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ANALYSIS OF MUSICAL-INSTRUMENT TONES

With computers we can not only analyse the sound of a musical instrument but also build up a synthesized copy of the sound. Comparison of real and synthetic tones tells which are the important parameters that lead to recognition of timbre.

JEAN-CLAUDE RISSET and MAX V. MATHEWS

POWERFUL NEW METHODS are now available for the study of tone quality of both existing and new instruments. The new methods involve both new tools (primarily computers) and new applications of experimental psychology (psychophysics and subjective judgments).

Starting from the classical physical analysis of an instrument we find some of the relevant physical parameters that may be important, such as formants and zeros of the spectrum, attack and decay rates and the way in which the spectrum varies with time. Then we can find precise values for these (or other) parameters with a computer analysis of digitized examples of the tone. We determine the significance of the parameters by producing simulated tones with computer synthesis; the parameters can be given any values, either typical or atypical of real instruments. Listening tests on our results for trumpet and violin tones show how successful we have been when we try to produce a tone indistinguishable from a real instrument, and we can discover from these tests which parameters are insignificant and can be eliminated from the description. Figure 1 illustrates the starting point of our analysis of trumpet tones.

This study of existing instruments and the variation of their important parameters appears to us to be a most promising source of new timbres.

Need for further analysis

Every musical instrument has its own capabilities and limitations. A clarinet can sustain a tone longer than a flute and is more agile than an oboe; a trumpet can yield considerably louder tones than a harpsichord, but it has a more limited pitch range. Musicians must take such limitations into account, but they often choose instruments primarily for their distinctive tone quality. The "tone quality"—also termed, with sometimes slightly different meanings, "tone color" or "timbre"—is generally defined as the attribute that enables the listener to identify the instrument producing the tone.[1]

For a long time physicists have performed analyses of musical-instrument tones, to find out the physical

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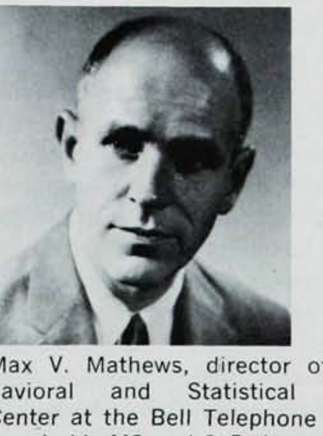

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TRUMPET PLAYER in the free-space room at Bell Telephone Laboratories. The mesh "floor" supports him in the center of this anechoic chamber. —FIG. 1

correlates of their tone quality. Many results of such analyses have been published,[2] and the general conclusion is that the tone quality is associated solely with the frequency spectrum of the waveshape. This conclusion is still widely accepted[1]: Even the reputed and recent treatise, *The Feynman Lectures on Physics,* gives no hint that there may be factors of tone quality other than "the relative amounts of the various harmonics."

Recently, new means of sound synthesis have helped to bridge the gap between physical description of tones and aural tone quality. Computer sound synthesis[3] makes it possible to synthesize virtually any sound from a physical description of that sound. This technique provides a way to check sound analyses: A successful analysis should yield a physical description of the sound from which one could synthesize a sound that, to a listener, is nearly indistinguishable from the original. We have tried to use the results of analyses of musical-instrument tones that are to be found in musical-acoustic treatises as input data for computer sound synthesis. In most cases we have obtained sounds that bear very little resemblance to the actual tones produced by the instrument chosen; in almost all cases the available descriptions of musical-instrument tones fail the fool-proof synthesis test. Hence the descriptions must be considered inadequate.

This failure points out the need for more detailed and more relevant analyses. Fortunately techniques are now available that make such analyses possible. Among these techniques we shall consider computer analysis and synthesis of sound.

Shortcoming of early analyses

Despite the considerable skill and ingenuity of scientists such as Hermann von Helmholtz or Dayton C. Miller, early analyses of musical-instrument tones have not given satisfactory results. We can now understand why.

First, only recently has progress in electrical engineering made it convenient to analyze rapidly evolving phenomena, an analysis that was hardly possible with equipment such as Helmholtz resonators, Henrici analyzers and stroboscopic analyzers.[4] These older instruments give steady-state analyses, yielding either the frequency spectrum averaged over some duration or the spectrum of a particular pitch period (assumed to repeat itself throughout the note). Helmholtz knew that Fourier sine-wave expansion was only one possible kind of analysis;[5] he even mentioned that "certain characteristic peculiarities in the tones of several instruments depend on the mode in which they begin and end"; yet he limited his investigations to "the peculiarities of the musical tones which continue uniformly," considering these to determine the "musical quality of the tone." Most of his followers do not appear to have felt the need to know the temporal evolution of a tone to be able to characterize it. Now computer synthesis shows that temporal changes bear strongly on tone quality. For instance, a rapid attack followed by a gradual decay gives a plucked quality to any waveform. The influence of time factors on tone quality is easy to demonstrate with a tape recorder; playing a piano tone backwards gives a quality very different from the original (yet both have the same spectra). Removing the initial segment of notes played by various instruments impairs the recognition of these instruments, as noted by Carl Stumpf[6] as early as 1910. Yet the importance of time factors in tone quality, quite familiar to musicians using tape manipulation[7] and computer synthesis of sound,[8] do not appear to be well publicized among physicists.[1] Clearly one must perform some kind of "running" analysis that follows the temporal evolution of the tones.

Attack transients

A few attempts have been made since 1930 to analyze the attack transients of instrument tones.[9] These transients constitute a most important part of the tone—in fact, many tones have no steady state. Yet their analysis has not produced a major step forward. The main difficulty lies in the intrinsic complexity of musical-instrument tones; the transients are complex and they are not quite reproducible from one tone to another, even for tones that sound very similar.[7] Most analyses have been restricted to a limited set of tones, and their authors have tended to generalize conclusions that may well be valid only for this set of tones. These shortcomings have produced many discrepancies in the literature and cast an aura of doubt on the entire body of acoustic data.

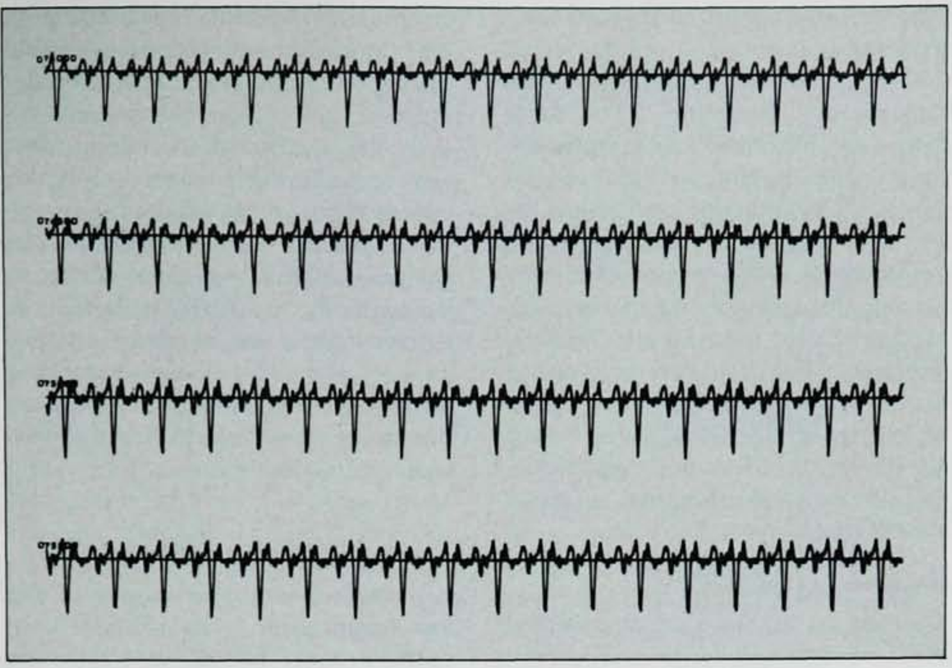
Complexity of instrument tones

The physical behavior of musical instruments is understood in its broad lines. The instrument responses have been studied, often very carefully, by distinguished physicists including Sir Chandrashekar Venkat Raman, celebrated for his investigations in infrared spectroscopy and Frederick A. Saunders, well known for his work on Russell–Saunders coupling.[10] The results in most cases are very detailed. In general the damping is low, and transients are not short compared to note duration. But the picture becomes quite confused when one takes the player into account.

Musical instruments are not usually operated by a standardized mechanical player but by human musicians who introduce intricacies, both intentionally and unintentionally. Even if a human player wanted to, he could not repeat a note as rigorously as a machine does. If he has good control of his instrument, he should be able to play two tones sounding nearly identical, but these tones can be substantially different in physical structure.[7] In general the player does not want to play all notes the same way. The score can require a violinist to play in many different ways: legato, staccato, sul ponticello (near the bridge), spiccato (with a bouncing bow), col legno (with the bow turned over, playing with the wood), con or senza sordini (with or without a mute).

Also the way a player interprets score markings depends both on his technique and his sense of style. Another player of the same instrument will be different; he may have learned a different technique and have a different stylistic understanding of the music. All these factors contribute to make his tones different from those of the first player. These closely mingled physical, physiological, psychological and aesthetic considerations complicate the task of the physicist seeking to analyze instrument tones for their characteristic features, but he must take then into account to reach relevant conclusions.

It is thus necessary to extract those significant features that are both regular and relevant from a complex physical structure. We need a way to control the psychoacoustical and musical relevance of the features extracted from the analysis, a requirement that could not be met in the past. The computer now makes it easy to include an aural control by synthesis in a reliable approach to the study of musical-instrument tones.



TRUMPET TONE shown as a computer oscillogram. Each line represents 50 msec duration. The tone, an F_4 with fundamental frequency varying around 350 Hz, is clearly quasi-periodic; pitch-synchronous analysis can therefore be applied. —FIG. 2

COMPUTER SOUND

Computers deal only with numbers, but sound waves can be converted to numbers and vice versa. A wave with frequencies from 0 to N Hz is properly described by a sequence of $2N$ "samples" per second. These samples are numbers that represent the amplitude of the wave at intervals of $1/2N$ sec. An analog-to-digital converter can sample a waveshape to give a string of numbers that can subsequently be analyzed by a computer. Conversely a computer can compute a string of numbers; a digital-to-analog converter can convert these numbers to pulses with amplitudes proportional to the numbers; the pulses can then be smoothed by an appropriate low-pass filter to obtain an audio output.[3]

The computer can be programmed to do various kinds of analysis, including "running analyses," that are necessary to understand the evolution of a tone. One such program, useful for quasi-periodic sounds, is called "pitch-synchronous analysis"[11] It consists basically of segmenting the wave shape into individual pitch periods and computing the Fourier spectrum for each of these periods. The computer thus produces a display on which one can follow the evolution, in time, of the individual harmonics. We have found this method useful for analysis of sounds that have a well defined pitch, such as speech vowels or violin and trumpet tones.

Computer synthesis

The problem of computer synthesis is to generate the appropriate sequence of numbers for a given sound. With a proper physical description of the sound one wants to synthesize, the computations can be done by a special program.[3] This program has been designed to be easy to use but very flexible. It gives the user the choice of designing his own "orchestra," that is, incorporating blocks of programming to specify the physical parameters he wants (for example, for vibrato, that is, frequency modulation, he has to set up a special block for this purpose). Then the user can describe a tone by numerical parameters punched into a computer card. The program will compute numerous samples for this tone from the few physical parameters characterizing it.

The technical limitations of computer sound processing are associated with the replacement of continuous quantities by discrete numbers, which is an inherent feature of digital representation. The finite rate at which the waves are sampled imposes a frequency-range limitation. The finite precision with which the samples are represented in the computer introduces a "quantization" distortion, or quantization noise. It is possible to reduce these limitations so that they are not objectionable to the ear; for instance, representing samples by four-significant-figure numbers will give a signal to quantization-noise ratio larger than 65 dB. A sampling rate of 40 000 Hz, which is now technically possible, produces frequencies up to 20 000 Hz.

Analysis and synthesis

We can now outline a reliable method that uses the computer to study instrumental tones. First, recordings of real instruments are analyzed—and in particular, although perhaps not exclusively, analyzed by computer. From the analysis we extract a physical description of the tone as a set of physical parameters, and then we synthesize tones with these parameters. Listeners judge how similar the synthetic and real tones sound, with results that indicate whether the physical description is sufficient. If it is not, additional analysis or interpretation of the analysis has to be performed, until we find proper parameters. Then systematic variations in the parameters (one at a time) enable listeners to evaluate the aural relevance of each of these parameters.

In this method the computer first serves as a flexible analysis tool; it also provides the synthesis that is necessary for a reliable aural control of the analysis; and thanks to the flexibility of the synthesis, it can efficiently help to schematize the results of the analysis in a manner that makes sense for the listener. We shall now give examples of this method for trumpet tones and violin tones.

TRUMPET TONES

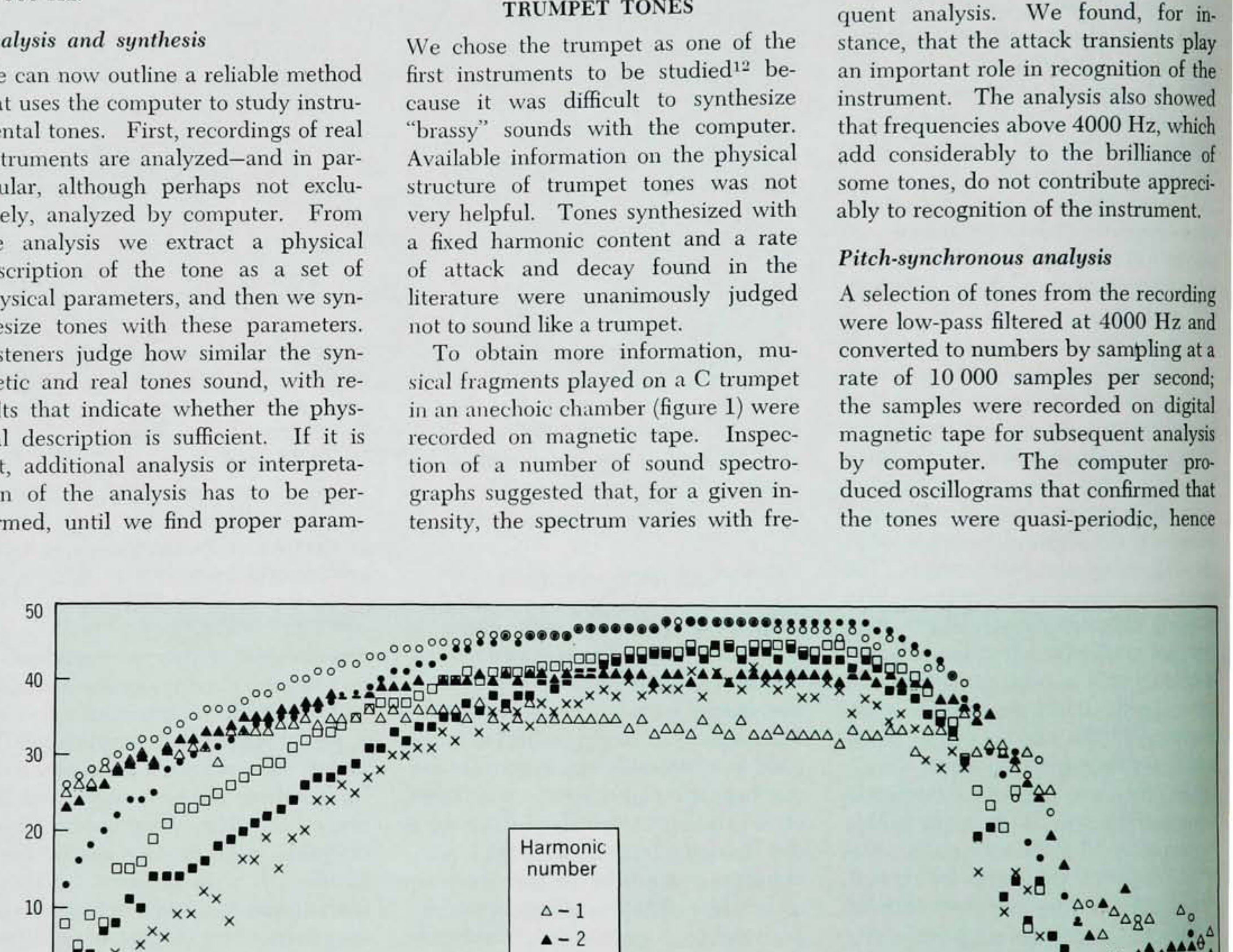
We chose the trumpet as one of the first instruments to be studied[12] because it was difficult to synthesize "brassy" sounds with the computer. Available information on the physical structure of trumpet tones was not very helpful. Tones synthesized with a fixed harmonic content and a rate of attack and decay found in the literature were unanimously judged not to sound like a trumpet.

To obtain more information, musical fragments played on a C trumpet in an anechoic chamber (figure 1) were recorded on magnetic tape. Inspection of a number of sound spectrographs suggested that, for a given intensity, the spectrum varies with frequency so as to keep a roughly invariant envelope. One says in this case that the spectrum has a "formant" structure; a formant corresponds to a portion of the spectrum where the envelope shows a peak. However, sound spectrograms failed to give precise enough information about attack and decay characteristics of the tones. Also it is tedious to get anything more precise than qualitative data about an evolving spectrum from a sound spectrograph. Thus a computer analysis was indicated.

Previously, tones selected from the recording had been filtered or spliced; listening to the results guided subsequent analysis. We found, for instance, that the attack transients play an important role in recognition of the instrument. The analysis also showed that frequencies above 4000 Hz, which add considerably to the brilliance of some tones, do not contribute appreciably to recognition of the instrument.

Pitch-synchronous analysis

A selection of tones from the recording were low-pass filtered at 4000 Hz and converted to numbers by sampling at a rate of 10 000 samples per second; the samples were recorded on digital magnetic tape for subsequent analysis by computer. The computer produced oscillograms that confirmed that the tones were quasi-periodic, hence



PITCH-SYNCHRONOUS ANALYSIS of a trumpet tone. These measurements are for a short tone (0.16 sec) with a fundamental frequency near 550 Hz (C#). Time, measured in units of one pitch period, runs from right to left because the pitch-synchronous programs give more information on the attack (on right of plot) when proceeding backward in time. Seven harmonics are shown here. —FIG. 3

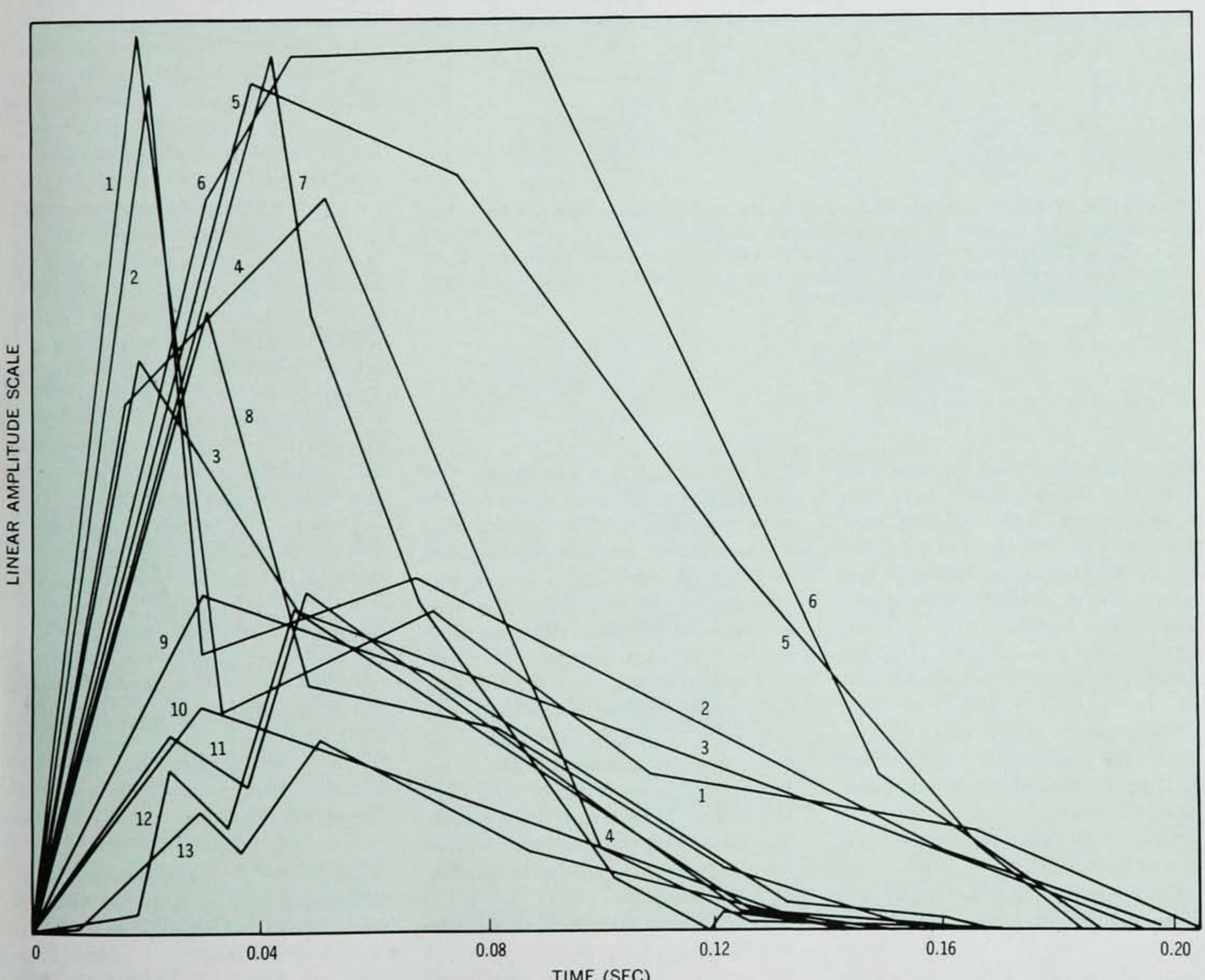
that pitch-synchronous analysis was applicable (figure 2); then it performed pitch-synchronous analysis, yielding displays with detailed information about trumpet tones (figure 3). The salient features are that the pitch-frequency display shows frequency glides at the beginning of the tone, periodic modulation at a rate of about 7 Hz (corresponding to vibrato) and quasi-random fluctuation at a much higher rate; the low-order harmonics build up faster than the high-order ones during the attack (typical values are 0.01 sec for the first harmonic, 0.03 sec for the fourth harmonic, 0.05 sec for the eighth harmonic; the overall attack time to reach the half-power amplitude is 0.015 to 0.030 sec), and similarly the low-order harmonics have a longer decay. During the steady state the spectrum undergoes minor changes, except when there is a notable change of level; an intensity increase is associated with an increased proportion of high-frequency harmonics.

Synthesis shows that the pitch-synchronous analysis displays adequately describe the tone. We could imitate a trumpet tone in the following manner: The evolution of the harmonics, as shown by the display, was approximated by more or less complex functions made up of linear segments; these functions were used to control the evolution of the harmonics in the synthesized tone. Figure 4 presents an example of these functions, showing how detailed the synthesis control can be. Tones synthesized like this were very similar to actual trumpets; when five synthesized tones were randomly interspersed with five actual tones, listeners were unable to distinguish which were the synthesized tones at better than the chance level of 50%. Highly trained musicians could do only slightly better (60%).

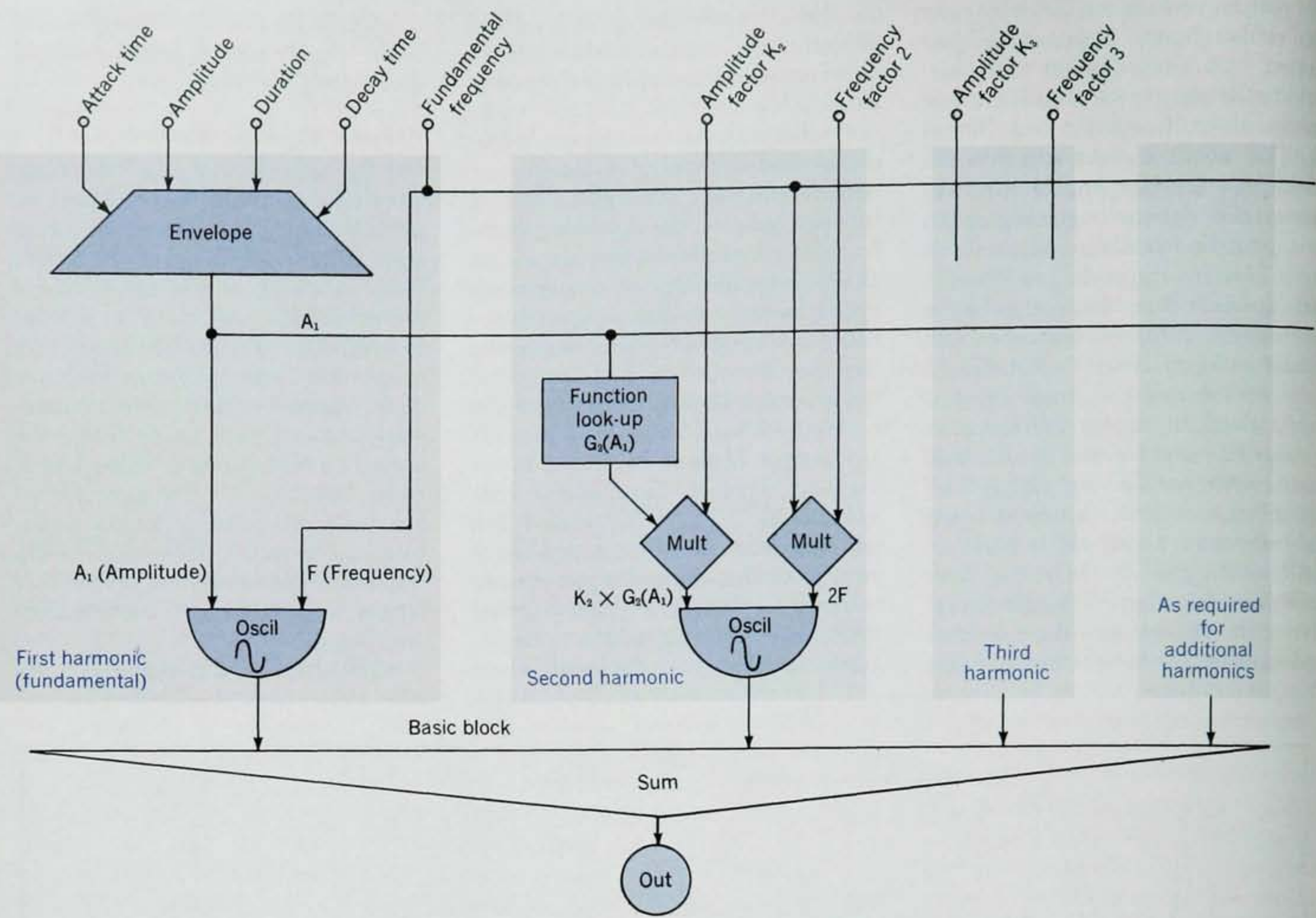
Relevant and irrelevant factors

But the specification of all the functions on figure 4 is very tedious, especially if they are different for every note. One would like to abstract, from these functions, a relevant sketch of the tones. Hence we made an attempt to synthesize trumpet-like tones in a simple way. By systematic variation of the various synthetic sound parameters, one at a time, we evaluated the subjective importance of these parameters. As a result a few physical factors were dismissed as aurally irrelevant, for example, short-term (fast) amplitude fluctuation. However, other factors were important, the principal ones being:

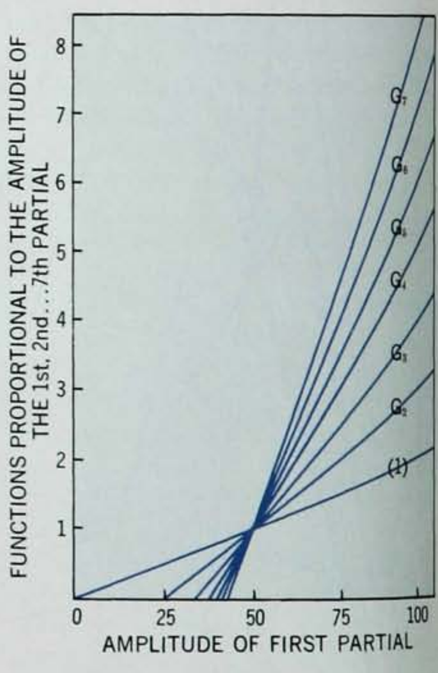
- The shape of the frequency spec-



LINE-SEGMENT FUNCTIONS that approximate the evolution in time of 13 harmonics of a D_4 trumpet tone lasting 0.2 sec. Functions like these, obtained by analysis of real tones, control the harmonic amplitudes of synthetic tones. —FIG. 4



BRASS-LIKE SOUNDS may be synthesized by the system shown above, with a harmonic content that depends on the first-harmonic amplitude according to the functions $A(1)$, G_2, . . . G_7 plotted on the right. Together with proper data for attack and decay times, this device produces synthetic trumpet tones good enough to confuse nearly all listeners; short sounds are especially successful. —FIG. 5



trum, which has a peak (formant) near 1500 Hz; louder sounds have relatively stronger high-frequency components. As the fundamental frequency is changed, the quality is kept more nearly constant if we use a spectrum with a formant structure rather than by simply keeping the same amplitude relations between the harmonics.

- The attack, which takes place in about 20 milliseconds, with faster build up of low-order harmonics than high-order ones.
- The high-rate quasi-random frequency fluctuation.

From a proper specification of only these features we have been able to synthesize brass-like sounds. Some sounds (especially the short ones) were so close to the real trumpet tones that even musically expert listeners could not differentiate reliably between real and synthetic. Thus, we believe that the features mentioned above are the most important to the listener; they certainly made possible the synthesis of brass-like sounds in a simple way (figure 5). Additional features may, in some conditions, also constitute cues for the identification. To duplicate trumpet sounds with a fidelity sufficient to confuse all listeners, one may have to specify more details: a more accurate spectrum with an extended frequency range beyond 4000 Hz, a good decay and perhaps a function controlling the frequency and introducing, at times, glides to simulate an intonation slip.

Some factors may have virtually no audible influence in certain conditions yet be important in other conditions. For instance we found that details of the attack were more audible in long sustained tones than in brief tones. Thus some of the above-mentioned conditions may have to be qualified and refined. Also, it appeared that most listeners made their judgment on the basis of some particular property. For instance they assumed that the real tones should be rougher, harsher or more complex than the synthetic ones. Although we have not yet studied this effect systematically, we believe that by emphasizing this property in the synthetic tones, one could bias the subjects towards a more than chance proportion of wrong answers in identifying the real and the synthetic tones.

VIOLIN TONES

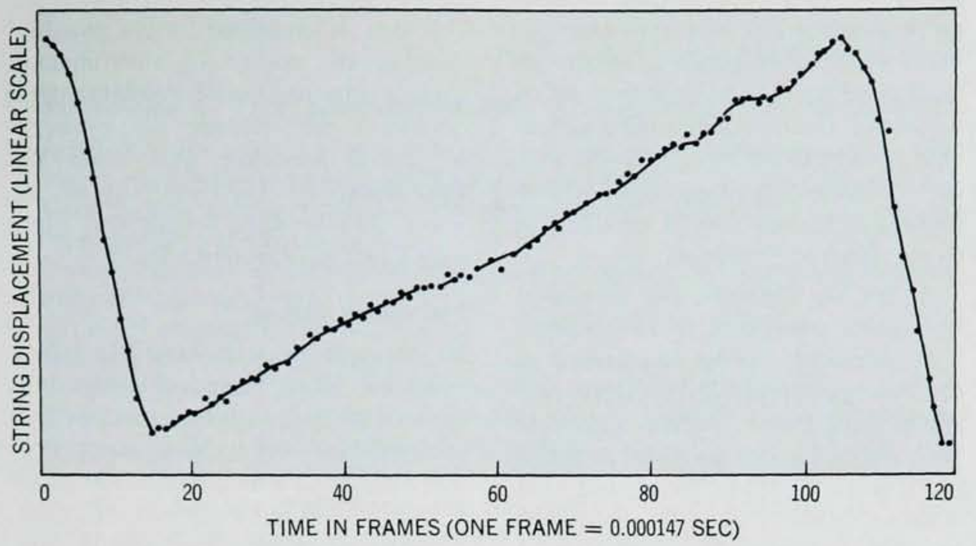
The study of violin tones[13] is less complete and less successful than the trumpet study in terms of synthesizing realistic tones, but it illustrates some other points about tone analysis.

Spectral analyses of violin tones by computer were carried out in a manner similar to the trumpet analyses. In addition to the expected formant peaks, they showed an unexpectedly large number of minima that appeared to be zeros in the spectrum. Some zeros are predicted by classic bowing theory, which says, for example, that if the bow is positioned one twelfth of the way down the string, every twelfth harmonic should be missing. But every third or fourth harmonic was missing, which corresponds to an unlikely bowing position.

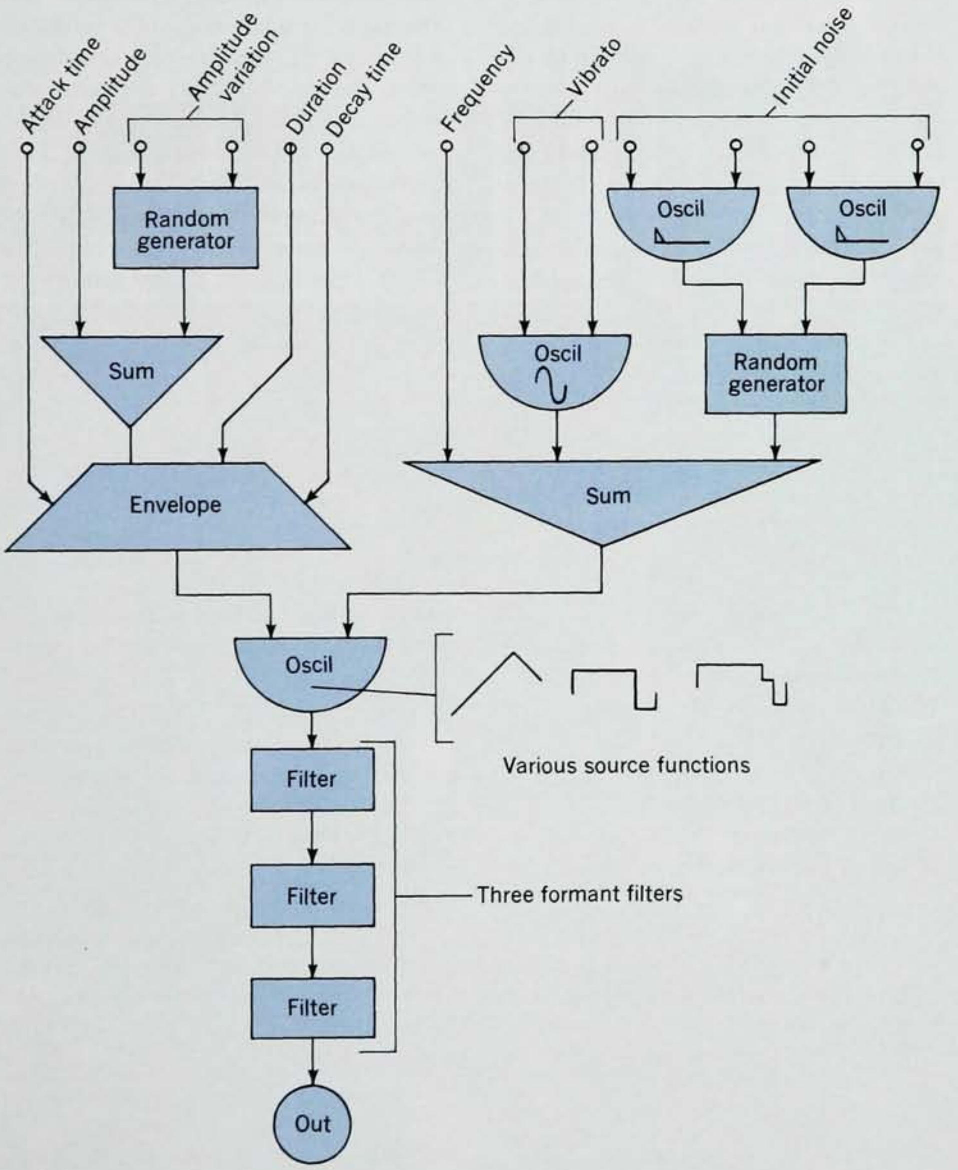
High-speed photography of the motion of a cello string yielded the classic triangular motion of the string under the bow shown in figure 6. The bow catches the string and pulls it to one side, after which the string separates from the bow and flies back to its original position. However, the flyback time of the triangle, T_F, was greater than that predicted by the position of the bow along the string, thus introducing additional zeros into the spectrum of the string motion. The increased flyback time indicates that some mechanism, other than reflections from the bridge and nut, determines the time at which the string slips from the bow.

The cello photographs do not quantitatively predict the zeros in the violin spectrum; furthermore we were unable to photograph violin-string motion because of its small amplitude. However, we could simulate a violin with appropriate zeros (and formants) with the instrument shown in figure 7. The excitation-oscillator waveshape was either a triangle or the derivative of a triangle, both having the correct zeros. The resulting sounds were unquestionably from the "string family." They sounded more like a viola than a violin.

The instrument provides for a vibrato, typical of violin players. In ad-



STRING-AND-BOW INTERACTION, as determined from high-speed photographs. This cello C string, 71 cms long, was bowed at a point 5.7 cms from the bridge. The period (104 frames) is 15.3 msec and flyback (steep portions of curve at left and right ends of plot) takes place in 1.91 msec (13 frames). The ratio of flyback time to total period (0.125) is greater than theory predicts. —FIG. 6



STRING-TONE SIMULATION. Typical parameters for this synthesis are: rise time, 250 msec; decay time, 100 msec; random amplitude variation, $\pm 20\%$ of mean amplitude; vibrato, $\pm 1\%$ of center frequency at 6.4 vibrations per sec; random frequency modulation, begins at $\pm 100\%$ of center frequency of tone and decreases linearly to 1% of center frequency, 87 msec after the start of the tone. —FIG. 7

dition the frequency control includes an initial random-frequency component, which corresponds to erratic vibration when the string is first set in motion by the bow and adds a noticeable quality to all tones. If the random component is exaggerated, a tone strikingly characteristic of a beginning string player is produced.

As for the trumpet, the simulated instrument enabled us to make subjective judgments of the importance to the tone quality of all the factors represented in figure 7. We concluded that these factors provide a useful simulation of "string" tone, but that additional factors must be discovered to yield an impeccable match to a given violin.

OTHER WORK

Harvey Fletcher and his collaborators[14] have recently carried out a very similar approach without a computer. While they did not go very far in examining the effect of various parameters—probably because of the lack of flexibility of their equipment—they obtained most valuable results, in particular the finding that warmth in piano tones is correlated with inharmonicity of the waveshape. Yet we believe that this kind of study would be much easier with a computer, which affords great precision and flexibility; this view is confirmed by the growing number of studies of instrumental tones performed with the help of a computer, for example the work of M. David Freedman and James W. Beauchamp at the University of Illinois, and William Strong and Melville Clark in Massachusetts.[15]

Instrument making

So far instrument making has relied essentially on old empirical instrument-makers' know-how rather than on scientific data.[16] As we have seen, previous analysis data on instrumental tones is to a large extent irrelevant. But the insight now being gained should help instrument makers to use a more scientific approach. Of course, studies are necessary to correlate variations in instrument design and building with variations in the physical parameters of the sound produced.[10] But after this correlation is done, computer synthesis will make it possible to determine which design will yield the most desirable sound without having to build the experimental instrument. This synthesis can also benefit designers of new instruments, and in particular of electronic instruments, for which the physical parameters of the sound can easily be correlated with the design. Procedures using computer synthesis help to bridge the gap between physical description (closely related to what the instrument maker can change) and psychoacoustical effect (strongly related to what the musician desires), thus making possible rapid trial-and-error tests instead of extremely slow and uncertain empirical modifications.

Computer music

We must mention here that the computer is a promising musical instrument in itself.[17] To take advantage of its unlimited possibilities, the user must gain psychoacoustical knowledge: He must know which physical parameter is responsible for the effect that he wants to achieve. While our emphasis here is on the contribution of these studies to understanding musical-instrument tones, it may be said that the results have also benefited the field of computer music. They have increased our knowledge of important, unimportant, or undesirable physical parameters, and they have provided ways to synthesize brass-like or string-like sounds economically. The musical value of the computer does not lie, of course, in its ability to duplicate exactly what a real instrument can do, but rather in yielding an extended repertory of sounds, including and going beyond the classes of sounds of actual instruments.

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