KTH ROYAL INSTITUTE
OF TECHNOLOGY

DT2212: Music Acoustics

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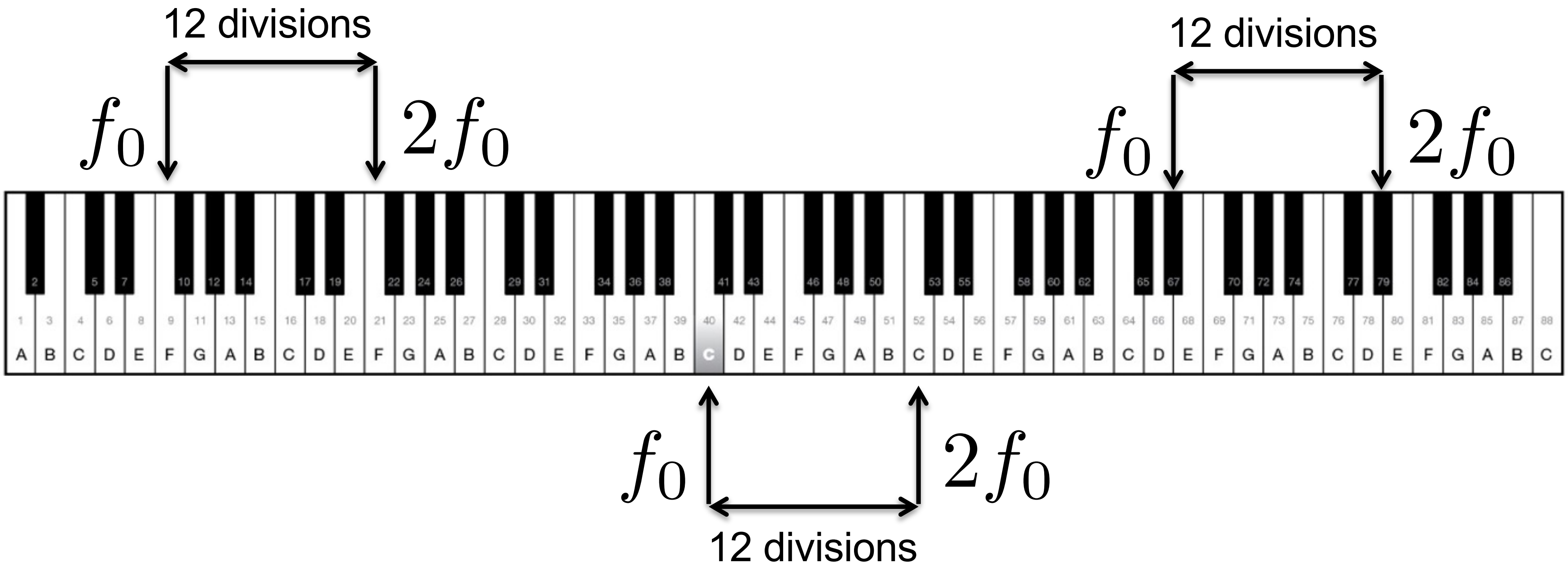
Lecture 4: Tuning + temperament
Reading: Appendix D in Hartmann

Question:


12 divisions
f_0
$2f_0$
12 divisions
f_0
$2f_0$
A B C D E F G A B C D E F G A B C D E F G A B C D E F G A B C D E F G A B C D E F G A B C D E F G A B C
f_0
$2f_0$
12 divisions

Consider the monochord

Dan Bau (Vietnam)

https://youtu.be/hpEGAkzbns0

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https://youtu.be/hpEGAkzbns0

Diddly Bow

https://youtu.be/tpSfvDwwoqM

“Alternative” Uses

Monochord Table

Pure relaxation through contactless touch

Stop the wheel, unarmor yourself, recharge your batteries and be ready for something new – with our sound massage table, we have perfected the feeltone idea of "feel the sound" for various areas of application. The body experiences pleasant sound waves, the mind experiences deep relaxation and the soul arrives "at home".

Read more

https://bit.ly/2Fgd15B

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Notions

Length of the plucked string relates to pitch we hear.

This pitch increases as the string length decreases.

We can describe the relationship between two pitches (interval) by the ratio of the lengths of the strings (given both strings are the same length and tuned to the same pitch):

- 1:1 – “unison” → $f_0:f_0$
- 2:1 – “octave” → $2f_0:f_0$
- 3:2 – “fifth” → $3f_0:2f_0$
- 4:3 – “fourth” → $4f_0:3f_0$

“2:1” means one string has twice the length of the other.

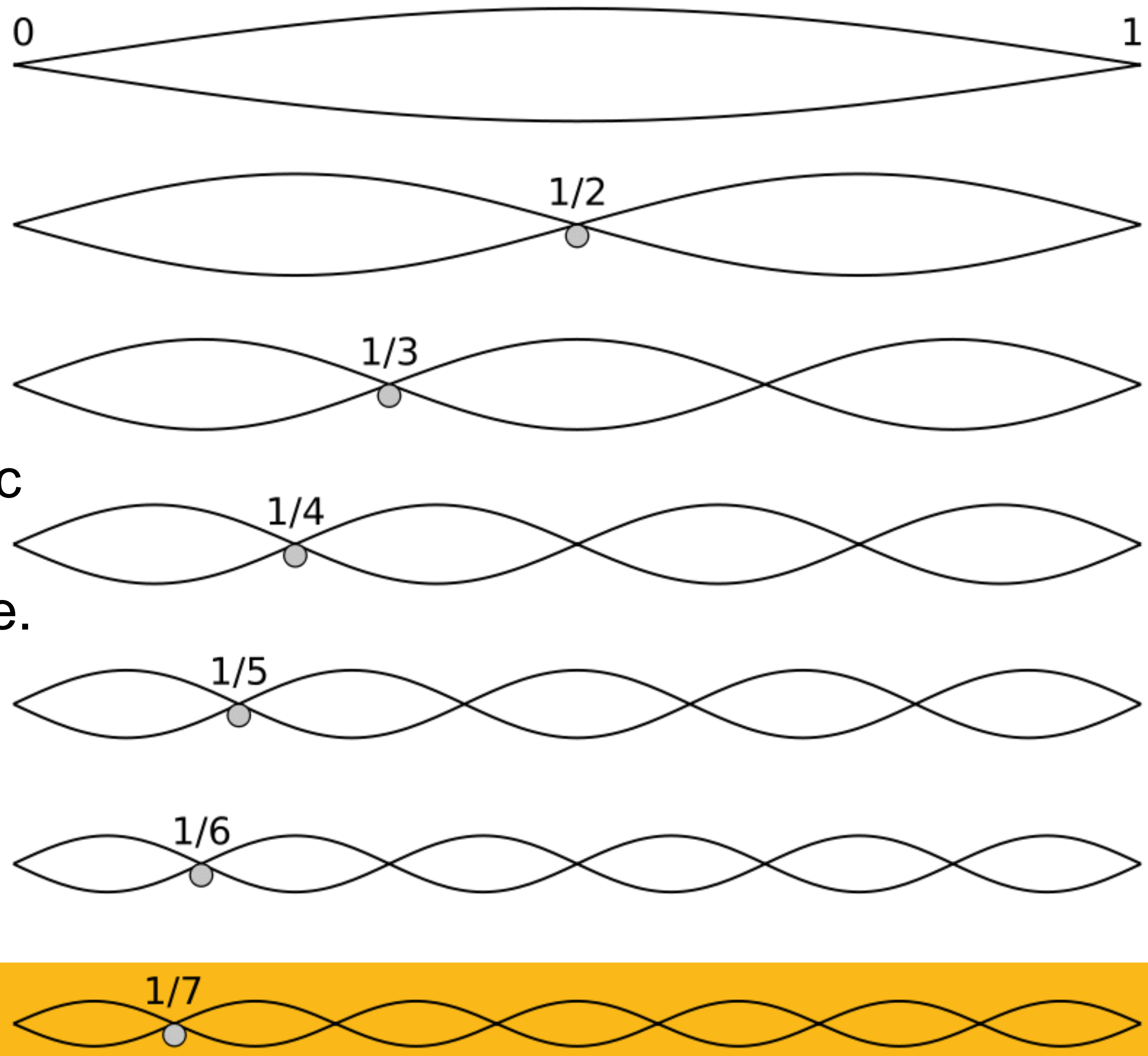
“3:2” means three times the length of one string is twice the length of the other (or equivalently, one string is 2/3 as long as the other).

Harmonic series

We know that the string motion at the terminations must be zero.

So, only certain wavelengths are permitted!

Frequency of nth harmonic is nf_0, where f_0 is the frequency of the first mode.



Example

An ideal string with the first mode frequency (fundamental) f_0 = 100 Hz will also have modes with frequencies: 200, 300, 400, 500, 600, 700, 800, … Hz

What are the *interval* relationships between these frequencies?

Interval	Frequency ratio
Unison	1:1
Minor 2nd	16:15
Major 2nd	9:8
Minor 3rd	6:5
Major 3rd	5:4
Perfect 4th	4:3
Tritone	45:32
Perfect 5th	3:2
Minor 6th	8:5
Major 6th	5:3
Minor 7th	16:9
Major 7th	15:8
Octave	2:1
Minor 9th	32:15
Major 9th	9:4
Minor 10th	12:5
Major 10th	5:2

Overtone Series
(Chart of harmonics on C)

• Darkened notes denote approximate pitches. These tones are considered out of tune.

Intervallic relationships

String 1: has f_0 = 100 Hz so will vibrate with freqs:

100, 200, 300, 400, 500, 600, 700, 800, 900 … Hz

String 2: has f_0 = 200 Hz so will vibrate with freqs:

200, 400, 600, 800, … Hz

String 3: has f_0 = 150 Hz so will vibrate with freqs:

150, 300, 450, 600, 750, 900, … Hz

String 4: has f_0 = 133.33 Hz so will vibrate with freqs:

133.33, 266.66, 399.99, 533.32, 666.66, 799.99… Hz

What are the *intervallic* relationships between string 1 and the others?

Intervallic relationships

Octave 2:1

String 1: has f_0 = 100 Hz so will vibrate with freqs:

100, 200, 300, 400, 500, 600, 700, 800, 900 … Hz

String 2: has f_0 = 200 Hz so will vibrate with freqs:

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What are the *intervallic* relationships between string 1 and the others?

Intervallic relationships

Perfect 5^{th}
3:2

String 1: has f_0 = 100 Hz so will vibrate with freqs:

100, 200, 300, 400, 500, 600, 700, 800, 900 … Hz

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What are the *intervallic* relationships between string 1 and the others?

Intervallic relationships

Perfect 4^{th}
4:3

String 1: has f_0 = 100 Hz so will vibrate with freqs:

100, 200, 300, 400, 500, 600, 700, 800, 900 … Hz

String 2: has f_0 = 200 Hz so will vibrate with freqs:

200, 400, 600, 800, … Hz

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Richness of intervals

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Why do some of these intervals sound “richer” than others?

Monochord demo

Richness of intervals

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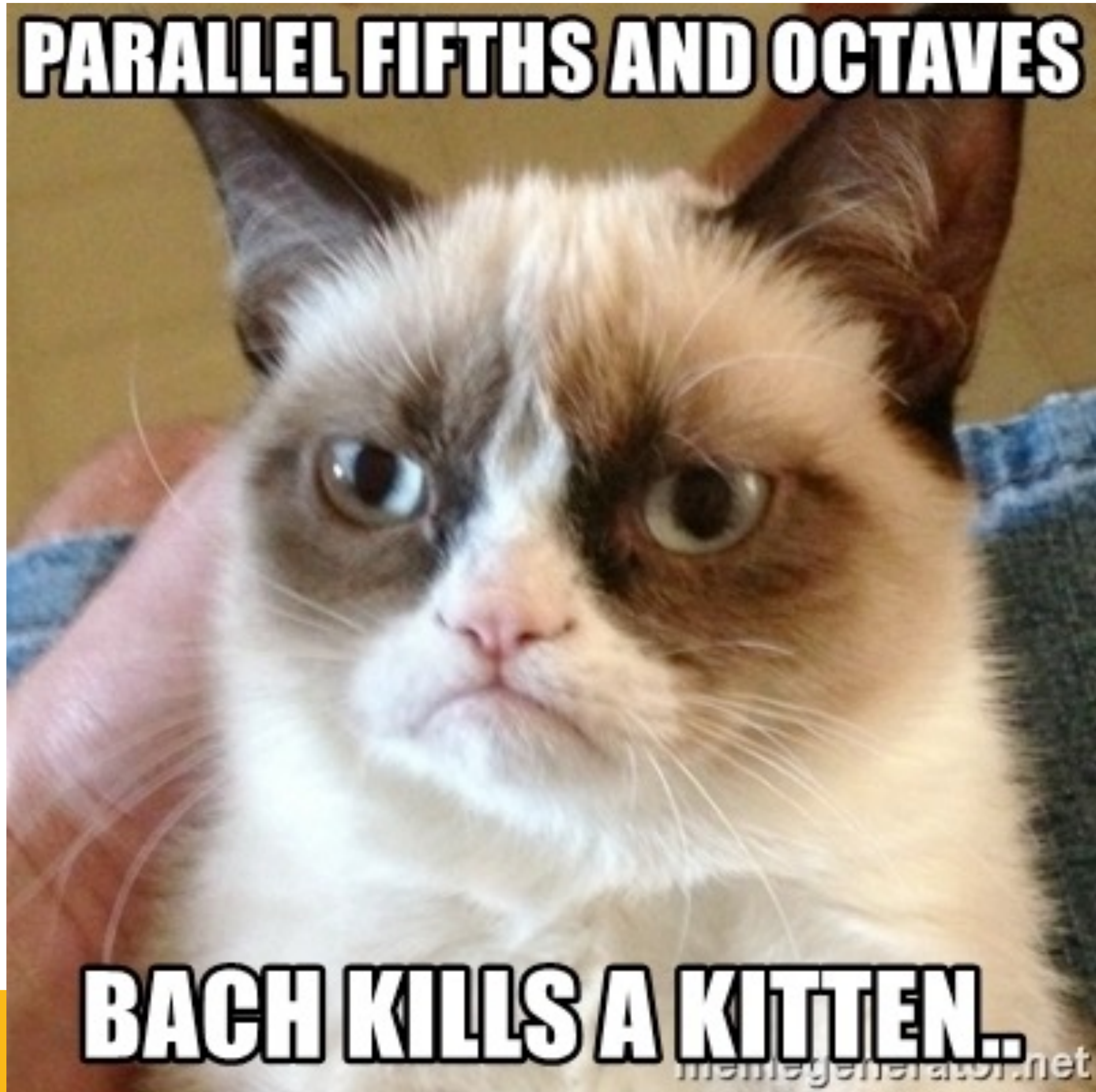
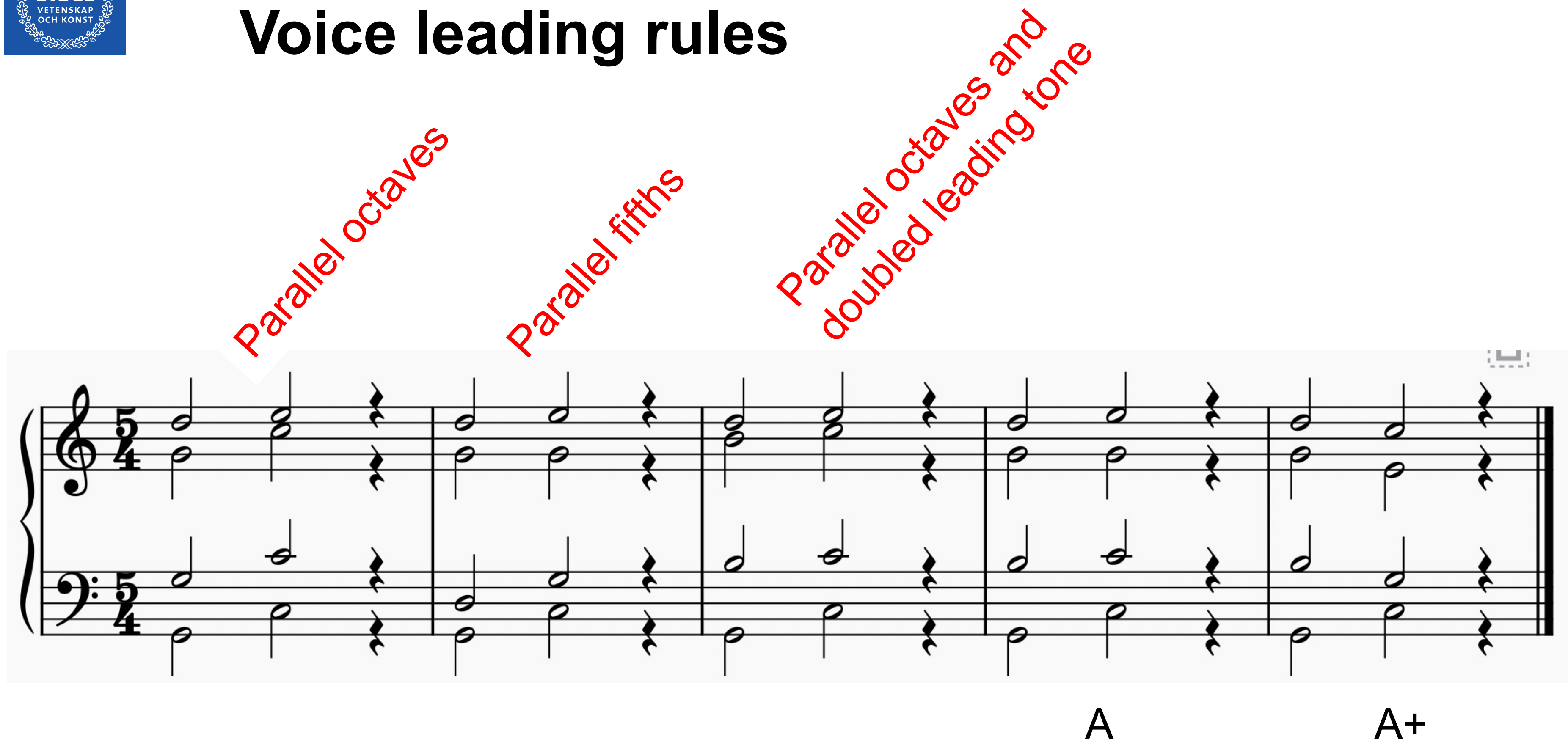
Voice leading rules



Voice leading rules



Units of pitch measurement

We want to relate frequencies, e.g., "how much is f_x above f_y?"

Define 1:1 (unison) interval as 0 cents (¢)

Define 2:1 (octave) interval as 1200 ¢

$$I(f_x, f_y) = 1200 \log_2(f_x/f_y)$$

$$f_x = f_y 2^{I/1200}$$

How many cents is the just noticeable difference (JND) in fundamental frequency (1 Hz @ 300 Hz)?

TABLE 18.1 Consonant interval sizes according to the Pythagorean hypothesis. For most purposes, it is quite adequate to round these off to the nearest cent. Note the four inversion pairs: For each the product of ratios is 2:1, or the sum in cents is 1200.

Musical Interval	Frequency Ratio	Size in Cents
Unison	1:1	0
Octave	2:1	1200
Fifth	3:2	701.95
Fourth	4:3	498.05
Major third	5:4	386.31
Minor sixth	8:5	813.69
Minor third	6:5	315.64
Major sixth	5:3	884.36
*	7:4	968.83
*	11:8	551.32

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Major [illegible]	[illegible]	[illegible]
Minor [illegible]	[illegible]	[illegible]
Minor third	6:5	315.64
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$$I(3f_x, 2f_x) = 1200 \log_2(3f_x/2f_x) = 1200 \log_2(1.5)$$

Division of the octave

Pythagorean approach: *rational numbers are beautiful*

- Decide on the frequency of “unison” (0 ¢)
- 2:1 is the “octave” (1200 ¢)
- 3:2 is the “fifth” (701.95 ¢)
- 4:3 is “fourth” (498.05 ¢)
- etc.

Notice!

- One octave below fourth is 498.05 – 1200 = -701.95 ¢, a fifth below unison (inversion pair)

Tuning by fifths

Choose frequency of the string 1, which is 0 ¢

Tune string 2 a fifth above 1: 0 + 702 = 702 ¢

Tune string 3 a fifth above 2: 702 + 702 = 1404 ¢

Tune string 3 one octave lower: 1404 – 1200 = 204 ¢

Tune string 4 a fifth above 3: 204 + 702 = 906 ¢

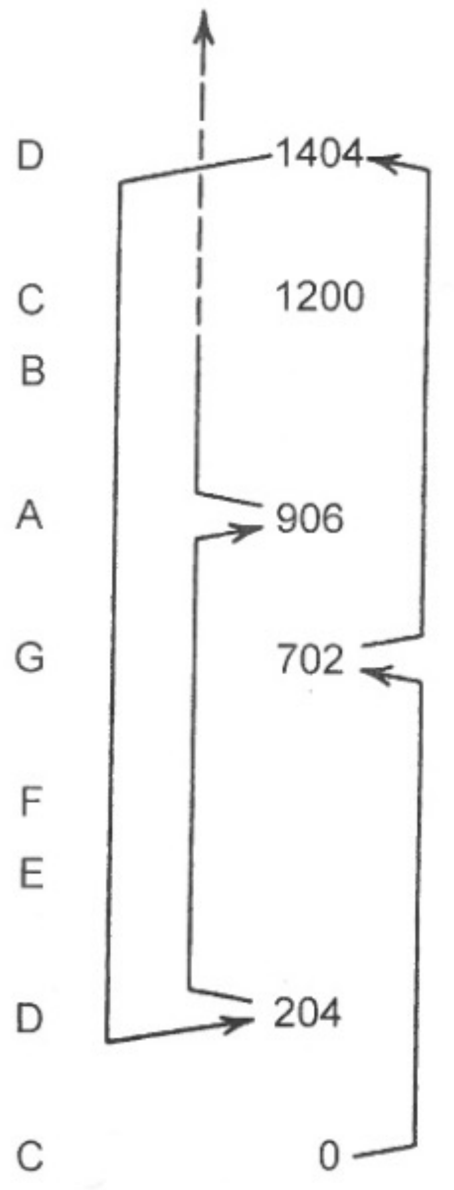
Tune string 5 a fifth above 4: 906 + 702 = 1608 ¢

Tune string 5 one octave lower: 1608 – 1200 = 408 ¢

And so on …

What is going to happen at string 13?

- Continue …



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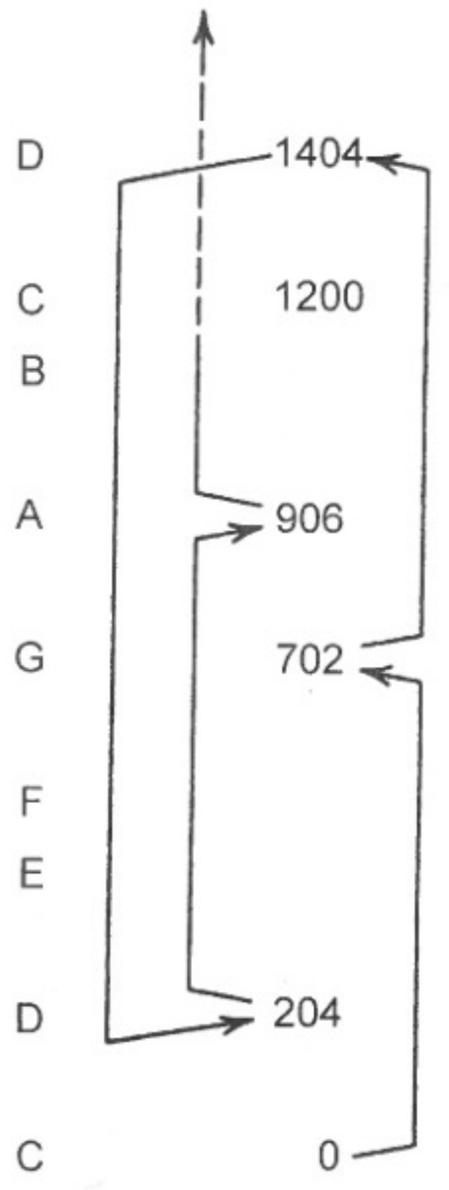


Tuning by fifths

Choose frequency of the string 1, which is 0 ¢

…

Tune string 11 a fifth above 10: 318+702 = 1020¢

Tune string 12 a fifth above 11: 1020+702 = 1722¢

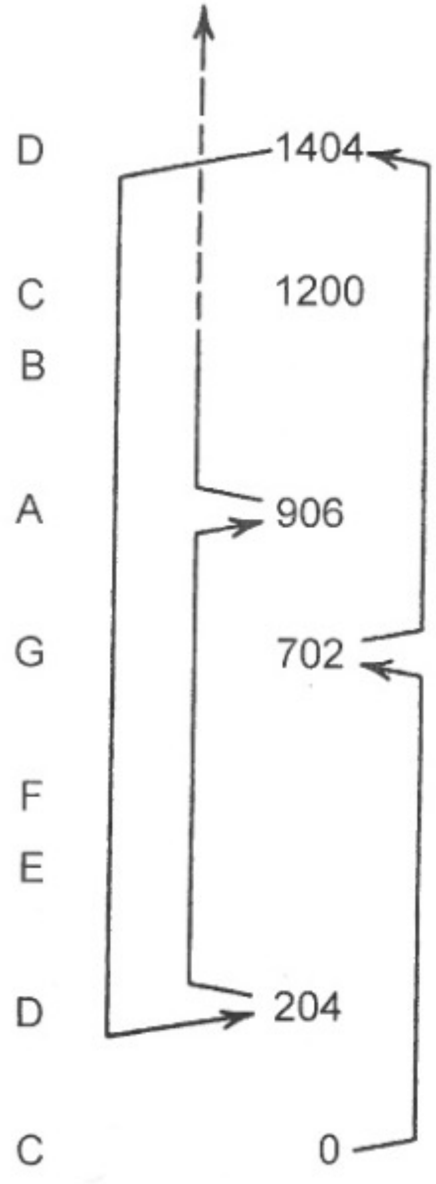
Tune string 12 one octave lower: 1722 – 1200= 522¢

Tune string 13 a fifth above 12: 522+702 = 1224¢

Uh oh. We overshot by 24¢ (actually 23.5¢)

Easier: 1200*log2(3/2)*12 – 7*1200 = 23.46 ¢

This is called the ***ditonic comma***



Tuning by (beatless) major thirds

We can tune by beatless major thirds

Frequency ratio: 5:4 (1200*log2(5/4) = 386.31¢)

A string with f_0 = 100 Hz will vibrate with freqs:
100, 200, 300, 400, 500, 600, 700, 800, 900 … Hz

A string with f_0 = 125 Hz will vibrate with freqs:
125, 250, 375, 500, 625, 750, 875, 900… Hz

But this results in 3*386.31¢ = 1158.93¢ < 1200 by 41.07¢.

This is called the ***lesser diesis***

Tuning by minor thirds

We can tune by minor thirds

Frequency ratio: 6:5 (1200*log2(6/5) = 315.64¢)

A string with f_0 = 100 Hz will vibrate with freqs:
100, 200, 300, 400, 500, 600, 700, 800, 900,1000 … Hz

A string with f_0 = 120 Hz will vibrate with freqs:
120, 240, 360, 480, 600, 720, 840, 960, 1080, 1200… Hz

But this results in 4*315.64¢ = 1262.5¢ < 1200 by 62.5¢.

This is called the ***greater diesis***

Strategies

We can tune some pitches by fifths, and others by major thirds.

We can impose some formal rules, e.g., all intervals must be composed of small whole numbers, like powers of prime numbers 2, 3, and 5 (“5-limit just intonation”)

We can tune by fifths and **spread** around the “leftovers” in a variety of ways (*tempering*).

What about spreading these leftovers *equally*?

12-tone Equal Temperament (12-TET)

Tune by a uniform division of the octave into 12 parts!

- Semitones are separated by 100¢ (frequency ratio: 2119:2000)

$$f = f_0 2^{n/12}$$

This is called *equal temperament*

Every fifth is mistuned -2¢ → -2 * 12 = 24¢ (the *ditonic comma*)

This means the following for all triads

- All major thirds are +14¢ different from pure (5:4)
- All minor thirds are -16¢ different from pure (6:5)

Examples

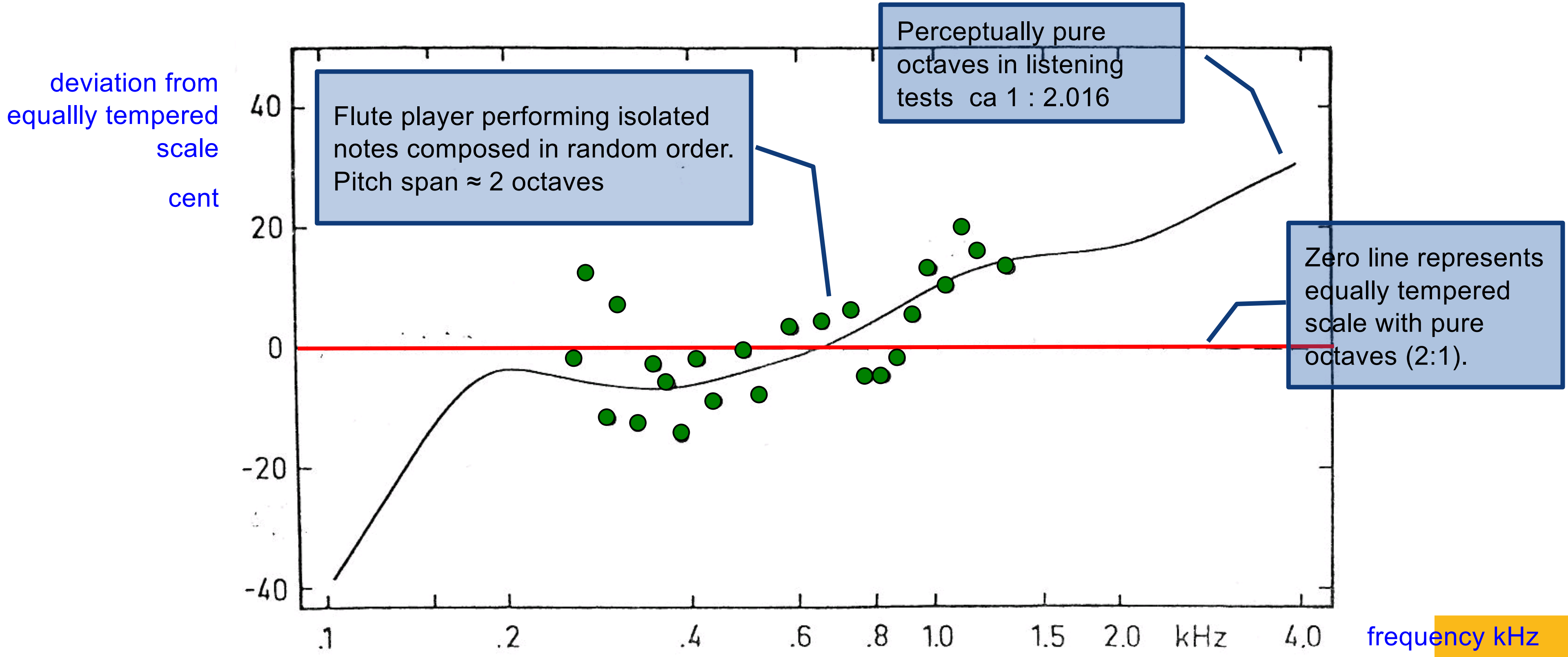
	Equal Temperament	Pythagorean (comma at 7^{th} sd)
Major scale		
Parallel fifths		
Thirds (Tenths)		
Bach (exc)		

Preference in tuning

⇒ We don't like pure octaves (2:1) in melodic intervals

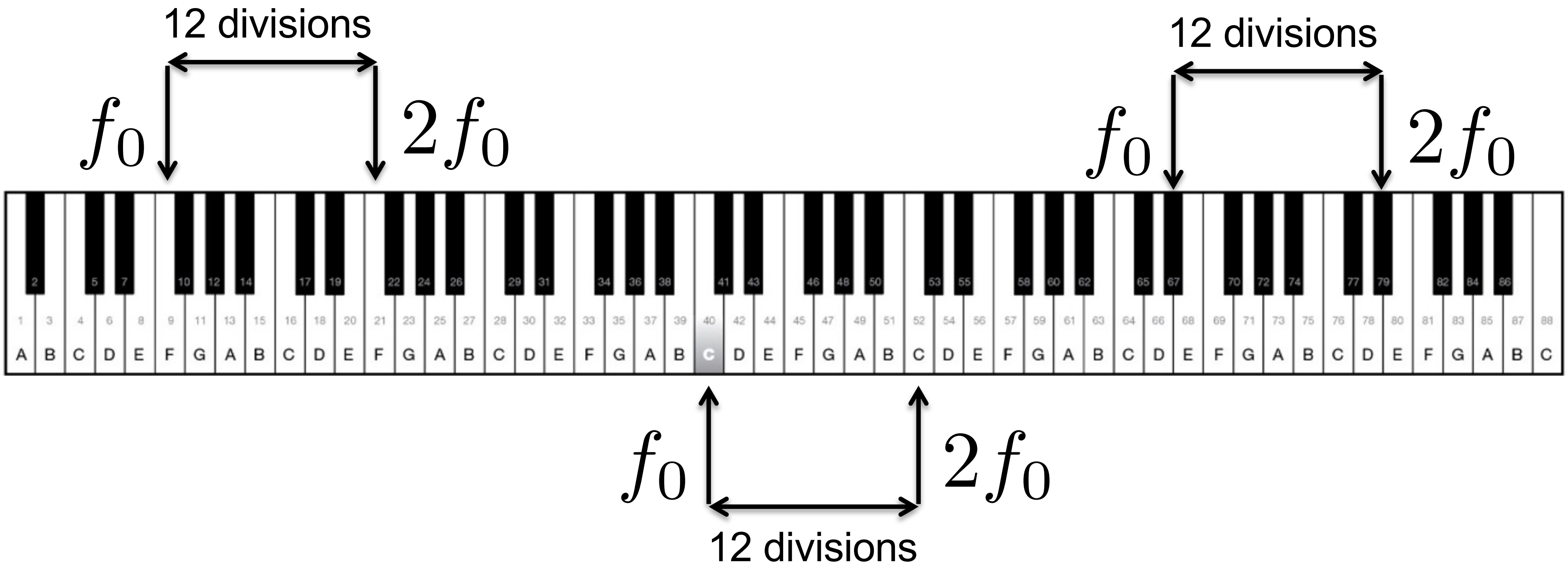
1 : 2.016 (+14 cent) sounds good

This flute player stretches the intervals to make them 'sound in tune.'



Demo: http://www.speech.kth.se/music/performance/Texts/high_sharp.htm

Question:



12-TET is not everything

Link to Erik Nathanial Gustafsson

Before next lecture

Read:

1. Chapter 9 in Hartmann