Cooperative Coevolutionary Negatively Correlated Natural Evolution Strategy

基于合作协同的负相关自然进化策略

计算机科学与技术
陈驿来

Southern University of Science and Technology

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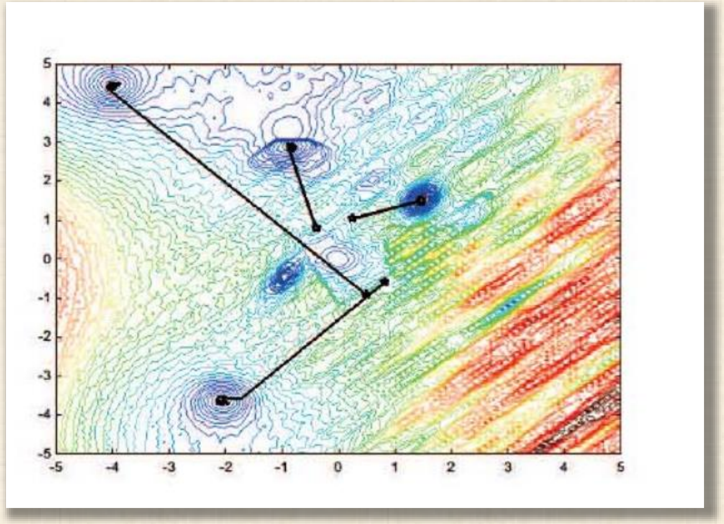
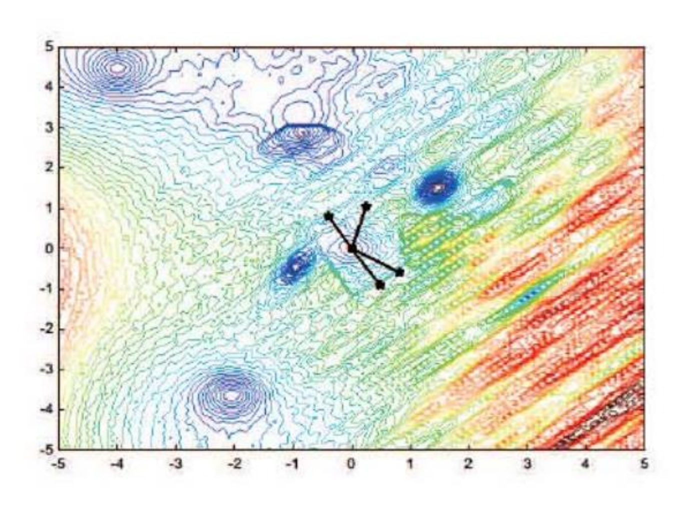
Background & NCNES framework

Negatively Correlated Search





TANG K, YANG P, YAO X. Negatively correlated search[J]. IEEE Journal on Selected Areas in Communications, 2016, 34(3): 542-550

Negatively Correlated Search (NCS) 是一种进化算法，通过迭代方式寻找最优解。其核心思想是生成多样化的种群，在搜索空间中进行负相关的探索，从而更有效地找到最优解。

缺点：根据直觉构思，缺乏数学解释

NCNES framework

NCNES通过数学公式推导，来定义了NCS的具体搜索过程：在种群的初始化和每次种群的生成时使用了概率分布，每次迭代的目标是更新概率分布的参数。

$$p(\theta_i) = \mathcal{N}(\boldsymbol{m}_i, \boldsymbol{\Sigma}_i)$$

目标函数：J

适应度模型：F

多样性模型：D

迭代目标：通过对目标函数求梯度来更新概率分布的参数

$$J = F + D = \sum_{i=1}^{\lambda} f(\theta_i) + \sum_{i=1}^{\lambda} d(p(\theta_i))$$

$$F = \sum_{i=1}^{\lambda} \int f(x) p(x|\theta_i) dx = \sum_{i=1}^{\lambda} f(\theta_i)$$

$$D = \sum_{i=1}^{\lambda} \sum_{j=1, i \neq j}^{\lambda} -C(p(\theta_i), p(\theta_j)) = \sum_{i=1}^{\lambda} d(p(\theta_i))$$

$$C(p(\theta_i), p(\theta_j)) = -\log\left(\int \sqrt{p(x|\theta_i) p(x|\theta_j)}\, dx\right)$$

YANGP,YANGQ,TANGK,etal.Parallel exploration via negatively correlated search[J]. Fron tiers of Computer Science, 2021, 15:1-13.

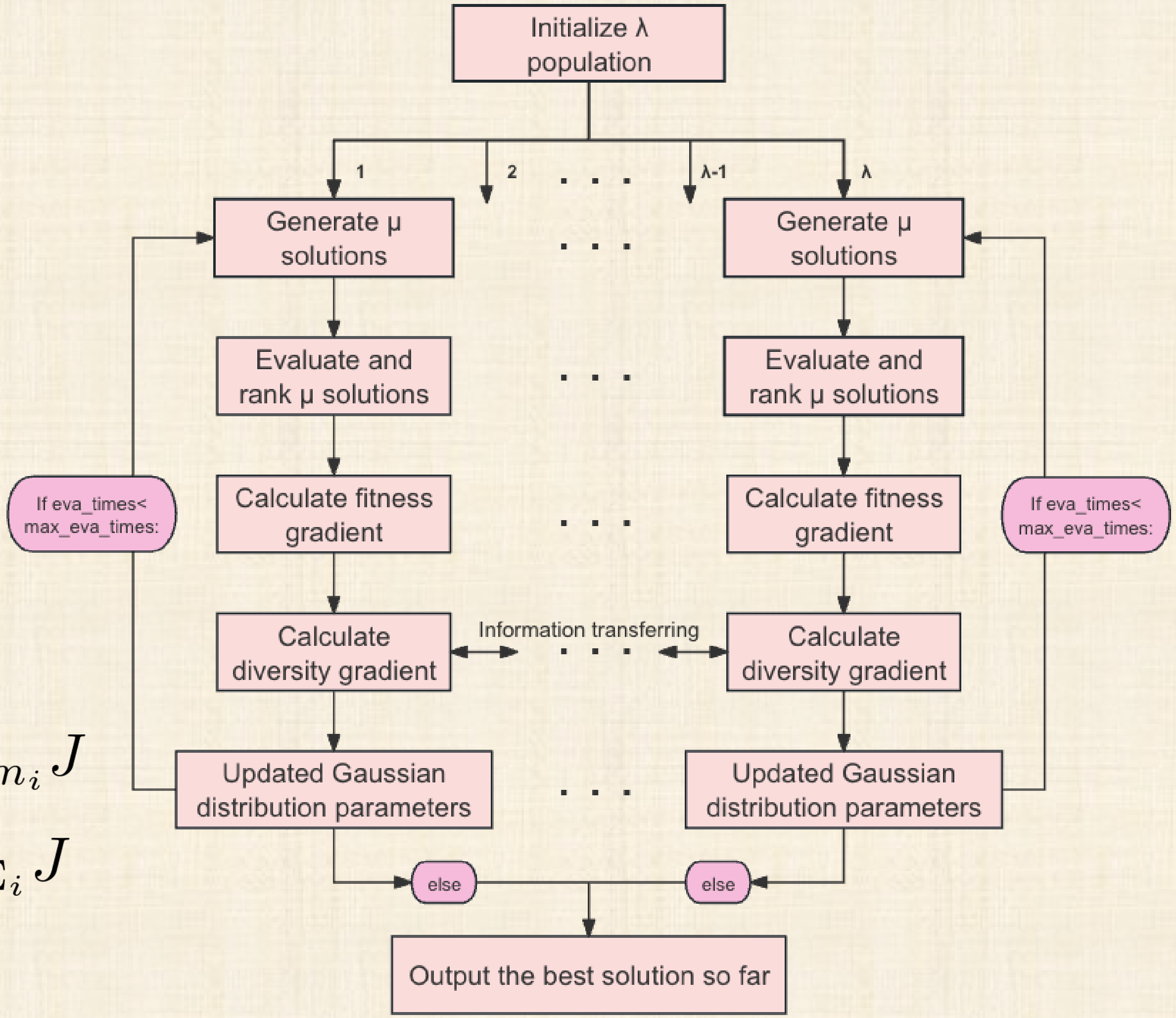
Bhattacharyya

NCNES flow chart

对于新NCS框架的实例化：将高斯分布带入适应性模型和多样性模型，每次迭代过程中更新高斯分布参数。

$$m_i = m_i + \eta_m \cdot \nabla_{m_i} J$$

$$\Sigma_i = \Sigma_i + \eta_\Sigma \cdot \nabla_{\Sigma_i} J$$



Pseudo-code

初始化种群

初始化学习率，权重值

对于每个种群

评估

排序

计算Utility

对于均值计算fitness梯度

对于方差计算fitness梯度

对于均值计算diversity梯度

对于方差计算diversity梯度

括号里值与标准差成反比，容易过大，用Fisher矩阵归一化

Algorithm 1 Pseudo-code of proposed NCNES

for $i = 1$ to λ **do**

  Initialize a Gaussian distribution for the ith Search Process as $N(m_i, \Sigma_i)$

end for

$T_{\text{cur}} = 0;$

while $T_{\text{cur}} < T_{\max}$ **do**

  $\eta_m \leftarrow \eta_m^{\text{init}} \cdot \frac{e - e^{\frac{T_{\text{cur}}}{T_{\max}}}}{e-1};$

  $\eta_\Sigma \leftarrow \eta_\Sigma^{\text{init}} \cdot \frac{e - e^{\frac{T_{\text{cur}}}{T_{\max}}}}{e-1};$

  $\varphi \leftarrow \varphi\text{init} \cdot e^{-\frac{e^{T_{\text{cur}}}}{T_{\max}} \cdot e^{-1}}$

  for $i = 1$ to λ **do**

    Generate μ solutions $\mathrm{x}_i^k \leftarrow N(m_i, \Sigma_i), \forall k = 1, \ldots, \mu$

    Evaluate the fitness $f(x_i^k), \forall k = 1, \ldots, \mu;$

    $T_{\text{cur}} \leftarrow T_{\text{cur}} + \mu;$

    Update x^* as the best solution ever found;

    Rank the kth solution in terms of its fitness $f(x_k)$ as $\pi(k), \forall k = 1, \ldots, \mu;$

    Set $U_i(\pi(k)) = \frac{\max\left\{0, \log\left(\frac{\mu}{2}+1\right) - \log(\pi(k))\right\}}{\sum_{k=1}^{\mu} \max\left\{0, \log\left(\frac{\mu}{2}+1\right) - \log(k)\right\}} - \frac{1}{\mu};$

    $\nabla_{m_i} f \leftarrow \frac{1}{\mu} \sum_{k=1}^{\mu} \Sigma_i^{-1}(x_i^k - m_i) \cdot U_i(\pi(k));$

    $\nabla_{\Sigma_i} f \leftarrow \frac{1}{2\mu} \sum_{k=1}^{\mu} \left(\Sigma_i^{-1}(x_i^k - m_i)(x_i^k - m_i)^T \Sigma_i^{-1} - \Sigma_i^{-1}\right) \cdot U_i(\pi(k));$

    $\nabla_{m_i} d \leftarrow \frac{1}{4} \sum_{j=1}^{\lambda} \left(\frac{\Sigma_i + \Sigma_j}{2}\right)^{-1} (m_i - m_j);$

    $\nabla_{\Sigma_i} d \leftarrow \frac{1}{4} \sum_{j=1}^{\lambda} \left(\left(\frac{\Sigma_i + \Sigma_j}{2}\right)^{-1} - \frac{1}{4}\left(\frac{\Sigma_i + \Sigma_j}{2}\right)^{-1} (m_i - m_j)(m_i - m_j)^T \left(\frac{\Sigma_i + \Sigma_j}{2}\right)^{-1} - \Sigma_i^{-1}\right);$

    $F_{m_i} \leftarrow \frac{1}{\mu} \sum_{k=1}^{\mu} \Sigma_i^{-1}(x_i^k - m_i)(x_i^k - m_i)^T \Sigma_i^{-1};$

    $F_{\Sigma_i} \leftarrow \frac{1}{4\mu} \sum_{k=1}^{\mu} \left(\Sigma_i^{-1}(x_i^k - m_i)(x_i^k - m_i)^T \Sigma_i^{-1} - \Sigma_i^{-1}\right)\left(\Sigma_i^{-1}(x_i^k - m_i)(x_i^k - m_i)^T \Sigma_i^{-1} - \Sigma_i^{-1}\right)^T;$

    $m_i \leftarrow m_i + \eta_m \cdot F_{m_i}^{-1}(\nabla_{m_i} f + \varphi \cdot \nabla_{m_i} d);$

    $\Sigma_i \leftarrow \Sigma_i + \eta_\Sigma \cdot F_{\Sigma_i}^{-1}(\nabla_{\Sigma_i} f + \varphi \cdot \nabla_{\Sigma_i} d);$

  end for

end while

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Cooperative Coevolutionary Framework

NCNES的缺点

- 随着搜索空间的增加（解的维度），性能表现显著下降。

引入Cooperative Coevolution 框架

- 分治思想，将每个解根据维度拆分为不同低维度的子问题，分别优化。
- 目的：使不同的子问题尽可能相互独立互不干涉, 降低相关性，从而增加优化效率。
- 如何评估低维度的子问题？使用不同的补充向量（确保负相关性）。

Pseudo-code

1.将原先的单个解分为M个（子解）

2.对于每个待优化的当前子解，通过分布函数产生具体参数

3.将当前子解补充完整（使用父代的对应维度的均值）

4.对于合并的解个体进行NCNES的评估和优化

Algorithm 3 CC-NCNES

Initialize λ solutions x_i randomly, $i = 1, \ldots, \lambda$.
while stop criteria are not met **do**
  Divide the D-dimensional problem into M sub-problems.
  for $j = 1$ to M **do**
    for $i = 1$ to λ **do**
      Generate an offspring solution $x'_{i,j}$ from the distribution $p_{i,j} \sim N(x_{i,j}, \Sigma_{i,j})$.
      Complement $x'_{i,j}$ as $[x'_{i,j}; v_{i,j}]$, where $v_{i,j}$ is the mean of the remaining dimensions of the parent
      Take $f([x'_{i,j}; v_{i,j}])$ as the new individual and do the NCNES steps to evaluate the qualities of $x'_{i,j}$.
    end for
  end for
  Update the distribution function of the dimension corresponding to this round
end while

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Experiments

Experiments

NCNES复现并在CEC2017测试集上的表现（低维）：

Table 2 CEC 2017 Benchmark

Serial Number	Function Description	Search Range	Optimum
$F1$	Shifted and Rotated Bent Cigar Function.	[-100, 100]	100
$F3$	Shifted and Rotated Zakharov Function.	[-100, 100]	300
$F4$	Shifted and Rotated Rosenbrock's Function.	[-100, 100]	400
$F5$	Shifted and Rotated Rastrigin's Function.	[-100, 100]	500
$F6$	Shifted and Rotated Expanded Scaffer's F6 Function.	[-100, 100]	600
$F7$	Shifted and Rotated Lunacek Bi-Rastrigin's Function.	[-100, 100]	700
$F8$	Shifted and Rotated Non-Continuous Rastrigin's Function.	[-100, 100]	800
$F9$	Shifted and Rotated Levy Function.	[-100, 100]	900
$F10$	Shifted and Rotated Schwefel's Function.	[-100, 100]	1000
$F11$	Hybrid Function 1.	[-100, 100]	1100
$F12$	Hybrid Function 2.	[-100, 100]	1200
$F13$	Hybrid Function 3.	[-100, 100]	1300
$F14$	Hybrid Function 4.	[-100, 100]	1400
$F15$	Hybrid Function 5.	[-100, 100]	1500
$F16$	Hybrid Function 6.	[-100, 100]	1600
$F17$	Hybrid Function 7.	[-100, 100]	1700
$F18$	Hybrid Function 8.	[-100, 100]	1800
$F19$	Hybrid Function 9.	[-100, 100]	1900
$F20$	Hybrid Function 10.	[-100, 100]	2000
$F21$	Composition Function 1.	[-100, 100]	2100
$F22$	Composition Function 2.	[-100, 100]	2200
$F23$	Composition Function 3.	[-100, 100]	2300
$F24$	Composition Function 4.	[-100, 100]	2400
$F25$	Composition Function 5.	[-100, 100]	2500
$F26$	Composition Function 6.	[-100, 100]	2600
$F27$	Composition Function 7.	[-100, 100]	2700
$F28$	Composition Function 8.	[-100, 100]	2800
$F29$	Composition Function 9.	[-100, 100]	2900
$F30$	Composition Function 10.	[-100, 100]	3000

旋转、位移

Table 3 NCNES on CEC 2017 Benchmark D = 10

Serial Number	Best	Worst	Mean	variance
$F1$	5.8147E+02	3.0335E+04	8.1647E+03	4.1988E+03
$F2$	**2.0000E+02**	3.8725E+04	**2.0076E+02**	2.2938E+02
$F3$	**3.0848E+02**	3.0922E+02	**3.0862E+02**	5.5929E-02
$F4$	**4.0257E+02**	4.0886E+02	4.0851E+02	5.5075E-00
$F5$	**5.0000E+02**	**5.0000E+02**	**5.0000E+02**	**4.9958E-07**
$F6$	**6.2122E+02**	6.4587E+02	6.2750E+02	3.7227E+00
$F7$	**7.1681E+02**	7.2886E+02	7.2041E+02	4.3002E+00
$F8$	**8.0000E+02**	**8.0000E+02**	**8.0000E+02**	2.5532E-09
$F9$	9.4236E+02	1.9038E+03	1.0064E+03	7.7422E+00
$F10$	**1.0013E+03**	**1.0010E+03**	**1.0072E+03**	**1.2067E+01**
$F11$	4.9412E+04	6.3254E+04	5.2397E+04	1.1421E+03
$F12$	1.4339E+03	9.9384E+03	3.1044E+04	1.4634E+03
$F13$	**1.3309E+03**	7.3298E+03	4.7792E+03	4.7515E+03
$F14$	2.4715E+03	8.2932E+04	7.7572E+04	4.9593E+04
$F15$	**1.5010E+03**	1.5293E+03	**1.5174E+03**	2.5552E+01
$F16$	**1.6047E+03**	**1.6021E+03**	**1.16125E+03**	**4.6921E+00**
$F18$	**1.8204E+03**	1.8657E+04	8.7459E+03	6.5805E+03
$F19$	**1.9376E+03**	1.9855E+03	1.9489E03	8.4205E+00
$F21$	**2.1000E+03**	**2.1000E+03**	**2.1000E+03**	**4.3271E-03**
$F22$	**2.2000E+03**	**2.2000E+03**	**2.2000E+03**	**2.6050E-05**
$F23$	**2.3000E+03**	**2.3000E+03**	**2.3000E+03**	**2.0114E-03**
$F24$	2.8784E+03	2.8801E+03	2.8798E+03	8.1228E-01
$F25$	**2.5000E+03**	**2.5000E+03**	**2.5000E+03**	**2.5767E-03**
$F26$	2.9672E+03	2.9673E+03	2.9672E+03	6.4629E-05
$F27$	**2.7000E+03**	**2.7031E+03**	**2.7014E+03**	**2.5546E+00**
$F28$	3.8025E+03	3.9521E+03	3.8715E+03	7.4365E+03

NCNES复现并在CEC2022测试集上的表现（低维）：

表 4 CEC 2022 Benchmark

Serial Number	Function Description	Search Range	Optimum
$F1$	Shifted and Rotated Zakharov Function.	[-100, 100]	300
$F2$	Shifted and Rotated Rosenbrock's Function.	[-100, 100]	400
$F3$	Shifted and Rotated Expanded Scaffer's F6 Function.	[-100, 100]	600
$F4$	Shifted and Rotated Non-Continuous Rastrigin's Function.	[-100, 100]	800
$F5$	Shifted and Rotated Levy Function.	[-100, 100]	900
$F6$	Hybrid Function 1.	[-100, 100]	1800
$F7$	Hybrid Function 2.	[-100, 100]	2000
$F8$	Hybrid Function 3.	[-100, 100]	2200
$F9$	Composition Function 1.	[-100, 100]	2300
$F10$	Composition Function 2.	[-100, 100]	2400
$F11$	Composition Function 3.	[-100, 100]	2600
$F12$	Composition Function 4.	[-100, 100]	2700

表 5 NCNES on CEC 2022 Benchmark D = 10

Serial Number	Best	Worst	Mean	variance
$F1$	3.2195E+03	5.2663E+04	4.3101E+03	8.7391E+02
$F2$	**4.0701E+02**	**4.0723E+02**	**4.0712E+02**	1.3621E-01
$F3$	**6.0000E+02**	**6.0000E+02**	**6.0000E+02**	**1.1504E-12**
$F4$	**8.1554E+02**	**8.2231E+02**	**8.1983E+02**	**2.7654E-00**
$F5$	**9.0000E+02**	**9.0000E+02**	**9.0000E+02**	**6.6536E-09**
$F6$	3.4482E+04	4.2531E+04	3.8994E+04	3.4928E+03
$F7$	**2.0100E+03**	**2.0431E+03**	**2.0242E+03**	**4.8023E+00**
$F8$	**2.2225E+03**	**2.5674E+03**	**2.2422E+03**	1.1012E+02
$F9$	2.6680E+03	2.6690E+03	2.6684E+03	**2.9461E-01**
$F10$	**2.6221E+03**	**2.8453E+03**	**2.6275E+03**	**4.7241E+00**
$F11$	**2.6005E+03**	2.8632E+03	**2.7815E+03**	1.3976E+02
$F12$	2.8664E+03	2.8665E+03	2.8664E+03	**2.1092E-02**

- 相当一部分的Function都优化到了最优值，但是其最差值，均值可能还没来得及在迭代结束前达到最优，从而导致较大的方差，这些都表明NCNES的性能与主流的优化算法比较，还有上升空间。
- 另外，注意到F9和F12，函数在没有达到最优解时提前收敛了，很小的方差表明了函数在每次训练时都陷入了相同的局部最优，这意味着NCNES在多样性指标上，对于全局搜索的比重需要增加，后期需要去优化这个平衡fitness和diversity的动态权重。

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NCNES/CC-NCNES复现并在CEC2017测试集上的表现（高维）：

Table 7 NCNES on CEC 2017 Benchmark D = 30

Serial Number	Best	Worst	Mean	variance
$F1$	5.8152E+09	8.0213E+09	6.6773E+09	7.5142E+08
$F2$	1.1645E+05	1.6321E+05	1.357E+05	7.7314E+03
$F3$	**9.888E+02**	**1.3425E+03**	**1.2298E+03**	1.5408E+02
$F4$	2.533E+03	4.231E+06	3.179E+04	3.0434E+02
$F5$	**5.0000E+02**	**5.0000E+02**	**5.0000E+02**	**3.9213E-03**
$F6$	1.6989E+05	3.1621E+05	2.2461E+05	2.2426E+05
$F7$	1.1075E+03	1.3211E+03	1.153E+03	2.5382E+02
$F8$	**8.1869E+02**	**8.3263E+02**	**8.2184E+02**	**2.4415E-00**
$F9$	7.477E+04	9.2131E+04	8.3454E+04	3.5473E+02
$F10$	2.0865E+04	3.1298E+05	2.4754E+05	3.1935E+04
$F11$	1.0532E+07	3.8231E+08	1.6550E+07	2.6097E+07
$F12$	1.7975E+07	9.3584E+07	6.189E+07	2.224E+07
$F13$	2.919E+07	8.6466E+07	5.20E+07	1.730E+07
$F14$	8.4644E+04	5.3356E+05	3.1680E+05	2.2005E+05
$F15$	1.4921E+04	2.0137E+04	1.7094E+04	1.533E+03
$F16$	3.240E+05	9.3142E+07	3.881E+07	3.8410E+07
$F17$	1.56E+05	1.1209E+06	2.27E+05	1.051E+05
$F18$	1.9196E+05	5.2389E+05	2.501E+05	4.8926E+04
$F19$	3.150E+03	3.6142E+03	3.338E+03	1.06E+02
$F20$	6.143E+03	6.754E+03	6.515E+03	2.6853E+02
$F21$	**2.401E+03**	2.411E+03	2.410E+03	5.737E-00
$F22$	6.849E+03	1.2385E+04	8.069E+03	6.3581E+02
$F23$	5.172E+03	6.0502E+03	5.552E+03	2.3051E+02
$F24$	3.043E+03	3.213E+03	3.145E+03	4.0913E+01
$F25$	3.419E+03	3.421E+03	3.425E+03	3.8312E+00
$F26$	3.196E+03	3.1542E+03	3.219E+03	1.244E+01
$F27$	3.208E+03	2.226E+03	3.214E+03	4.4224E+00
$F28$	1.1107E+08	3.1289E+09	1.9667E+09	8.8536E+08

Table 8 CC-NCNES on CEC 2017 Benchmark D = 30

Serial Number	Best	Worst	Mean	variance
$F1$	4.639E+10	7.1856E+10	5.169E+10	4.209E+09
$F2$	**6.789E+04**	**9.866E+04**	**8.333E+04**	9.251E+03
$F3$	5.163E+03	1.211E+04	8.944E+03	1.822E+03
$F4$	4.3246E+04	7.5542E+04	5.305E+04	5.189E+03
$F5$	**5.0002E+02**	**5.0011E+02**	**5.0003E+02**	**3.1910E-03**
$F6$	1.7288E+06	1.9881E+06	1.8324E+06	7.7238E+04
$F7$	**1.330E+03**	1.8666E+03	1.417E+03	4.6203E+01
$F8$	**8.3418E+02**	**8.4001+02**	**8.3942E+02**	3.021E+00
$F9$	7.305E+03	7.813E+03	7.766E+03	2.2005E+02
$F10$	7.957E+05	3.1222E+06	2.274E+06	9.310E+05
$F11$	3.4614E+09	5.2376E+08	4.4774E+09	7.2617E+08
$F12$	2.429E+09	7.5662E+06	5.1387E+09	1.471E+06
$F13$	6.9032E+05	1.8642E+06	1.4286E+06	4.49E+05
$F14$	9.8705E+08	1.5233E+09	1.52128E+09	4.9080E+08
$F15$	4.7308E+07	2.0120E+08	1.9192E+08	1.213E+08
$F19$	**2.143E+03**	**2.451E+03**	**2.265E+03**	7.2452E+01
$F20$	**2.699E+03**	**2.919E+03**	**2.810E+03**	7.0991E+01
$F21$	**2.219E+03**	**2.2375E+03**	**2.2260E+03**	2.464E+00
$F22$	3.296E+04	3.842E+03	3.733E+04	3.4289E+03
$F23$	2.437E+04	2.8324E+03	2.703E+03	1.990E+03
$F24$	5.339E+03	6.5231E+03	6.312E+03	6.2237E+02
$F25$	3.988E+03	4.068E+03	4.001E+03	8.1512E+00
$F26$	**2.963E+03**	**3.002E+03**	**2.977E+03**	4.521E+01
$F27$	**3.012E+03**	3.243E+03	**3.185E+03**	5.0184E+00
$F28$	3.2105E+07	1.0025E+09	8.2586E+08	6.0178E+08

结合了CC框架之后在相同的迭代次数下NCNES较多在更多的Function上更为接近最优值。
并且更小的标准差表明其可以在相同条件下获得更稳定的解。

CC-NCNES比NCNES更具有优势的体现:

Table 9 Main advantages of CC-NCNES on CEC 2017 Benchmark D = 30

Serial Number	CC-NCNES Best	CC-NCNES Mean	NCNES Best	NCNES Mean
$F2$	**6.789E+04**	**8.333E+04**	1.1645E+05	1.357E+05
$F5$	5.0002E+02	5.0003E+02	5.0000E+02	5.0000E+02
$F8$	8.3418E+02	8.3942E+02	8.1869E+02	8.0237E+02
$F9$	**7.305E+03**	**7.766E+03**	7.477E+04	8.3454E+04
$F19$	**2.143E+03**	**2.265E+03**	3.150E+03	3.338E+03
$F20$	**2.699E+03**	**2.810E+03**	6.143E+03	6.515E+03
$F21$	**2.219E+03**	**2.2260E+03**	2.401E+03	2.410E+03
$F24$	5.339E+03	6.312E+03	3.043E+03	3.145E+03
$F26$	**2.963E+03**	**2.977E+03**	3.196E+03	3.219E+03
$F27$	**3.012E+03**	**3.185E+03**	3.208E+03	3.214E+03
$F28$	**3.2105E+07**	**8.2586E+08**	1.1107E+08	1.9667E+09

CC-NCNES在大多数测试函数上（例如F2, F9, F19, F20, F21, F26, F27, F28）的“Best”和“Mean”值都明显优于NCNES。这表明CC-NCNES在这些函数上能够找到更好的解决方案，并且具有更稳定的性能。

4

Conclusions

本项目从数学原理上实现了NCS（负相关搜索）称为NCNES。他同时建模并最大化下一代种群的多样性模型（用于探索）和适应度模型（用于优化）。两个模型都可以通过对每个搜索过程执行梯度下降来最大化。并使其在Benchmark2017和Benchmark2022上测试，获得了较好的结果，但是在优化效率和全局最优的搜索上还有上升空间。

另一篇论文《Evolutionary Reinforcement Learning via Cooperative Coevolutionary Negatively Correlated Search》提出了合作进化负相关搜索，用于训练百万个连接权重的神经网络，主要思路是将决策变量分成多个独立组，并分别用进化算法解决较小的子问题（即决策变量组）来扩展进化算法。

在拓展现有NCS算法的基础上，本项目进一步结合了Cooperative Coevolutionary框架，将单个高维度解分解为多个低维子解进行负相关优化。这种策略不仅显著提升了高维问题的求解效率，同时也在复杂度和计算资源方面表现出色，使得我们能够在更短的时间内找到更优的解决方案。

参考文献

[1] TANGK, YANGP, YAOX.Negatively correlated search[J]. IEEE Journal on Selected Areas in Communications, 2016, 34(3):542-550

[2] YANGP,ZHANGH,YUY,etal. Evolutionary reinforcement learning via cooperative coevolutionary negatively correlated search[J]. Swarm and Evolutionary Computation, 2022, 68:100974

[3] YANGP,YANGQ,TANGK,etal.Parallel exploration via negatively correlated search[J]. Fron tiers of Computer Science, 2021, 15:1-13.

[4] YANGZ,TANGK,YAOX.Largescaleevolutionary optimization using cooperative coevolution [J]. Information sciences, 2008, 178(15):2985-2999

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