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Parametric Electric Guitar Synthesis

The electric guitar is one of the most common musical instruments today. Several synthesis algorithms have been created to synthesize its sound (Sullivan 1990; Karjalainen et al. 2006; Pakarinen, Puputti, and Välimäki 2008; Smith 2008). However, these previous synthesis models lack a few features that contribute to, or alter, the characteristic electric guitar tone. Firstly, the magnetic pickup used in electric guitars alters the sound radically. The magnetic pickup acts as a bandpass filter and causes a comb filter–like effect as a function of its position (Jungmann 1994). In addition, the response of the magnetic pickup has been found to be inharmonic, as are the vibrations of the string. Secondly, when using a distortion effect, the details of the attack and the sustain part of a tone are brought to an audible level, which is not the case with acoustic instruments. This is caused by the radically increased gain (0–120 dB) and the inherent compression of the distortion effect. Thirdly, the instrument can be played very expressively: The player has total control of the plucking event (plucking angle, force, and width) and can bend the string to alter the pitch.

This article proposes a new parametric synthesis model that enables the creation of these sonic features. The synthesis model is based on the waveguide method (Jaffe and Smith 1983; Smith 1992; Karjalainen, Välimäki, and Tolonen 1998) with novel alterations. A new excitation model is introduced that recreates the plectrum scrape and the first displacement pulse created during the plucking event. The excitation model also accounts for different plucking forces, and the angle of the virtual plectrum is controllable. The string model is a time-varying one, where the parameter controlling the decay time is varied and the amount of pitch glide is mapped to different plucking forces. The magnetic pickup model recreates the level and timbre changes that occur when the distance of the magnetic pickup to the strings is altered. Moreover, the inharmonicity of the magnetic pickup is modeled with a cascade of allpass filters. Sound examples are available on-line (Penttinen, Välimäki, and Lindroos 2011).

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Electric Guitar Sound Analysis

A Fender Stratocaster (American Deluxe model) guitar, equipped with three magnetic pickups and a Fishman Powerbridge piezoelectric under-saddle pickup, was used in this study. The piezoelectric pickup was used for the excitation and string-sound analysis, because it offers a broader bandwidth than does a magnetic pickup and hence a clearer "view" of the plucking event and the vibrations in the string. The output signals of the guitar were fed into a PC through a sound card (Edirol UA-101, $f_s = 44.1$ kHz, 16 bits) with the ability to record the piezo and the magnetic pickup signals simultaneously in separate channels, as shown in Figure 1.

Excitation Analysis

Electric guitar strings are usually excited with the fingertips, fingernails, or a plectrum. Most commonly the electric guitar is excited with a plastic plectrum. Hence, this work concentrates on excitations done with a plectrum. As shown in Figure 2, the beginning of the electric guitar

Figure 1. Electric guitar tone signal of an open 6th string ($f_0 = 82.5$ Hz), with plucking point $d = 32.6$ cm, as a function of time in the piezoelectric under-saddle pickup (top) and the magnetic neck pickup (bottom).

Figure 2. The beginning of the piezoelectric pickup signal on the open 6th string. The plucking point is at the midpoint between the nut and the bridge.

Figure 3. Magnitude response of three excitation pulses (each averaged from ten pulses) with different playing dynamics on the open 4th string. The maximum of each magnitude response has been normalized to 0 dB. The maximum of the magnitude response for the ff *pluck is 7.4 dB higher than the* mf *pluck and 15 dB higher than the* pp *pluck.*

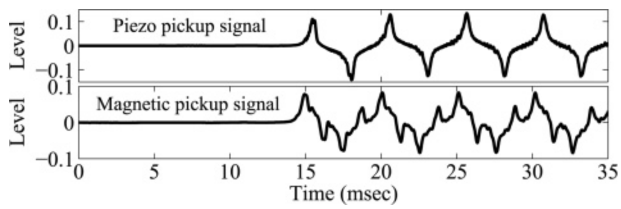


Figure 1.

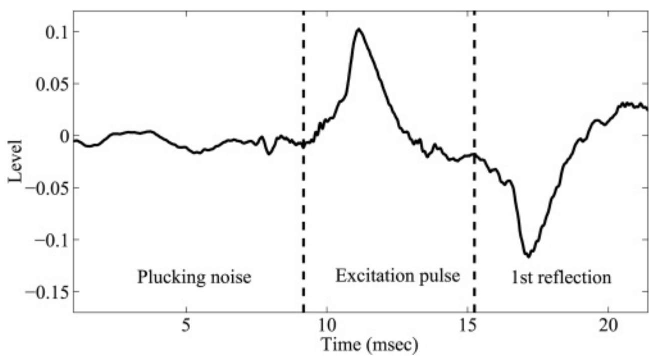
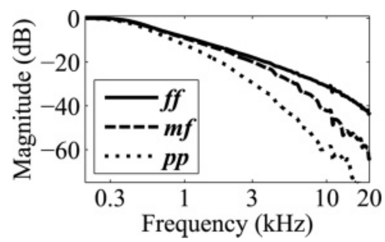


Figure 2.



tone can be split into three parts: plucking noise, excitation pulse, and the first reflection from the nut. The plucking noise is generated while the plectrum slides over the string but before the string has been released. This plucking noise is audible, especially when the guitar is played through a distortion unit. Two transverse waves travel in opposite directions from the plucking point when the string is released (Fletcher and Rossing 1991; Penttinen and Välimäki 2004). The wave travelling directly from the plucking point to the under-saddle pickup is the second part of the excitation. The third part of the excitation is the first reflection, the wave reflecting from the nut and travelling to the under-saddle pickup.

The angle between the plectrum and the string, commonly perpendicular to the top of the guitar, may vary during playing. A change in the plucking angle affects the duration and level of the excitation noise. The larger the angle is between the string and plectrum, the longer the excitation noise is, and the higher the excitation noise level is compared to the excitation pulse. The longer noise duration can be explained by the plectrum sliding over the string for a longer time period before the release.

Altering the plucking dynamics affects not only the level of the tone but also the shape of the excitation pulse's magnitude response (Jaffe and Smith 1983; Laurson et al. 2001). Figure 3 shows how the level of the high frequencies is more pronounced for stronger plucks, which is clearly seen when the maxima of all magnitude spectra have been normalized to 0 dB. For example, at 3 kHz the level of pianissimo (*pp*) and mezzoforte (*mf*) plucks are 11.3 dB and 1.8 dB lower, respectively, than a fortissimo (*ff*) pluck.

Plucking the strings with a thumb (without using a thumbnail) drastically attenuates the high frequencies of the excitation pulse, as can be seen from Figure 4. In addition, the level of the plucking noise is significantly lower in plucks with the thumb than in plucks with a plectrum.

Players can change the timbre of the guitar by changing the plucking position. How the plucking position affects the spectrum of guitar tones is well known (Fletcher and Rossing 1991): The harmonics that have a node at the plucking point are not excited and will have a low amplitude. However, inharmonicity of the string has to also be taken into account when modeling the plucking point accurately. As shown in Figure 5, the spectrum of an electric guitar tone is slightly inharmonic and thus modeling the plucking point with purely harmonic relations is sufficient for the first few partials only.

String Vibration Analysis

Plucking dynamics have an effect on string vibration as well as on the excitation pulse's magnitude response. The effect of plucking dynamics was

Figure 4. Difference of magnitude spectra of an ff *pluck with a plectrum and an* ff *pluck with a thumb.*

Figure 5. Magnitude spectrum (solid) of an open 6th string (f_0 = 82.5 Hz) when plucked at the midpoint of the string. Dashed vertical lines indicate the position of the even harmonics when they have purely harmonic relations. Notice the linear frequency axis.

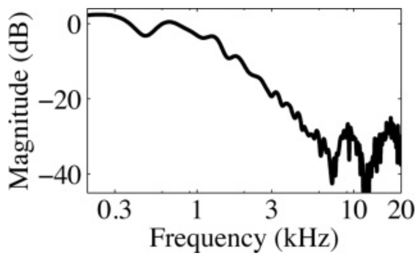


Figure 4.

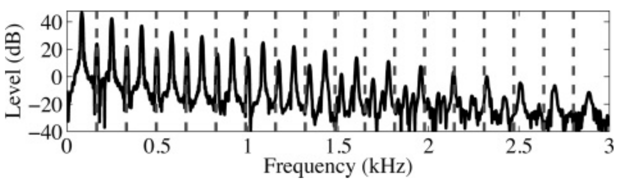


Figure 5.

examined in addition to the attenuation of the harmonics, the beating phenomenon, and the inharmonicity of the strings.

As a starting point, the attenuation of the harmonics was analyzed with an iterative algorithm (Erkut et al. 2000). It was noted that a forceful pluck causes a rapid change in the decay rate of harmonics, as can be seen in Figure 6 (top), where the envelope of the first harmonic (i.e., the fundamental frequency) of the first open string ($f_0 = 330$ Hz) plucked with a playing dynamic level *ff* is shown. This phenomenon occurs also for *pp* and *mf* plucks but is not as dramatic. The decay characteristics of a tone can be expressed with the instantaneous time to decay 60 dB, T_{60} (t). The initial T_{60} of the first harmonic is about 4 seconds, as shown in Figure 6 (middle). After 2.5 seconds the T_{60} of the tone increases to 10 seconds. The instantaneous T_{60} is calculated at a discrete point on the envelope as

$$T_{60}(t) = \frac{-\Delta t}{\Delta L} 60\,\mathrm{dB} \quad (1)$$

where Δt is the time difference between two consecutive samples, and ΔL the level difference. The analysis window used, that is, the length of Δt, is 16 times the length of the period of f_0 (e.g., when $f_0 = 330.5$ Hz, the analysis window is 2,135 samples, or 48 msec) with an overlap factor of 75 percent. Figure 6 (bottom) shows a calculated loop-gain coefficient $g(t)$ as a function of time from the $T_{60}(t)$ time shown in Figure 6 (middle). The loop-gain can be calculated from the formula

$$g(t) = 10^{-\frac{3}{T_{60}(t) f_0}} \quad (2)$$

where $T_{60}(t)$ is calculated for the first harmonic and f_0 is the frequency of the first harmonic. The loop-gain coefficient is discussed subsequently in the String Model section.

In addition to affecting the decay rates of harmonics, forceful plucking of an electric guitar string causes a pitch glide (Valette 1995; Tolonen, Välimäki, and Karjalainen 2000). The initial pitch glide of the strings was estimated with the YIN algorithm (de Cheveigné and Kawahara 2002) for three different plucking dynamics. All open strings were measured and the most extreme pitch glide (0.75 semitones) was measured on the open 6th string. As seen in Figure 7, the pitch glides of tones plucked with different forces follow the same trajectory accurately, after they have been synchronized to make the same pitch occur at the same time. Moreover, the point where the pitch glide starts to follow the trajectory depends on the plucking force. The pitch glide for the *ff* tone is much more prominent than for the *mf* and *pp* tones.

Inharmonicity coefficients of the strings (Fletcher, Donnell, and Stratton 1962) were estimated with an automatic estimation algorithm (Rauhala, Lehtonen, and Välimäki 2007) for each string and fret. The inharmonicity coefficient B can be calculated as

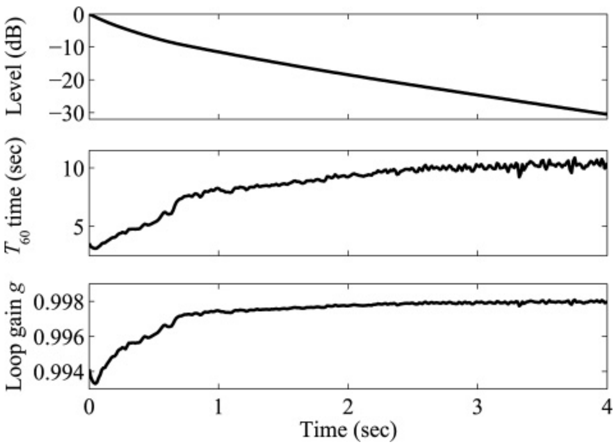
$$B = \frac{\pi^3 Q d^4}{64 l^2 T} \quad (3)$$

where Q is Young's modulus, d is the diameter of the string, l is its length, and T is its tension. We observed that, by ignoring a change in the string tension T in Equation 3 when the string is pushed against the fretboard, the inharmonicity coefficient of the string as a function of the fret number can be expressed as

$$B_n = B_0 2^{n/6} \quad (4)$$

where n is the fret number, B_n is the inharmonicity coefficient of the string at fret number n, and B_0 is

Figure 6. (Top) Temporal envelope of the first harmonic of the ff *tone played on the 1st string ($f_0 = 330$ Hz). (Middle) Instantaneous* T_{60} *time estimated from the envelope of the first harmonic as a function of time. (Bottom) Loop gain* $g(t)$ *of the one-pole filter calculated from the instantaneous* T_{60} *estimated using a 48-msec-long sliding window with an overlap factor of 75 percent.*



the inharmonicity coefficient of the open string. As can be seen in Figure 8, the approximation without taking into account the shift in string tension is quite accurate.

String vibrations have two transversal directions of polarization, horizontal and vertical (Weinreich 1977). Admittances of the bridge and the nut (for open strings) or fret (for stopped strings) are different for the string polarizations, which causes a slight deviation between the fundamental frequencies of each polarization (Capleton 2004; Woodhouse 2004). The frequency differences between fundamental frequencies of the polarizations were analyzed from a beating appearing in the temporal envelopes of the first harmonics. The amplitude of the resultant of two tones with slightly different frequencies, f_0 and $f_0 + \Delta f$, varies at frequency Δf (Rossing, Moore, and Wheeler 2002). One can see in Figure 9 the envelope of the first harmonic of the open 6th string, where the time difference between the first and the second notch is about 5 seconds. Thus the frequency difference Δf between the polarizations is $\Delta f = 1/5$ sec $= 0.2$ Hz.

Magnetic Pickup Analysis

The differences between the frequency response of the piezoelectric under-saddle pickup and the frequency responses of the three magnetic pickups were measured for each string individually and expressed as transfer functions. Synchronous recording of piezo and magnetic pickup signals enabled the calculation of the transfer functions. To create a broadband excitation, the wound strings were excited by scratching the string longitudinally with a plectrum. The unwound strings were similarly excited by sliding a metal stick along the string. Again, as in the case of the plucking-point filter, it was observed that, owing to the string stiffness, the notches in the transfer functions from the piezoelectric pickup to the magnetic pickups are not in harmonic relation. As can be seen in Figure 10, frequencies f_1 and f_2 of the first two notches are 4.1 kHz and 8.5 kHz, respectively. The inharmonicity coefficient B, calculated from the frequencies f_1 and f_2, corresponds to the inharmonicity coefficient of the string.

Figure 10 also shows how the distance between the strings and the pickup affects the transfer function. Increasing the distance decreases the overall output level of the magnetic pickup. For example, on the 1st string ($f_0 = 330$ Hz), the output level of the magnetic bridge pickup at 2.4 kHz is lowered by 9 dB when the distance is changed from 1.8 mm to 4.3 mm.

Electric Guitar Synthesis

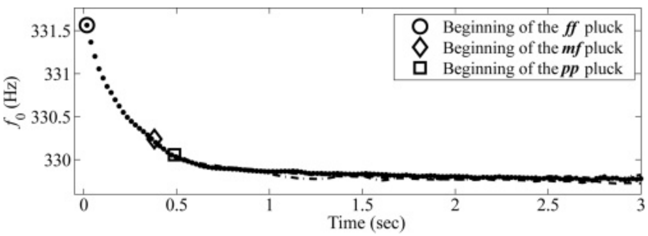
The electric guitar model proposed in this article is based on earlier research (Jaffe and Smith 1983; Sullivan 1990; Karjalainen, Välimäki, and Tolonen 1998; Karjalainen et al. 2006; Välimäki et al. 2006; Smith 2008). Figure 11 shows the proposed model in four parts: a parametric data segment, an excitation model, a dual-polarization string model, and a pickup model. The parametric data segment stores all the data measured from the guitar that are needed to control the model. The excitation model, the string model, and the magnetic pickup model are discussed in the following sections.

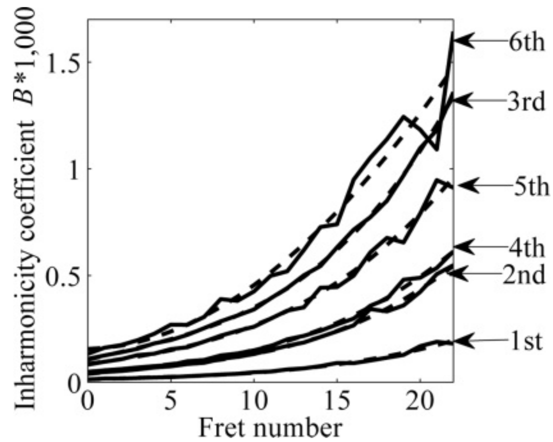
Excitation Model

A new way to model the excitation of a single delay loop string model is proposed. As suggested

Figure 7. Initial pitch glide of an ff *(dotted), an* mf *(dashed), and a* pp *(dot-dashed) tone played on the 1st open string (f_0 = 329.8 Hz). Note that only the* ff *pluck trajectory starts at 0 sec, but the other two trajectories have been time-shifted to match the trajectories.*

Figure 8. Inharmonicity coefficients for the tone of each the six strings estimated with the automatic estimation algorithm (solid lines) (Rauhala, Lehtonen, and Välimäki 2007) and approximated with Equation 4 (dashed lines).





in the Excitation Analysis section, the excitation model consists of three parts: the plucking noise, the excitation pulse, and the first reflection (i.e., the plucking-point model). The block diagram of the plucking-noise and excitation-pulse model is shown in Figure 12. The first part of the excitation, the excitation noise, is modeled with a burst of low-pass–filtered white noise. The second part of the excitation, the excitation pulse, is modeled with a third-order integrating filter (Heylen and Hawksford 1993; Heylen 1995) and a low-pass filter. After the generation of the excitation noise and the pulse, these signals are combined and then filtered with a plucking-position filter.

The first part of the excitation is modeled with white noise, which is filtered with a time-varying low-pass filter, the transfer function of which is

$$L_{\mathrm{p}_1}(z) = g(1+a)/(1+az^{-1})$$

The cutoff frequency of the low-pass filter is time-invariant and depends on the fundamental frequency of the plucked tone, but the gain is increased linearly from zero to the desired value. The length of the analyzed noise burst varies between 20 and 130 milliseconds and the maximum level is 10 to 40 dB below the level of the excitation pulse. These variations are mainly caused by changes in the plectrum angle.

The excitation pulse is modeled with a third-order integrating filter and a low-pass filter. The integrating filter $H(z) = H_{\mathrm{FIR}}(z)/H_{\mathrm{IIR}}(z)$ consists of two sections, a sparse finite-impulse response (FIR) part $H_{\mathrm{FIR}}(z) = h_0 + h_1 z^{-d_1} + \cdots + h_6 z^{-d_6}$ and an infinite-impulse response (IIR) part $H_{\mathrm{IIR}}(z) = (1 - z^{-1})^3$. The FIR filter has seven taps manually synchronized with the rapid changes in the excitation. The values of the taps are optimized with a least-square algorithm (Heylen and Hawksford 1993; Heylen 1995) for each string. The excitation pulse is generated by feeding the sparse FIR part through the IIR part, which consists of three integrators, as presented in Figure 12. After the integrating filter, the excitation pulse is filtered with a low-pass filter

$$L_{\mathrm{p}_2}(z) = g(1+a)/(1+az^{-1})$$

to model the plucking dynamics (Jaffe and Smith 1983; Laurson et al. 2001). Figure 13 (top) shows the taps with the measured and the modeled pulses in the time domain. Figure 13 (bottom) compares the magnitude responses of the sampled and the modeled excitation (same pulses as in Figure 13 (top)), showing a good match.

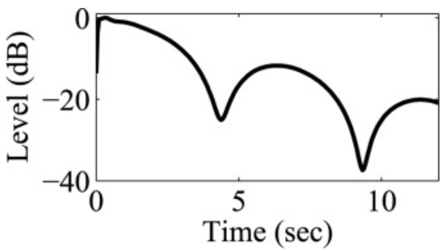
The plucking point is modeled with a feed-forward comb filter (Jaffe and Smith 1983), modified to reproduce the stretching of the notches toward high frequencies. The frequency-dependent delay consists of an integer-length delay line, a fractional delay $F_1(z)$, and a dispersive allpass filter, which can be implemented with a second-order allpass filter $A_{\mathrm{d}_1}(z) = (a_2 + a_1 z^{-1} + z^{-2})/(1 + a_1 z^{-1} + a_2 z^{-2})$ (Rauhala and Välimäki 2006). Figure 14 shows the block diagram of the plucking-point filter.

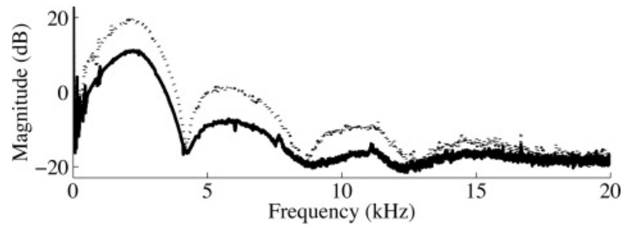
String Model

The dual-polarization string model is based on the single delay loop model (Jaffe and Smith 1983; Smith 1992; Karjalainen, Välimäki, and Tolonen 1998) with a time-varying low-pass filter and delay line

Figure 9. Amplitude envelope of the first harmonic on the 6th string ($f_0 = 82.5$ Hz) as a function of time. The time interval between notches is about 5 seconds, thus making a frequency difference between horizontal and vertical polarizations to be $f_{\text{difference}} = 0.2$ Hz.

Figure 10. Transfer functions measured from the piezoelectric pickup to the magnetic bridge pickup on the 1st string. The distance between the pickup and the strings is $d_1 = 4.3$ mm (solid) and $d_2 = 1.8$ mm (dotted). Notice the linear frequency axis.





(see Figure 15). The inharmonicity of the string is modeled with one or four second-order allpass filters (Rauhala and Välimäki 2006). Another design for a dispersive allpass filter has recently been proposed by Abel, Välimäki, and Smith (2010).

In earlier string models (Jaffe and Smith 1983; Smith 1992; Karjalainen, Välimäki, and Tolonen 1998), the frequency-dependent loss filter has been time-invariant. However, the decay rates of the harmonics vary with time, especially in the beginning of tones with strong plucks, as was evident in Figure 6. Therefore, a time-varying low-pass filter is necessary ($L_{p_3}(z)$ in Figure 15) to recreate the time-varying decay rates of the harmonics. The transfer function of the proposed time-varying loss filter is

$$H(z) = g(t)\frac{1+a}{1+az^{-1}} \tag{5}$$

where $g(t)$ is the time-varying loop gain and coefficient a determines the corner frequency of the filter. A time-invariant version of this one-pole loop filter has been previously used in several studies (Välimäki et al. 1996). The time-varying loop gain $g(t)$ follows a trajectory obtained from measurements, for instance, the trajectory shown in Figure 6 (bottom) (for the first open string and *ff* pluck). This time-varying decay phenomenon is due to the two transversal (vertical and horizontal) polarizations coupled at the bridge (Weinreich 1977).

An integer-sized delay (z^{-L} in Figure 15) is used for coarse tuning, and a third-order Lagrange interpolator (Laakso et al. 1996) for fine tuning (filter $F_2(z)$ in Figure 15). The pitch-glide phenomenon (Valette 1995), which is especially characteristic of strong plucks (as seen in Figure 7), is modeled by altering the length of the delay line during the tone.

The same pitch drift trajectory is used for the string models of both polarizations. Different plucking forces follow the same pitch glide trajectory (as seen in Figure 7), but the initial frequency depends on the plucking force. Energy compensation of the string model (Pakarinen et al. 2005) is not necessary in this model because the required delay length variations are not significant in the pitch glide phenomenon.

The beating of harmonics due to the two transversal polarizations of the string (Weinreich 1977) is modeled with two parallel, slightly mistuned string models (Jaffe and Smith 1983; Karjalainen, Välimäki, and Tolonen 1998). As was discussed in the String Analysis section, the deviation between the fundamental frequencies of the polarizations (Capleton 2004; Woodhouse 2004) is obtained from the beating interval of the first harmonic (shown in Figure 9). Moreover, as the magnetic pickups are not equally sensitive to the vertical and the horizontal vibrations of the string, the mixing balance between the polarizations was set to 1/10 after Horton and Moore (2008).

The inharmonicity of the strings is modeled with second-order allpass filters (Rauhala and Välimäki 2006)

$$A_{d_2}(z) = [(a_2 + a_1z^{-1} + z^{-2})/(1 + a_1z^{-1} + a_2z^{-2})]^n$$

$n = \{1, 4\}$, depending on the fundamental frequency of the string. The inharmonicity filter remains unchanged when f_0 is altered during a tone. The inharmonicity of the tones with a high fundamental frequency $f_0 > 330$ Hz is modeled with four allpass filters ($n = 4$), and for low tones with $f_0 \leq 330$ Hz the inharmonicity is modeled with one allpass filter ($n = 1$).

Figure 11. Block diagram of the proposed electric guitar model.

Figure 12. Block diagram of the plucking-noise and excitation-pulse model.

Figure 13. For the open 6th string (top) recorded excitation pulse (solid), the sparse FIR part of the integrating filter (impulses) and modeled excitation pulse, where the FIR part has been fed through the integrators (dashed line). (Bottom) Magnitude responses of measured (solid line) and modeled (dashed line) excitation pulses.

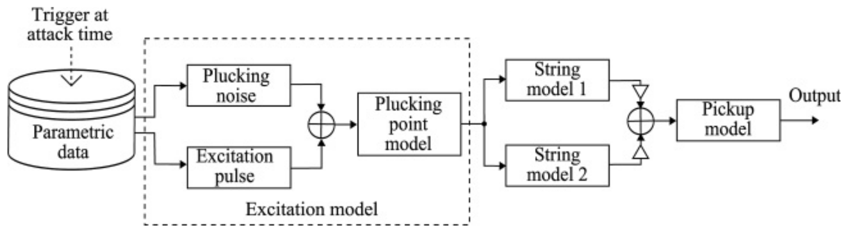


Figure 11.

Pickup Model

The proposed pickup model is based on the feedforward filter (Sullivan 1990; Karjalainen, Välimäki, and Tolonen 1998; Karjalainen et al. 2006) with a novel improvement to simulate the stretching of the notches toward high frequencies in the magnitude response of the magnetic pickup. The model also simulates the pickup placement and the distance from the strings. Figure 16 shows the block diagram of the proposed model.

The feedforward chain of the model consists of four cascaded filters. An integer-sized delay z^{-L} and a fractional delay $F_3(z)$ are used for accurate pickup placement modeling. Because the locations of the magnetic pickups are fixed, no time variance is needed for pickup location modeling. Thus, the fractional delay is realized with a first-order Thiran allpass filter (Laakso et al. 1996), used in some guitar models for string tuning (Jaffe and Smith 1983; Sullivan 1990). The stretching of the notches toward high frequencies in the magnitude response of the magnetic pickup is caused by the inharmonicity of the strings, and is modeled with a second-order Thiran allpass filter (Rauhala and Välimäki 2006)

$$A_{d_3}(z) = (a_2 + a_1 z^{-1} + z^{-2})/(1 + a_1 z^{-1} + a_2 z^{-2})$$

A similar filter is used in this study to simulate the string dispersion in the string model. The Thiran allpass filters add extra delay (from 2 to 26 samples) to the delay line, which has to be taken into account in the calculations of the fractional delay and the integer-sized delay. A low-pass filter $L_{p_4}(z)$ in the feedforward chain is used to lower the depth of the notches toward high frequencies. The one-pole low-pass filter was found to be adequate for this purpose. The corner frequency of the one-pole filter depends on the distance between the string and the pickup.

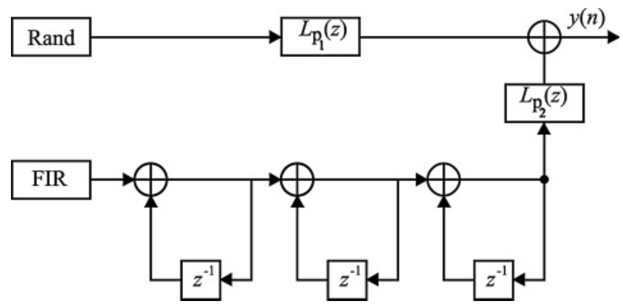


Figure 12.

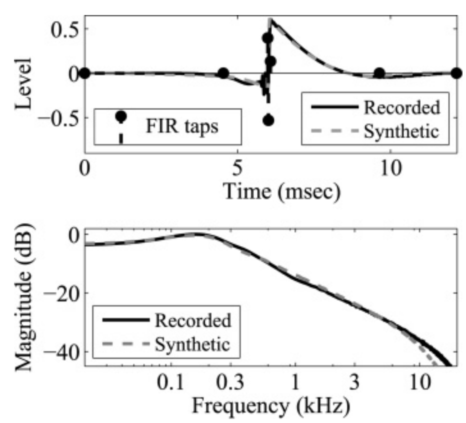


Figure 13.

The low-pass characteristic of the magnetic pickups (Jungmann 1994; Karjalainen, Välimäki, and Tolonen 1998; Penttinen 2002; Karjalainen et al. 2006) is modeled with a second-order shelving filter (Zölzer 1997) of the form

$$L_{p_5}(z) = (b_0 + b_1 z^{-1} + b_2 z^{-2})/(1 + a_1 z^{-1} + a_2 z^{-2})$$

Moreover, the distance between the strings and the pickup, as well as the particular string used,

Figure 14. Structure of the inharmonic plucking-point filter.

Figure 15. Structure of the string model with a time-varying one-pole filter.

Figure 16. Dispersive magnetic pickup model.

Figure 17. Measured and modeled transfer function from the piezo pickup to the magnetic neck pickup on the 3rd string. The distance between the string and the pickup is d = *1.8 mm. Notice the linear frequency axis.*

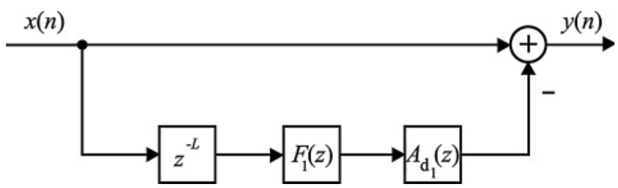


Figure 14.

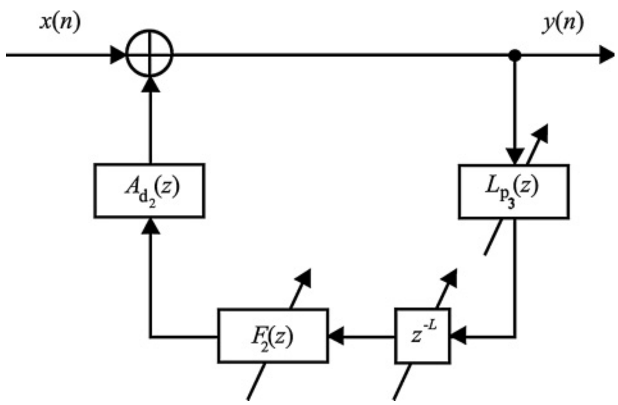


Figure 15.

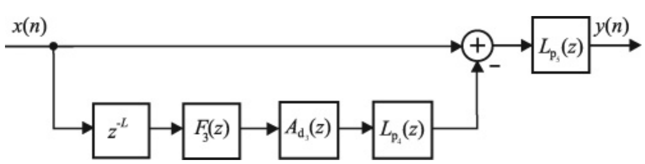


Figure 16.

influence the corner frequency and the gain coefficient of the shelving filter.

The modeled and measured magnitude response of the magnetic pickup is shown in Figure 17. The model presented is accurate for a string vibrating with relatively low amplitudes. However, with large amplitudes of string vibration nonlinearities in the magnetic field of the magnetic pickups should be considered (Horton and Moore 2008).

Synthesis Parameters

This section provides example values for parameters of the proposed electric guitar model. All the examples are for the open 6th string. Tables 1, 2, and 3 provide values for the excitation model shown in Figure 11. Table 1 gives values of the $H_{\mathrm{FIR}}(z)$ filter used in the excitation pulse generator, shown in Figure 12. Table 2 gives the coefficients for the dynamic plucking filter $L_{p_2}(z)$, shown in Figure 12. Values for the inharmonic plucking-point filter $A_{d_1}(z)$, shown in Figure 14, are given in Table 3. Tables 4 and 5 present the filter coefficients for the three pickup models (bridge, middle, and neck pickups), shown in Figure 16.

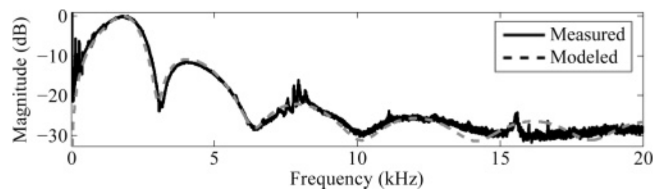


Figure 17.

Table 1. $H_{\mathrm{FIR}}(z)$ Filter Values of the Excitation Pulse Generator for the 6th String, as Shown in Figure 12

n	d_n	h_n
0	0	$2.081 \cdot 10^{-6}$
1	200	$5.956 \cdot 10^{-5}$
2	264	$-4.613 \cdot 10^{-1}$
3	265	$6.167 \cdot 10^{-1}$
4	268	$-1.557 \cdot 10^{-1}$
5	425	$7.467 \cdot 10^{-5}$
6	536	$-6.079 \cdot 10^{-6}$

Table 2. Coefficients for the Plucking Dynamics Filter $L_{p_2}(z)$ for the 6th String, as Shown in Figure 12

	g	a
Plectrum *ff*	1	0
Plectrum *pp*	0.18	−0.63
Thumb *ff*	1	−0.98

Table 3. Coefficients for the Inharmonic Plucking-Point Model for the 6th String, as Shown in Figure 14

Plucking Point (mm)	*Delay (samples)*	a_1	a_2
100	51.77	0.674686	−1.63220
326	221.58	0.769937	−1.74993

Table 4. Parameters for the Filters in the Feedforward Chain of the Pickup Model, as Shown in Figure 16, for the Bridge, Middle, and Neck Pickups for the 6th String, Where *h* is the Distance Between the String and the Pickup

Pickup, distance	*Delay (samples)*	g	a	a_1	a_2
Bridge, $h = 1.8$ mm	25.54	1	−0.650000	0.628145	−1.57085
Middle, $h = 3.5$ mm	60.53	1	−0.704255	0.711621	−1.67902
Neck, $h = 6.5$ mm	102.40	1	−0.800000	0.793192	−1.77724

Table 5. Parameters of the Low-Pass Filter $L_{p_5}(z)$ for the Bridge, Middle, and Neck Pickup Models, as Shown in Figure 16, Where *h* is the Distance Between the String and the Pickup

Pickup, distance	b_0	b_1	b_2	a_1	a_2
Bridge, $h = 1.8$ mm	2.22843	−0.381599	0.393900	−1.43896	0.564965
Middle, $h = 3.5$ mm	1.80455	−0.748276	0.364759	−1.38417	0.533409
Neck, $h = 6.5$ mm	1.29681	−1.11006	0.394504	−1.30878	0.492594

Conclusion

The effects of the excitation and magnetic pickups on the electric-guitar sound were analyzed, and signal-processing models for them were proposed. A complete electric-guitar synthesis algorithm based on the digital waveguide approach was developed incorporating these novel techniques and other appropriate improvements.

The excitation signal produced by plucking a steel string with a plectrum was measured using a piezoelectric pickup. A parametric excitation model consisting of three parts was then proposed: The first is a filtered noise burst, the second is a parametric simplified pulse that is reproduced with an integrating filter, and the third is the inharmonic plucking-point filter.

The proposed magnetic pickup model is founded on a special feed-forward comb filter in which a frequency-dependent delay is implemented using an allpass filter. The frequency-dependent delay filter is needed to simulate the dispersive wave propagation on the vibrating steel string, which causes the notches of the comb filter to be nonuniformly spaced in frequency.

In addition to these novelties, an improvement to the waveguide string model was introduced: A time-varying loop gain helps to emulate the two-stage decay of electric guitar tones. The gain variation over time is easily calibrated by smoothing a short-term averaged envelope of the first harmonic. Inharmonicity and beating appearing in electric guitar tones are also accounted for in the proposed synthesis model. Supporting sound examples are provided on-line (Penttinen, Välimäki, and Lindroos 2011) and on the forthcoming Winter 2011 *Computer Music Journal* DVD.

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