KTH Datavetenskap
och kommunikation

DT2212 Music Acoustics

Lab I

GUITAR

Prepare for the lab carefully by

1. Reading
   - excerpt from Fletcher & Rossing: The Physics of Musical Instruments, Ch. 9 Guitars
   - handout 'Strings' by Askenfelt

2. Study this lab-PM and solve the preparatory problems so you understand what to do.

Name: Bartosz Brzuchalski, Yilai Chen (Group 9) Date: 06/02/2025

Preparatory exercises ☐ OK

Lab ☐ OK

Lab. ass. signature:………………………

The guitar

Eine kleine Einführung

The classical guitar consists of a body, a neck with fingerboard, and six strings. The frets on the fingerboard are used to adjust the effective length of each string, which in turn determines the pitches we perceive. A plucked string vibrates freely with a combination of its vibration modes and exerts a vibratory force on the bridge. The overtones form an almost harmonic series due to the low inharmonicity of guitar strings (low stiffness).

The vibrations of the body consist of a combination of vibration modes driven by the force of the vibrating string at the bridge. The amplitudes of the body modes are determined by (1) how close its frequency is to that of an overtone, and (2) how close the excitation point at the bridge lies to a nodal line of that particular mode. It follows that the excitation of the body differs significantly depending on which pitch is played. The overtone frequencies change much between notes (the smallest change is 6%, corresponding to a semitone step), while the mode frequencies of the body remain fixed as well as the position of the nodal lines. They are inherent properties of the guitar body.

A body mode with high vibration amplitude does not necessarily need to be prominent in the radiated sound. The efficiency in converting the vibrations into radiated sound differs much between modes. The main parameter in determining the radiation efficiency is the net change in volume of displaced air during a vibration period. It may well happen that a particular mode has a large vibration amplitude but the contribution to the sound radiation and perceived sound is relatively small, and vice versa.

What to do

In this lab you will study the main body modes of a guitar by modifying (perturbing) the frequencies and amplitudes of the modes one by one. In this way it is possible to estimate how a particular mode contributes to the overall sound and timbre.

You will encounter some of the difficulties in relating measured physical properties of an instrument (vibrations and sound) to perceived features like timbre (tone quality). Some of the main design properties of the guitar will be changed in steps, documented by measurements, and evaluated by test playing and listening.

You will also experiment with a rigidly terminated string (monochord), and test how the physical and perceptual parameters change depending on the plucking process.

Reflection

The acoustical instruments have developed over long time periods. From a musical perspective, they have in a sense been brought to perfection in their roles as sound-producing devices. Their development, driven by trial and error over long periods of time, has in several respects reached an end. Their capabilities are exploited optimally by professional musicians. From an engineering perspective, a paradox becomes evident when studying these 'optimized' musical instruments. A small change in a certain construction parameter may give a large perceived change, while another change hardly influences anything at all perceptually. The connection between physical/acoustical properties of a musical instrument and perception is not at all self-evident. It is challenging task to make a specification of instrument quality in physical terms.

Lab

A. String physics
String spectrum for different excitations
Inharmonicity of guitar & piano strings

B. Guitar body modes
Mobility and sound radiation
Perturbation of Helmholtz resonance and top plate modes
Effective mass

C. Perceptual estimates
Excitation by plucking
Perturbations of guitar body

Preparatory exercises

0. Read about guitars in the excerpt from Fletcher & Rossing: *The Physics of Musical Instruments*. Read about strings in the handout Strings. Both readings are available in Canvas.

A. String physics

a) Calculate the string tension (static force on the bridge) for each string *and* the total force. Assume the angle of each string into the base of the bridge to be 30 degrees.

String	**E_4** homogen	**H_3** homogen	**G_3** homogen	**D_3** wrapped	**A_2** wrapped	**E_2** wrapped
Fundamental freq. (Hz)	330	247	196	147	110	82.4
Diameter (mm)	0.70	0.83	1.03	0.75	0.93	1.07
Linear density (g/m)	0.417	0.593	0.892	2.04	3.45	5.33
Spec. admittans $1/Z_0$ (m/Ns)	5.59	5.25	4.33	2.56	1.97	1.97

The string length of the lab guitar is 648 mm.

b) The just noticeable difference (JND) in pitch is about 6 cent for a melodic interval.
- What is the corresponding change in tension for the high E_4 string if its length were to stay constant? Answer in % and N.
- What is the corresponding change in string length if its tension were to stay constant? Answer in % and mm.

c) The low E_2 string is wrapped. What would the diameter need to be if it was homogenous and made of the same material as the three top strings?
What would be the pros and cons of a homogenous string?

d) Sketch the spectrum for the string mode amplitudes u_n of a plucked string according to Eq. (2) below for a β of your own choice

B. Body modes - original and perturbed.

a) The guitar body encloses an air volume, which is 'breathing' through the sound hole. This gives rise to a vibration mode by combination of the compliance of the air volume and the mass (inertia) of the air plug in the sound hole (Helmholtz resonance). The resonance frequency is just above 100 Hz for guitars of normal size.

Compute the frequency of the Helmholtz resonance ('air mode') A0 for the lab guitar

$$f_{A0} = \frac{c}{2\pi}\sqrt{\frac{S}{\ell_e V}} \qquad (1)$$

where V is the body volume (12.6 dm^3), S is the area of the sound hole (diameter 88 mm), l_e is the effective length of the sound hole and c is the speed of sound (assume room temperature). Note that the effective length requires some attention. A certain amount of air just outside the sound hole moves passively in synchrony with the air plug in the hole (both at the outer and

A.

a) $f_0 = \frac{1}{2L}\sqrt{\frac{S}{\rho_\ell}} \rightarrow S = (2Lf_0)^2 \rho_\ell$, $F_{tr} = S \sin\alpha$:

E_4:	$S = 76{,}27\,N$	$F_{tr} = 38{,}13\,N$
H_3:	$S = 60{,}77\,N$	$F_{tr} = 30{,}38\,N$
G_3	$S = 57{,}55\,N$	$F_{tr} = 28{,}78\,N$
D_3	$S = 74{,}04\,N$,	$F_{tr} = 37{,}02\,N$
A_2	$S = 70{,}11\,N$	$F_{tr} = 35{,}06\,N$
E_2	$S = 60{,}78\,N$	$F_{tr} = 30{,}39\,N$

b) 6 cents : $f_2 / f_1 = 1{,}003472$

E_4: $f_{0,1} = 330 \cdot 1{,}003472 = 331{,}14567\,Hz$

$f_0 = \frac{1}{2L}\sqrt{\frac{S}{\rho_\ell}} \rightarrow S = 4 f_0^2 L^2 \rho_\ell$, $L = \frac{1}{2f_0}\sqrt{\frac{S}{\rho_\ell}}$

$S_{330} = 76{,}27\,N$, $S_{331{,}1} = 76{,}80\,N$

$\rightarrow \Delta S = 0{,}53\,N = 0{,}7\,\%$

$L_{330} = 0{,}648\,m$, $L_{331{,}1} = 0{,}6457\,m$

$\rightarrow \Delta L = 2{,}24\,mm = 0{,}35\,\%$

c) $\rho_l = A\rho \rightarrow f = \frac{1}{2L}\sqrt{\frac{S}{A\rho}}$

(For E_4)

$$\rho = \frac{S}{4f^2L^2A} = \frac{76{,}27}{4 \cdot 330^2 \cdot 0{,}648^2 \cdot (0{,}35 \cdot 10^{-3})^2 \pi} =$$

$$= 1083{,}5 \text{ kg/m}^3$$

For E_2:

$$d = 2 \cdot \sqrt{\frac{S}{4f^2L^2\rho\pi}} = 2 \cdot \sqrt{\frac{60{,}78}{4 \cdot 82{,}4^2 \cdot 0{,}648^2 \cdot 1083{,}5 \cdot \pi}} =$$

$$= 2{,}5 \text{ mm}$$

Homogenous string:

Pros: Easier to manufacture, especially for high frequencies, require less tension relative to frequency.

Cons: Thicker strings.

d) $u_n = \frac{2h}{\pi^2 n^2} \cdot \frac{1}{\beta(1-\beta)} \cdot \sin(n\pi\beta)$

Take $h = 1$, $\beta = 0{,}5 \rightarrow u_n = \left|\frac{4\sin(\frac{n\pi}{2})}{\pi^2 n^2}\right|$

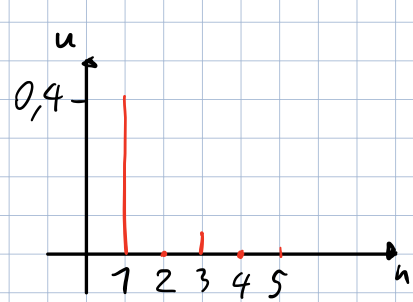
$u_1 = 0{,}405$

$u_2 = 0$

$u_3 = 0{,}045$

$u_4 = 0$

$u_5 = 0{,}016$



B.

a) $f_{A0} = \frac{c}{2\pi} \sqrt{\frac{S}{l_e V}}$

$V = 12{,}6\ dm^3 = 0{,}0126 m^3$

$S = 0{,}044^2 \pi\ m^2 \approx 0{,}006 m^2$

$c = 346 m/s$

$l_e = 0{,}003 + 2 \cdot 0{,}85 \cdot 0{,}044 =$
$= 0{,}0778\ m$

$f_{A0} = 137{,}2 Hz$

b) $S_1 = S \cdot 0{,}75 \approx 0{,}00456 m^2$

$f_{A0,1} = 118{,}8\ Hz \rightarrow \Delta f = 18{,}4\ Hz$

c) Decrease volume by putting things in the sound hole.

inner side) and makes the effective length of the sound hole longer than the top plate thickness (3 mm). This *end correction* can be approximated by

$\Delta l = 0.85a$ for one side of a hole with radius a in a baffle (plate).

b) If a quarter (¼) of the sound hole is covered by a piece of felt, how large will be the shift in frequency for A0?

c) Suggest a way of raising the Helmholtz frequency temporarily (without destroying the guitar) which you can try during the experiments.

Lab instructions

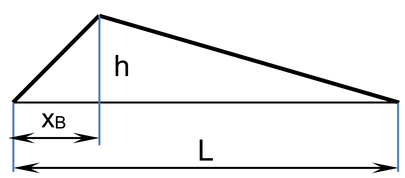
A. String physics

The *ideal string* is homogenous with constant cross section and constant linear density (mass per unit length). Further, it is thin and offers no resistance to bending (no stiffness). The ideal string gives a strictly harmonic spectrum with partial frequencies $f_n = nf_0$ with partial number n and fundamental frequency f_0. Real strings are not perfectly homogenous and have stiffness. As a consequence, real strings generate somewhat inharmonic spectra.

String spectrum

An ideal string with length L and plucked at distance x_B from the termination point by pulling it sideways a short distance h before releasing creates mode amplitudes u_n (displacement)

$$u_n = \frac{2h}{\pi^2 n^2} \cdot \frac{1}{\beta(1-\beta)} \cdot \sin(n\pi\beta) \qquad (2)$$



where $\beta = x_B/L$ is relative plucking distance

The equation shows that the mode amplitudes
- decrease fast for higher mode number n
- depend on the plucking position β ('amplification factor' for all n)
- will be zero when the sin-function has zeros.

Inharmonicity

The stiffness of the string shifts the mode frequencies slightly upwards ('stretched' spectrum) compared a strictly harmonic spectrum:

$$f_n = nf_0\sqrt{1+n^2B} \qquad (3)$$

The inharmonicity coefficient B depends on Young's modulus E, string diameter d, length L and tension S

$$B = \frac{\pi^3 d^4 E}{64 S L^2} \qquad (4)$$

For wrapped strings it is the diameter of the inner core, which enters into B (the tension is applied to the core only).

B can be determined by measuring two mode frequencies f_m and f_n

$$B = \frac{K-1}{m^2-n^2K} \qquad \text{with } K = \frac{n^2}{m^2}\frac{f_m^2}{f_n^2} \qquad (5)$$

Using the monochord strung with a guitar and a piano string, you will study the spectrum of each string when excited by a pluck. You will measure the transversal motion of the string using a capacitive vibration detector, recorded using an audio interface into your computer (use Audacity). Use MATLAB scripts we have already written to perform a spectral analysis of the recording, and to answer the following questions.

A1
For the guitar and piano strings, determine the fundamental frequency, pitch, linear density and tension.

A2
Pluck the piano string and verify the three factors in Eq. 2 step by step. That is, hold h constant and change β. Then hold β constant and change h.

A3
Pluck the guitar string with a plectrum or wire at different dynamic levels (*pp* – *mf* – *ff*) and correlate the changes in spectrum with the perceived sounds.

A4
Determine the inharmonicity coefficient for each string. Tip: Inharmonicity grows rapidly with partial number.

A. Results

A1

GUITAR

Fundamental frequency 86,1 Hz Pitch F_2

Linear density 5,741 g/m String tension 71,5 N

PIANO

Fundamental frequency 172,3 Hz Pitch F_3

Linear density 3,508 g/m String tension 174,9 N

Constant h, different β: the amplitudes of the peaks vary. For example amplitude of u_2 higher for β = 1/2 than β = 1/4.

A2

For constant h, different β:
Amplitudes of partials vary. For example, when β = 1/2, the amplitude u_2 is lower than for β = 1/4.

For constant β, different h:
The amplitudes of the partials have the same relationship to each other but the average amplitude for all partials increases with higher h.

A3

General influence of dynamic level

Level of higher frequency partials increases with dynamic level.

Comparison spectral content and perceived sound

Pluck with *pp* gives $L_{pf_1}=45$, $L_{pf_2}=35$, $L_{pf_3}=35$, $L_{pf_4}=29$, $L_{pf_5}=28$ [dB]

Pluck with *mp* gives $L_{pf_1}=55$, $L_{pf_2}=43$, $L_{pf_3}=47$, $L_{pf_4}=38$, $L_{pf_5}=44$ [dB]

Pluck with *mf* gives $L_{pf_1}=62$, $L_{pf_2}=57$, $L_{pf_3}=59$, $L_{pf_4}=46$, $L_{pf_5}=52$ [dB]

Pluck with *ff* gives $L_{pf_1}=66$, $L_{pf_2}=57$, $L_{pf_3}=58$, $L_{pf_4}=59$, $L_{pf_5}=63$ [dB]

$$f_n = n f_0 \sqrt{1+n^2 B} \rightarrow B = \frac{\left(\frac{f_n}{n f_0}\right)^2 - 1}{n^2}$$

A4

GUITAR

Low E string partial N:o ...1......... frequency ...87,5 Hz.........

Low E string partial N:o ...2......... frequency ...175,0 Hz......... $B_1 = 0$

Low E string partial N:o ...3......... frequency ...265,1 Hz......... $B_2 = 0{,}002$

Low E string partial N:o ...4......... frequency ...350,0 Hz......... $B_3 = 0$

Low E string inharmonicity coefficient B = ...$B_{avg} \approx 7{,}4 \cdot 10^{-4}$...........

PIANO

Piano string partial N:o ...1......... frequency ...175,0 Hz.........

Piano string partial N:o ...2......... frequency ...347,2 Hz......... $B_1 = -0{,}004$

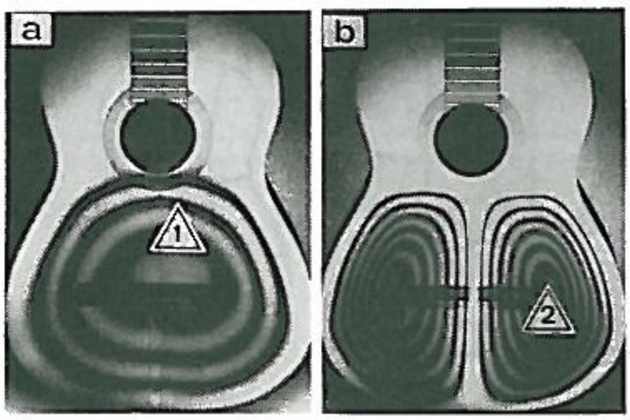
Piano string partial N:o ...3......... frequency ...525,0 Hz......... $B_2 = 0$

Piano string partial N:o ...4......... frequency ...697,2 Hz......... $B_3 = -5 \cdot 10^{-4}$

Piano string inharmonicity coefficient B = ...$B_{avg} \approx -1{,}5 \cdot 10^{-3}$...........

B. Guitar body modes – original and perturbed

In this section some of the modes of the guitar body are investigated: the Helmholtz resonance A0, and two body modes with large motions in the top, T1 and T2. A convenient way of observing the mode frequencies is to measure the mechanical mobility $Y = v/F$, where F is excitation force and v the resulting vibration velocity. When the driving frequency is close to a mode frequency the displacement as well as the velocity amplitudes will be large, and the mobility takes a high value, corresponding to peaks in the mobility curve (see figure at bottom of page). A practical problem is to drive with a periodic force at one point and measure the vibration velocity at the same position.

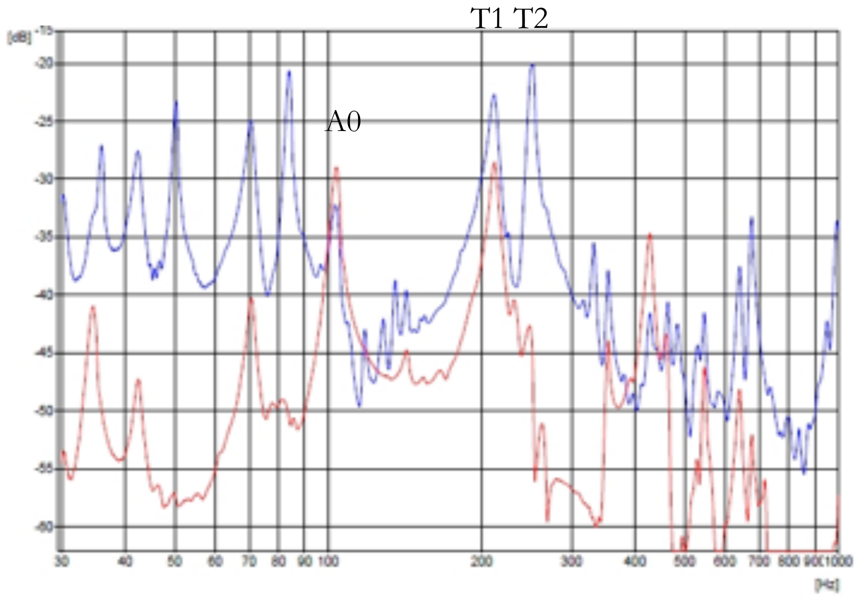


Vibration modes in a guitar top, glued to the ribs but without back. T1 = 185 Hz and T2 =287 Hz. Interference holography pictures, which show the vibration amplitude as a topographical map. The triangles indicate driving points.

Method

Support the guitar on foam rubber cushions. Put an accelerometer at a suitable measuring point on the guitar using wax. Put a small magnet on top of the accelerometer. Position an electrical coil just above the magnet with a small air gap (1 mm) in between. The magnet-coil acts as an electrodynamical driving system (like a loudspeaker) and generates a force on the guitar, which is proportional to the current in the coil. A sinusoidal current gives a sinusoidal force. Use the program *Tombstone* for driving and analysis by excitation with sinusoidal frequency sweeps.

The radiated sound is measured using a shortcut with the microphone inside the guitar instead of in the room. It can be shown that the sound pressure inside a cavity is proportional to the sound pressure at a large distance from the object. The intuitive explanation is that the microphone inside senses net changes in volume as pressure variations. Large net volume variations correspond to efficient sound radiation and consequently high sound pressure at a distance.



Mobility measurement of the lab guitar (blue, driving point 2, above) and sound pressure inside the guitar body (red). The modes ('resonances') show up as peaks in mobility and sound pressure. The Helmholtz resonance A0 and the two lowest top plate modes T1 and T2 are seen as prominent peaks.

B1

Measure some mobility curves for the guitar. Select suitable driving points to enhance/suppress different modes. Determine the frequencies for the Helmholtz resonance A0 and top plate modes T1 and T2.

B2

a) Perturb A0 by covering ¼ of the sound hole, and the entire hole, respectively, with a piece of felt.

b) Perturb T1 with a small mass (~~16~~ 20 g) and a large mass (125 g) fastened with wax fastened at critical positions.

c) Perturb T2 with two small masses (4.8 g) fastened with wax at critical positions.

B3

Which modes radiate efficiently?

B4

Where are the nodal lines located for T1 and T2, and which parts vibrate in and out of phase? Drive with a fixed frequency and use a second hand-held accelerometer to sample the vibrations amplitudes of the top.

B. Results

	A0	T1	T2	
B1	118,1	245,2	288,8	Hz
B2				
a) A0 ¼ hole covered	101,3			Hz
A0 entire hole covered	95,9			Hz
b) T1 small perturb. 16 g		220,9		Hz
T1 large perturb. 125 g		203,0		Hz
c) T2 two small masses 4.8 g			261,5	Hz
B3				
Efficient sound radiation	Yes	Yes	No	Yes/no

B4 Sketch T1 and T2 mode shapes (nodal lines and phases+/-).

T1

T2