Probability

Module 3b

Conditional Probability and Independence

Learning Objectives

- Defining and understanding conditional probability
- Using independence to make novel probability calculations

Conditional Probability Notation

P(A|B) = Probability event A occurs given that event B occurs

Formally, $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Example:
A = Getting a sum of 7 on the roll of two dice
B = First die rolls a 4

$P(A|B) = \;\;?$

$P(A \cap B) = \frac{1}{36}$ (i.e. 4,3)

Definition $\Rightarrow$ $P(A \cap B) = P(A|B)P(B)$ or $P(B|A)P(A)$ which forms the basis for **the law of total probability—very powerful law in circumstances where sample space can be broken up into mutually exclusive events. To be discussed in Module 3c**

Independent Events

Two events A and B are said to be independent if:

P (A | B) = P(A | ~B) = P(A), and P(B | A) = P(B | ~A) = P(B).

In other words, two events are independent if the occurrence (or non-occurrence) of one event does not change the probability that the other will occur.

Is each pair of events independent or dependent?

- Example 1: A = heads on first toss of a fair coin, B = tails on second toss of that coin.
    - **P(B) = 0.5**
    - **P(B|A) =**
    - **Are A and B independent?**
- Example 2: A = a MISM student is 200 cm tall, B = that individual is a women.
    - **P(B) = 0.33**
    - **P(B|A) =**
    - **Are A and B independent?**
- Example 3: A = heads on first toss of a fair coin, B = tails on first toss of that coin.
    - **P(B) = 0.5**
    - **P(B|A) =**
    - **Are A and B independent?**

Independent Events Continued

Important Rule: $\mathbf{P(A \cap B) = P(A \mid B)\, P(B)}$

If A and B are independent:

$P(A \cap B) = P(A)\, P(B)$ because $P(A|B) = P(A)$.

More generally, for independent events $A_1..A_n$:

$P(A_1 \cap \ldots \cap A_n) = P(A_1)\, P(A_2) \ldots P(A_n)$.

Note: mutually exclusive events are NOT independent

Independence Illustrated—The Space Shuttle Challenger Explosion

Richard Feynman's Famous Application of Independence

Probability of a catastrophic failure per launch:

$P(C) = 0.0069$

Probability of a safe launch:

$P(\sim C) = 1 - P(C) = 1 - 0.0069 = 0.9931$

What is the probability of at least 1 catastrophic failure in 100 launches?

Let (At least 1) = At least 1 failure in 100 launches

P(At least 1) = 1 - P(~At least 1)

Assuming approx. independence of P(C),

P(~At least 1) = P(none)=

$.9931 * .9931 \ldots * .9931 = (0.9931)^{100} = 0.5$

Thus, P(At least 1) = 1 - 0.5 = 0.5

"NASA officials see the risk of catastrophe as roughly 1 in 145 missions for each ship and worry that another disaster is almost inevitable." (NYT 1/28/1996)

Discussion Item & Tests of Understanding

- While the space shuttle was in operation, NASA engineers and contractor made many adjustments to the shuttle's mechanical systems, monitoring sensors, telemetry etc.. Thus, from launch-to-launch the probability of a catastrophic failure was not constant and depended on monitoring information from prior launches. In your judgment does this invalidate the the Feynman calculation?
- If you roll a six-sided die three times, calculate the probability of getting:
    a. 2,2,2
    b. 1,3,4
- A six-sided die is rolled, and a coin is flipped.
  Find P(roll 1 or 2 **and** get heads)
- A six-sided die is rolled, and a coin is flipped.
  Find P(roll 1 or 2 **or** get heads)

Tests of Understanding (cont.)

- Two squadrons of 4 fighter jets each are sent out on a dangerous mission. Suppose the probability of each of the jets in squadron A being shot down is .1 and for squadron B that probability is .2 for each jet. What is the probability of all jets returning safely in squadron A? in squadron B? In both squadrons?