KTH ROYAL INSTITUTE
OF TECHNOLOGY

DT2212: Music Acoustics

Bob L. T. Sturm (TMH)

bobs@kth.se

Lecture 2: Modes of vibration, percussion
Chapters 2, 3, 26 in Hartmann
Chapters 1, 2, 3 in Fletcher and Rossing

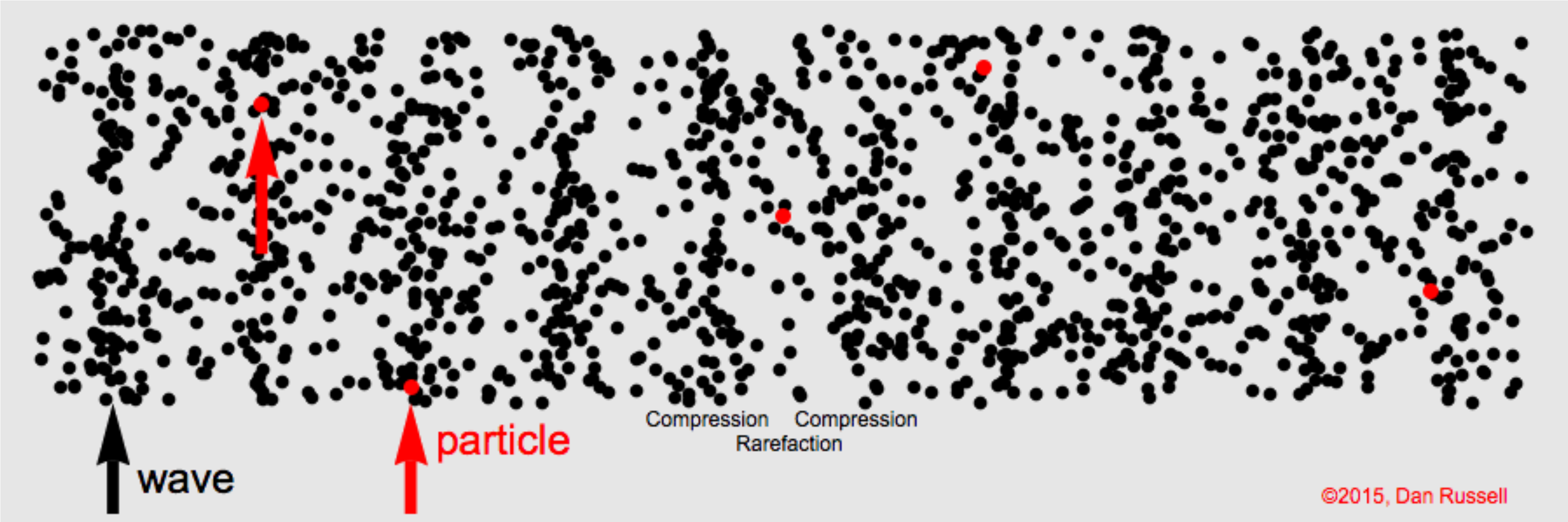
Musical sound is all around us

What is it? How does it work?

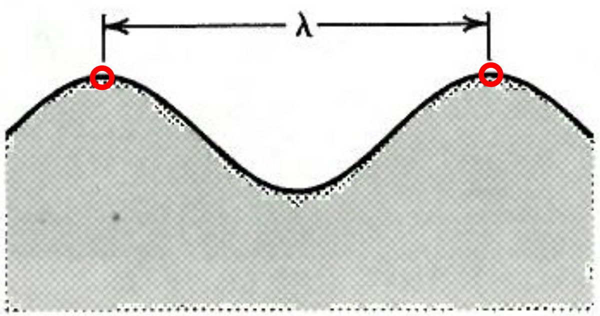
Motivating questions

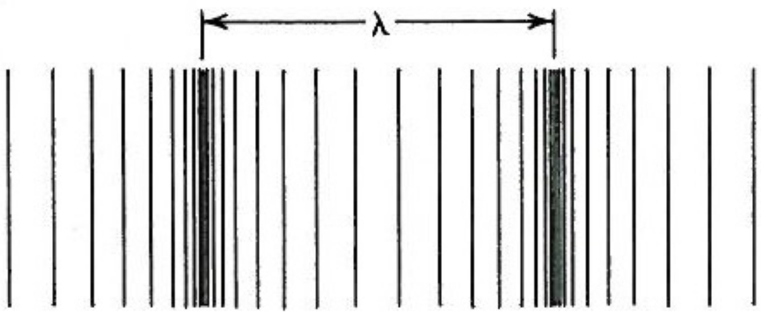
How does this thing work?
What defines its acoustic properties?
Why does it sound the way it does?

Sound is vibration

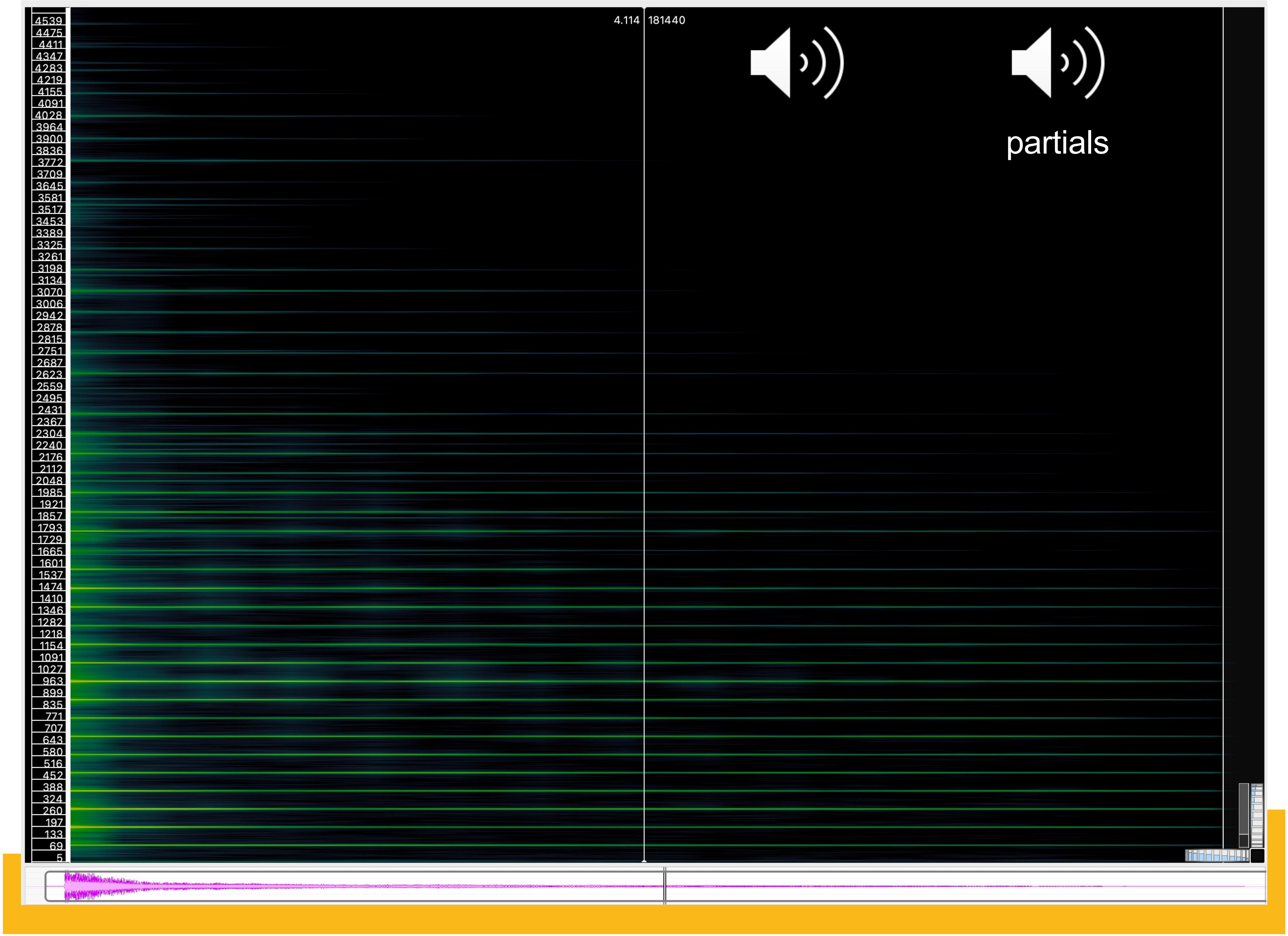


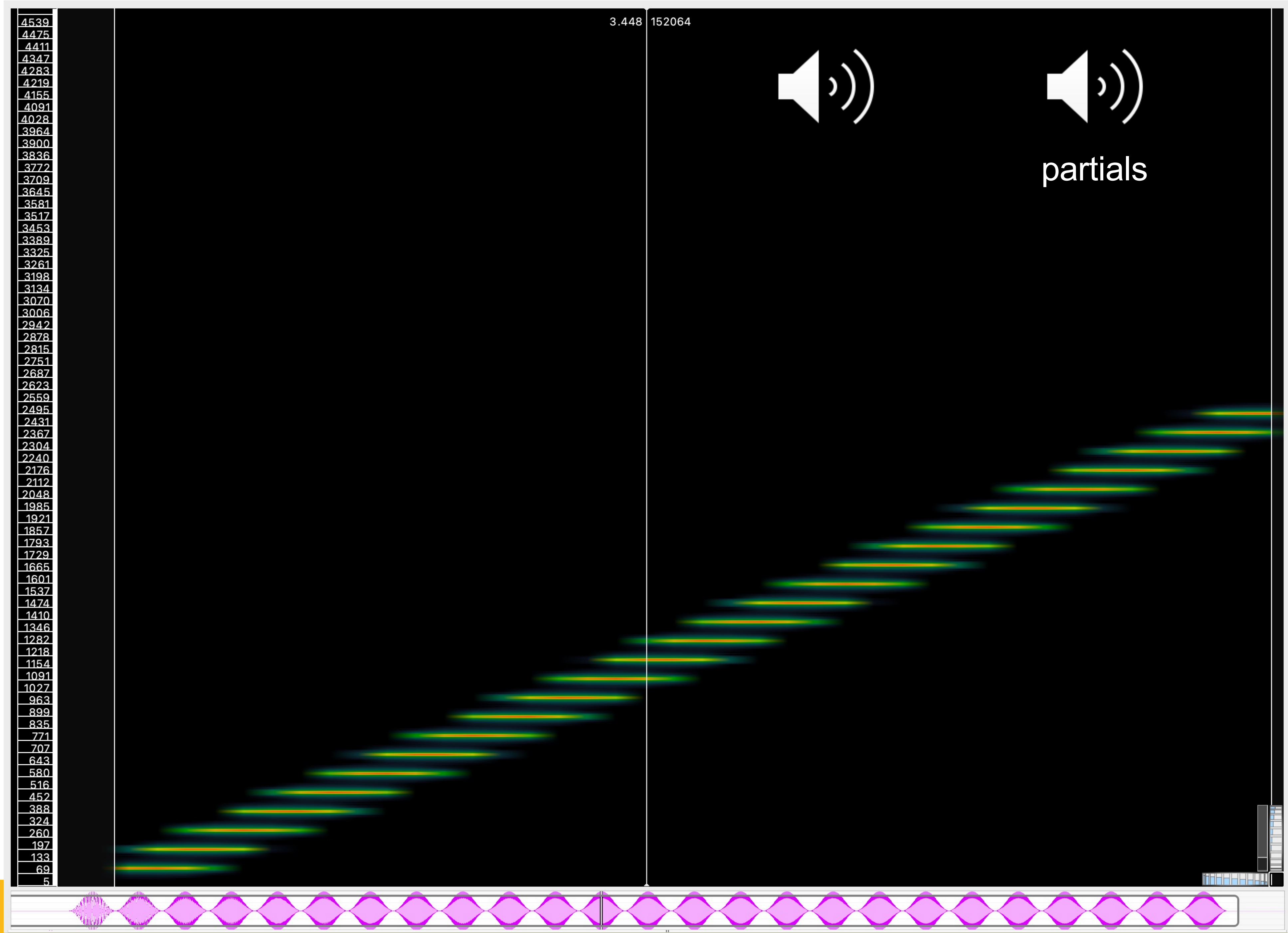
Displacement is in direction of wave travel instead of orthogonal to it.



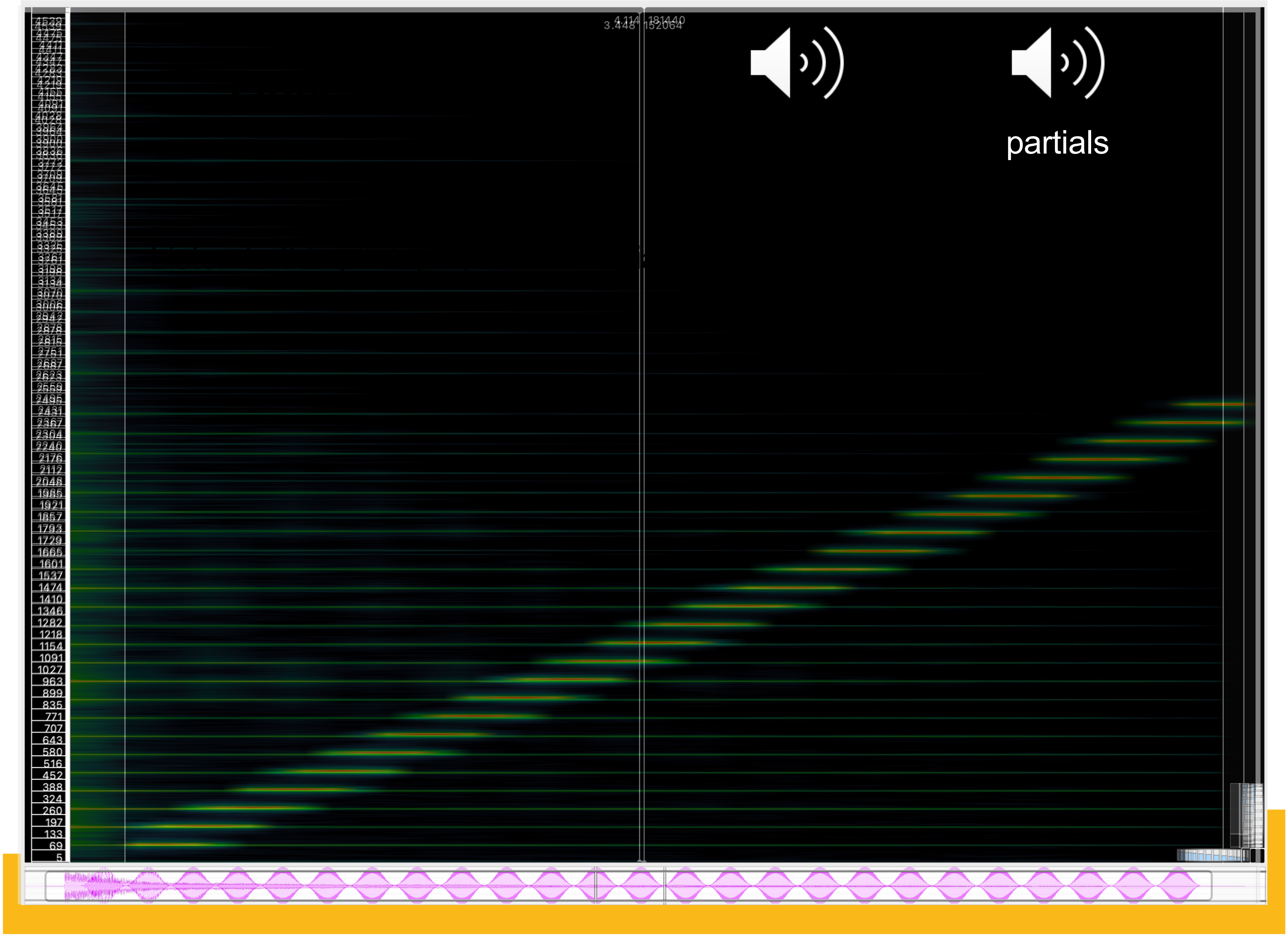


Listen to this note played on the piano


4.114
181440
partials



4539
4475
4411
4347
4283
4219
4155
4091
4028
3964
3900
3836
3772
3709
3645
3581
3517
3453
3389
3325
3261
3198
3134
3070
3006
2942
2878
2815
2751
2687
2623
2559
2495
2431
2367
2304
2240
2176
2112
2048
1985
1921
1857
1793
1729
1665
1601
1537
1474
1410
1346
1282
1218
1154
1091
1027
963
899
835
771
707
643
580
516
452
388
324
260
197
133
69
5
3.448
152064
partials

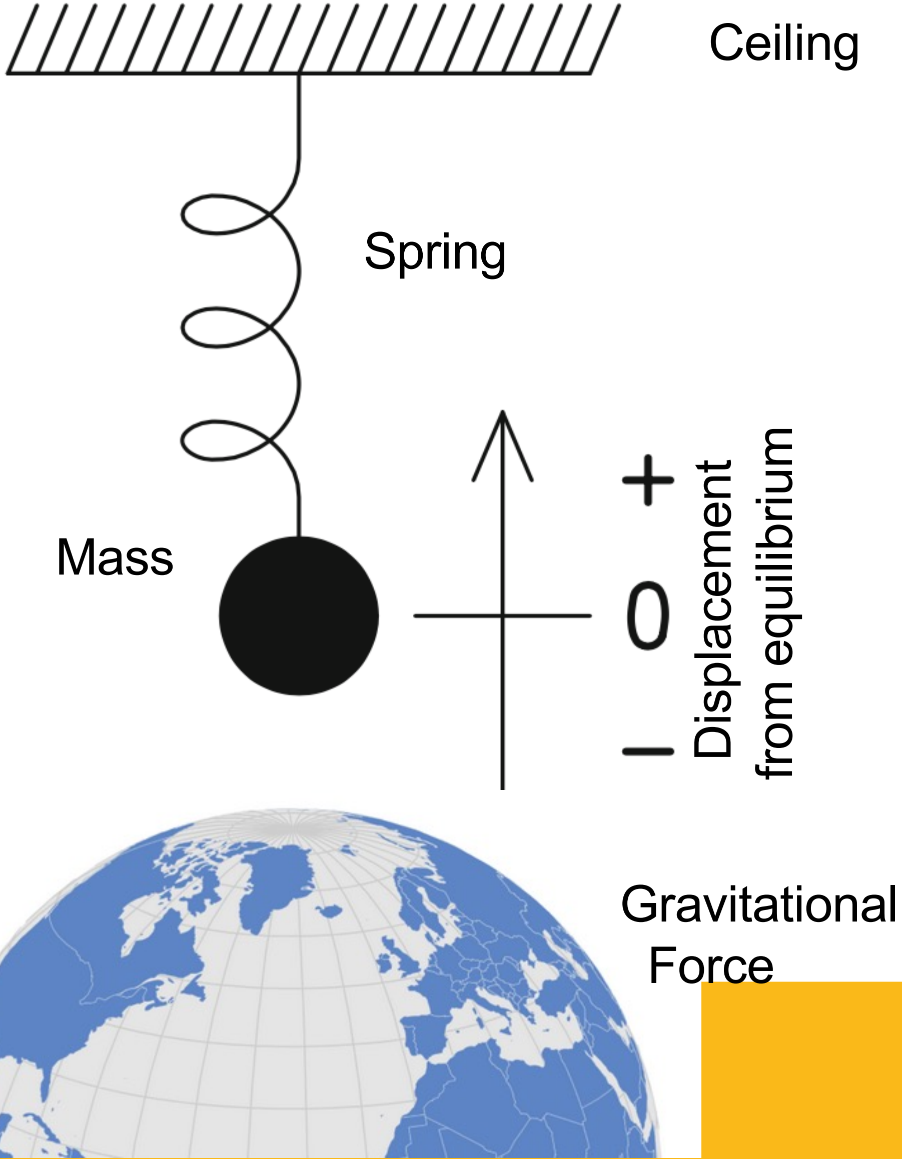

partials

Question:

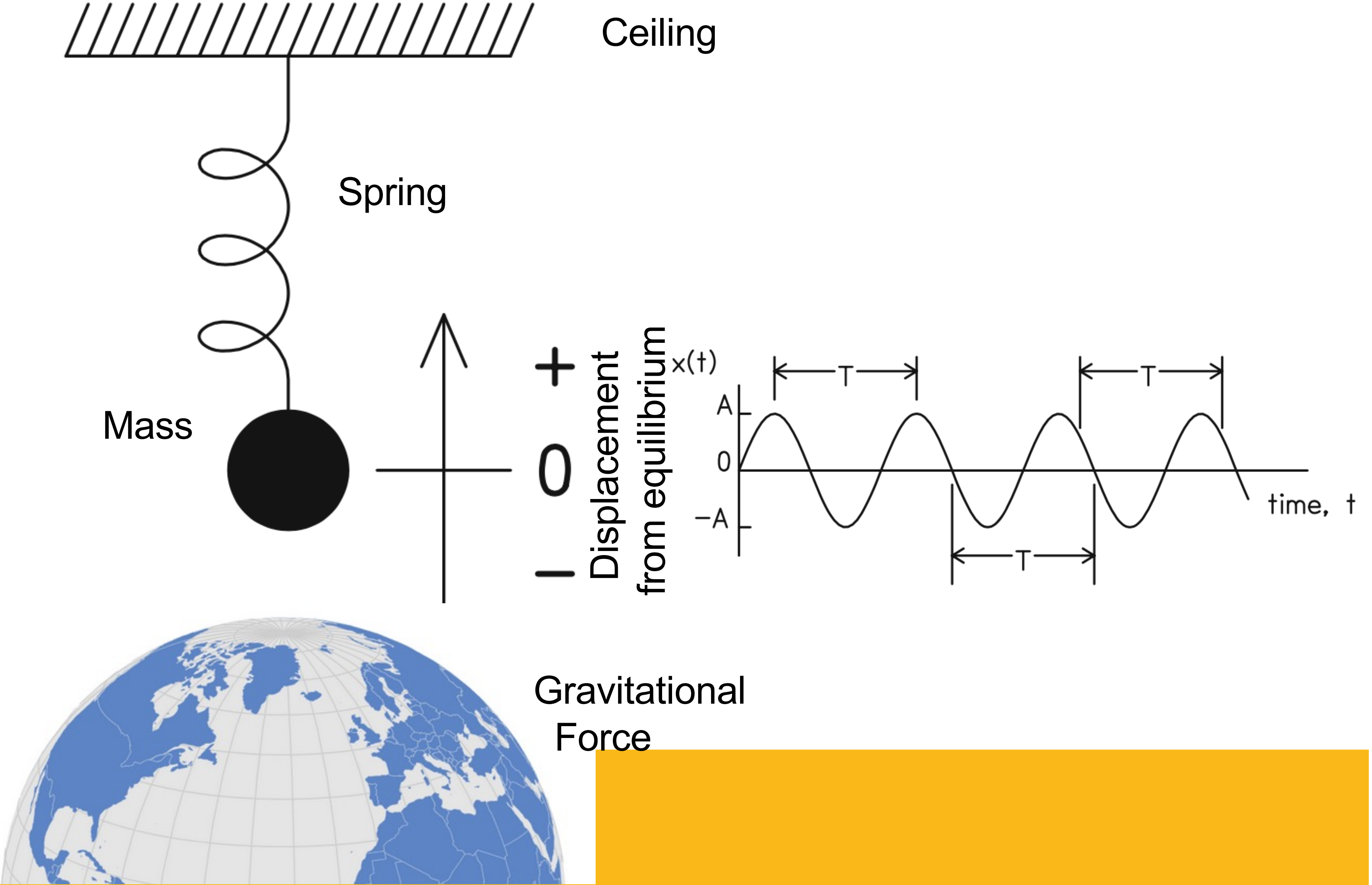
Listen to this note played on the piano

Why are all these frequencies in that one note?

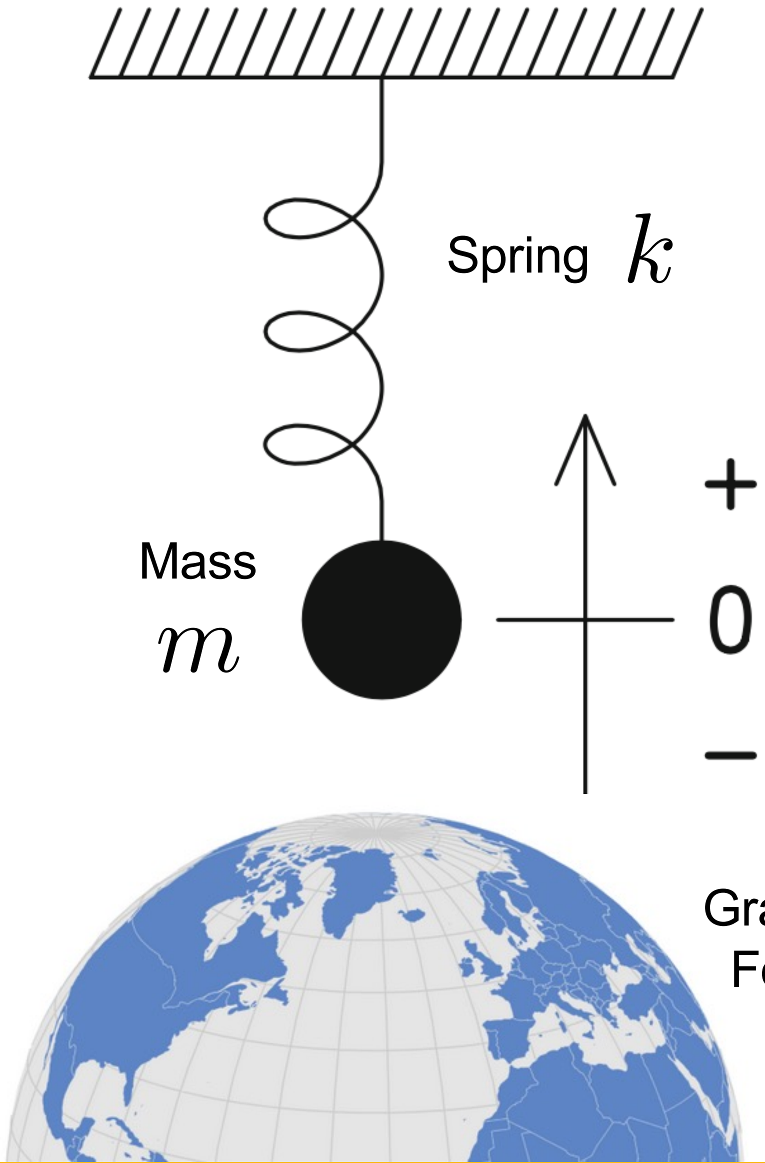
We need to understand vibration



We need to understand vibration



We need to understand vibration



Recall Newton's 2nd

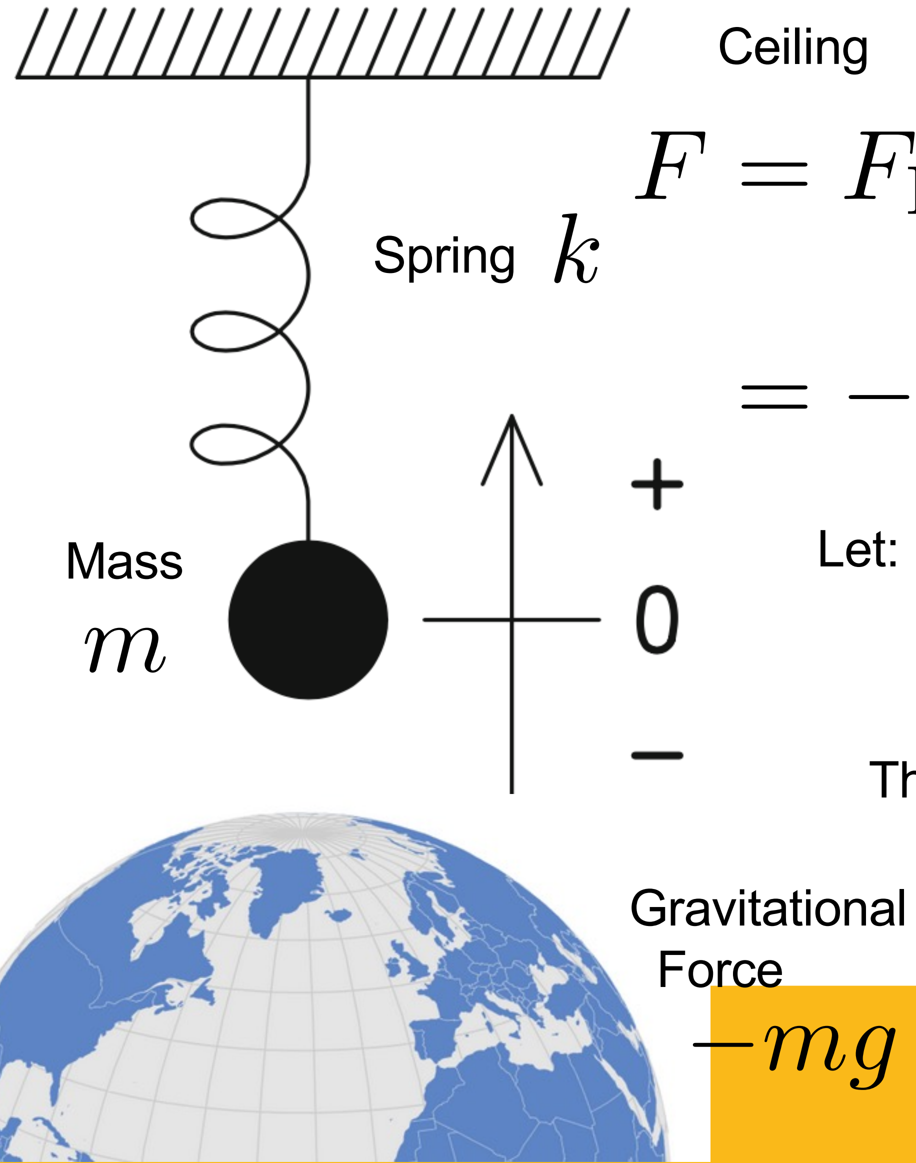
$$F = ma = m\frac{d^2x}{dt^2}$$

$$F_{\text{spring}} = -kx$$

Hooke's law

$$F_{\text{Earth}} = -mg$$

We need to understand vibration



$$F = F_{\text{Earth}} + F_{\text{spring}}$$

$$= -mg - kx = m\frac{d^2x}{dt^2}$$

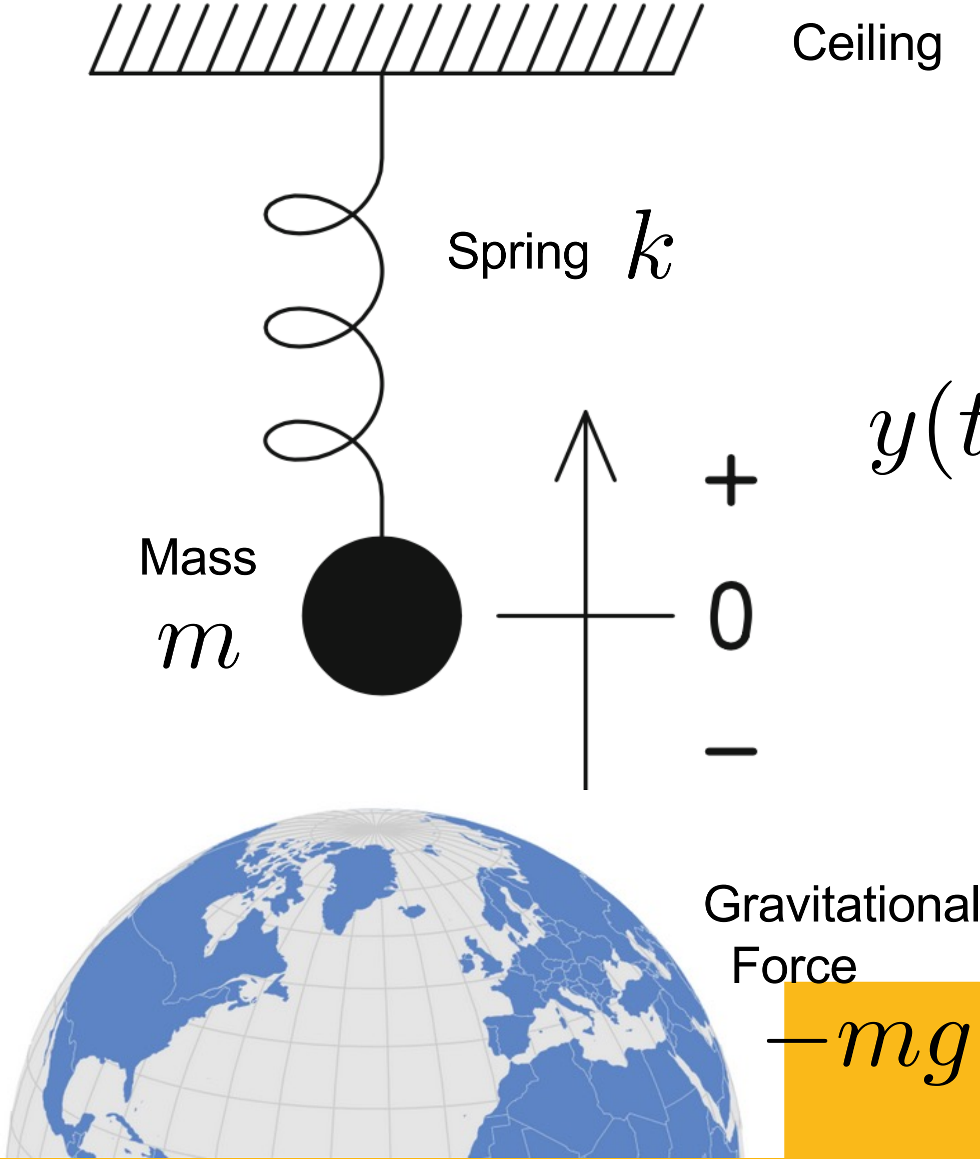
Let:

$$y = x + mg/k$$

Then:

$$m\frac{d^2y}{dt^2} = -ky$$

We need to understand vibration



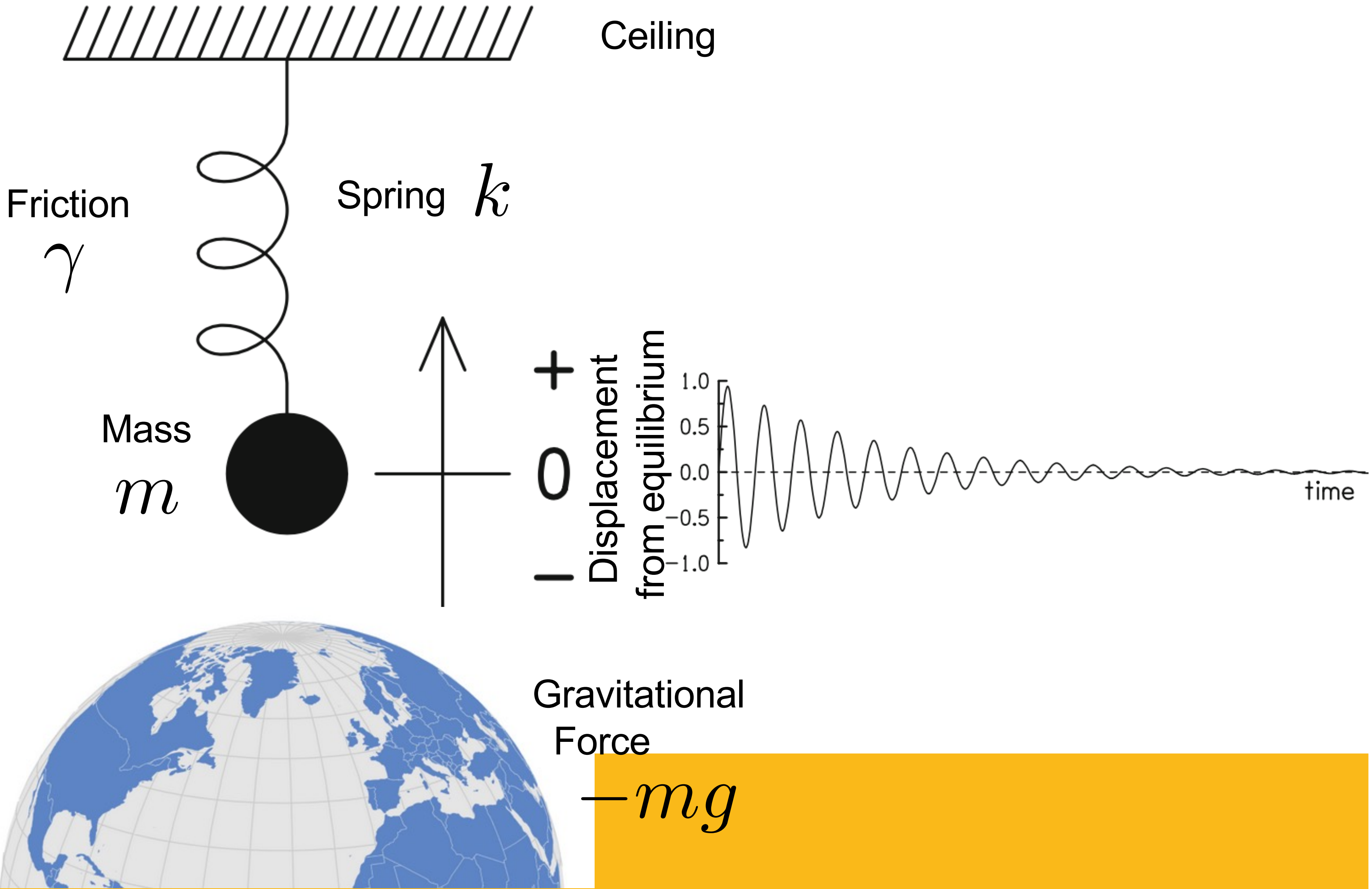
$$m\frac{d^2y}{dt^2} = -ky$$

$$y(t) = A\cos(\omega t + \phi)$$

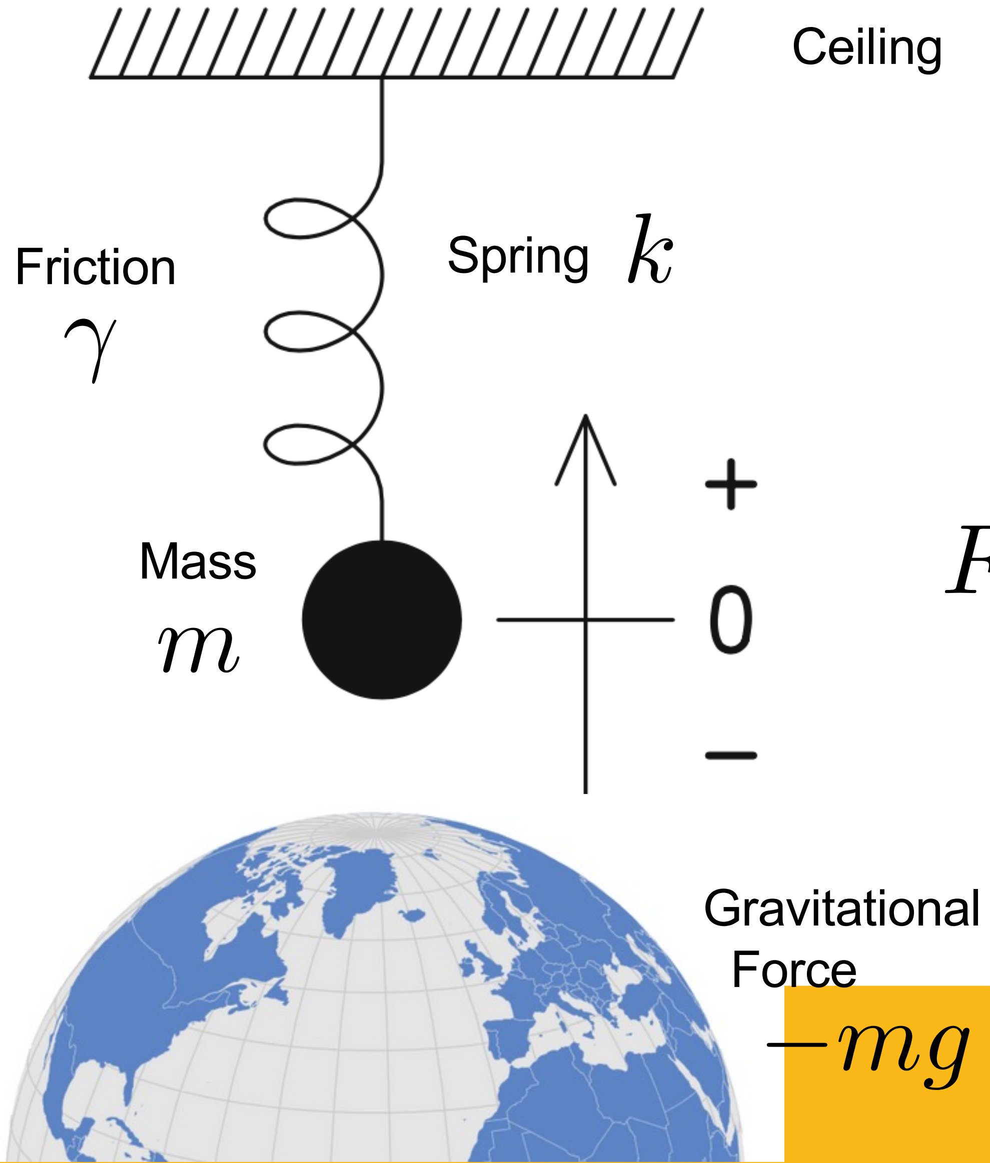
$$\omega = \sqrt{\frac{k}{m}}$$

A, ϕ depend on initial conditions

We need to understand vibration



We need to understand vibration

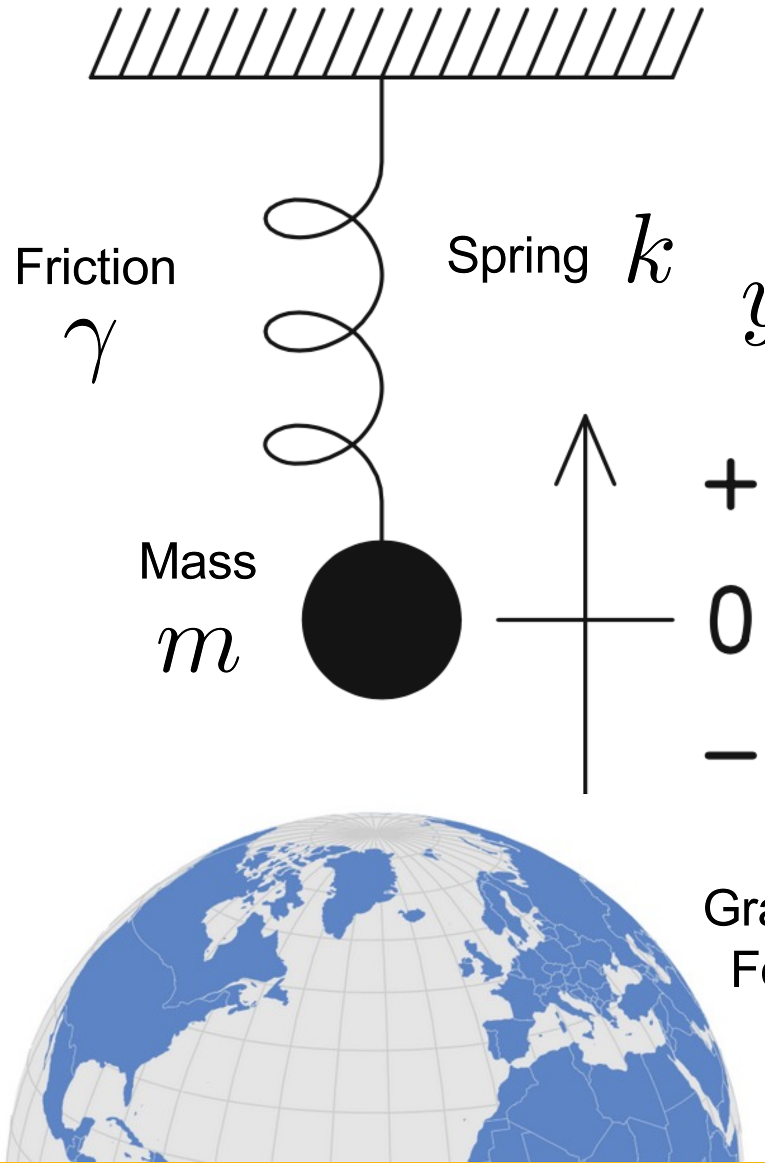


$$F_{\text{spring}} = -kx$$

$$F_{\text{Earth}} = -mg$$

$$F_{\text{friction}} = -\gamma \frac{dx}{dt}$$

We need to understand vibration

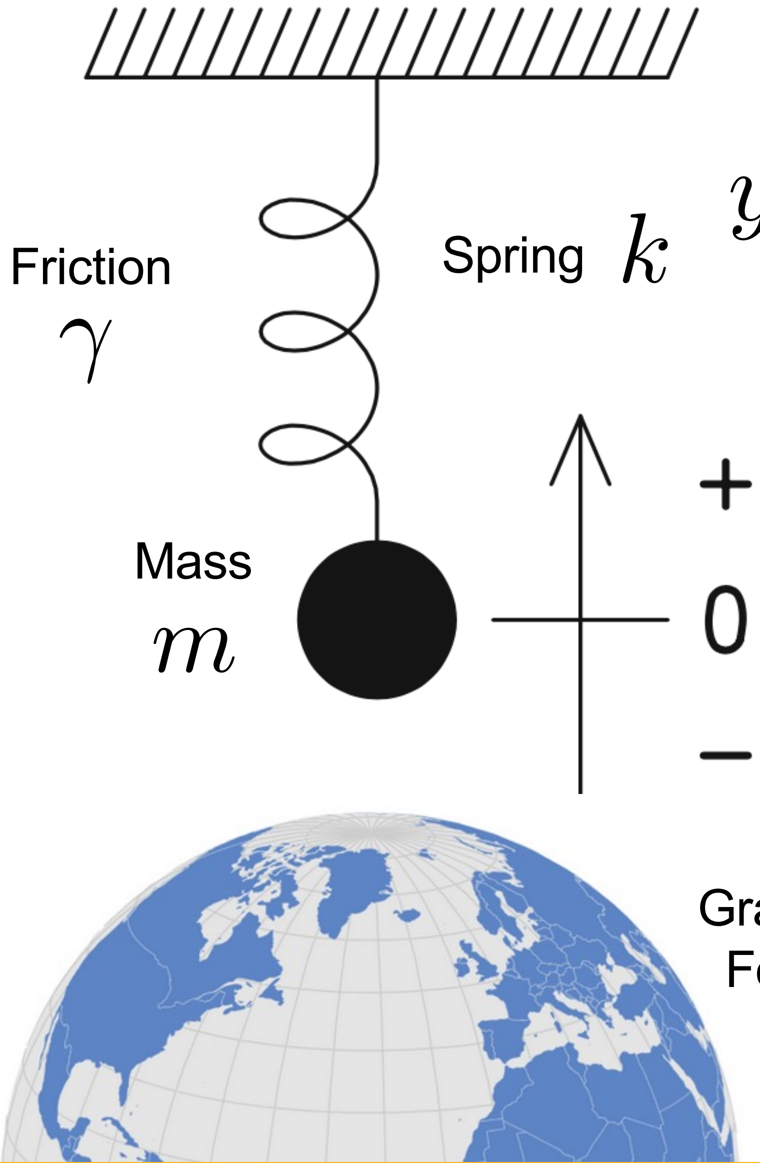


$$y(t) = A_\gamma(t)\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$A_\gamma(t), \phi$ depend on initial conditions

We need to understand vibration



$$y(t) = A_\gamma(t)\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

This system has one *natural mode*

- Up-down motion
- Frequency ω
- Damping γ

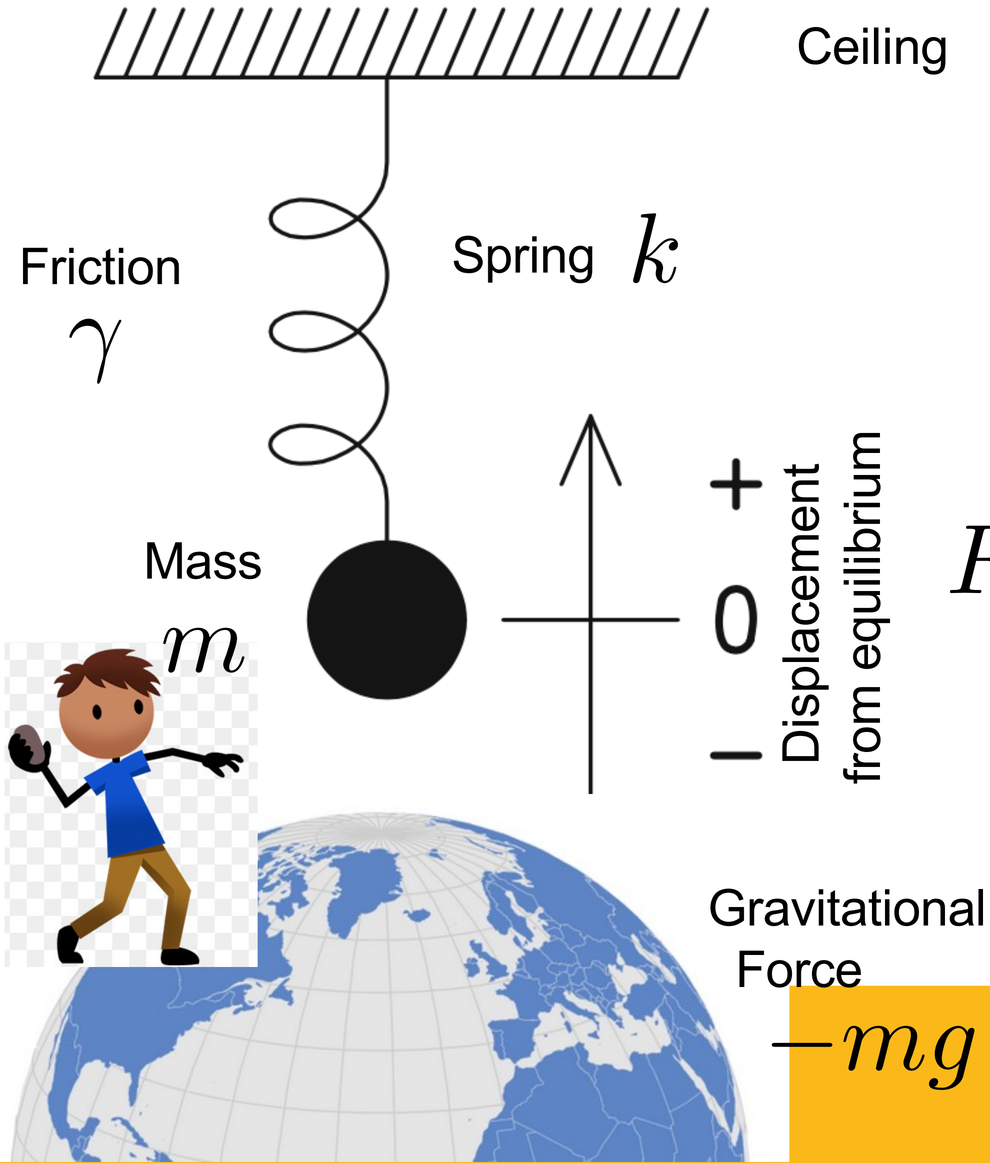
Significance of natural modes

- A system of coupled oscillators having N masses has N natural modes.
- Each natural mode is described by three things:
  - Shape
  - Frequency
  - Damping
- In any natural mode, each mass is undergoing simple harmonic motion (*sinusoidal*) — with some masses having zero amplitude → called a "node"
- *Any* motion of the system can be described as a linear combination (superposition) of the *natural modes* of the system!

Resonance

Resonance occurs when a natural mode of a mechanical system is driven by an external force.

We need to understand vibration



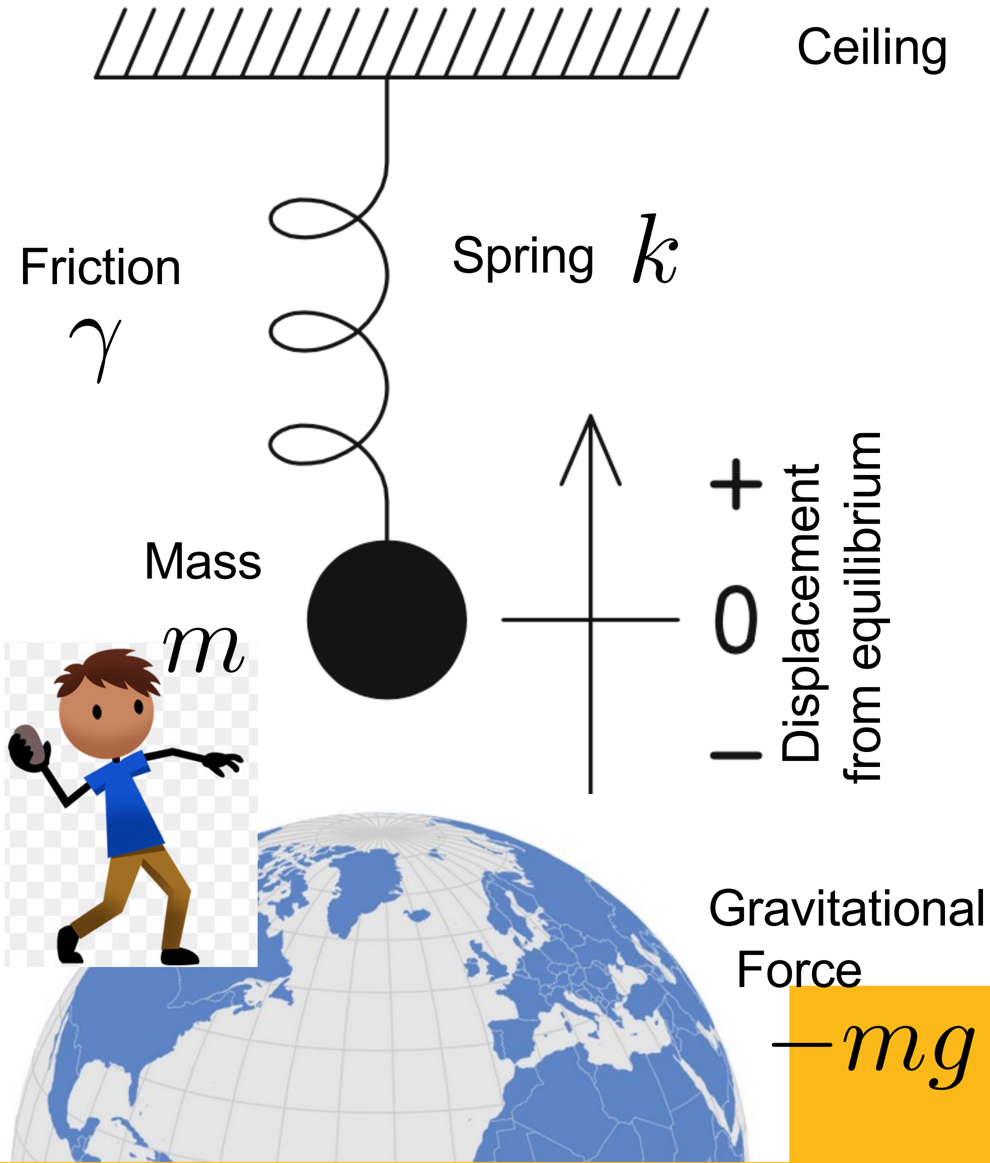
$$F_{\text{spring}} = -kx$$

$$F_{\text{Earth}} = -mg$$

$$F_{\text{friction}} = -\gamma \frac{dx}{dt}$$

$$F_{\text{unruly child}}(t)$$

We need to understand vibration



$$F_{\text{unruly child}}(t)$$

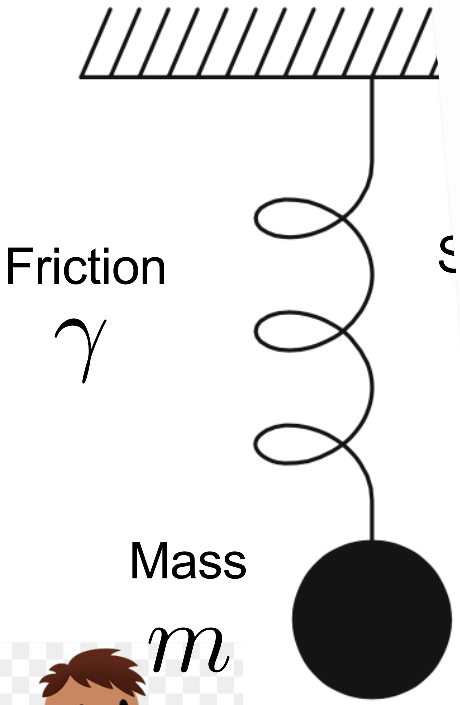
$$\omega = \sqrt{\frac{k}{m}}$$

If this unruly child continues to excite this system with a frequency equal to that of the natural mode:

D.I.S.A.S.T.E.R.

e.g., *Tacoma Narrows Bridge*
https://youtu.be/esfpcnQW6qs

We nee



kx

ng

$\frac{dx}{dt}$

$t)$

The mass, the spring, the damper, and an unruly child

Dr. Bob L. Sturm, Universitetlektor
School of Electronic Engineering and Computer Science
KTH, Sweden

January 16, 2024

A mass is hung vertically from a spring attached to a fixed and rigid board, with a damper that can be turned on or off. Denote the mass as m, the spring constant k, half damping coefficient γ, and time t. With the damper turned off, and no other forces acting on the system other than gravity and the restorative force of the spring, the differential equation describing the system is

$$m\frac{d^2x(t)}{dt^2} + kx(t) = 0. \tag{1}$$

When the damper is turned on, and there exist other forces acting on the mass, e.g., an unruly child, the differential equation describing the system is

$$m\frac{d^2x(t)}{dt^2} + 2\gamma\frac{dx(t)}{dt} + kx(t) = F_{\text{unruly child}}(t) \tag{2}$$

where $F_{\text{unruly child}}(t)$ is pretty much self-explanatory. We want to describe the motion of the mass under this variety of conditions.

1 Displacement and release of mass from rest without damping

Alone in the lab before any child comes for a visit, we pull the mass a distance A from equilibrium and release it from rest. The relevant differential equation and initial conditions are:

$$\frac{d^2x(t)}{dt^2} + \frac{k}{m}x(t) = 0; \qquad x(0) = A, \qquad \left.\frac{dx(t)}{dt}\right|_{t=0} = 0. \tag{3}$$

The form of the solution $x(t)$ is such that it looks like its second derivative. Since the exponential function has this property, let us pose the solution as

$$x(t) = ce^{\theta t}. \tag{4}$$

Taking the first and second derivative of this and plugging into (3) gives

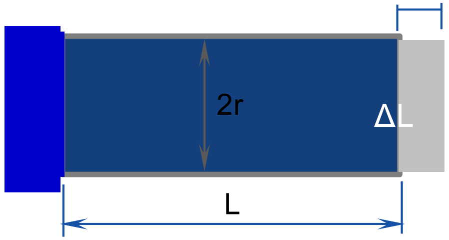
$$c\theta^2 e^{\theta t} + \frac{k}{m}ce^{\theta t} = c\left(\theta^2 + \frac{k}{m}\right)e^{\theta t} = 0. \tag{5}$$

Helmholtz Resonator

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{\pi\ r^2}{V\ L_{eff}}}$$

End correction *ΔL* due to a vibrating air plug at neck opening

$L_{eff} = L + \Delta L$



$$L_e = L + 0.85r \text{ (baffled end)}$$

$$L_e = L + 0.62r \text{ (cut end)}$$

Fletcher & Rossing pg 14

https://theonlinemetronome.com/free-online-tuner-for-musicians.html

Other Helmholtz Resonators

Ghatam

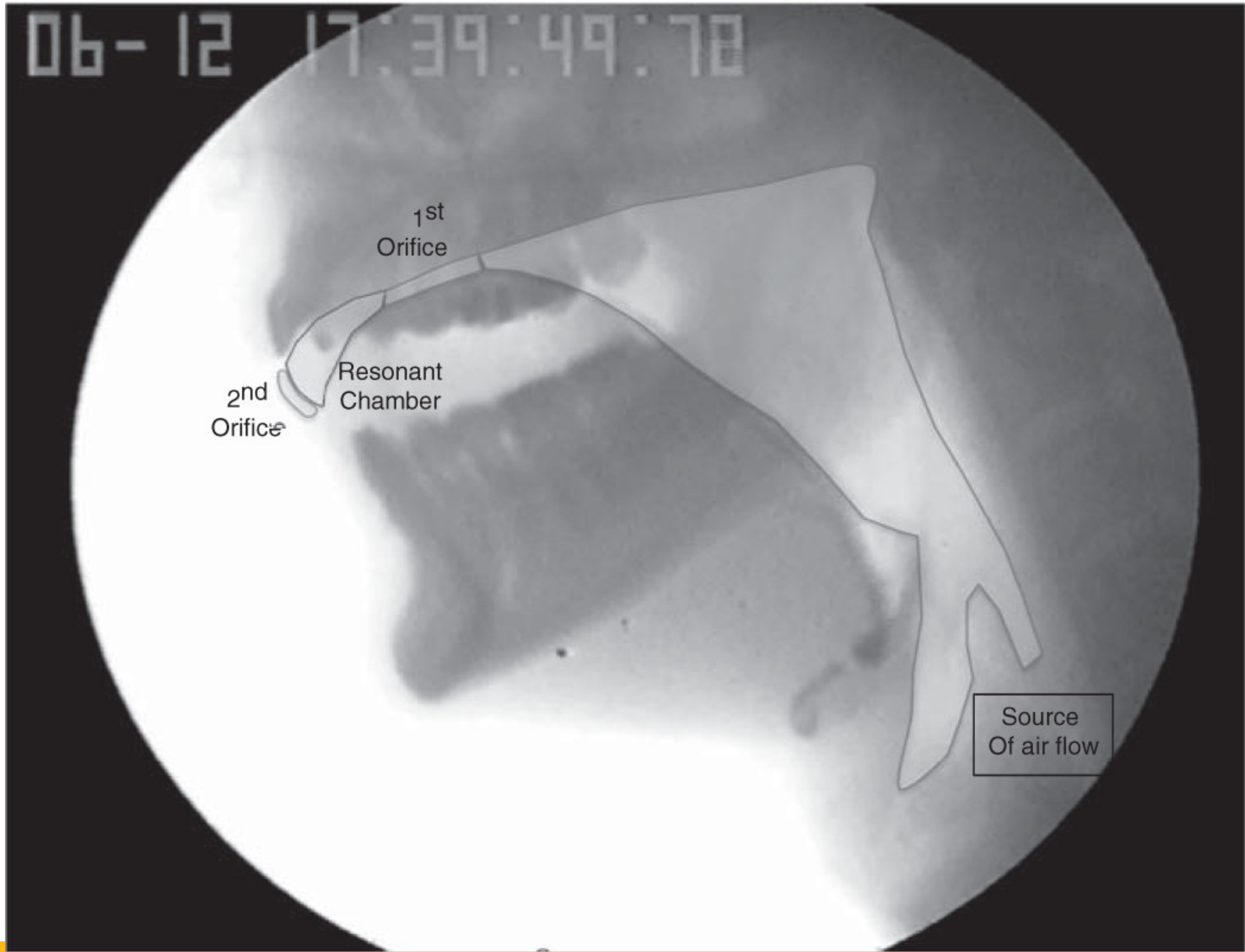
https://youtu.be/owirfUKg9eM

Other Helmholtz Resonators

Udu

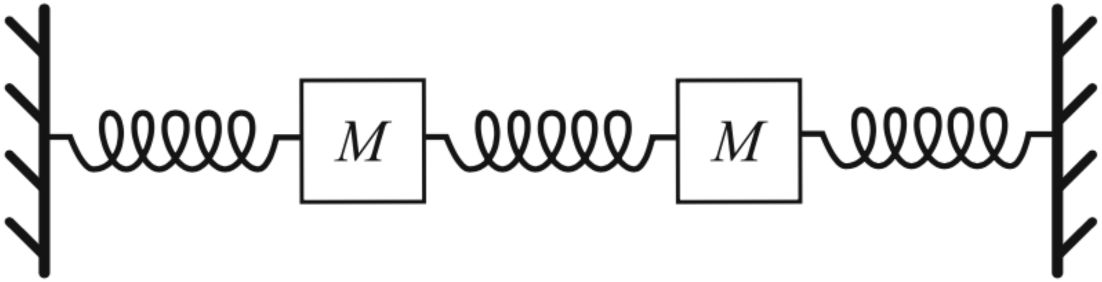
https://youtu.be/0ZUZaKUBs1M

Other Helmholtz Resonators



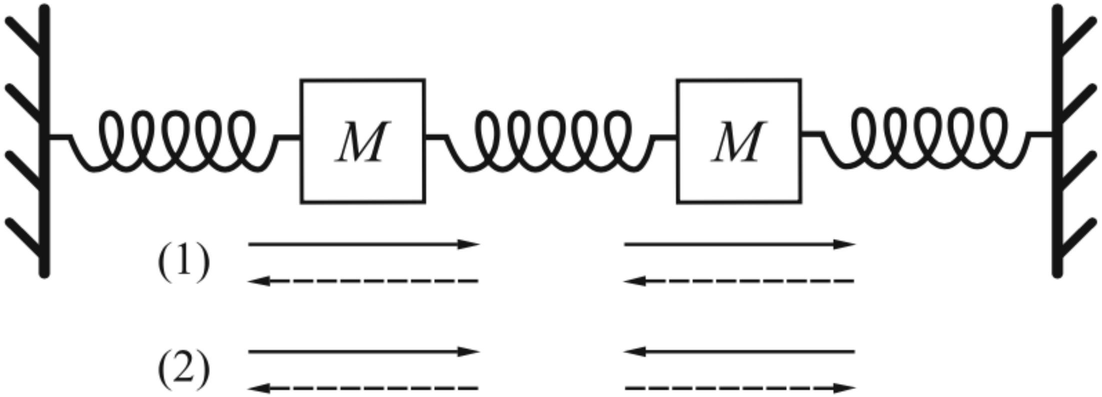
https://journals.physiology.org/doi/full/10.1152/japplphysiol.00902.2016

More complex vibration



How many modes does this system have?

More complex vibration



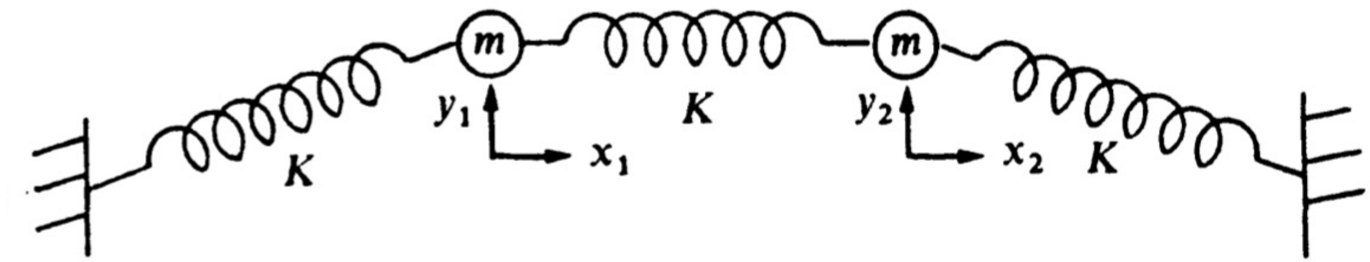
This *system* of two masses has two *modes* of vibration (allowing only horizontal motion).

What if there are *n* masses and n+1 springs?

What happens when $n \rightarrow \infty$?

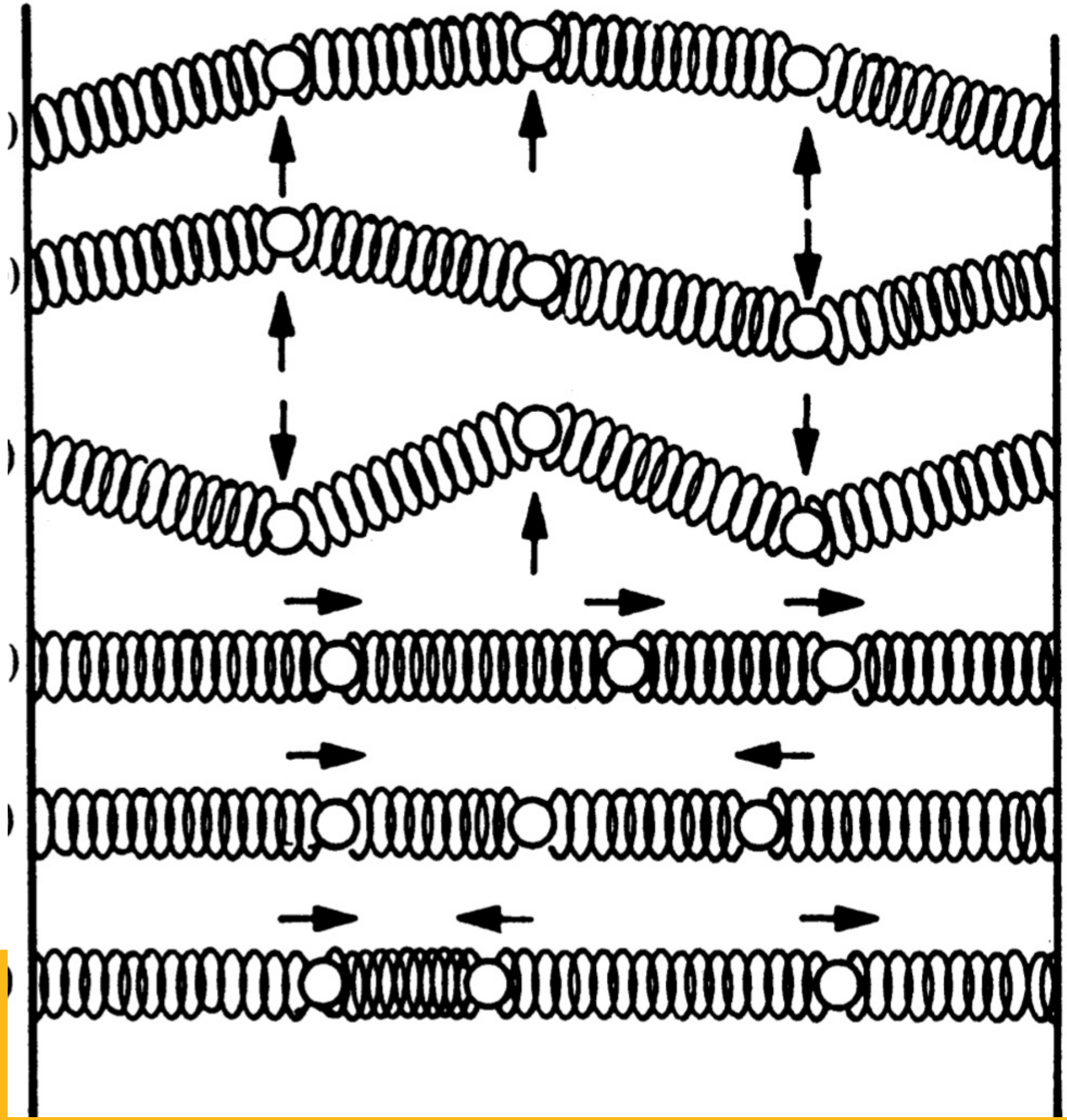
Demo: http://www.falstad.com/coupled

More complex vibration

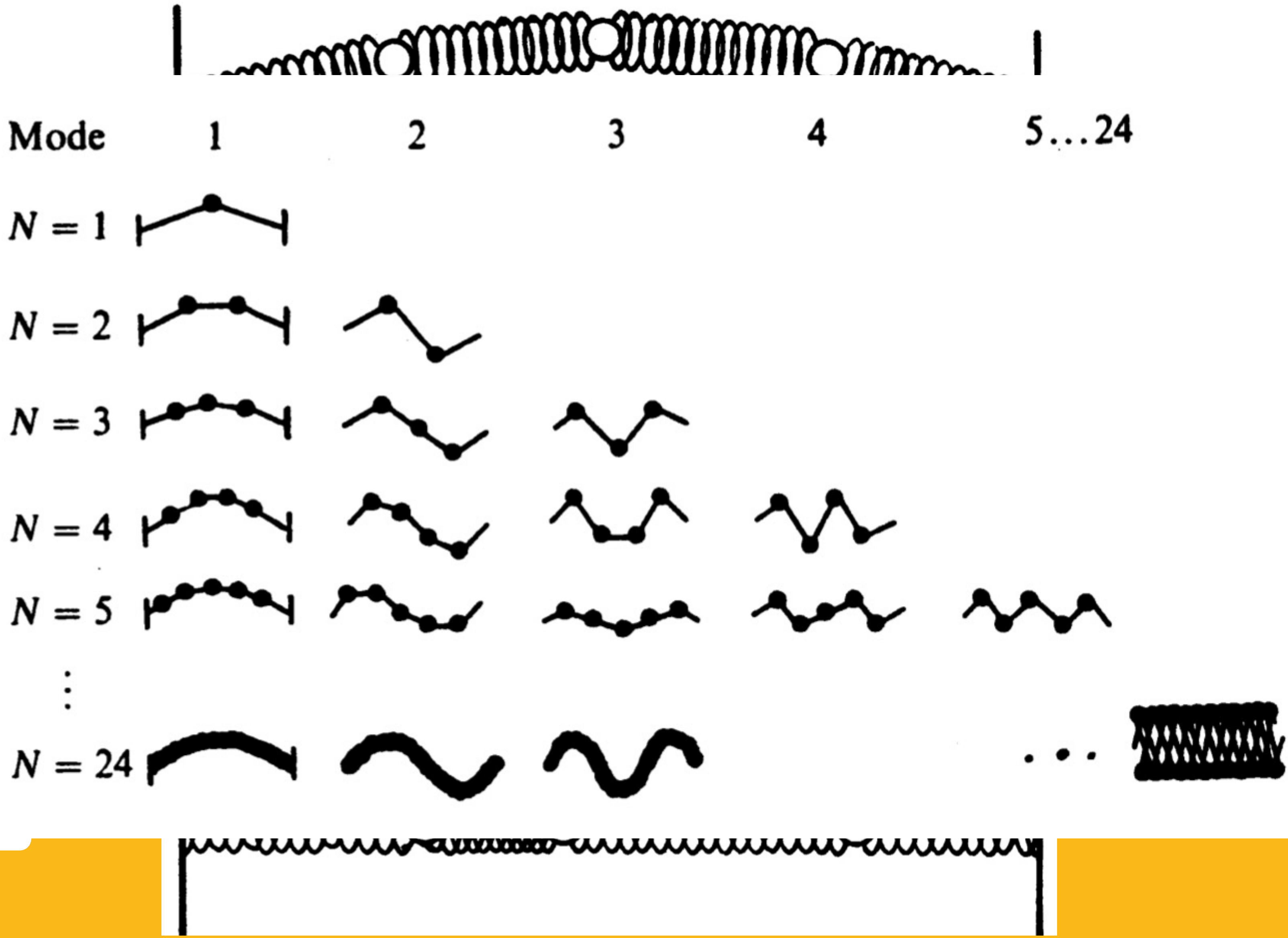


How many modes does this system have?

More complex vibration

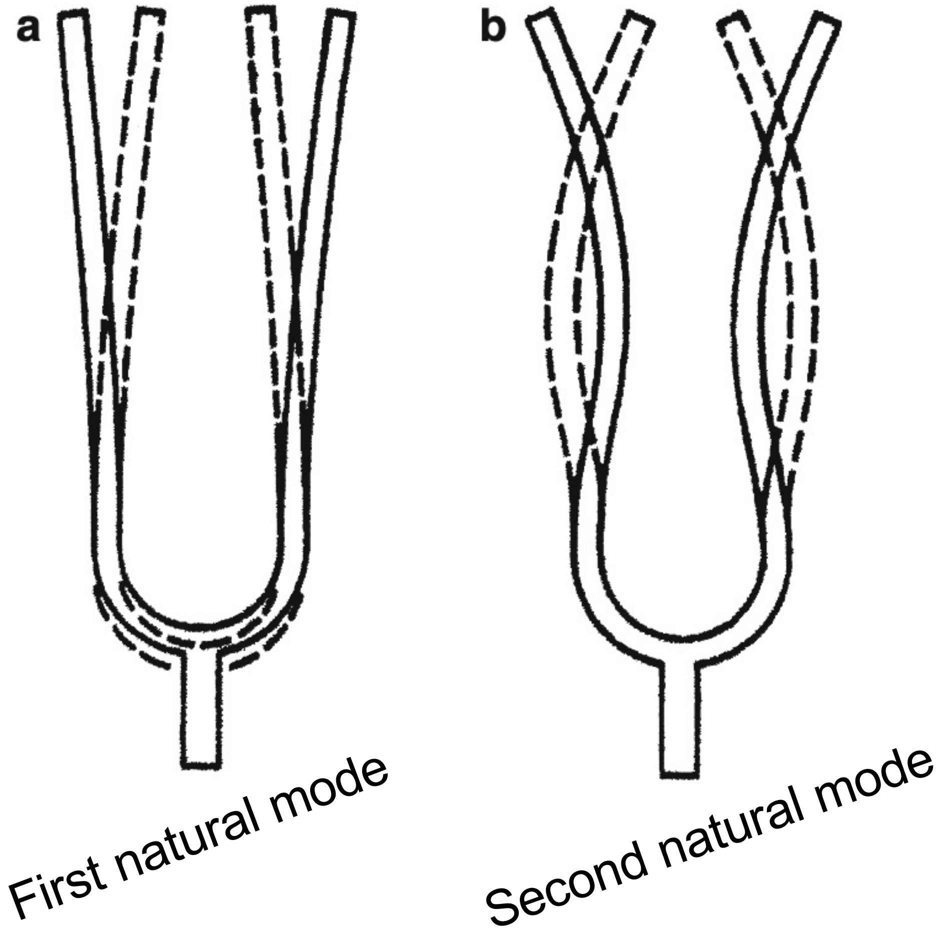


More complex vibration

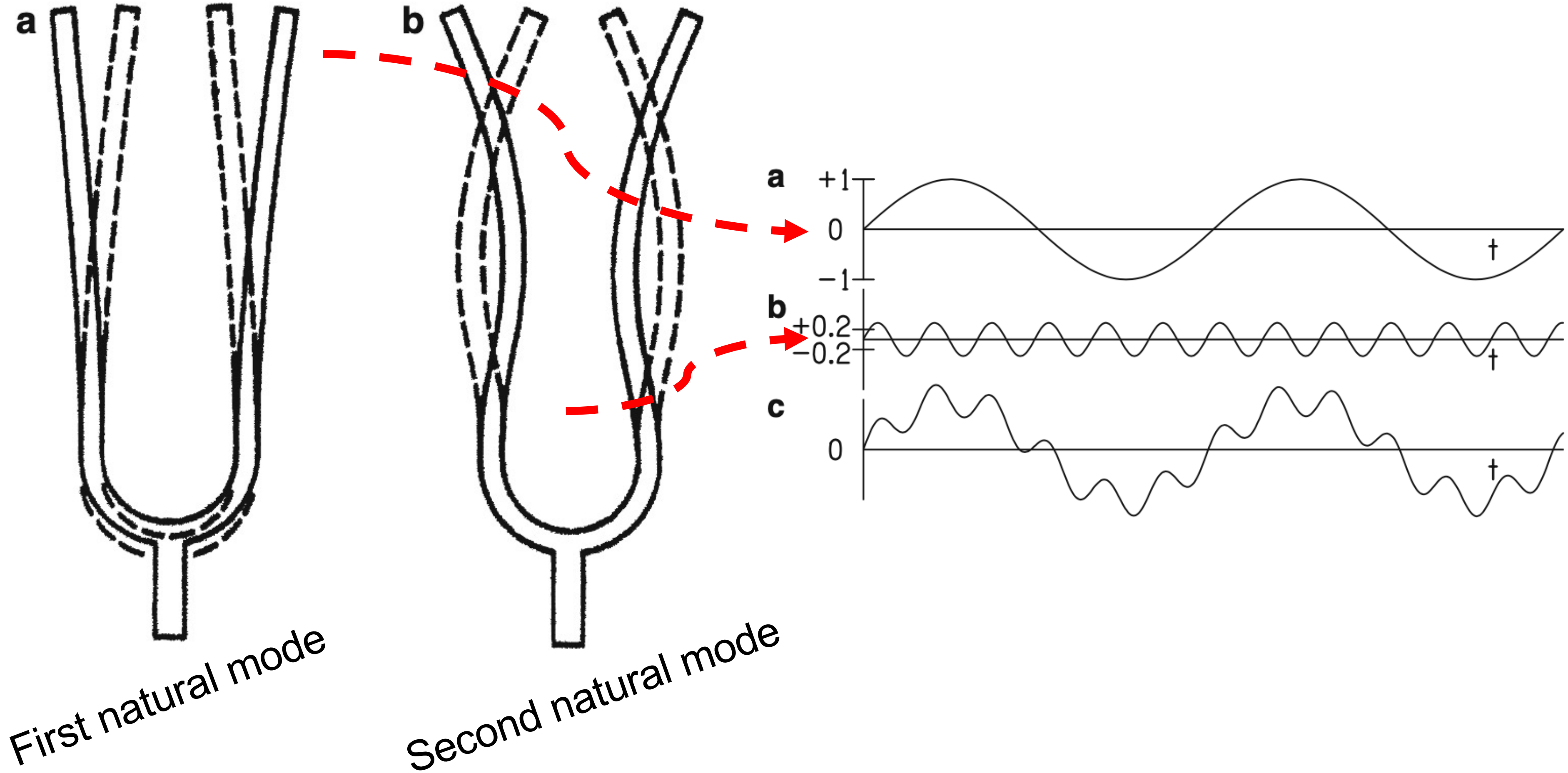


The tuning fork

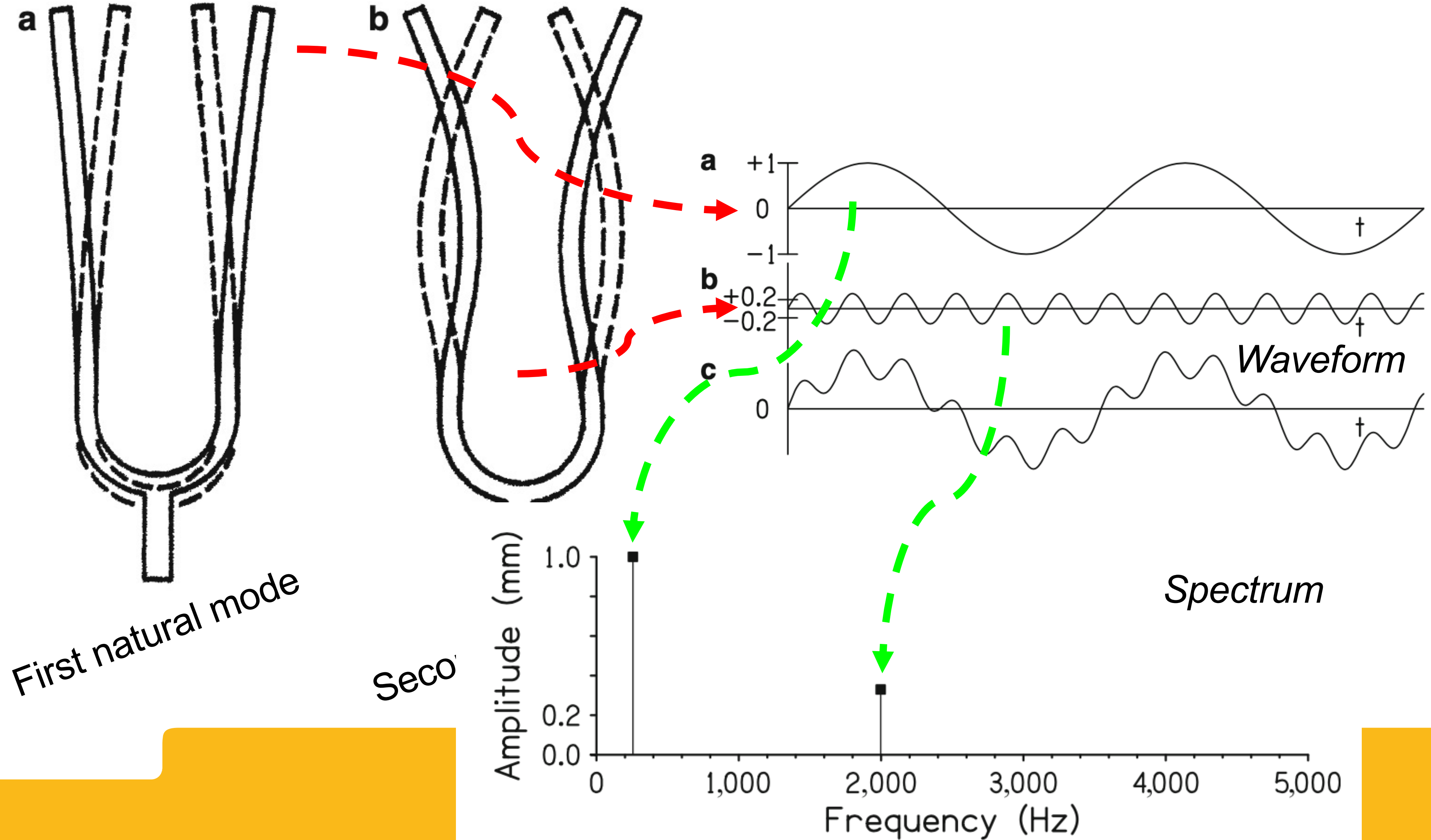
The tuning fork



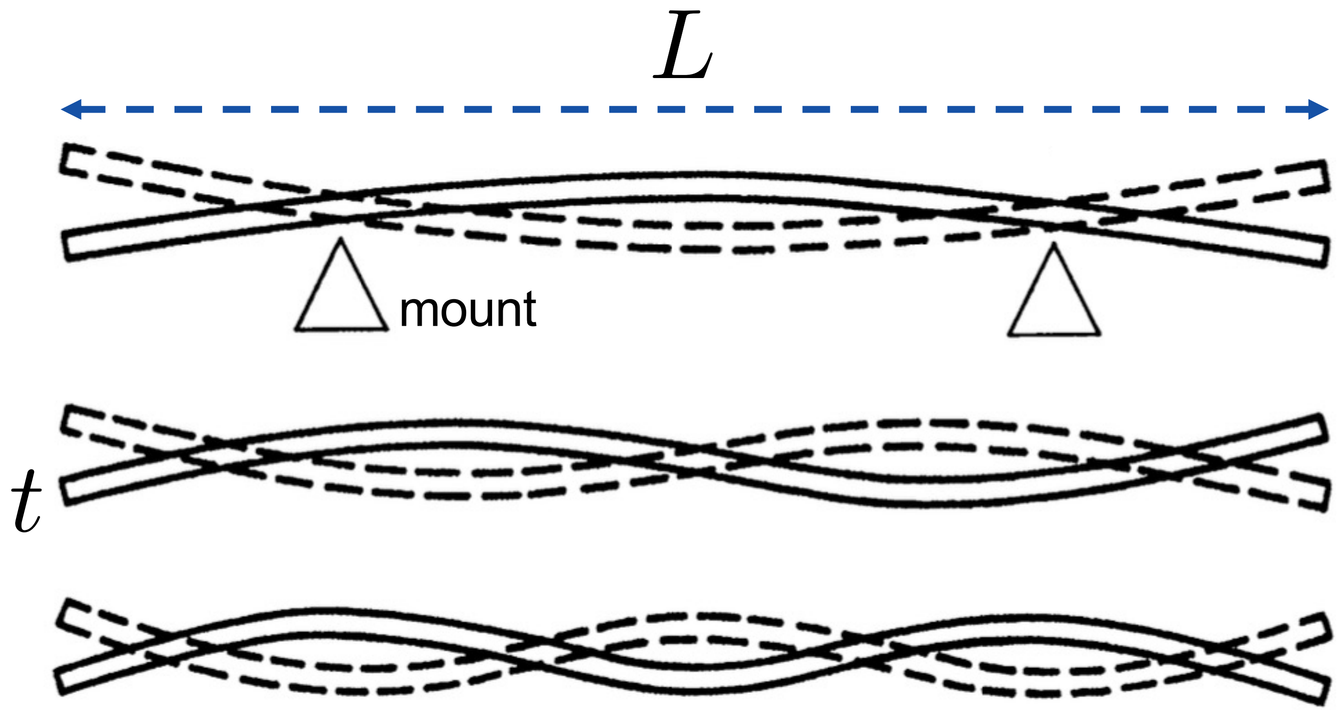
The tuning fork



The tuning fork



The vibrating bar (free at endpoints)



mode	relative frequency
1	f_o
2	$2.76\ f_o$
3	$5.4\ f_o$

Frequency of mode n: *Hartmann (pg. 270)*

$$f = 0.113 \frac{v_L\, t}{L^2} (2n+1)^2$$

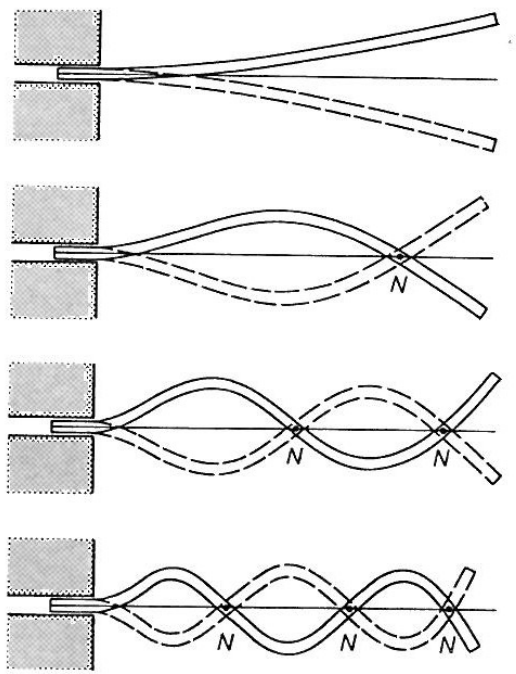
Demo: http://www.falstad.com/barwaves

v_L is speed of sound in the bar

Table 6.1. Elastic properties of materials [5]

Material	Density $\rho(\mathrm{kg/m^3})$	Young's modulus $E(\mathrm{N/m^2})$		Sound velocity $v(\mathrm{m/s})$		Reference
Aluminum	2700	7.1×10^{10}		5150		Kinsler et al., 1982
Brass	8500	10.4×10^{10}		3500		
Copper	8900	10.4×10^{10}		3700		
Steel	7700	19.5×10^{10}		5050		
Glass	2300	6.2×10^{10}		5200		
Wood		$\parallel$ grain	$\perp$ grain	$\parallel$ grain	$\perp$ grain	
Brazilian rosewood	830	1.6×10^{10}	2.8×10^{9}	4400	1800	Haines, 1979
Indian rosewood	740	1.2×10^{10}	1.7×10^{9}	4000	1500	
African mahogany	550	1.2×10^{10}	1.2×10^{9}	5000	1500	
European maple	640	1.0×10^{10}	2.2×10^{9}	4000	1800	
Redwood	380	0.95×10^{10}	0.96×10^{9}	5000	1600	
Sitka spruce	470	1.3×10^{10}	1.3×10^{9}	5200	1700	

The vibrating bar (fixed at one end)



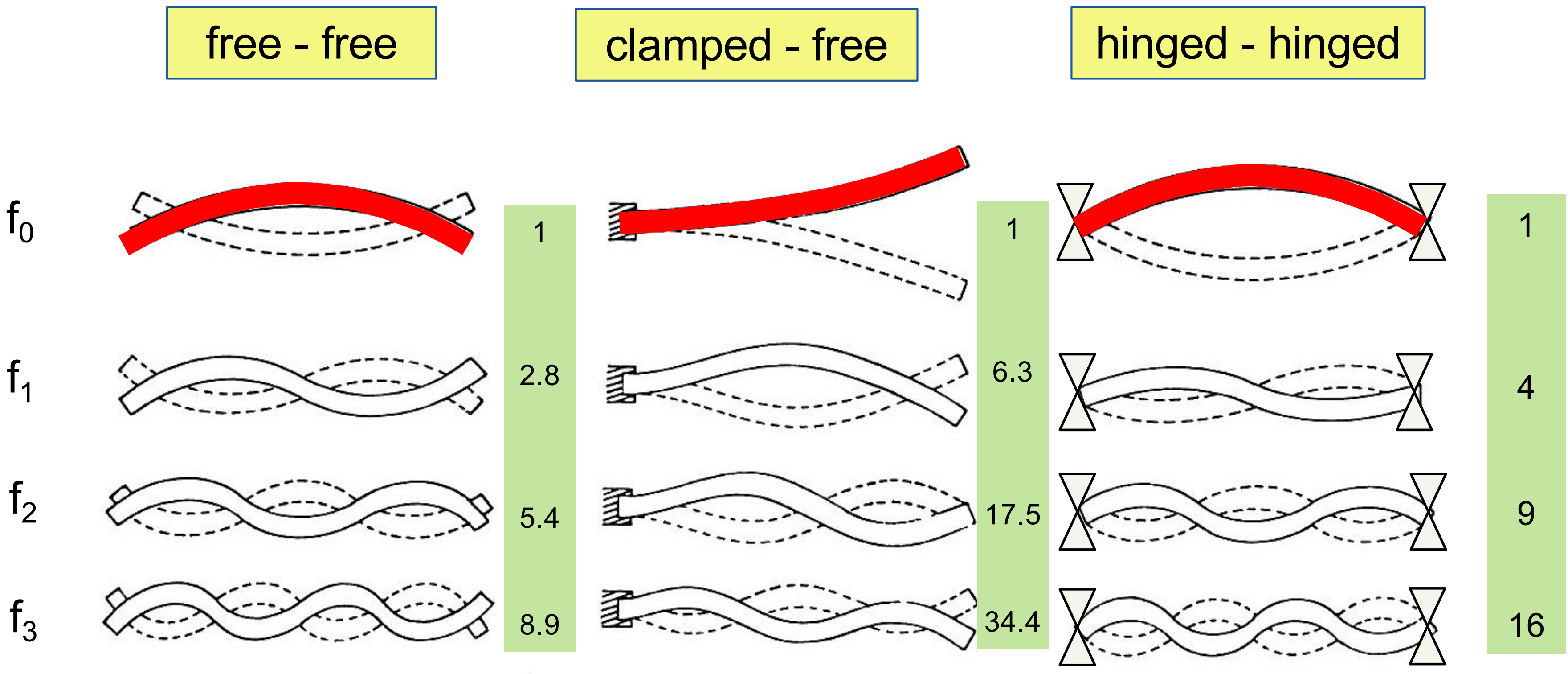
mode	relative frequency
1	f_o
2	$6.3\ f_o$
3	$17.6\ f_o$
4	$34.4\ f_o$

The mode shapes are not sinusoids

inharmonic mode frequencies

Demo: http://www.falstad.com/barwaves

The vibrating bar



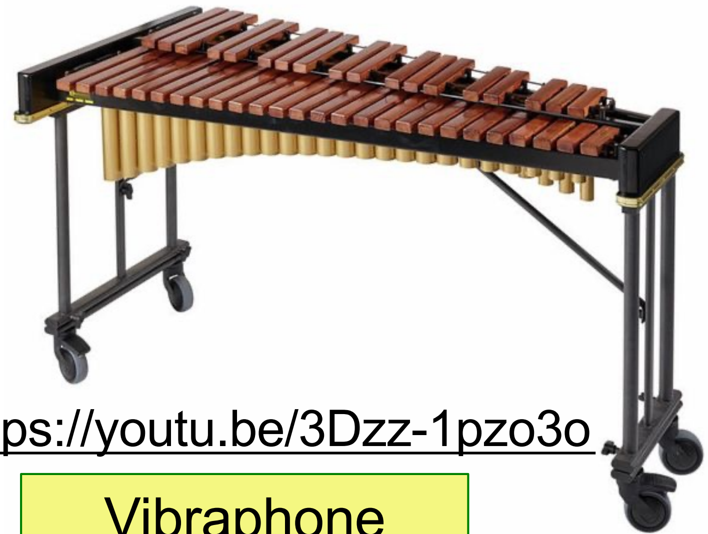
This is the only one used in percussion instruments

Instruments with vibrating bars

Marimba

https://youtu.be/5YkK-cHnw8k

Xylophone



https://youtu.be/3Dzz-1pzo3o

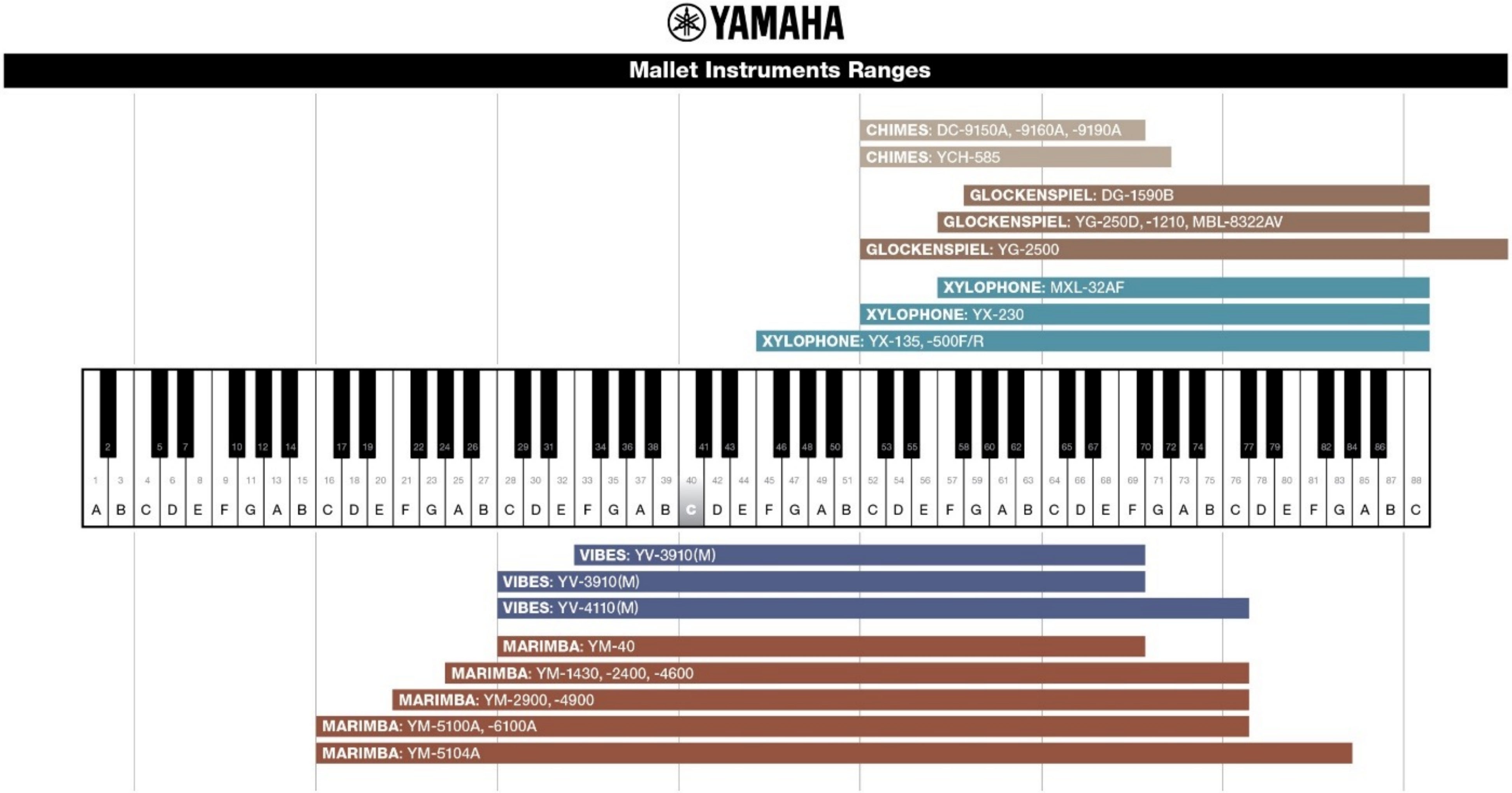
Glockenspiel

https://youtu.be/CyHBkwPguP4

Vibraphone

https://youtu.be/SWi5EGygvEc

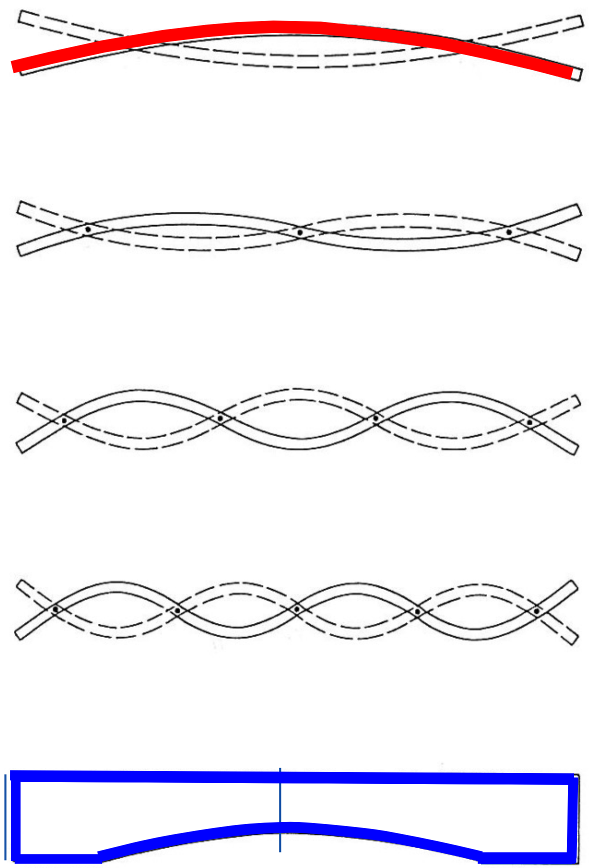
Instrument ranges



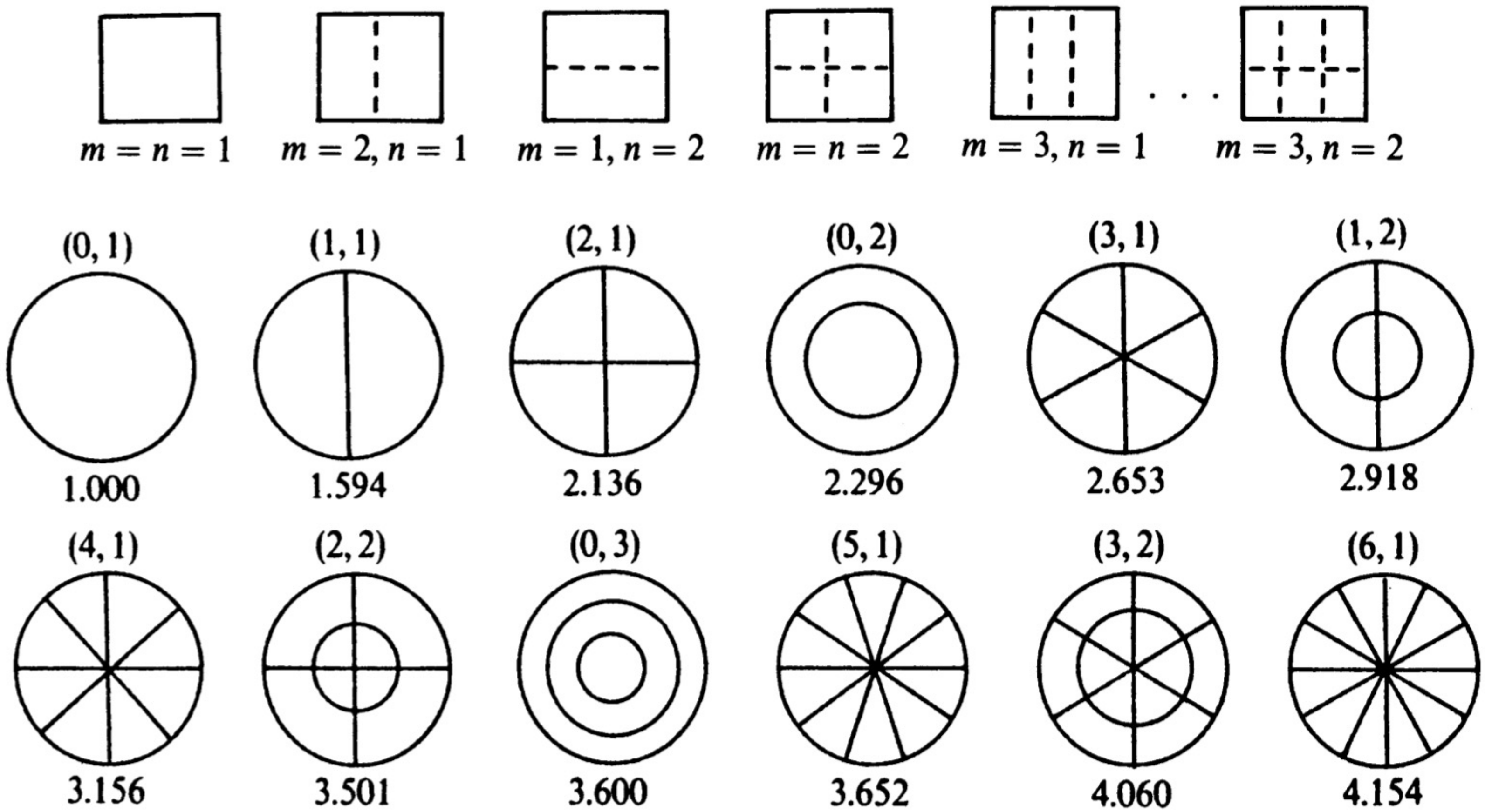
https://hub.yamaha.com/drums/percussion/whats-the-difference-between-marimba-and-xylophone/

The marimba ”trick”: Undercut

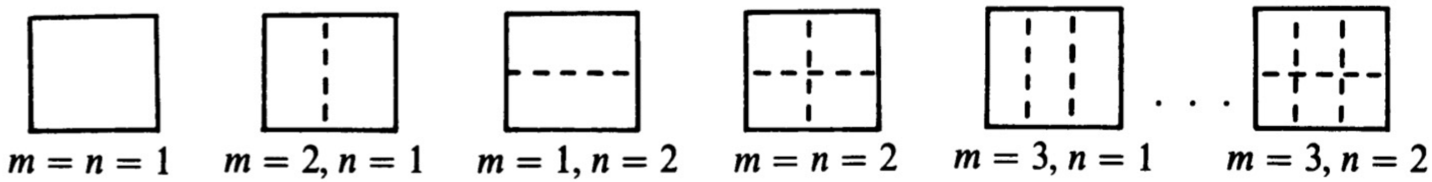
Mode frequencies before undercut		Mode frequencies with undercut
1		1
2.8		≈ 4
5.4		≈ 10
8.9		≈ 20

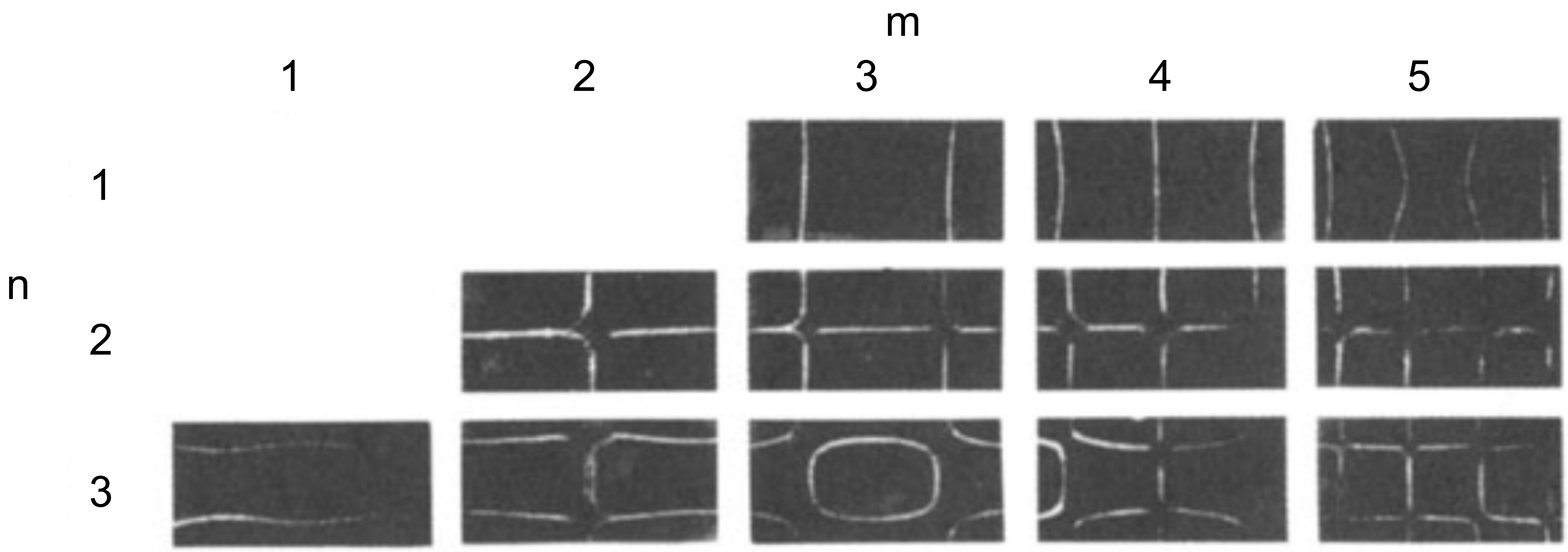


Modes of ideal membranes



Modes of ideal membranes

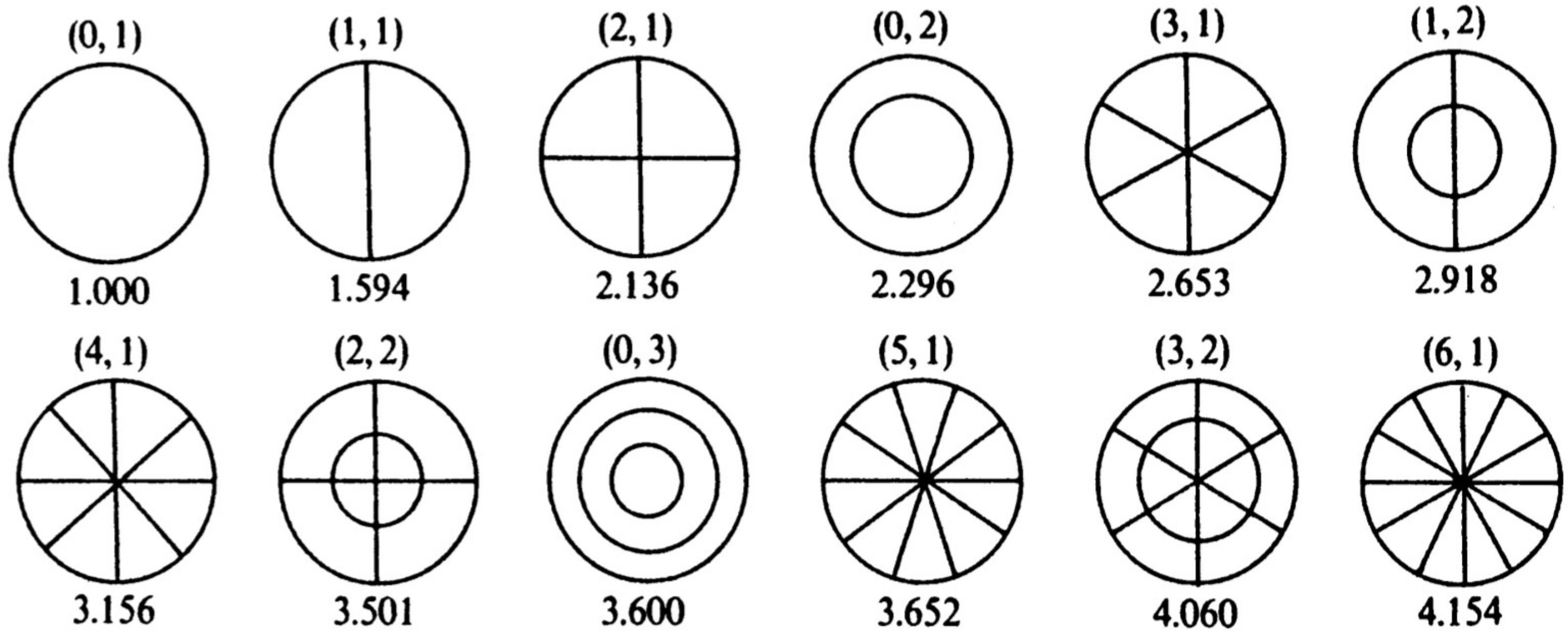




Fletcher and Rossing pg. 73

https://youtu.be/wMIvAsZvBiw

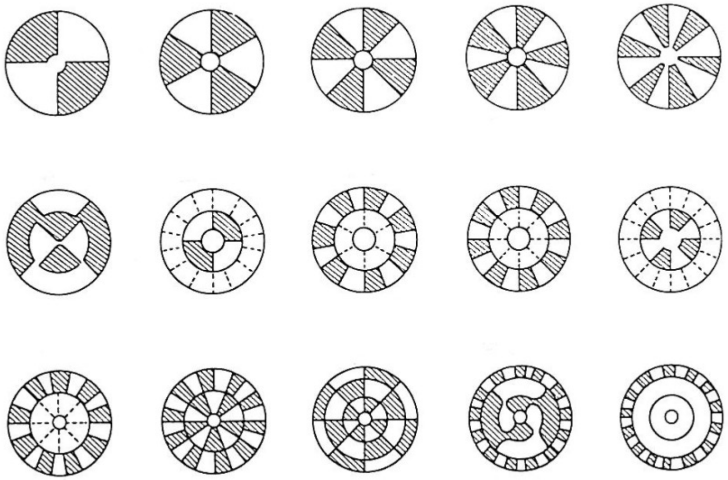
Modes of ideal membranes



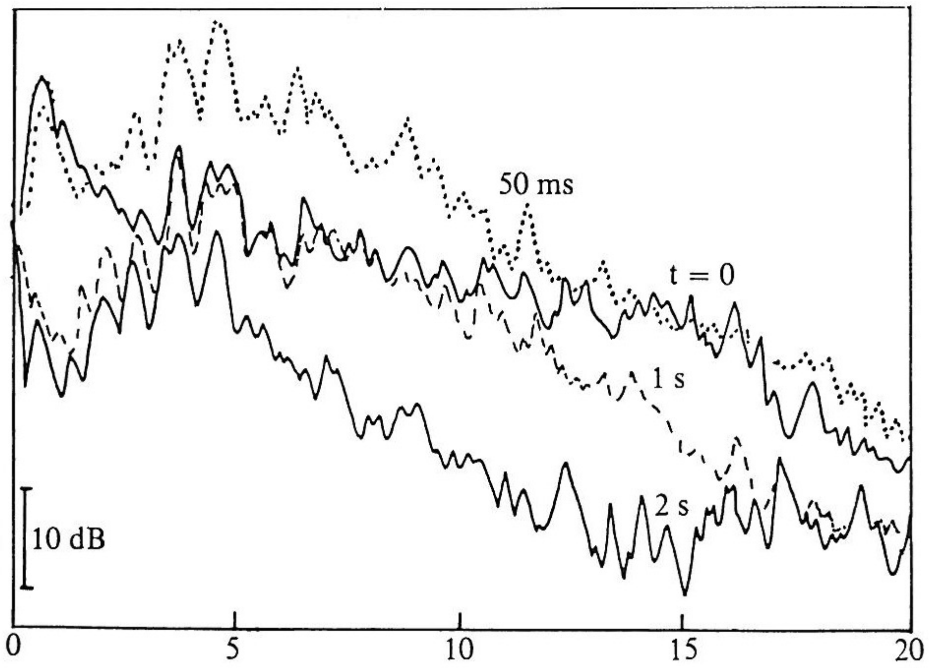
(m,n) = (# diametrical nodes, # elliptical nodes)

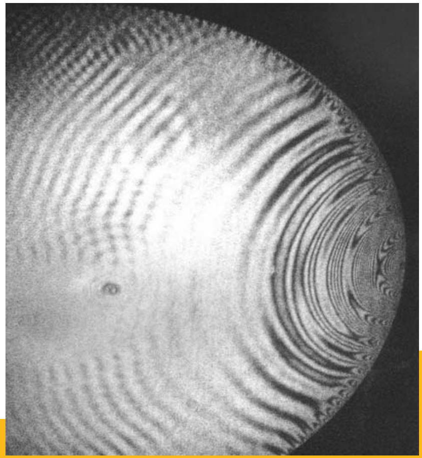
Cymbals

Modal shapes



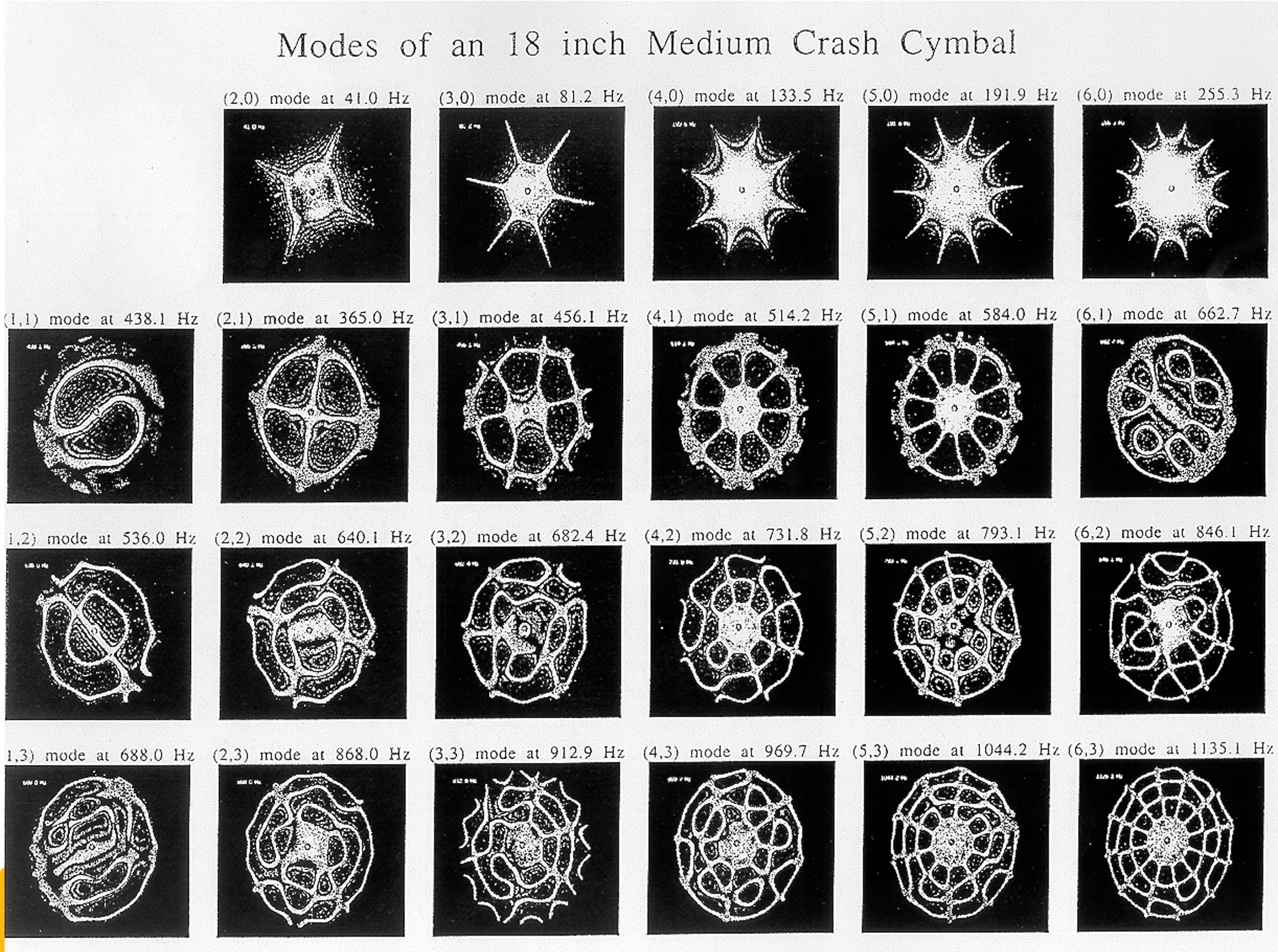
Spectra at different times



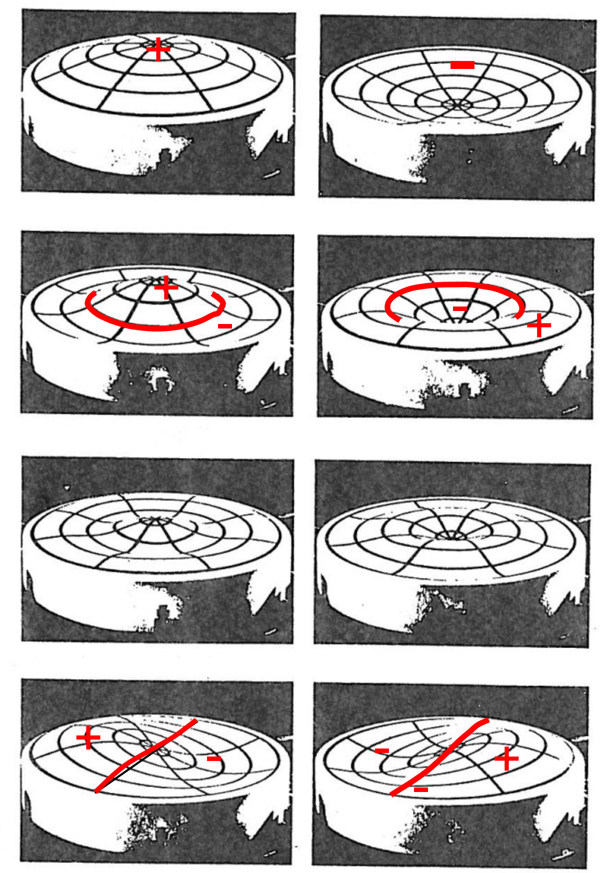


Propagating bending waves of an impacted, tinkling cymbal at 240 μs after impact start, recorded using pulsed holographic Interferometry. Molin&Zipser2004

Chladni patterns on a cymbal



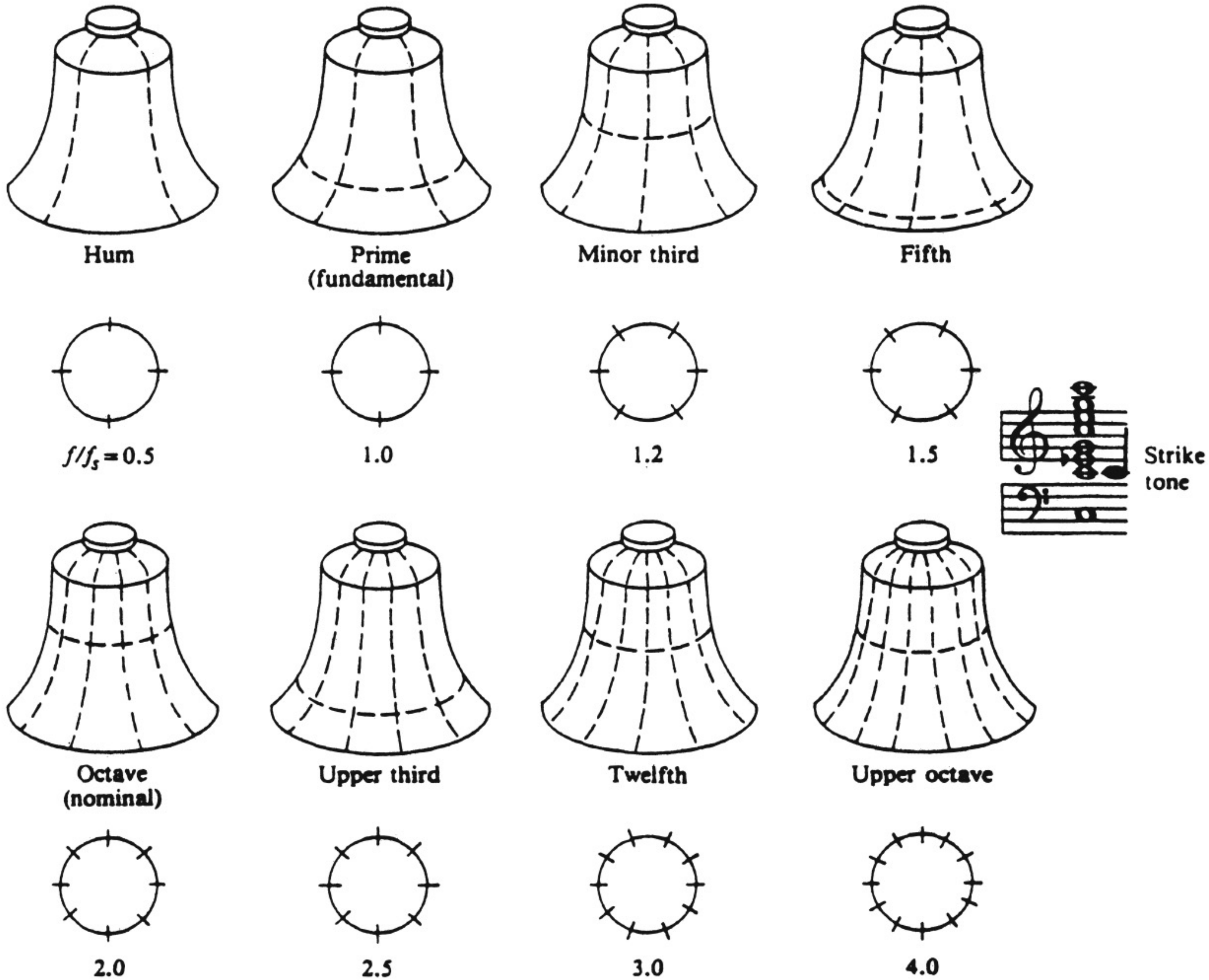
The vibrating membrane (fixed at edge)



mode	frequency
1	f_o
2	$2.3\ f_o$
3	$3.6\ f_o$
4	$1.6\ f_o$

Demo: http://www.falstad.com/circosc

The vibrating bell



Summary

- The response of any dynamic system to an excitation can be expressed as a linear combination (a.k.a., *superposition*) of its natural modes.
- A natural mode is characterized by:
  1. modal shape (vibration pattern)
  2. modal frequency (resonance frequency)
  3. damping (the decay of the mode)
- These are determined by the distribution of mass in the system and the stiffness.

Summary

- We can describe modal frequencies in relation to the frequency of the *fundamental* mode f_0.
- If all modal frequencies are related to f_0 by an integer multiple (*harmonic*), the motion of the system will always be periodic, independently of how it is excited.
- There are only two vibrating systems with harmonic modal frequencies:
  1. Strings (without stiffness)
  2. Pipes (with constant or conically varying cross section)

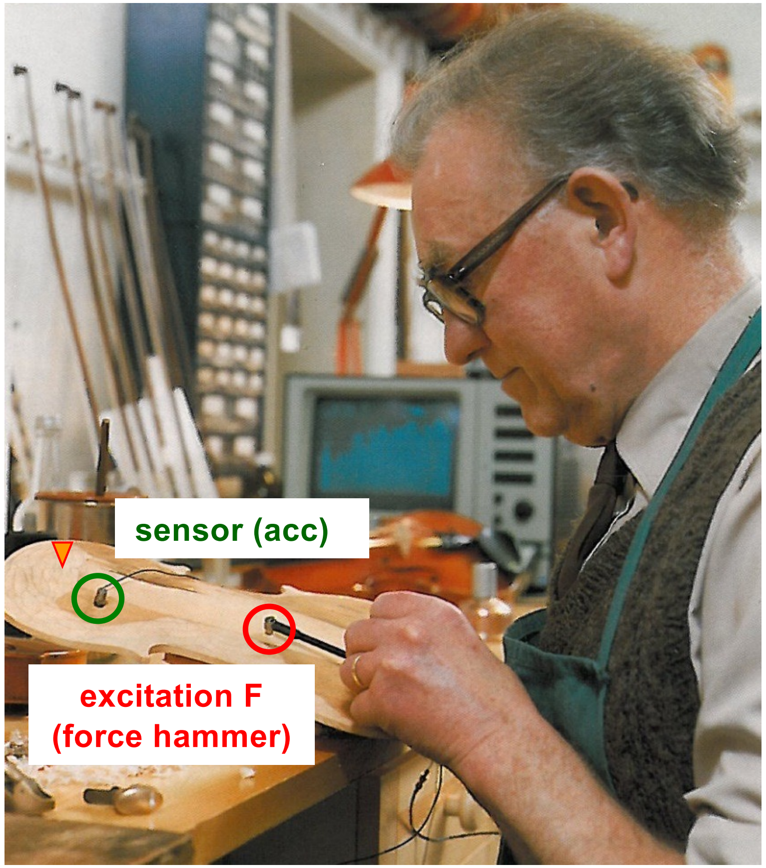
How do we determine the modes?

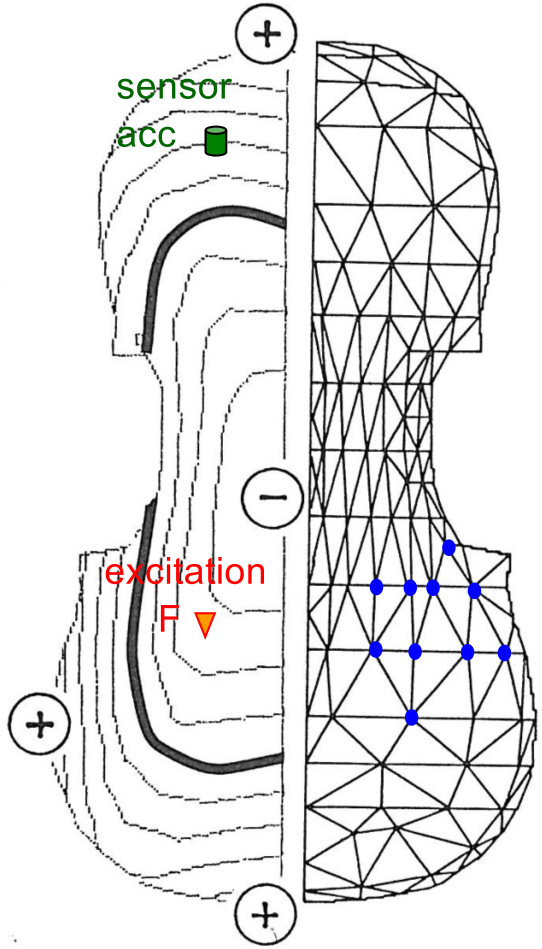
- One way is by deriving a mathematical model of the system and derive the values from theory.
  - This is in all but the simplest cases difficult to impossible!
- Another is by constructing a digital model of the entire system with masses, springs and dampers, and then simulating it (finite element modeling, or FEM).
  - This is easier, but requires computational power!

How do we determine the modes?

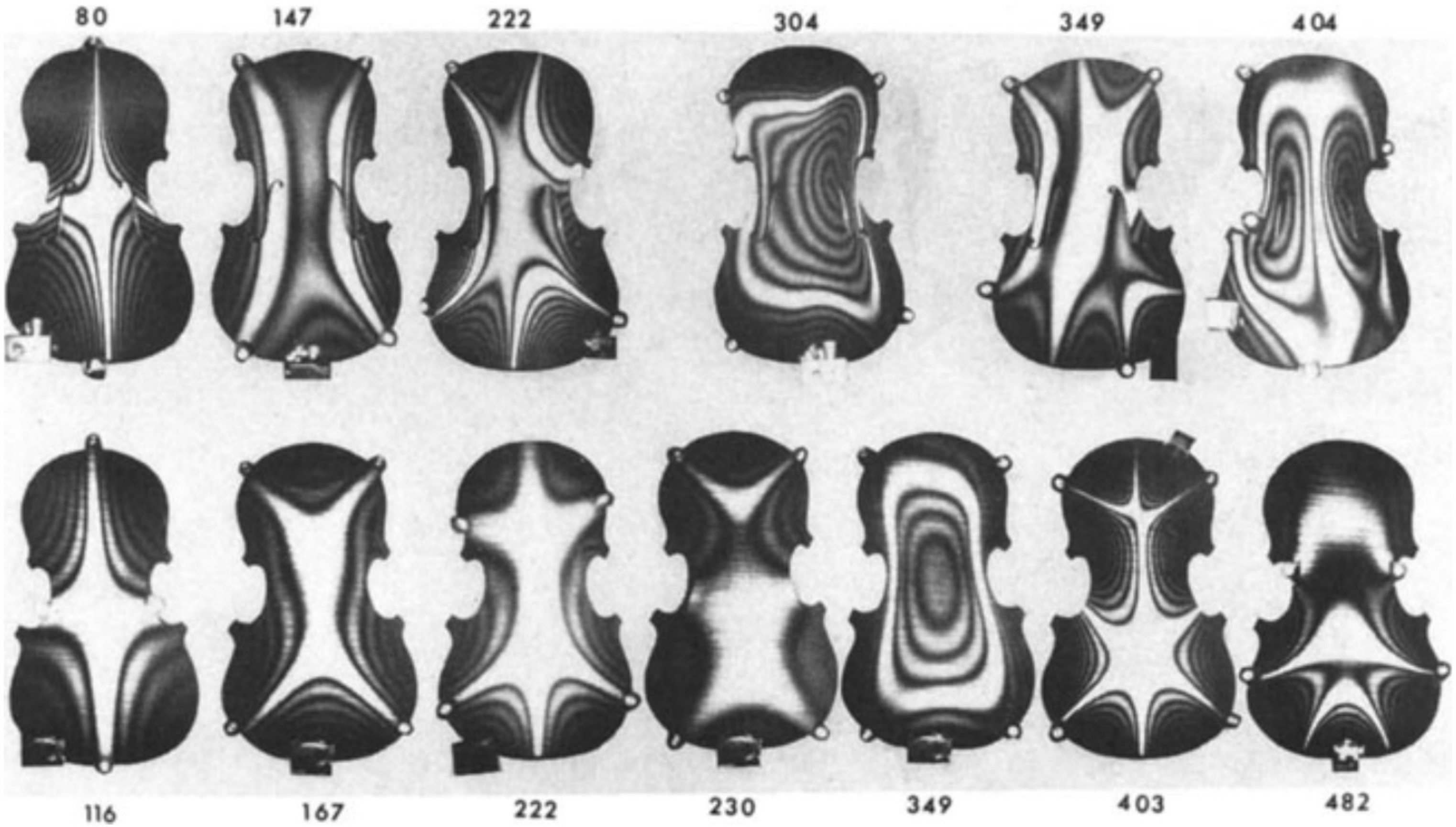
- Another way is by “instrumentation”!
  - Ie. Experimentation and measurement
- *Think*: What is a mode?
  - A way that a system ”likes” to move.
  - Let’s excite the system with different frequencies and watch for ones that make a bigger response than others (resonances).

How do we determine the modes?





How do we determine the modes?



Fletcher and Rossing, pg. 293

Before next lecture

Read:

1. Hartmann Chapters 11–14