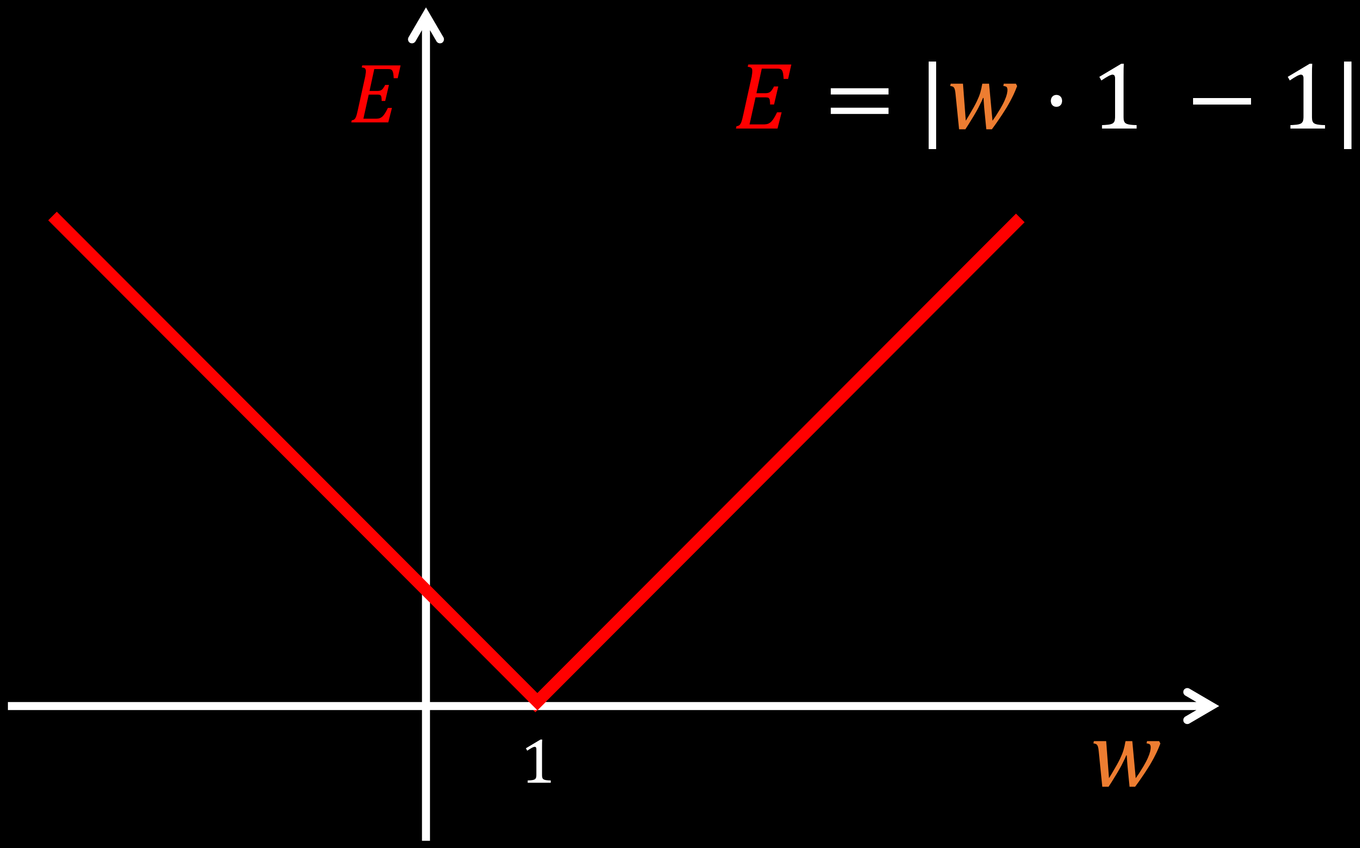
Supervised Learning
지도학습

www.desmos.com

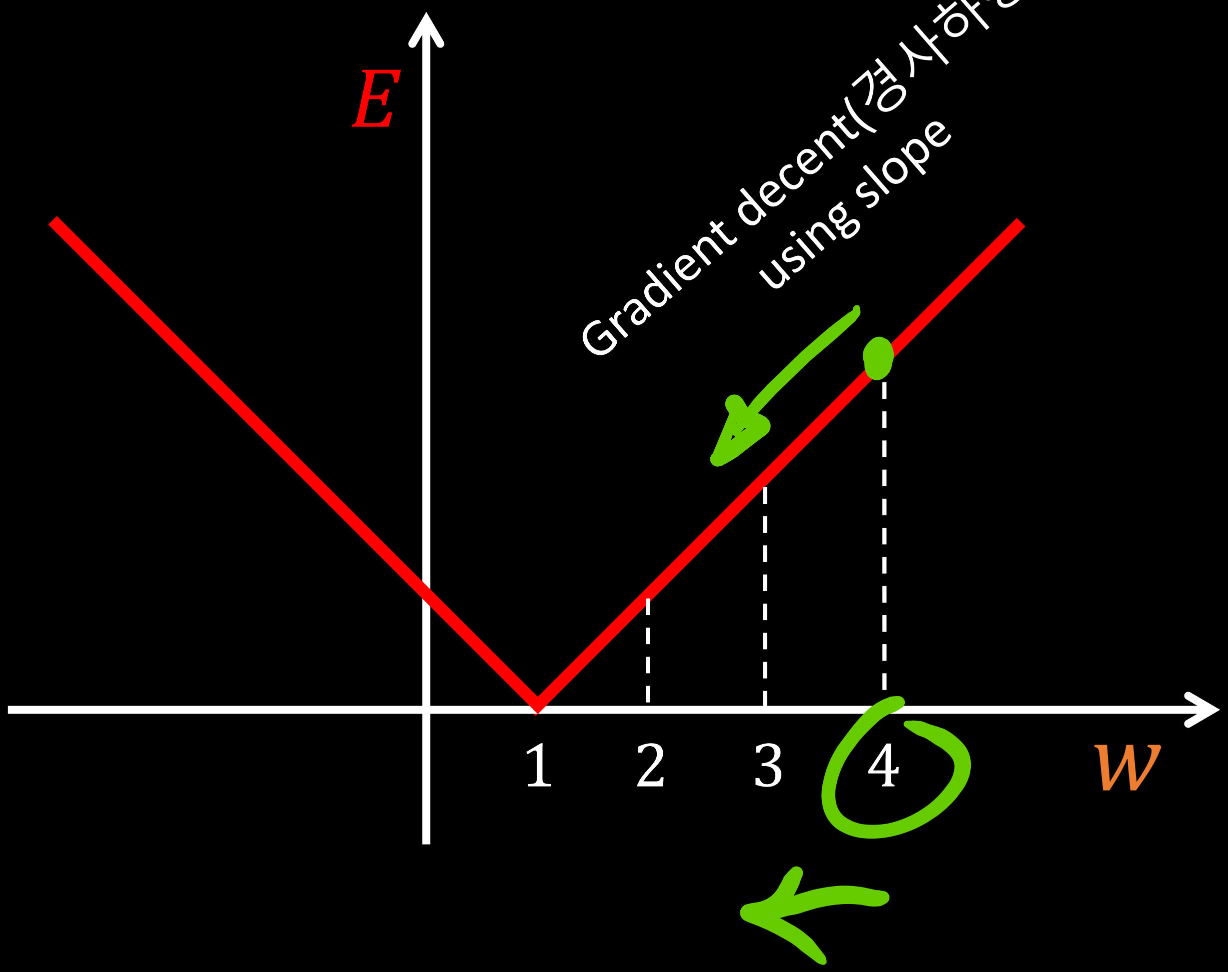
1. Mark (1, 1)
2. $h = w \cdot x$
3. $E = w \cdot 1 - 1$
4. $E = |w \cdot 1 - 1|$
5. (w, E)

desmos

Error Function of w



How to update w

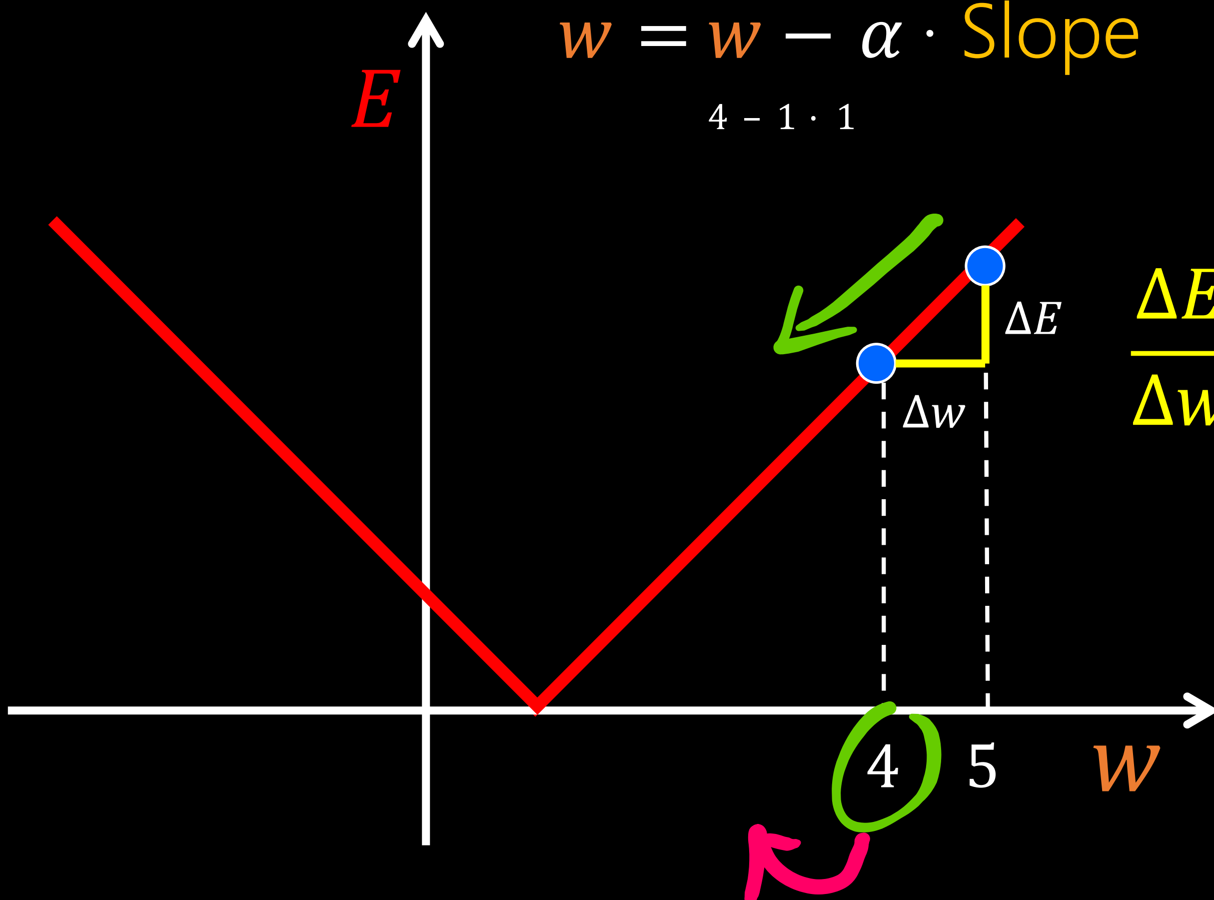


How to update w

learning rate (ex, 1)

$$w = w - \alpha \cdot \text{Slope}$$

$$4 - 1 \cdot 1$$



$$\frac{\Delta E}{\Delta w} = \frac{+1}{+1}$$

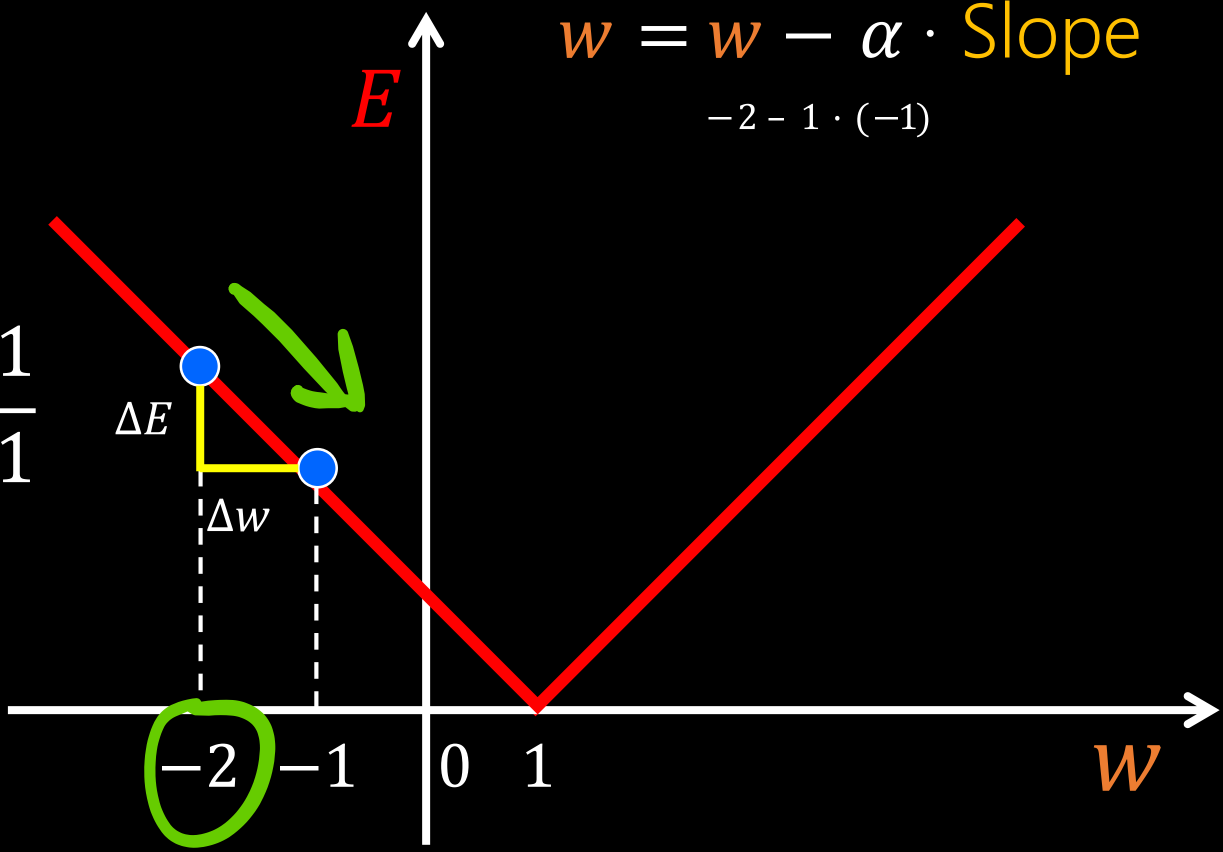
How to update w

learning rate (ex, 1)

$$w = w - \alpha \cdot \text{Slope}$$

$$-2 - 1 \cdot (-1)$$

$$\frac{\Delta E}{\Delta w} = \frac{-1}{+1}$$



$$w = 4, \alpha = 1, Slope = 1$$

$$w = w - \alpha \cdot \text{Slope}$$

$$4 - 1 \cdot 1 \longrightarrow 3$$ Error E = 2

$$3 - 1 \cdot 1 \longrightarrow 2$$ Error E = 1

$$2 - 1 \cdot 1 \longrightarrow 1$$ Error E = 0

$$w = -2, \alpha = 1, Slope = -1$$

$$w = w - \alpha \cdot \text{Slope}$$

$$-2 - 1 \cdot (-1) \longrightarrow -1$$ Error E = 2

$$-1 - 1 \cdot (-1) \longrightarrow 0$$ Error E = 1

$$0 - 1 \cdot (-1) \longrightarrow 1$$ Error E = 0

$$w = -2, \alpha = 2, Slope = -1$$

$w = w - \alpha \cdot \text{Slope}$

$-2 - 2 \cdot (-1) \longrightarrow 0$ Error E = 1

$0 - 2 \cdot (-1) \longrightarrow 2$ Error E = 1

$2 - 2 \cdot (1) \longrightarrow 0$ Error E = 1

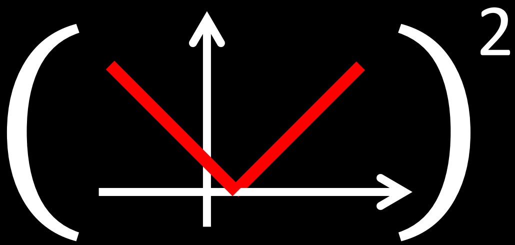
$0 - 2 \cdot (-1) \longrightarrow 2$ Error E = 1

$2 - 2 \cdot (1) \longrightarrow 0$ Error E = 1

Issues in the absolute error

- Always the same slope in the error graph regardless of the value of w
- Therefore, the same speed in movement
- Not guarantee to get the w value which gives 0 error or almost 0

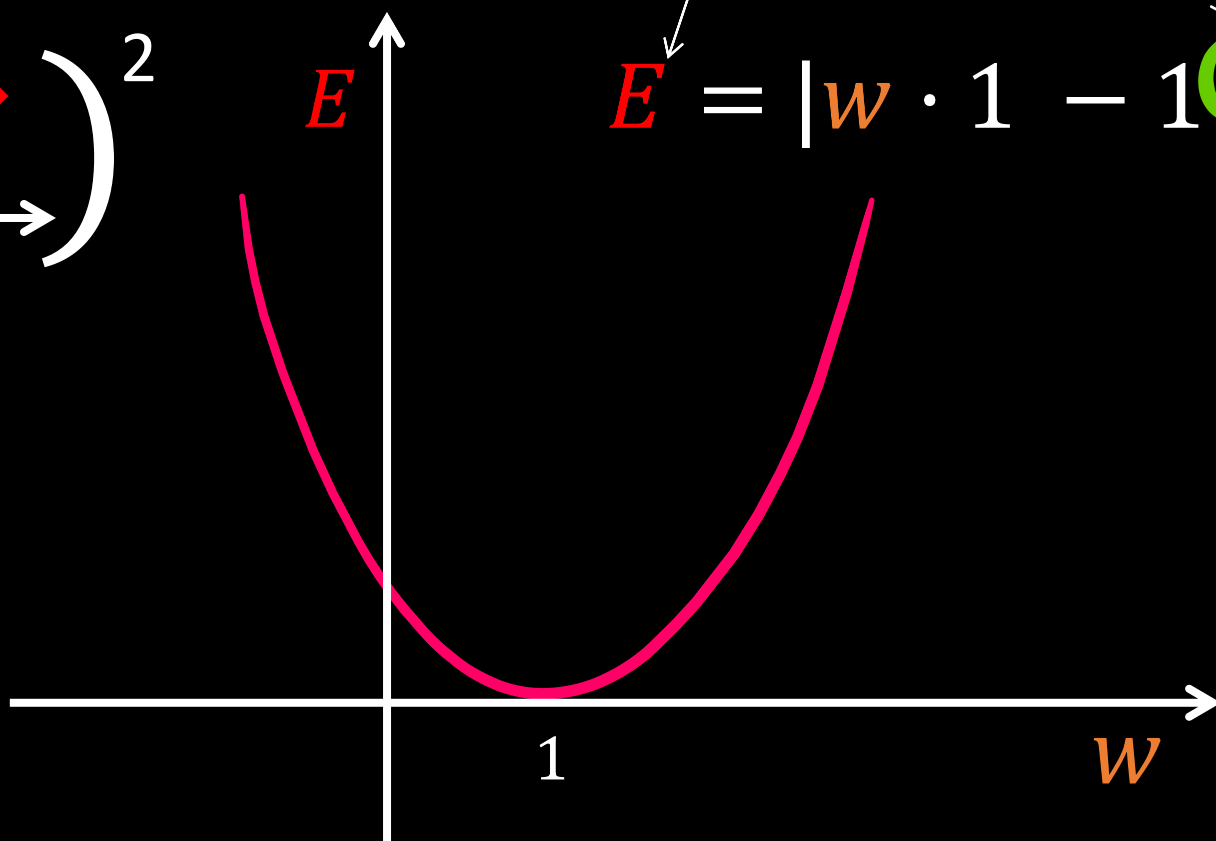
Square Error



error

square(제곱)

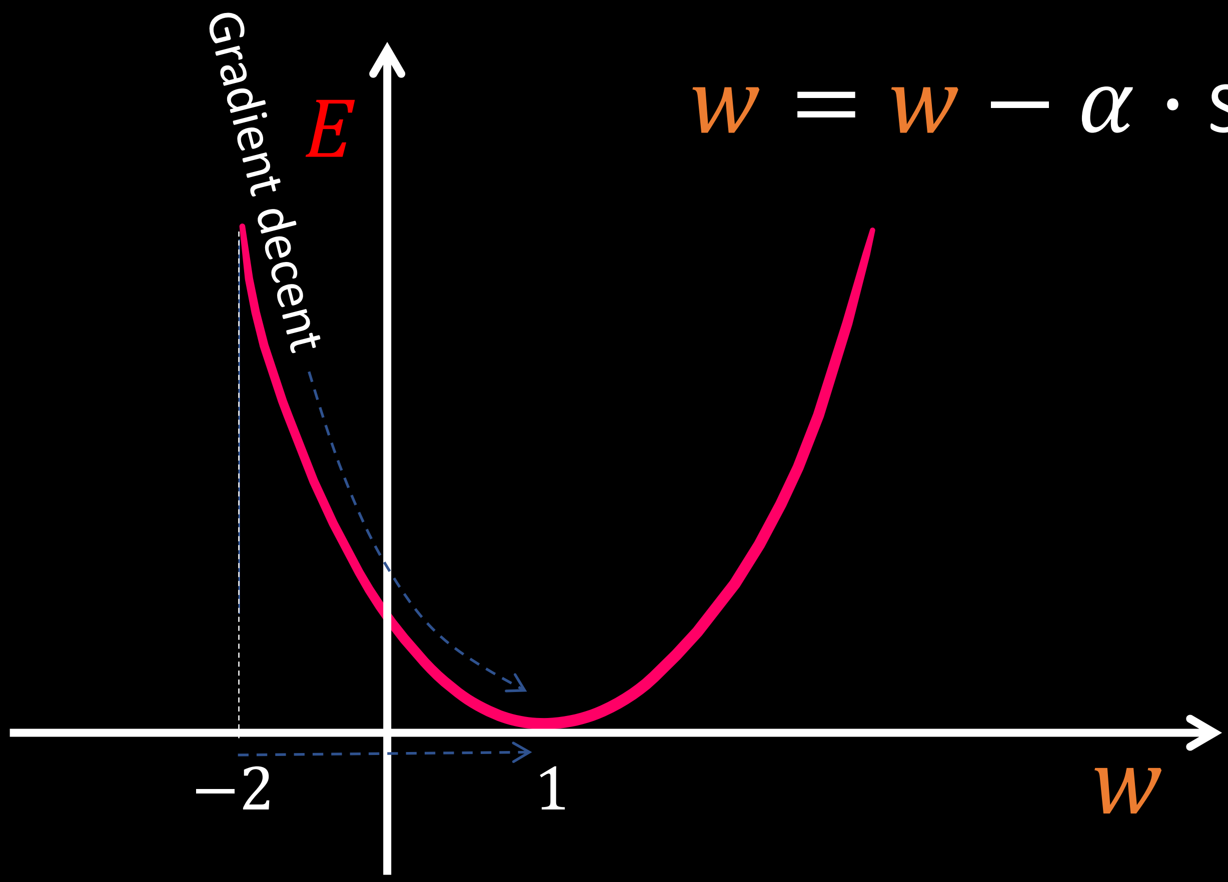
$$E = |w \cdot 1 - 1|^2$$



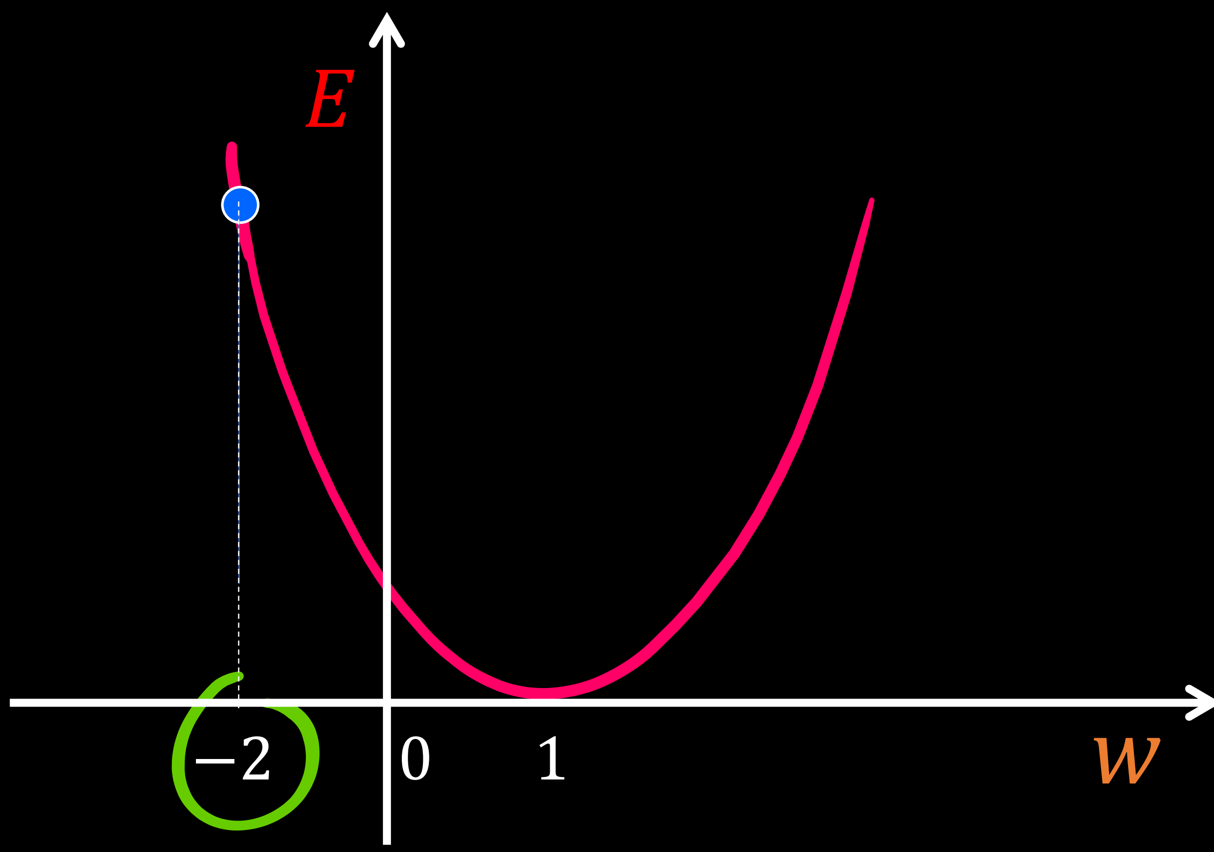
How to update w

$$\frac{\Delta E}{\Delta w}$$

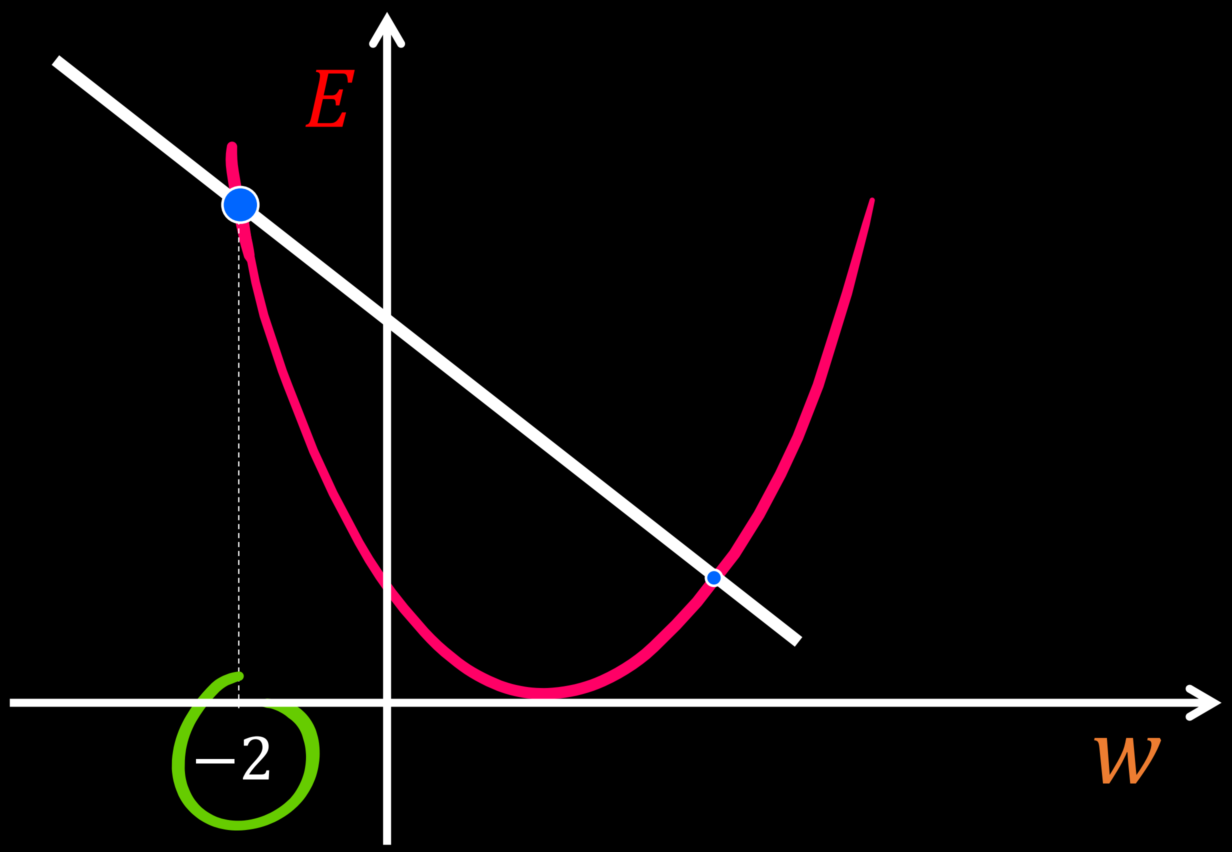
$$w = w - \alpha \cdot \text{Slope}$$



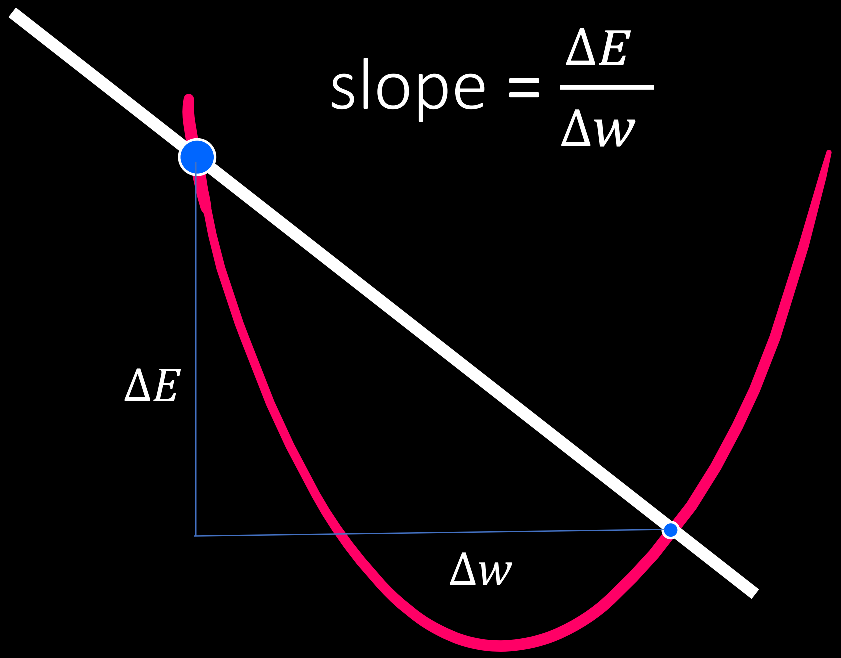
How to update w



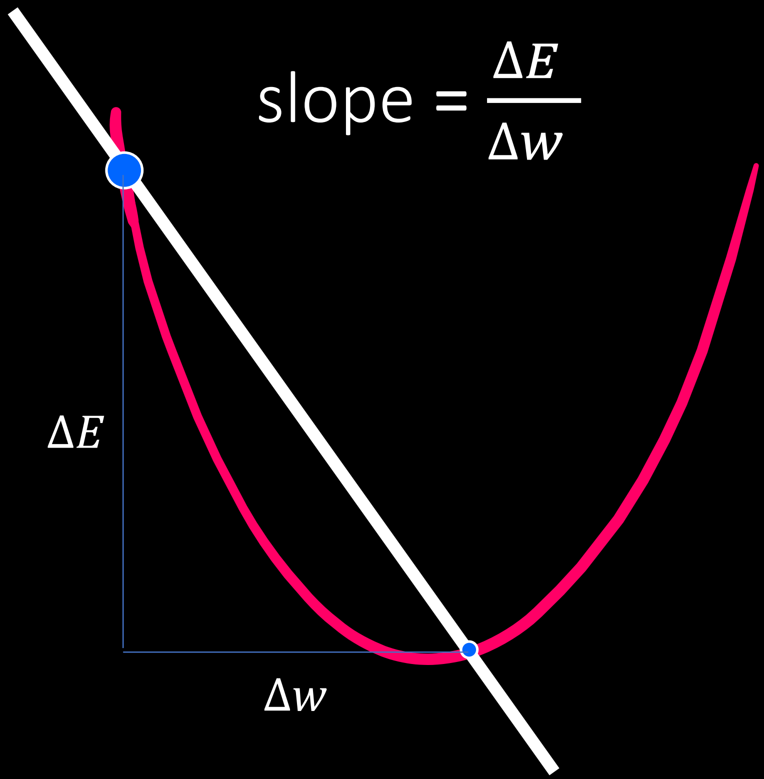
How to update w



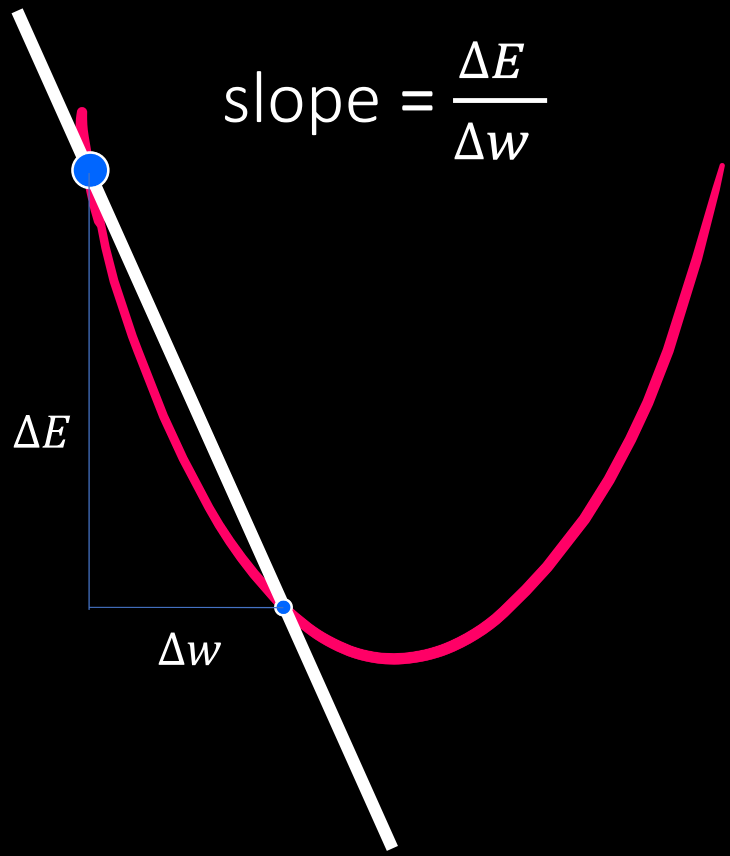
How to update w



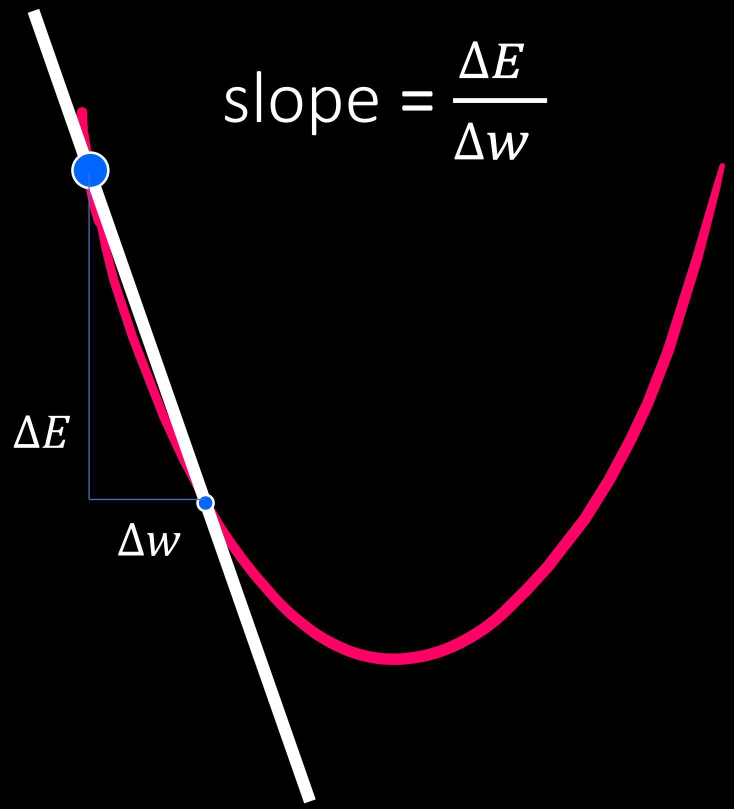
How to update w



How to update w

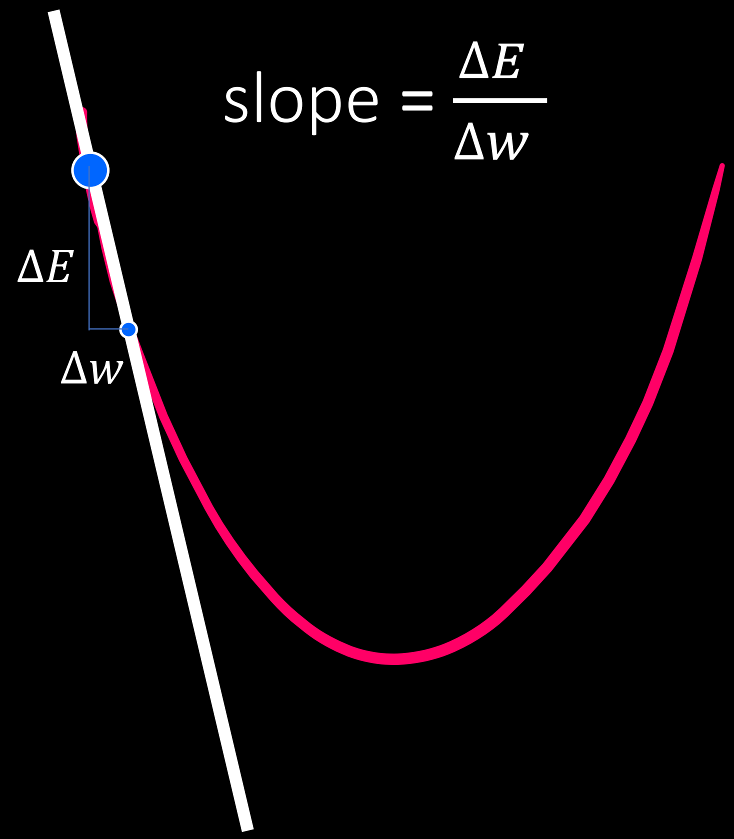


How to update w



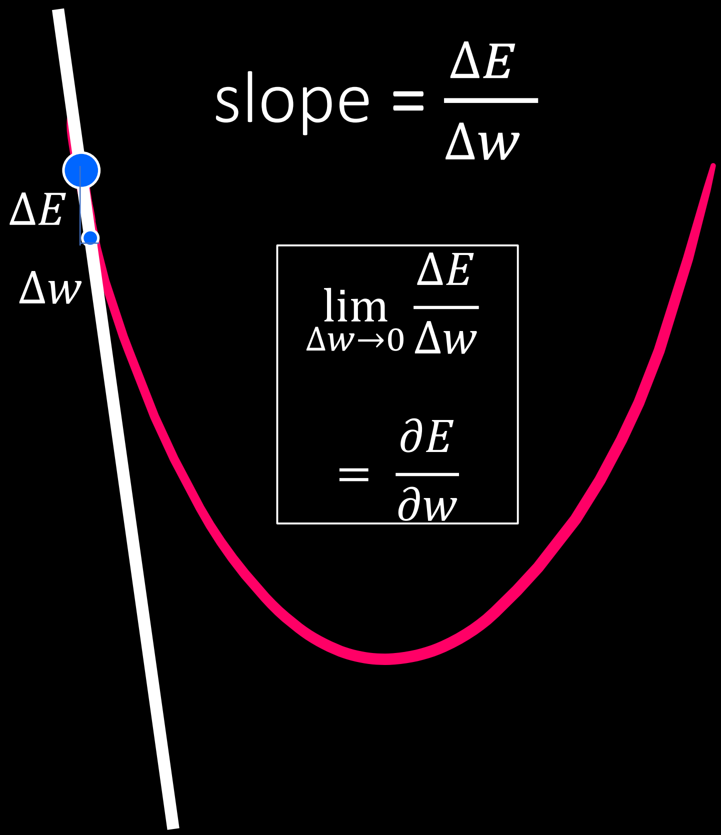
slope = $\frac{\Delta E}{\Delta w}$

How to update w



How to update w

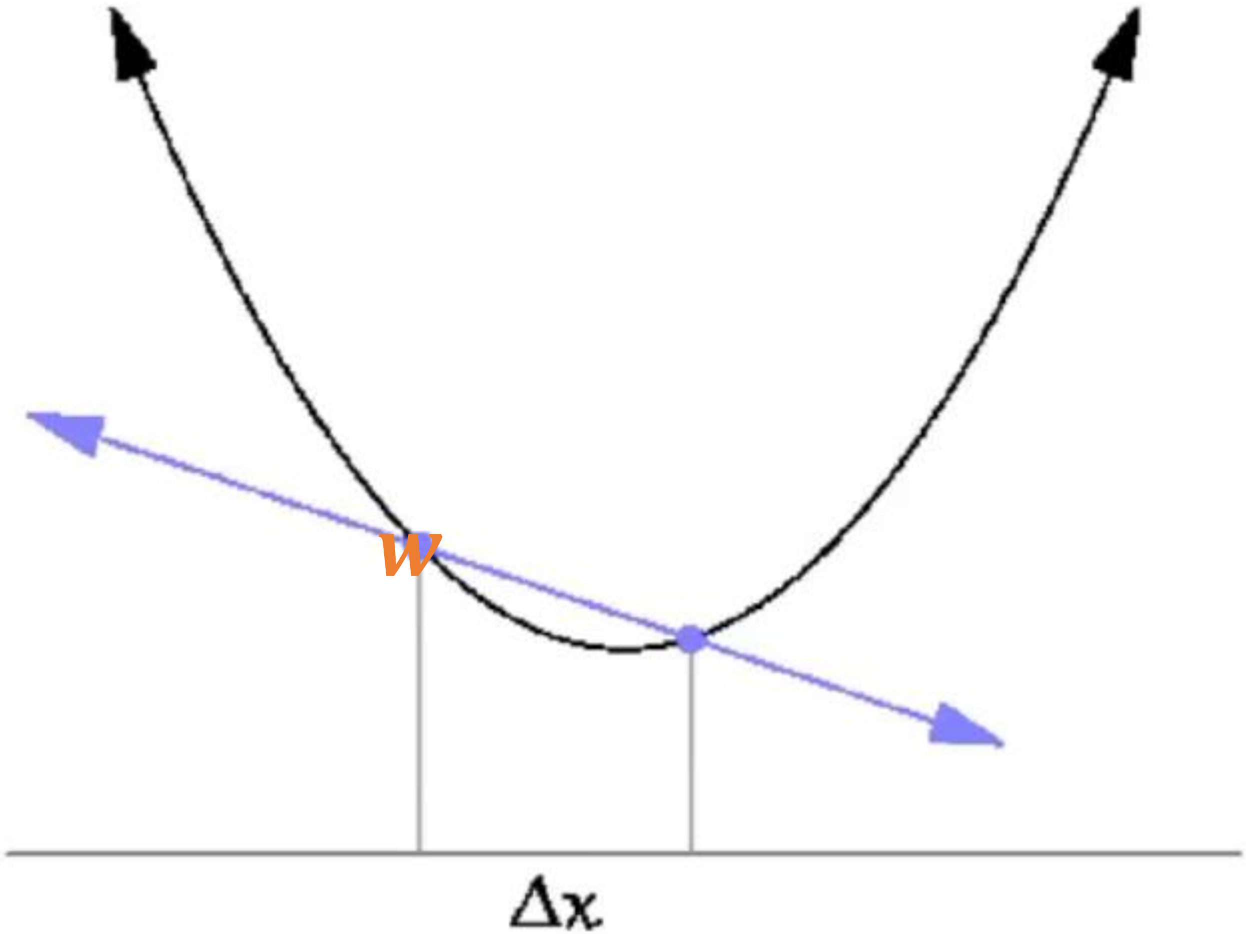
접선·Tangent line



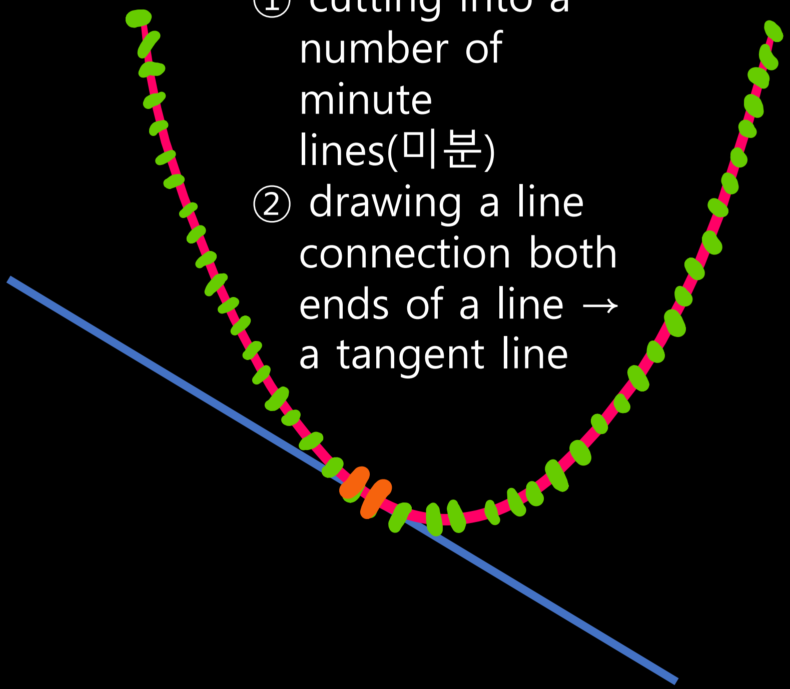
$$\text{slope} = \frac{\Delta E}{\Delta w}$$

$$\boxed{\begin{aligned} &\lim_{\Delta w \to 0} \frac{\Delta E}{\Delta w} \\ &= \frac{\partial E}{\partial w} \end{aligned}}$$

How to update w



How to update *w*



Numerical differentiation

① cutting into a number of minute lines(미분)
② drawing a line connection both ends of a line → a tangent line

$$\lim_{\Delta w \to 0} \frac{\Delta E}{\Delta w}$$

$$= \frac{\partial E}{\partial w}$$

= 미분

How to update w

$$w = w - \alpha * \text{Slope}$$

$$w = w - \alpha \frac{\partial E}{\partial w}$$

$\alpha =$ learning rate (ex, 0.1)

Advantages

- Fast movement from both sides and fine tuning at the valley(center) area
- Different slope/gradient according to the value of w
- Steep slope means that the error is big and w is far from the optimal area.
- We can get the slope(gradient) at any place(differentiable).

In case of AE

- Always the same slope in the error graph regardless of the value of w
- Therefore, the same speed in the movement
- Not sure to get the w value which gives 0 error or almost 0
- No way to guess the current value of w
- Not differentiable when w is 1

Multiple Data

For 3 instances of data

x_i	y_i
1	1
2	2
3	3

$$E = \frac{1}{3}\sum_{i=1}^{3}(wx_i - y_i)^2$$

desmos

Add (2, 2), (3, 3)

$$E = \frac{1}{3}\sum_{i=1}^{3}(wx_i - y_i)^2$$

Draw (w, E)

Multiple Data

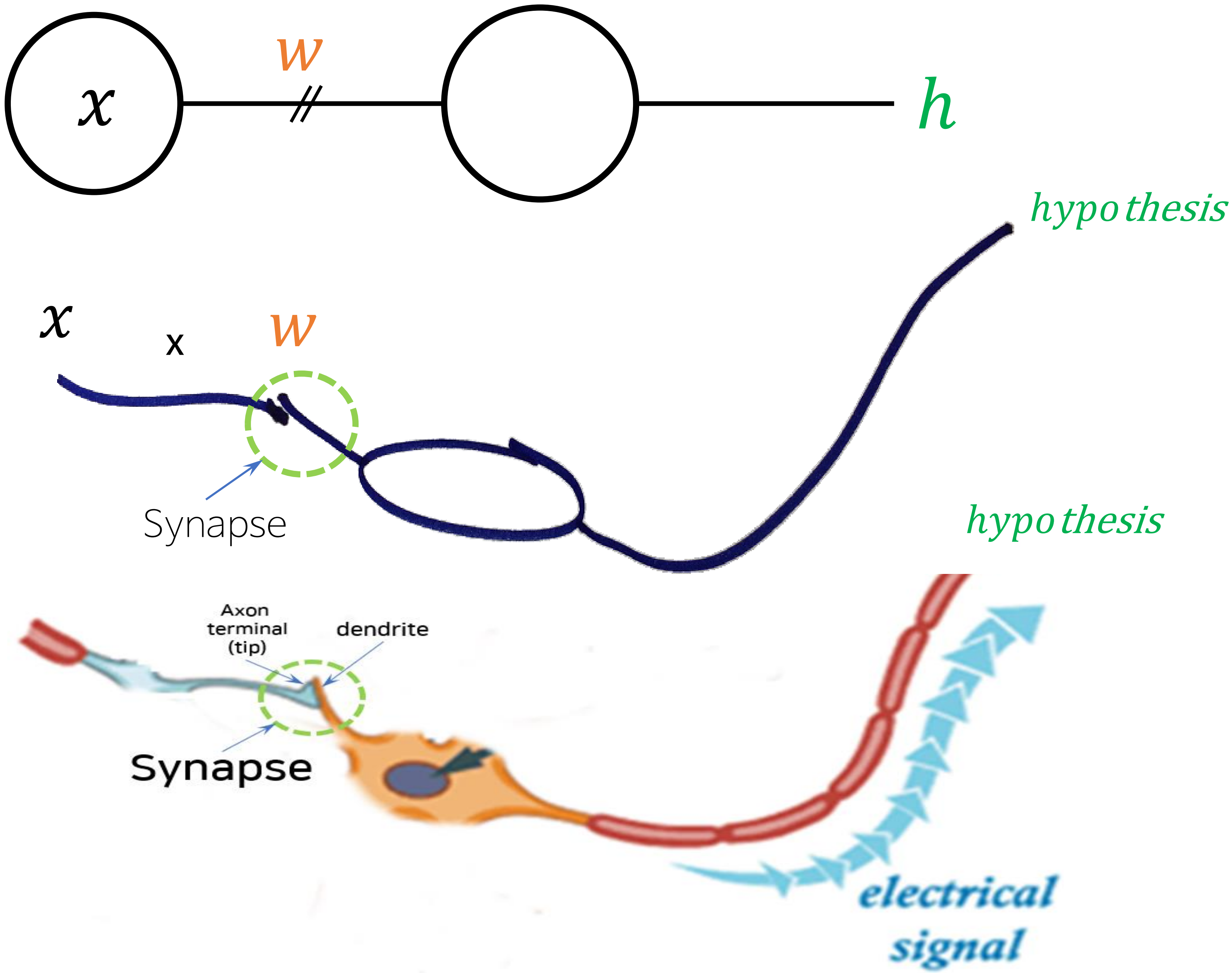
In case of m instances,

Mean Square Error

An answer by a neuron

$$E = \frac{1}{m}\sum_{i=1}^{m}(wx_i - y_i)^2$$

Ground truth



x
w
h
hypothesis
x
x
w
Synapse
hypothesis
Axon terminal (tip)
dendrite
Synapse
electrical signal

The meaning of slope

Steep slope $\frac{\Delta E}{\Delta w}$ $\frac{8}{1}$

If we change w, then the error E change drastically.

Gentle slope $\frac{\Delta E}{\Delta w}$ $\frac{1}{10}$

Even if we change w, the error E changes just a little bit.

$$\lim_{\Delta w \to 0} \frac{\Delta E}{\Delta w}$$

Slope/Gradient

$$= \frac{\partial E}{\partial w}$$

The influence of w change on error E

(Q) Compute the influence

$$E = (wx - y)^2$$

Let's assume that data (x, y) is $(1, 1)$, then compute the influence of w change on E when w is equal to 3.

Method1 numerical gradient

$$E = (w \cdot 1 - 1)^2$$

w: 3 -> E: 4

w: 3.00001 -> E: 4.00004

Δw=0.00001

ΔE=0.00004

$$\frac{\Delta E}{\Delta w} = \frac{0.00004}{0.00001} = 4$$

Slope =

Influence of w change = 4

Method2 derivative, differential equation

$$E = (w \cdot 1 - 1)^2$$

$$\lim_{\Delta w \to 0} \frac{\Delta E}{\Delta w} = \frac{\partial E}{\partial w} = \frac{\partial}{\partial w}(w \cdot 1 - 1)^2$$

$$= 2(w \cdot 1 - 1)$$

Therefore, when $w = 3$,
the gradient is $2(3 - 1) = 4$

How to update w (Learning)

1. Initialize w with a random value (ex, 4)

Loop

2. Get influence(slope) of w on E
3. To decrease the error, update w using below eq:

Parameter Tuning

$$W = W - \alpha * \text{slope}$$

4. Go to step 2

TensorFlow

- Machine learning framework by Google
- Tuning parameters including w automatically instead of us
- Define w inside of TensorFlow to be tuned (managed) by it
- Hypothesis and cost_function(E)

Linear Regression using TF

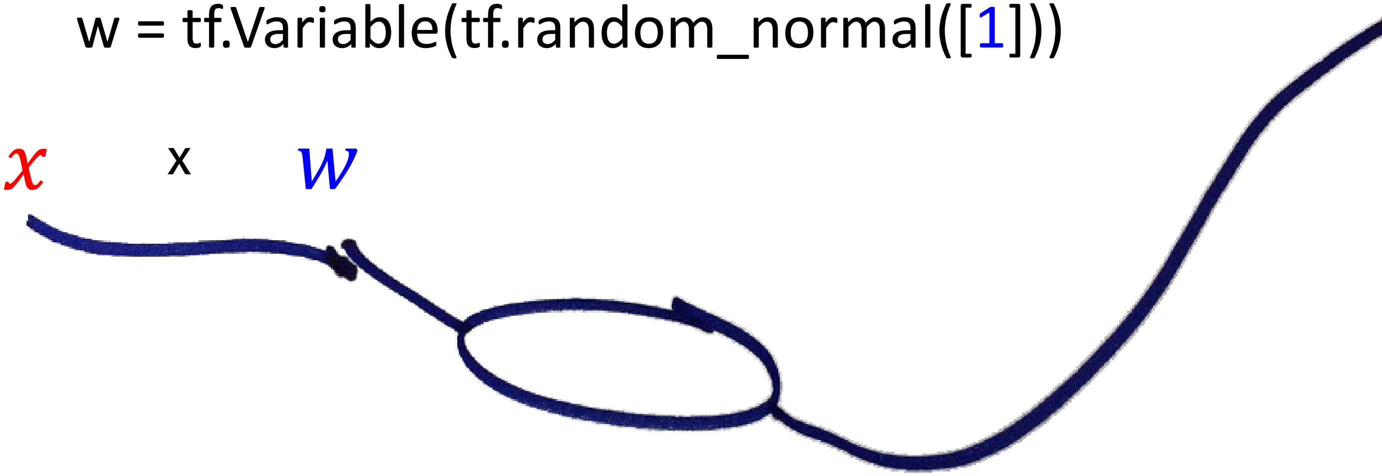
③

w = tf.Variable(tf.random_normal([1]))

hypo=$w * x$

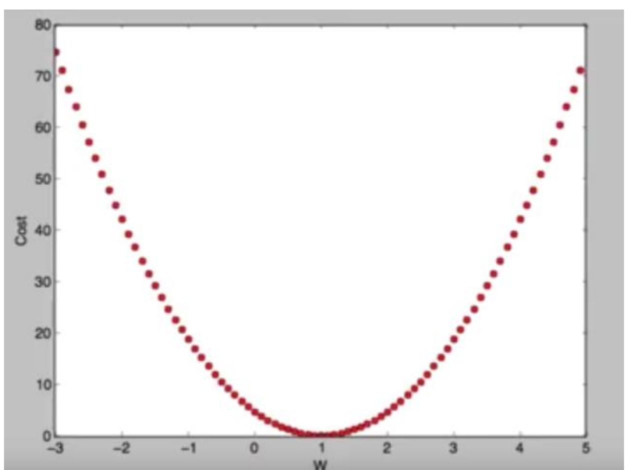
h ④

x x w



① x = [1]

y = [1] ②



cost_function = (hypo – y) ** 2

⑤

$$E = (\text{hypo} - y)^2$$

Download myml.git

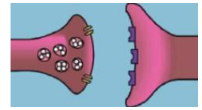
https://github.com/yungbyun/myml.git

1) Run DOS prompt
2) git clone https://github.com/yungbyun/myml.git
3) Open using PyCharm (File | Open...)

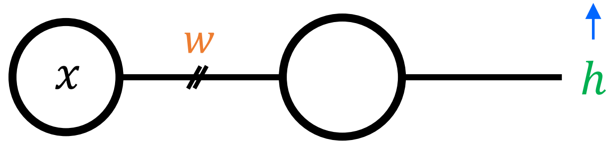
Lab 01.py

Finding w in linear regression

```python
import tensorflow as tf
```



$$E = |w \cdot x - y|$$




```python
#----- training data
x_data = [1]
y_data = [1]
```

```python
#----- a neuron / neural network
w = tf.Variable(tf.random_normal([1]))
hypo = w * x_data
```

train operation to update w to minimize error(E)

```python
#----- learning
cost = (hypo - y_data) ** 2

train = tf.train.GradientDescentOptimizer(learning_rate=0.01).minimize(cost)

sess = tf.Session()
sess.run(tf.global_variables_initializer())

for i in range(1001):
    sess.run(train) #1-run, 1-update of w -> 1001 updates

    if i % 100 == 0:
        print('w:', sess.run(w), 'cost:', sess.run(cost))
```

```python
#----- testing(prediction)
x_data = [2]
print(sess.run(x_data * w))
```

`sess.run(train)`

How to update w in TensorFlow

Computation Graph

loss/error function

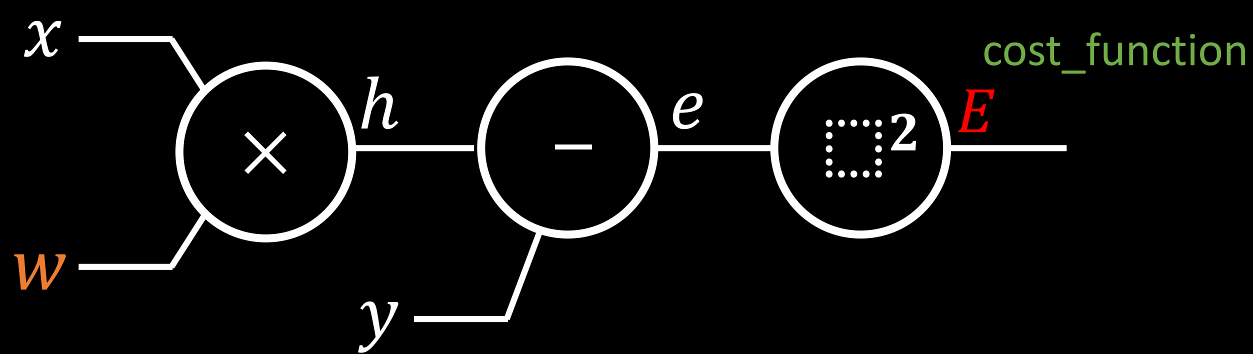
$$E = (wx - y)^2$$

- The part representing a neuron
- Where is a synapse?
- Which one is an input data?
- The output of a neuron
- Find a correct answer or ground truth.
- Find hypothesis.
- Imagine E having many inputs.

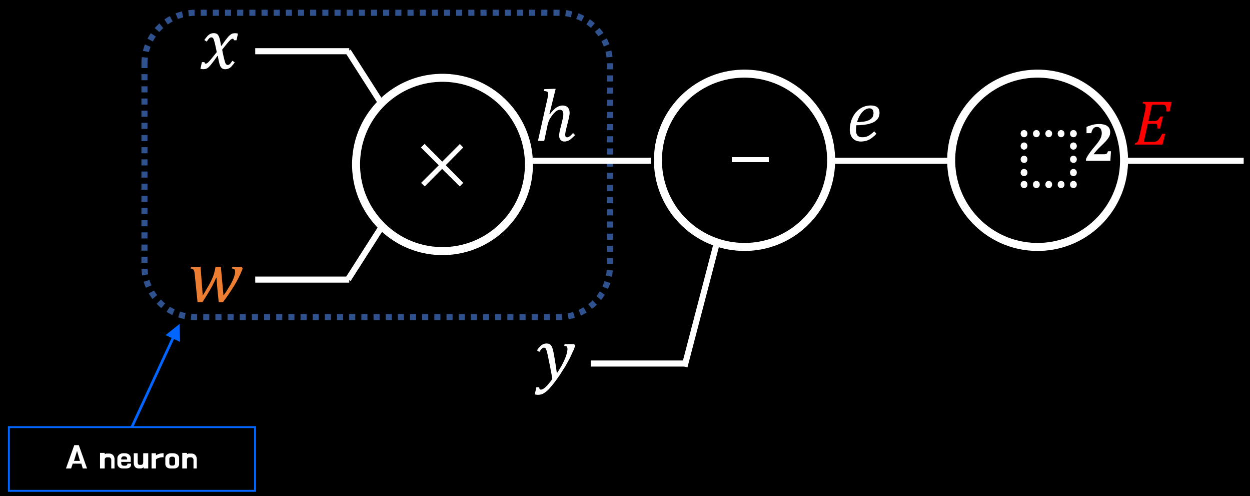
Computation Graph

$$E = (w \cdot x - y)^2$$

hypo = w * x
cost_function(E) = (hypo - y) ** 2)

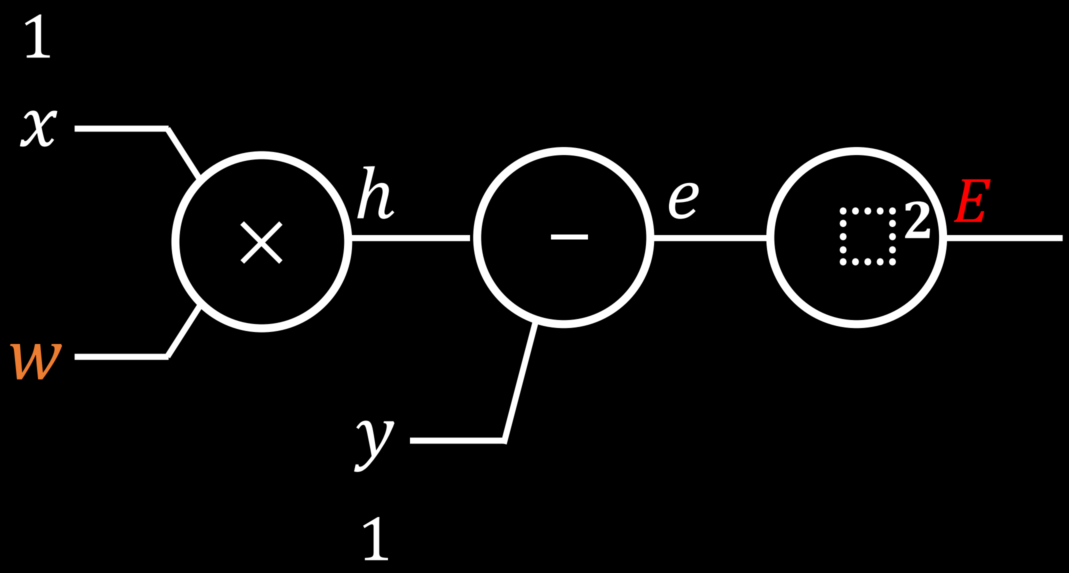


Computation Graph

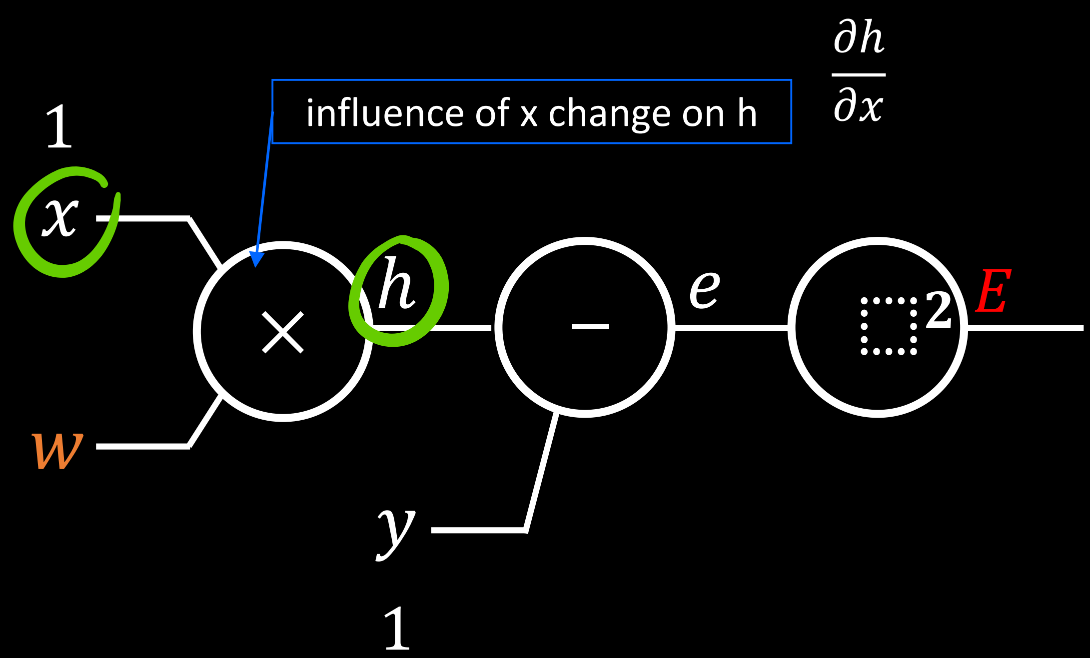


Computation Graph

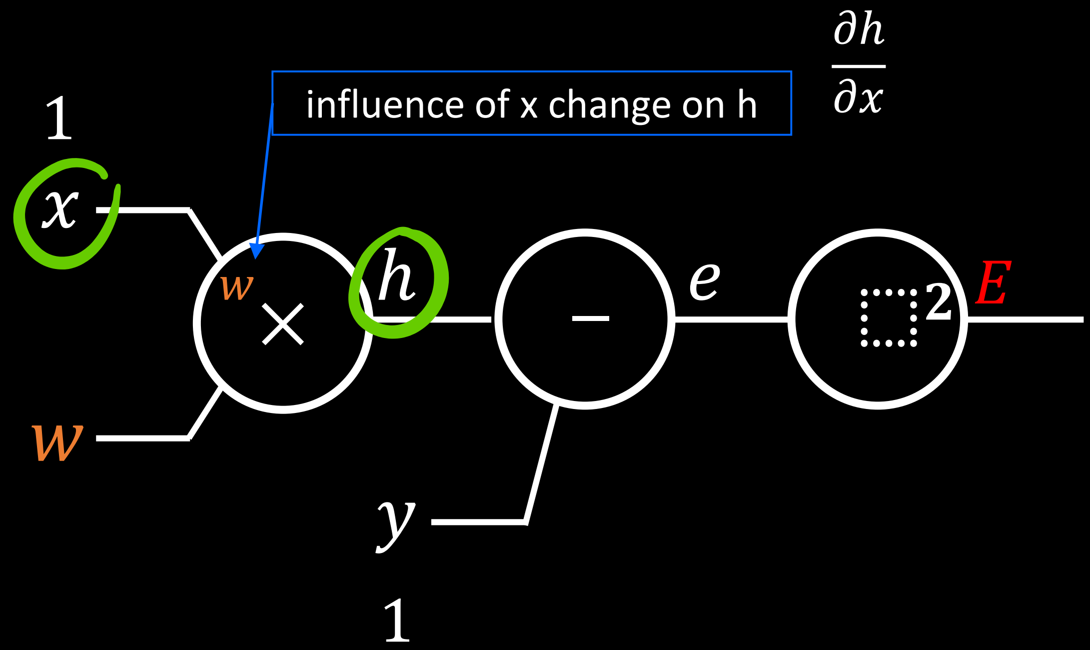
Data (x, y) −> $(1, 1)$



Computation Graph

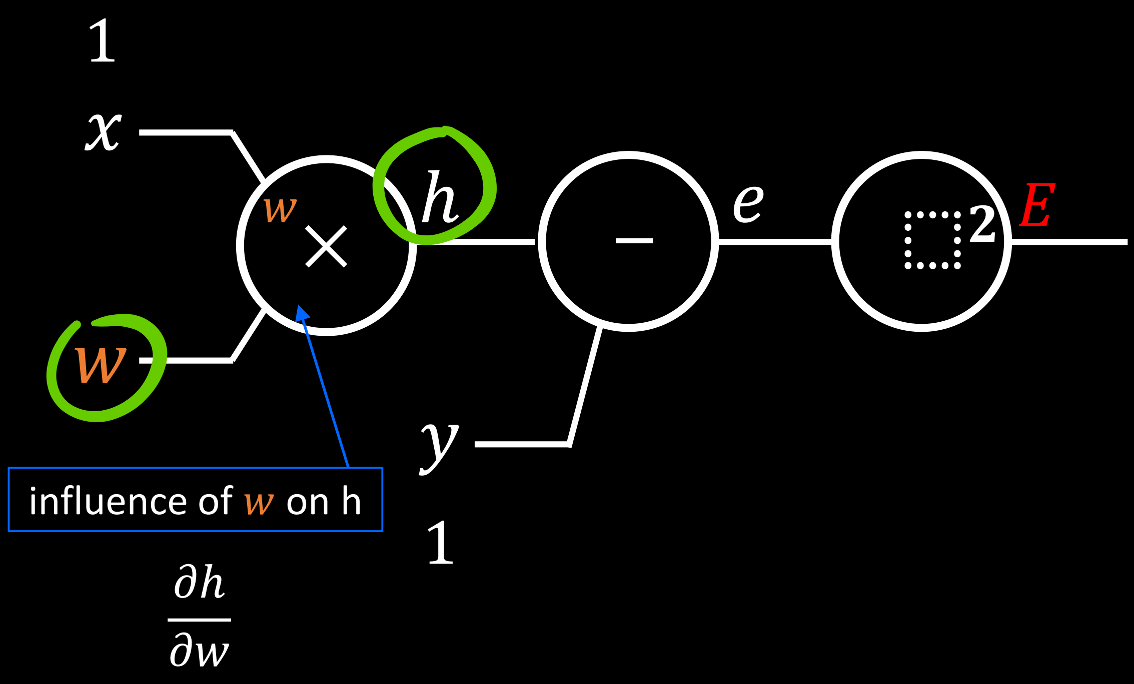


Computation Graph

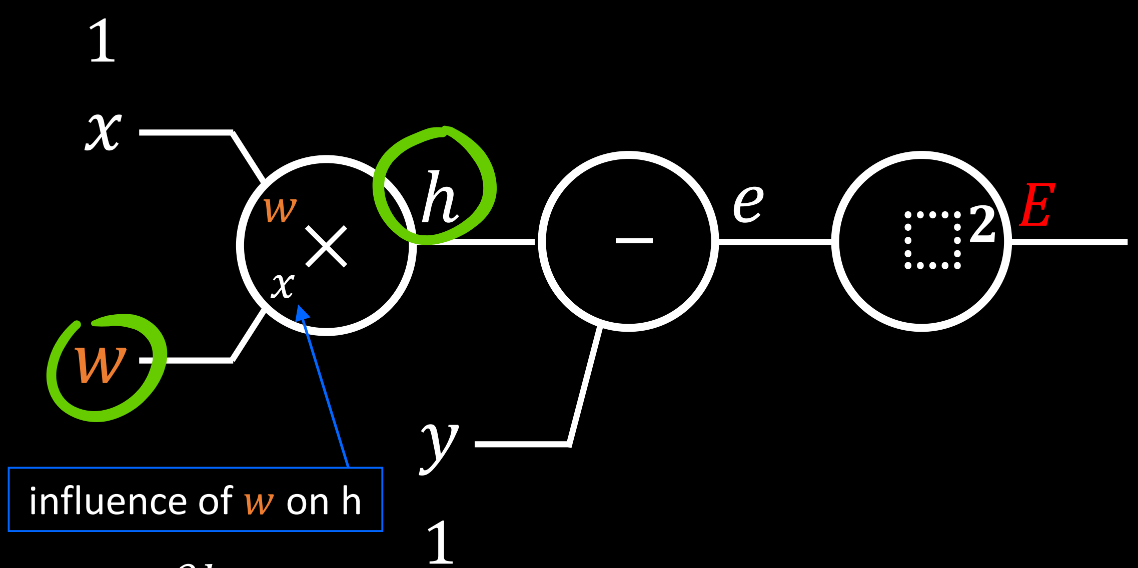


Local gradient

Computation Graph

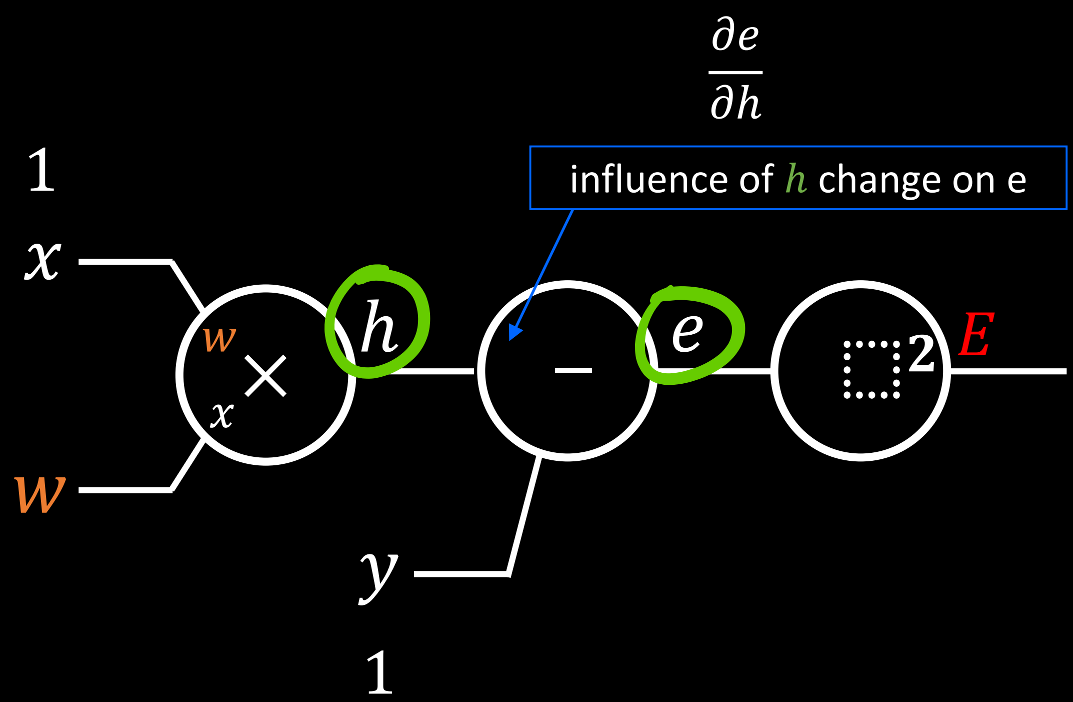


Computation Graph

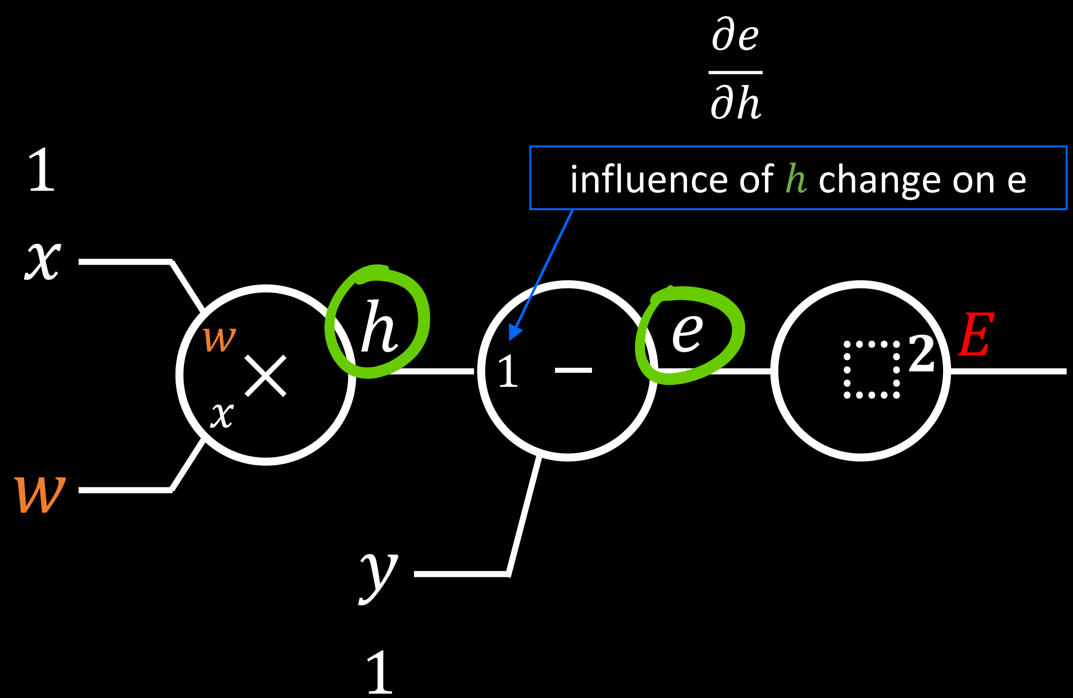


$$\frac{\partial h}{\partial w}$$

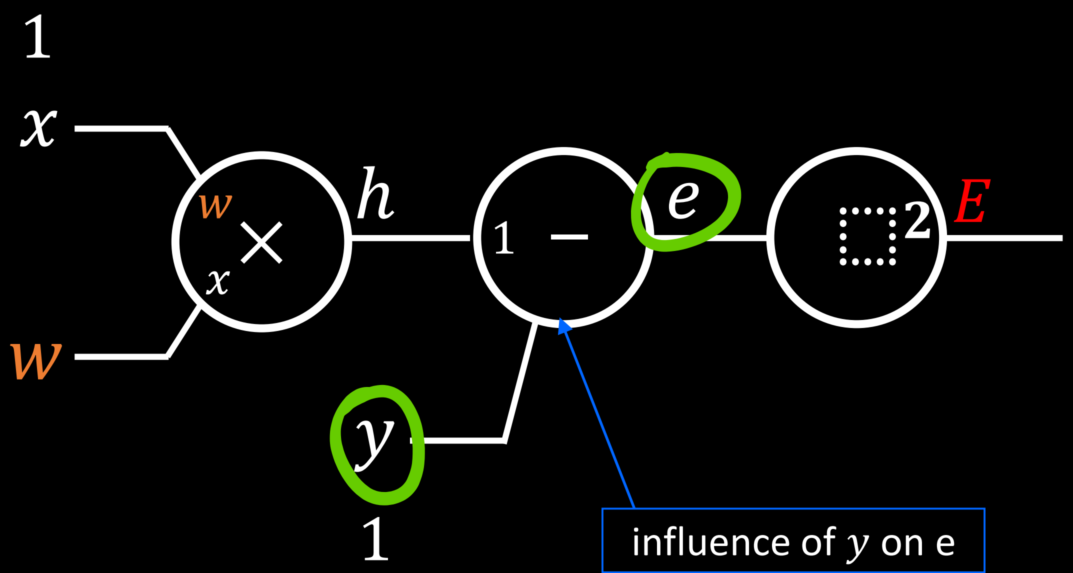
Computation Graph



Computation Graph

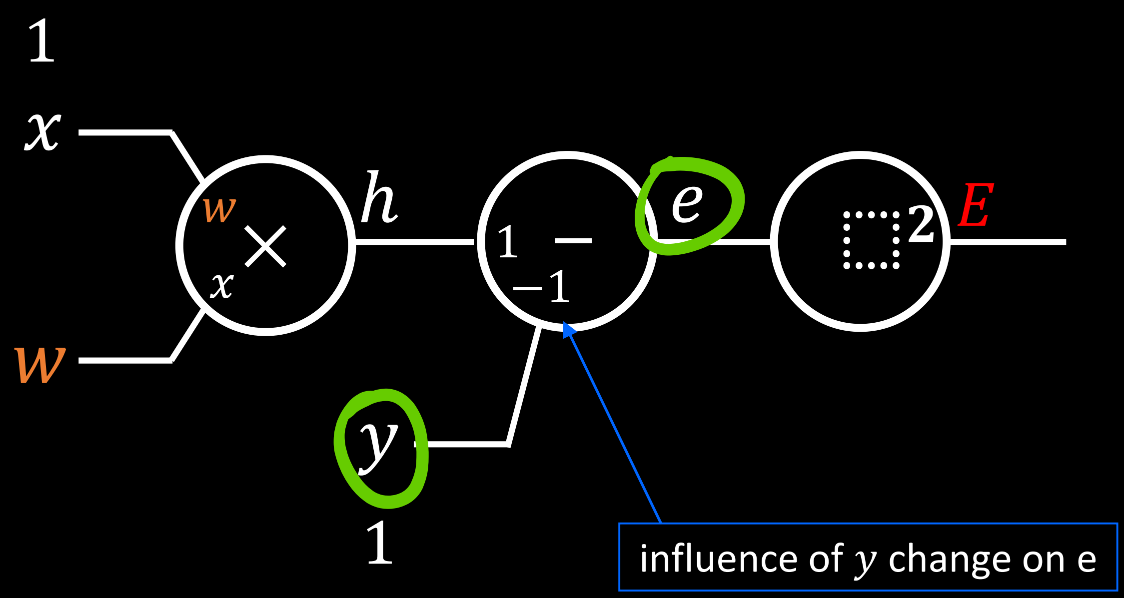


Computation Graph



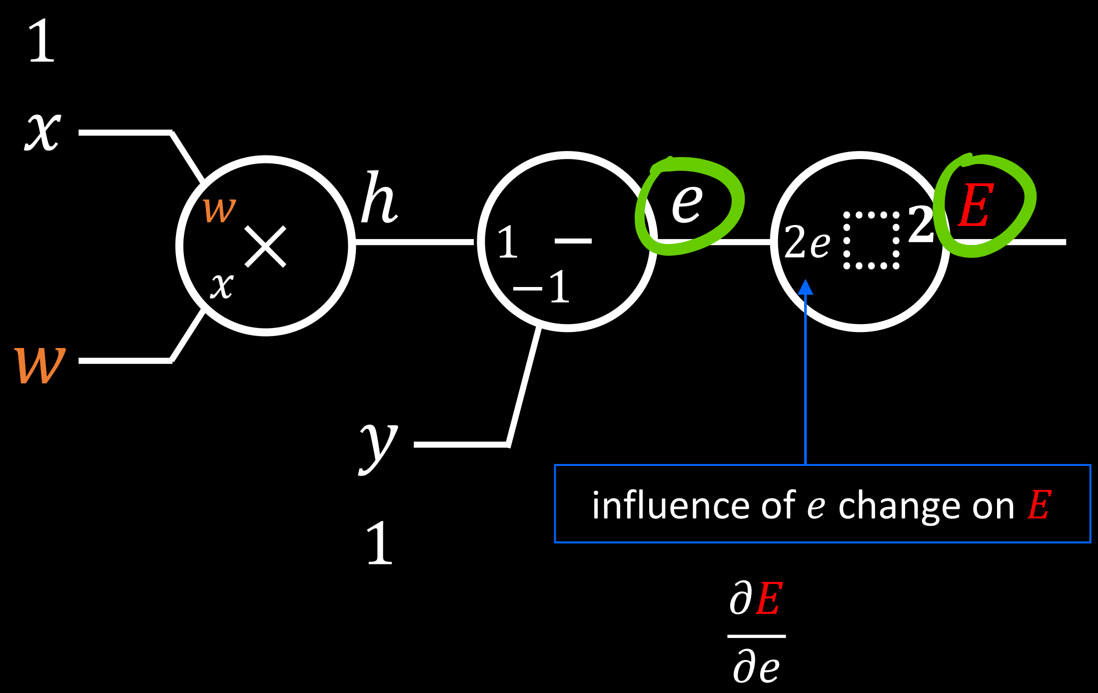
$$\frac{\partial e}{\partial y}$$

Computation Graph

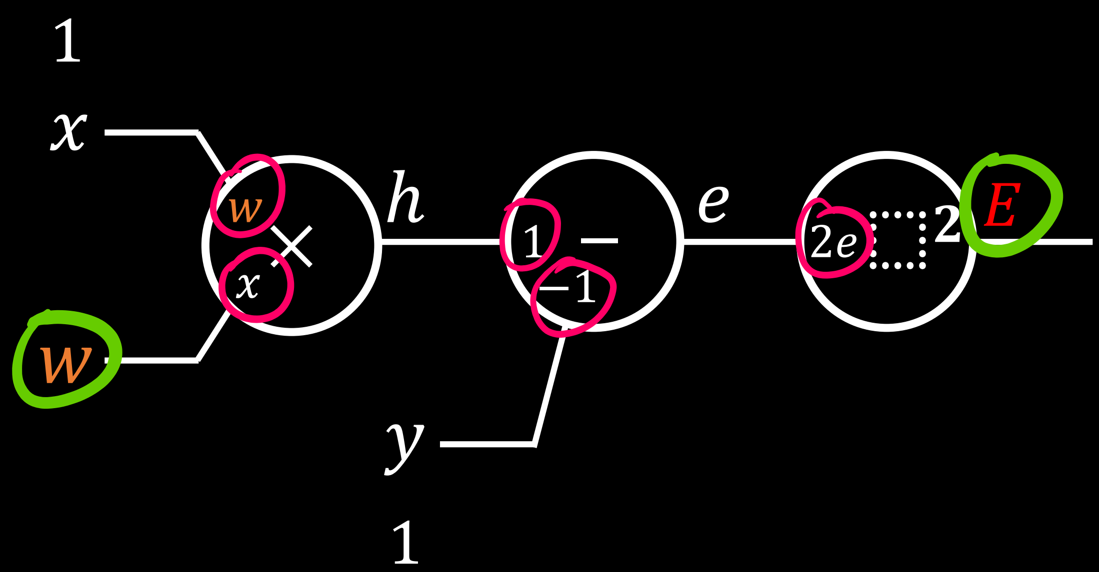


$$\frac{\partial e}{\partial y}$$

Computation Graph

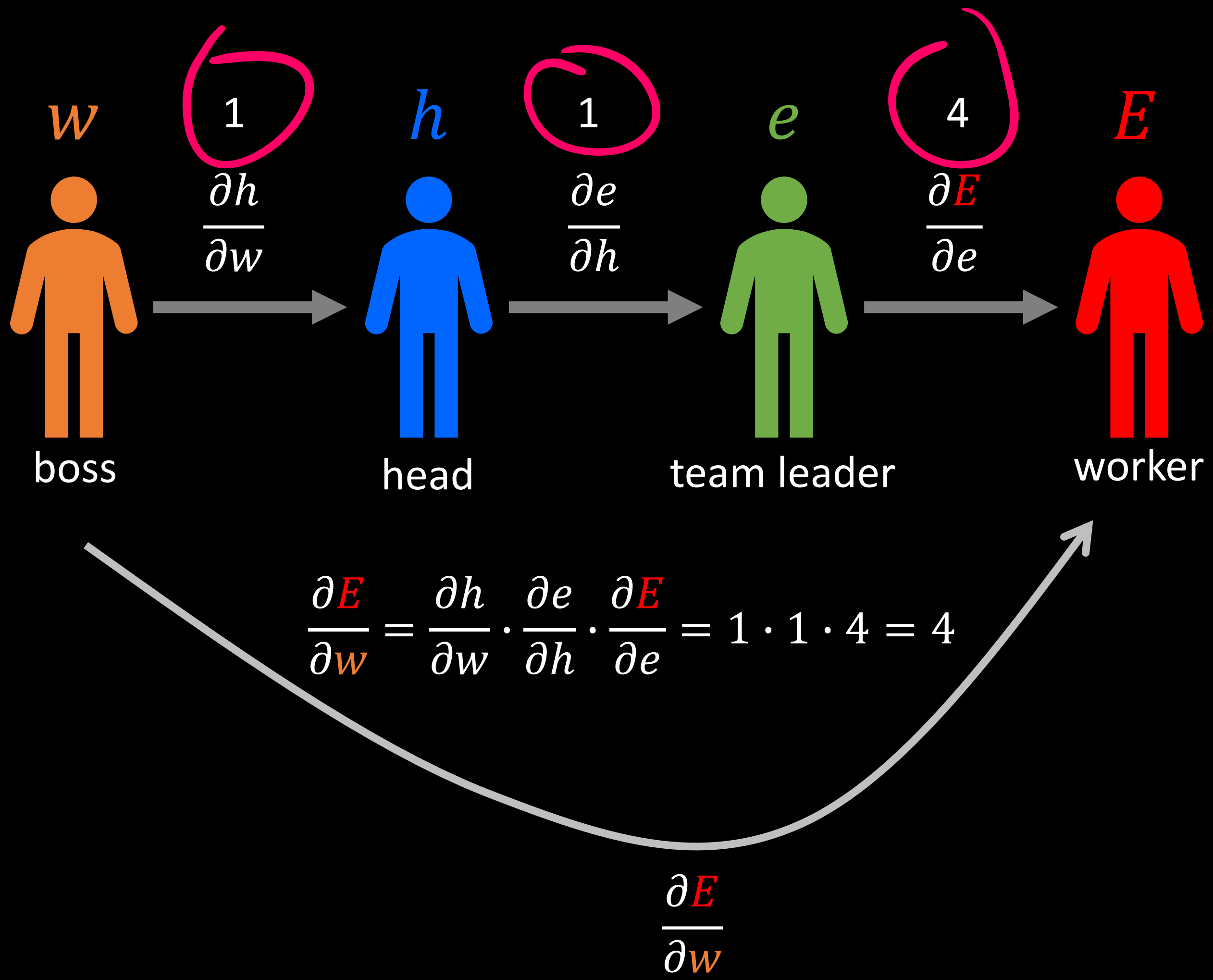


5 Local Gradients in gates



How can we get the influence of w change on E?

Influence between persons



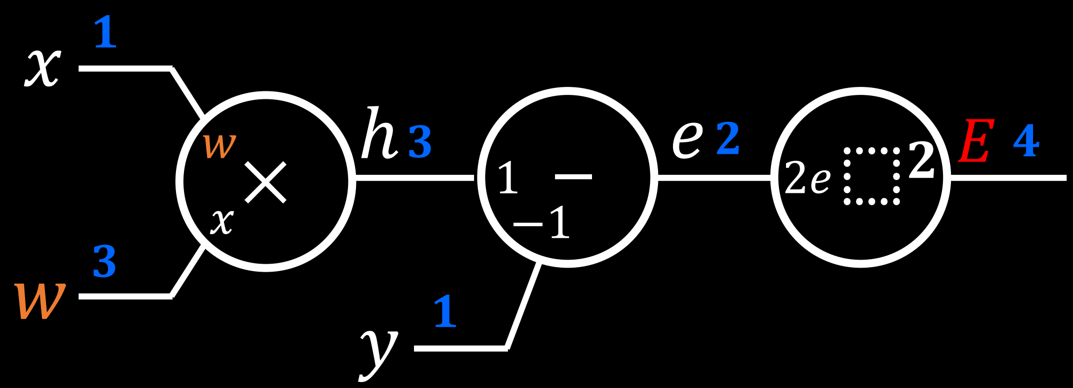
The influence of w change on E

$$\frac{\partial E}{\partial w} = \frac{\partial h}{\partial w} \times \frac{\partial e}{\partial h} \times \frac{\partial E}{\partial e}$$

Chain rule!

Forward propagation

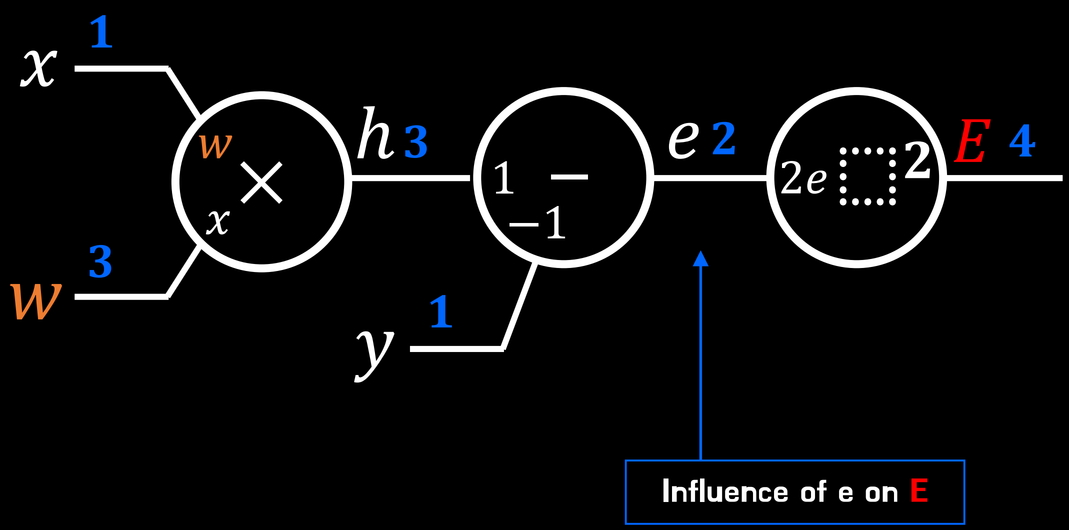
If $(x, y) = (1, 1)$ and $w = 3$, then compute E.



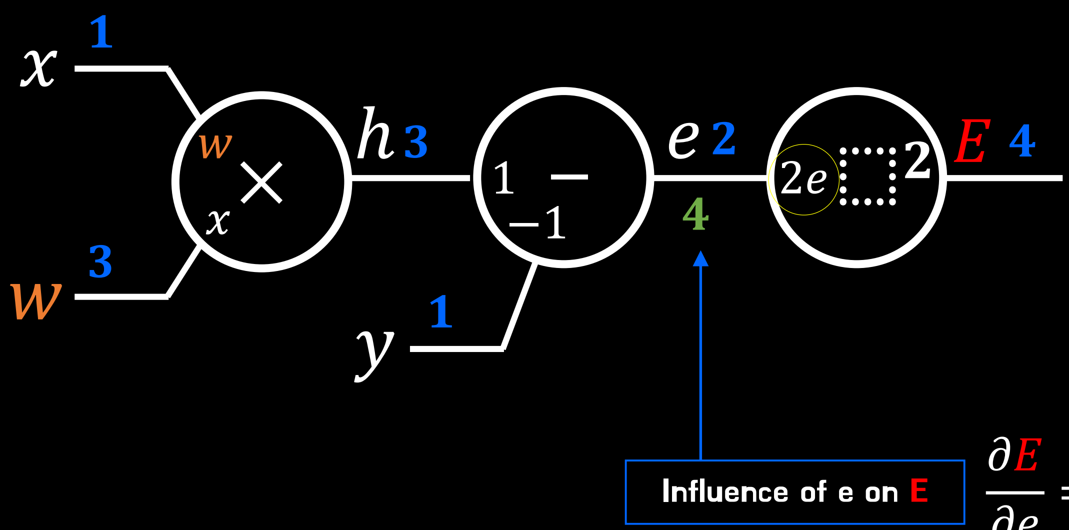
Error is big (4) ,
so let's update w

using back-propagation.

Back-propagation

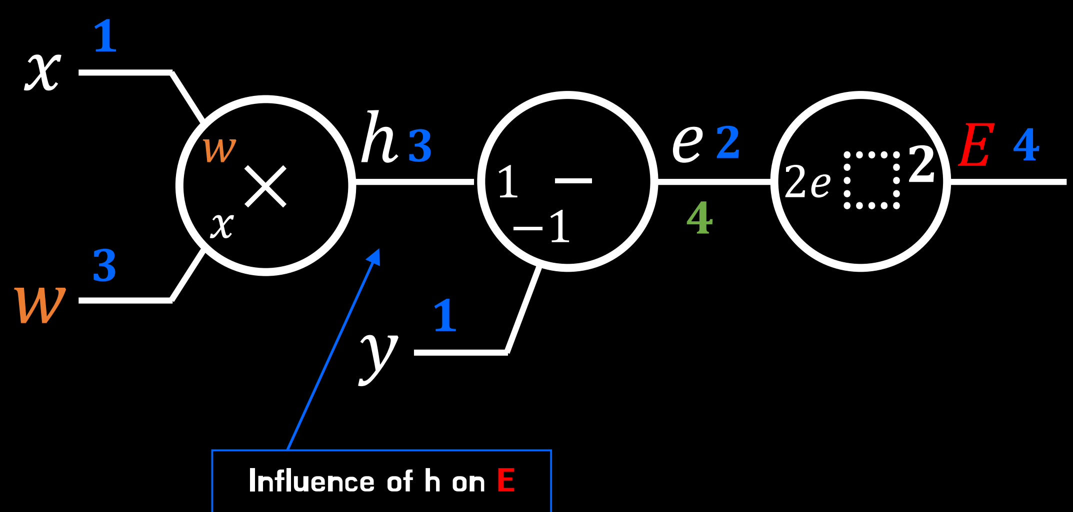


Back-propagation



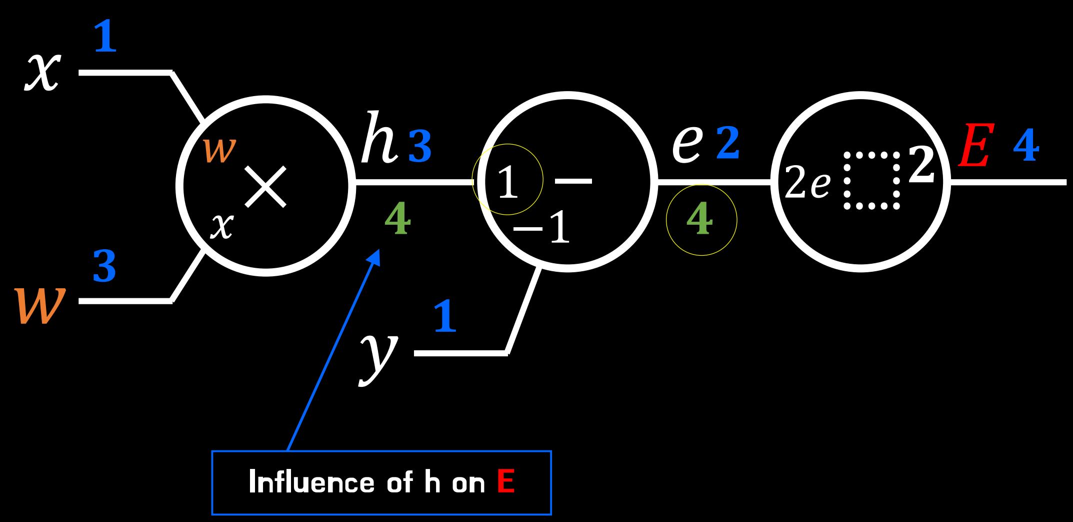
$$\frac{\partial E}{\partial e} = 2e$$

Back-propagation

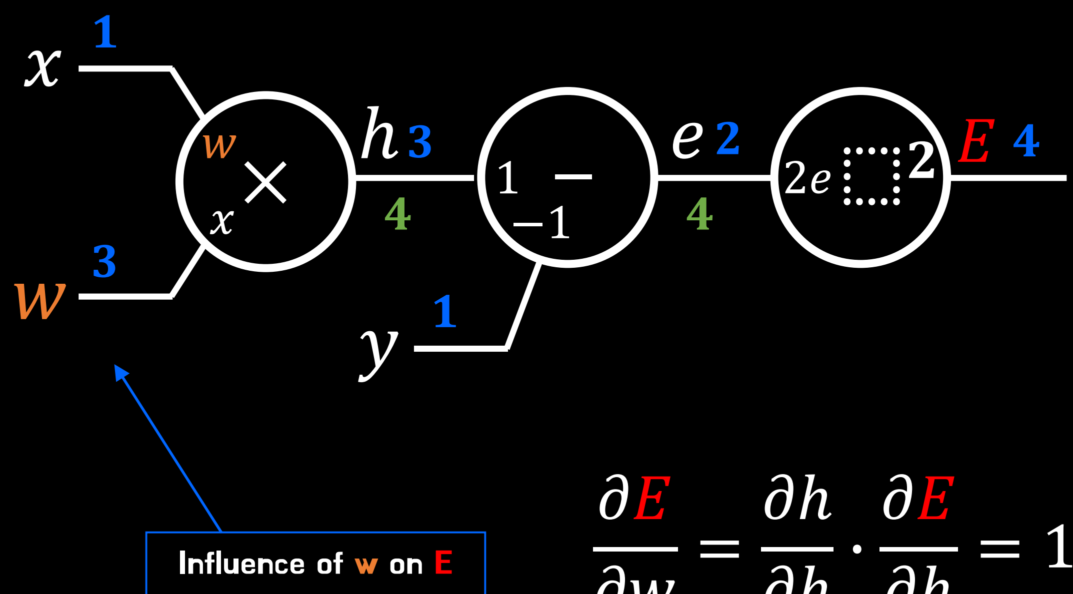


$$\frac{\partial E}{\partial h} = \frac{\partial e}{\partial h} \cdot \frac{\partial E}{\partial e} = 1 \cdot 4$$

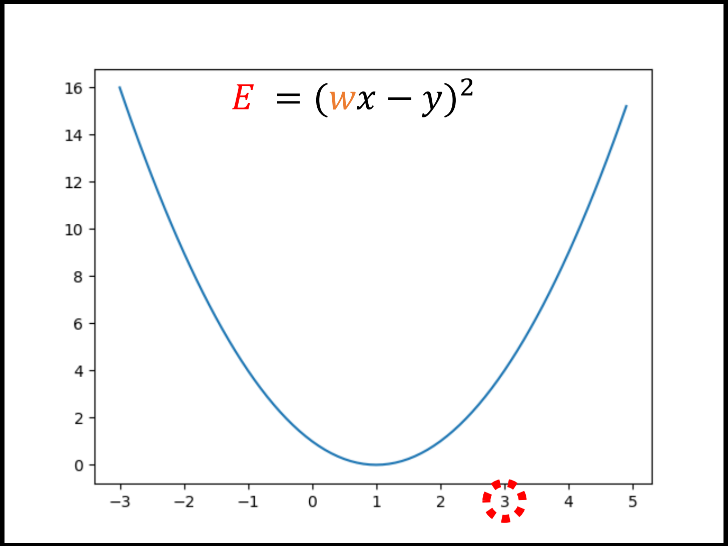
Back-propagation

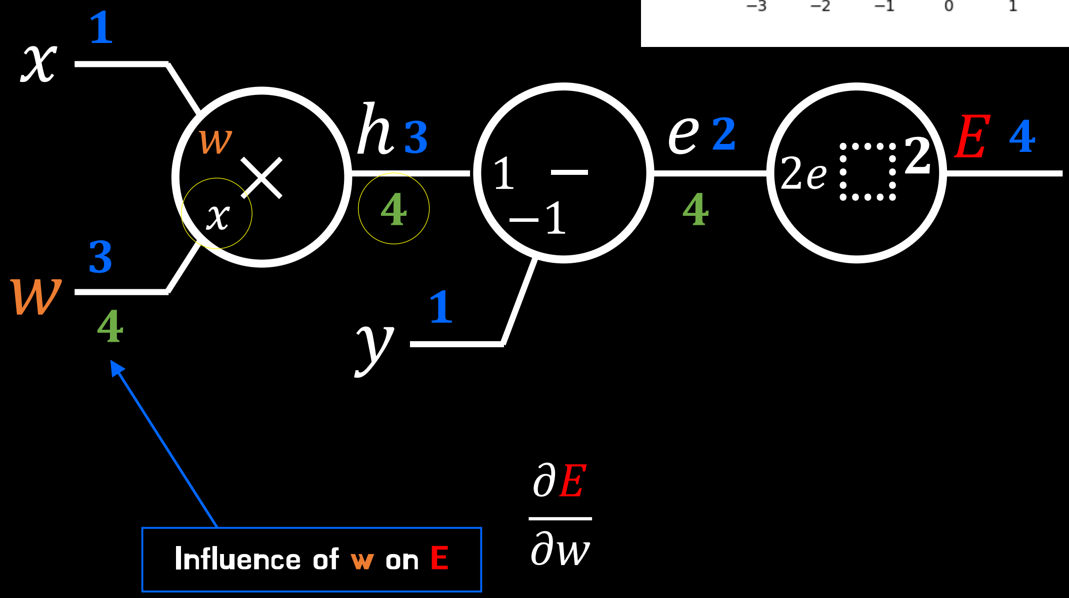


Back-propagation



$$\frac{\partial E}{\partial w} = \frac{\partial h}{\partial h} \cdot \frac{\partial E}{\partial h} = 1 \cdot 1 \cdot 4$$





$$\frac{\partial E}{\partial w}$$

Back-propagation, the process to apply chain rules.

$$\frac{\partial E}{\partial w}$$

$$\frac{\partial E}{\partial w}$$

$$w = 3 - 0.1 * 4$$

$$w = 2.6$$

Tuned parameter after 1 step learning

After enough number of steps,
the parameter w will be optimized.

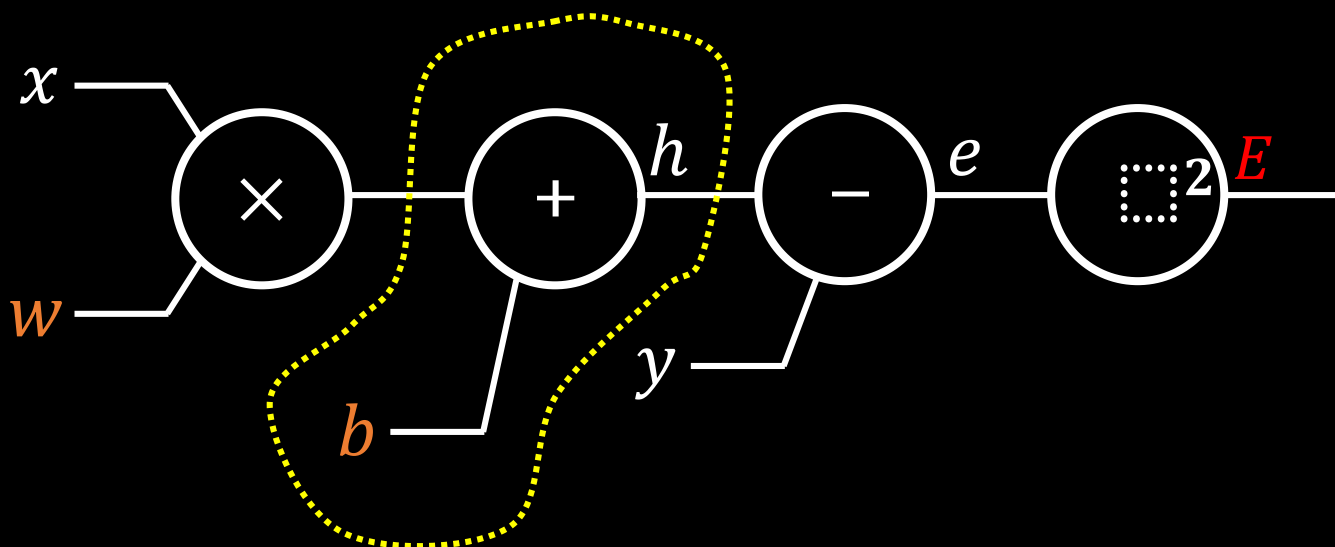
```python
import tensorflow as tf

#----- training data
x_data = [1]
y_data = [1]

#----- a neuron / neural network
w = tf.Variable(tf.random_normal([1]))
hypo = w * x_data

#----- learning
cost = (hypo - y_data) ** 2

train = tf.train.GradientDescentOptimizer(learning_rate=0.01).minimize(cost)

sess = tf.Session()
sess.run(tf.global_variables_initializer())

for i in range(1001):
    sess.run(train) # 1-run, 1-update of w -> 1001 updates

    if i % 100 == 0:
        print('w:', sess.run(w), 'cost:', sess.run(cost))

#----- testing(prediction)
x_data = [2]
print(sess.run(x_data * w))
```

train operation to update w to minimize cost(error)

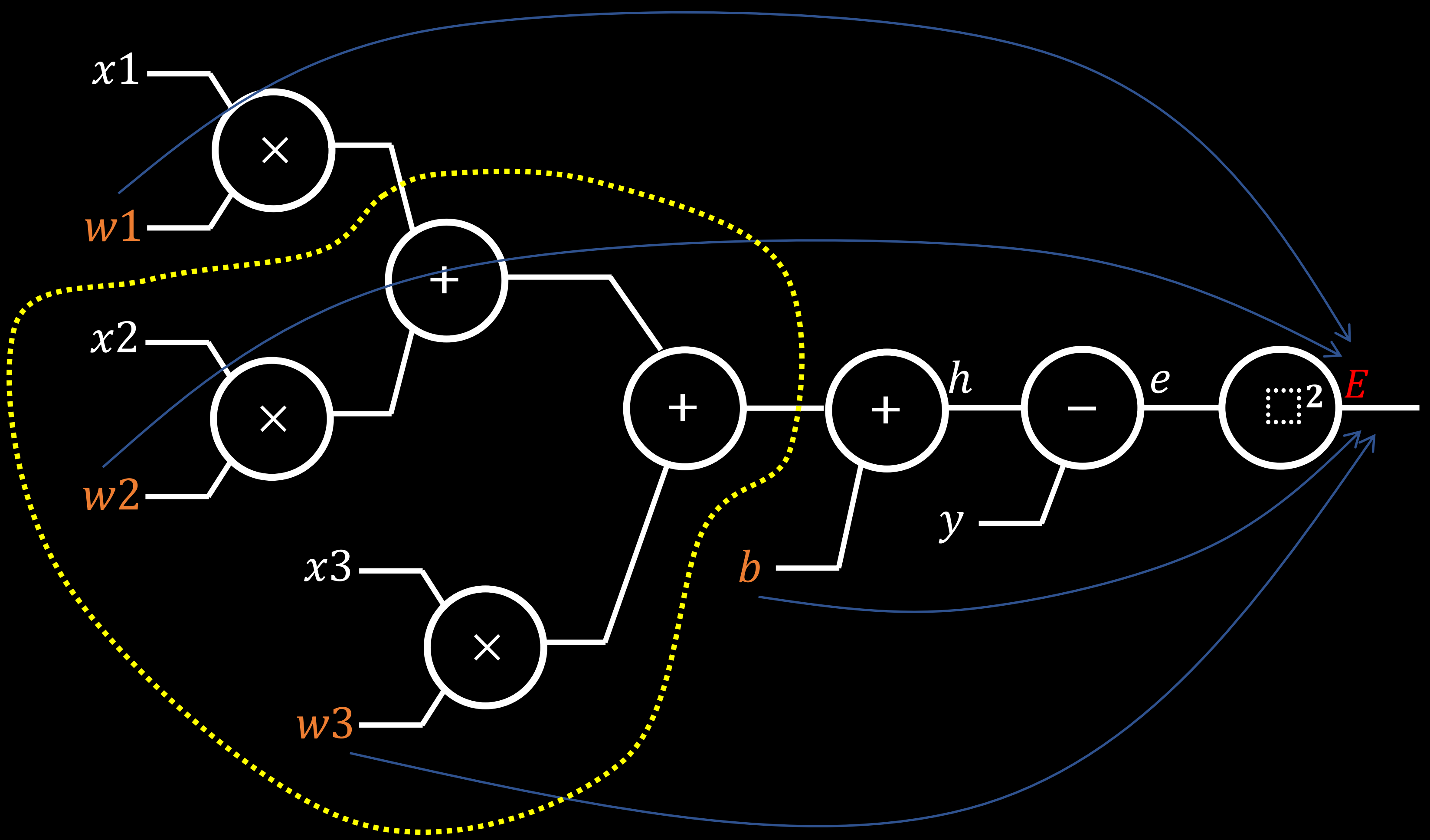
Extension of the Graph

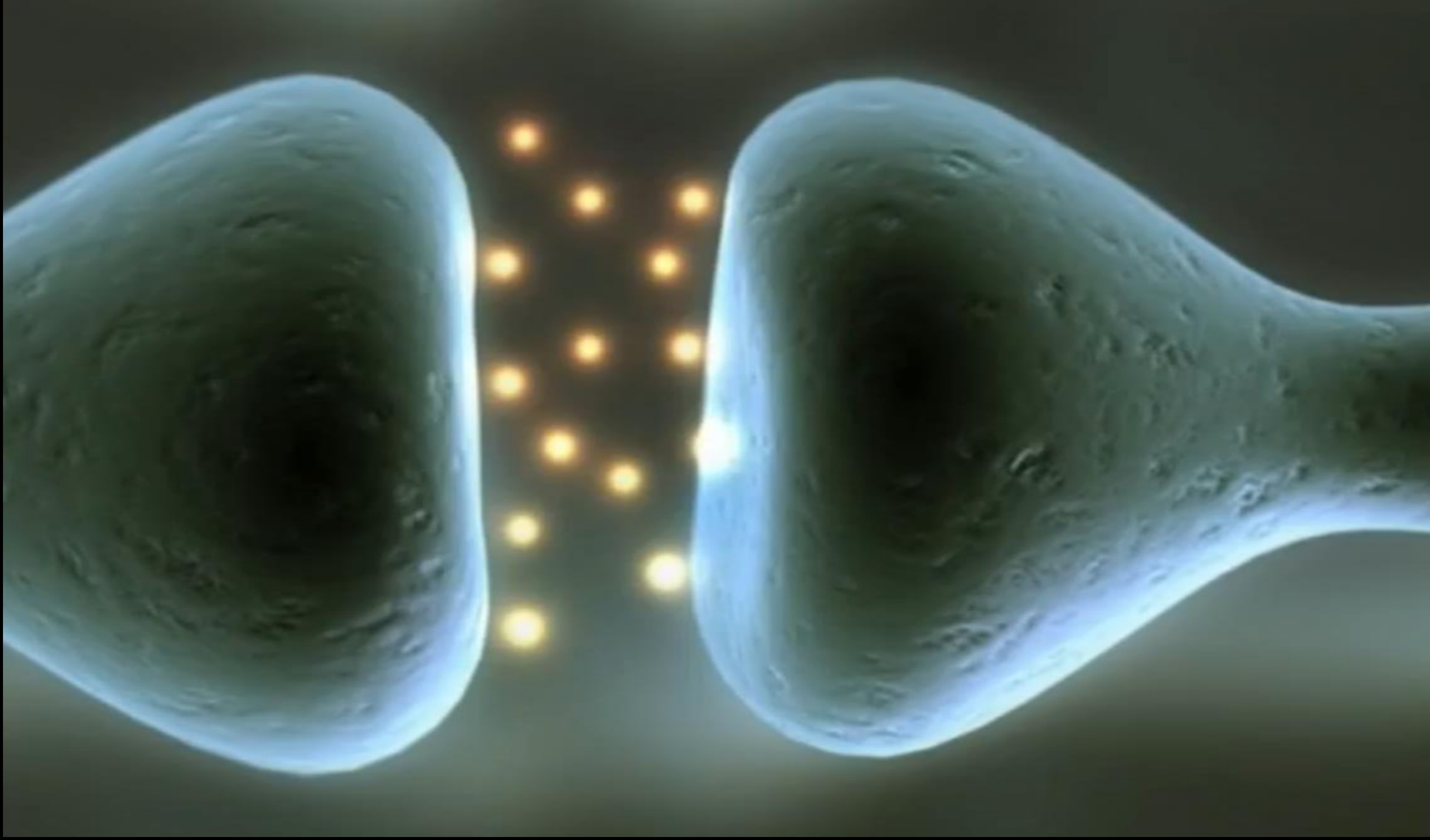
- adding bias b (one more plus gate)
- a neuron with 3 inputs (2 more + gate)
- two neurons

$$E = ((wx \boxed{+ b}) - y)^2$$



$$E = ((w1x1 \boxed{+ w2x2 + w3x3} + b) - y)^2$$

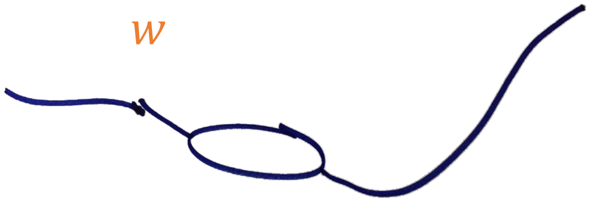


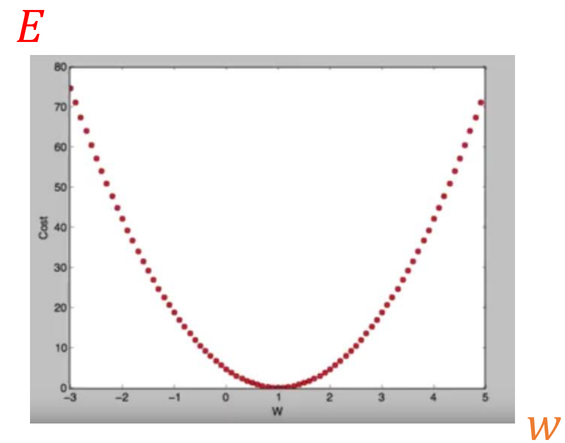


Learning, making the connection better

Cost(Error) graph

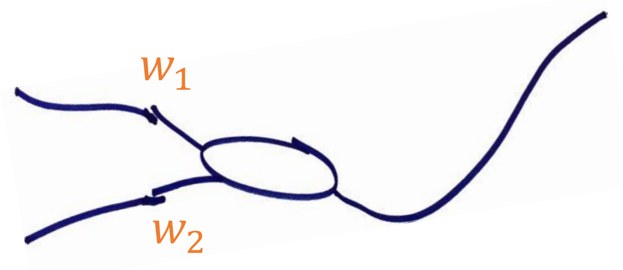
$$E = (w \cdot 1 - 1)^2$$

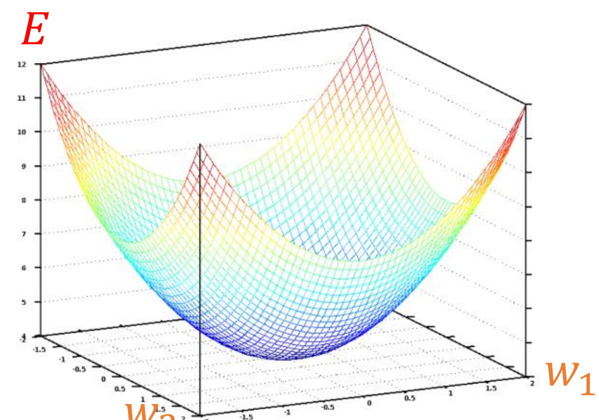




convex function

$$E = (\boldsymbol{w_1} \cdot 1 + \boldsymbol{w_2} \cdot 1 - 1)^2$$





convex function

Lab 02.with_bias.py

Parameter tuning including bias

Lab 03.py

Using multiple data

Lab 04.py

Training a neuron having multiple inputs