KTH ROYAL INSTITUTE
OF TECHNOLOGY

DT2212: Music Acoustics

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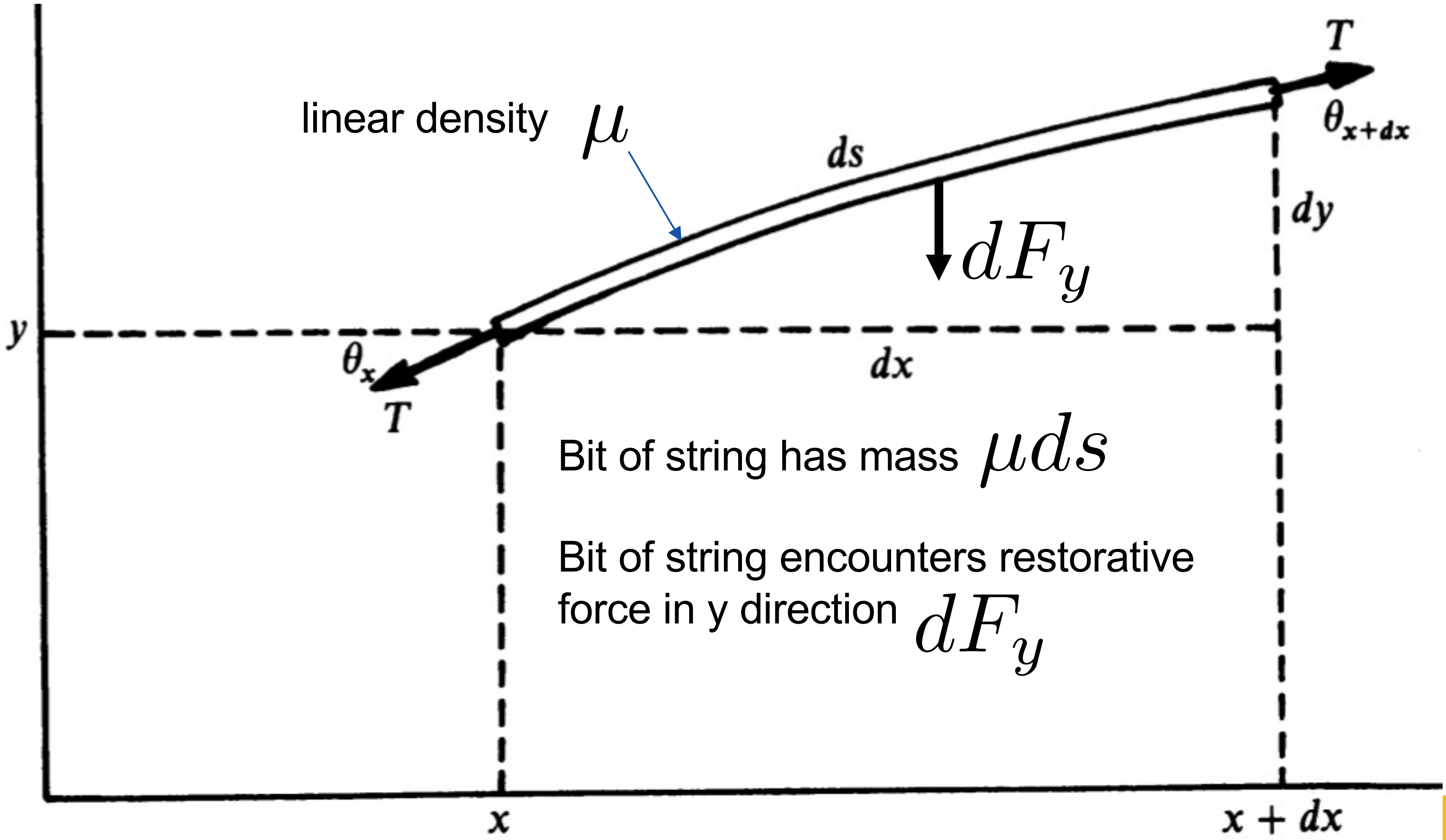
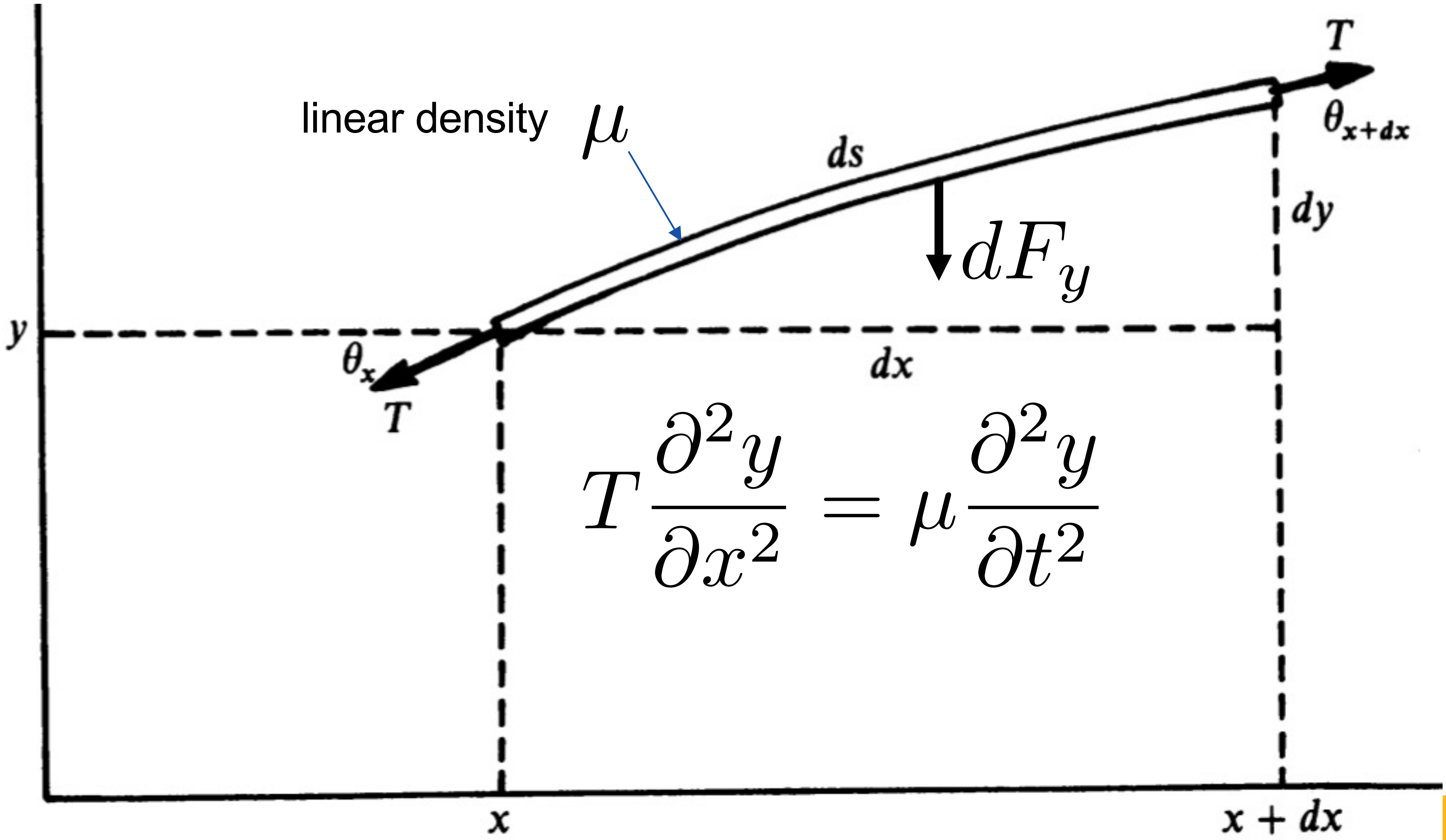
Lecture 6: Guitar and Piano
Reading: Ch 25.1 Hartmann

Ideal string from a physical perspective



There is force in x direction too (see Fletcher and Rossing 2.14)

Ideal string from a physical perspective



$$T\frac{\partial^2 y}{\partial x^2} = \mu\frac{\partial^2 y}{\partial t^2}$$

Wave equation for ideal string: *Solution*

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

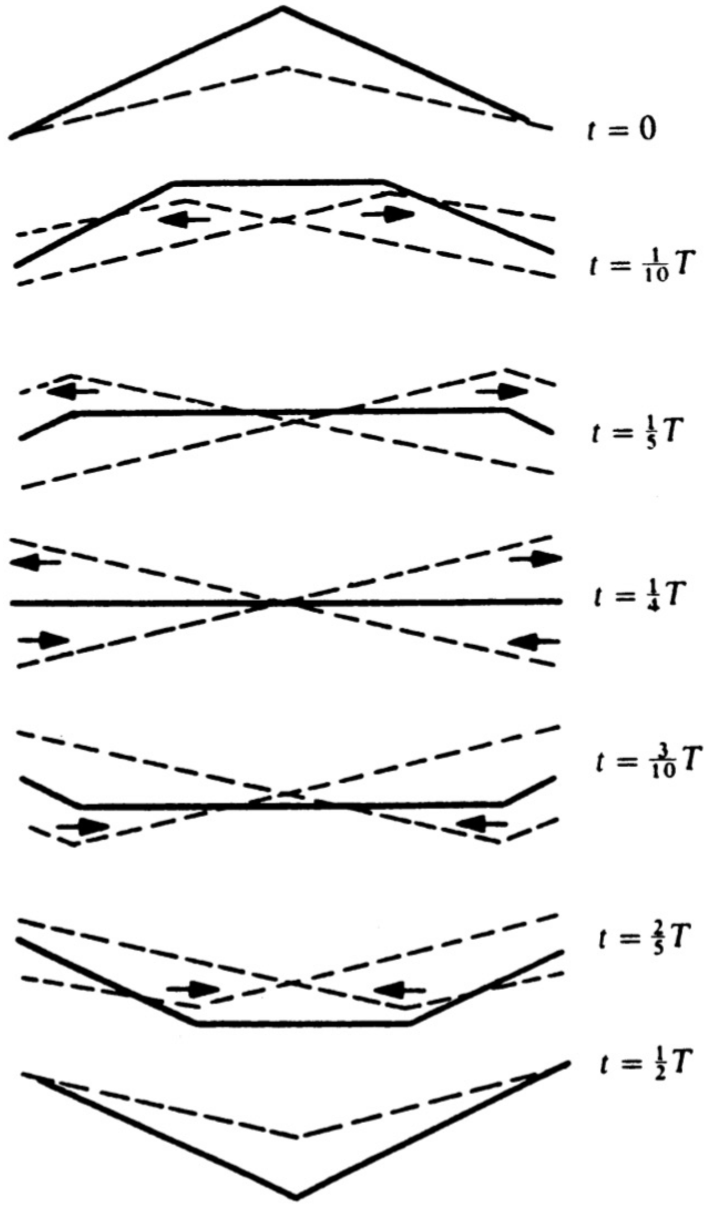
Bernoulli's solution

$$y(x,t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}x\right)\cos\left(\frac{n\pi}{L}ct + \phi_n\right)$$

$$\omega_1 = \frac{\pi c}{L} = \frac{\pi}{L}\sqrt{T/\mu}$$ Fundamental (angular) frequency

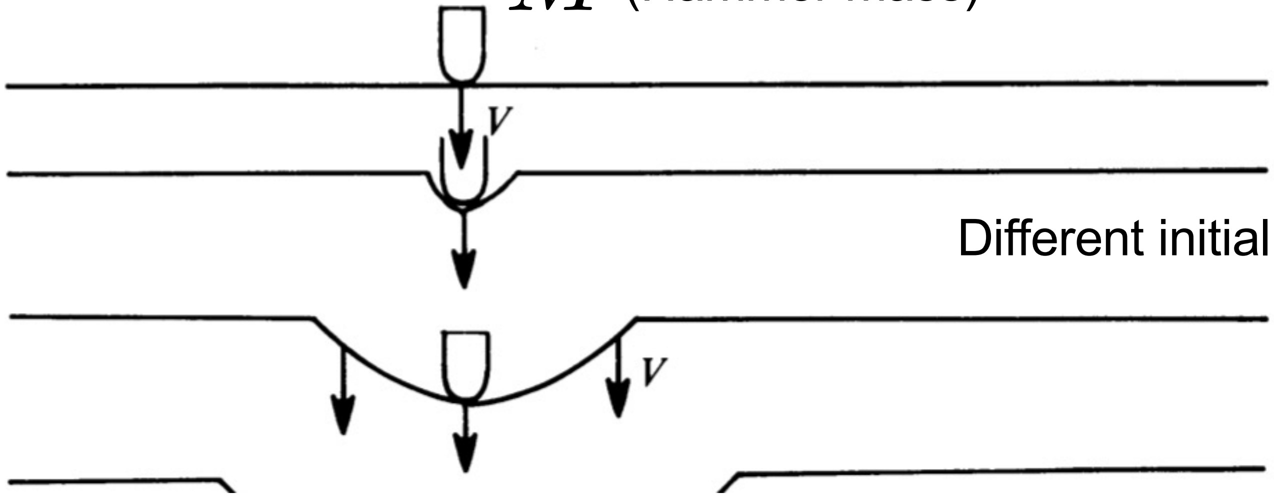
$$f_1 = \omega_1/2\pi = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Plucked Ideal String



Struck Ideal String

M (Hammer mass)



Different initial conditions!

$$y(x,t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}x\right) \cos\left(\frac{n\pi}{L}ct + \phi_n\right)$$

The coefficients will be different between the plucked and struck string.

Strings in the real world

1. **Not perfectly flexible (resistance to bending)**

$$T\frac{\partial^2 y(x,t)}{\partial x^2} - ESK^2\frac{\partial^4 y(x,t)}{\partial x^4} = \mu\frac{\partial^2 y(x,t)}{\partial t^2}$$

Tension

Young's modulus

Cross-sec. area

Radius of gyration

density

Strings in the real world

1. **Not perfectly flexible**
2. **Not perfectly terminated**
3. **Damping is occurring**

https://www.falstad.com/loadedstring

Laboratory 1 preparatory exercises

Important equations:

$$v^2 = T/\mu$$

Velocity of transverse wave

String tension

Linear density of string

ATTN: *Use m, kg, s units*

Laboratory 1 preparatory exercises

Important equations:

$$v^2 = T/\mu$$

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = \frac{n}{2}\sqrt{\frac{T}{m/L}}$$

Frequency of nth harmonic

String length

String mass

ATTN: *Use m, kg, s units*

Laboratory 1 preparatory exercises

Important equations:

$$v^2 = T/\mu$$

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = \frac{n}{2}\sqrt{\frac{T}{m/L}}$$

$$m = V\rho$$

String volume x density

ATTN: *Use m, kg, s units*

Laboratory 1 preparatory exercises

Important equations:

$$\frac{f_1}{f_0} = 2^{I/1200}$$

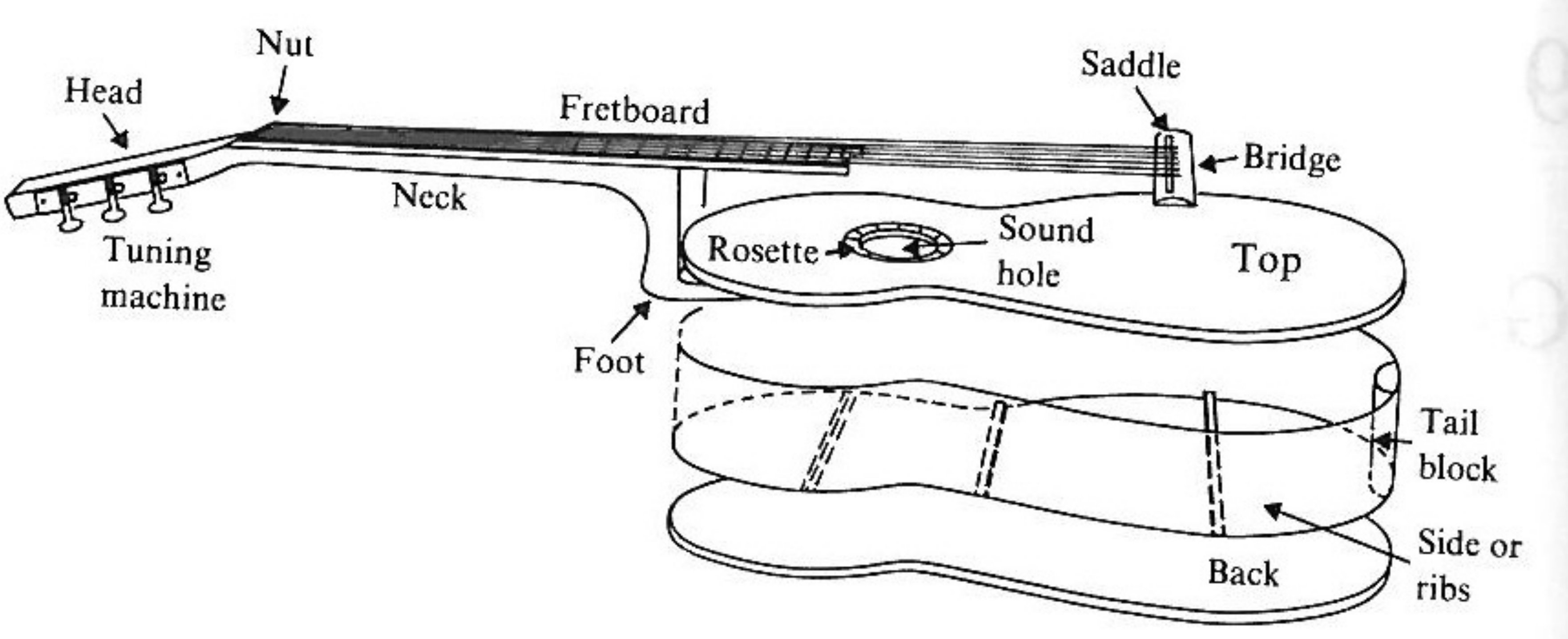
Ratio of frequencies

Interval between them (cents)

ATTN: *Use m, kg, s units*

Guitars and Pianos!

Parts of the modern guitar



History of the guitar

Lineage includes instruments from Egypt & Mesopotamia

1425 BC

History of the guitar

Lineage includes instruments from Egypt & Mesopotamia

Antonio Torres 1817-1892 Spanish luthier and musician "Father of the modern guitar"

Torres designed guitars with better loudness and tone

€140,000

Underside of top

The ribs stiffen the top and equalizes the stiffnesses along and across the grain.

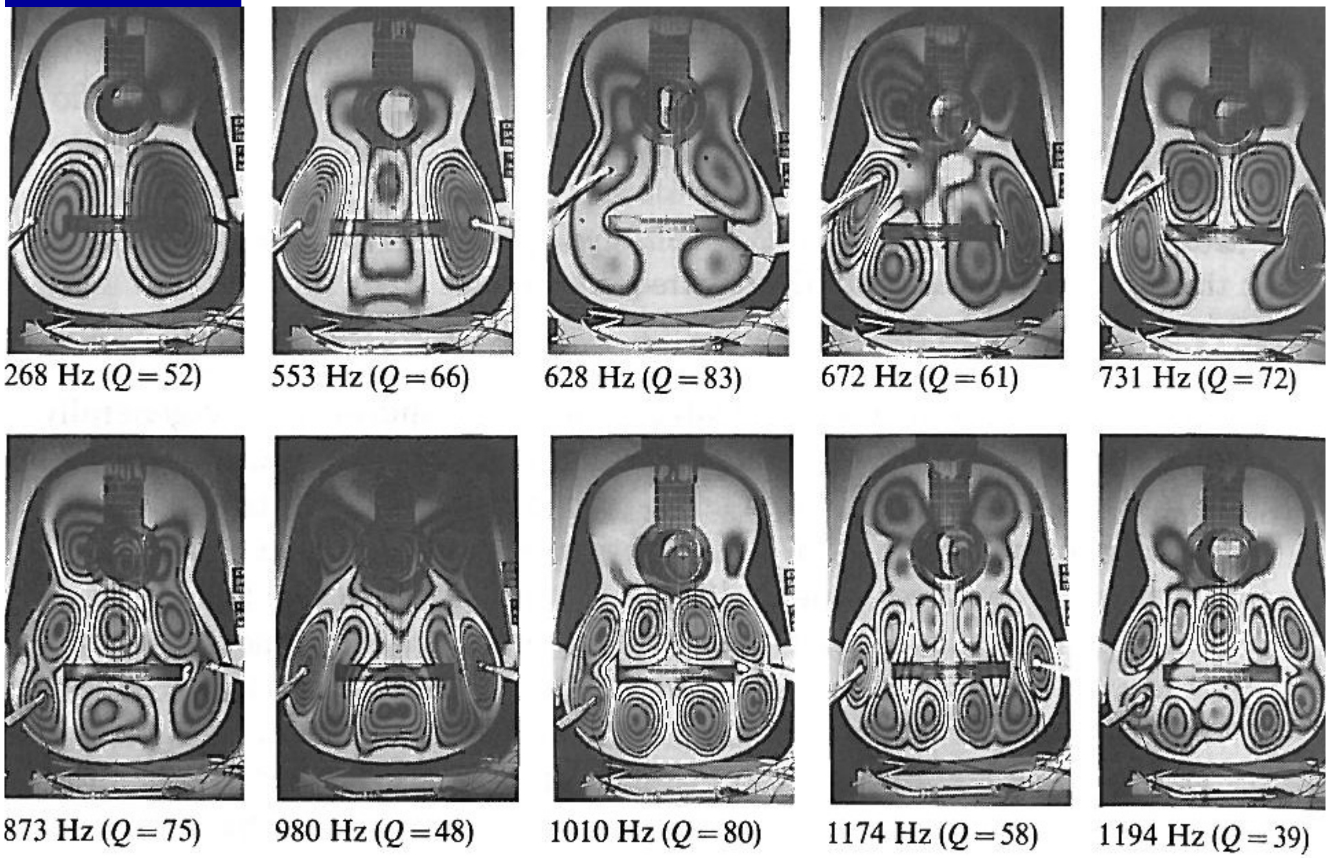
The guitar maker's 'magic' secret. Everything is tried

Torres original

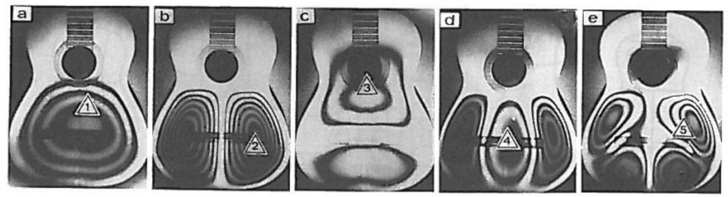
Modes of top

mode T2

Mode shapes & frequencies



another guitar



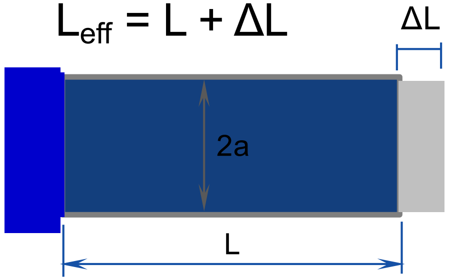
mode T1 T2

Recall the Helmholtz Resonator

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{\pi a^2}{V\, L_{eff}}}$$

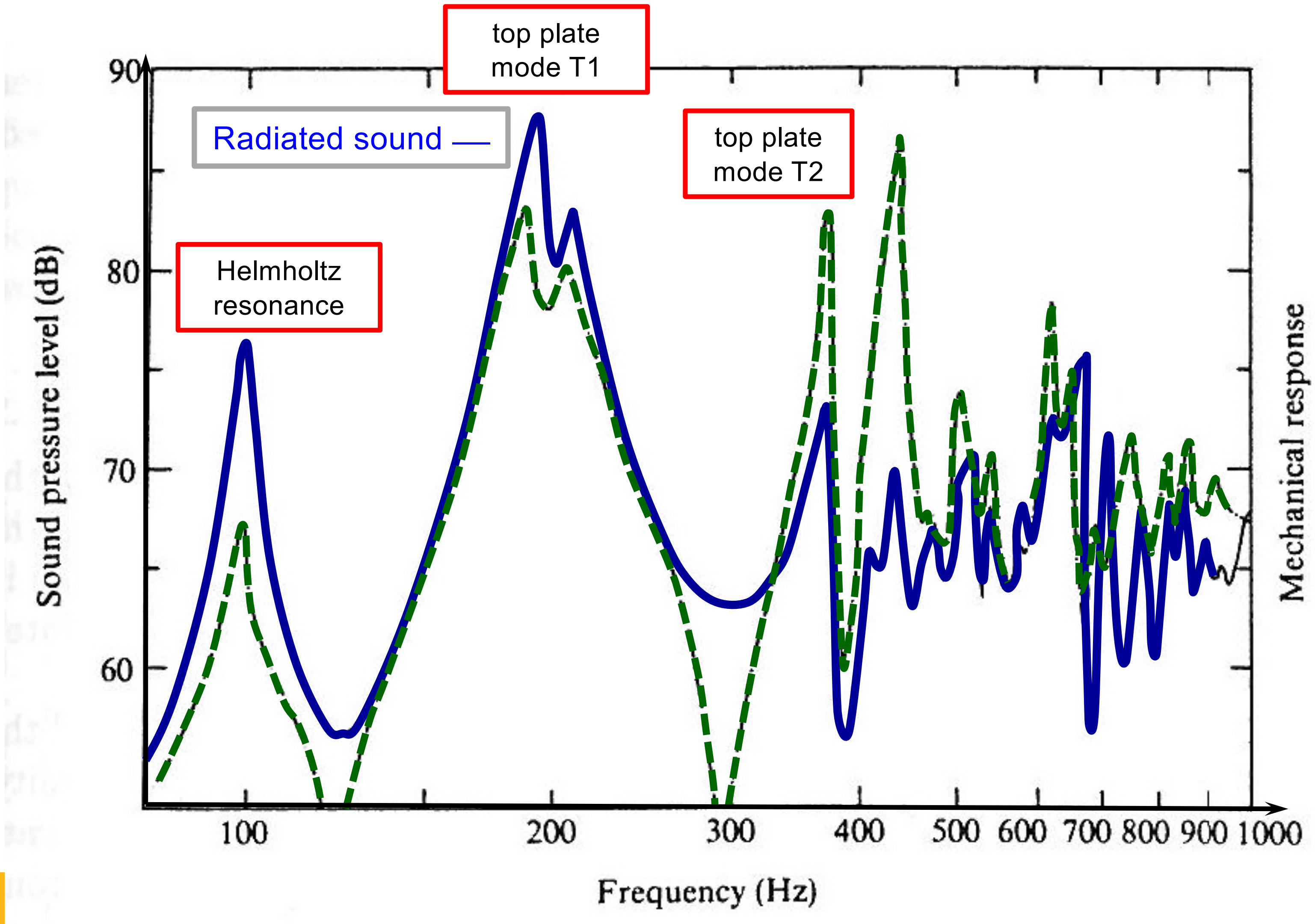
End correction ΔL due to a vibrating air plug at neck opening

$L_{eff} = L + \Delta L$

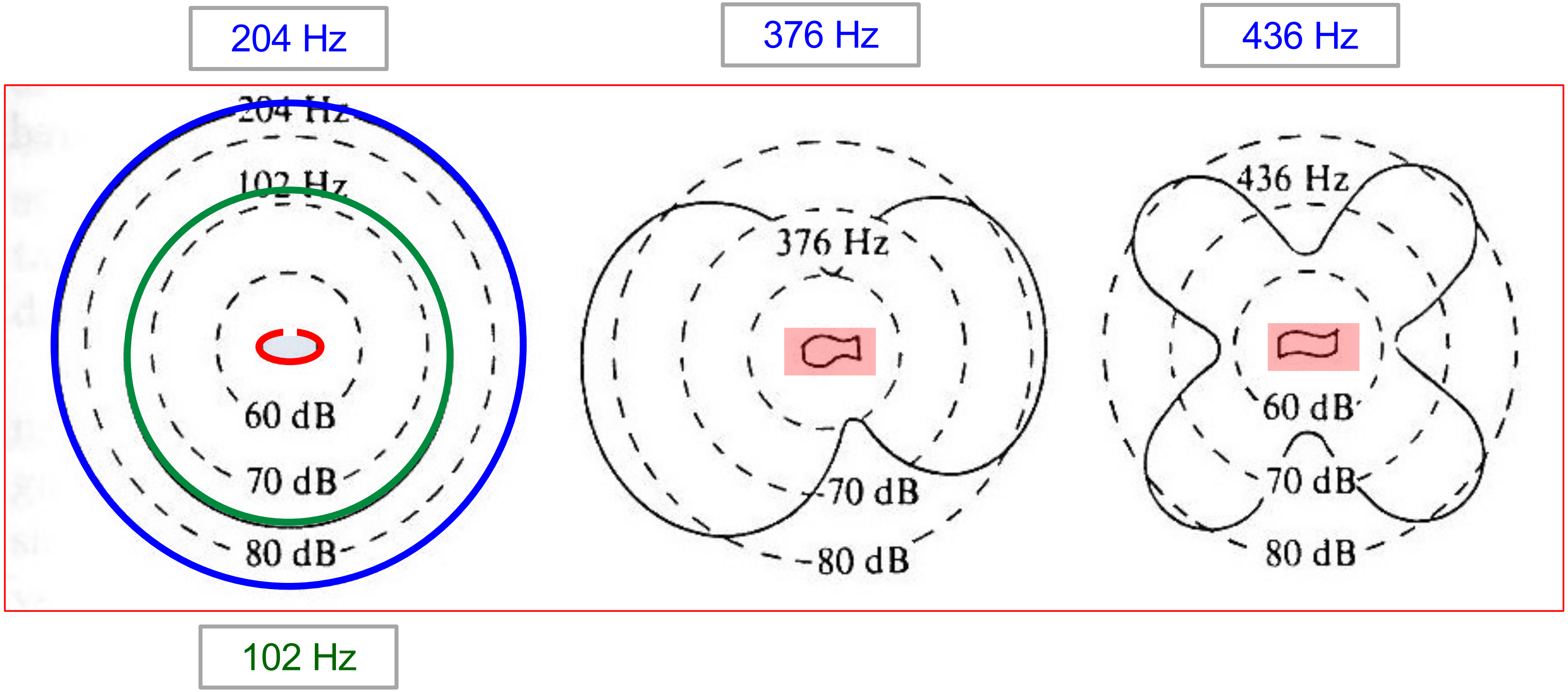


$$L_e = L + 0.85a \ \text{(baffled end)}$$
$$L_e = L + 0.62a \ \text{(cut end)}$$

Body vibrations and radiated sound

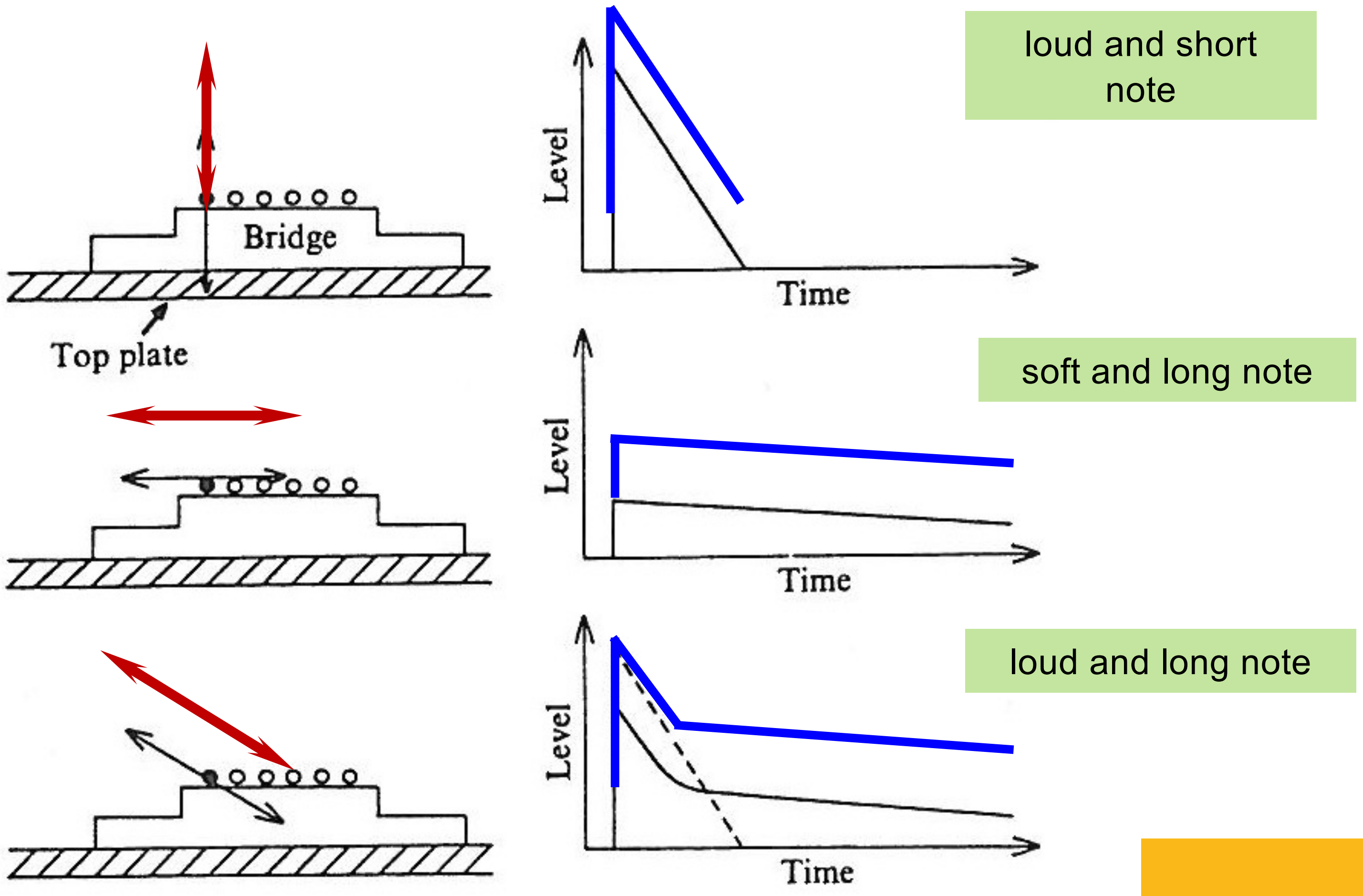


Body modes and sound radiation



Sound radiation patterns at four resonant frequencies of a Martin D-28 folk guitar. In all three patterns, the sound hole is pointing downward. (From "Sound Radiation from Classical and Folk Guitars," by J. Popp and T.D. Rossing).

Pluck direction and decay



Frets

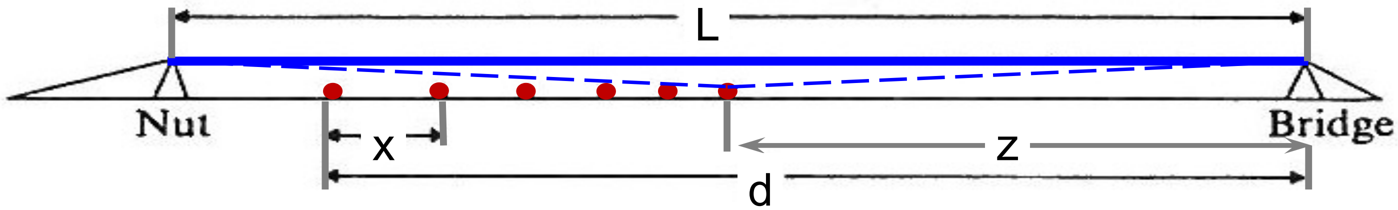
Where do we place these?

Fret Placement: Marker's Rule of Thumb

”The distance *x* to the next fret should be 1/18 of the remaining string length *d*.”

(d – x)/d = 1 - 1/18 = 17/18

Close to a semitone step (100 cent -> 2^(1/12) ≈ 1.05946):
1/1.05946 = 17/18.01



<u>Complication:</u> Compensation for the increase in tension when the string is pressed down (due to the elongation) is necessary. The trick is to make the distance *z* from the 12th fret (octave) to the bridge slightly longer than L/2.

Other kinds of fret placement

http://www.metatonalmusic.com/necks.html

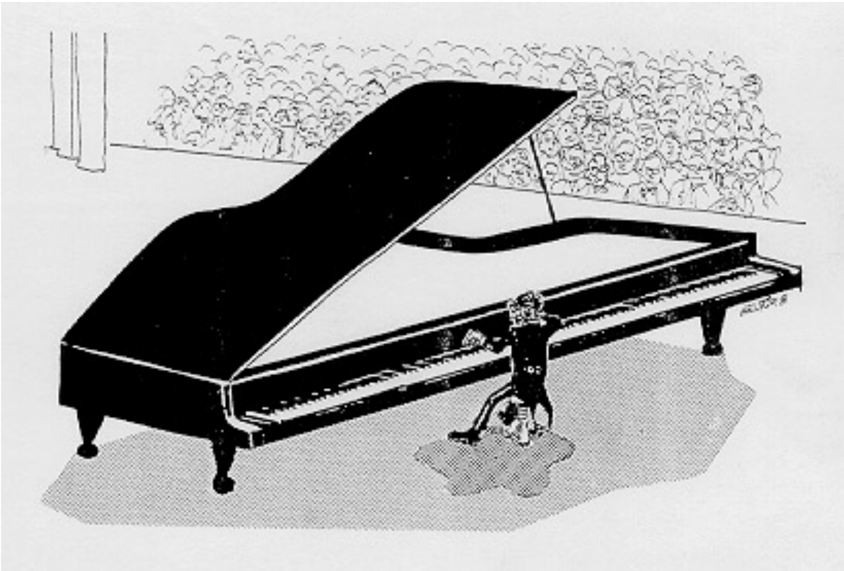
Other kinds of fret placement

https://youtu.be/kyQaSFgnVI8

Who says you need frets?

https://youtu.be/wko1BQvEsb4

The Piano (Piano forte)



The piano soloist's view of the world

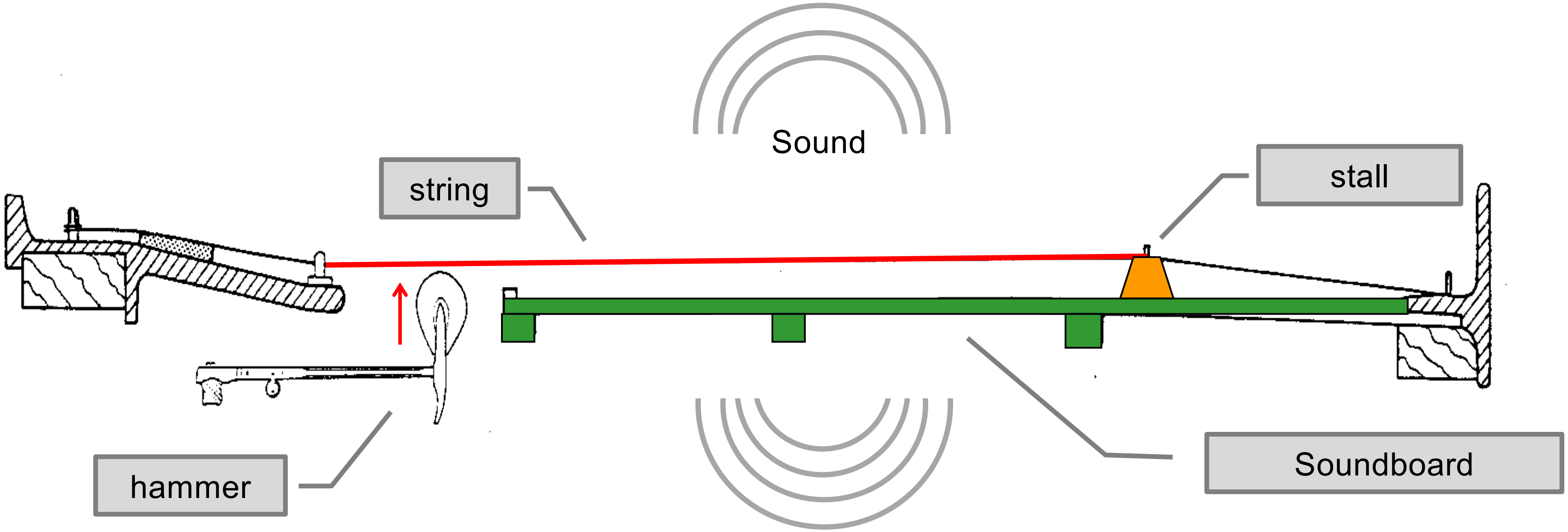
The Piano forte

Bartolomeo Cristofori
”Gravicembalo col piano e forte”
Florens 1709

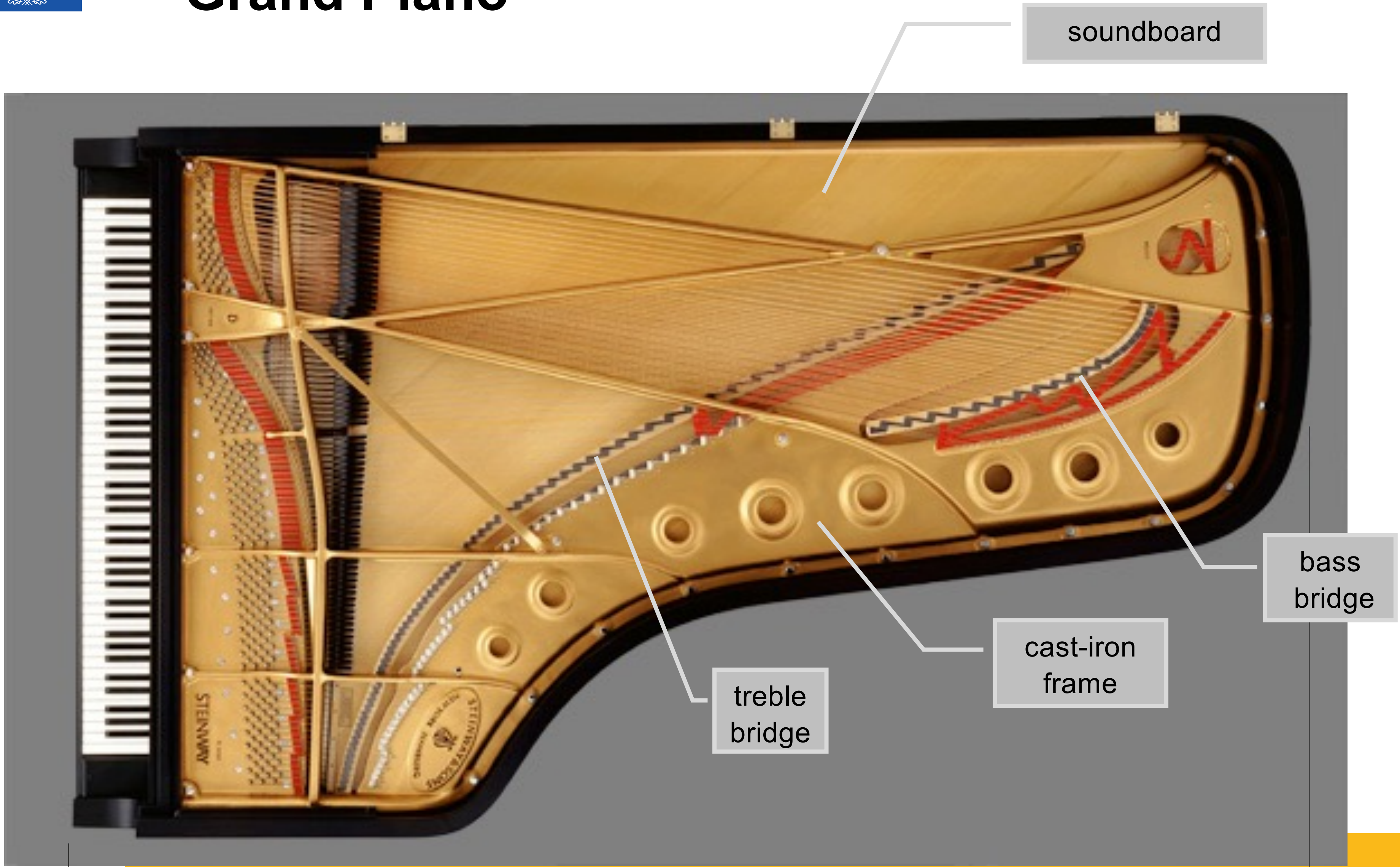
Cristofori 1720

Fazioli Pianoforti 1999

Piano Mechanics

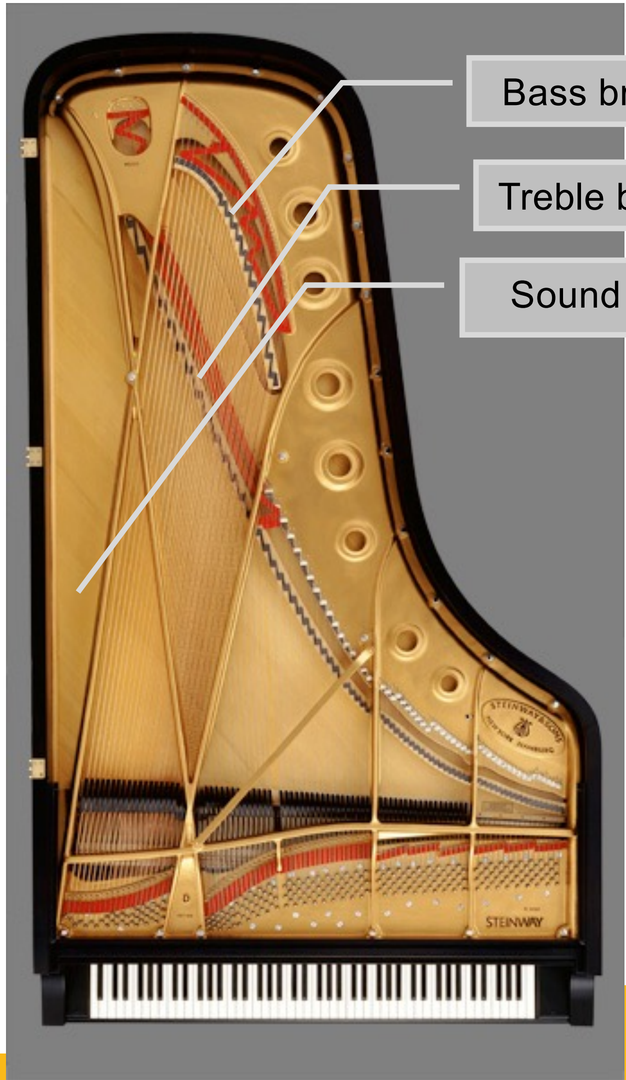


Grand Piano



concert size 2.74 m, 500 kg

Piano Parts

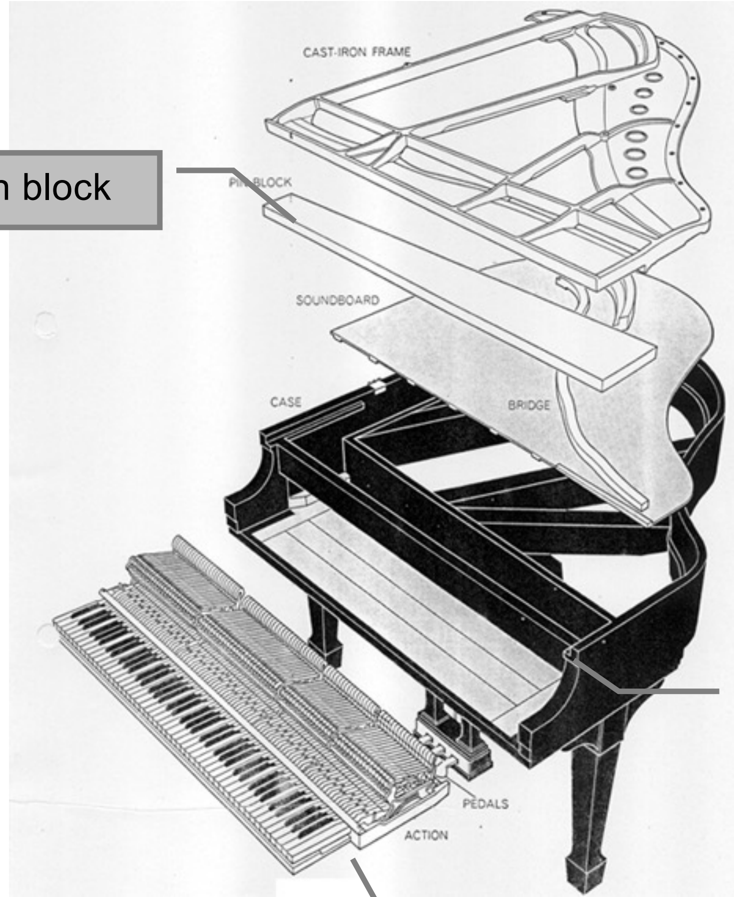


Bass bridge

Treble bridge

Sound board

Pin block

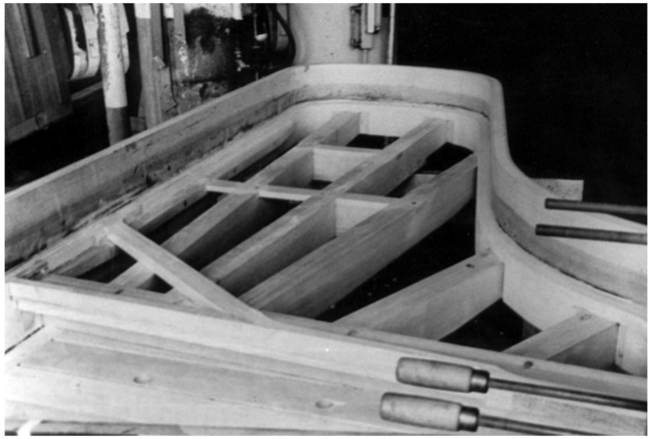


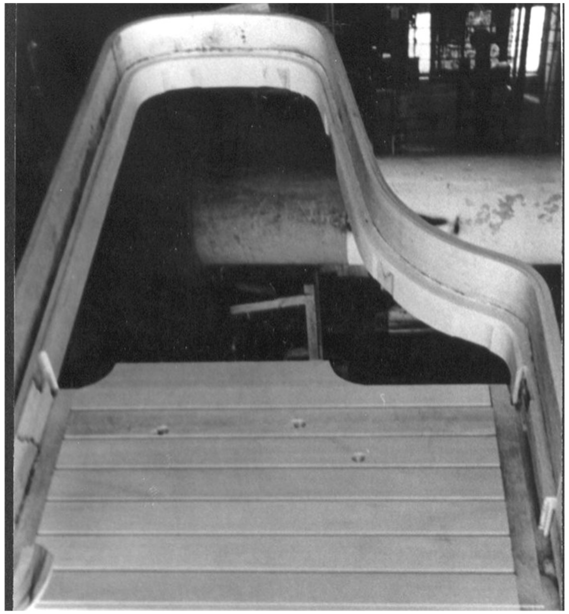
Furniture

Key mechanics

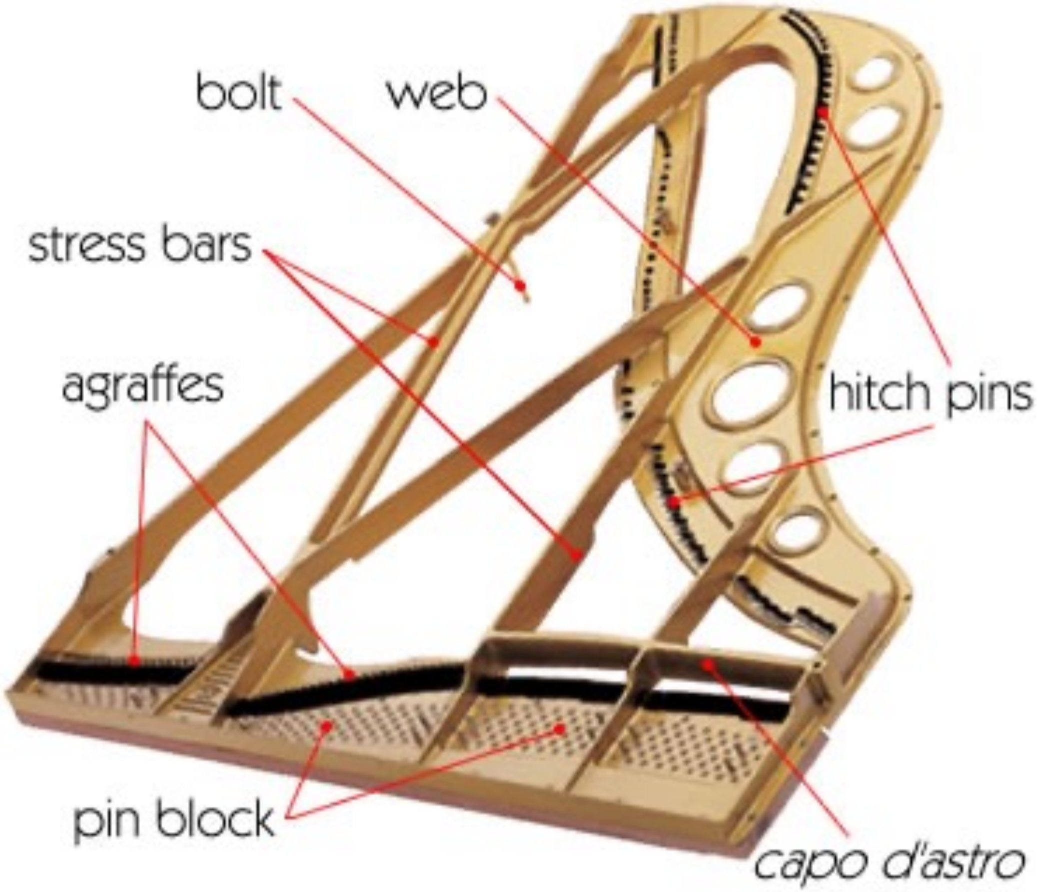
The Piano Deconstructed
http://piano.christophersmit.com

The Frame



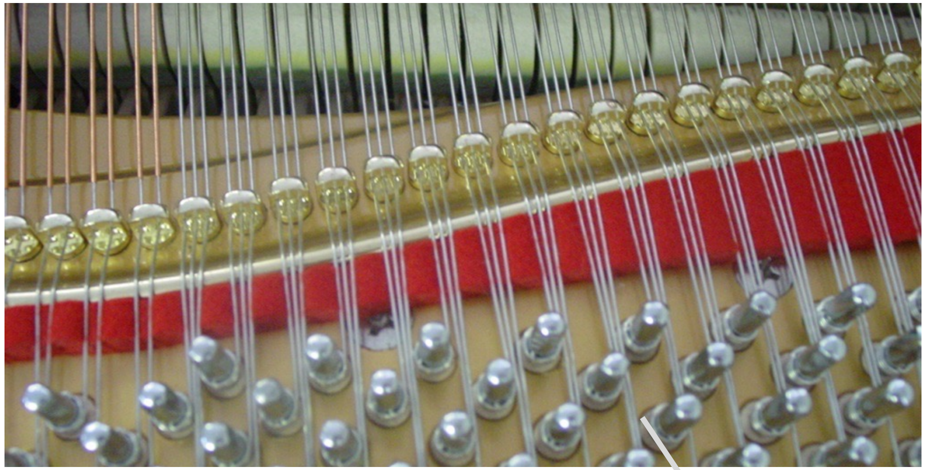


Cast-Iron Frame



Total tension from all strings is about 200,000 N (20 tons). The iron frame relieves the wooden construction from all this force, which provides a stable instrument and good tuning.

String attachments

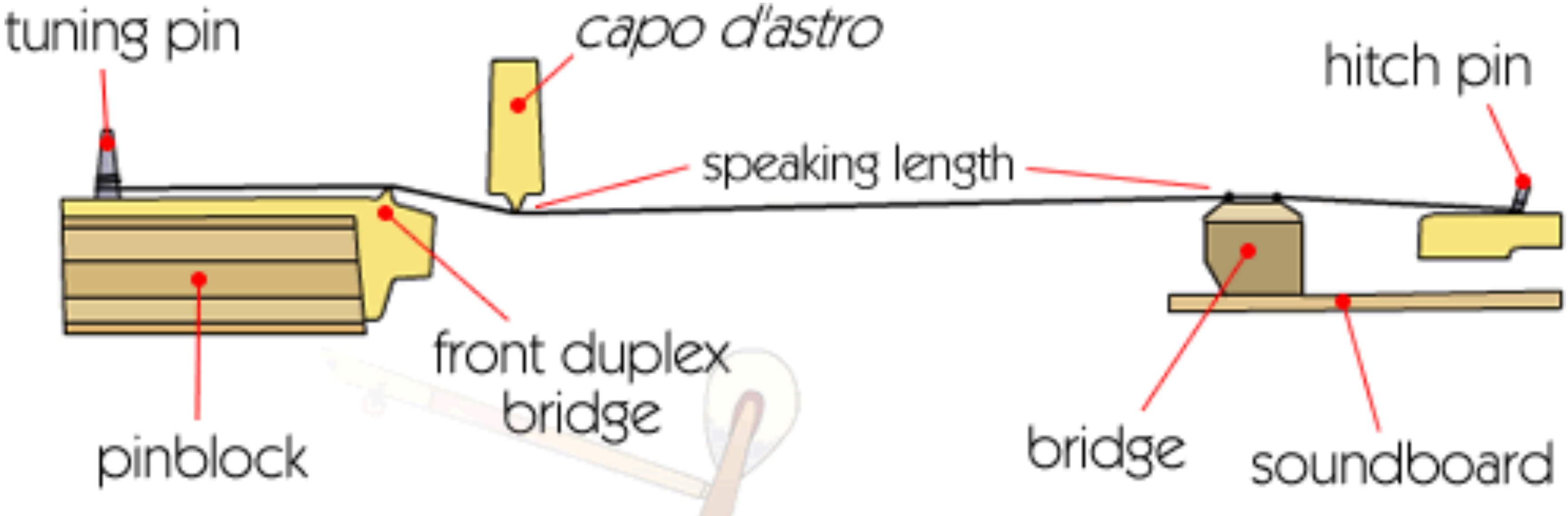


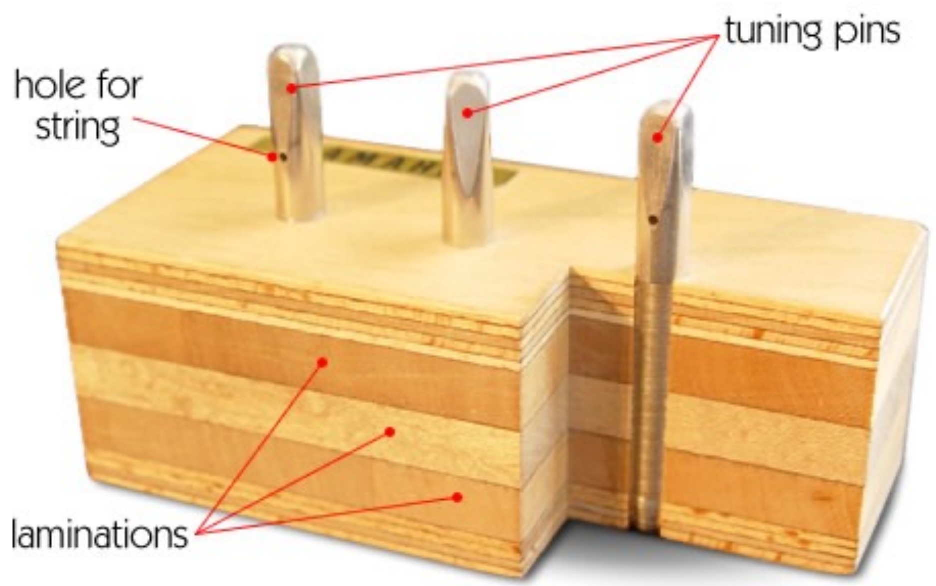
Tuning Nail

Agraff

Capo d’ astro

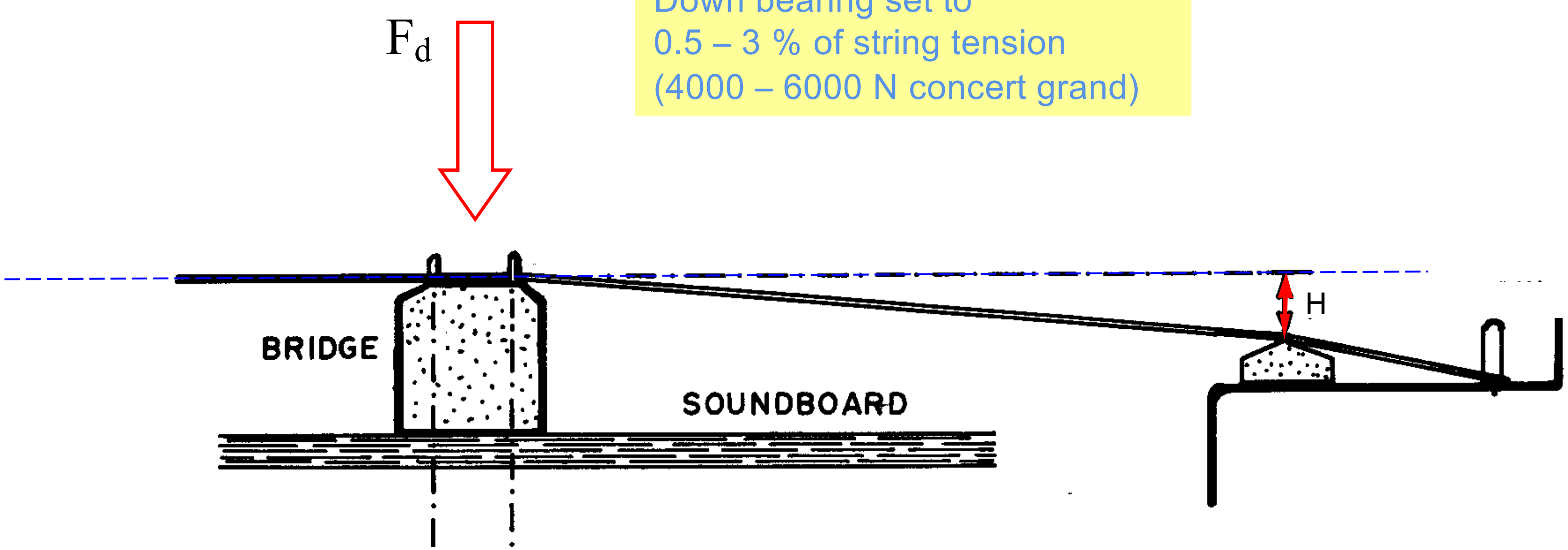
Stringing

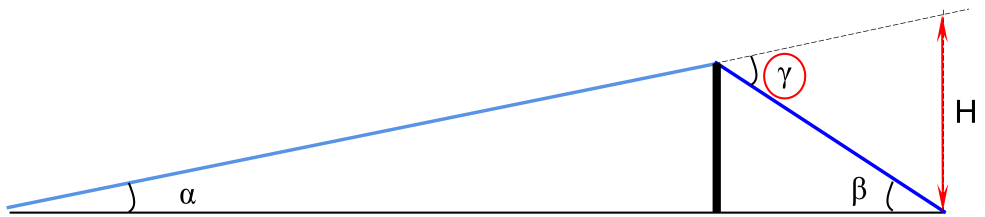

tuning pin
capo d'astro
hitch pin
speaking length
front duplex
bridge
pinblock
bridge
soundboard


tuning pins
hole for
string
laminations

Down-bearing Tension

Down bearing set to
0.5 – 3 % of string tension
(4000 – 6000 N concert grand)





Down bearing angle

$$\gamma = \alpha + \beta$$

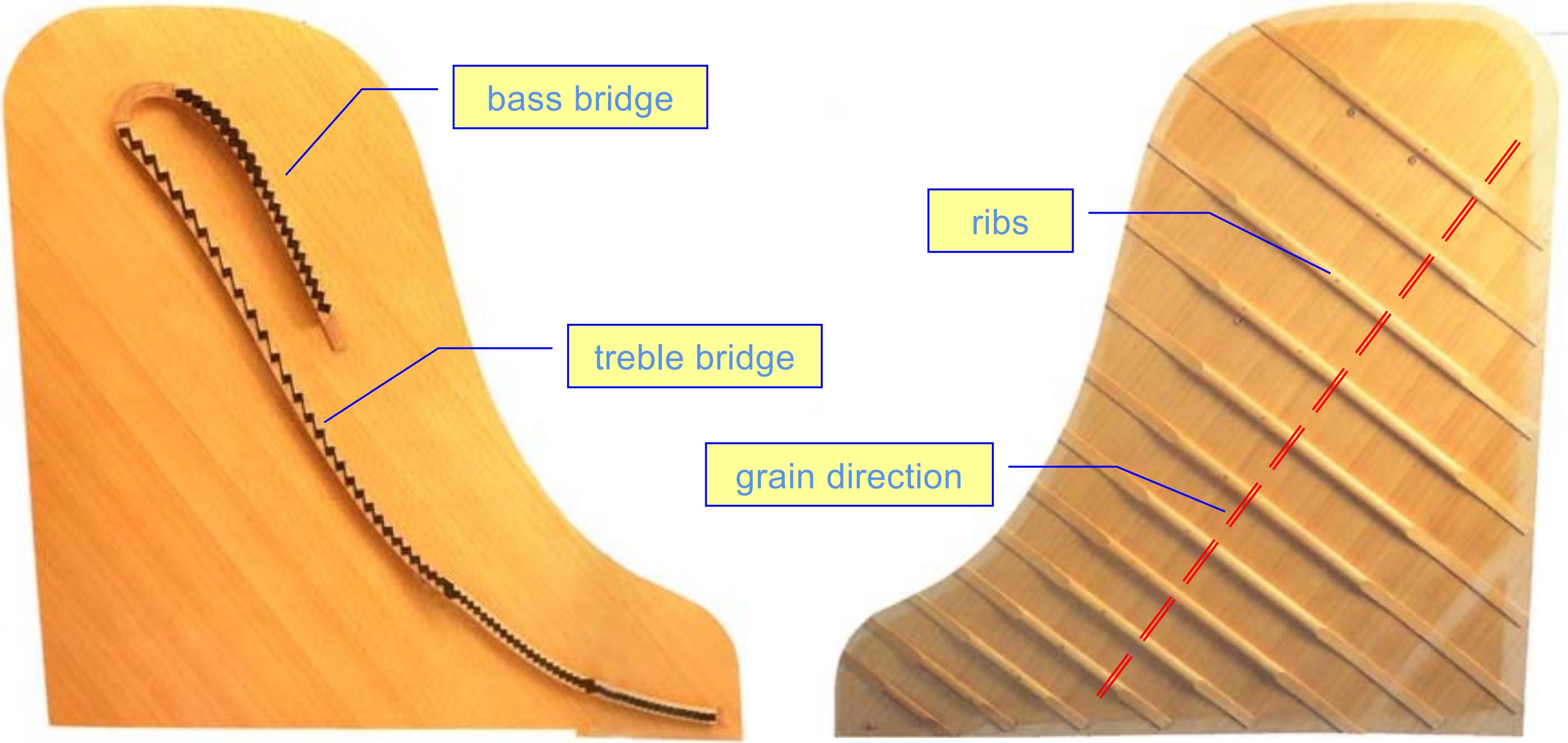
Down bearing

$$F_d = T\,(\sin\alpha + \sin\beta) \approx T\,(\alpha + \beta) = T\,\gamma$$

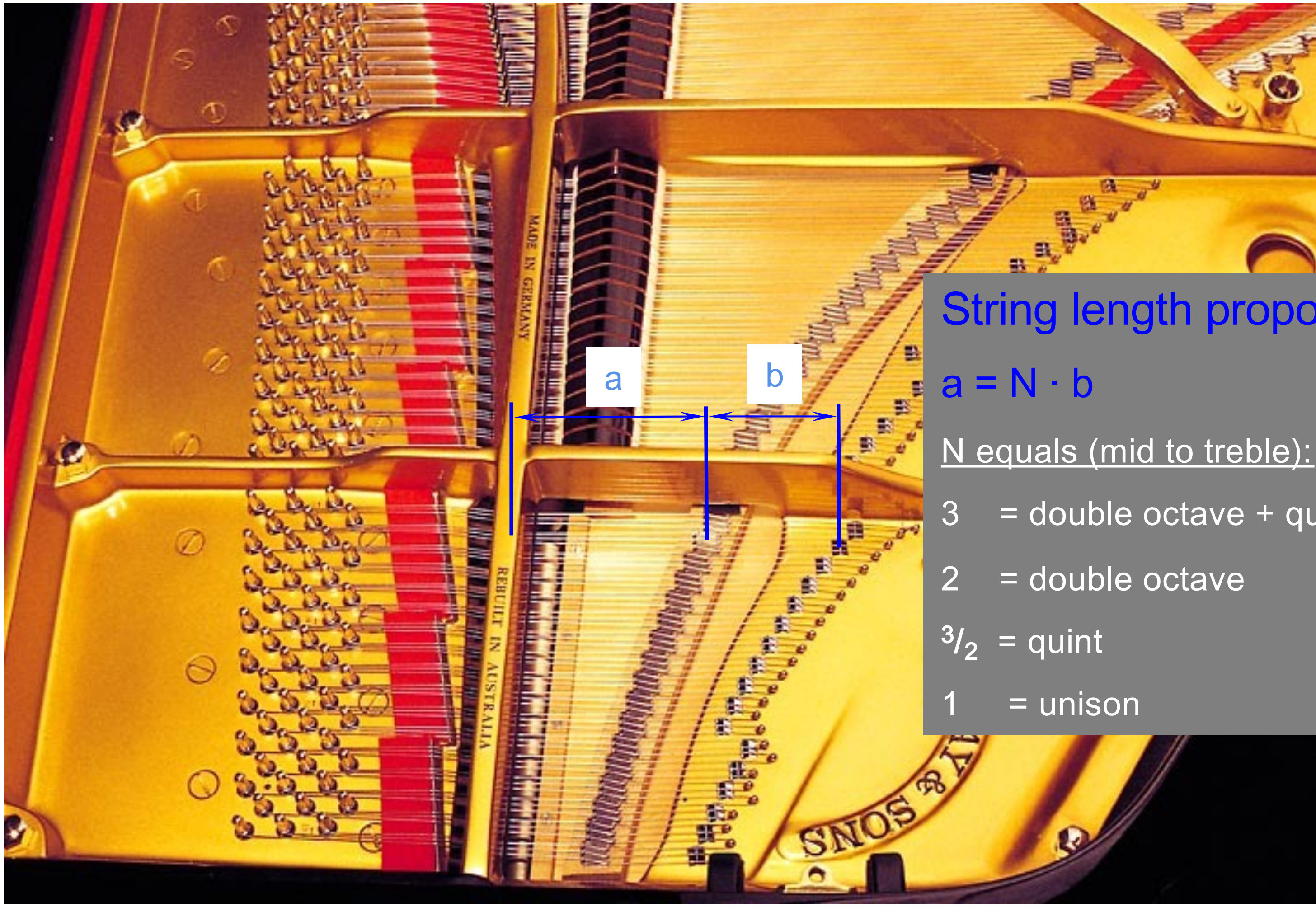
Sound boards

“The most important part of a piano.”

- ‘tone wood’ = quarter-sawn spruce
- density 0.4 kg/m3
- Young´s modulus: 12-13 GPa along the grain
  < 0.4 GPa cross the grain
- ribs add cross-grain stiffness
- tapered (9 – 6 mm)
- crown (dome shape at top)



Duplex stringing



String length proportions

$a = N \cdot b$

<u>N equals (mid to treble):</u>

3 = double octave + quint

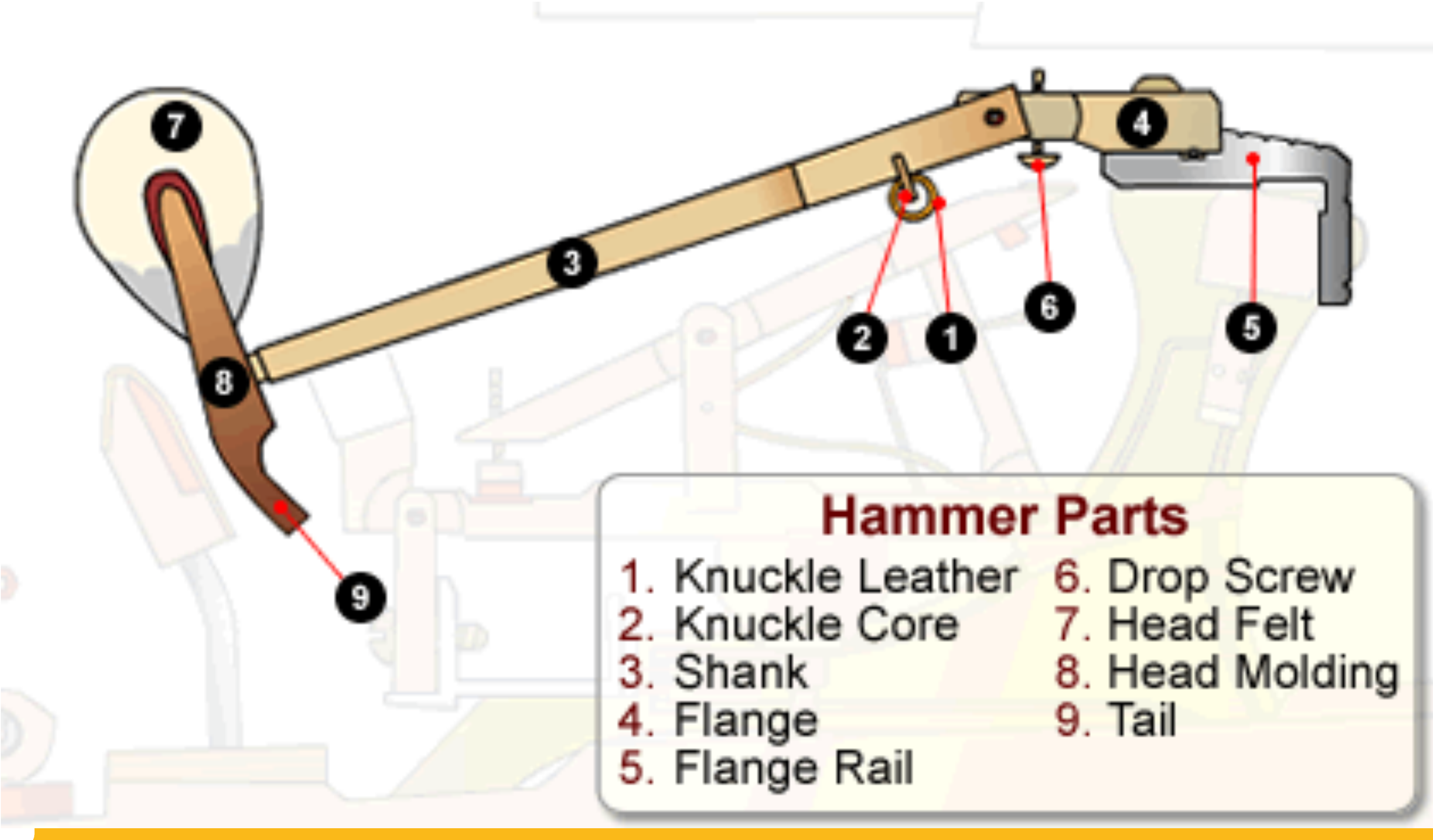
2 = double octave

$^3/_2$ = quint

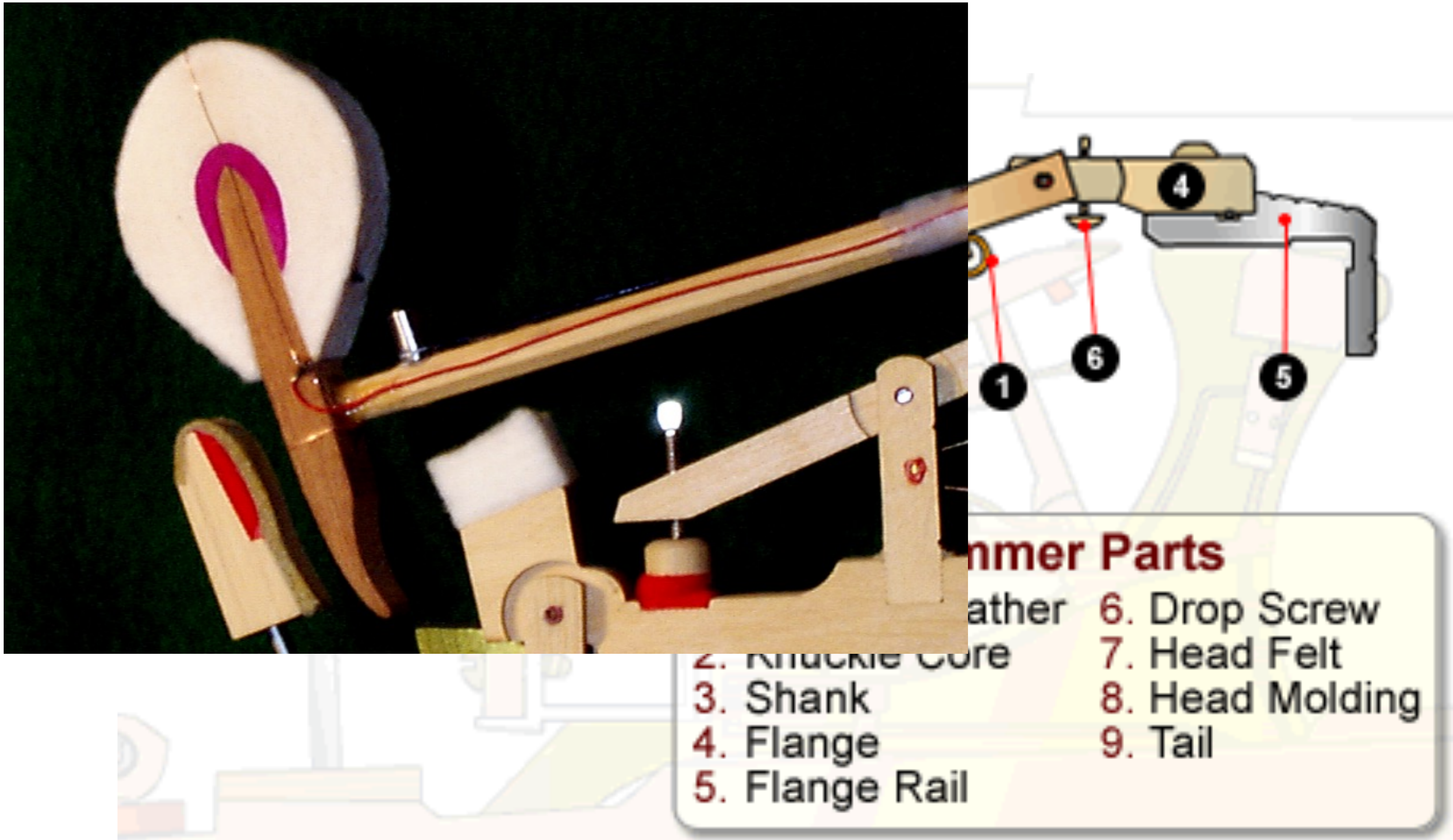
1 = unison

Hammer Anatomy

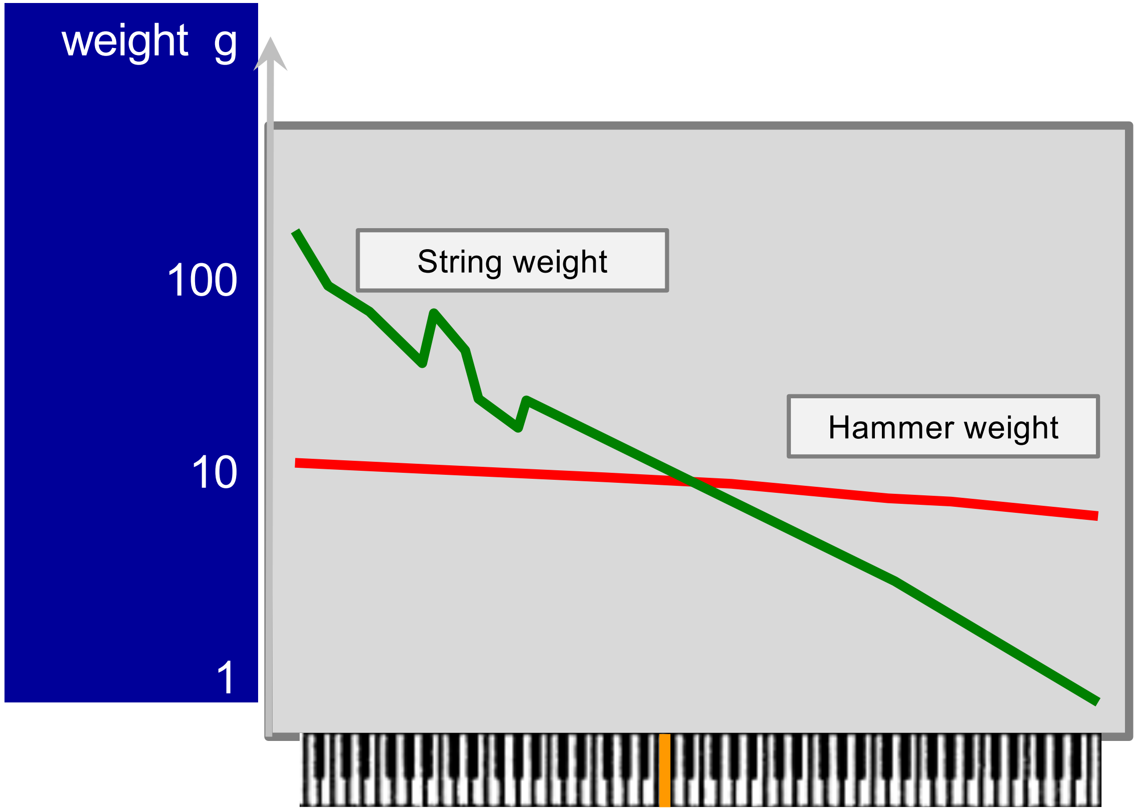
Made of hard pressed felt



Hammer Anatomy

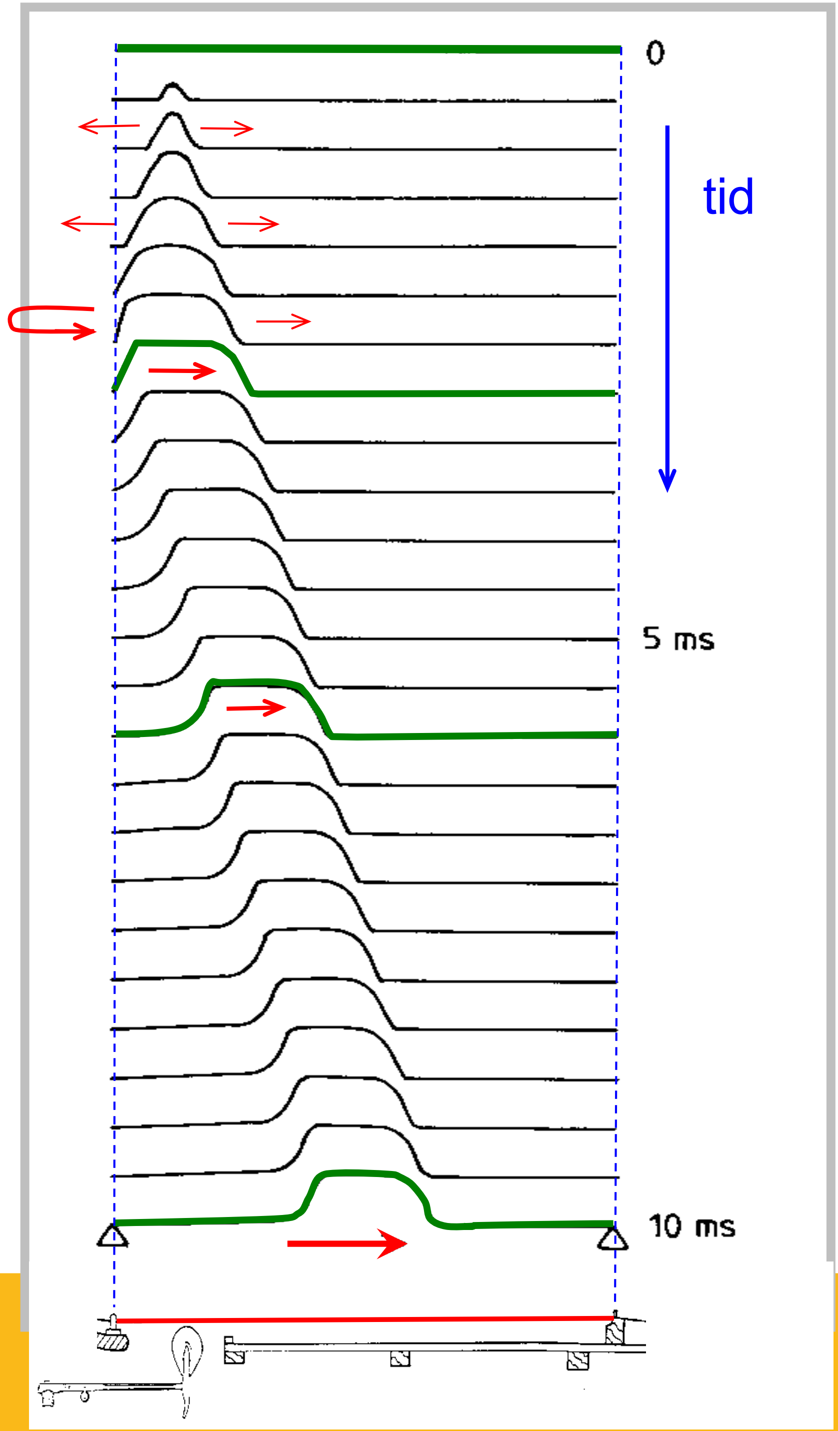


Weights of Hammers and Strings



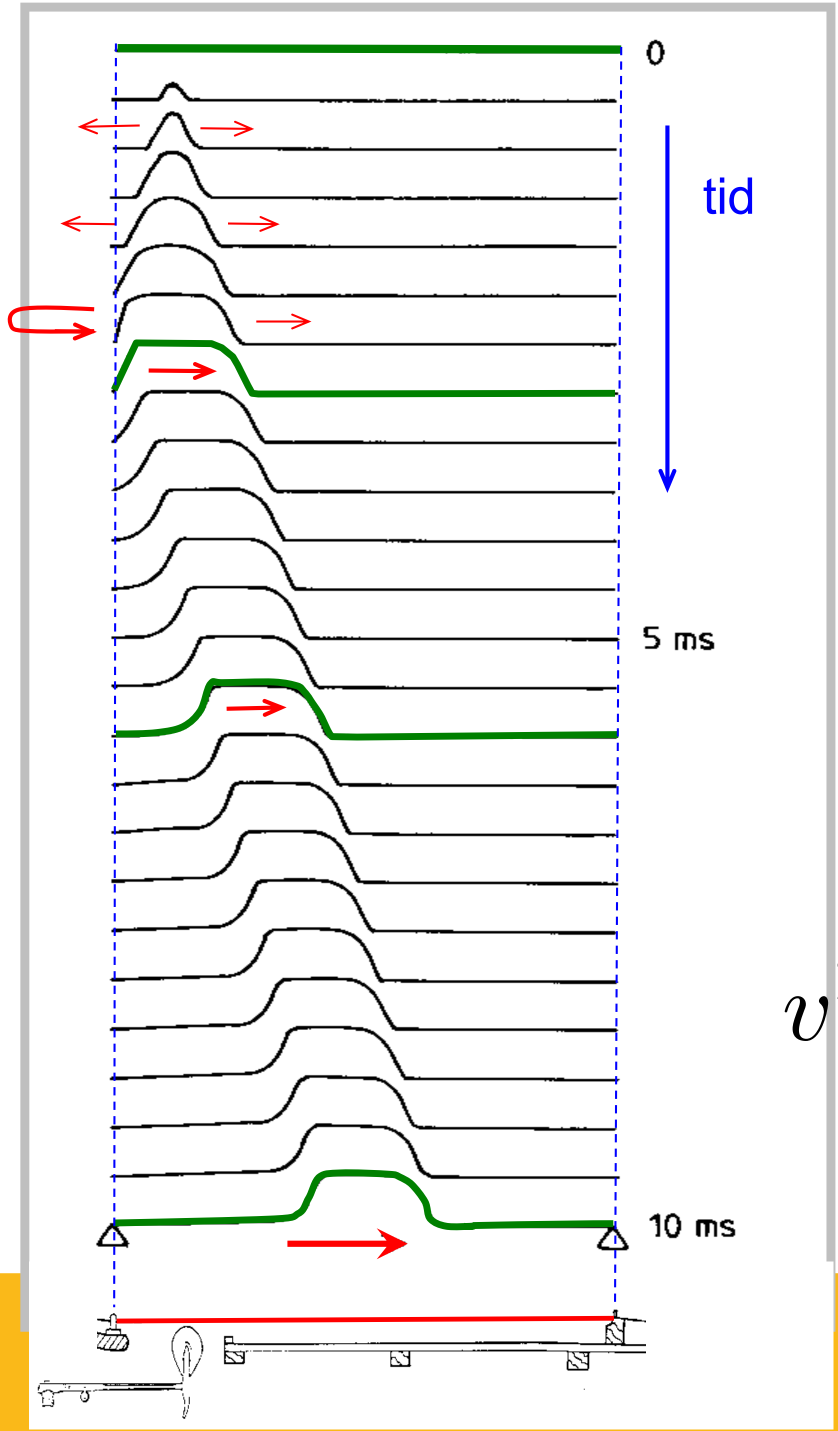
The ratio of the weight of the hammer and the string determines the hammer–string contact time

In the middle register, the hammer (red) and the strings (green) are about the same weight.
In the treble, the hammer is heavy in relation to the strings.
In the bass, the hammer is light in relation to the strings.



When the hammer strikes the string, a wave (pulse) is created that propagates on the string and is reflected back and forth between the agraff and the bridge.

What is the round trip time?



When the hammer strikes the string, a wave (pulse) is created that propagates on the string and is reflected back and forth between the agraff and the bridge.

What is the round trip time?

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$T = 4L^2 f_1^2 \mu$$

$$v^2 = T/\mu = 4L^2 f_1^2$$

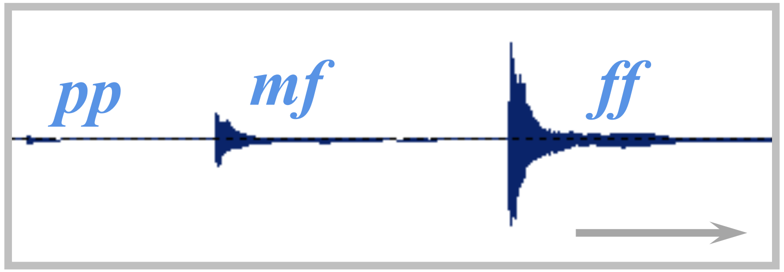
$$t_{\mathrm{trip}} = \frac{2L}{v} = \frac{1}{f_1}$$

Dynamics

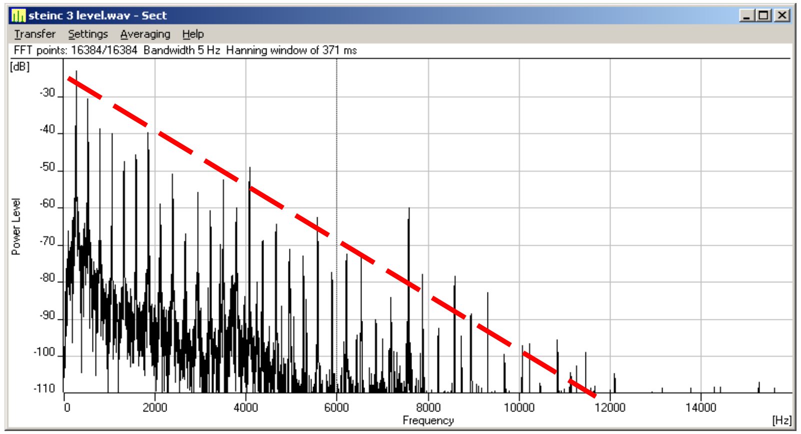
The spectral content of the piano tone grows rapidly with increasing force. This adds higher partials, contributing to timbre brilliance.

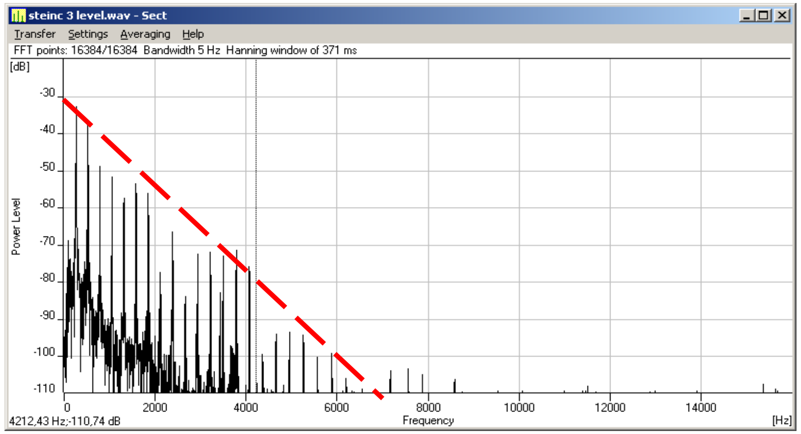
At *pp*, fewer than 10 partial tones are heard

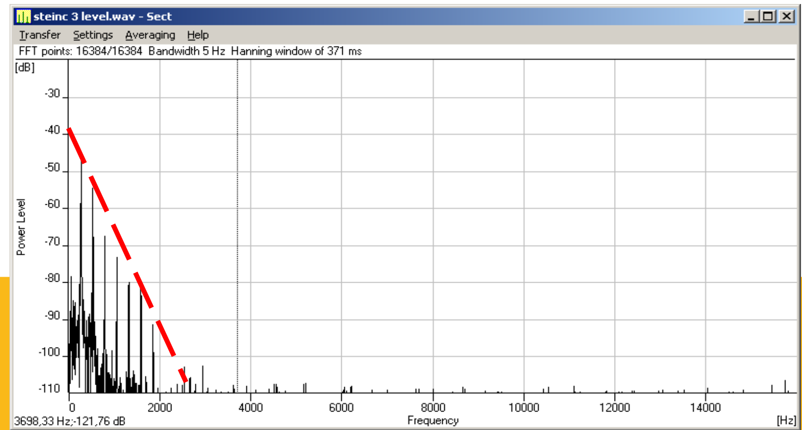
At *ff*, approx. 50 partial tones are heard, and the low tones become approx. 25 dB stronger compared to *pp.*



time





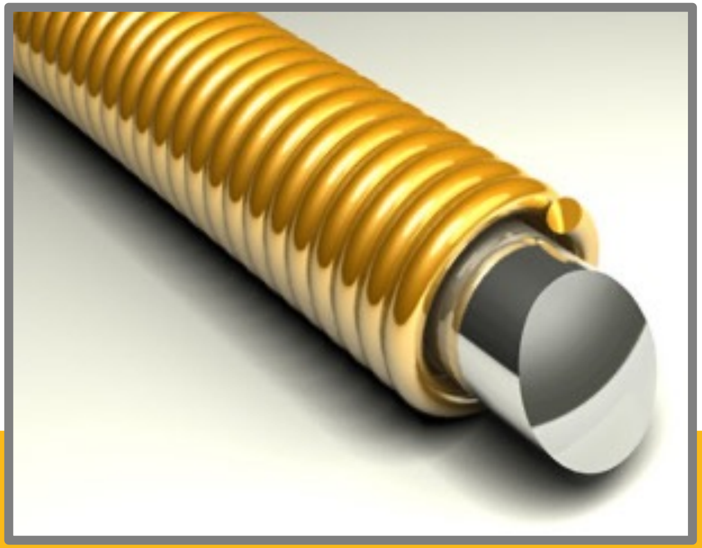


Inharmonicity

A typical piano string in the middle register (solid steel) has such inharmonicity that the overtones are related by

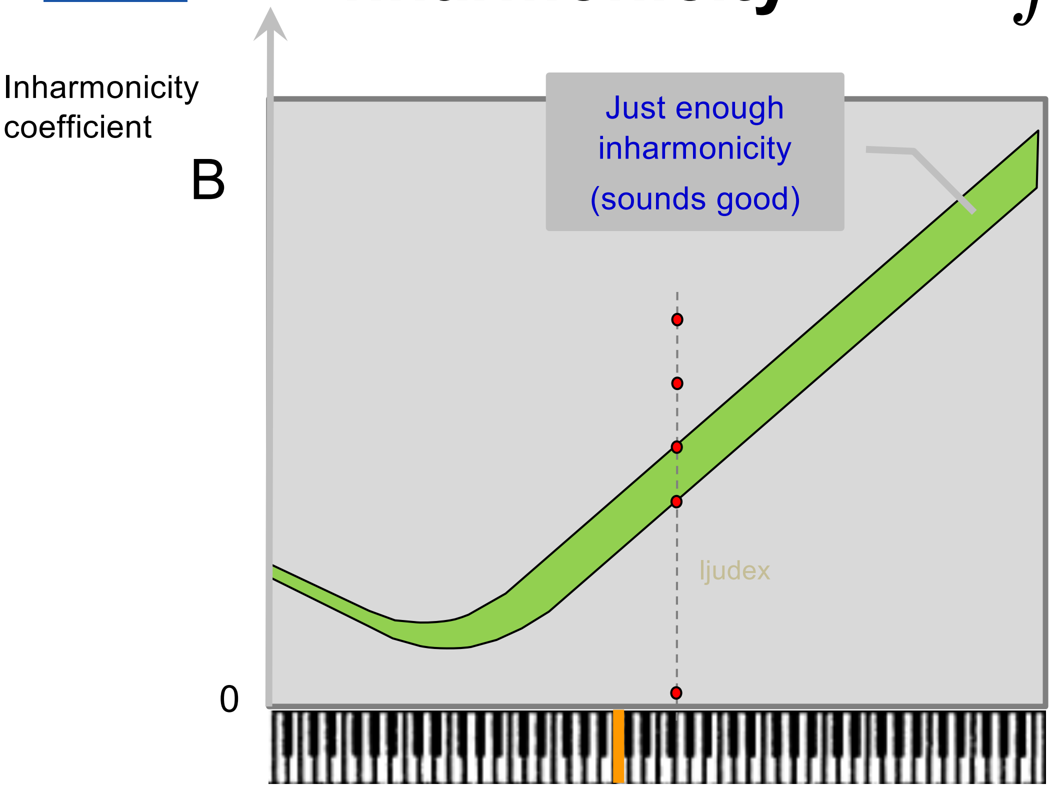
1 : 2.002 : 3.005 : 4.120.

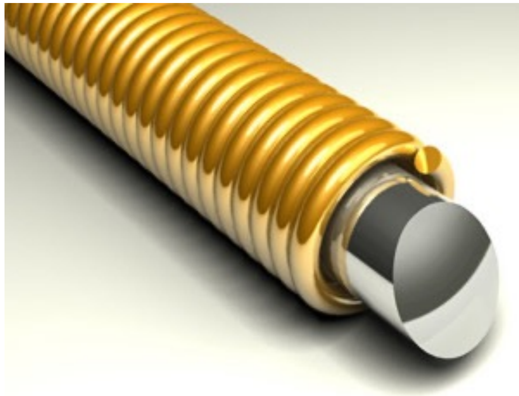
To reduce that impact, bass strings are wrapped with a thin copper thread to increase their mass but not their stiffness.



Inharmonicity

$$f_n = n f_0 \sqrt{1 + n^2 B}$$





5 grades of inharmonicity (x3)
(First three tones are pure harmonic sound, B=0)

A thin, long, and tightly tensioned string gets low inharmonicity

A thick, short and loosely strung string gets a high inharmonicity

Especially the length affects the inharmonicity strongly (quadratically).

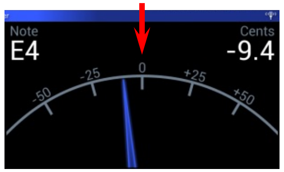
In the piano, the inharmonicity increases sharply from the middle register towards the treble. This is because the string length is getting shorter and shorter (almost halving for each octave).

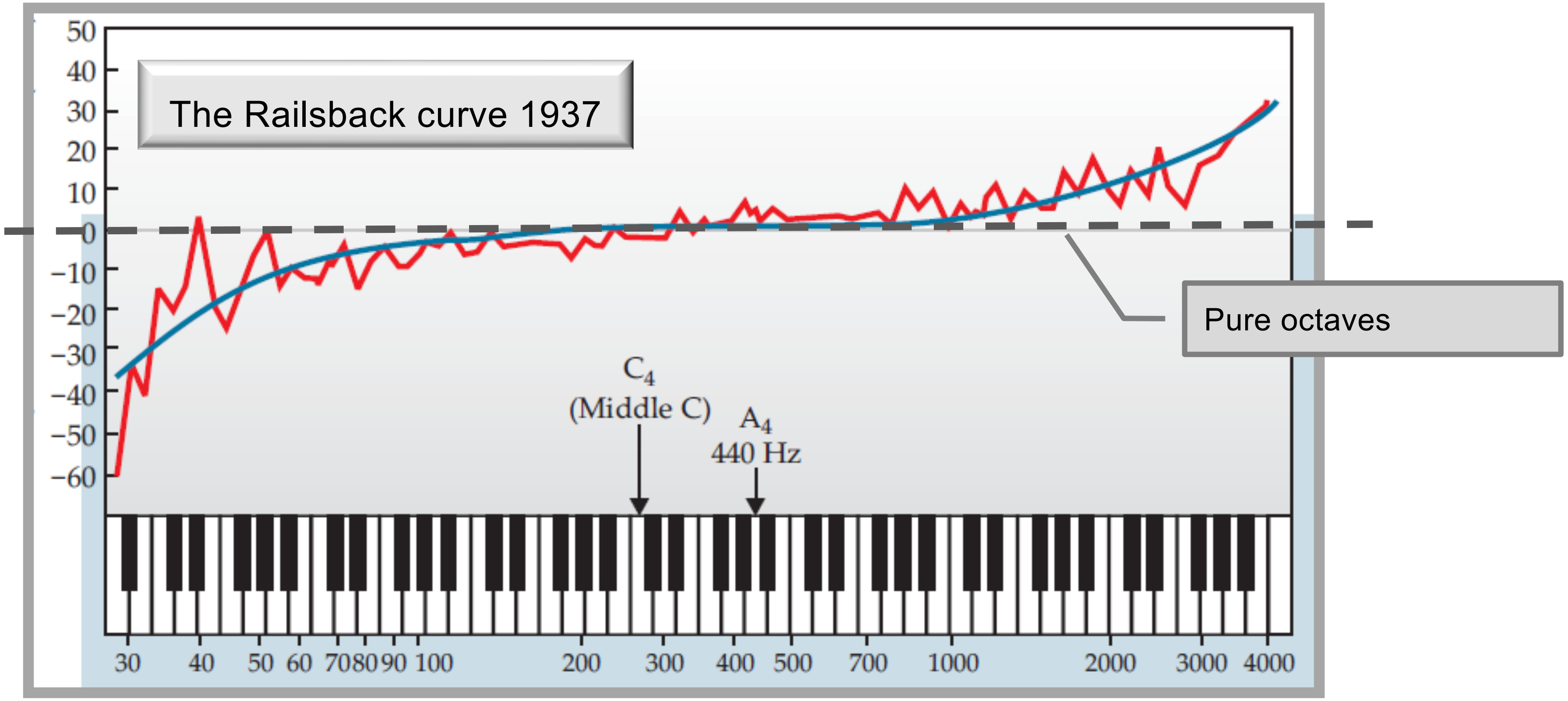
In the bass they try to keep the inharmonicity down through spun strings. Despite this, very long strings are required (concert grand piano 2.74 m).

Tuning Deviation from Equal Temperament

Deviation from equal temperament

cent

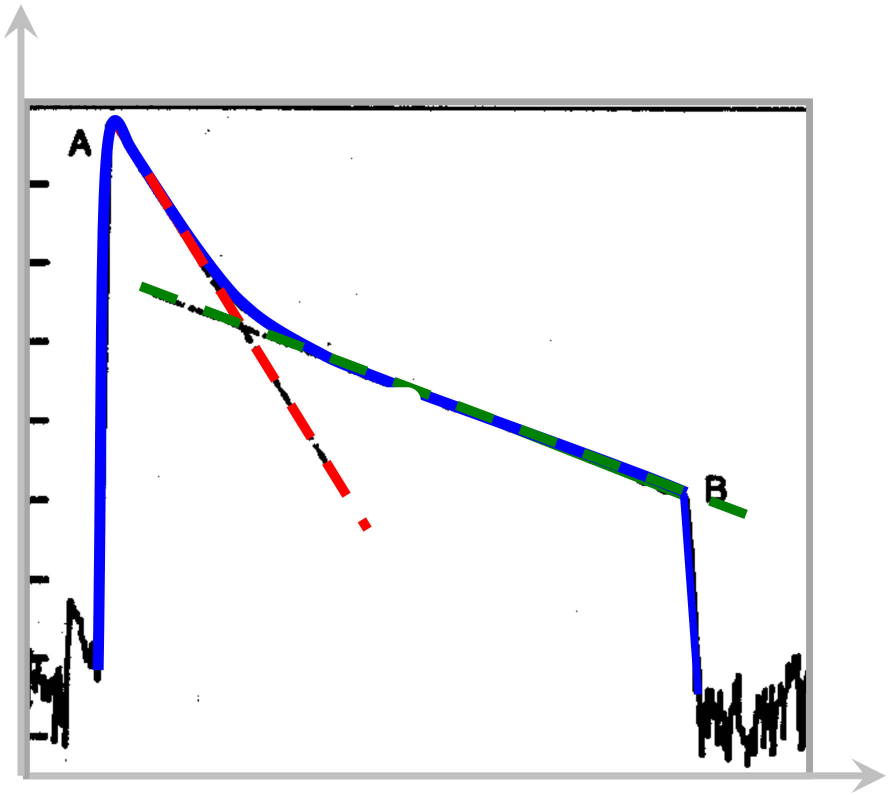




The octaves are 'stretched' (greater than 2:1) to reduce vibrations caused by string inharmonicity.

The treble is thereby tuned high and the bass low.

Piano note decay



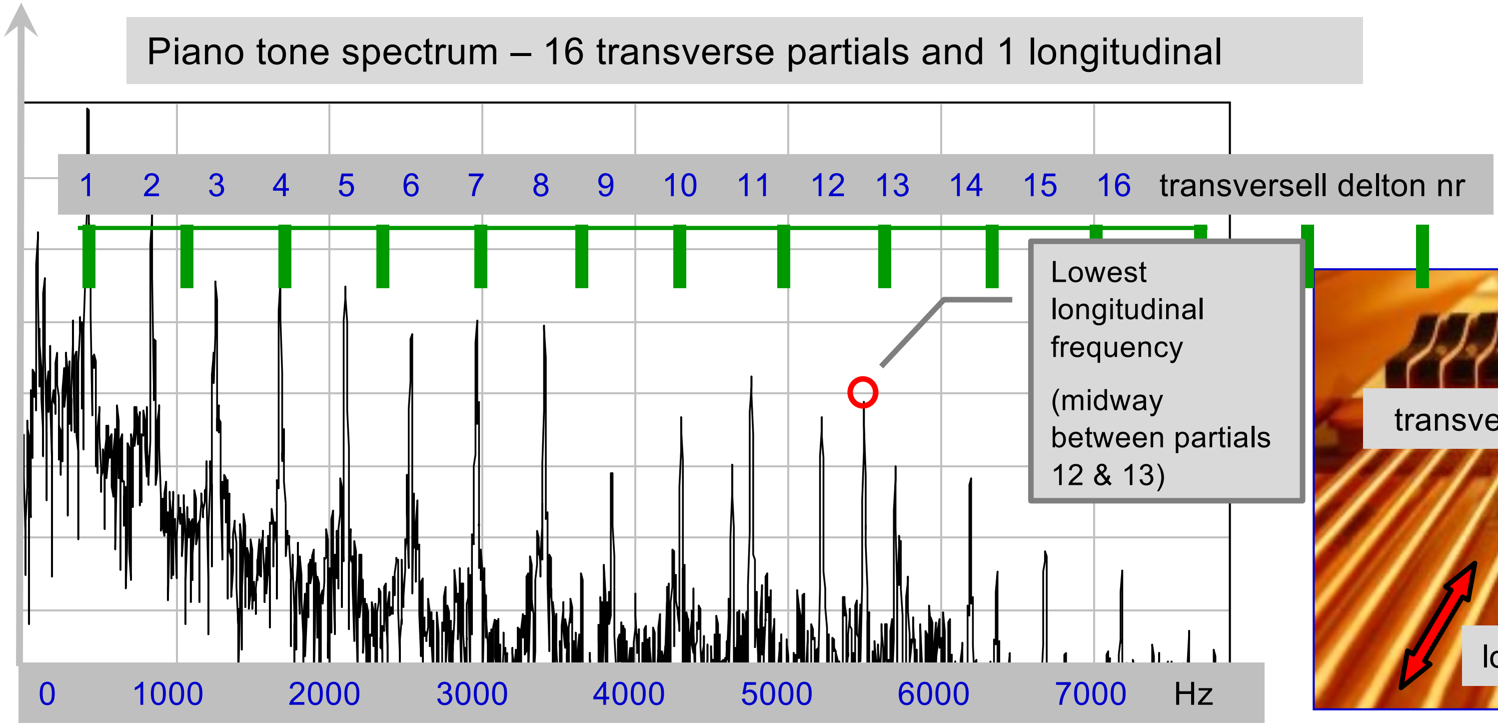
The decay of the piano tone consists of two parts

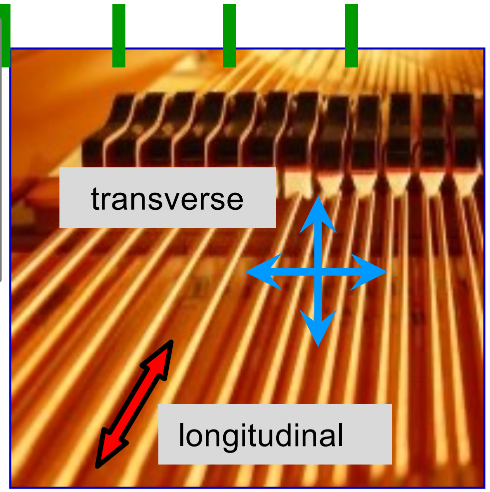
- an initial strong approach that fades quickly
- a sustained part that sounds slowly ('sustain')

Because of this, the tone is perceived as both strong and long.

Longitudinal vibrations

Piano tone spectrum – 16 transverse partials and 1 longitudinal





The piano string can swing in two different ways

- transverse (2 polarization directions: up-down, sideways) => 'regular' subtones
- longitudinal (in the string's longitudinal direction) => extra partials

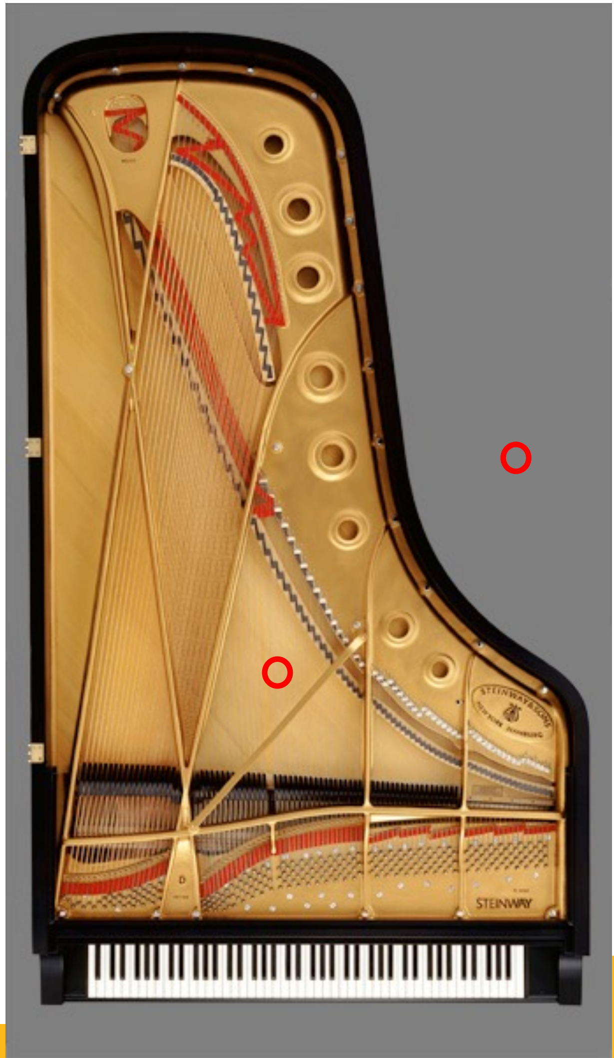
The frequencies of the longitudinal deltas do not fit at all into the (almost) harmonic spectrum generated by the transverse oscillations.

The longitudinal partials are audible and contribute to the piano sound.

For spun strings, the frequencies of the longitudinal deltas can be 'tuned' during manufacture by varying the diameter of the spinning wire.

6 bass strings for the same tone (G1 = 49 Hz) with different frequency on the first longitudinal subtone

Tone and Tone+Body



String & noise components
in radiated sound:

- string only (string velocity detector)
- radiated sound (mic)

Ljudex (3x)
String velocity – radiated sound

“Sonatas and Interludes” by John Cage (1948)

- Multimovement work for solo piano
- A landmark of modern American music

Maro Ajemian and John Cage

“Sonatas and Interludes” by John Cage (1948)

https://youtu.be/jRHoKZRYBIY

Magnetic Resonator Piano

http://instrumentslab.org/research/mrp.html

Before next lecture

Read:

1. Chapter 27 in Hartmann
2. Peruse Tolonen, Välimäki and Karljanian, “Evaluation of Modern Sound Synthesis Methods” (1998)