

Gauss-Jacobi (ADGH) versus Gauss-Seidel (ABELN..)

. _

Dampening to Stabilize an Unstable "Hog Cycle".

- Suppose inverse demand is p = 21 3q and supply is q = p/2 3
- Linear system is not diagonally dominant:

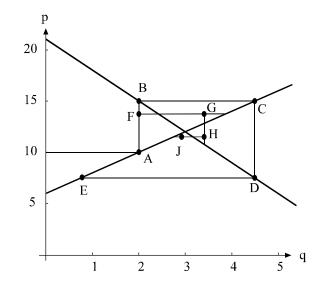
$$\begin{pmatrix} 1 & 3 \\ 1 - 2 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 21 \\ 6 \end{pmatrix} \tag{3.9.8}$$

• Gauss-Seidel is unstable:

$$p_{n+1} = 21 - 3q_n \tag{3.9.9a}$$

$$p_{n+1} = 21 - 3q_n$$
 (3.9.9a)
 $q_{n+1} = \frac{1}{2}p_{n+1} - 3$ (3.9.9b)

Judd Figure 3.4: Dampening an unstable hog cycle



Exatrapolation to Accelerate Convergence in a Game

- Assume firm two's reaction curve is $p_2 = 2 + 0.80p_1 \equiv R_2(p_1)$, and firm one's reaction curve is $p_1 = 1 + 0.75p_2 \equiv R_1(p_2)$.
- Equilibrium system is diagonally dominant
- Gauss-Seidel is the iterative scheme

$$p_1^{n+1} = R_1(p_2^n) \tag{3.9.12a}$$

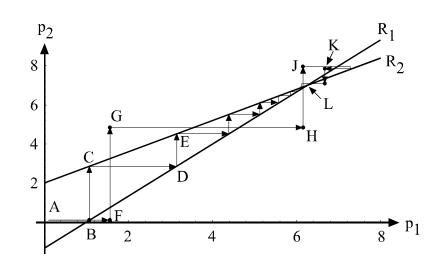
$$p_2^{n+1} = R_2 \left(p_1^{n+1} \right) \tag{3.9.12b}$$

• Accelerate (3.9.12). If $\omega = 1.5$, we arrive at faster scheme:

$$p_1^{n+1} = 1.5R_1(p_2^n) - 0.5p_1^n, (3.9.13a)$$

$$p_2^{n+1} = 1.5R_2 \left(p_1^{n+1} \right) - 0.5p_2^n.$$
 (3.9.13b)

Judd Figure 3.5 Accelerating a Nash equilibrium computation



~ ~