Probability

Module 3a

Probability Basics

Understanding Probability is Essential

Probability is a:

- Basis for statistical inference:
  - Margin of error on opinion poll is +/- 4%.
  - Difference between test scores is significant at 5% level.

- Key element of business:
  - Expected profit, risk, uncertainty, etc.

- Key element of operations management :
  - Setting inventory level, delivery cycle, response time.

And, our intuitions about probabilities are terrible!
"98% of individuals who do not make a return visit to a web site are first-time visitors."
98% of first-time visitors will not make a return visit to a web site."

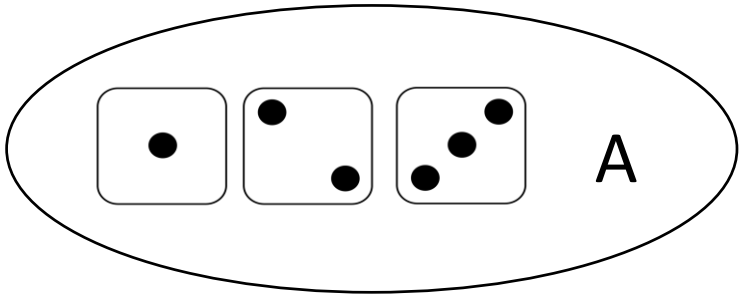
Learning Objectives

- Understand primary interpretations of probability
  - Relative frequency vs. proportion of population
- Interpret and utilize probability notation
- Apply fundamental probability rules to novel problems
  - Intersections - $P(A \cap B)$, i.e. the "probability of A and B."
  - Unions = $P(A \cup B)$, i.e. the "probability of A or B."
  - Complements

Basic Definitions

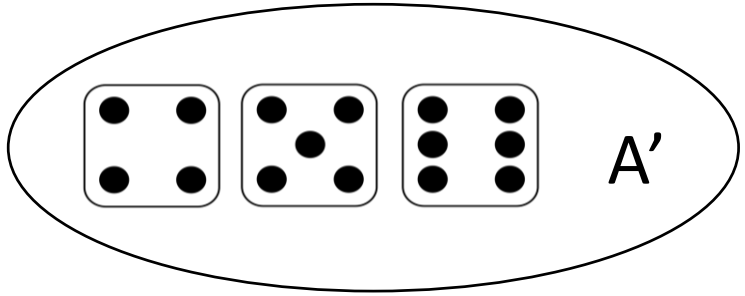
- **Probability of A**: P(A), a number between 0 and 1 indicating the likelihood of event A.
  - P(coin flip lands on heads) = ½
  - P(it will rain tomorrow) = 0.8

Event A: Roll 1, 2, or 3



- **Complement** of event A: the event that A does not occur, usually denoted by ~A, A^C, or A'.
  Important rule: **P(~A) = 1 – P(A).**

Complement of Event A:
Roll 4, 5, or 6



Two Interpretations of Probability

The probability of event A, P(A), can be interpreted in two important ways:

1) Relative Frequency:

$$P(A) = \lim_{n \to \infty} \frac{\#\text{ of times event A occurs in } n \text{ trials}}{n}$$

P(A) = 0.05 implies that given an infinite number of trials, event A will occur 5% of the time.

Example: The probability of a 1 in a fair 6-sided dice is 1/6.

2) Proportion of a population with characteristic A

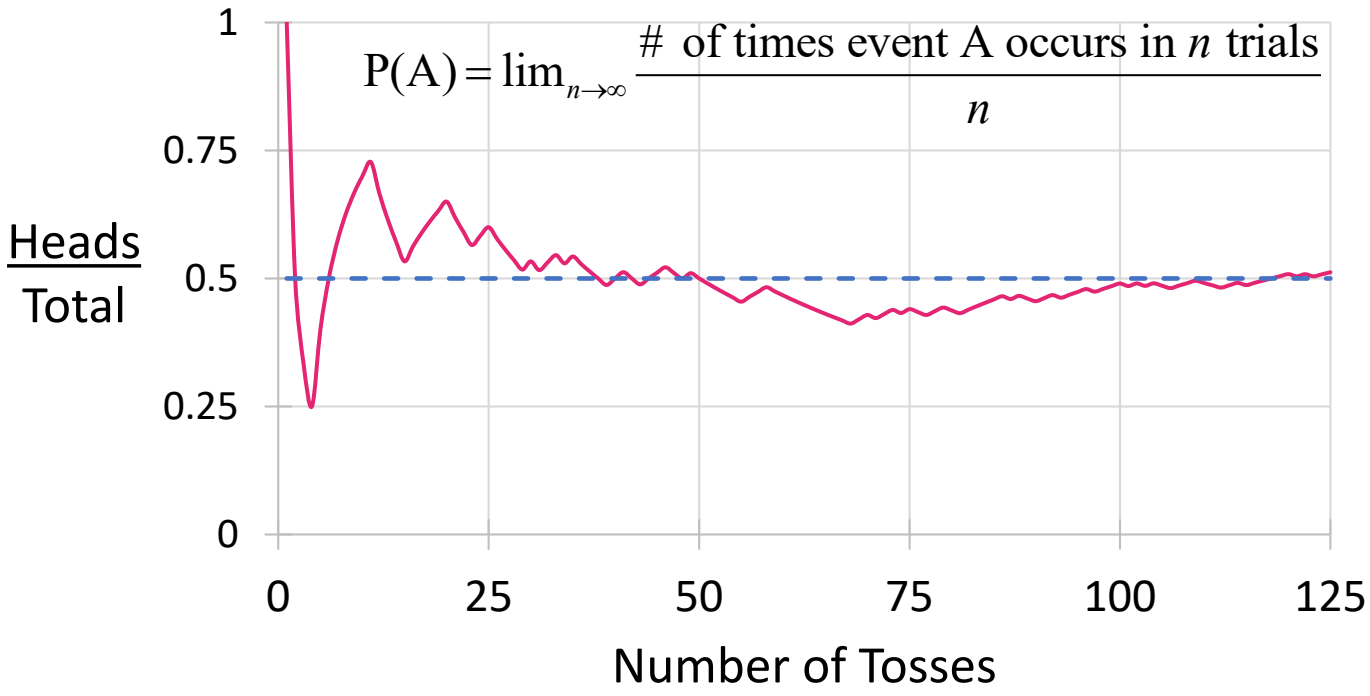
P(A) = 0.05 implies that 5% of the population has characteristic A.

Example: The probability that an American citizen has a degree beyond a bachelor's is 13.1%

Two Interpretations of Probability

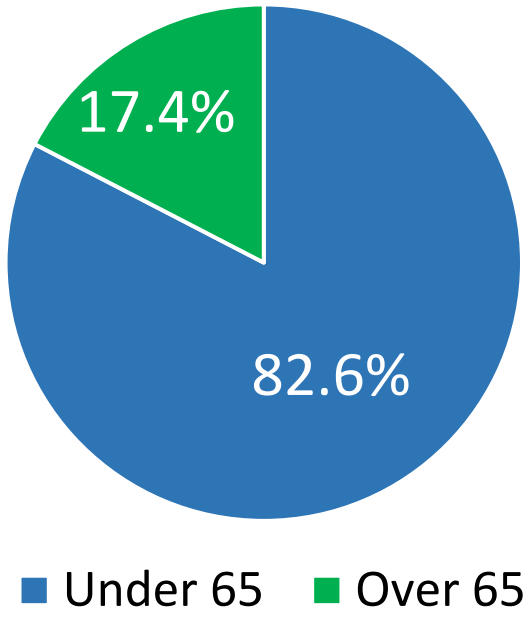
Relative Frequency

Frequency of Flipping Heads on a Coin



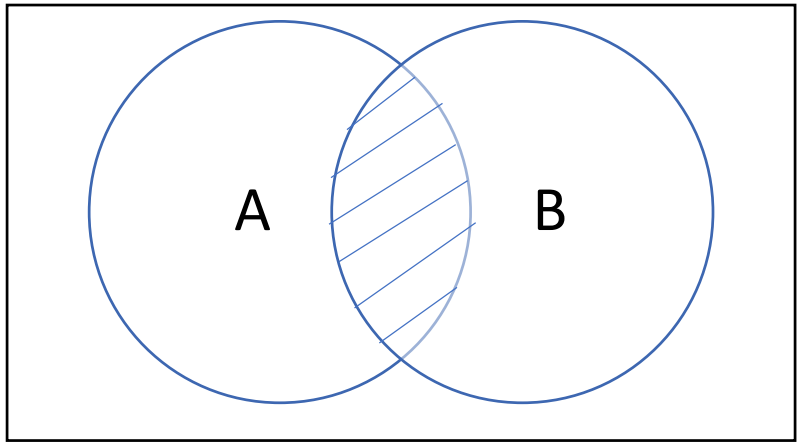
Proportion of Population with Some Specified Characteristic

Population of Pennsylvania

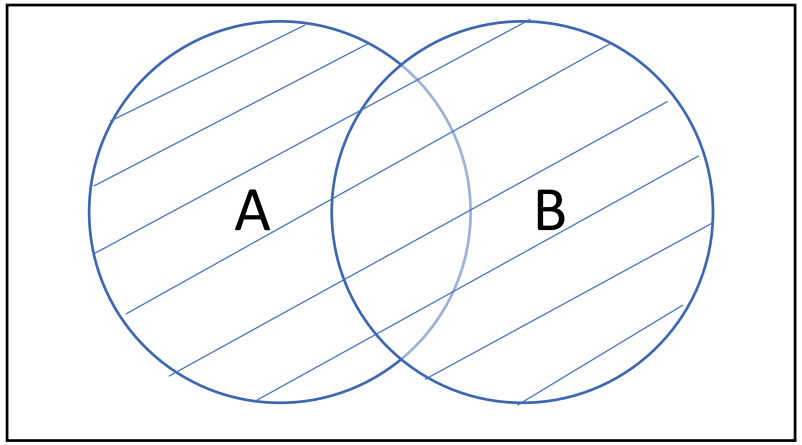


Combining Probabilities

- **P(A ∩ B)**, the probability of both events occurring simultaneously, i.e. the "probability of A and B."

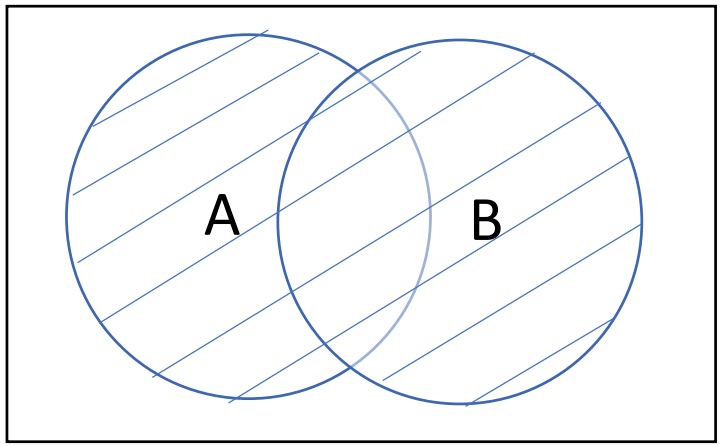


- **P(A U B),** The probability of at least one of the two events occurring, i.e. the "probability of A or B."



Calculating the Union of Two Events

Important rule: **P(A U B) = P(A) + P(B) - P(A ∩ B)**



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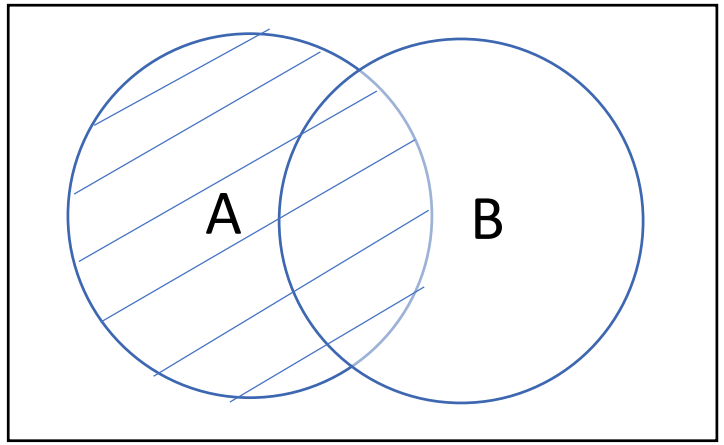
Example: x = roll of a six-sided die.

P({x is even} U {x ≥ 3}) =

$P(x\ is\ even) + P(x \geq 3) - P(4 \text{ or } 6)$

$= 0.5 + 0.67 - 0.33$

$= 0.84$



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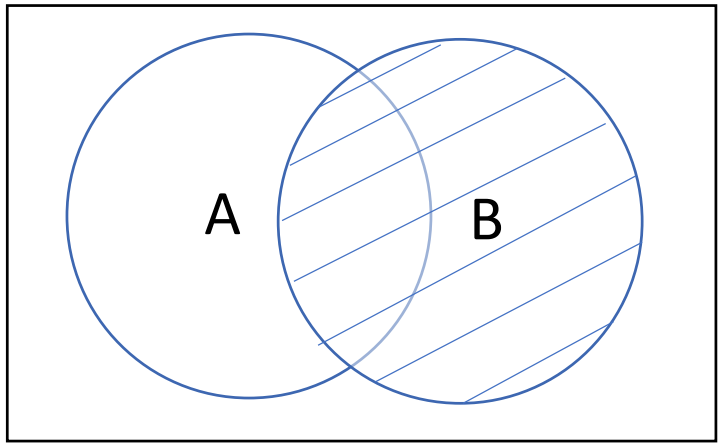

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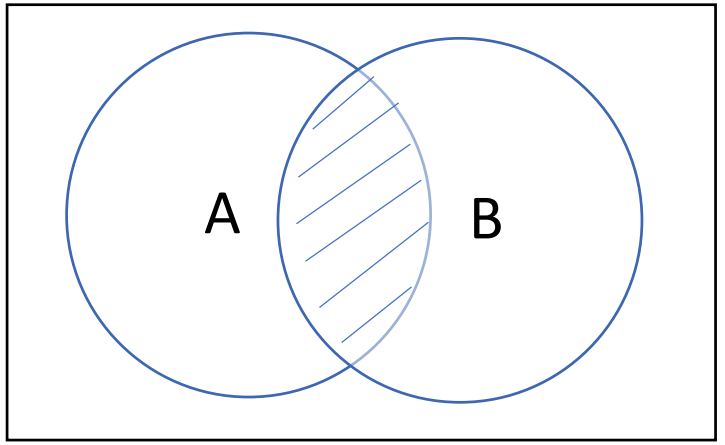
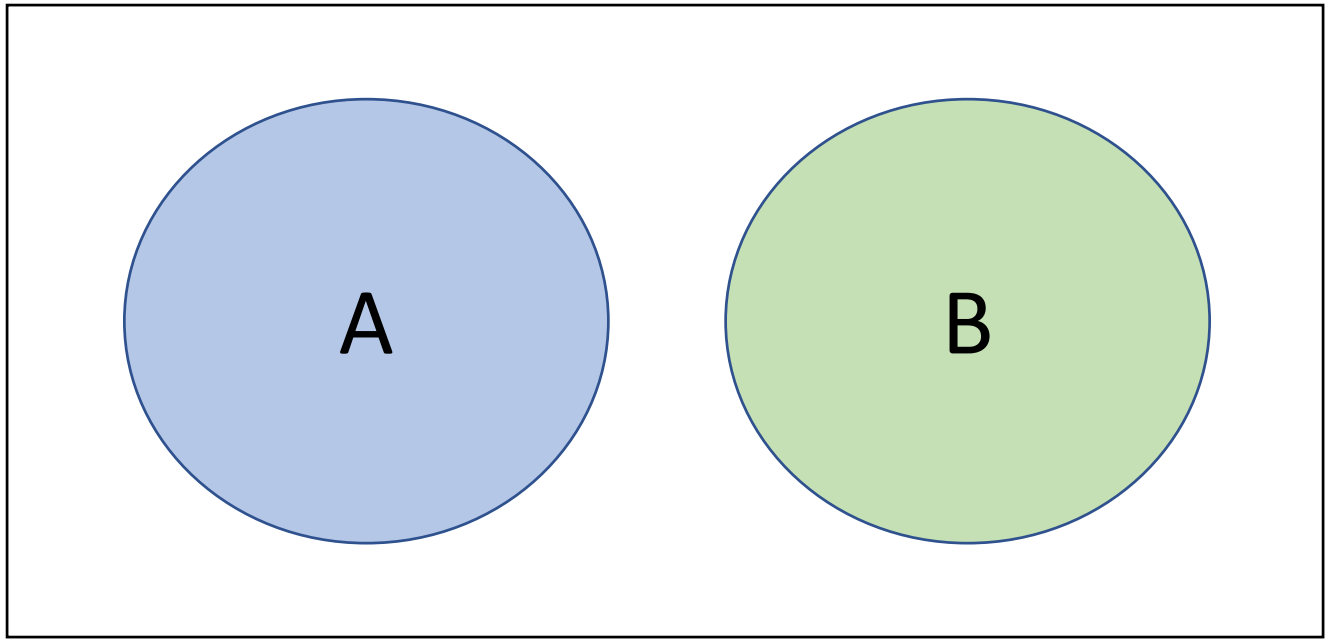


Mutually Exclusive Events

If two events are mutually exclusive**: P(A ∩ B) = 0**.

- For mutually exclusive events, P(A U B) = P(A) + P(B).
- Example: x = roll of a six-sided die. A = {x is even}, B = {x = 1}.



Tests of Understanding

A six-sided die is rolled.

Let: A = rolling an even number
B = rolling a number less than 4

1. Find $P(A \cap B)$

2. Find $P(A \cup B)$

3. $P(A) = 0.15$, what is $P(\sim A)$?

4. Two dice are rolled. Calculate:
   a) P(Sum = 2)
   b) P(Sum = 7)
   c) P(Sum > 9)